# Instability of Extremal Black Holes in AdS Supergravity


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Quantum Information, Quantum Field Theory, and Black holes

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- Black holes are parametrized by their **conserved charges**.

  Examples: angular momentum $J$ and electric charge $Q$.

- The **extremal limit**: the **minimal mass** for given charges.

- Two **conflicting** intuitions on extremal black holes:

  1. Lowest possible energy $\Rightarrow$ **ground state** of the system.

     Extremal black holes are **particularly stable**.

  2. Repulsive forces between constituents the **maximal** allowed by gravitational attraction.

     **Marginal** bound states: on **boundary to unstable phase**.

# Supersymmetry and Unitarity

- Guidance from **supersymmetry**: charges form an algebra.
- Quantum states form **representations** of the algebra.
- **Unitarity**: charges give **lower bound** on energy.
- **Saturation** of unitarity: **additional conditions** on charges.
- **Supersymmetric** black holes $\Leftrightarrow$ unitarity bound saturated.

# Paradigm: Instability is Generic

- **Supersymmetric** black holes are completely **stable**.
- They are **rare**: their charges satisfy an **additional condition**.
- **Non**-SUSY extremal black holes are **classically** unstable.

  Decay **mechanism** and **end-product** depend on details.
- **Near** extremal black holes are vulnerable as well.

  Moreover, **quantum effects** are strong (not discussed here).

# Example: 4D Black Holes in Flat Space

- Black hole entropy:

$$S = \frac{A}{4G_4} = \frac{\pi}{G_4}\left[\frac{J^2}{M^2} + \left(MG_4 + \sqrt{(MG_4)^2 - Q^2 - \frac{J^2}{M^2}}\right)^2\right]$$

- **Extremality** bound:

$$G_4 M^2 \geq \frac{1}{2}Q^2 + \sqrt{\frac{1}{4}Q^4 + J^2}$$

Black holes solutions exist **only** for these masses.

- **Supersymmetry**: condition **in addition** to extremality.

$$G_4 M^2 = Q^2 \ , \quad J = 0$$

# Superradiance

- **Hypothesis**:

  extremal black holes with $J \neq 0$ are **unstable**.

- Naïve emission rate of Hawking quanta:

$$\Gamma(\omega) = \frac{\sigma_{\mathrm{abs}}(\omega)}{e^{\beta(\omega - m\Omega)} - 1} \frac{d^4 k}{(2\pi)^4}$$

  $\Omega$ is the **rotational velocity**.

- The rate **diverges** at sufficiently low energy $\omega \leq m\Omega$.

- Physical interpretation: **superradiance**.

# The Black Hole Bomb

- Incoming particles reflected with **larger** amplitude

  $\Rightarrow$ energy is **extracted** from the black hole.

- A rotating BH **in a box**: energy reflected back into BH.

  So BH is **unstable**: the black hole **bomb**.

- Significant in astrophysics **if** extremely light scalars exist.

- This talk: perspectives in $AdS_3$ and $AdS_5$.

# Overview: Perspectives on Instability

- An extremal (not SUSY) black hole is **unstable**.

  What does it decay **to**?

- Given conserved charges, what is the SUSY ground state?

  When the constraint is **not** satisfied: **not** a BH.

- What is the **CFT mechanism** for the instability?

# Outline of Talk

- Part A: Extremal Black Hole **Thermodynamics**
- Part B: **Superfluidity** in Bulk

  The **spectrum** of scalar fields
- Part C: **Bose-Einstein** Condensation on the Boundary

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# Part A

# Extremal Black Hole Thermodynamics

# Black Holes in $\mathrm{AdS}_5 \times S^5$: Quantum Numbers

- Symmetry of theory: $SO(2,4) \times SO(6)$ + SUSY.
- $SO(2,4)$ representation of fields:
  **Conformal weight** $E$ and **angular momenta** $J_{a,b}$.
- $SO(6)$ representation of fields: **R-charges** $Q_I$ $(I = 1,2,3)$.
- So **asymptotic data** of black holes in $\mathrm{AdS}_5$:
  Mass $\mathrm{M} = \mathrm{E}$, 2 Angular momenta $J_{a,b}$, and 3 R-charges $Q_I$.
- The mass for **supersymmetric** black holes:

$$M_{\mathrm{SUSY}} = Q_1 + Q_2 + Q_3 + J_a + J_b$$

This presentation: $\frac{\pi}{4G_5}\ell_5^3 = \frac{1}{2}N^2$ and $\ell_5 = 1$.

# The Constraint on Charges

- The quantum numbers of all **supersymmetric** black holes in $AdS_5$ satisfy the **constraint**:

$$\left((Q_1 Q_2 + Q_2 Q_3 + Q_1 Q_3) - \frac{1}{2}N^2(J_a + J_b)\right)\left(\frac{1}{2}N^2 + (Q_1 + Q_2 + Q_3)\right) + \frac{1}{2}N^2 J_a J_b - Q_1 Q_2 Q_3 = 0$$

- For example, **rotation is mandatory** $J_{a,b} \neq 0$.

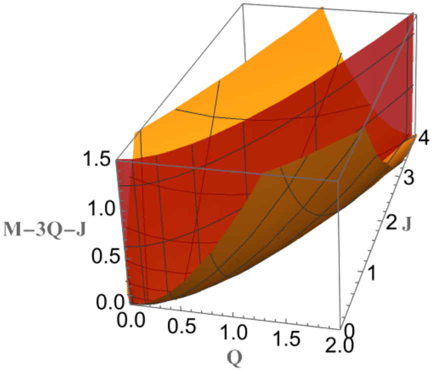
- Extremal BHs that violate the constraint have **excess** mass.

$$M - M_{\mathrm{SUSY}} \geq 0$$

# The Mass Excess: Above the BPS Bound

- Extremal BHs that violate the constraint have **excess** mass.

$$M - M_{\rm SUSY} \geq 0$$



# Instability Criterion of $AdS_5$ BH from the First Law

- Hypothesis: extremal nonSUSY BHs are **unstable**.

- The **first law** of black hole thermodynamics:

$$\begin{aligned} TdS &= dM - 2\Omega dJ - 3\Phi dQ \\ &= \underbrace{d(M - 2J - 3Q)}_{\text{mass excess}} + 2(1-\Omega)dJ + 3(1-\Phi)dQ \end{aligned}$$

- Emission of a **BPS particle**:

  $dJ \leq 0$ and $dQ \leq 0$ and mass excess preserved.

- Thermodynamically **favorable**: $dS > 0$ if $\Omega > 1$ or $\Phi > 1$.

# Instability Criterion: Extremal Black Holes

- Three types of **extremal** black holes:
  - Reissner-Nordström-like: constraint **positive** and $\Phi > 1$.
  - Kerr-Like: constraint **negative** and $\Omega > 1$.
  - BPS: constraint **satisfied** and $\Phi = \Omega = 1$.
- Conclusion: **all** nonBPS extremal BHs are **unstable**.
- Stability bound **strictly** above the extremality bound:

$$M_{\text{stability}} \geq M_{\text{ext}} \geq M_{\text{BPS}}$$

# Example: BTZ Black Hole in $AdS_3$

- $CFT_2$ with $\mathcal{N} = 4$ supersymmetry: $c_L = 6k_L$.
- Black hole "angular" momenta: $P$ within $AdS_3$, $J_L$ in $S^3$.
- The unitarity bound is saturated by BPS black holes:

$$E \geq P + J_L - \frac{1}{2}k_L = E_{\rm BPS}$$

- Black hole solutions exist only when

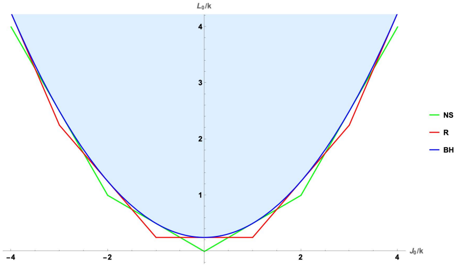
$$E \geq E_{\rm ext} = P + \frac{1}{2k_L}J_L^2 \geq E_{\rm BPS}$$

Extremal black holes have $E = E_{\rm ext}$.

- Condition that extremal are BPS: $E_{\rm ext} = E_{\rm BPS} \quad \Leftrightarrow \quad J_L = k_L$.

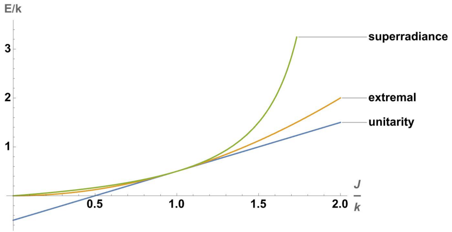
# Unitarity Bound in $AdS_3$

- BPS limit (green): linear in $J$ with range $0 \leq J \leq 2k_L$.
- Extremal black hole limit (blue).
- Red curve and $J$ in broader range: spectral flow.
- Key point: some energies do not permit a black hole.



# Superradiant Instability

- Non-SUSY extremal black holes are not stable.
- Thermodynamic potentials favor emission in $AdS_3$ and/or $S^3$.
- Emission of chiral primary **increases** black hole entropy.



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# Part B

# Superfluidity

# Instability: the Breitenlohner Freedman Bound

- Model: free scalar field with mass $m^2$ in $AdS_{d+1}$.
- The wave equation reduces to the Schrödinger equation:

$$-\frac{d^2\psi}{dz^2} + \left[\vec{k}^2 + \frac{1}{z^2}(m^2\ell^2 - \frac{1-d^2}{4})\right]\psi = \omega^2\psi$$

- Bound states in $1/r^2$ potential if coefficient is too negative.

  Instability: this corresponds to exponential time dependence.
- The **Breitenlohner-Freedman bound**:

$$m^2\ell_{d+1}^2 \geq -\frac{1}{4}d^2$$

# Scalars in $AdS_5$

- $\mathcal{N} = 4$ SYM $\underset{\text{AdS/CFT}}{\Leftrightarrow}$ $\mathcal{N} = 8$ SUGRA in bulk.

- 42 Scalars fields in **vacuum** of $\mathcal{N} = 8$ $AdS_5$ supergravity:
  - $t$ in $\mathbf{20}'$ of $SU(4)_R$: $m^2 = -4 \leftrightarrow \Delta = 2$
  - $\varphi$ in $\mathbf{10}_c$ of $SU(4)_R$: $m^2 = -3 \leftrightarrow \Delta = 3$
  - $\Lambda$ in $\mathbf{1}_c$ of $SU(4)_R$: $m^2 = 0 \leftrightarrow \Delta = 4$

- The BF stability bound in $AdS_5$:

$$m^2 \ell_5^2 \geq -\frac{1}{4} d^2 \underset{d=4}{=} -4$$

The $t$ scalars are **at** the BF bound in $AdS_5$.

# Kerr-Newman AdS as a Supergravity Solution

- Here the environment is an $AdS_5$ **black hole**, not the vacuum.

- Kerr-Newman-AdS solves Einstein-Maxwell-AdS theory.

  Now: **reinterpret** it as a solution to supergravity.

- $\mathcal{N} = 8$ SUGRA has $SU(4)_R$ symmetry $\Rightarrow$ 15 vector fields.

- Pick **background** vector field as the unique linear combination that permits **constant** scalars.

# Fluctuating Matter Fields in $AdS_5$

- Fluctuations around black hole: supergravity fields expanded to quadratic order around background.
- Symmetry breaking pattern: $SU(4)_R \to SU(3) \times U(1)$.
- Generally: matter fields are **charged** with respect to "the" gauge field in the black hole background.
- Also: **degeneracy** remains due to $SU(3)$ global symmetry.
- $\mathbf{20}'$ scalars $t \;\Rightarrow\;$ 8 neutral $t_-$ and 12 $t_+$ with charge $e = \pm 2$:

$$\mathbf{20}' \to \mathbf{8}_0 \oplus \left(\mathbf{3}_2 \oplus \bar{\mathbf{3}}_2 \oplus \mathbf{3}_{-2} \oplus \bar{\mathbf{3}}_{-2}\right)$$

# Attractor Flow

- The radial dependence in BH background: **attractor flow**.
- Very well developed in **ungauged** supergravity.
- Complicating factors in current context:
  - AdS vacua in **gauged** supergravity.
  - **Rotation**, or else supersymmetry is **not possible**.
  - **General extremal** case, not necessarily supersymmetric.
- Upshot: an effective mass in near horizon $AdS_2$ region (with squashed $S^3$ fibre).

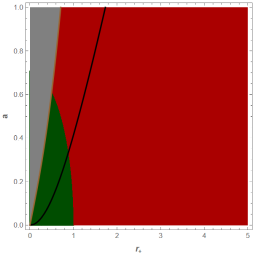
# Light Scalars in KNAdS Background

- The BF-bound in $AdS_2$:

$$m^2\ell_2^2 = -4\frac{\ell_2^2}{\ell_5^2} \geq -\frac{1}{4}d^2 \underset{d=1}{=} -\frac{1}{4}$$

- The fate depends on the BH parameters via the $AdS_2$ radius.

- Large unstable region **includes** many BPS black holes.



# Non-Minimal Couplings

- **All** of the $\mathbf{20}'$ scalars have **non-minimal** couplings.
- Kinetic terms for vectors in supergravity:

$$\mathcal{L} \sim -\mathcal{N}(\phi) F_{\mu\nu} F^{\mu\nu}$$

  Kinetic function $\mathcal{N}(\phi)$ depends on the scalar field.

- In $AdS_2$, this **Pauli Coupling** is an effective mass:

$$m^2_{\mathrm{Pauli}} = -p \cdot \frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$

  The coupling $p = +2$ for $t_+$ and $p = -2$ for $t_-$.

- Moreover, 8 neutral $t_-$ mix with 8 fluctuating gauge fields $a_-$.

# The Physics of Minimal Couplings

- **Minimal coupling** to the background vector field:

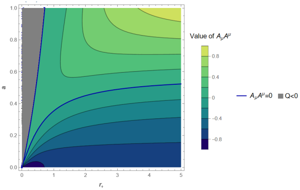
$$|(D_\mu + ieA_\mu)\phi|^2$$

For the 12 $t_+$ fields, the $U(1)_R$ charge is $e = 2$.

- In $\mathrm{AdS}_2$, the coupling gives an effective mass

$$m^2_{\mathrm{minimal}} = e^2 g^{\mu\nu} A_\mu A_\nu$$

It is **negative** in an electric background.



# Superfluidity

- The background electric potential gives **charged scalars** an **expectation value**.
- This is **superfluidity**
- Holographic superconductivity was much studied, but not embedded in full supergravity.
- Superconductivity applies when the BH is **under**rotating.

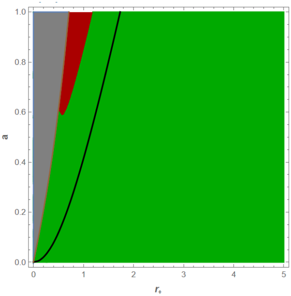
# The Fate of the Lightest Scalars: Charged Sector

- 12 charged $t_+$ also have Pauli couplings to $\mathcal{F}_{\mu\nu}$.

  The coupling $p = 2 > 0$ compensates the minimal coupling.

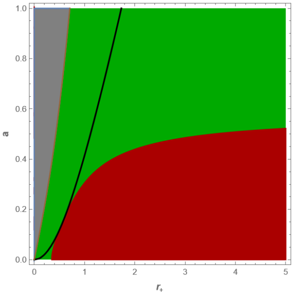
- **On balance**: the $t^+$ scalars are **stable** for BPS black holes.

- Also stable in Reissner-Nordström: **no superconductivity**

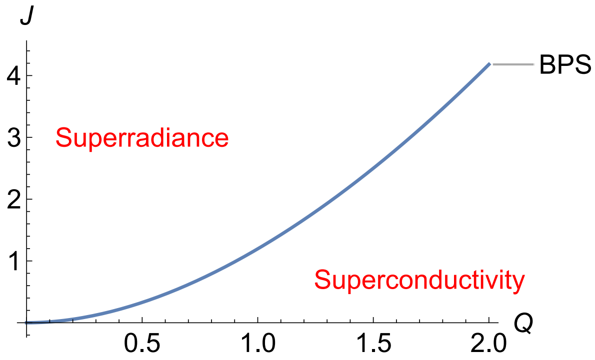


# The Pseudo Scalars $\varphi_1$ Condense

- $\mathbf{10}_c$ fields $\varphi$ have $m^2\ell_5^2 = -3$: **easily** stable in $AdS_5$ vacuum.
- In KN-$AdS_5$ background: two components $\varphi_1$ have $e = \pm 3$.
- This **large** minimal coupling drives **superconductivity**.



# Phases of Extremal Black Holes in $AdS_5$




- Instability near axes: **superradiance/superconductivity**.
- They extend **all the way** to the BPS line.

  The BPS line is a **phase boundary**.

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# Part C

# Bose-Einstein Condensation

# Toy Model

- **Toy model**: two chiral 1D bosons with charges $\pm 1$:

$$Z = \mathrm{Tr}\; e^{-\beta(H-\Omega J)} = q^{-\frac{1}{12}} \prod_{n=1}^{\infty} \frac{1}{1-q^n y} \frac{1}{1-q^n y^{-1}}$$

- **Exact** rewriting ($\tilde{y} = e^{-2\pi i \Omega}$ and $\tilde{q} = e^{-4\pi^2/\beta}$):

$$Z = \frac{\sinh \frac{\beta\Omega}{2}}{\sin \pi\Omega} e^{\frac{\pi^2}{3\beta} + \frac{\beta\Omega^2}{2}} \underbrace{\prod_{n=1}^{\infty} \frac{1}{(1-\tilde{y}\tilde{q}^n)(1-\tilde{y}^{-1}\tilde{q}^n)}}_{\text{Exp. suppression @ high T}}$$

- **Cardy's formula** for $c = 2$ SCFT with $R$ charge $J$:

$$\ln Z = \frac{\pi^2}{3\beta} + \frac{\beta\Omega^2}{2} \quad \Rightarrow \quad S = 2\pi\sqrt{\frac{1}{3}E - \frac{1}{6}J^2}$$

# The Black Hole Analogue

- Entropy at **high temperature**:

$$S = 2\pi\sqrt{\frac{1}{3}E - \frac{1}{6}J^2}$$

  Favorable to spread energy over **all** modes.

  Occupation #'s $N_n \ll 1$.

  The **black hole** description in the toy model.

- Given conserved charge $J$, minimal energy: $E_{\mathrm{min}} = \frac{1}{2}J^2$.

  The **extremal** black hole in the toy model.

# The Unitarity Bound

- For conserved charge $J$: **ground state energy** $E_{\rm gs} = J$.

  There are particles with $E = J = 1$ in the theory.

- For large charge, $E_{\rm gs} = J$ **violates** the bound $E_{\rm min} = \frac{1}{2} J^2$.

- Partition function (up to non-perturbative in $T$):

$$Z = \frac{\sinh \frac{\beta\Omega}{2}}{\sin \pi\Omega} e^{\frac{\pi^2}{3\beta} + \frac{\beta\Omega^2}{2}}$$

  Large angular momentum: $\Omega$ near 1: **prefactor matters**

# Bose Einstein Condensation

- For comparable small $\beta$ and $1-\Omega$:

$$E = \frac{\pi^2}{3\beta^2} + \frac{1}{\beta(1-\Omega)} \ , \ J = \frac{1}{\beta(1-\Omega)}$$

- Entropy:

$$S = 2\pi\sqrt{\frac{1}{3}(E-J)}$$

- **Bose Einstein Condensation**:

  **All** angular momentum in BPS modes with $E = J$ $(n = 1)$.

  **Remaining** energy $E - J$ spread across all other modes.

# Summary

- Thermodynamics of $AdS_5$ black holes is **intricate**.
- Charges of **supersymmetric** black holes must satisfy a **constraint**.
- **Extremal** nonSUSY black holes are unstable:

  Superradiant **overrotation** or superconducting **overcharging**