Kinematic Anisotropies and Pulsar Timing Arrays

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Based on
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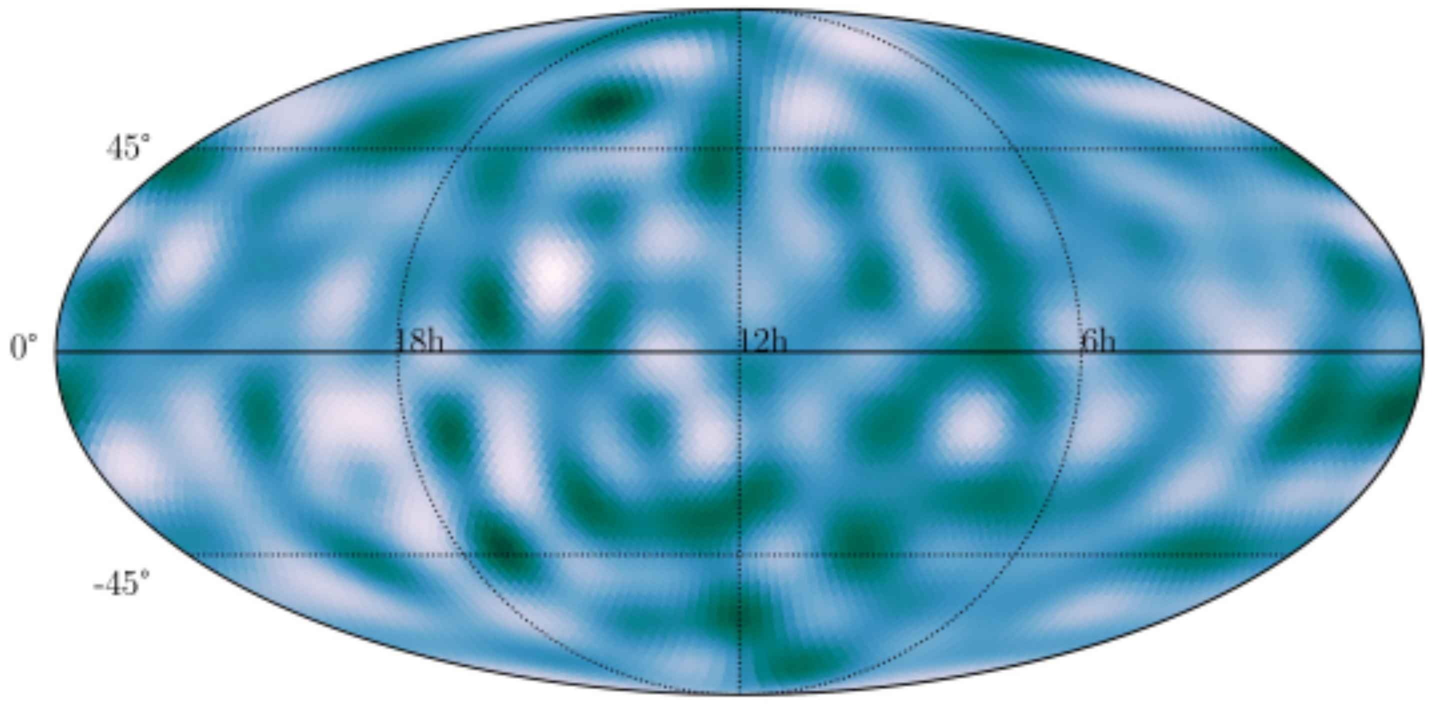
2406.04957
2412.14010

Stochastic gravitational wave background

- Several **astrophysical** and **cosmological** phenomena predict the existence of a stochastic background of gravitational waves

  – **Astro:** Merging of compact binaries in various range masses . . .

  – **Cosmo:** Cosmological inflation, cosmic strings, phase transitions . . .



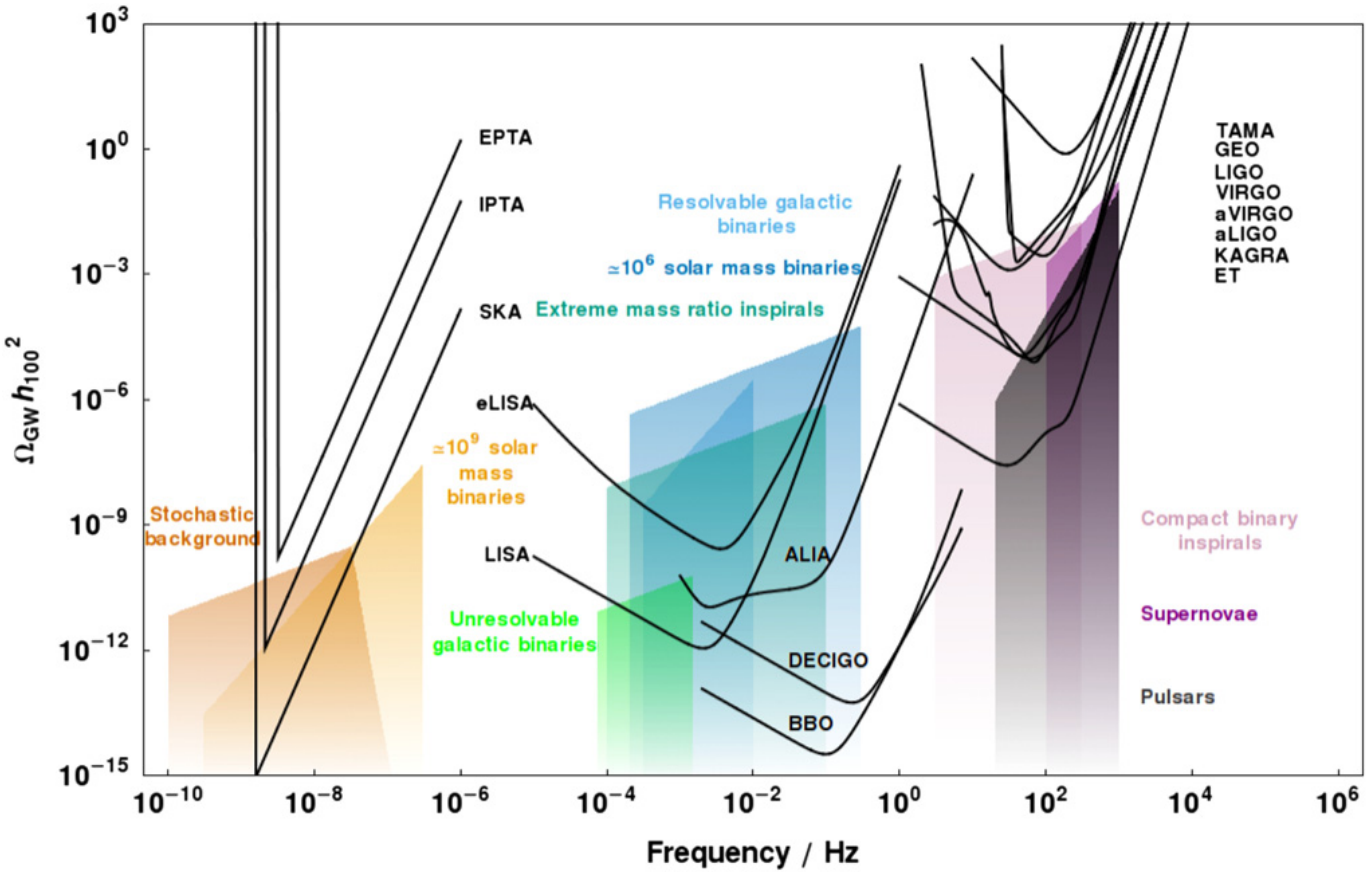
- Tasks

  – Detect it with sufficiently high significance

  – Find methods for **distinguishing among different sources**

This talk: SGWB anisotropies

Stochastic gravitational wave background

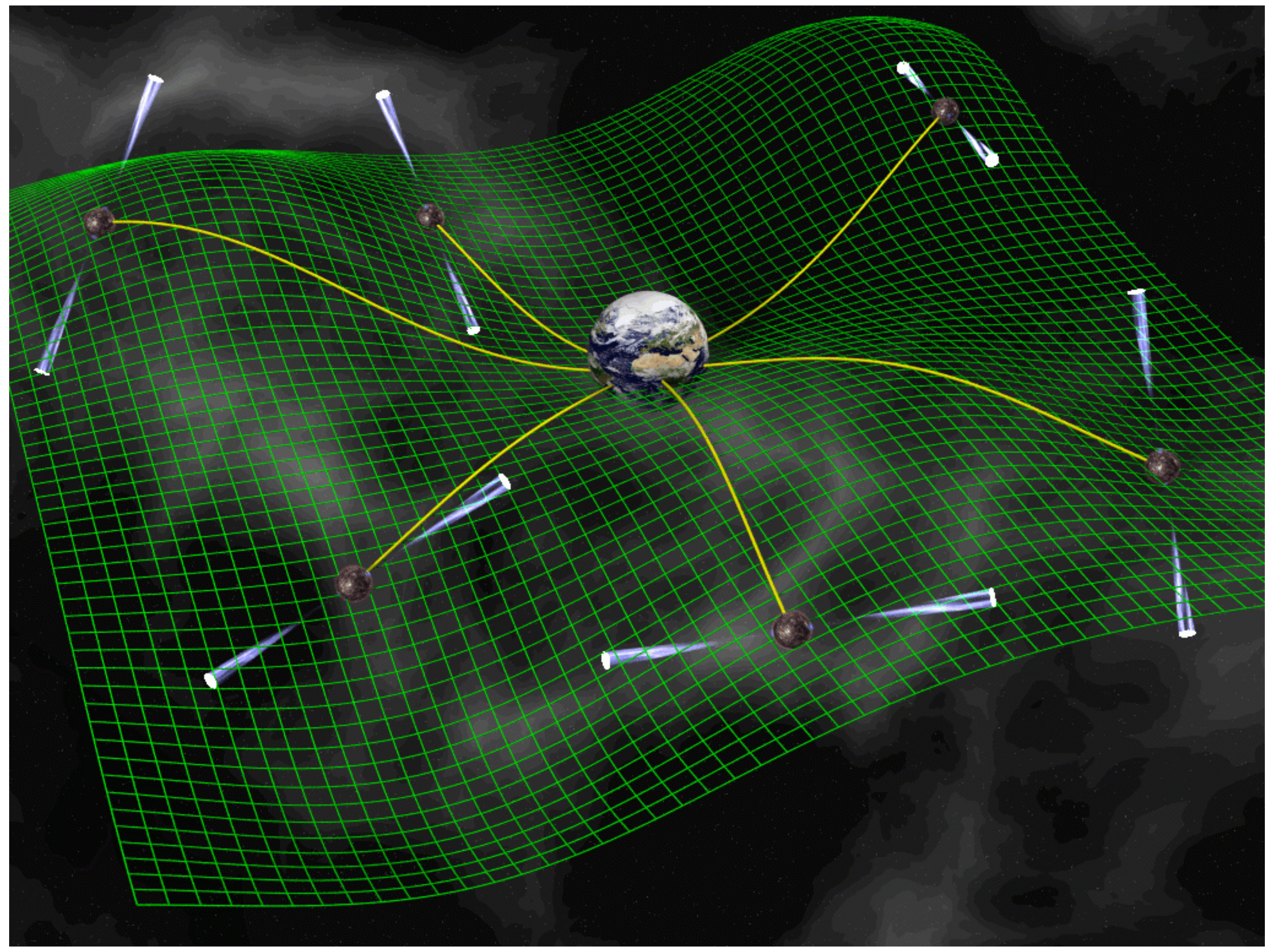


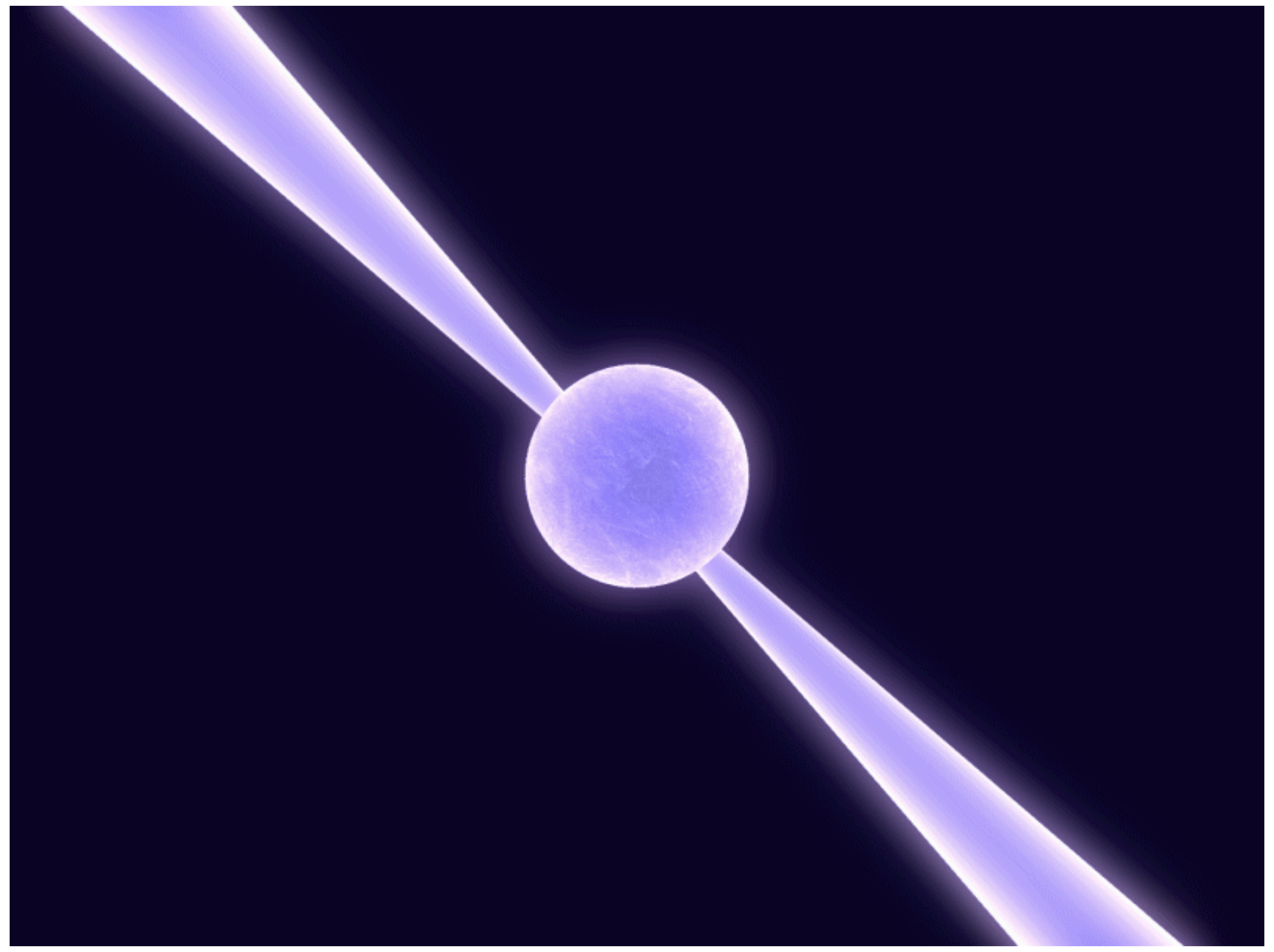
$$\Omega_{\rm GW} = \frac{1}{\rho_c}\frac{d\rho_{\rm GW}}{d\ln k}$$

[Moore et al]

Pulsar Timing Arrays and Gravitational Waves

- Pulsars are **rapidly rotating neutron stars**:
  extremely precise astrophysical clocks given their period is almost constant in time.





- The **Time of Arrival** of emitted light to earth is sensitive to deformations of space-time between pulsar and earth.

'Cosmic lighthouse'

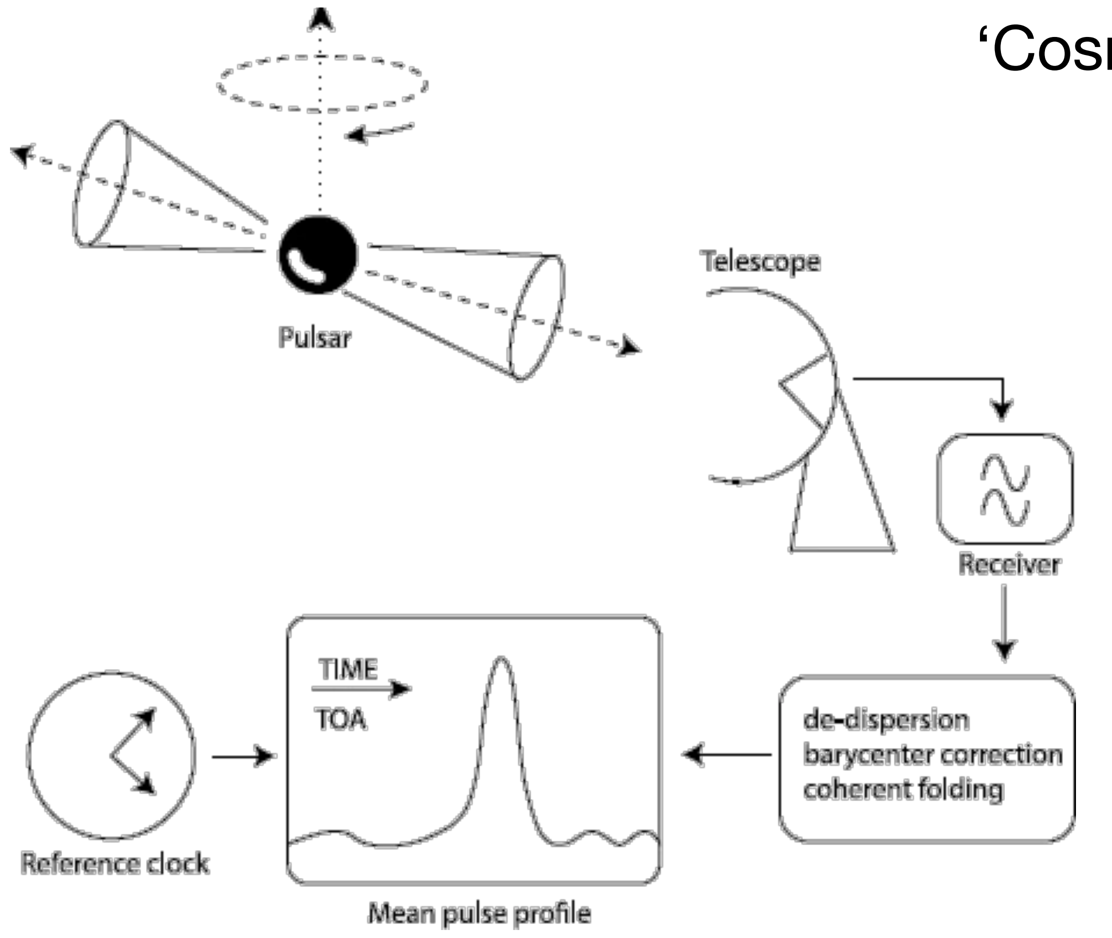


Image: D. Lorimer

Pulsar Timing Arrays and Gravitational Waves

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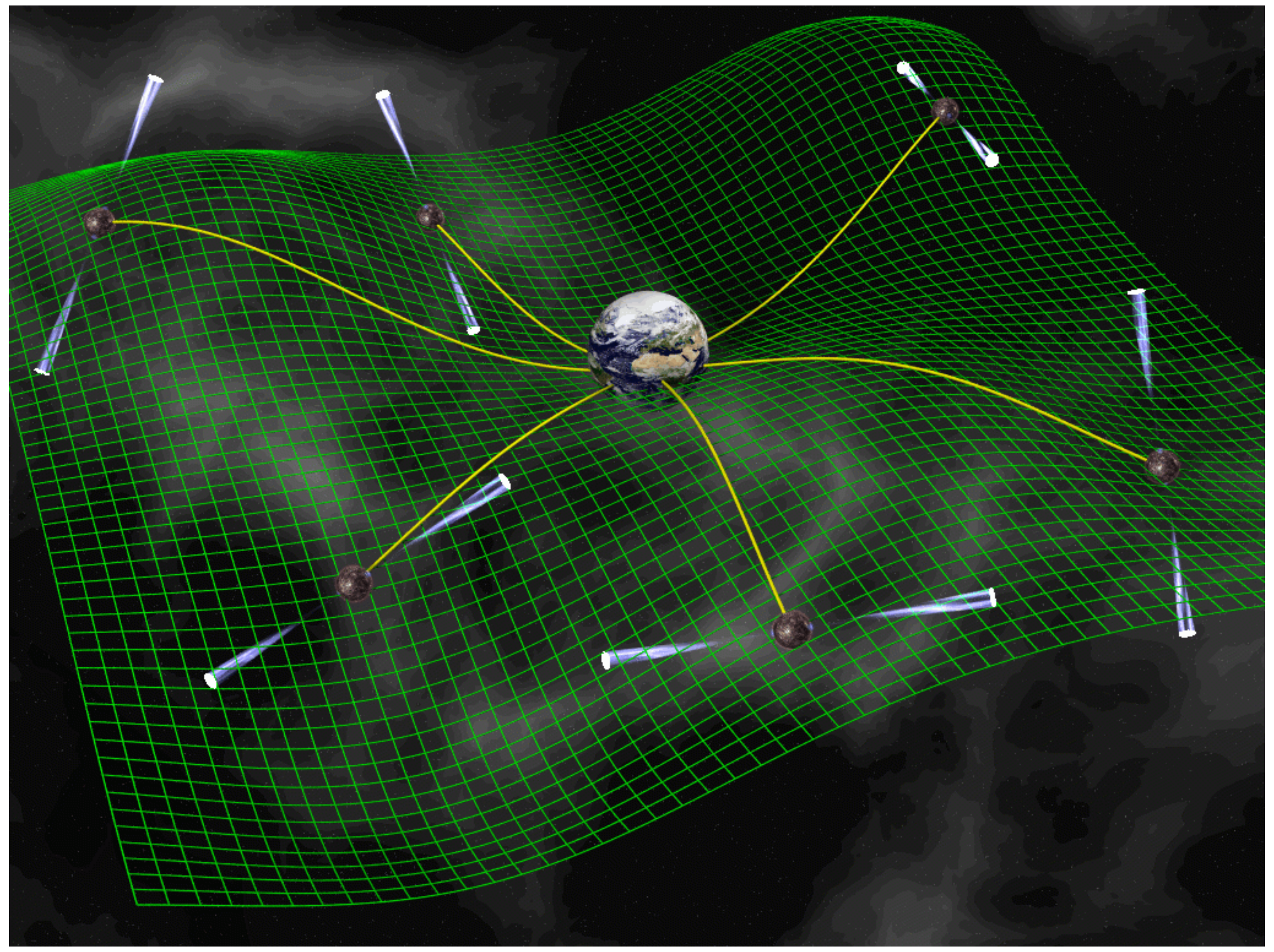

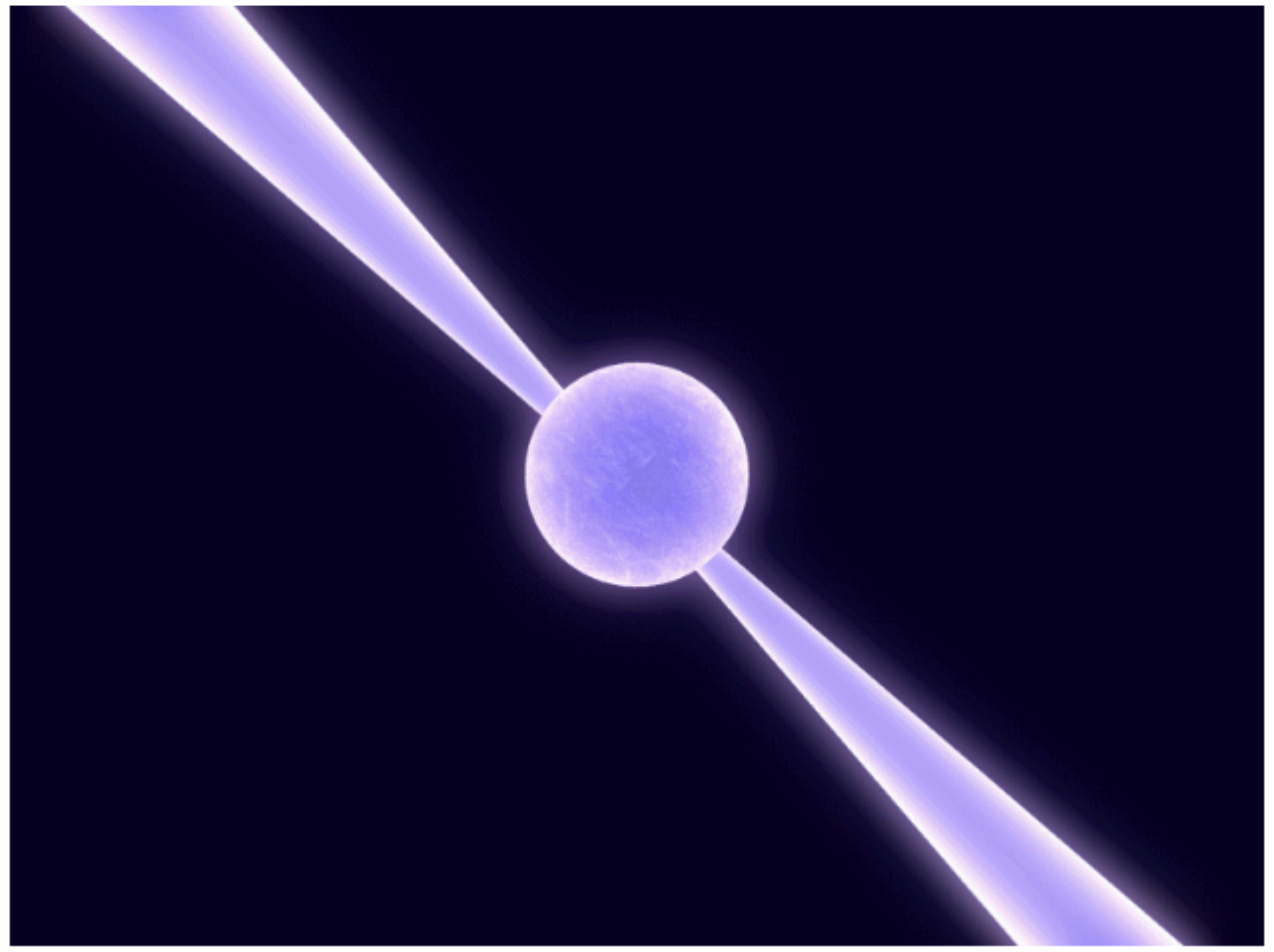

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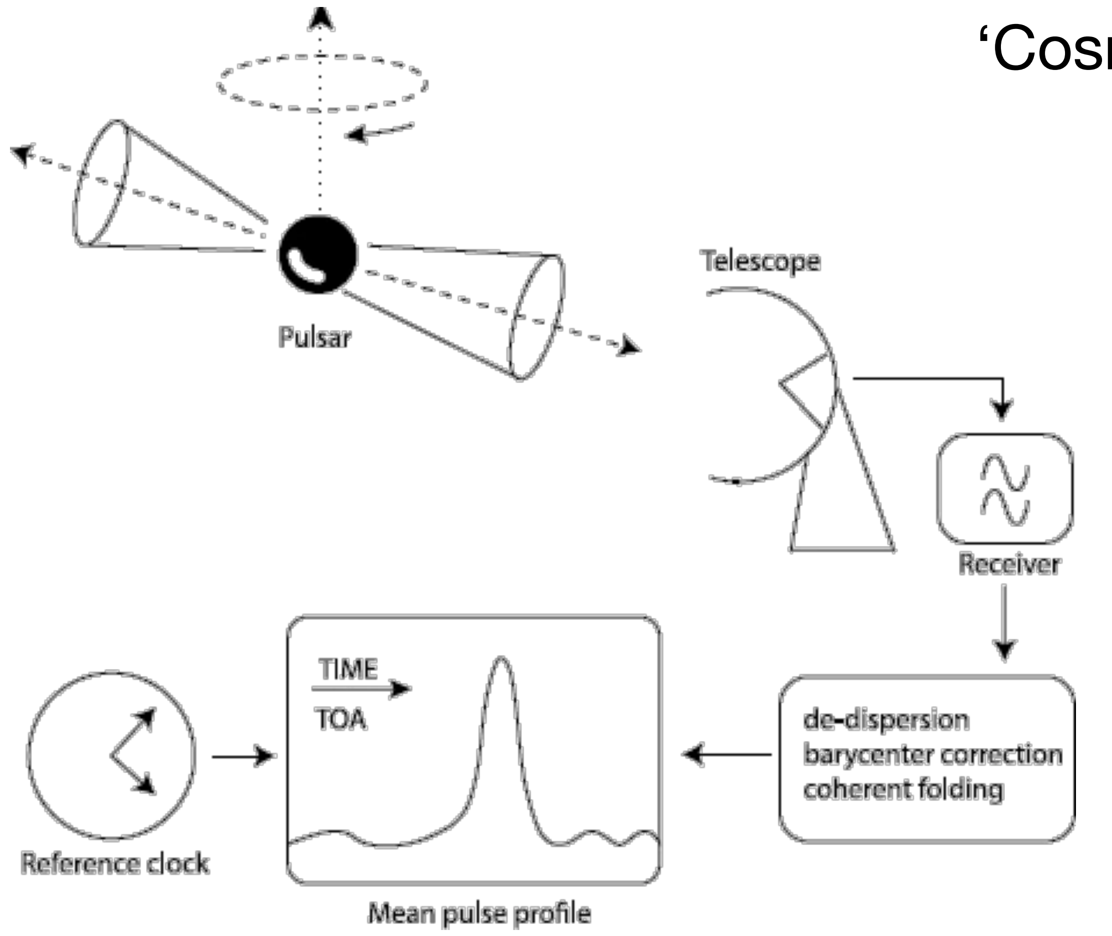

Image: D. Lorimer

A **change** in observed period of pulsar can be attributed to the presence of a gravitational wave. It is the same principle of terrestrial interferometers – but with astronomically long arms! For this reason the frequency detected is in the nano-Hertz band.

Pulsar Timing Arrays and Gravitational Waves

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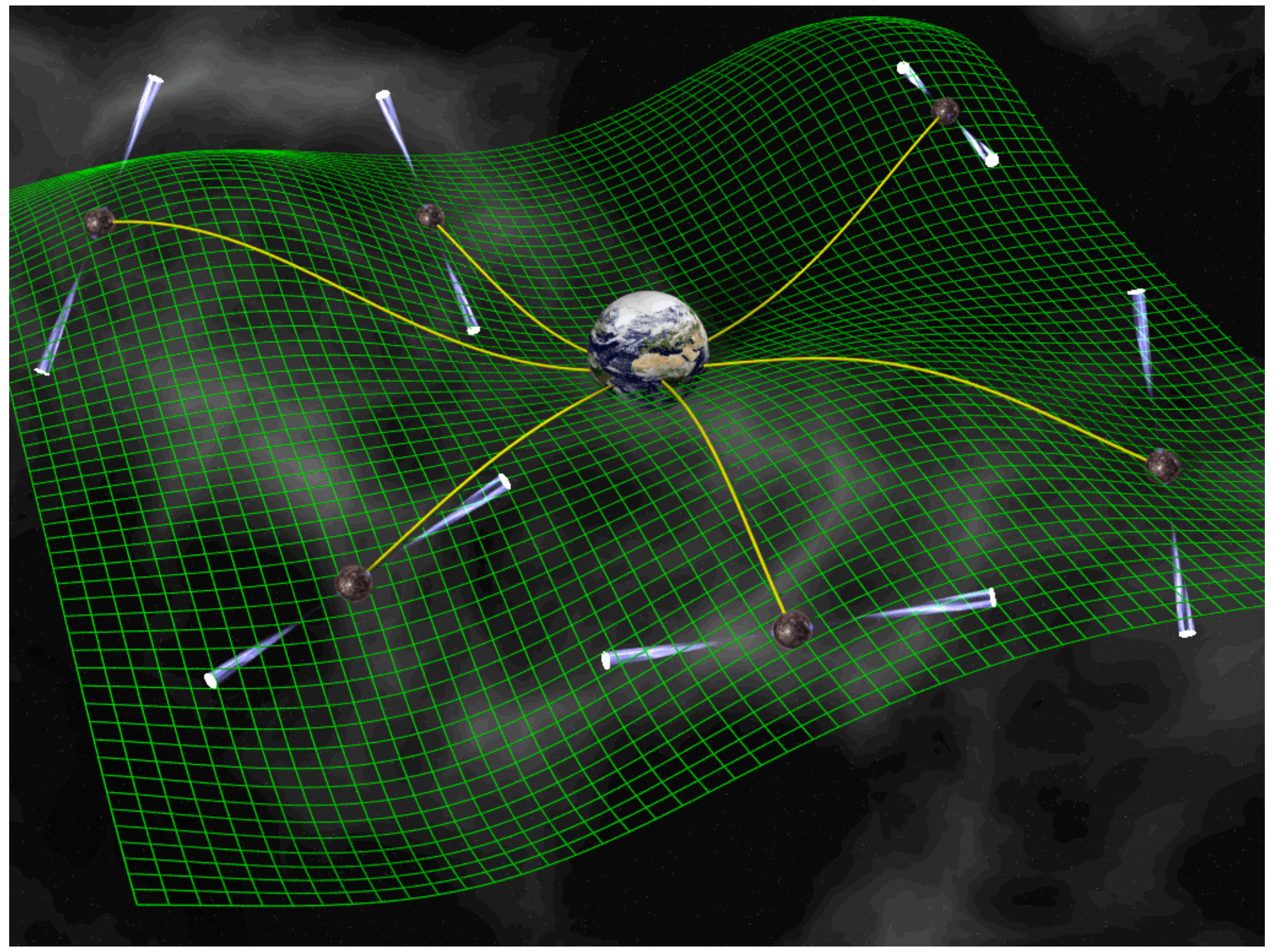
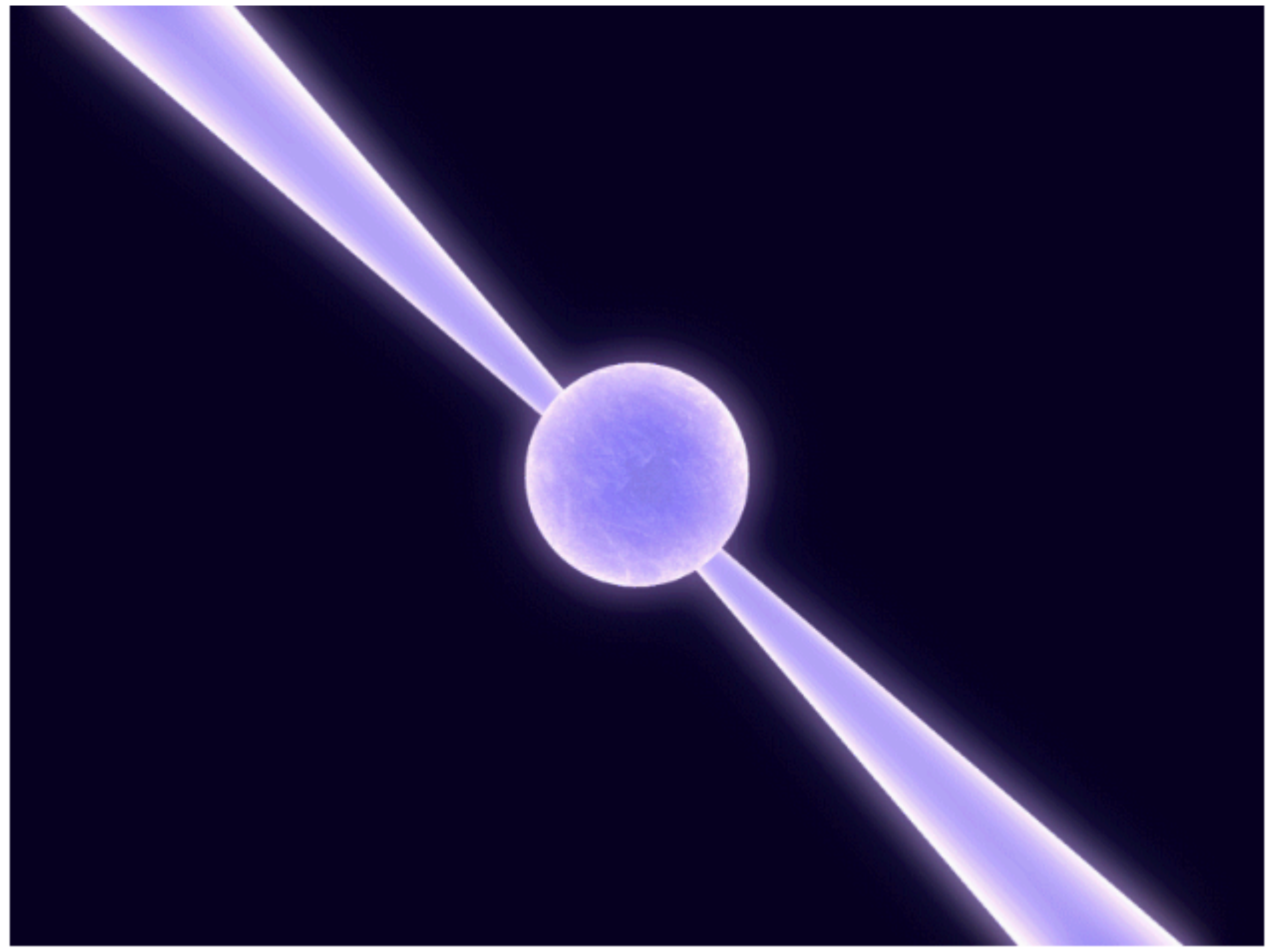
  extremely precise astrophysical clocks given their period is almost constant in time.





'Cosmic lighthouse'

- The **Time of Arrival** of emitted light to earth is sensitive to deformations of space-time between pulsar and earth.

- To reduce noise and avoid spurious signals, it is necessary to monitor several pulsars

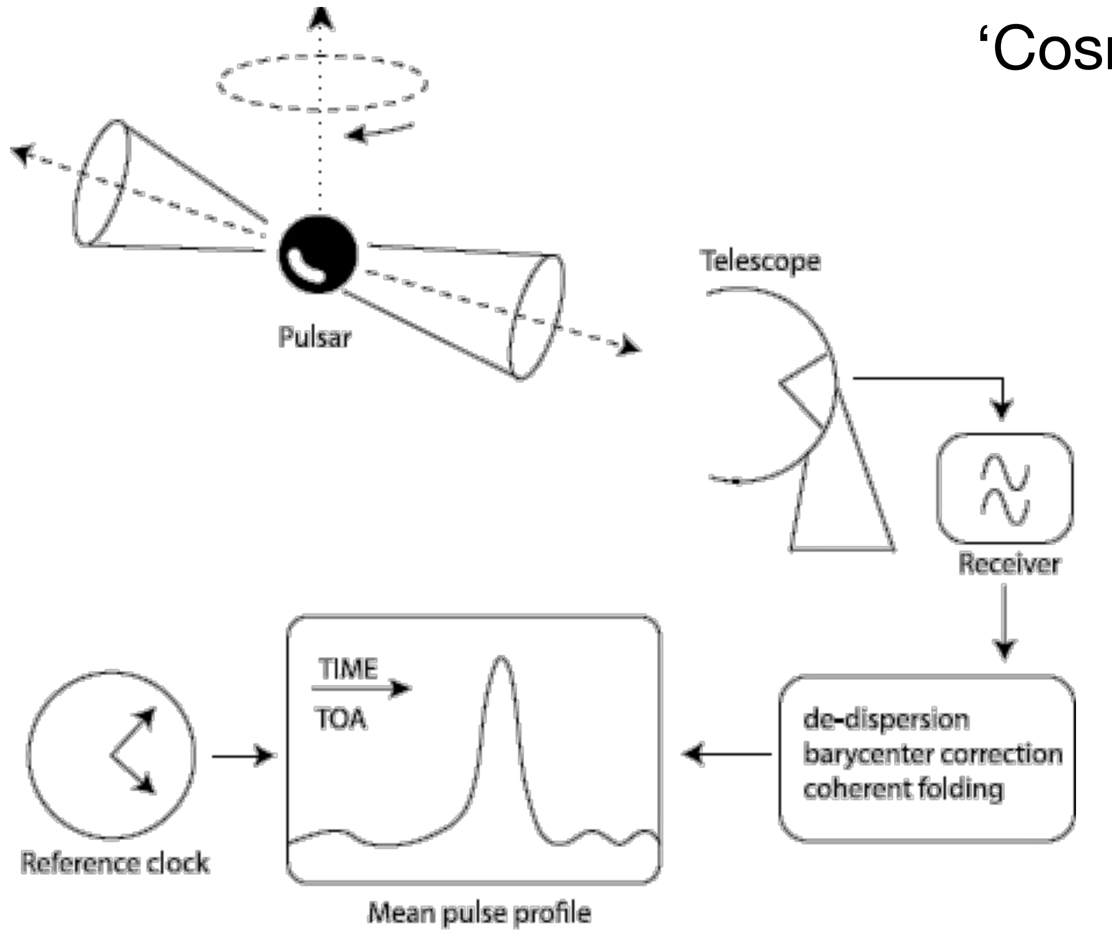
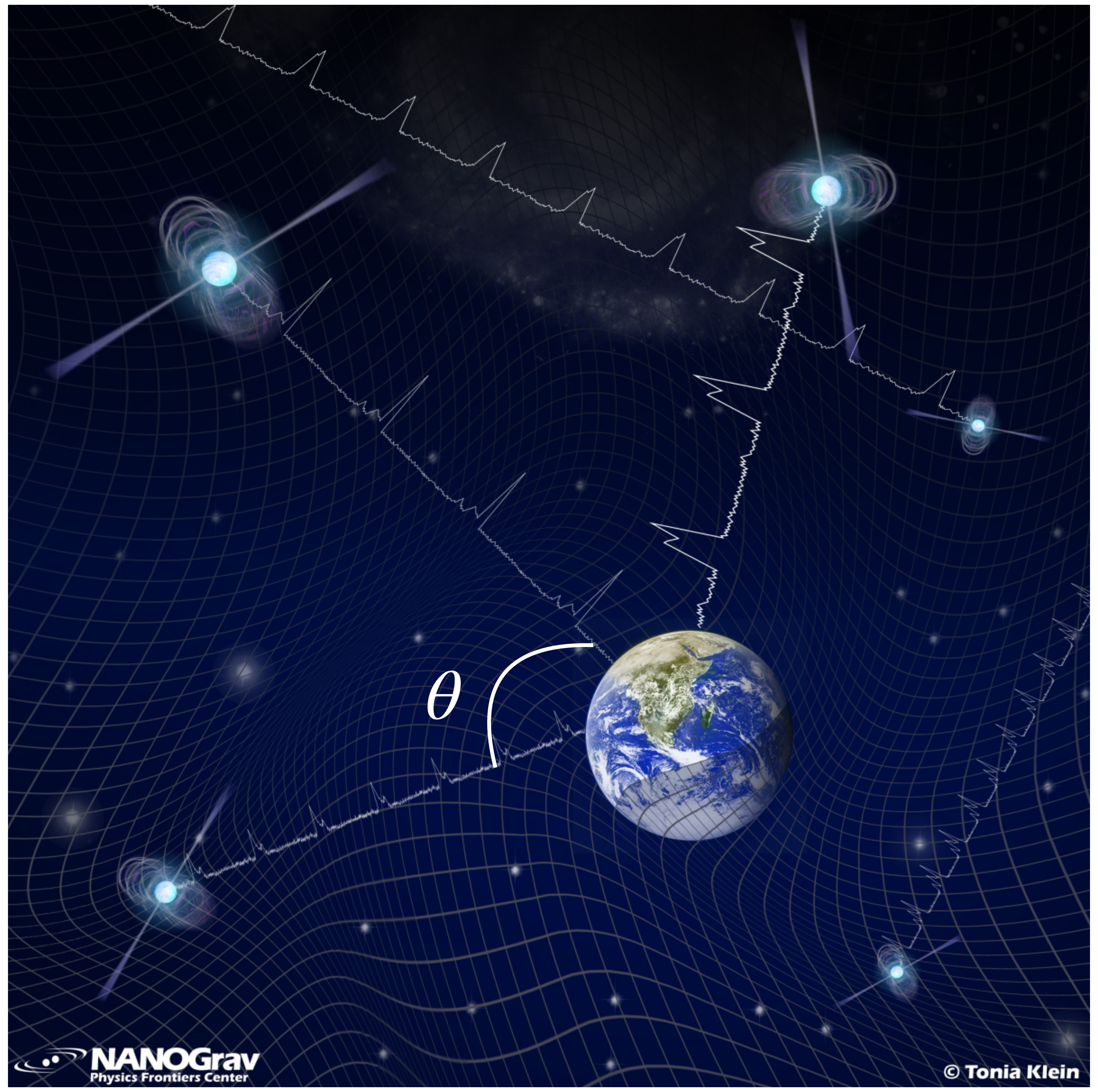


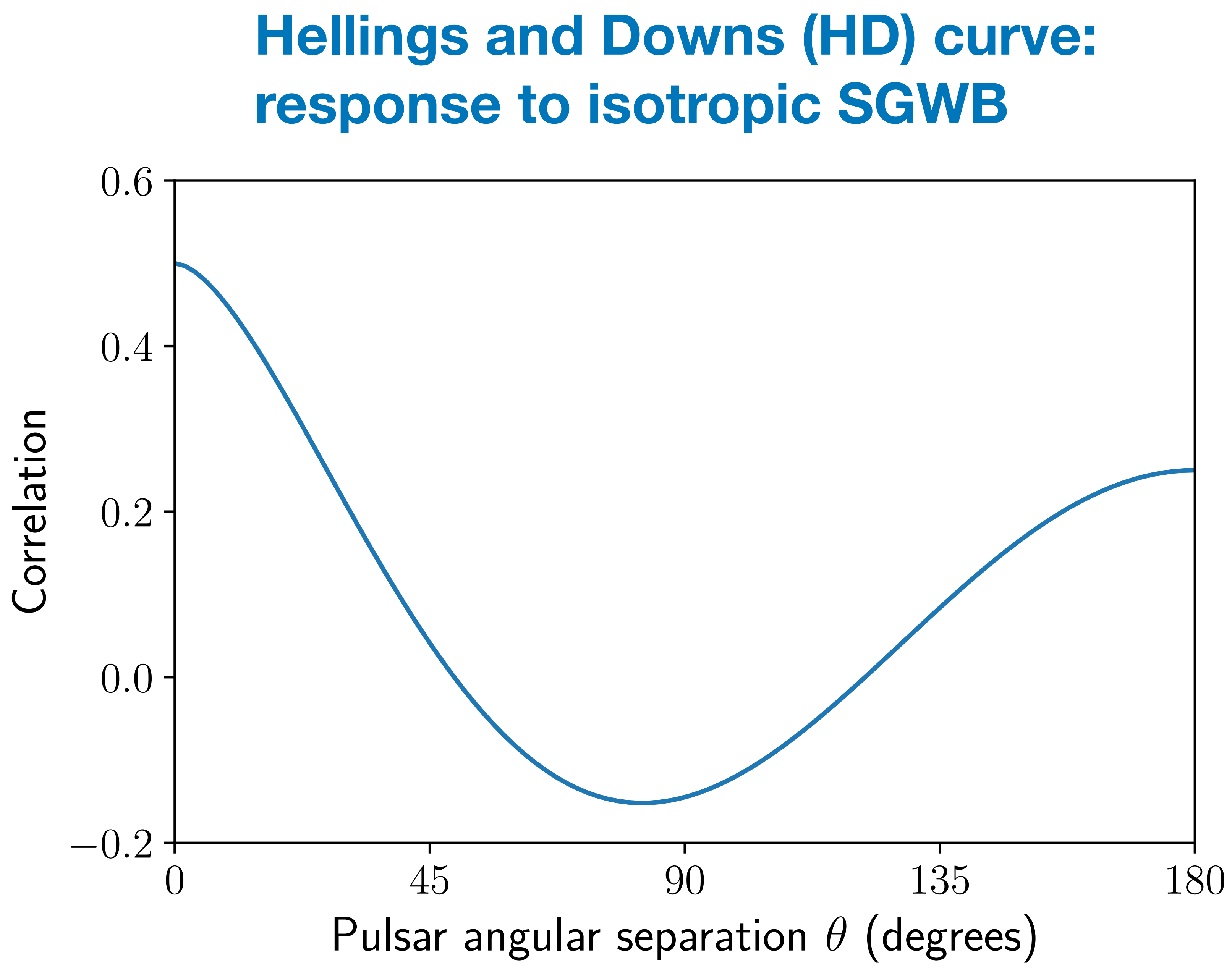
Image: D. Lorimer

Pulsar Timing Arrays and Gravitational Waves

Recently, several PTA collaborations found relatively strong evidence for a signal compatible with stochastic gravitational wave background.

E.g. NANOGrav monitored 67 pulsars for a 15 years period. They found the characteristic angular correlation between signals detected with different pulsars, as predicted by General Relativity. This is called Hellings-Downs curve.



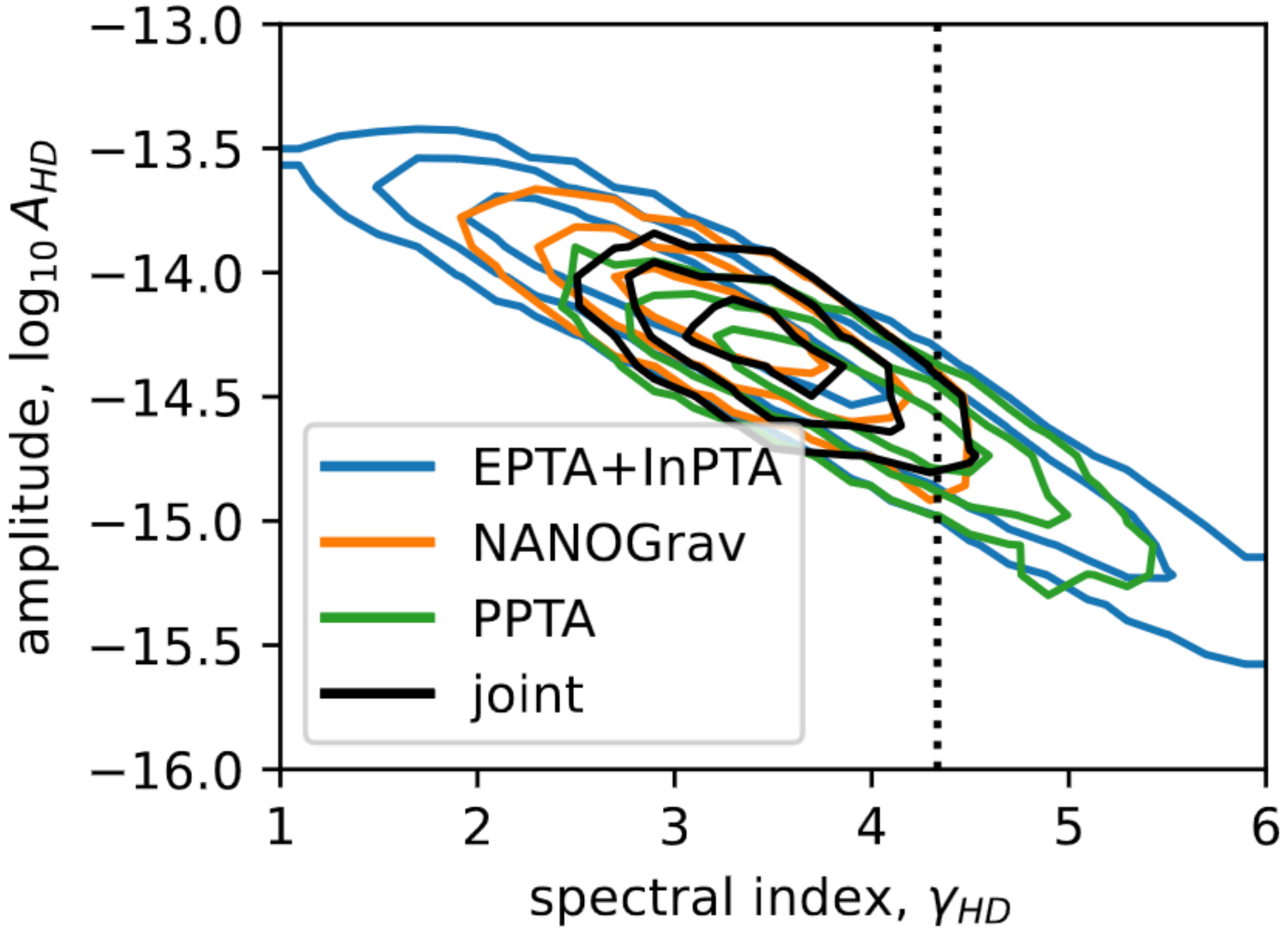


Pulsar Timing Arrays and Gravitational Waves

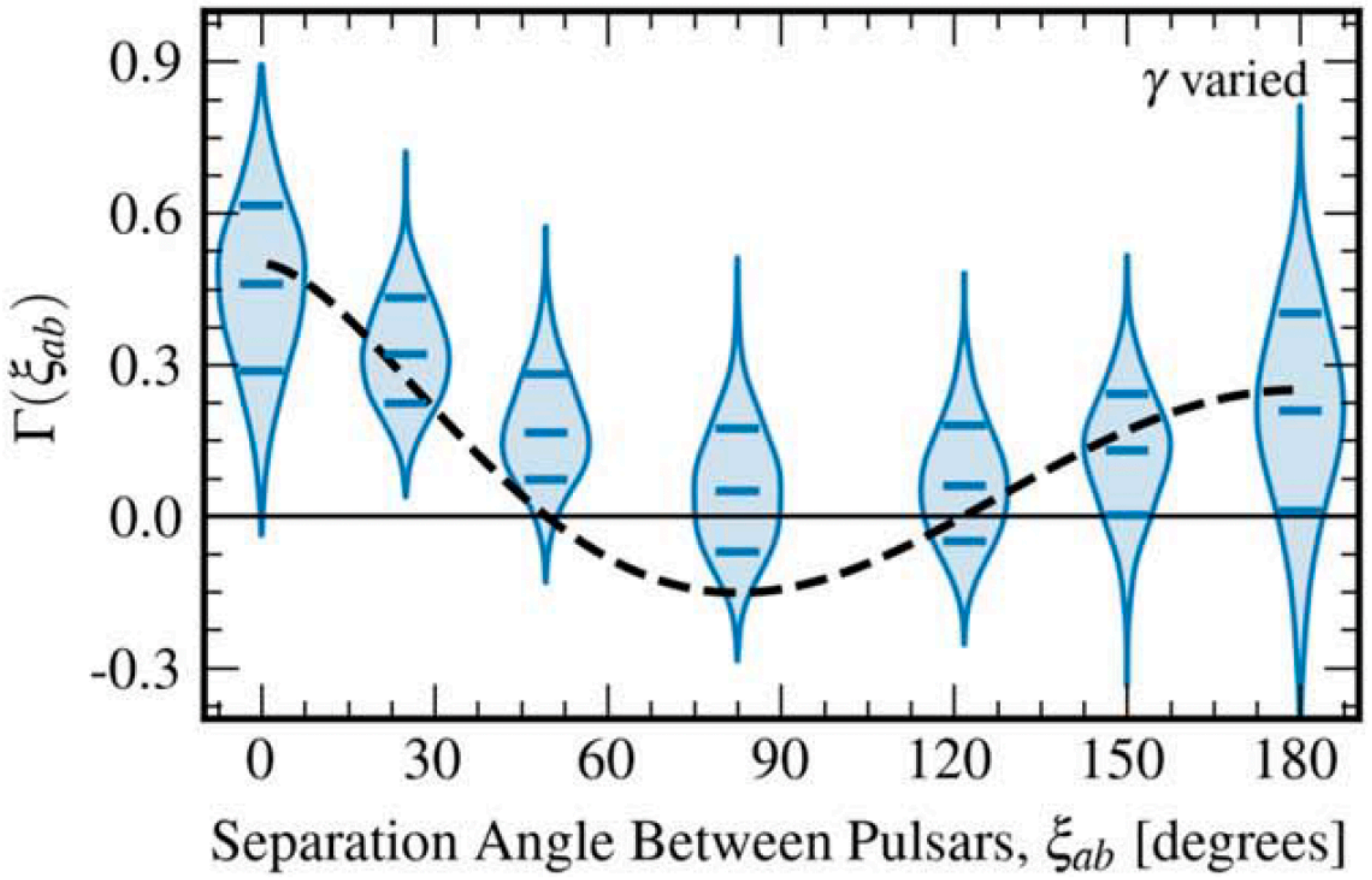
- Common spectrum process detected by NANOGrav, EPTA, PPTA, InPTA, CPTA
- HD correlations detected with ~$3 - 4\sigma$ significance

$$\Omega_{\rm GW}(f) = \frac{2\pi^2 f^2}{3H_0^2} A^2 \left(\frac{f}{f_{\rm ref}}\right)^{2\alpha}$$

$$\gamma \equiv 3 - 2\alpha.$$



arxiv: 2309.00693



NANOGrav 15 year analysis

$$\Omega_{\rm GW} = \frac{1}{\rho_c}\frac{d\rho_{\rm GW}}{d\ln k} \simeq (5 \pm 2) \times 10^{-9}$$

The slope though is not well measured, presently compatible with several possibilities.

The Hellings-Downs curve

- How do you actually compute HD? You do angular integrations, which need some care...

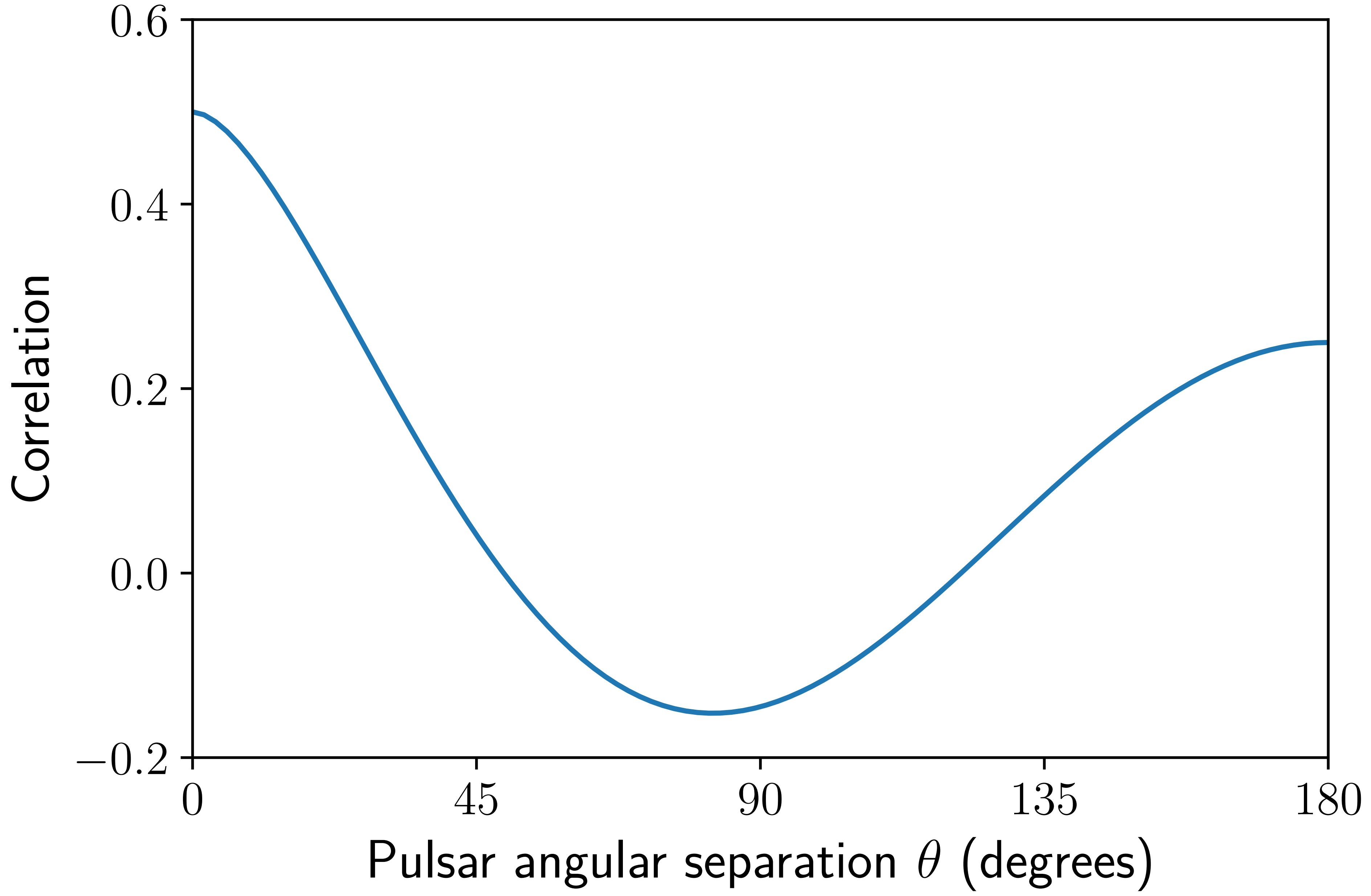
$$\Gamma^I_{ab}(f) = \frac{1}{2\pi\,\bar{I}(f)} \int d^2\hat{n} \left(D^\lambda_a(\hat{n}) D^{\lambda'}_b(\hat{n})\, \delta_{\lambda\lambda'} \right) I(f, \hat{n})$$

$$D^\lambda_a(\hat{n}) \equiv D^{ij}_a(\hat{n})\, \mathbf{e}^\lambda_{ij}(\hat{n})$$

$$D^{ij}_a \equiv \frac{\hat{x}^i_a\, \hat{x}^j_a}{2(1+\hat{n}\cdot\hat{x}_a)}$$

$$\Omega_{\rm GW} = \frac{4\pi\, f^3}{3H_0^2}\, I$$

Hellings and Downs (HD) curve: response to isotropic SGWB

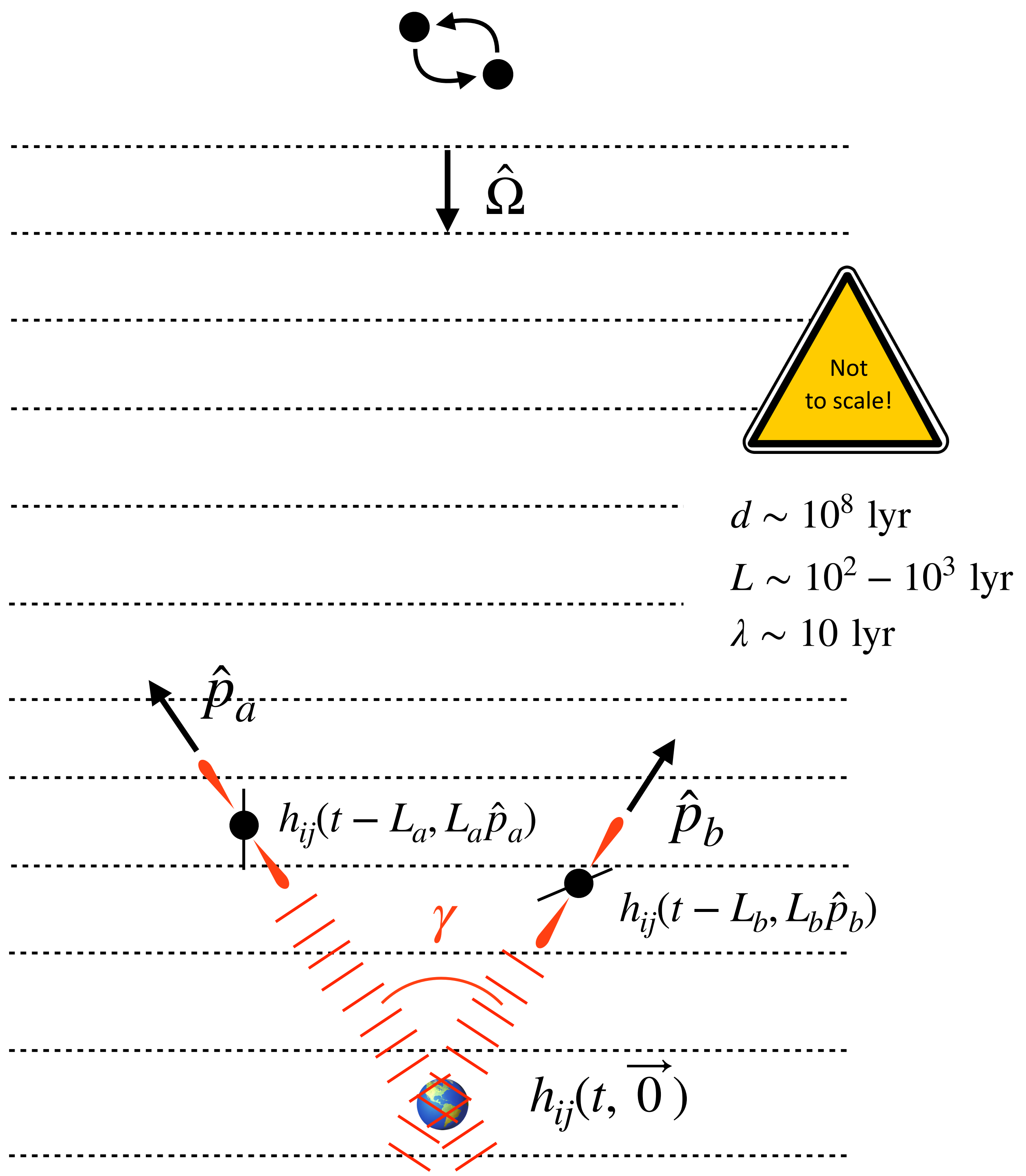


$$\Gamma^{\rm HD}_{ab} = \frac{1}{3} - \frac{y_{ab}}{6} + y_{ab} \ln y_{ab}$$

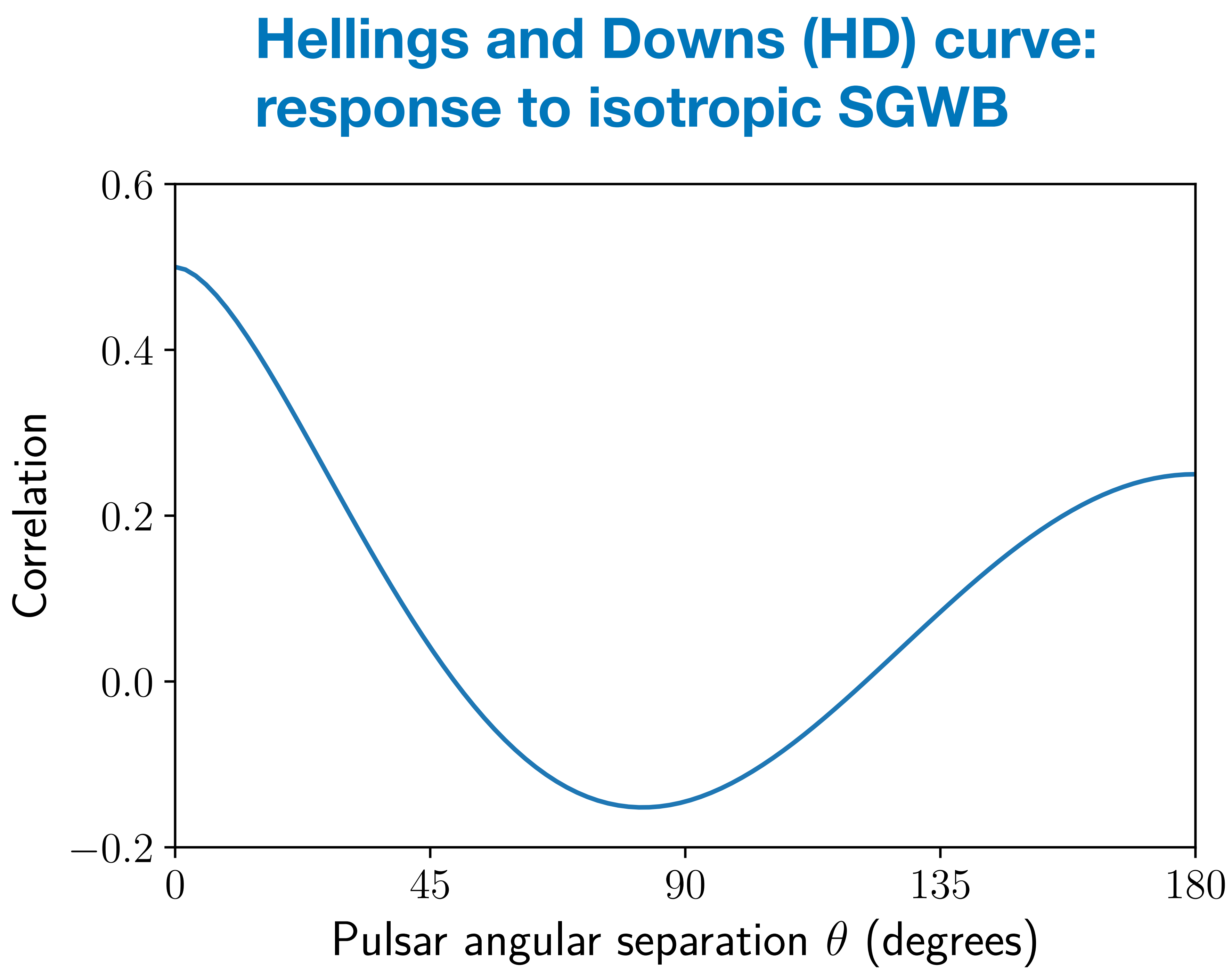
$$y_{ab} = \frac{1-\hat{x}_a \hat{x}_b}{2} = \frac{1-\cos\zeta}{2}$$

The Hellings-Downs curve

- Why isn't the HD angular correlation symmetric wrt $\theta = 90°$?



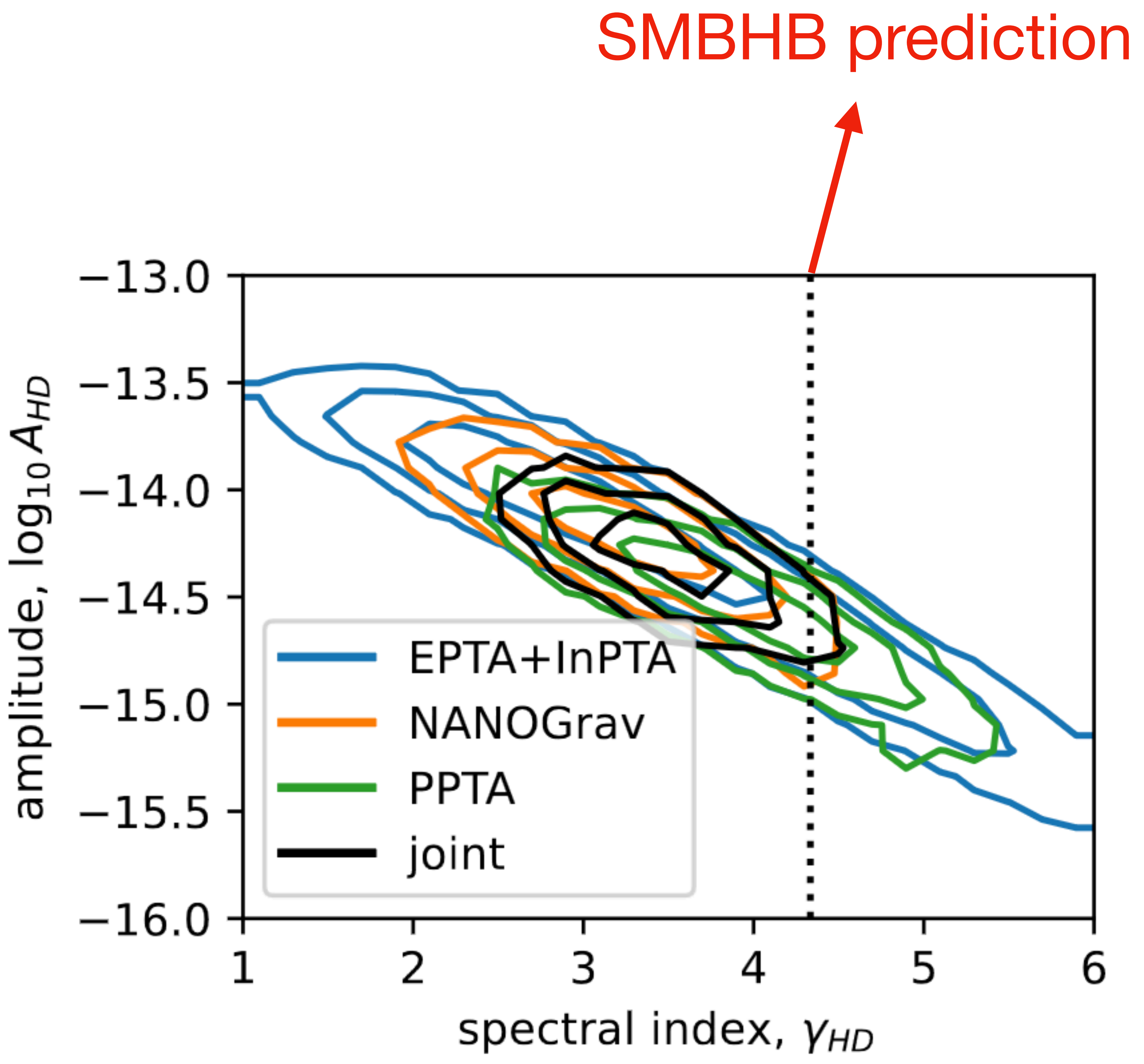
[Romano-Allen]



Because for some small angles light pulses partially swim with the current of the GW, for some larger angles they swim against the current

Stochastic gravitational wave background

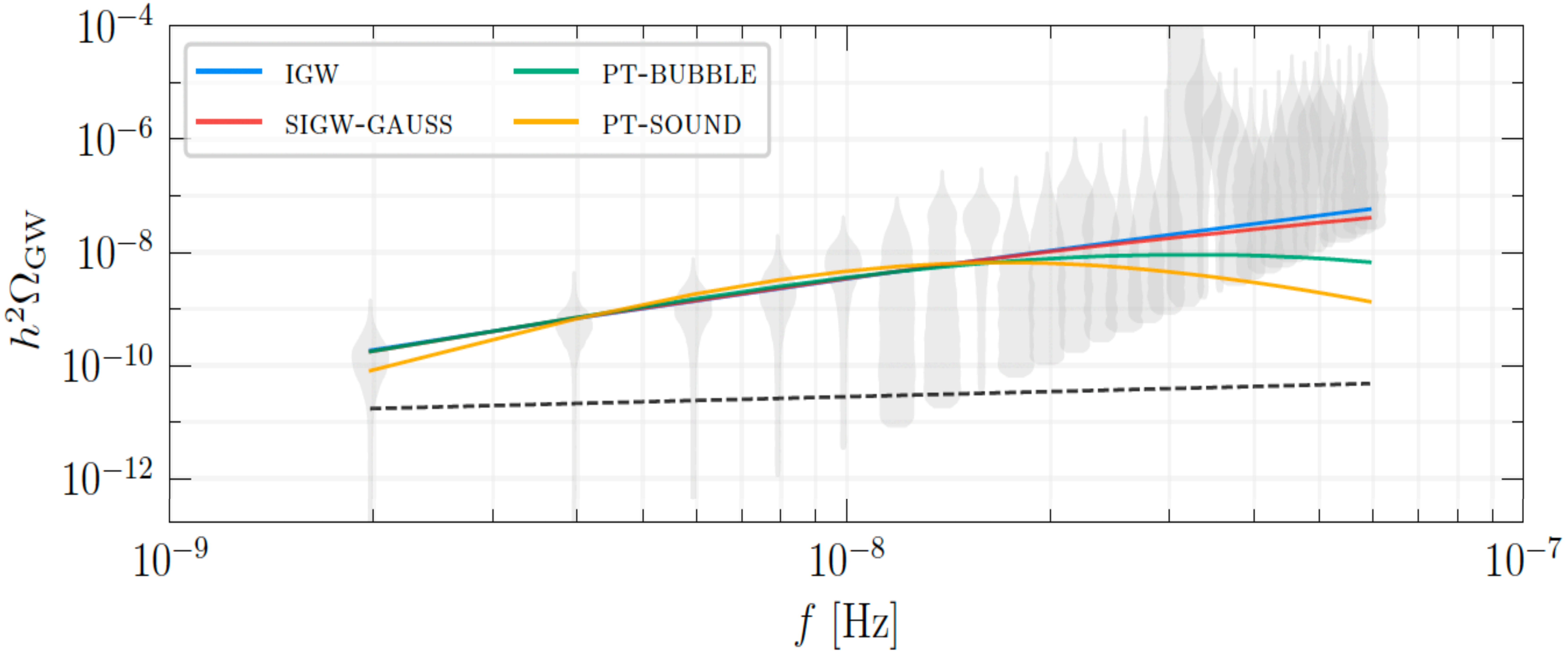
Supermassive BH mergers predicted to produce amplitude $\sim 10^{-15}$ and spectral index $\gamma = 13/3$ [Phinney (2001), Sesana et al. (2008)+]



Strong contender!

Stochastic gravitational wave background

Or is it from the early universe?



arXiv: 2306.16219

Inflation

Large density perturbations <—> PBH

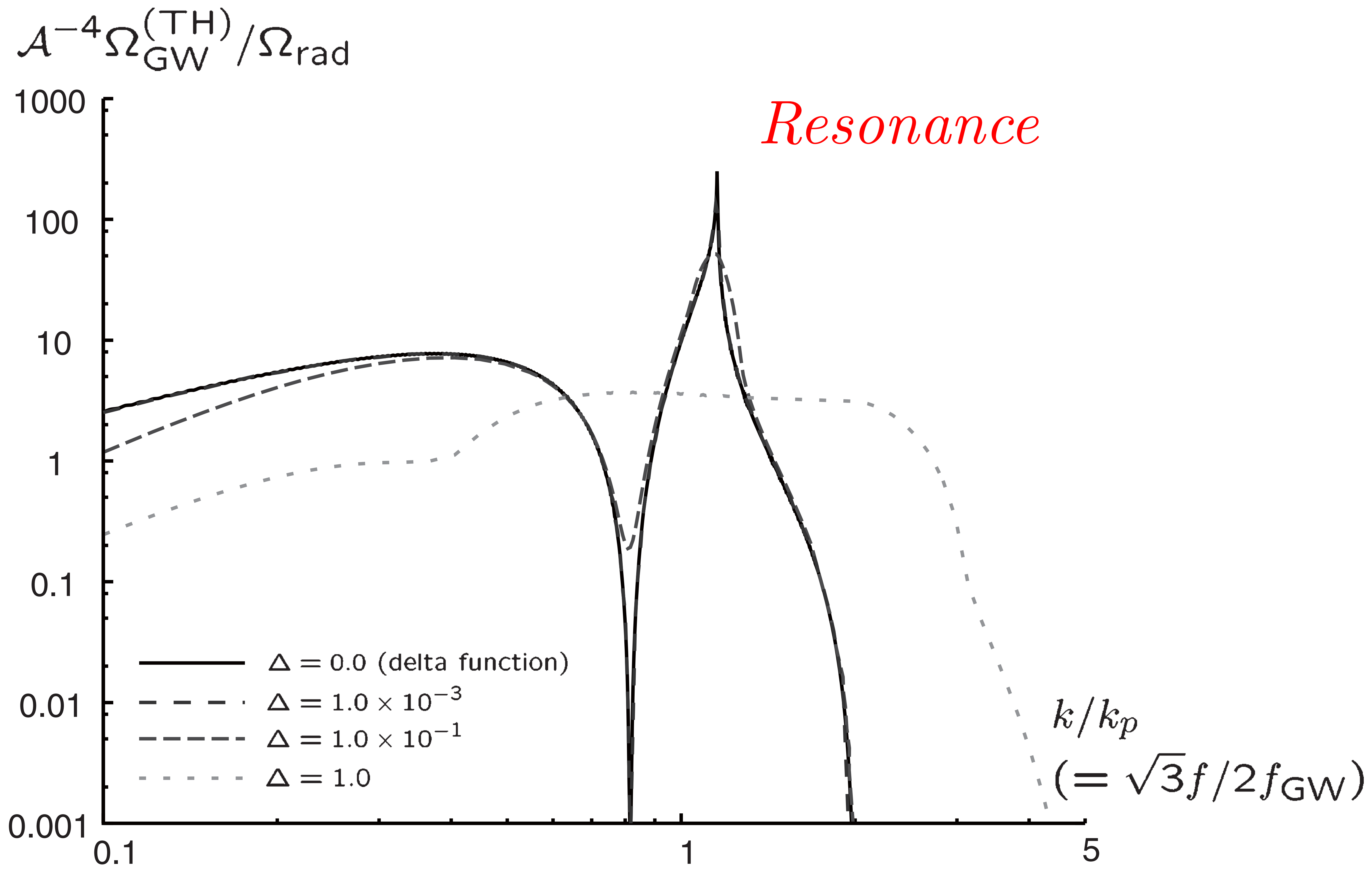
Phase Transitions

Additional possibilities studied in
[arXiv: 2306.16219, 2306.16227 + more]

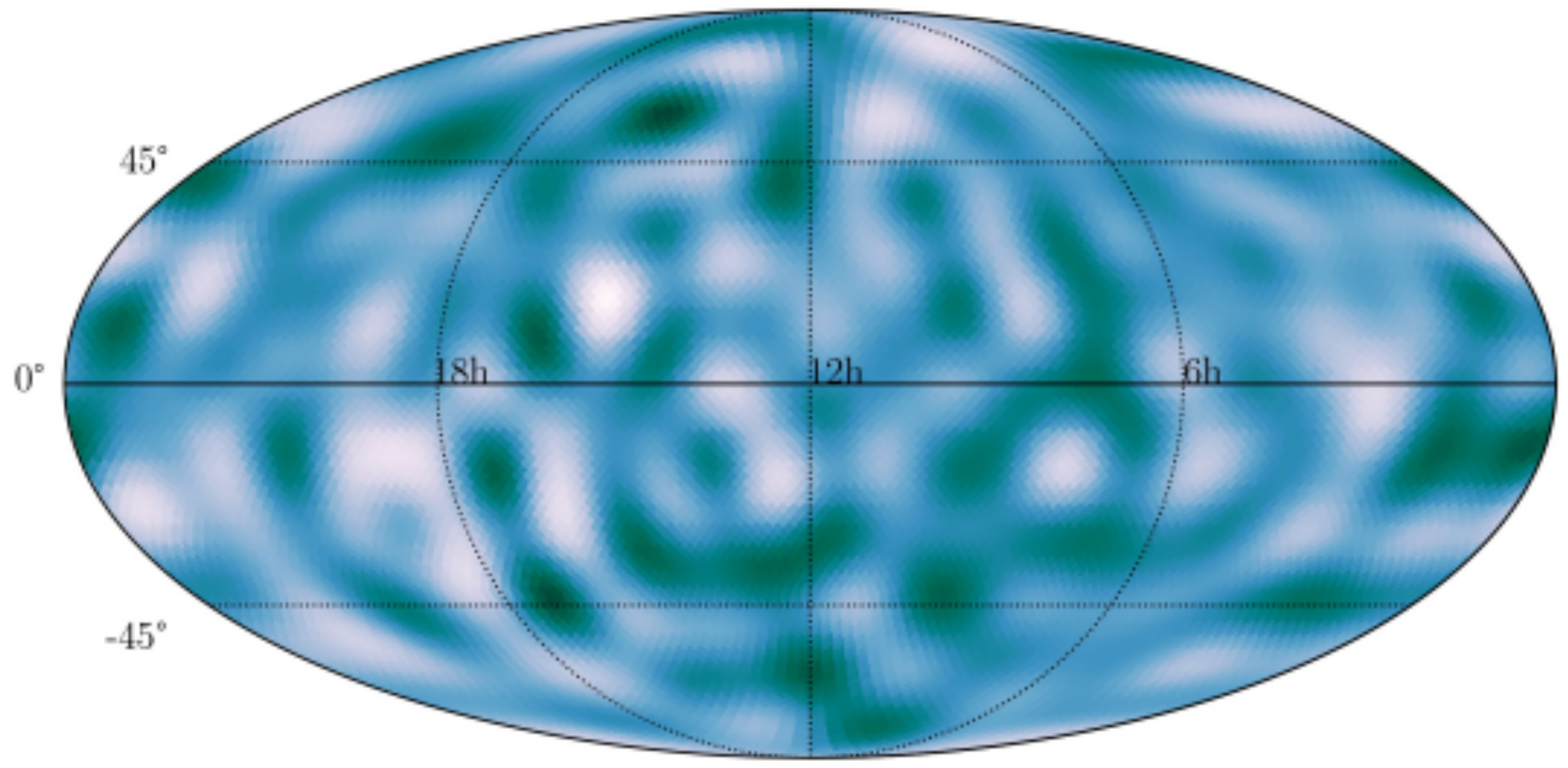
Example: scalar induced GW

Amplification of primordial GW induced by a peak in the curvature perturbation spectrum.

[Ananda et al, Baumann et al, Saito-Yokoyama,…]



What next? Detect the anisotropies of the SGWB



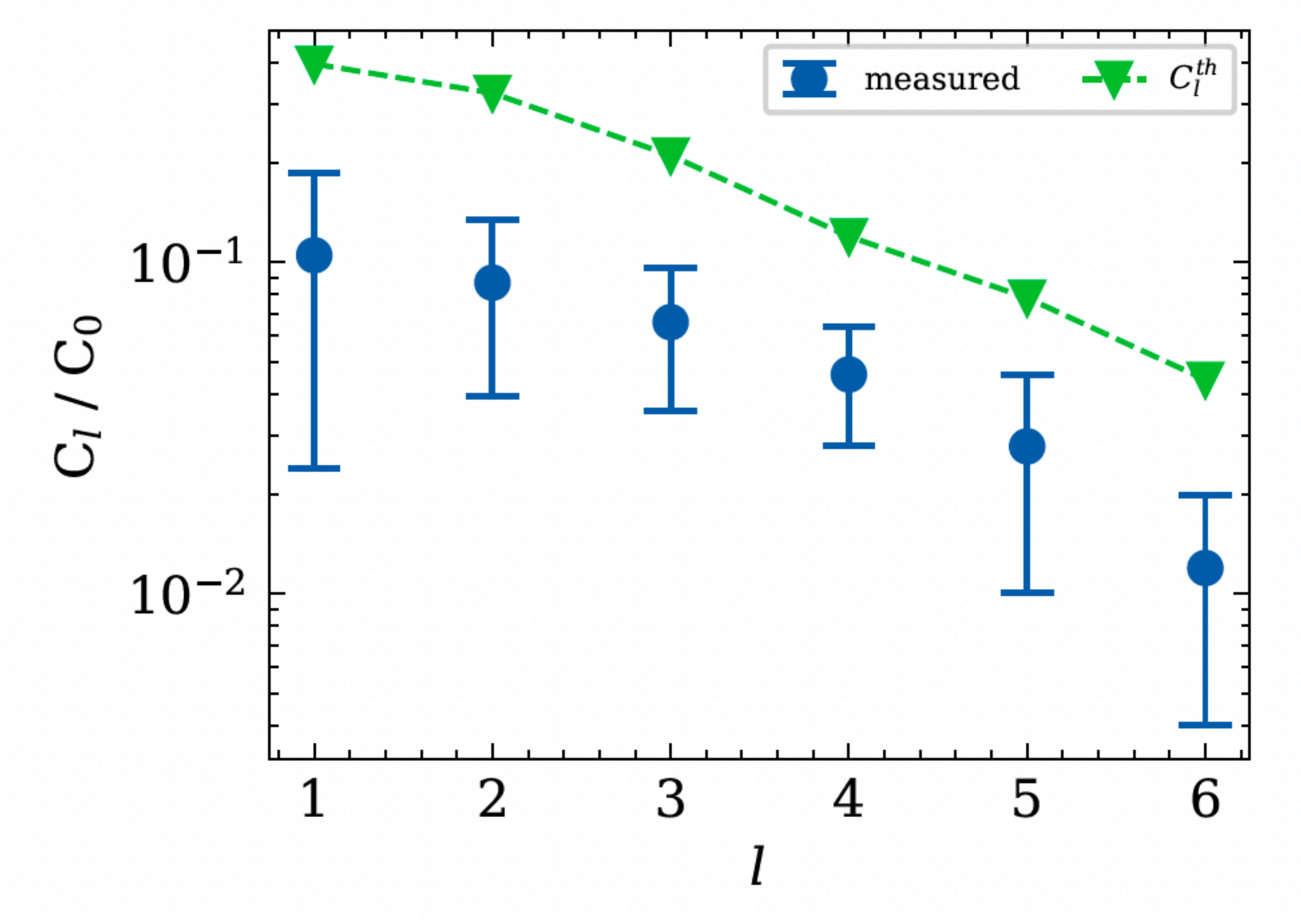
- Both astro and cosmo SGWB, as detectable by PTA, are expected to have intrinsic anisotropies, which depend on their sources.

  For astro, they might be as large as $\frac{\Delta\Omega_{\rm GW}}{\bar{\Omega}_{\rm GW}} \simeq \mathcal{O}(10^{-2})$. For cosmo, $\frac{\Delta\Omega_{\rm GW}}{\bar{\Omega}_{\rm GW}} \leq \mathcal{O}(10^{-5})$.
  [Sato Polito-Kamionkowski], [Alba-Maldacena, Contaldi, Bartolo…GT]

  No detection so far but with extra data and more time of observation a detection might be forthcoming, in case of astro SGWB.

SGWB Anisotropies

Currently PTA data is consistent with isotropy

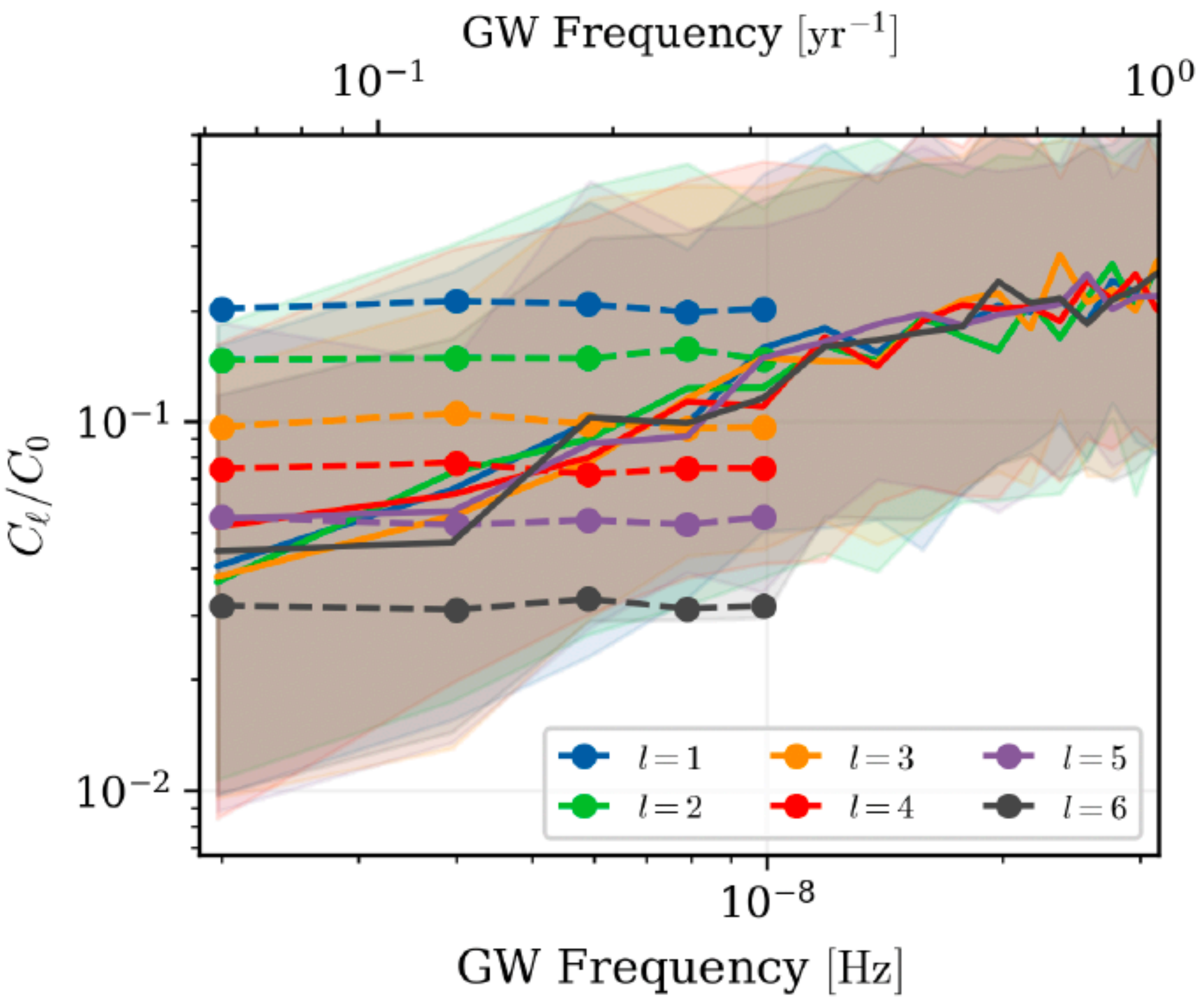


NG15: Search for Anisotropy in the Gravitational Wave Background

SMBHB Anisotropies

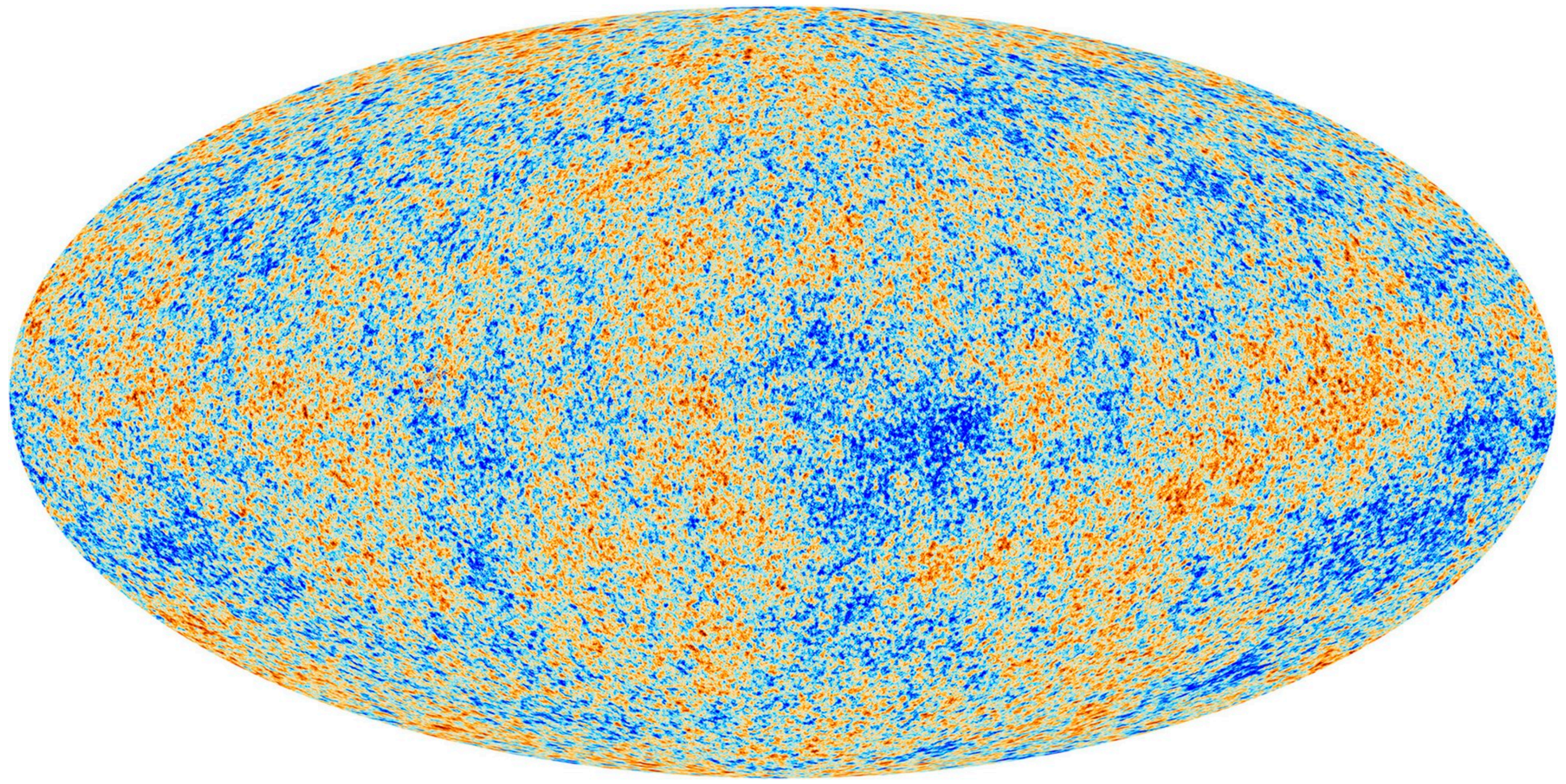
Estimates vary, but **SMBHB** anisotropies are expected to be large

[Mingarelli et al. 2013; Taylor & Gair 2013; Mingarelli et al. 2017), Sato-Polito & Kamionkowski (2023) + more]

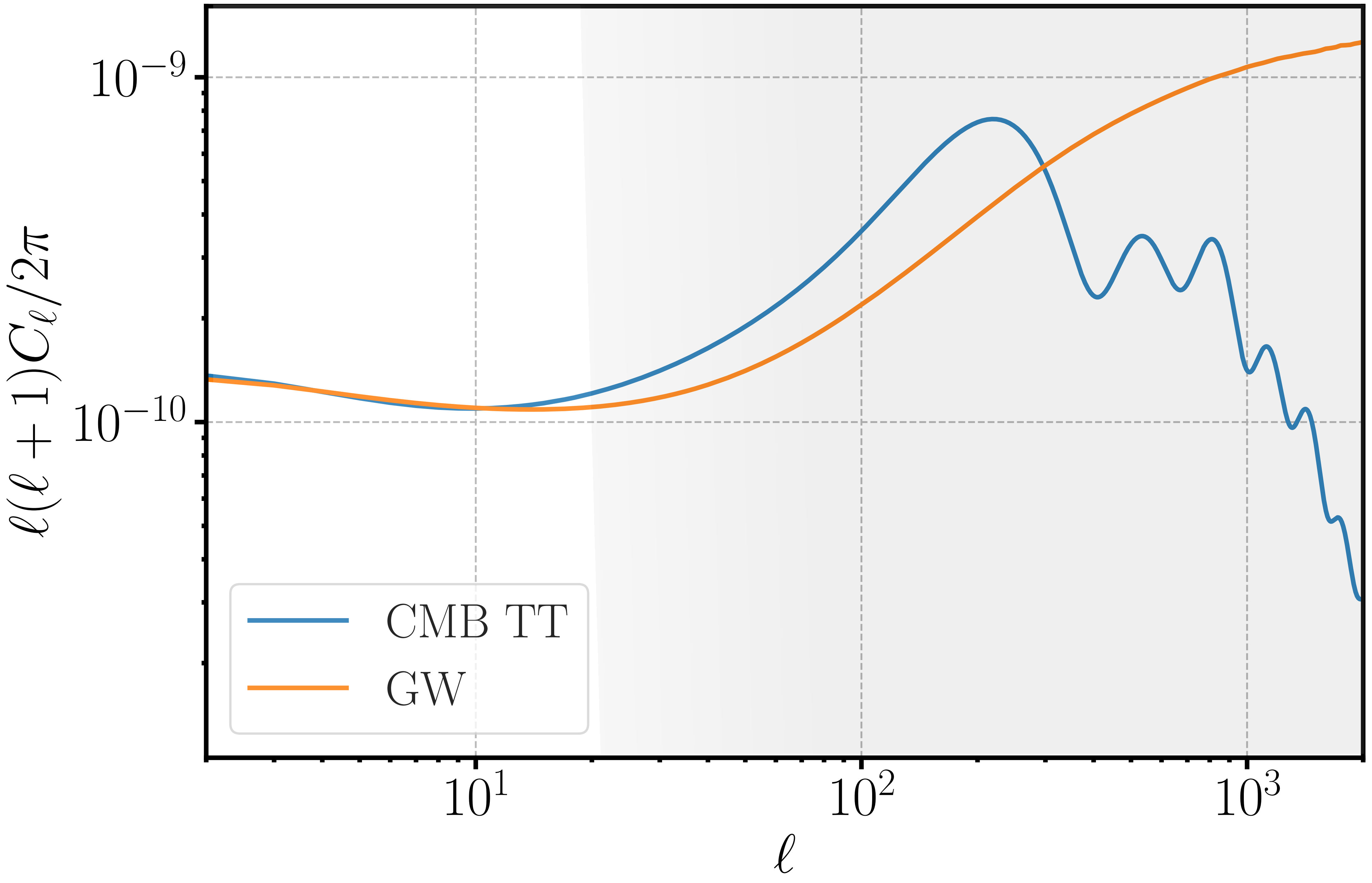


NANOGrav 15-year Anisotropic Gravitational-Wave Background

Cosmological SGWB anisotropies



CMB observations indicate large scale inhomogeneity at the 10^{-5} level



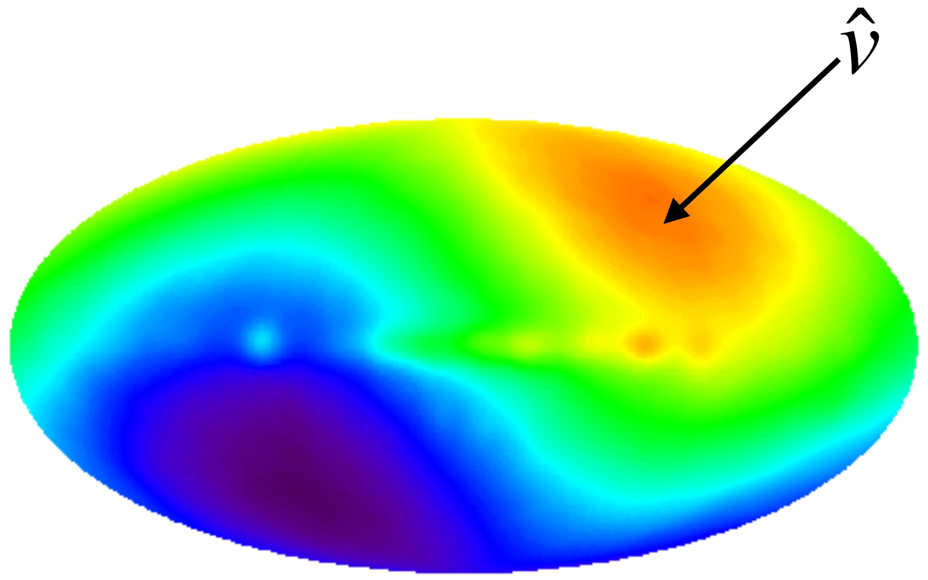
In general, cosmological SGWB anisotropies are expected to be small

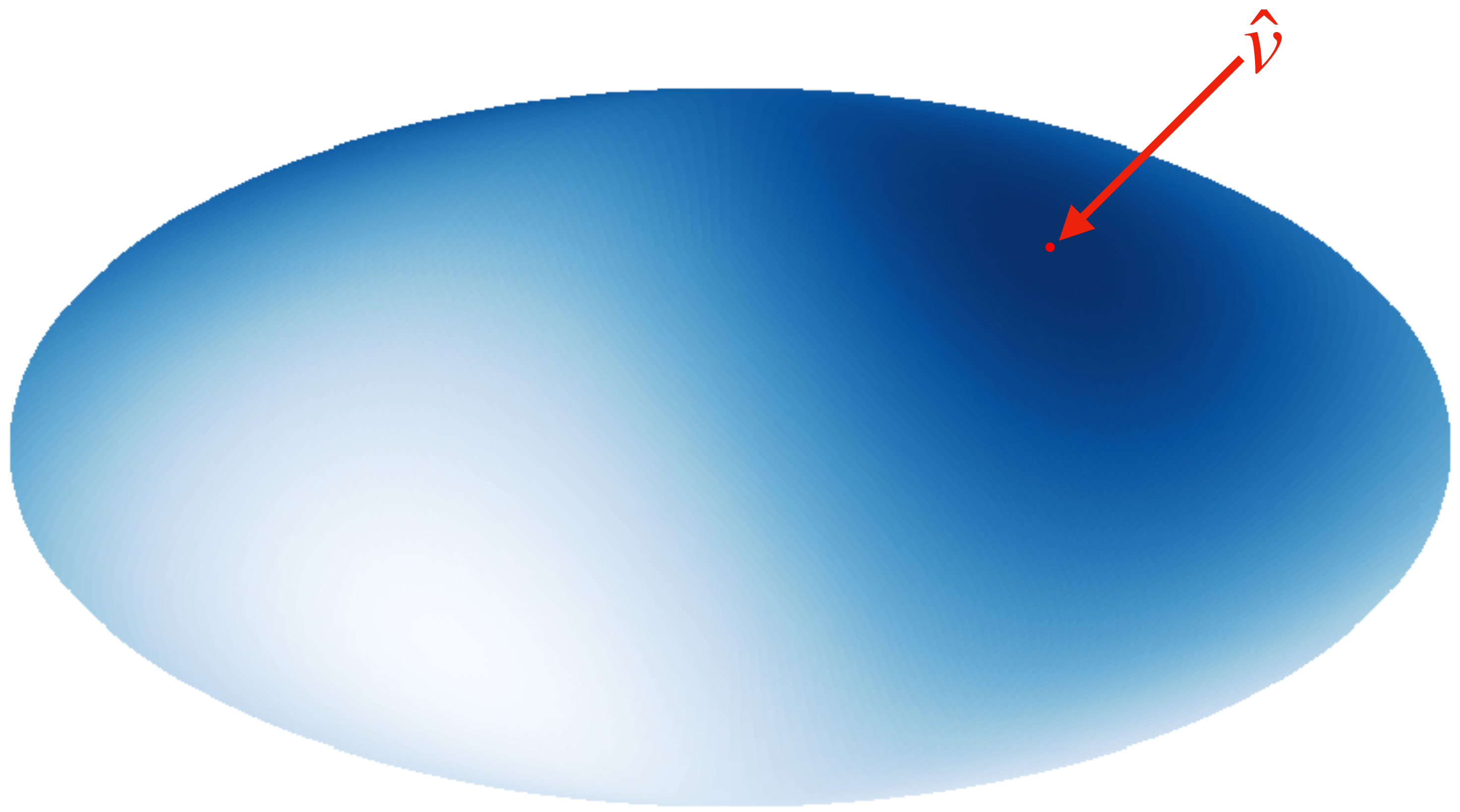
See [review by LISA CosWG (2022)](#)

Kinematic anisotropies of the SGWB with PTA

- For cosmo SGWB, we do expect a large **Doppler anisotropy** due to our relative motion wrt SGWB source rest frame

very similar to CMB!





- Motion towards $(l, b) = (264°, 48°)$ with velocity $\beta = v/c = 1.23 \times 10^{-3}$ (galactic co-ordinates)

- Recent $\sim 3 - 4\sigma$ tension between magnitude of CMB and LSS dipole, directions roughly consistent

Kinematic anisotropies of the SGWB with PTA

- For cosmo SGWB, we do expect a large **Doppler anisotropy** due to our relative motion wrt SGWB source rest frame

$$\left.\frac{\Delta\Omega_{\rm GW}}{\bar{\Omega}_{\rm GW}}\right|_{\rm Doppler} \simeq \mathcal{O}(10^{-3})$$

- Especially interesting as **independent probe** of intensity and direction of our speed wrt SGWB rest frame

- PTA measurements of Doppler effects are also sensitive to modified gravity (circular polarization, extra scalar dofs) hence they provide additional tests of gravity

Kinematic anisotropies of the SGWB with PTA

- Convenient to express in terms of GW intensity

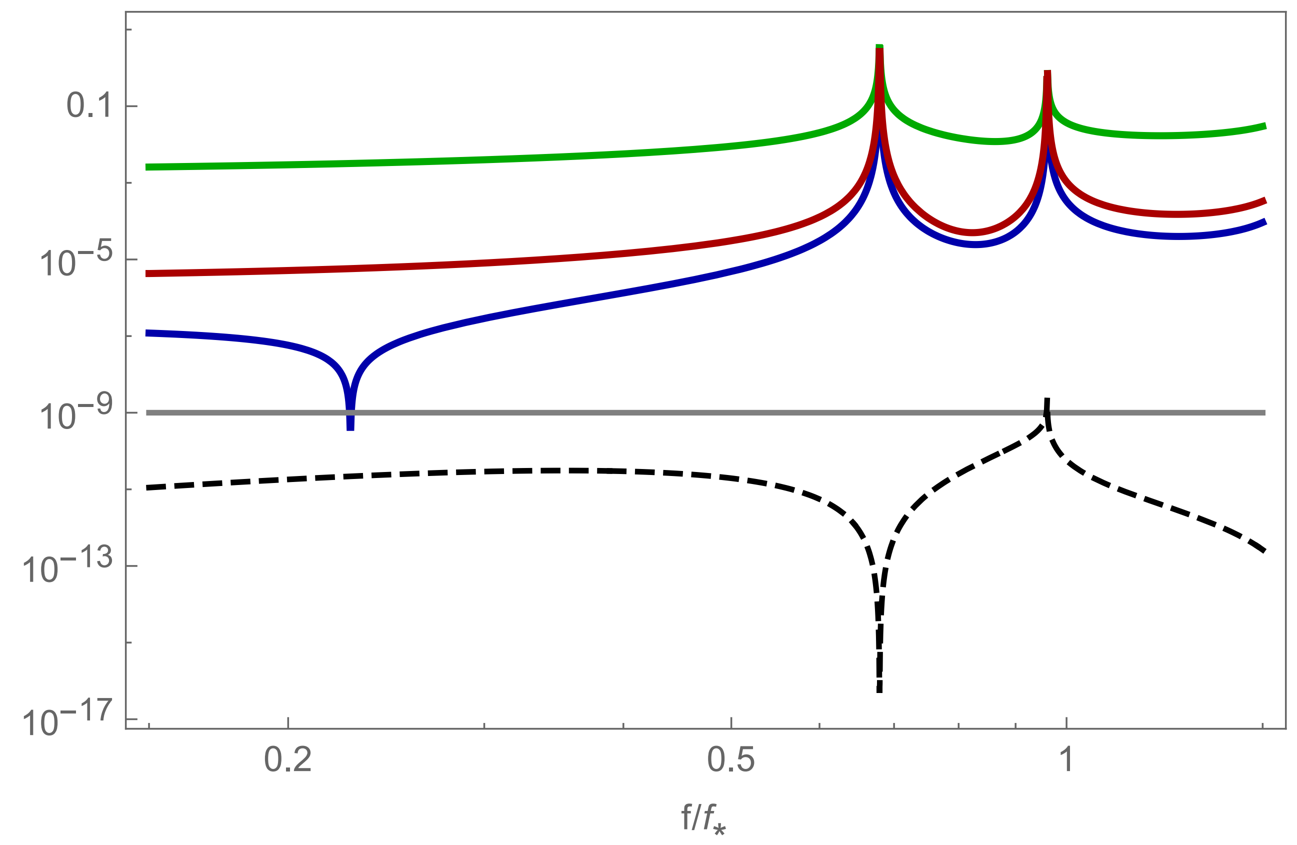
$$\Omega_{\rm GW} = \frac{4\pi\, f^3}{3H_0^2}\, I$$

- Using conservation of graviton number in geometrical optics approx, one gets, from an initially isotropic intensity: [Cusin, GT]

$$I(f,\hat{n}) = \mathcal{D}\, \bar{I}(\mathcal{D}^{-1}\, f)$$

with

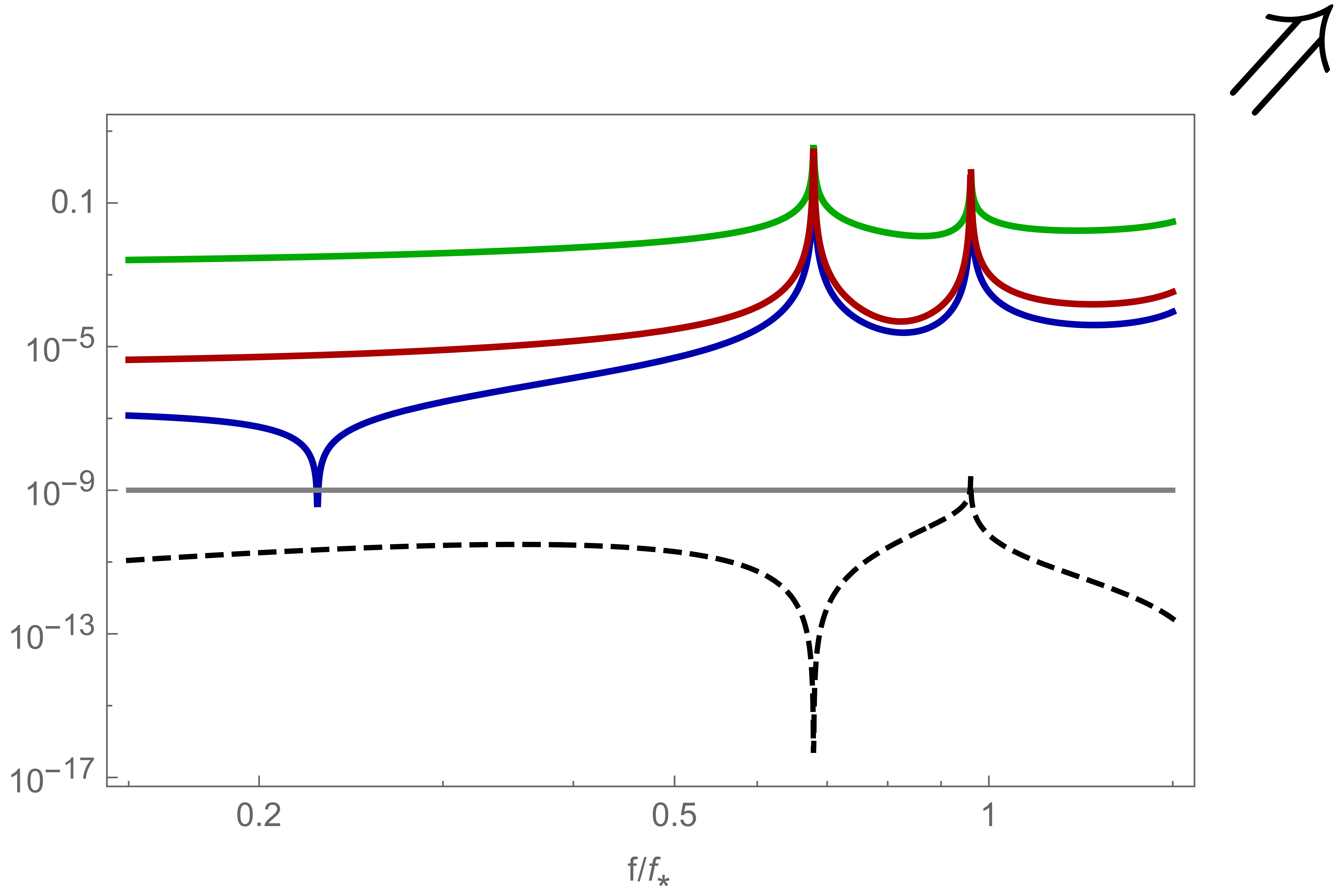
$$\mathcal{D} = \frac{\sqrt{1-\beta^2}}{1-\beta\,\hat{n}\cdot\hat{v}}$$



- One can also get similar expressions for circular polarization, or intensity in extra dofs, and make forecasts for detection with PTA

Kinematic anisotropies of the SGWB with PTA

The size of kinematic anisotropies
depend on the scale



Kinematic anisotropies of the SGWB with PTA

First task: develop theory

Kinematic anisotropies of the SGWB with PTA

- First task: Develop the theory. Derive the PTA response functions to kinematic anisotropies.
  - Modification of HD correlations, due to extra effects of our motion wrt SGWB.
  - More complicated angular integrals to perform, but with some tricks can be done analytically.
  - Correlations now depend also on **relative position of pulsars wrt** $\hat{v}$, not only on angle between pulsars. They also depend on the (possible) presence of extra GW polarizations.

    [Anholm et al, Mingarelli et al, GT]

Kinematic anisotropies of the SGWB with PTA

- First task: Develop the theory. Derive the PTA response functions to kinematic anisotropies.

$$\Gamma_{ab}(f) = \left[1 - \frac{\beta^2}{6}\left(1 - n_I^2 - \alpha_I\right)\right]\Gamma_{ab}^{(0)} + \beta\,(n_I - 1)\,\Gamma_{ab}^{(1)} + \frac{\beta^2}{2}\left(2 - 3n_I + n_I^2 + \alpha_I\right)\Gamma_{ab}^{(2)}\,,$$

$$\Gamma_{ab}^{(0)} = \frac{1}{3} - \frac{y_{ab}}{6} + y_{ab}\ln y_{ab}$$

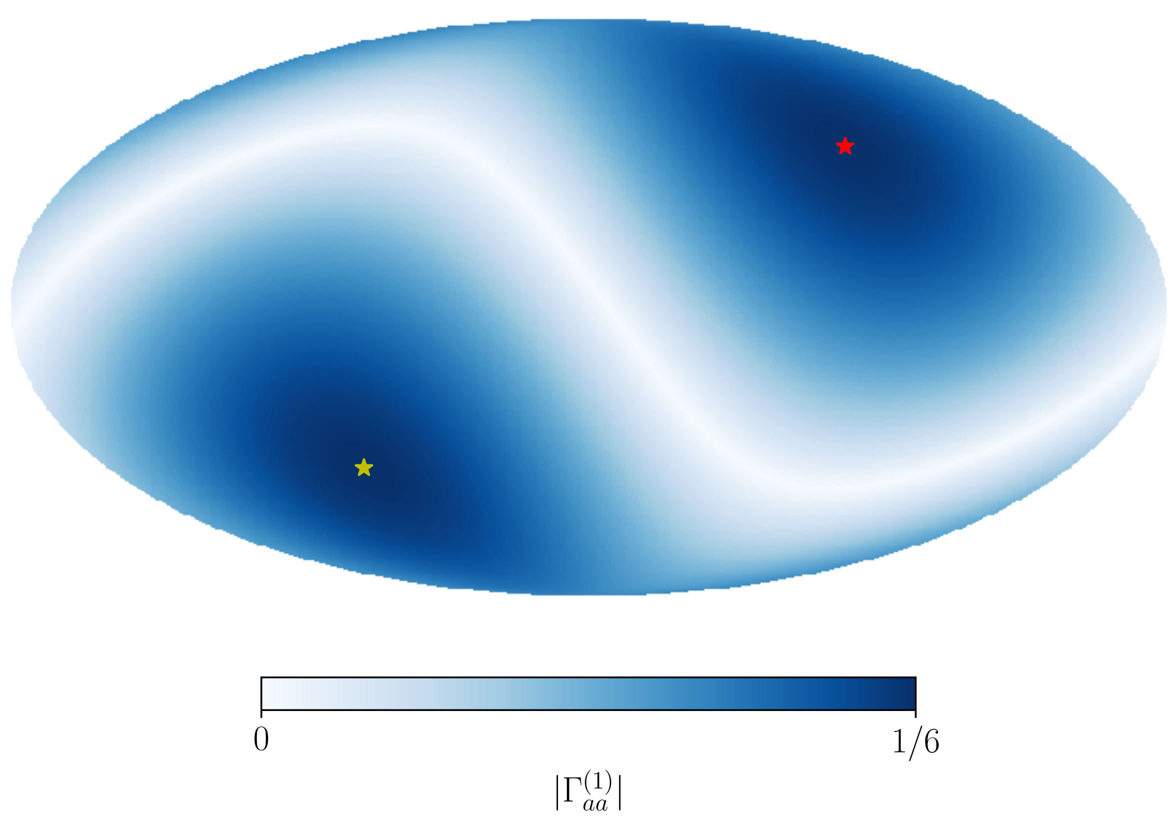
$$\Gamma_{ab}^{(1)} = \left(\frac{1}{12} + \frac{y_{ab}}{2} + \frac{y_{ab}\ln y_{ab}}{2(1-y_{ab})}\right)\left[\hat{v}\cdot\hat{x}_a + \hat{v}\cdot\hat{x}_b\right]\,,$$

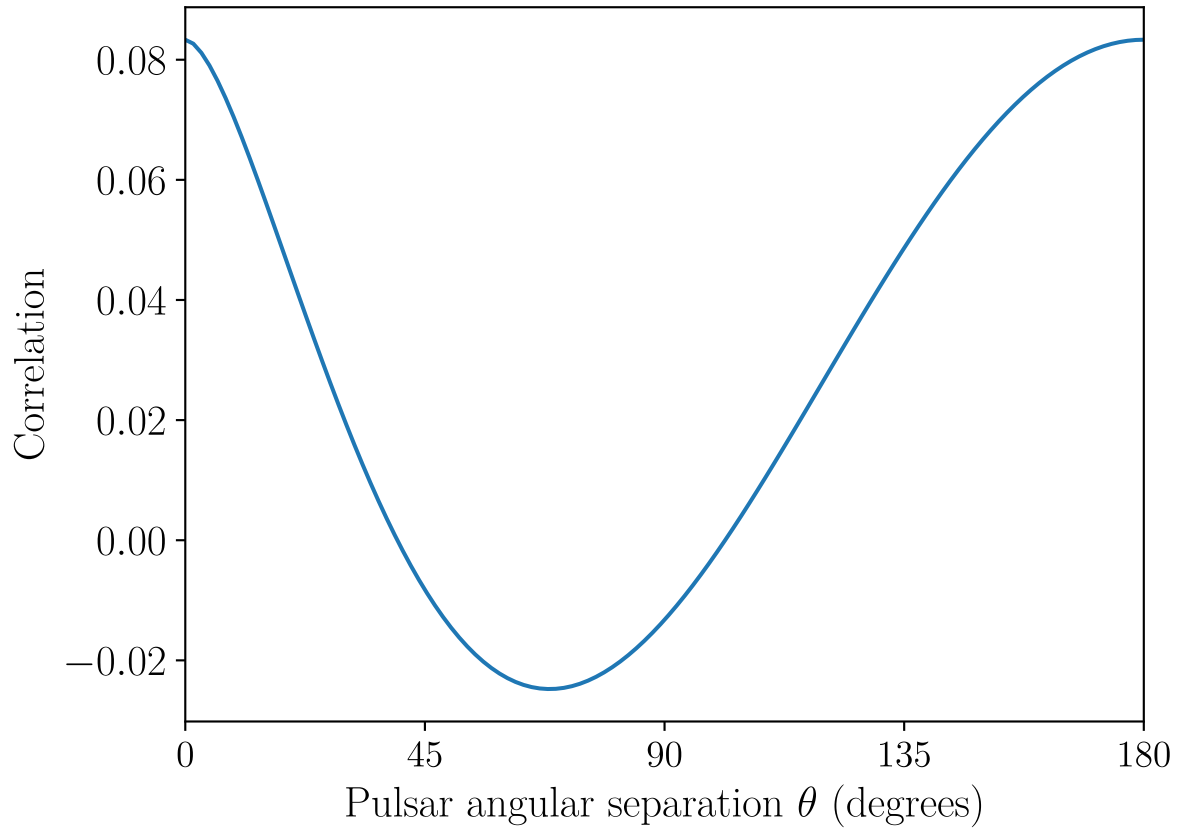
$$n_I = \frac{d\,\ln I}{d\,\ln f} \quad , \quad \alpha_I = \frac{d\,n_I}{d\,\ln f}$$

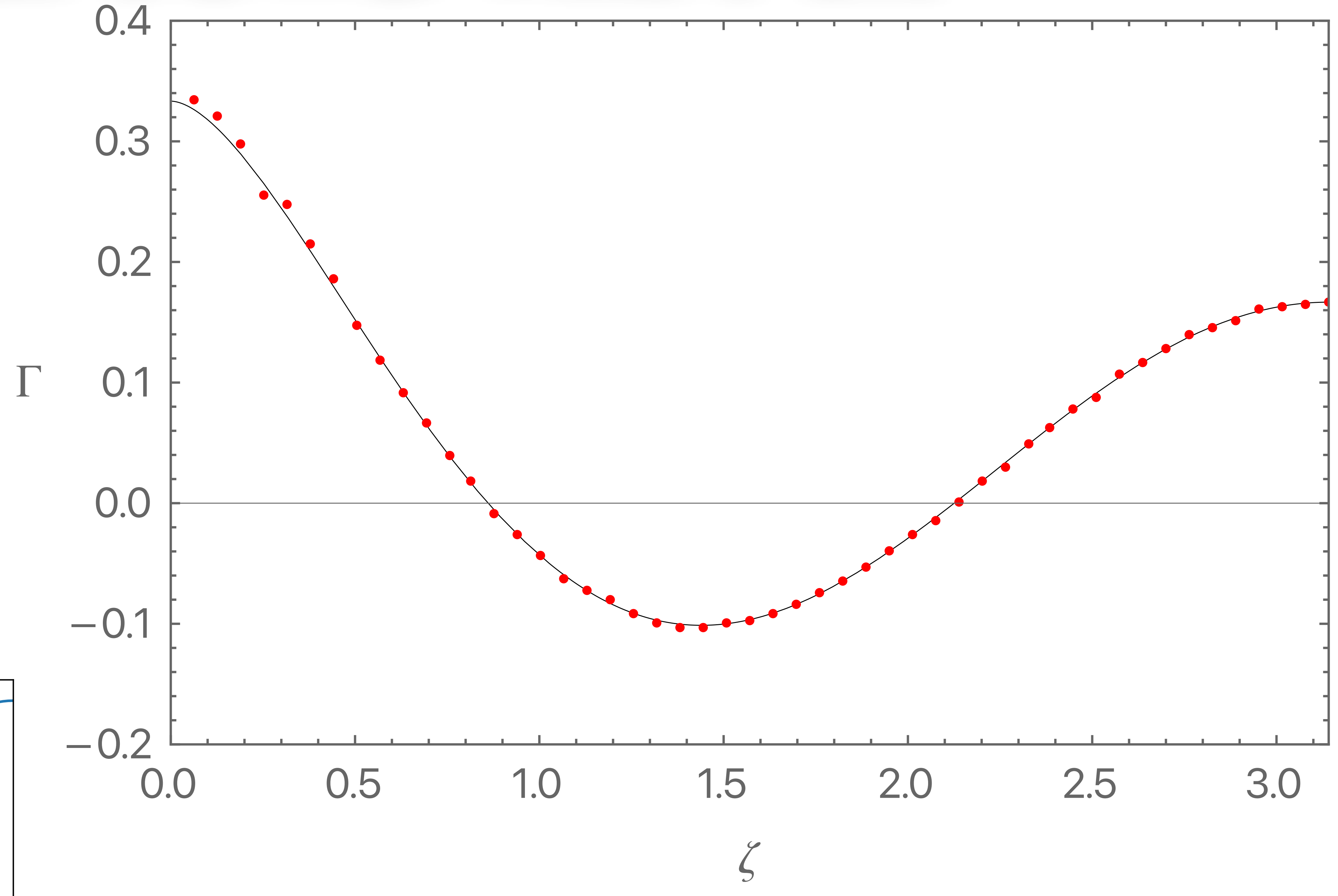
Kinematic anisotropies of the SGWB with PTA

$$\Gamma^{(0)}_{ab} = \frac{1}{3} - \frac{y_{ab}}{6} + y_{ab}\ln y_{ab}$$

$$\Gamma^{(1)}_{ab} = \left(\frac{1}{12} + \frac{y_{ab}}{2} + \frac{y_{ab}\ln y_{ab}}{2(1-y_{ab})}\right)\left[\hat{v}\cdot\hat{x}_a + \hat{v}\cdot\hat{x}_b\right] ,$$







Circular polarisation

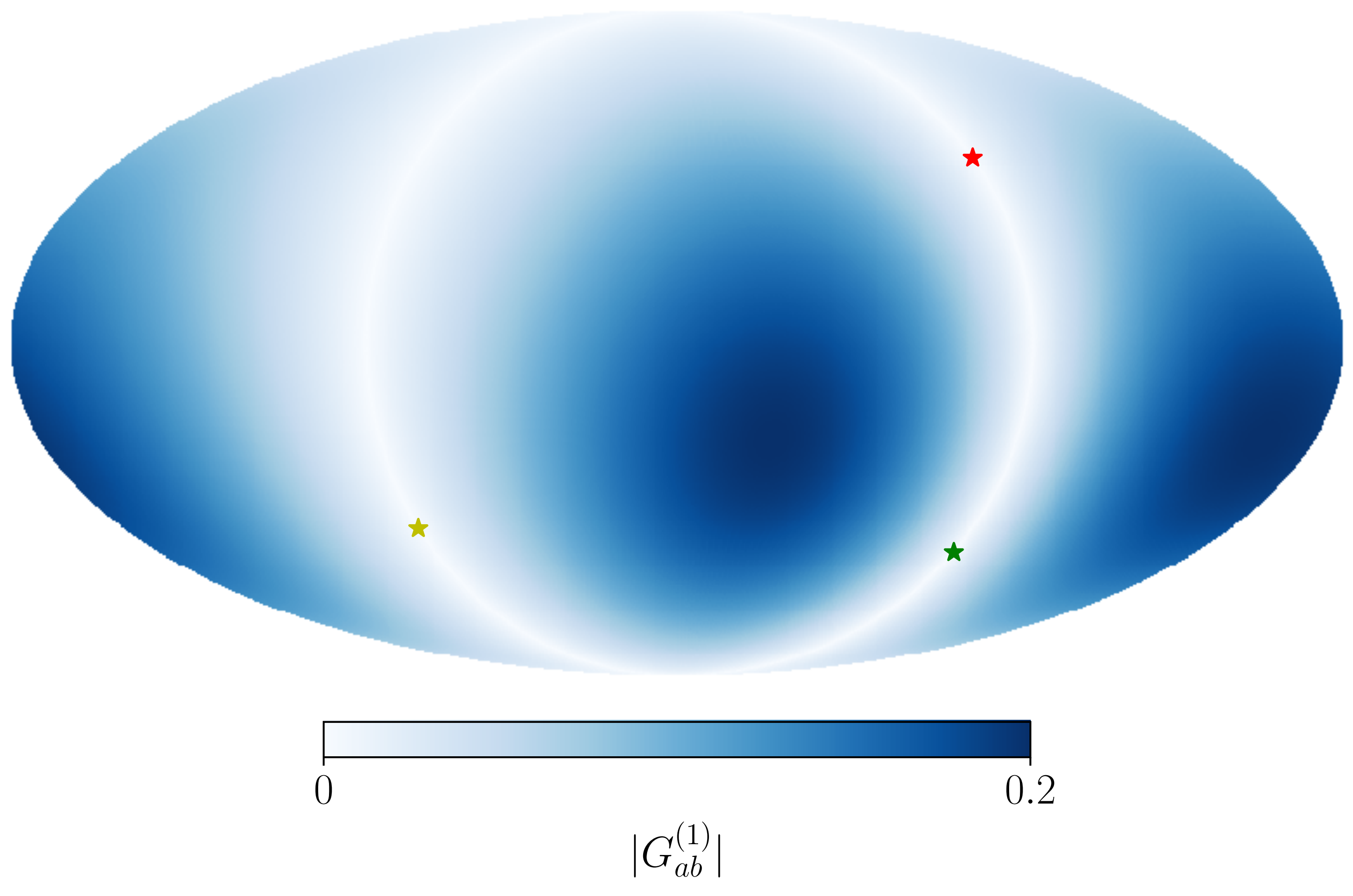
Cosmological sources e.g. GW from axion-gauge fields [Unal et al. 2023 + more]

PTA blind to circular polarisation monopole — planar detector

PTA response begins at dipole

$$\Gamma^{V}_{ab} = \beta\ (n_V - 1)\ G^{(1)}_{ab}\ V$$

$$G^{(1)}_{ab} = -\left(\frac{1}{3} + \frac{y_{ab}\ln y_{ab}}{4(1-y_{ab})}\right)\left[\hat{v}\cdot(\hat{x}_a \times \hat{x}_b)\right]$$



Kinematic anisotropies of the SGWB with PTA

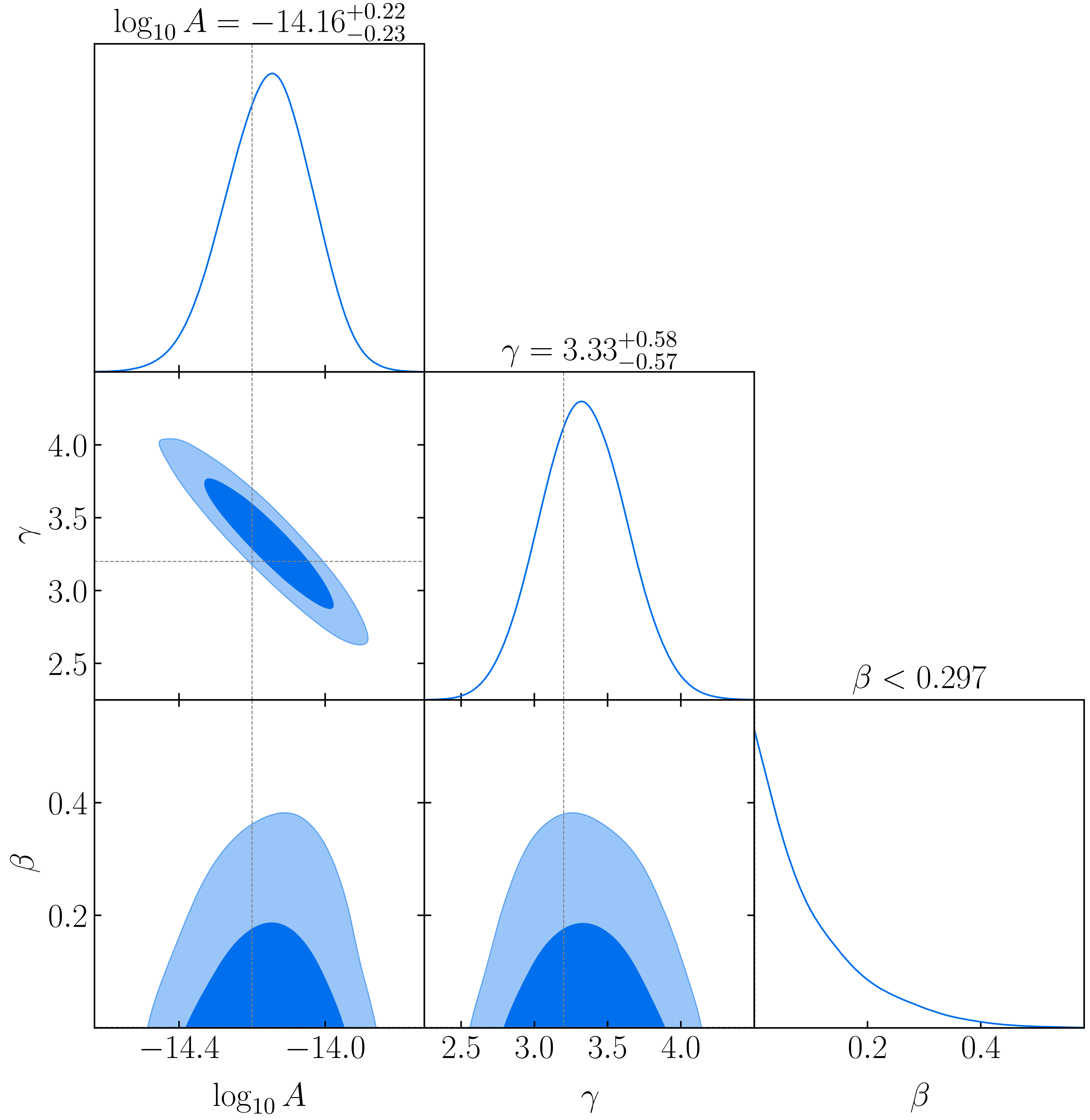
Second task: quantitative forecasts, and design new methods to extract info from data

Perspectives for Detecting Kinematic Anisotropies with PTA

- Take existing NANOGrav data, and model signal as power law

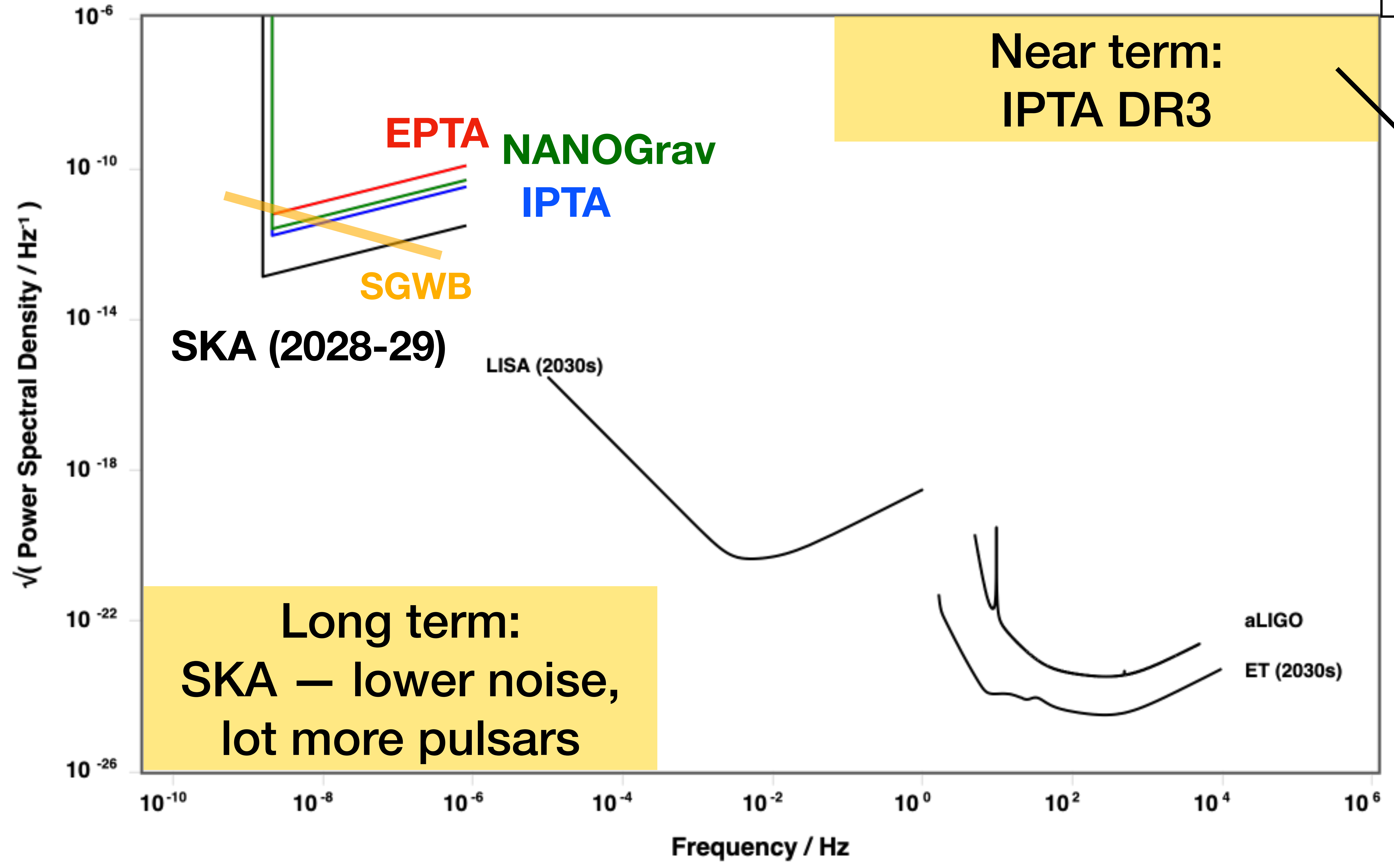
$$I(f) = \frac{A^2}{2f}\left(\frac{f}{f_\star}\right)^{3-\gamma}$$

Use NANOGrav likelihood and methods in ENTERPRISE packages.



Perspectives for Detecting Kinematic Anisotropies with PTA

$$\mathrm{SNR}_{\mathrm{iso}} \propto \sqrt{T N_{\mathrm{pair}}}\,\frac{I}{\sigma^2_{\mathrm{noise}}}$$



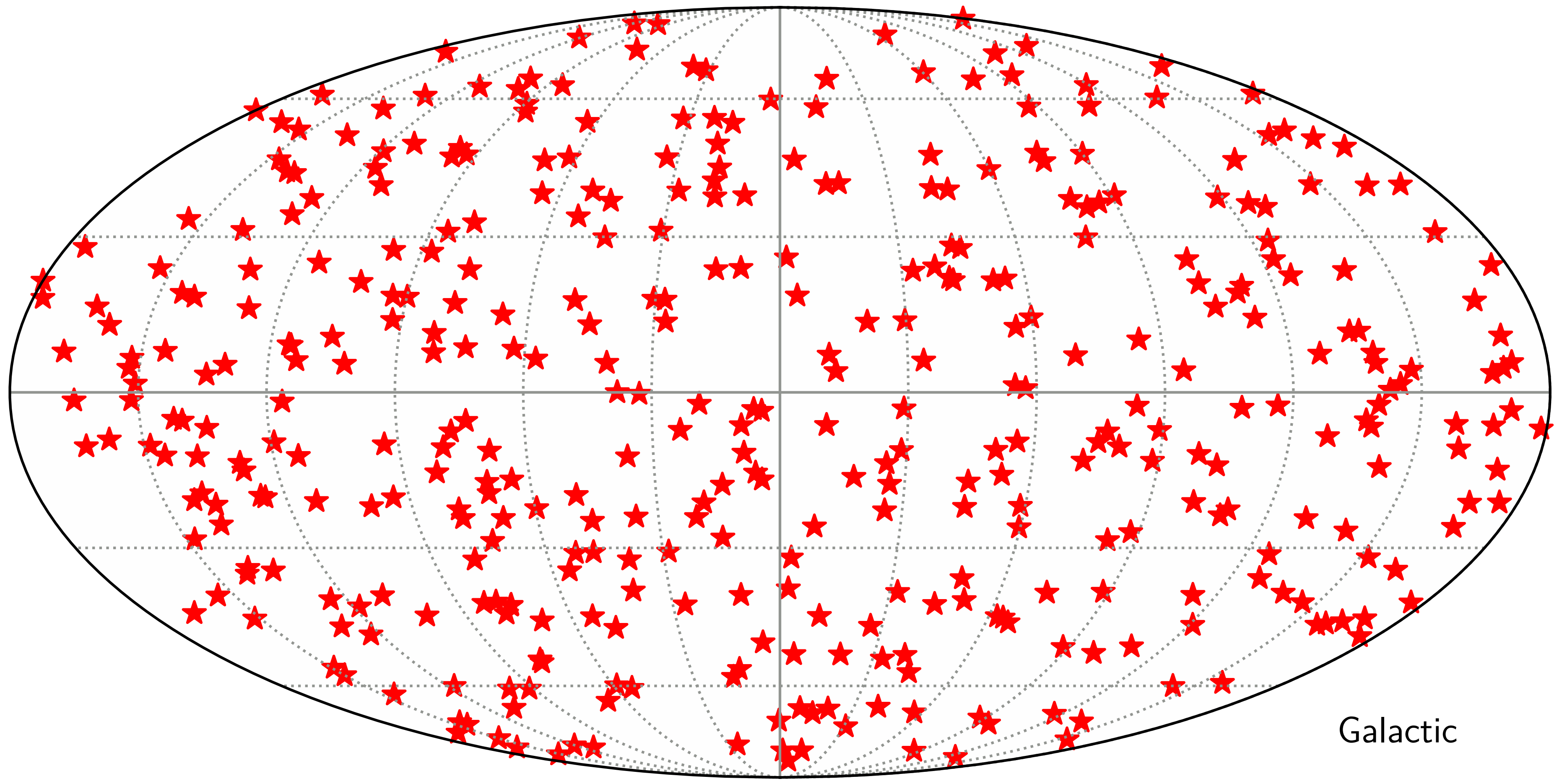
May be enough to detect SMBHB anisotropy

But unlikely to be enough for the kinematic dipole…

Forecasts: SKA era

Idealised scenario with $N \gg 100$ identical pulsars distributed uniformly

We make several simplifying assumptions —> most optimistic estimate



[Keane et al. (2015), Janssen et al. (2015)]

Forecasts: SKA era

Assuming a Gaussian likelihood in the timing residual cross-spectra

A, B = pairs of pulsars

$$-2\ln\mathcal{L} = \sum_f \sum_{AB} \left(\hat{\mathcal{R}}_A - \frac{\Gamma_A \cdot I}{(4\pi f)^2}\right) C_{AB}^{-1} \left(\hat{\mathcal{R}}_B - \frac{\Gamma_B \cdot I}{(4\pi f)^2}\right)$$

$N_{\rm pair} \times N_{\rm pair}$ covariance matrix

Forecasts: SKA era

Weak signal Fisher matrix

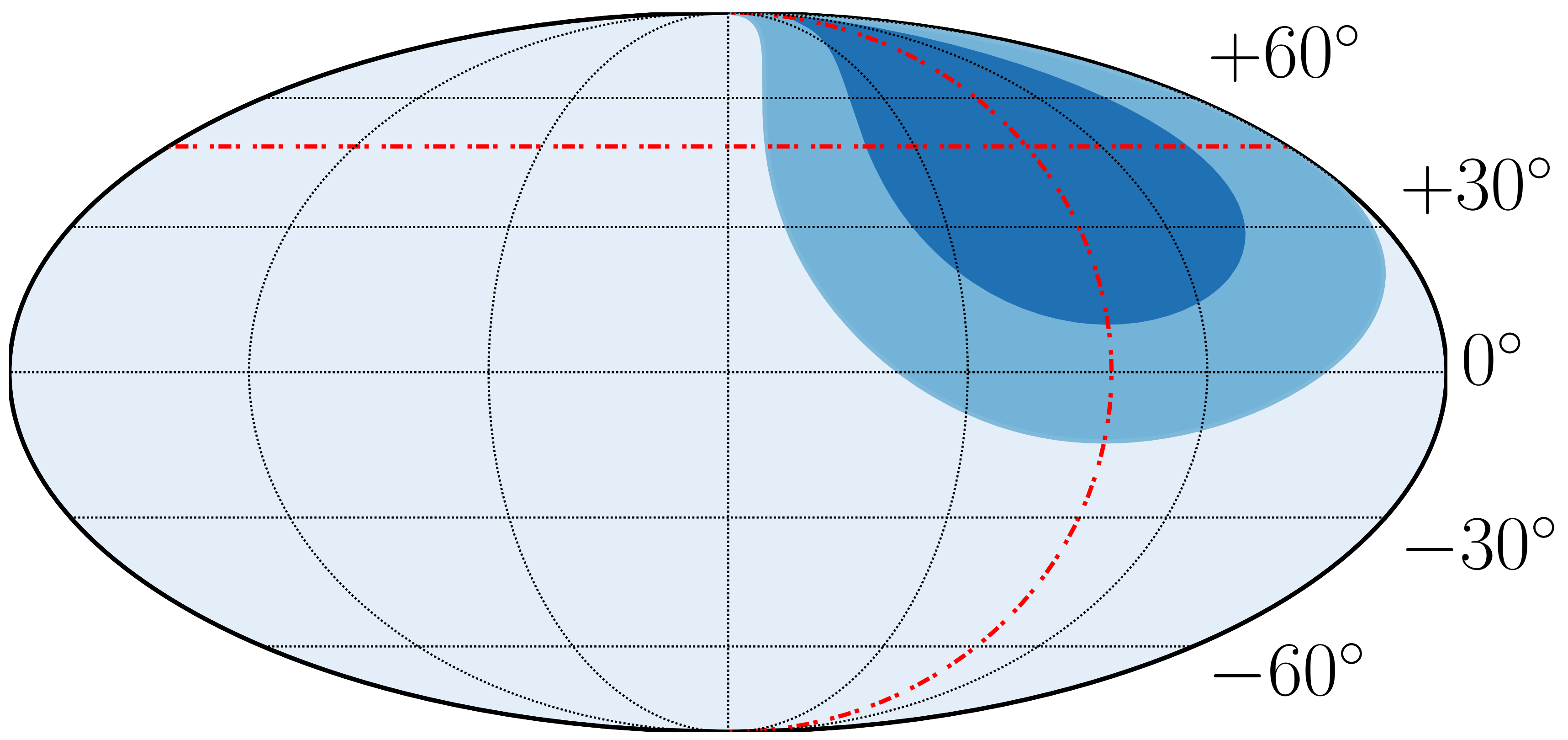
We extend results of Haïmoud, Smith & Mingarelli (2020)

$$\Delta\theta_i = \sqrt{(\mathcal{F}^{-1})_{ii}}, \quad \vec{\theta} = \{\beta, \theta, \phi\} \longrightarrow$$ dipole magnitude and direction

$$\mathcal{F}_{ij} \propto \frac{2T}{S_N^2} N_{\rm pair} \times \begin{bmatrix} \frac{I_0^2(1-n_I)^2 F_1}{3} & 0 & 0 \\ 0 & \frac{F_1 I_0^2(1-n_I)^2\beta^2}{3} & 0 \\ 0 & 0 & \frac{F_1\, I_0^2(1-n_I)^2\beta^2 \sin^2\theta}{3} \end{bmatrix}, \qquad F_1 \approx F_0/7$$

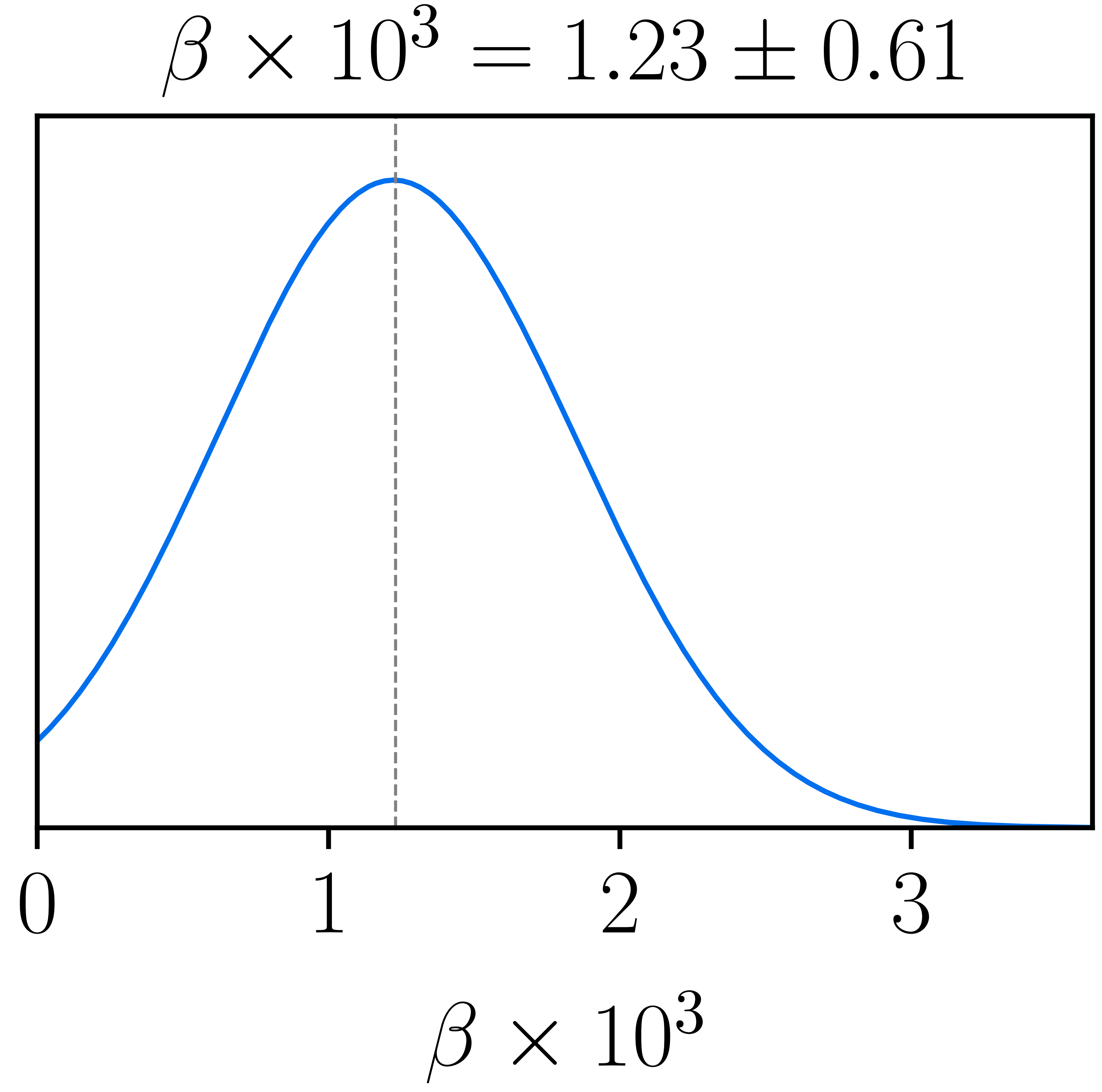
Forecasts: SKA era

Weak signal results



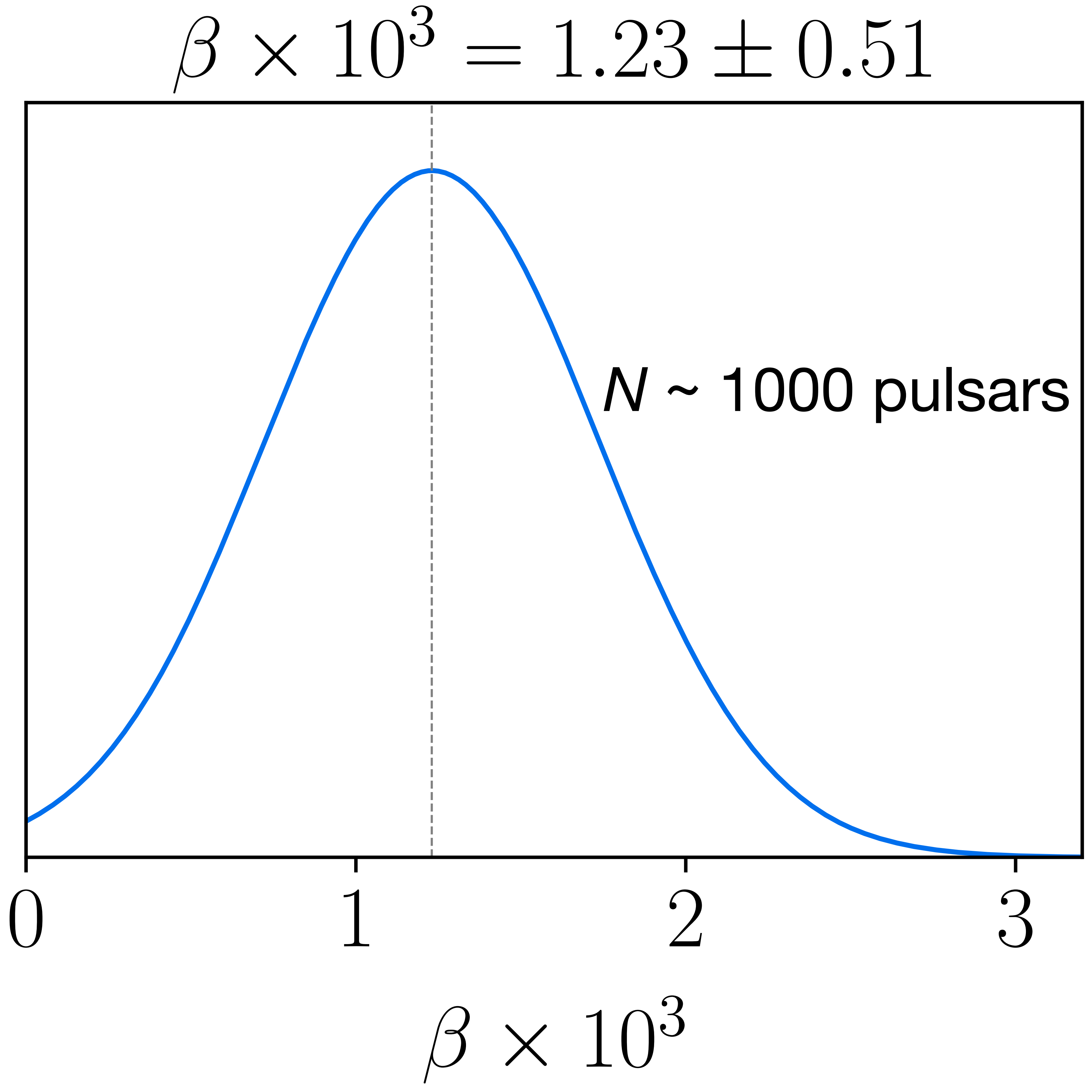
~30° degree localisation of dipole direction

Challenging even with ~4000 pulsars



Forecasts: SKA era

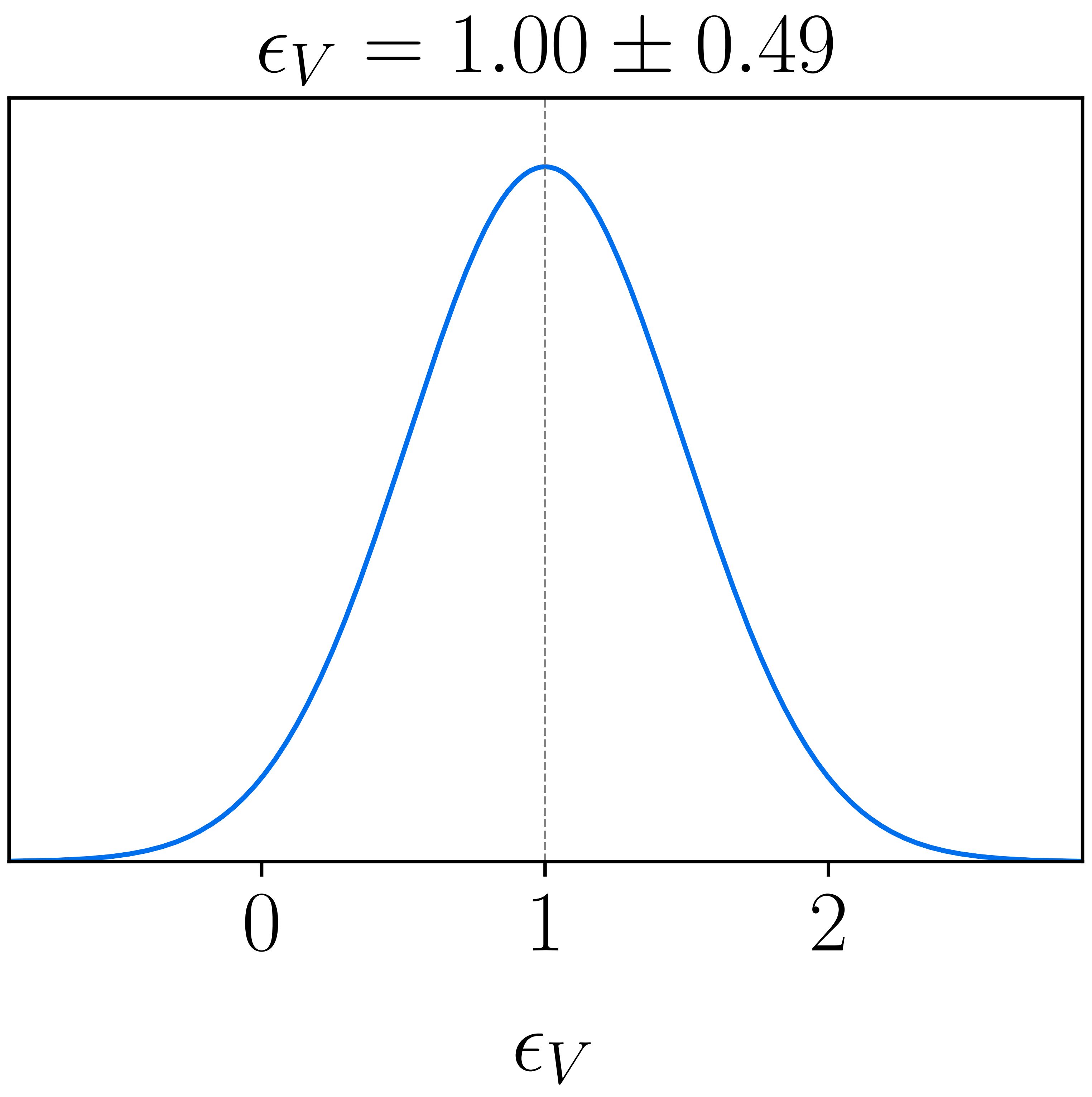
Strong signal regime



Detection will be challenging even for futuristic experiments

See also Depta et al. (2024) for strong signal results

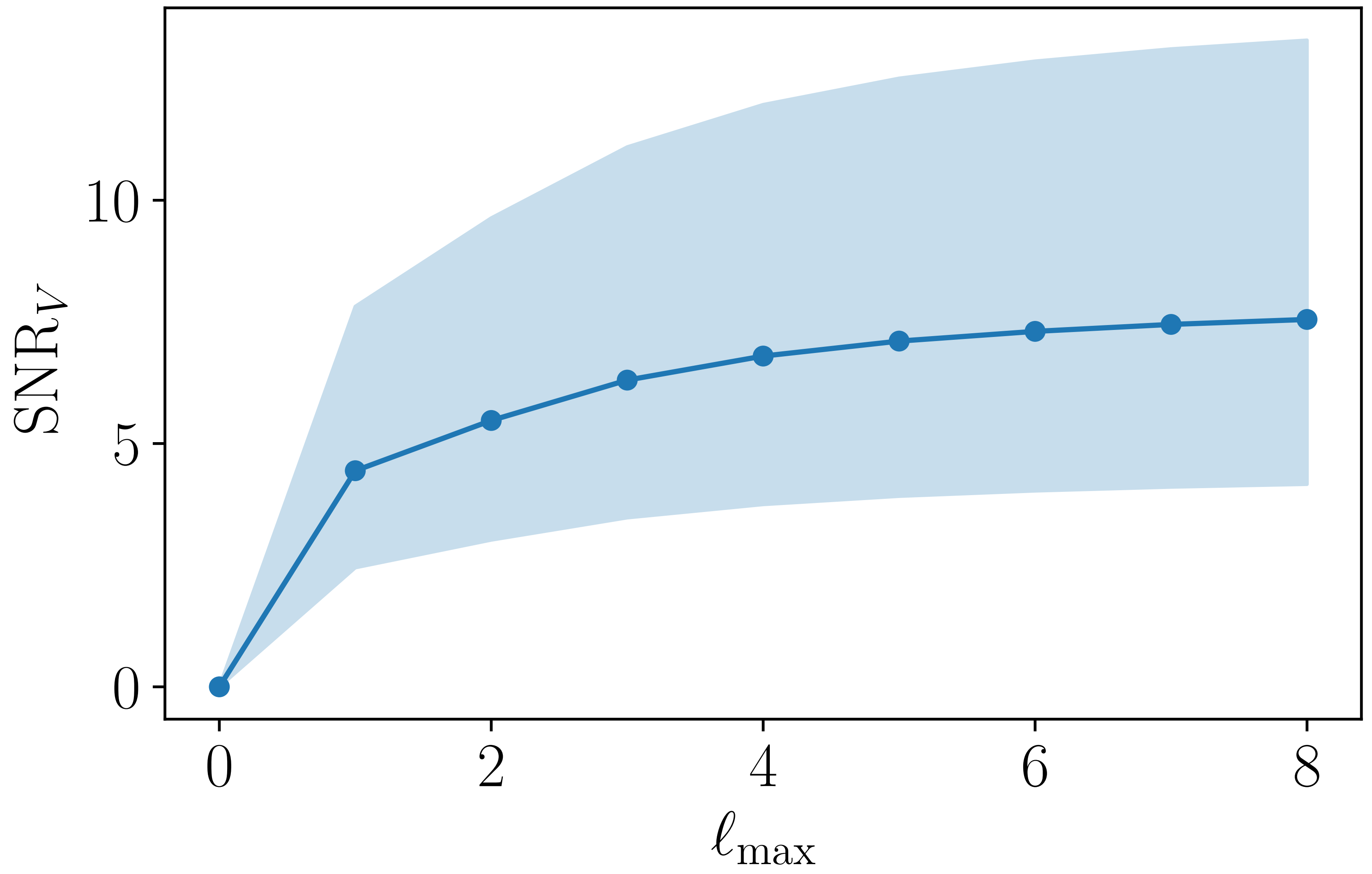
Circular polarisation (for general anisotropies)



Degree of circular polarisation

$$\epsilon_V = \frac{V}{I}$$

Unconstrained by current data
(again for cosmo SGWB)

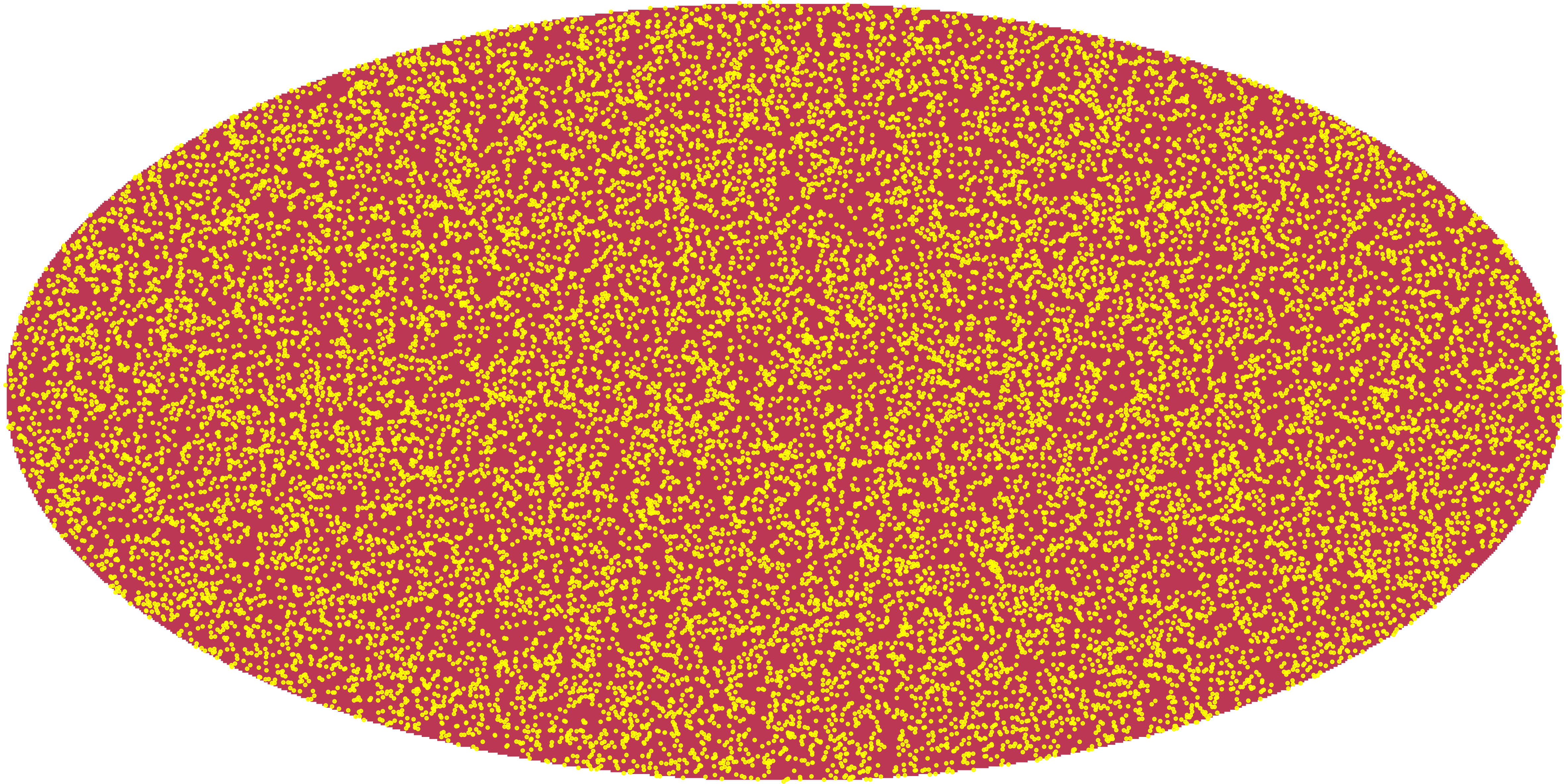


Near maximal polarisation may be detected with SKA ($N_{\rm psr} \gtrsim 10^3$)

Astrometry and SGWB

Precision astrometry with a large number of stars as a SGWB detector

[see Book, Flanagan (2010) for a review]



Gaia has $N \sim 10^9$ observed over 10 years with $\mathcal{O}(mas)$ precision. Already used to put constraints on low-frequency SGWB [Darling et al. 2018; Aoyama et al. 2021; Jaraba et al. (2023)]

Astrometry x PTA

Cross-correlations

The angular deflections and timing residuals induced by the SGWB are correlated

$$K_i^{(0)}(\vec{n}, \vec{x}) = \frac{16\pi}{3} \frac{(1-2y)\, n_i - x_i}{4y(1-y)} \left(2y - 2y^2 + 3y^2 \ln(y)\right)$$

Pulsar direction ($\vec{n}$)

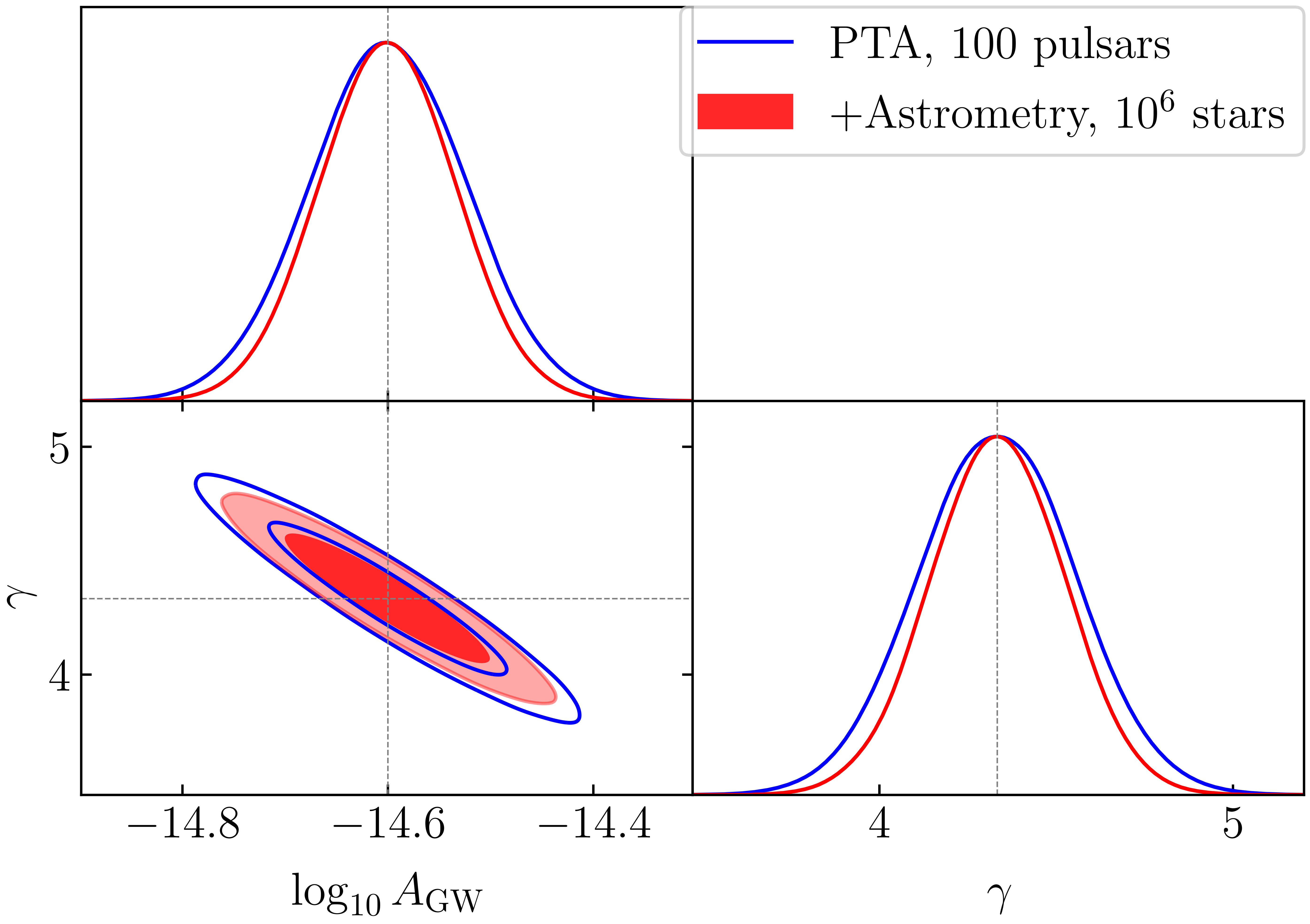
Star direction ($\vec{x}$)

Can cross-correlating Astrometry with PTA help?

Astrometry x PTA

Power-law

~10 % improvement over current PTA constraints

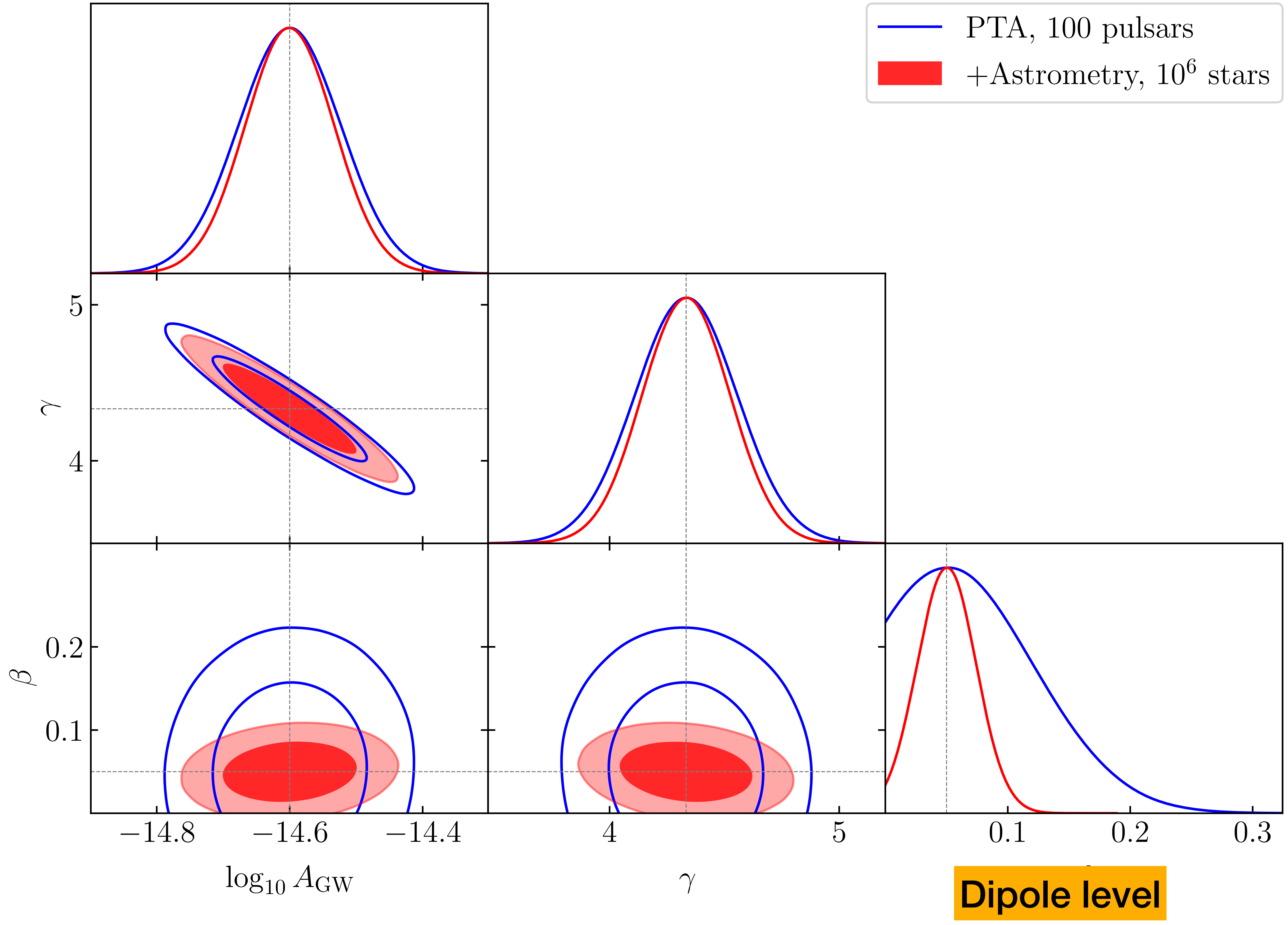


Astrometry x PTA

Dipole anisotropy

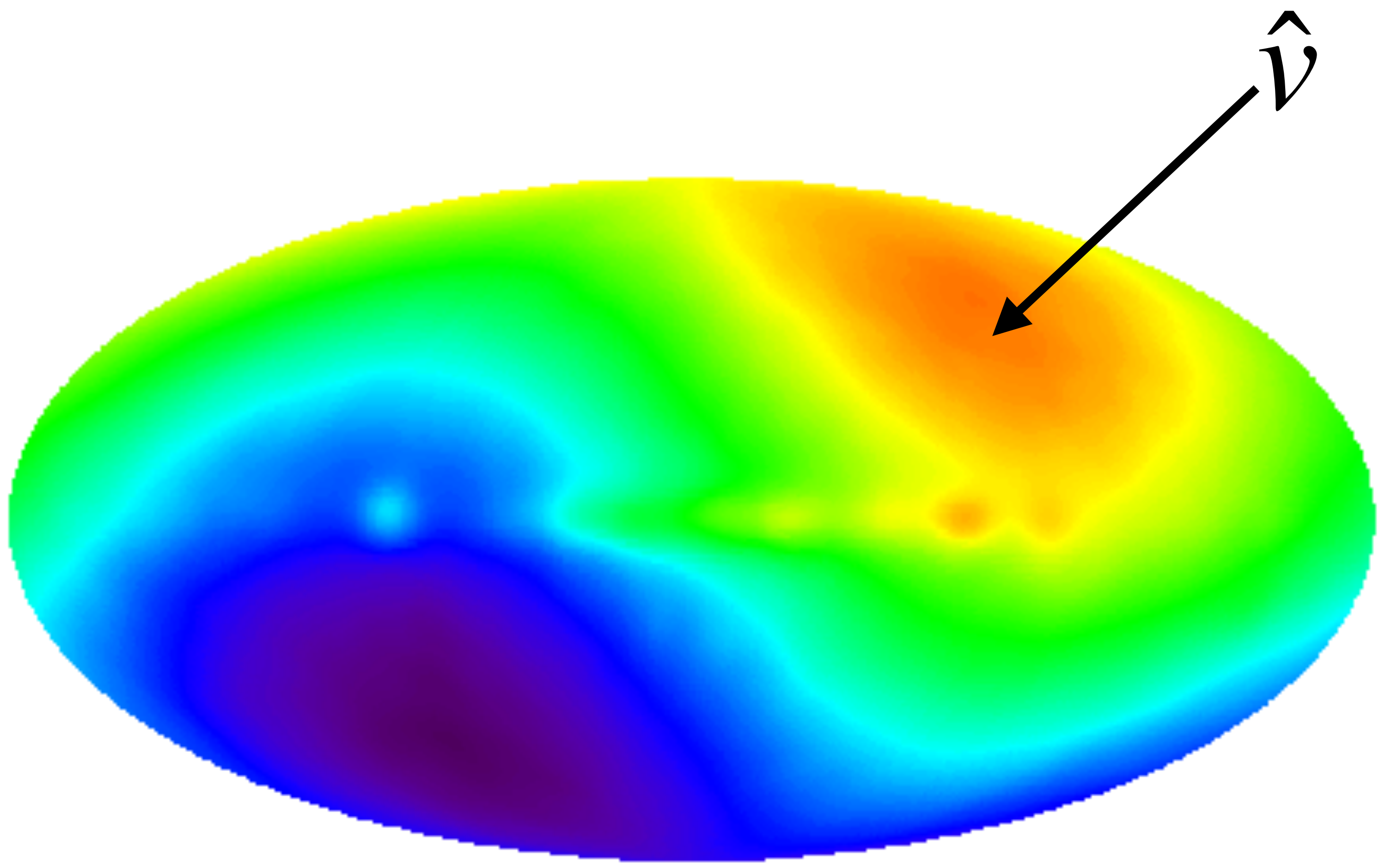
Minimum detectable dipole anisotropy relative to monopole ~ 0.05.

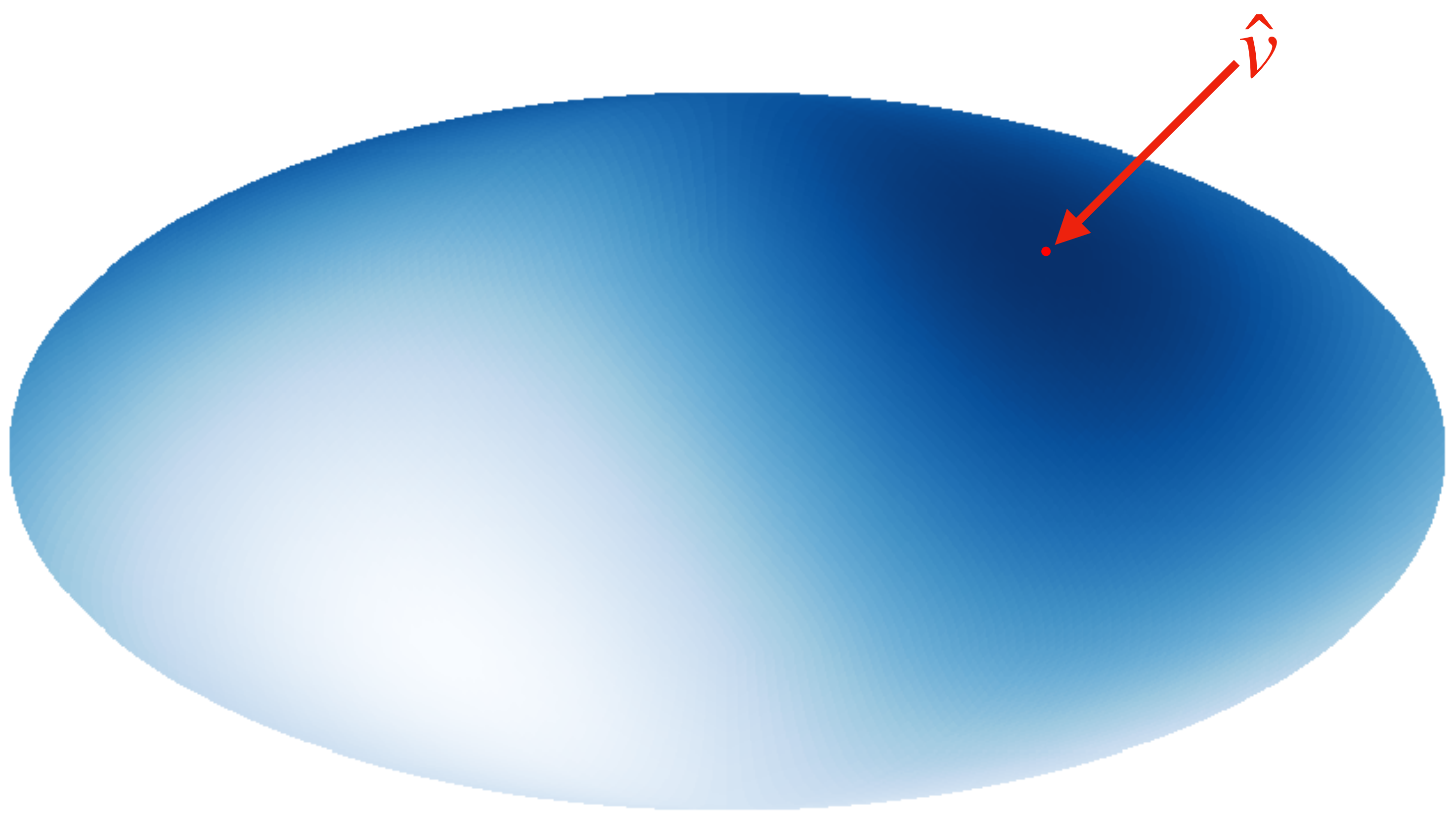
Current PTA level ~ 0.1



Conclusions

- If the SGWB is cosmological, it will present kinematic anisotropies with an amplitude much larger than intrinsic ones (like CMB)
- We theoretically characterized the PTA response to such anisotropies, and made initial forecasts for their detection
- More work to better characterize the prospects of detection, with more refined noise characterization, and with more systematic analysis of how the measurements depend on SGWB properties



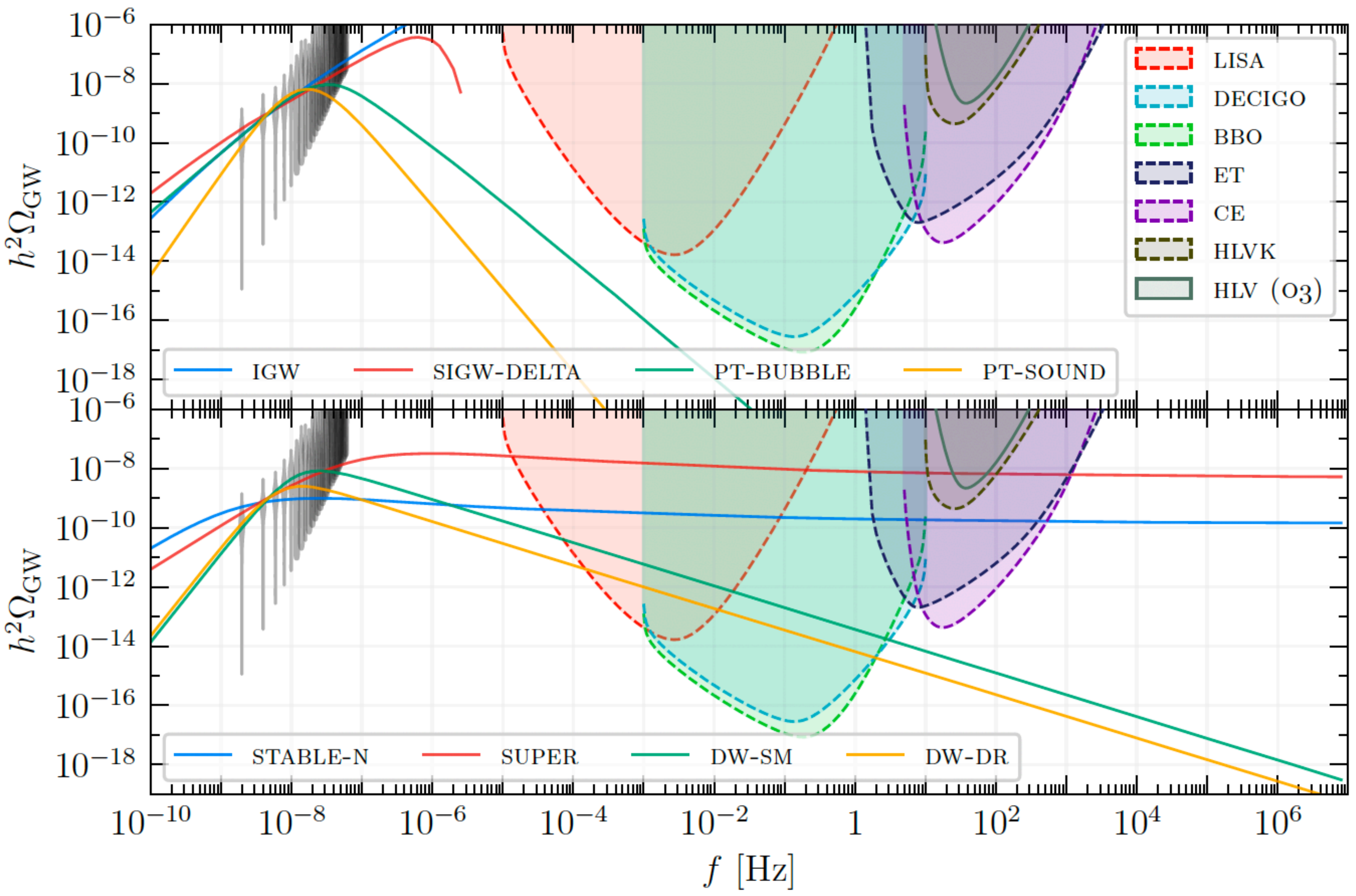


EXTRA SLIDES

SGWB induced timing residual

$$\delta t_p^{\mathrm{GW}}(t) = \frac{1}{2}\hat{p}_i\hat{p}_j \int_{t-D_p}^{t} dt'\, h_{ij}(t', (t-t')\hat{p})$$

$$\delta t_p^{\mathrm{GW}}(f) = \frac{\hat{p}^i\hat{p}^j}{4\pi i f} \int d^2\hat{n}\, \frac{h_{ij}(f,\hat{n})}{(1+\hat{n}\cdot\hat{p})}$$



arXiv: 2306.16219

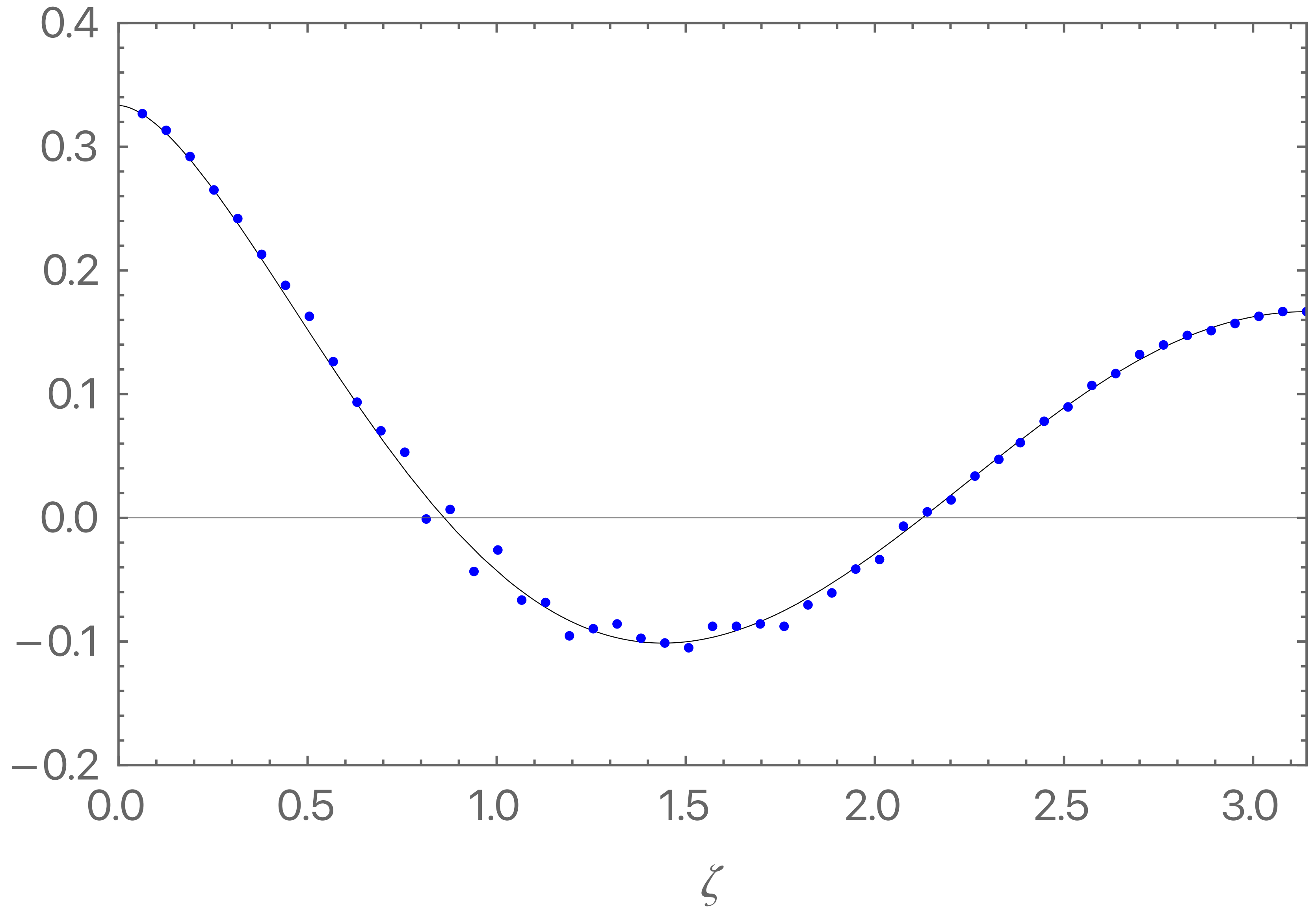
$$y_{ab} = \frac{1 - \hat{x}_a \hat{x}_b}{2} = \frac{1 - \cos\zeta}{2}$$

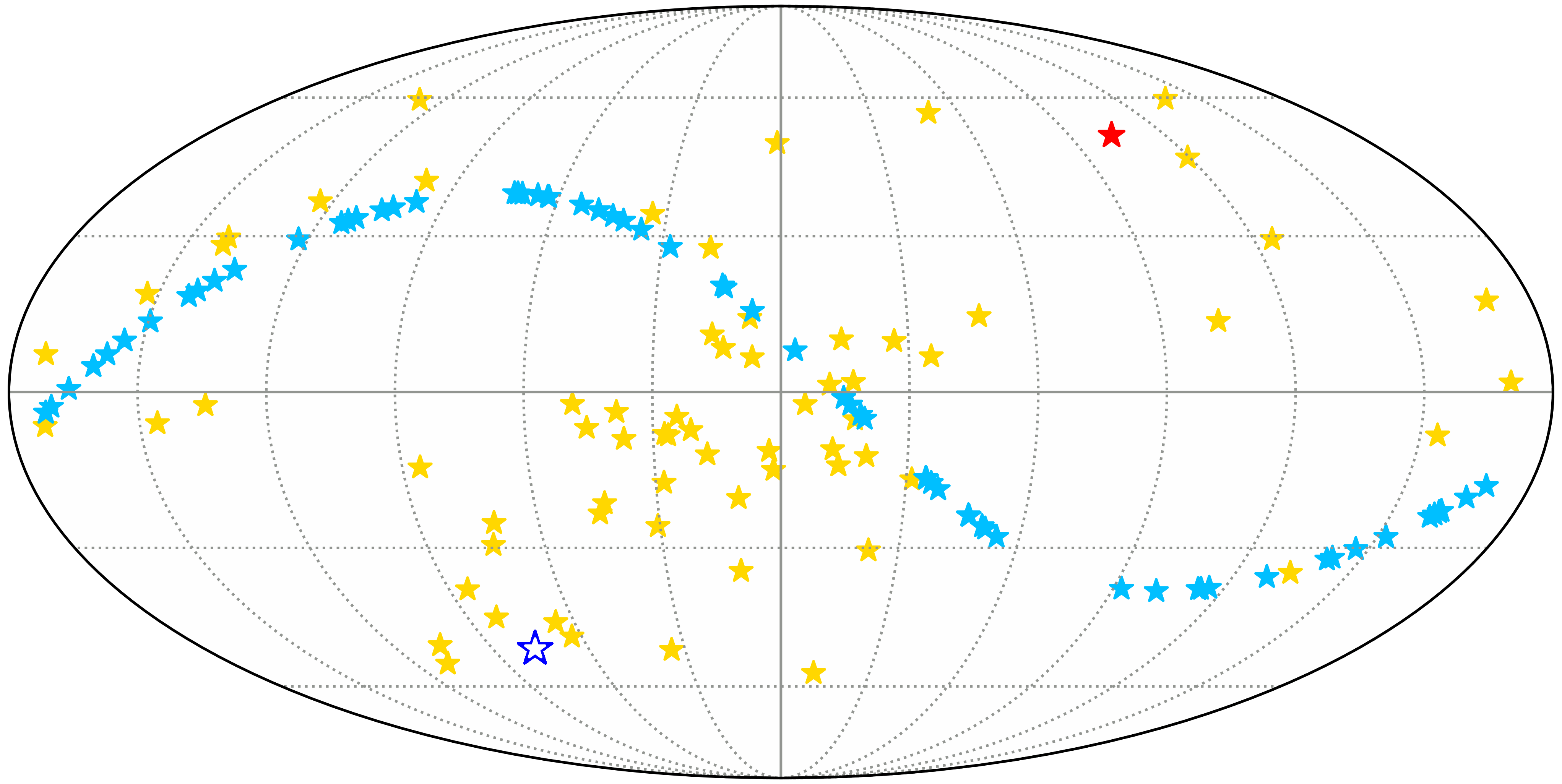
$$\Gamma_{ab}(f) = \left[1 - \frac{\beta^2}{6}\left(1 - n_I^2 - \alpha_I\right)\right] \Gamma_{ab}^{(0)} + \beta\,(n_I - 1)\,\Gamma_{ab}^{(1)} + \frac{\beta^2}{2}\left(2 - 3n_I + n_I^2 + \alpha_I\right)\,\Gamma_{ab}^{(2)}\,,$$

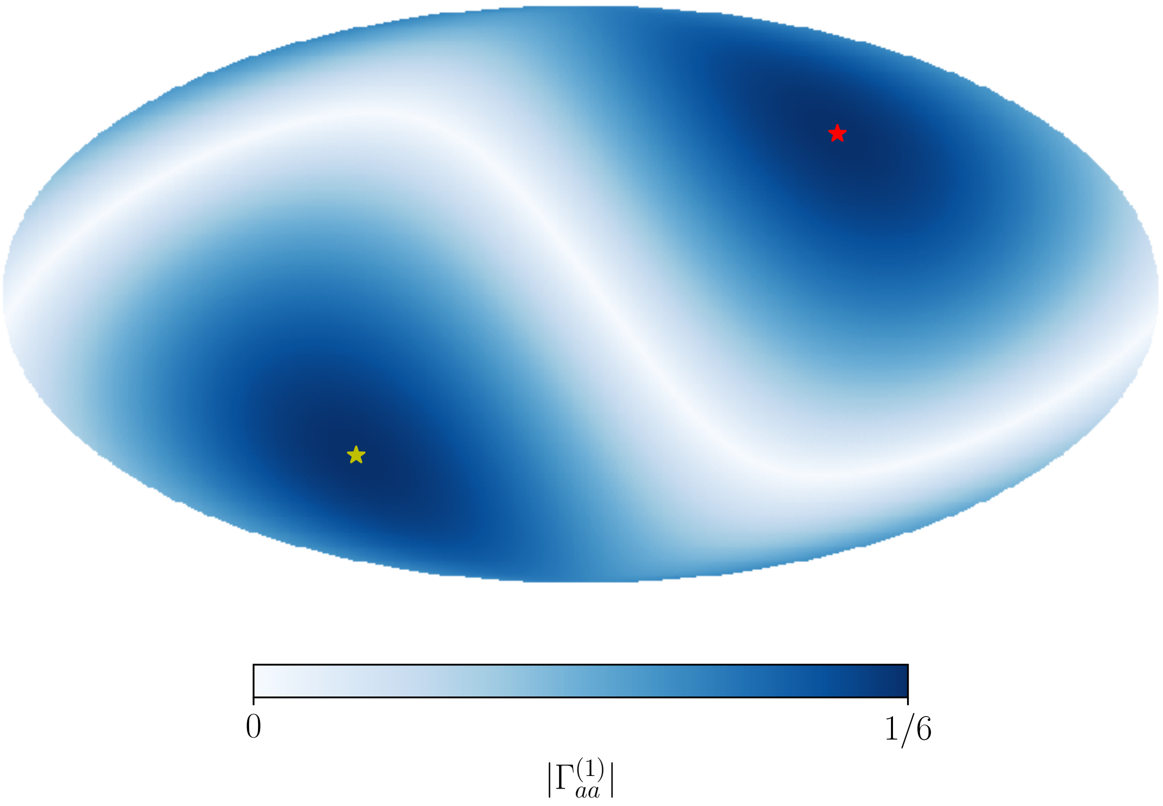
$$n_I = \frac{d\,\ln I}{d\,\ln f} \quad , \quad \alpha_I = \frac{d\,n_I}{d\,\ln f}$$

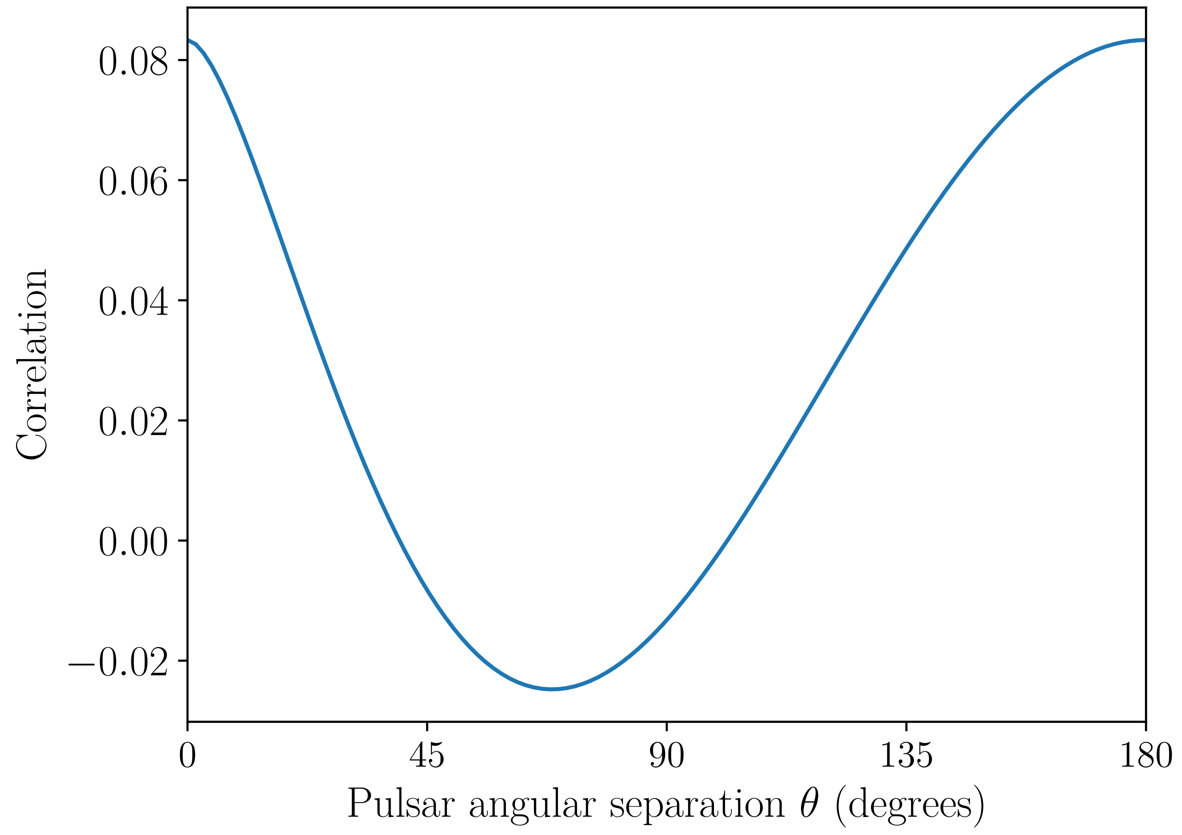
$$\Gamma_{ab}^{(0)} = \frac{1}{3} - \frac{y_{ab}}{6} + y_{ab}\ln y_{ab}$$

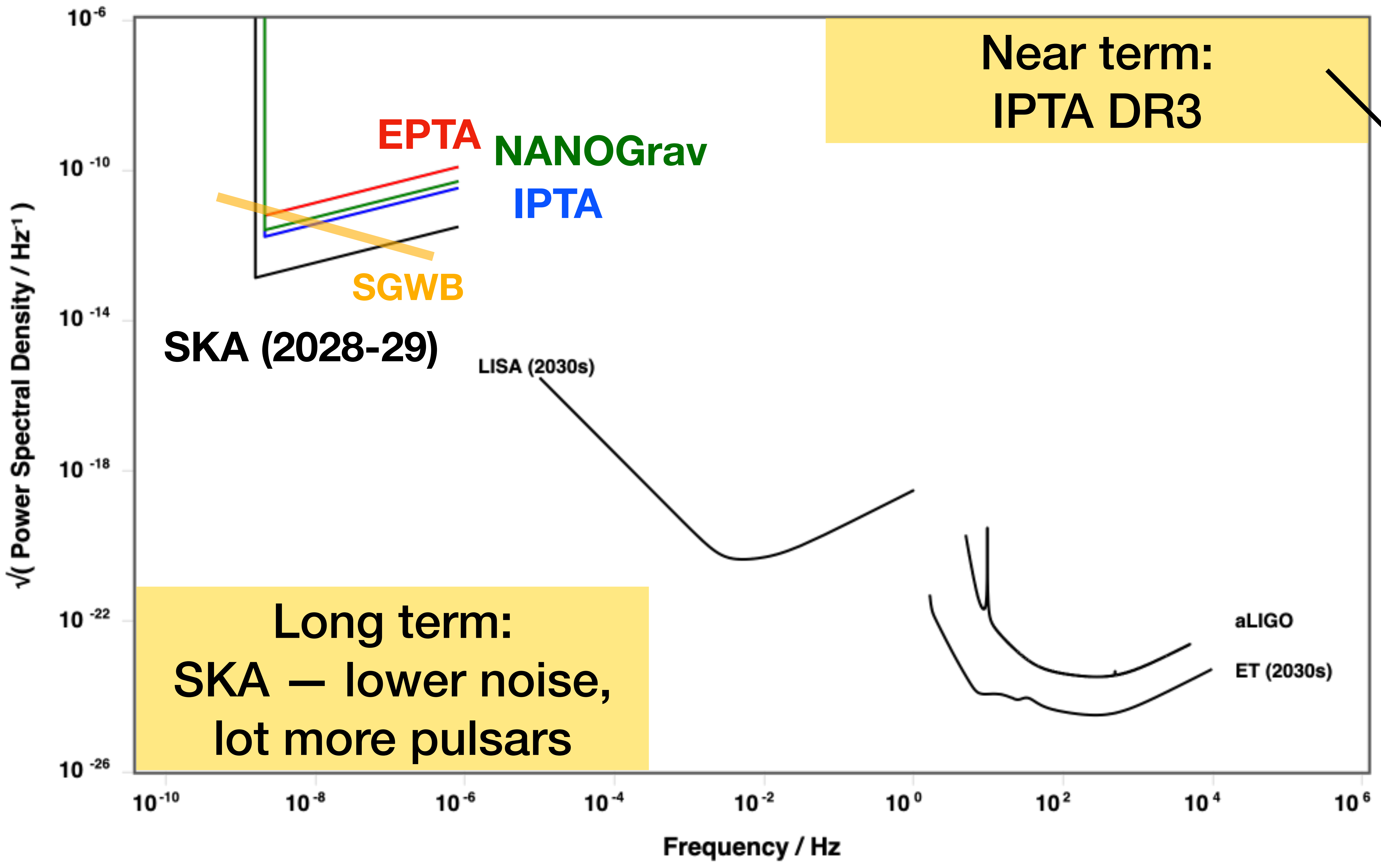
$$\Gamma_{ab}^{(1)} = \left(\frac{1}{12} + \frac{y_{ab}}{2} + \frac{y_{ab}\ln y_{ab}}{2(1 - y_{ab})}\right)\,[\hat{v}\cdot\hat{x}_a + \hat{v}\cdot\hat{x}_b]\,,$$


0.4
0.3
0.2
0.1
0.0
−0.1
−0.2
0.0
0.5
1.0
1.5
2.0
2.5
3.0
ζ



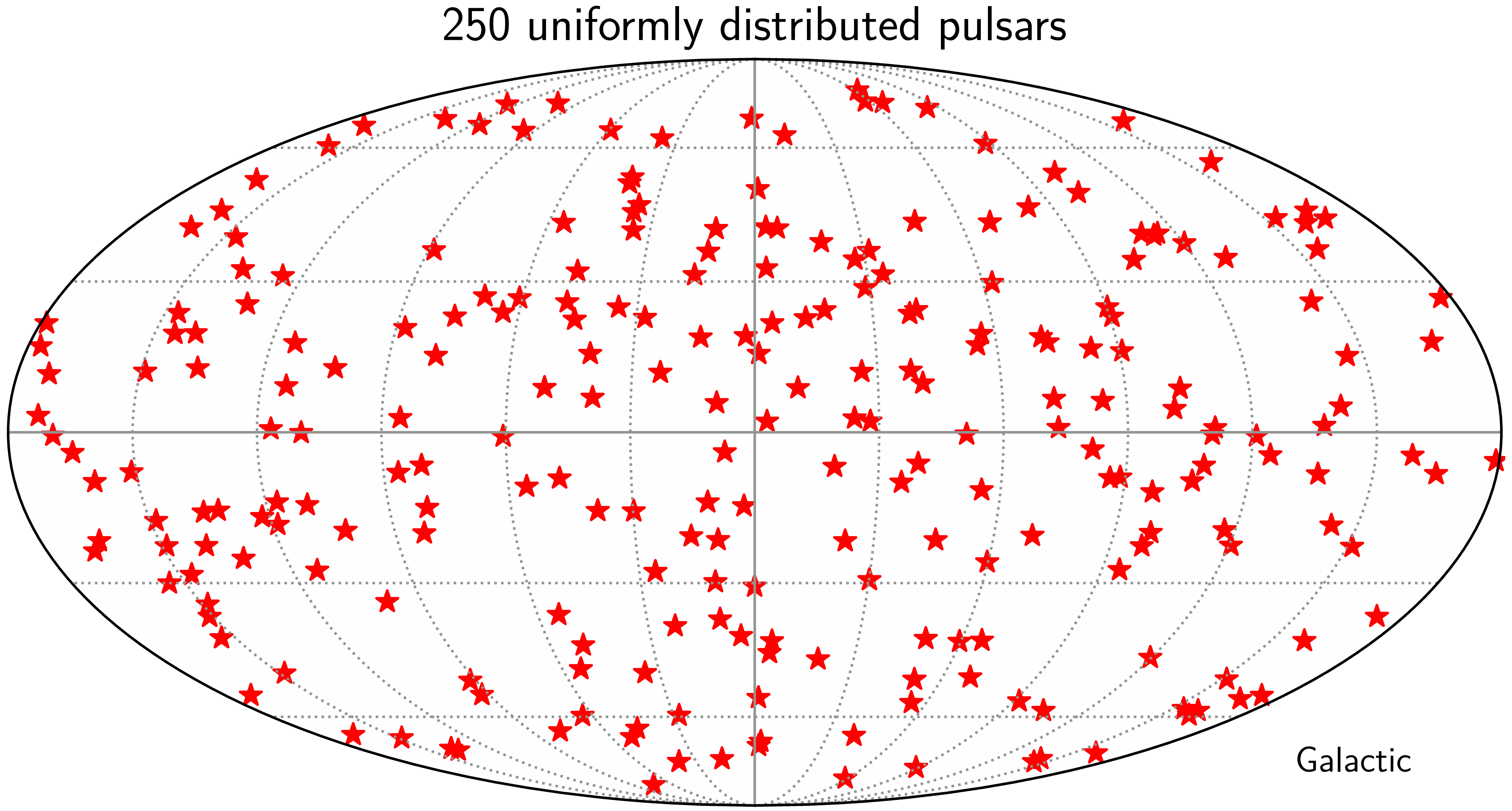

0
1/6
$|\Gamma^{(1)}_{aa}|$

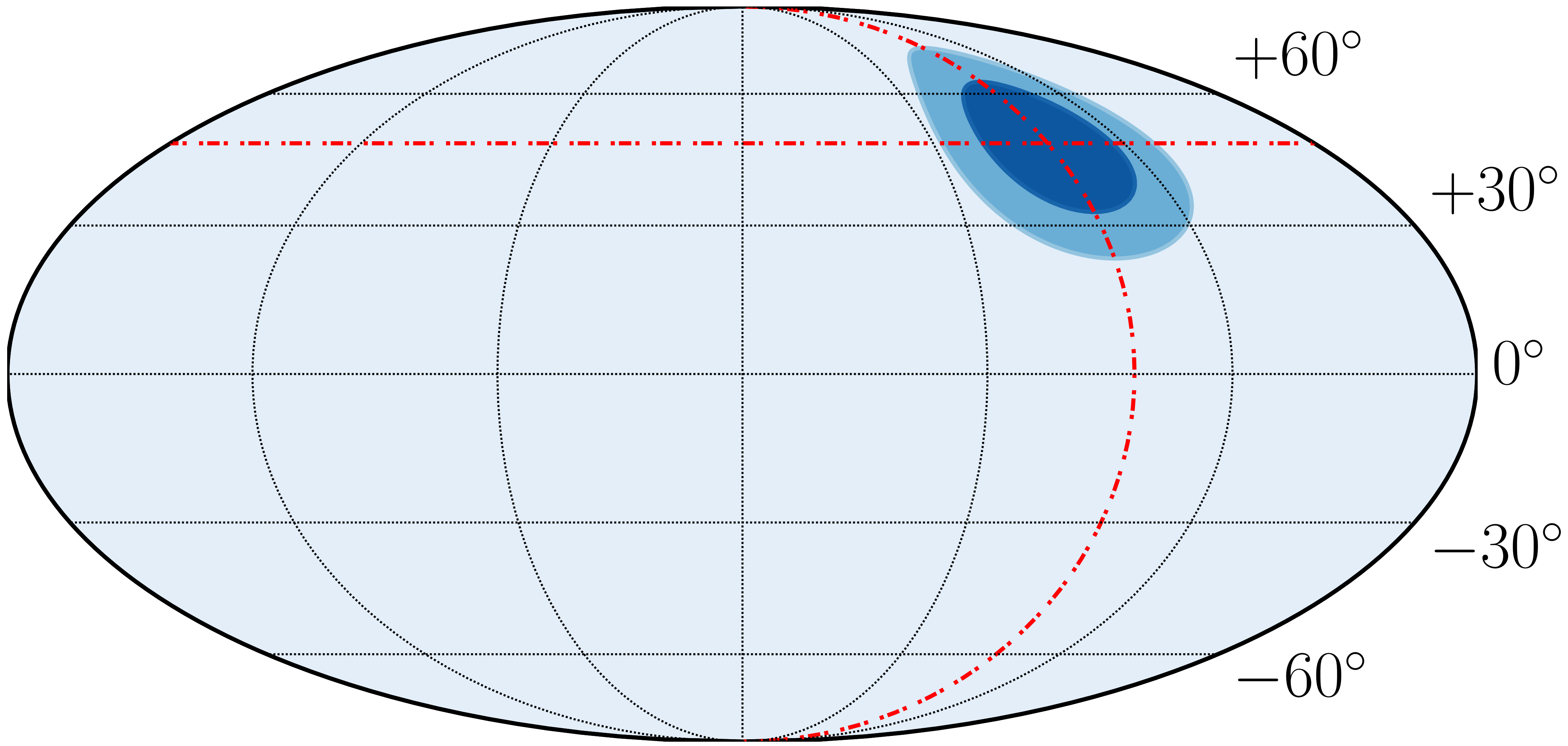

0.08
0.06
0.04
0.02
0.00
−0.02
Correlation
0
45
90
135
180
Pulsar angular separation θ (degrees)



May be enough
to detect SMBHB
anisotropy

But unlikely to be enough
for the kinematic dipole…





$$\Delta\beta \approx \left\lfloor \frac{20}{\mathrm{SNR}_{\mathrm{iso}}^2 (1 -} \right.$$

~20° degree localisation of dipole

$$\beta = (1.2 \pm 0.2) \times 10^{-3}$$

$$\mathrm{SNR}_{\mathrm{iso}} \propto \sqrt{T N_{\mathrm{pair}}} \frac{I}{\sigma_{\mathrm{noise}}^2}$$