Goldman-Hodgkin-Katz Model
GHK Equation

permeability.

NPE
Critical ~~NPE~~ does not really account for membrane properties

~~Assumption~~. Reversal potential of the membrane taking into account multiple ions that all membrane is permeable to

Assumptions
① NPE holds within the membrane (aqueous pore)
(Fails with complex barriers)

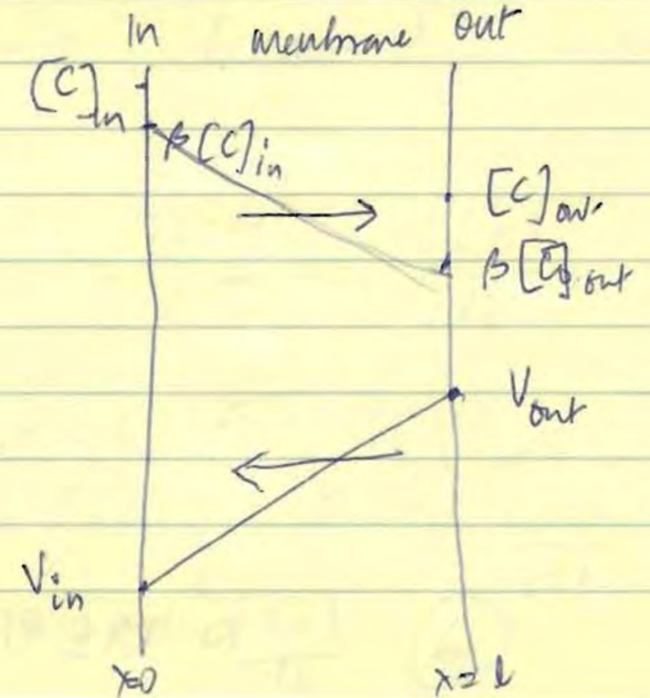
② Electric field within the membrane is constant
$$E = -\frac{dv}{dx} = -V/l$$

③ Ions do not interact with each other as they move across the membrane

Based on 1st Assumption and using NPE

$$I = -\left(\nu z^2 F[C]\frac{dv}{dx} + \nu z RT \frac{d[C]}{dx} \right)$$

$$\frac{dv}{dx} = \frac{V}{l}$$



Remember that the −ve sign indicates I flows down the voltage gradient. However here ∴

First term is +ve

Second term is −ve

$$I = \nu z^2 F[c]\frac{V}{l} - \nu zRT\frac{d[c]}{dx} \qquad (2)$$

we use one trick.

$$y = I - \frac{\nu z^2 F[c]\,V}{l} \qquad (3)$$

$$\frac{dy}{dx} = \frac{dI}{dx} - \frac{\nu z^2 F}{l}\,V\,\frac{d[c]}{dx} \qquad (4)$$

$\frac{dI}{dx} = 0$ because I is in steady state

$$\frac{dy}{dx} = \frac{-\nu z^2 FV}{l}\,\frac{d[c]}{dx} \qquad (5)$$

Substitute (2) in (3)

$$y = \nu z^2 F[c]\frac{V}{l} - \nu zRT\frac{d[c]}{dx} - \frac{\nu z^2 F}{l}V[c]$$

$$y = -\nu zRT\frac{d[c]}{dx} \qquad (6)$$

Substituting for $\frac{d[C]}{dx}$ [illegible] in ⑥

using ⑤

$$\frac{d[C]}{dx} = \frac{-l}{\nu z^2 F(V)} \frac{dy}{dx} \quad (7)$$

using ⑦ in ⑥

$$y = \frac{+\nu z RT\,l}{\nu z^2 F(V)} \frac{dy}{dx}$$

$$y = \frac{RTl}{zFV} \frac{dy}{dx} \quad (8)$$

$$\int_0^l dx = \frac{RTl}{zFV} \int_{y(C_{x=0})}^{y(C_{x=l})} \frac{dy}{dy}$$

$$l = \frac{RTl}{zFV} \Big|_{y(C_{x=0})}^{y(C_{x=l})} \ln y \quad (9)$$

Remember $y = I - \frac{\nu z^2 F[C]V}{l}$

If $x=0$, $[C]_{x=0} = \beta[C_s]_i$ at $x=l$ $[C]_{x=l} = \beta[C]_o$ (10)

$$\cancel{I = \frac{RTl}{zFl} \ln \frac{I - \nu z^2 FV\beta [C]_0}{l}}$$

using (10)

$$l = \frac{RTl}{zFV} \ln \frac{I - \frac{\nu z^2 FV\beta [C_o]}{l}}{I - \frac{\nu z^2 FV\beta [C_i]}{l}} \quad (11)$$

$$\frac{zFV}{RT} = \ln \frac{Il - \nu z^2 FV\beta [C]_o}{Il - \nu z^2 FV\beta [C]_i} \quad (12)$$

let call $\frac{zFV}{RT} = \xi$

(12) become

$$e^{-\xi} = \frac{IL - \nu z^2 FV\beta [C_i]}{IL - \nu z^2 FV\beta [C_o]} \quad (13)$$

Solve for I

$$IL e^{-\xi} - \nu z^2 FV\beta [C]_o e^{-\xi} = IL - \nu z^2 FV\beta [C]_i \quad (14)$$

$$I\left(l e^{-\xi} - l\right) = \vartheta z^2 F V \beta [C]_o e^{-\xi} - \vartheta z^2 F V \beta [C]_i$$

multiplying with -ve

$$I\left(l - l e^{-\xi}\right) = \vartheta z^2 F V \beta [C]_i - \vartheta z^2 F V \beta [C_o] e^{-\xi}$$

$$I = \frac{\vartheta z^2 F V \beta}{l} \frac{[C_i] - [C_o] e^{-\xi}}{(1 - e^{-\xi})}$$

Remember $P = \frac{\beta \vartheta R T}{l F}$ $\quad \xi = \frac{z V F}{R T}$

$$\boxed{I = P z F \xi \left[\frac{[C]_i - [C]_o e^{-\xi}}{(1 - e^{-\xi})}\right]}$$

GHK Current Equation for I and V

we had have seen I-V Relationships before
Ohms law → linear relationship
is this a linear → NO

$$I_{tot} = I_{out} - I_{in}$$

where $I_{out} = P z F \xi \dfrac{[C]_i}{[1 - e^{-\xi}]}$

$$I_{in} = -P z F \xi \frac{[C]_o e^{-\xi}}{[1 - e^{-\xi}]}$$

Case I $\frac{[C]_o}{[C_i]_i} = 1$ or $[C]_o = [C]_i$

$$I = PzF\xi\,[C_o]\,\frac{(1-e^{-\xi})}{(1-e^{-\xi})}$$

$$I = PzF\xi\,[C]_o$$

$$\boxed{I = \frac{Pz^2F^2}{RT}[C_o]\,V}$$

Ohms Law → linear Relationship

Case II

$\frac{[C_o]}{[C]_i} < 1$ → case of pottassium
→ outward rectified

~~we~~ ~~are~~ What are we talking about → IV Relationships
Case I was linear → slope was constant
Case II What is the ~~exact~~ slope going to look like
as membrane becomes more and more +ve ?
Slope is going to get faster or steeper with
membrane potential
As the membrane gets +ve more K^+ ions go out as
it is easier for them to escape.

Case III

$\frac{[Co]}{[Ci]} > 1$

Opp is true.