

Inflationary Models in String Theory

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Hearing beyond the standard model with cosmic sources of Gravitational Waves
8th January 2025

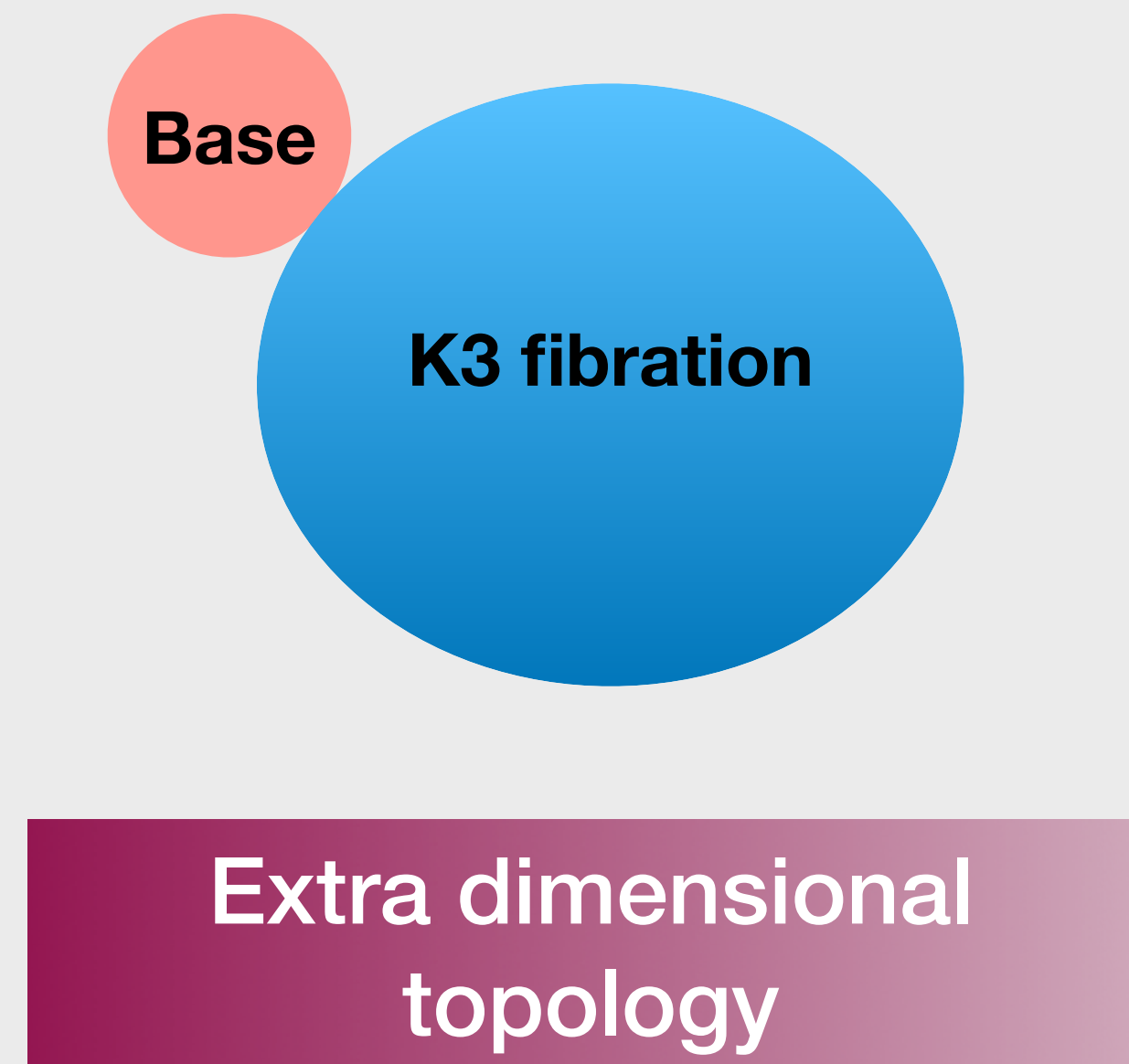
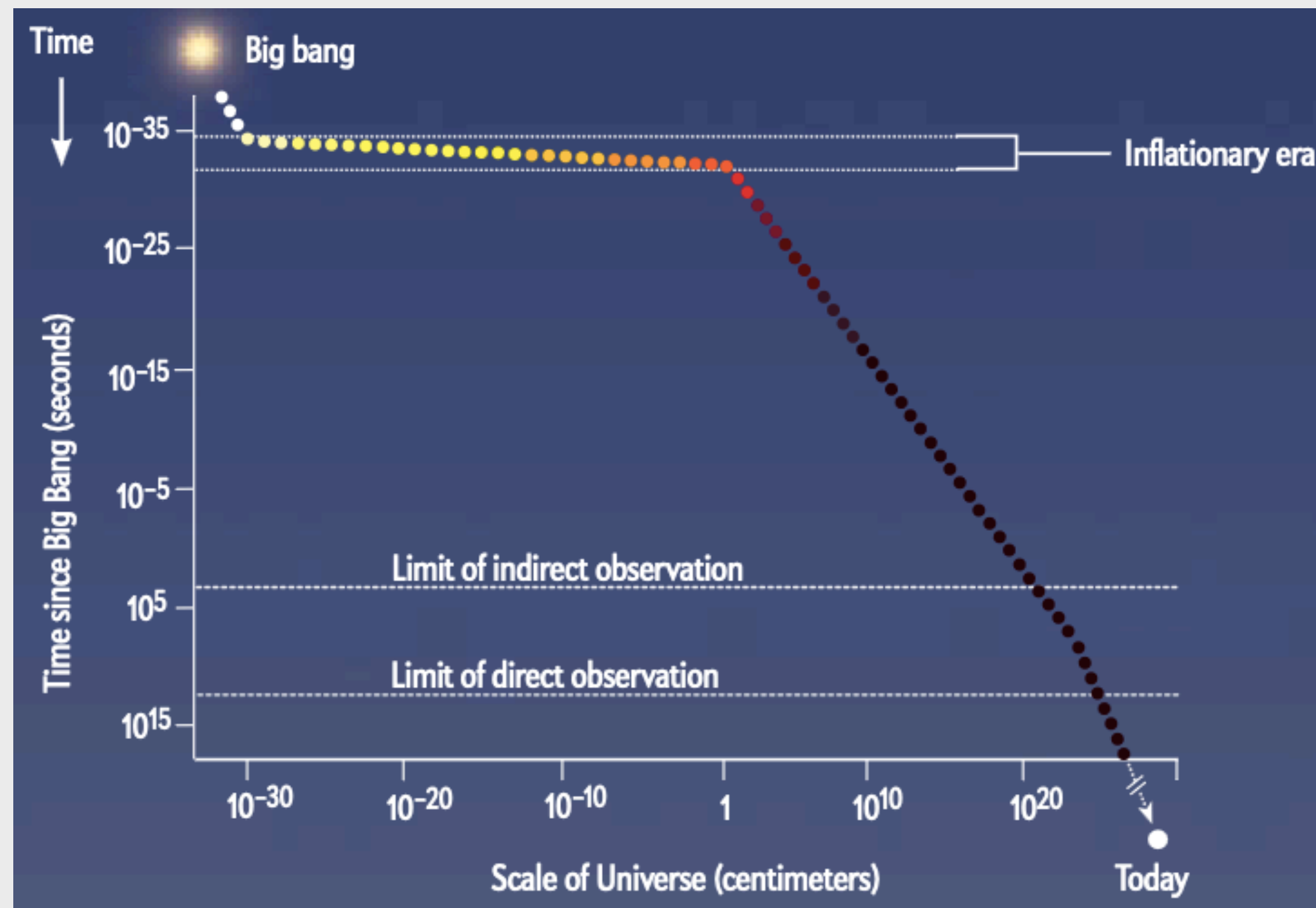


Keywords

 Inflation

 String Theory Model building

 Moduli stabilisation



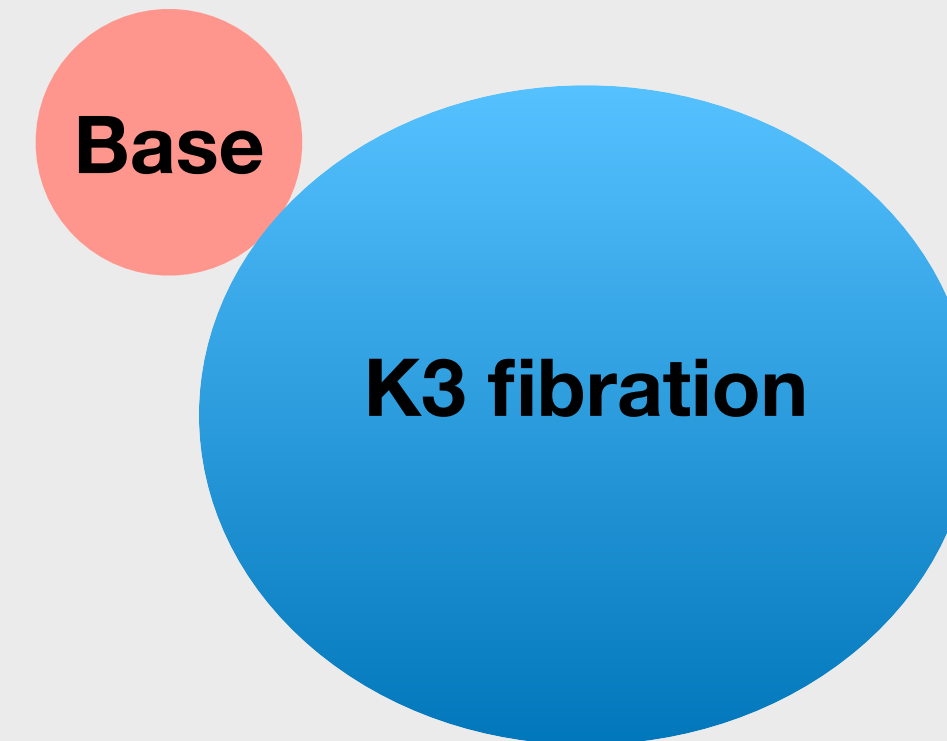
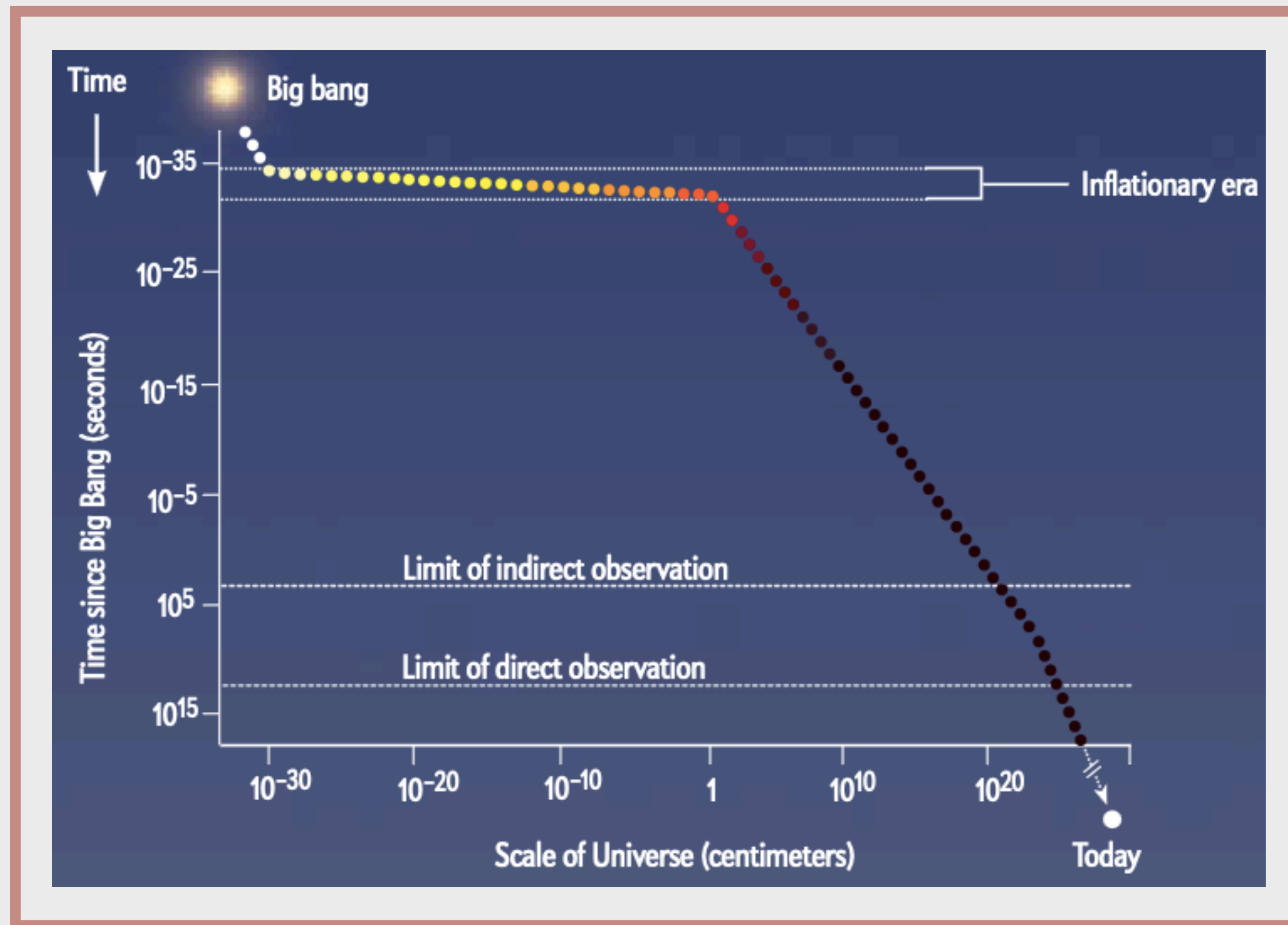
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[Ref: SriramKumar's, Ivonne's, Gianmassimo's talk]

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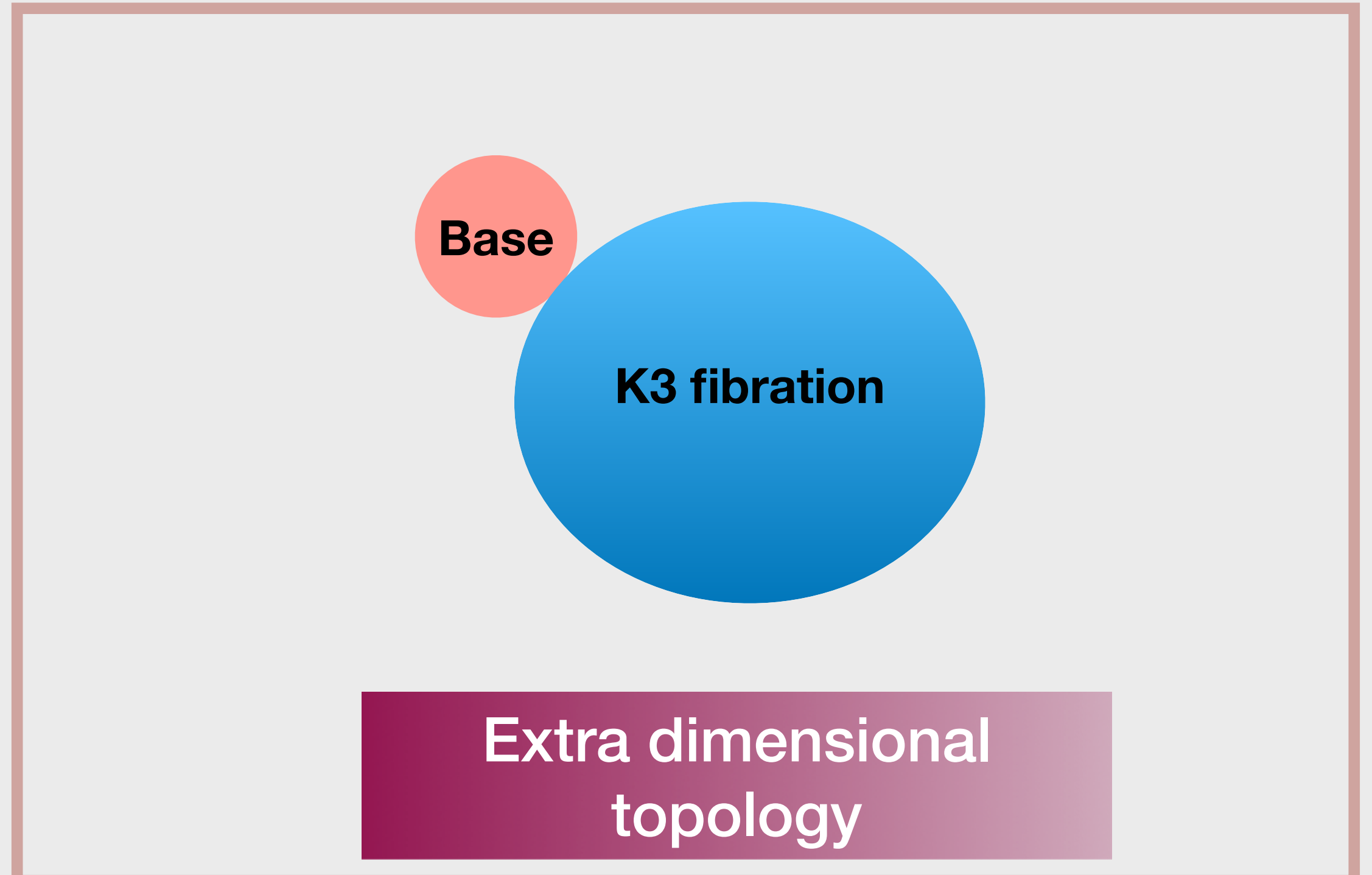
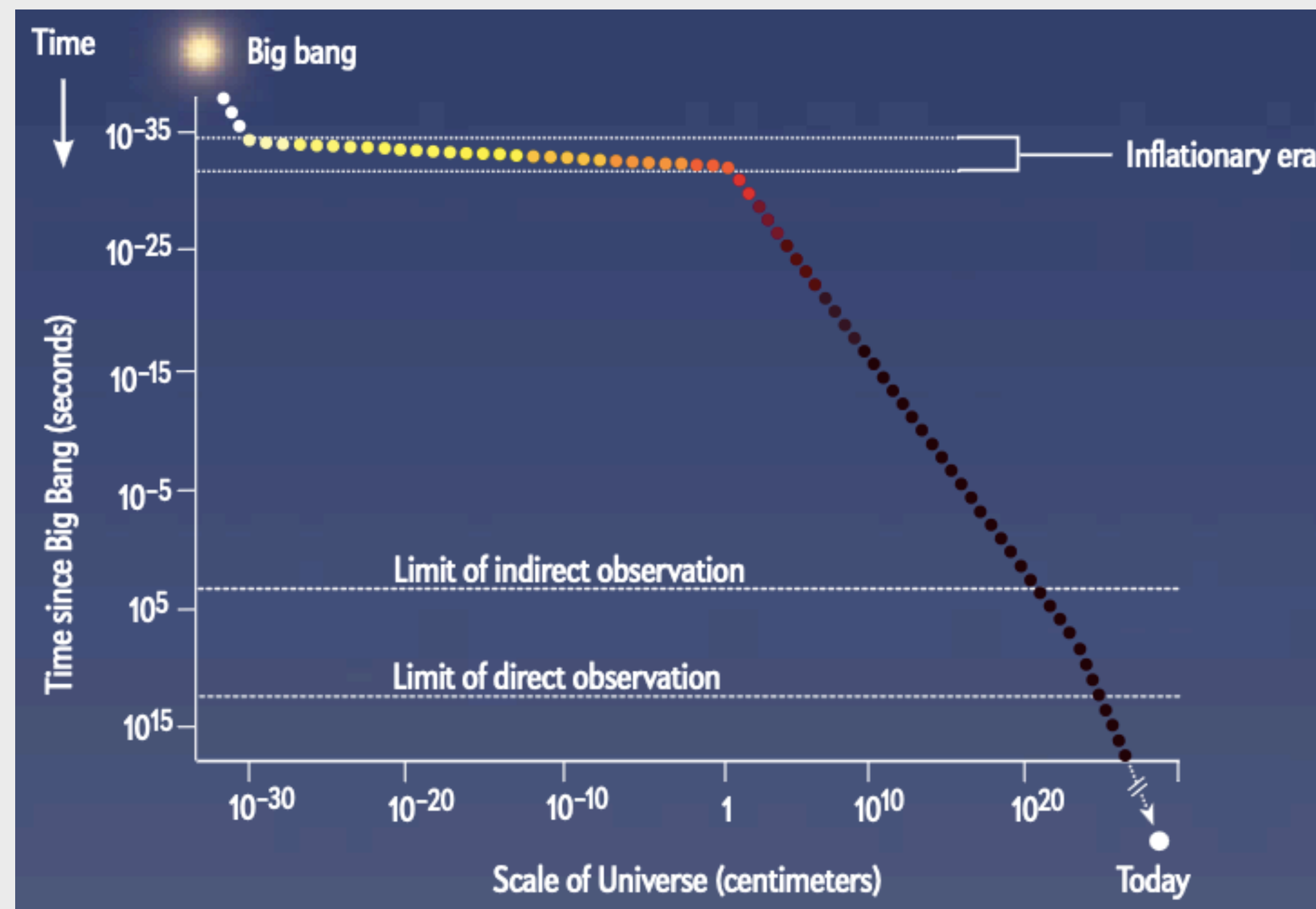
Extra dimensional
topology

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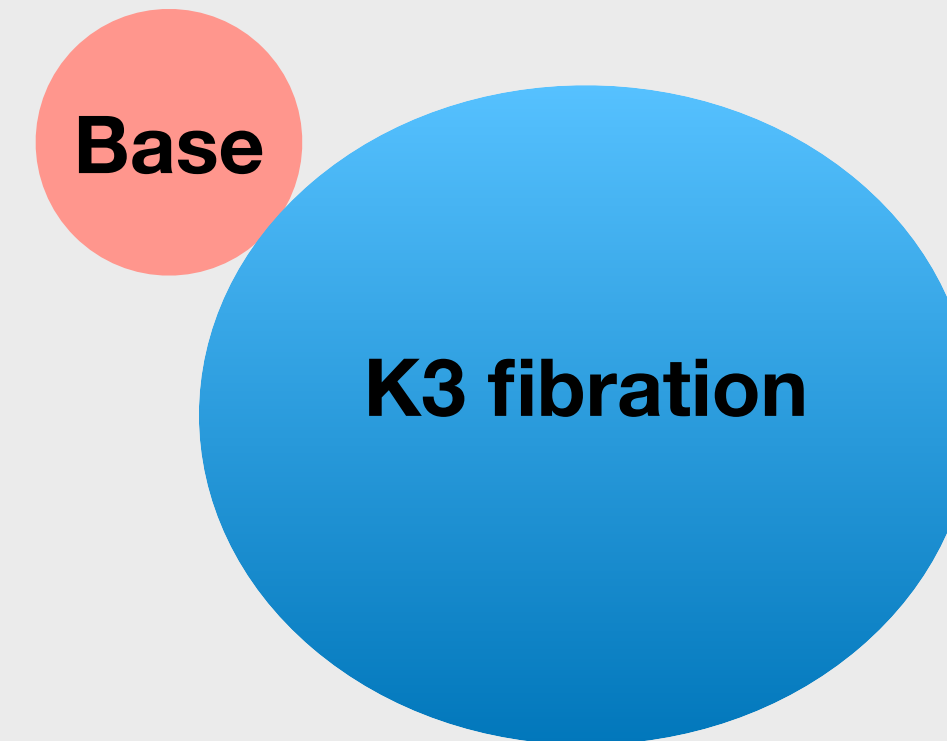
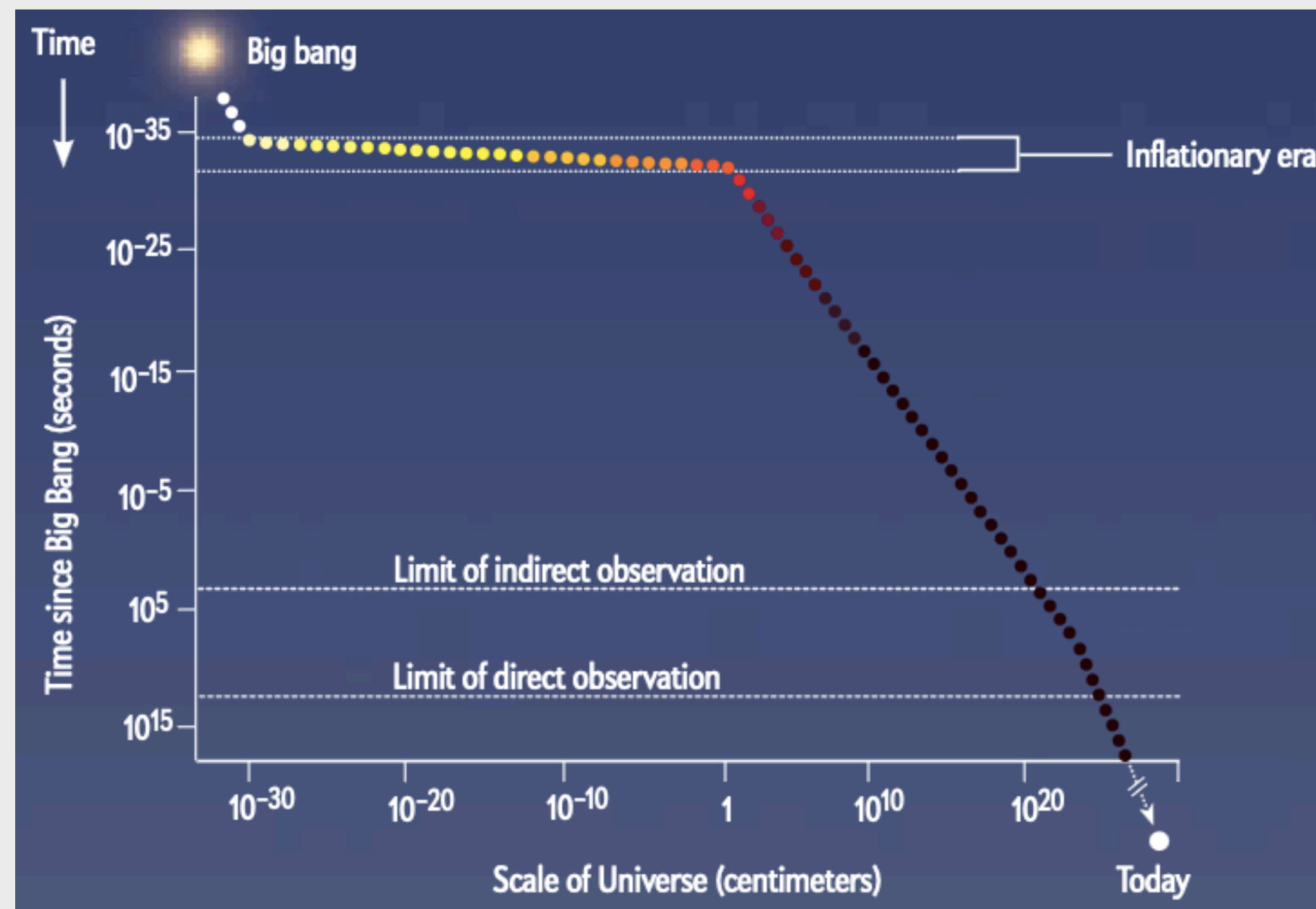
📌 Inflation →

Volume Inflation

Fibre Inflation

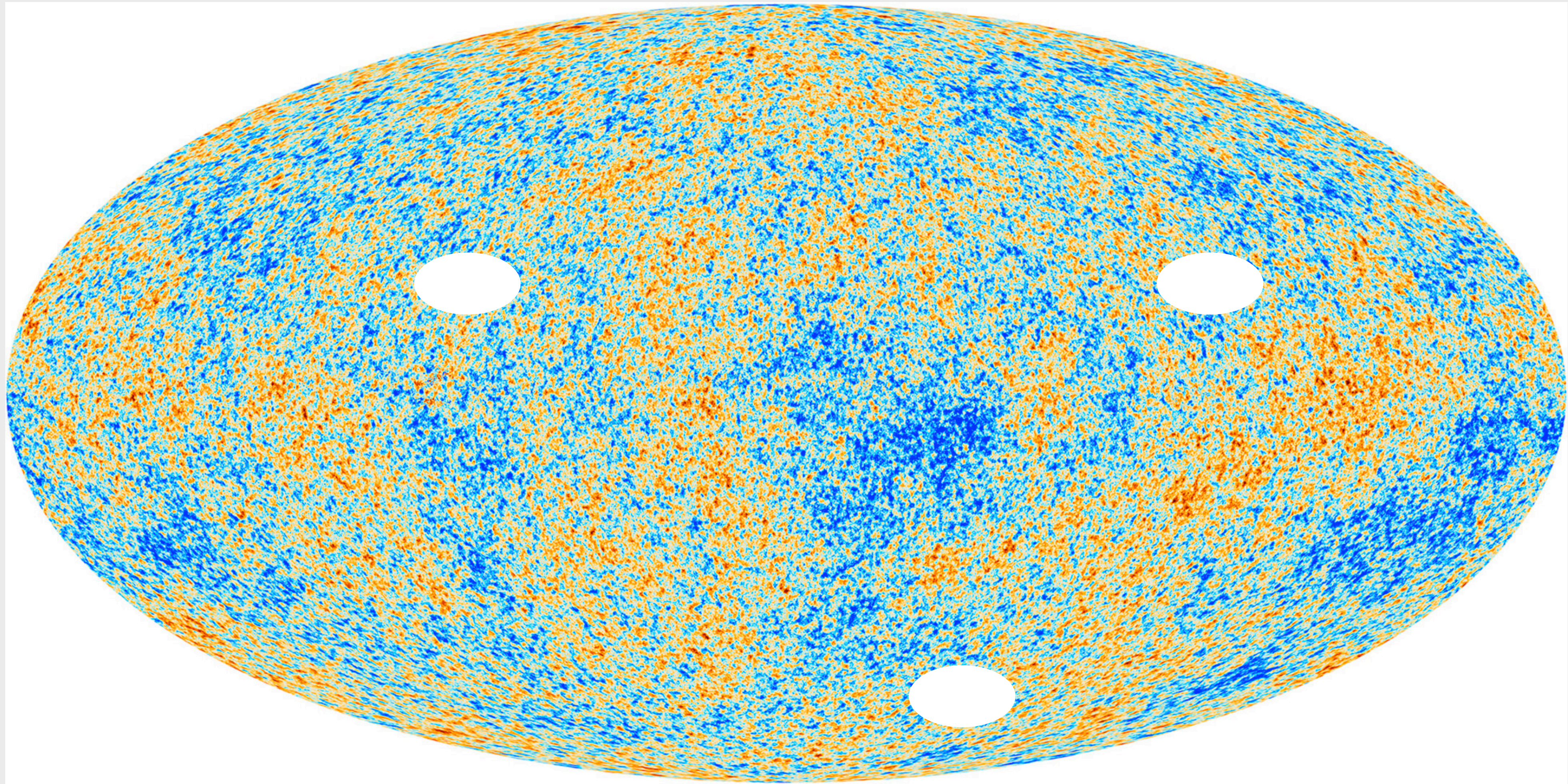
📌 String Theory Model building

📌 Moduli stabilisation



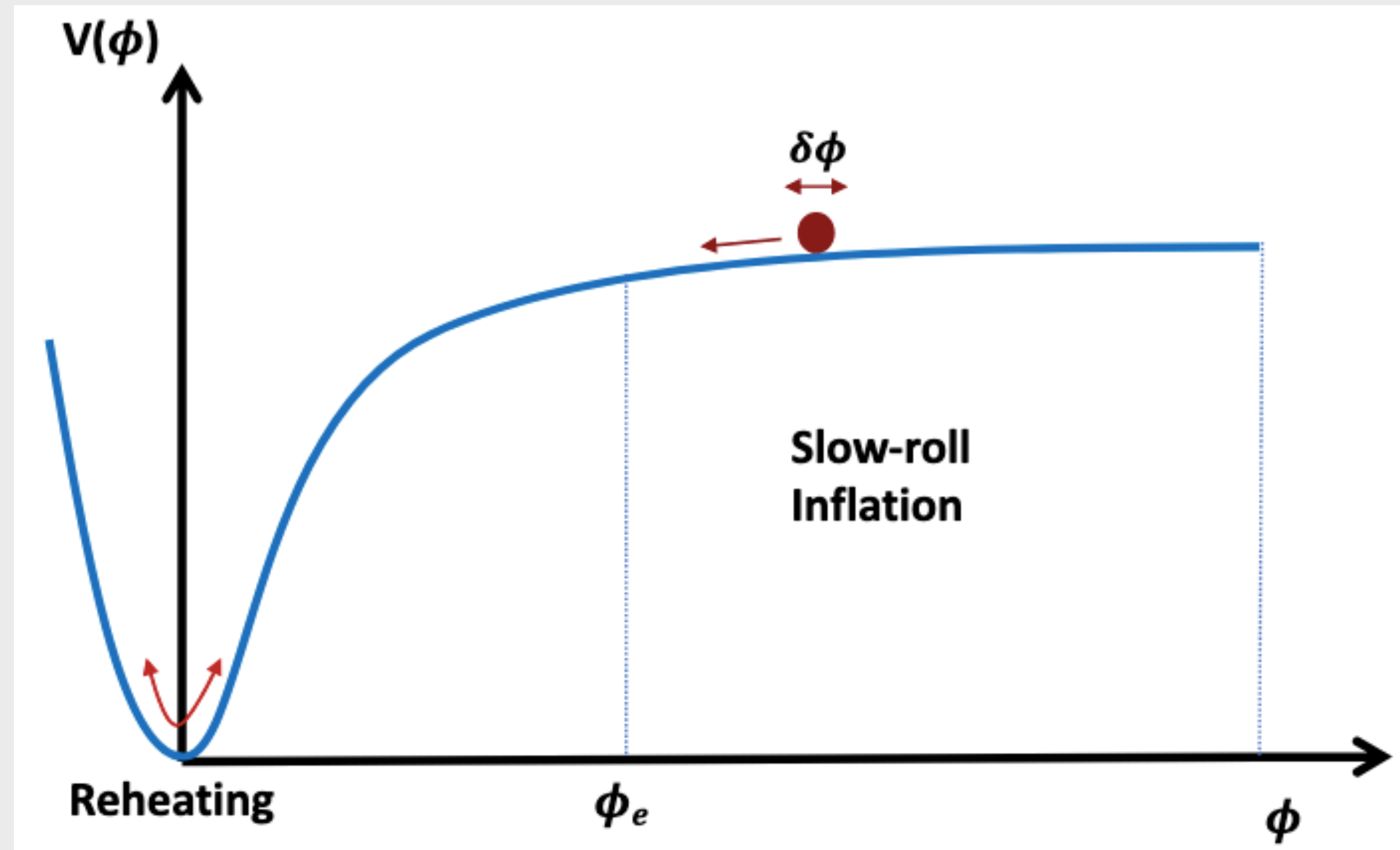
Extra dimensional topology

Why inflation



The **horizon problem**: antipodal points in the CMB sky share the same temperature modulo an anisotropy of 1 parts in a million despite not being in causal contact at the time of decoupling. [Ref: Subhodip's talk]

Minimally coupled scalar field driven inflation



Scalar field minimally coupled to gravity driving **slow-roll** inflation

Slow-roll inflation

EOM

$$\ddot{\phi} + 3H\dot{\phi} + V_{,\phi} = 0$$

Slow-roll parameters

$$\epsilon_H = -\frac{\dot{H}}{H^2}, \quad \eta_H = \frac{\dot{\epsilon}_H}{\epsilon_H H}, \quad \epsilon_V = \frac{1}{2} \left(\frac{V_{,\phi}}{V(\phi)} \right)^2, \quad \eta_V = \frac{V_{,\phi\phi}}{V(\phi)}$$

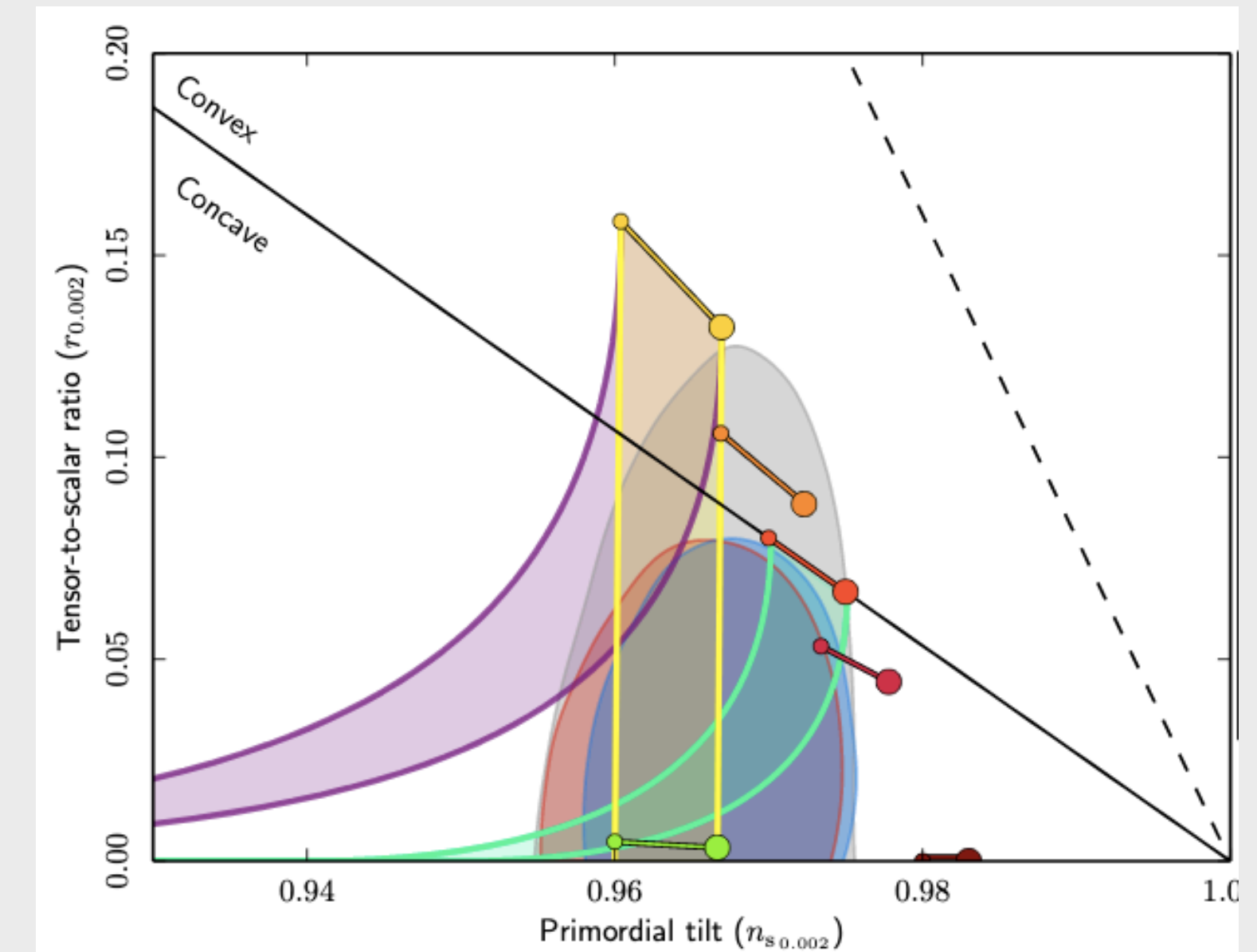
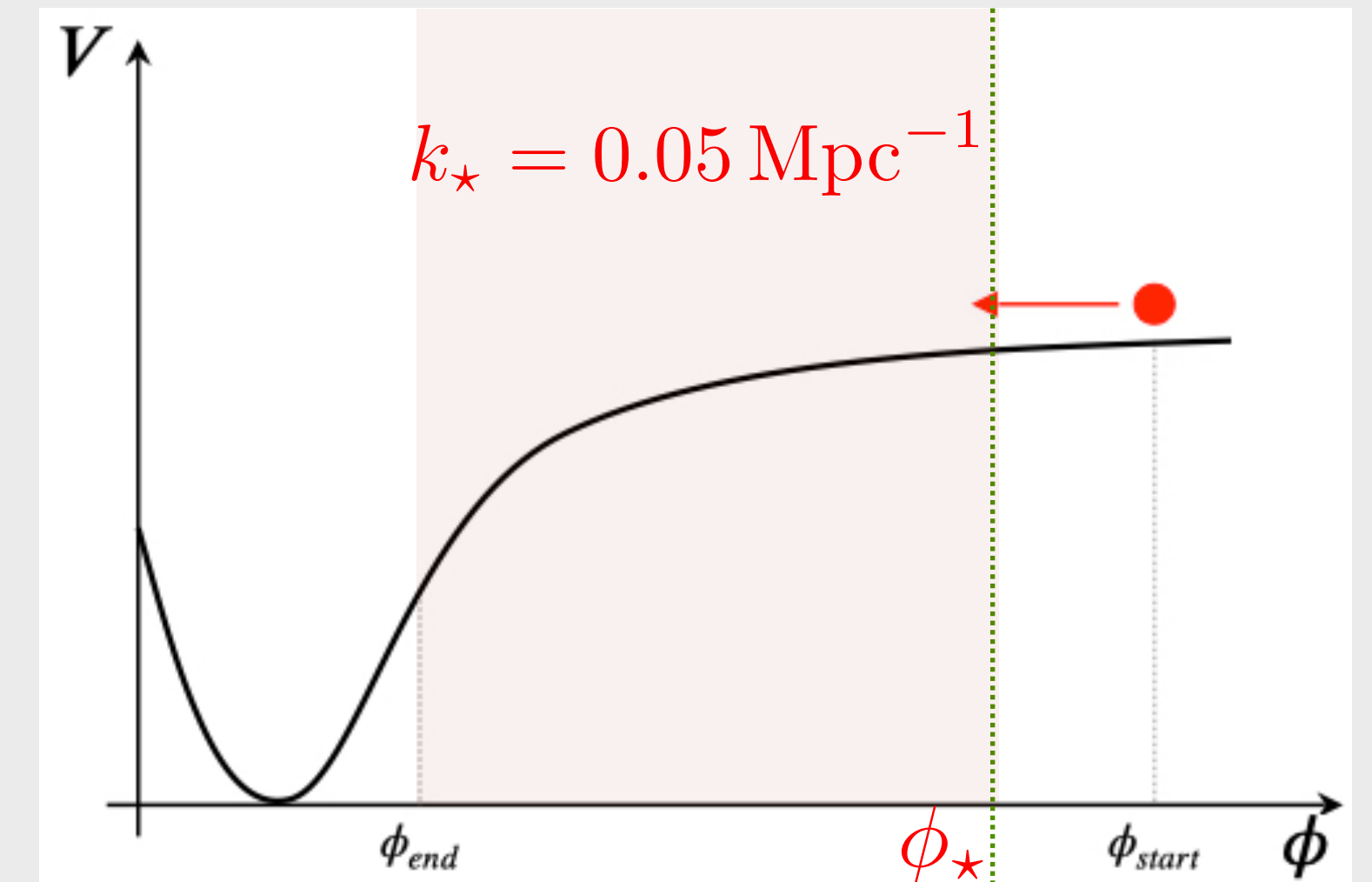
Cosmological parameters

$$P_s = \frac{V(\phi)^*}{24\pi^2 \epsilon_H^*} = 2.105 \pm 0.03 \times 10^{-9}$$

$$n_s = 1 - 2\epsilon_H^* - \eta_H^* \simeq 2\eta_V^* - 6\epsilon_V^* = 0.9649 \pm 0.0042$$

$$r = 16\epsilon_H^* \simeq 16\epsilon_V^* < 0.036$$

$$N(\phi) = \int H dt = \int_{\phi_{end}}^{\phi_*} \frac{1}{\sqrt{2\epsilon_H}} d\phi \simeq \int_{\phi_{end}}^{\phi_*} \frac{V(\phi)}{V'(\phi)} d\phi \simeq 60$$



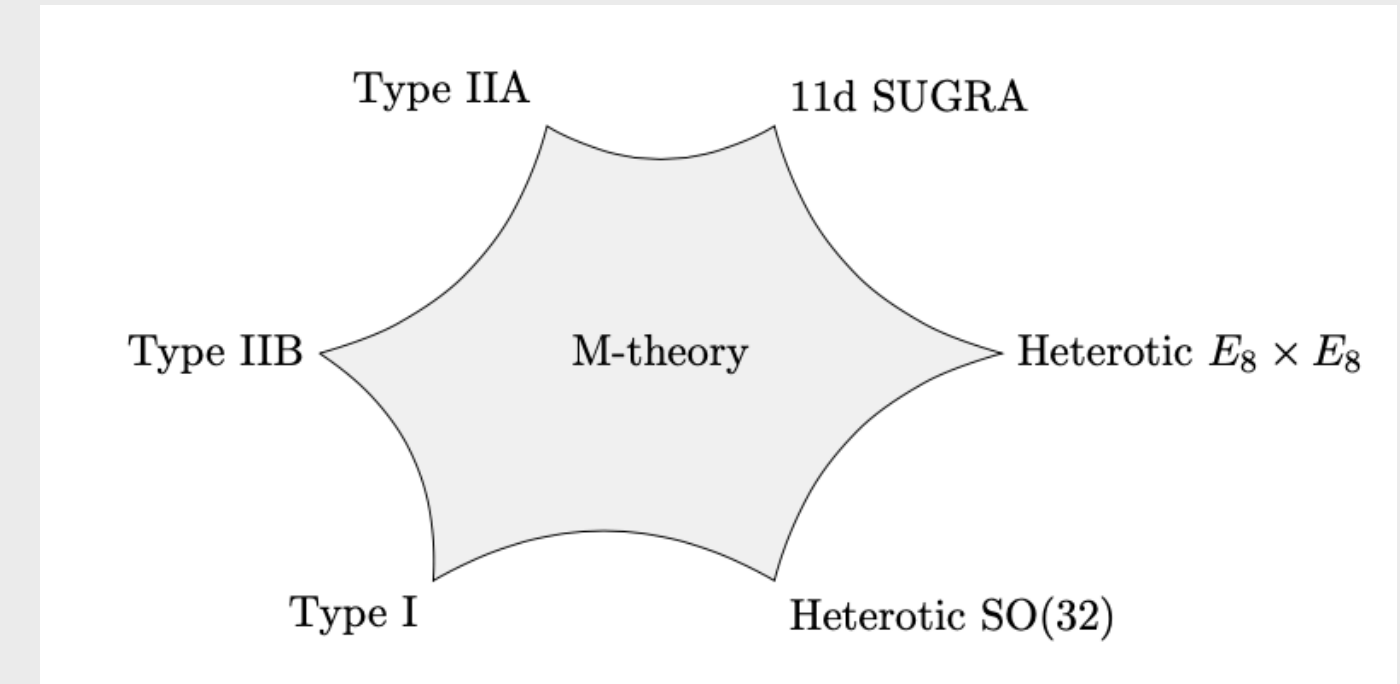
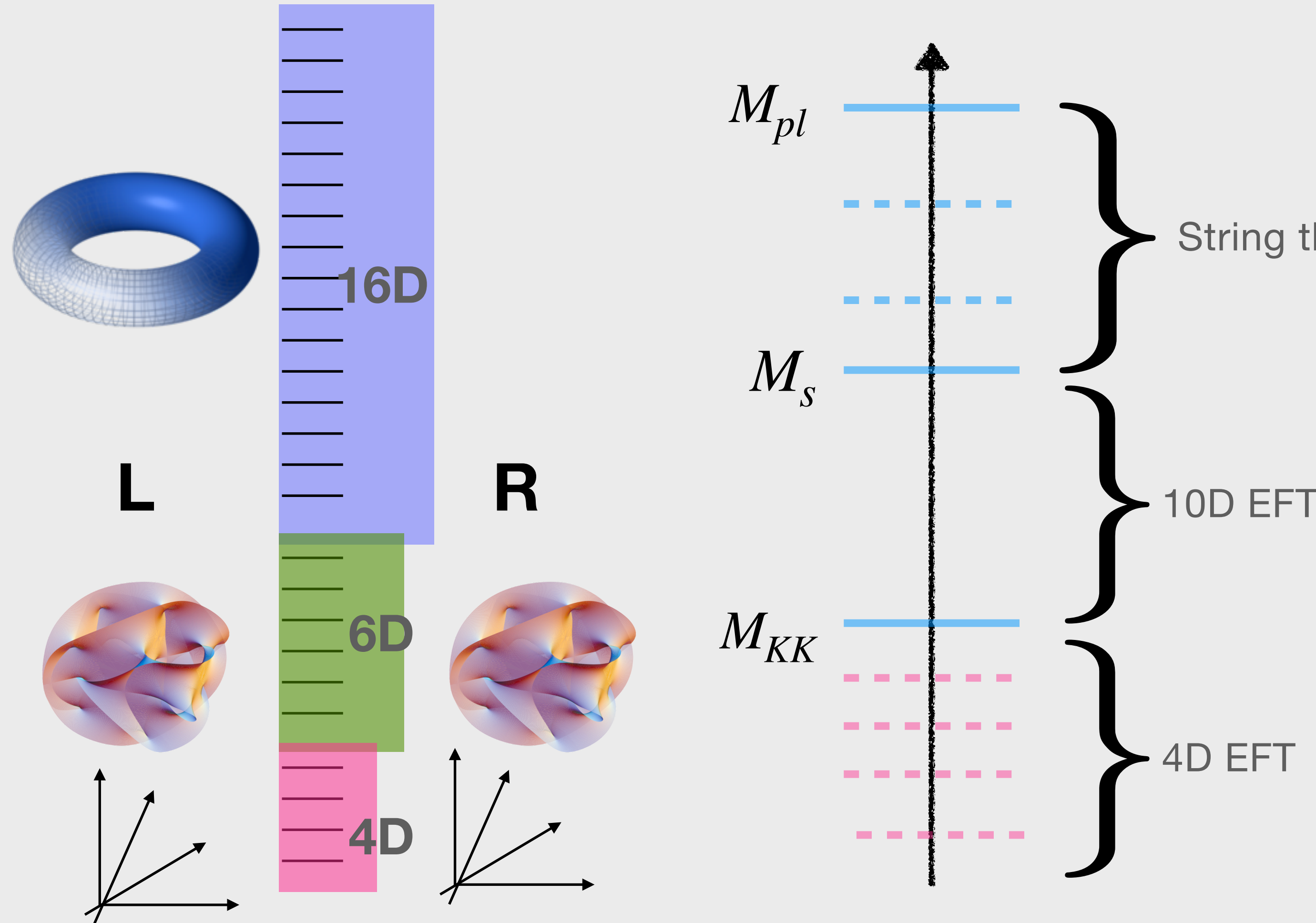
String Cosmology

Exploring String Inflation - rich structures dubbed in moduli

- 🎤 A unification of all the interactions? Leading candidate - **String Theory**
- 🎤 No experimental proof— connection between ST and Cosmology is essential in fundamental physics.
- 🎤 **Plethora of scalar fields** — “moduli” — **can act as inflaton** — light and has gravitational strength interactions [Ref: Sonia’s talk]
- 🎤 Understanding the **potential energy** functional is crucial — they have to be stabilised to avoid **fifth force**. [Ref: Wan’s talk]

String Compactification in a nutshell

To make contact with the observed world, 10d or higher dimensional string theory needs to be compactified down to 4d



$$S_{10}[C] \xrightarrow[\mathcal{M}_{10} \rightarrow \mathcal{M}_4 \times X_6]{\text{Compactification}} S_4[\varphi]$$

6d internal space: Topology, geometry, Fluxes, Local sources (D-branes, O-planes)

Compactify on a **Calabi-Yau 3-fold:** complex Ricci-flat Kähler manifolds with SU(3) holonomy/structure group

Current Focus: Type IIB String Theory

Moduli of Calabi-Yau 3-folds

The **geometric moduli** of a CY3 (X) are determined by the number of embedded **2- 3- spheres**. Massless deformation of the internal manifold, massless particles on 4D

		1		b_0	Kahler moduli (τ) — $h^{1,1}(X) = b_2 - 2$ cycles — related to the overall volume of the internal space.
	0		0	b_1	
0		$h^{1,1}$		b_2	Complex structure moduli (z^a) — $h^{1,2}(X) = \frac{b_3}{2} - 1$ — 3 cycles — related to the shape of the internal space.
1	$h^{1,2}$		$h^{1,2}$	b_3	
	0		$h^{1,1}$	b_4	Axio-dilaton (S) — related to the string coupling
		0		b_5	<input checked="" type="checkbox"/> Current Focus: U, S stabilised at their minima and perturbation around that minima to stabilise τ . Study inflation with a certain τ .
			1	b_6	Moduli must be stabilised! If left unstabilised then they can mediate fifth force or missing energy in the collider — not observed in nature.

b'_i s denote Betti numbers which are cohomology dimensions,

$$b_i = \dim_{\mathbb{R}} H_{dR}^i(X)$$

General idea: use fluxes to generate a potential and non-zero vev for moduli.

Outline of the talk

- 📌 Addressing **moduli stabilisation** problem
- 📌 Finding a **dS vacuum** in String Theory — based only on the **perturbative corrections**
- 📌 If found! — YES
- 📌 Examine their interesting cosmological implications— such as **Inflation**.

Type IIB effective Supergravity

Basic ingredients: Superpotential W and Kahler potential K .

The fluxes in type-IIB are: $F_3 = dC_2$ and $H_3 = dB_2$ and $G_3 = F_3 - SH_3$, giving us the **flux induced superpotential** of GVW type^a:

$$W_0 = \int \Omega_3(z_a) \wedge G_3$$

W -flatness conditions: $\mathcal{D}_{z^a} W = 0$, $\mathcal{D}_S W = 0$ \rightarrow (z^a, S) stabilised but **Kahler moduli not stabilised!**

Type -IIB theory's effective Kahler potential,

$$\mathcal{K}_0 = -\log(S + \bar{S}) - \log\left(-i \int \Omega_3 \wedge \bar{\Omega}_3\right) - 2 \log \mathcal{V}$$

Block diagonal form

The associated F-term scalar potential takes the following form,

$$V_F = e^{\mathcal{K}} \left(\sum_{I,J} \mathcal{K}^{I\bar{J}} \mathcal{D}_I W_0 \mathcal{D}_{\bar{J}} \bar{W}_0 - 3|W_0|^2 \right)$$

$I, J = S, z^a, T_i$

Dependence on the Kahler moduli drops out due to block diagonal form of Kahler metric

Road-map

Kahler moduli completely undetermined

$$\mathcal{K}_0 = -\log(S + \bar{S}) - \log\left(-i \int \Omega_3 \wedge \bar{\Omega}_3\right) - 3\log(T + \bar{T})$$

$$\mathcal{K}^{T\bar{T}} = \frac{(T + \bar{T})^2}{3} \quad \mathcal{D}_T W_0 = K_T W_0$$

$$V_{F_T} = e^{\mathcal{K}} \left(\mathcal{K}^{T\bar{T}} \mathcal{D}_T W_0 \mathcal{D}_{\bar{T}} \bar{W}_0 - 3|W_0|^2 \right) = 0 \quad \text{No scale structure}$$

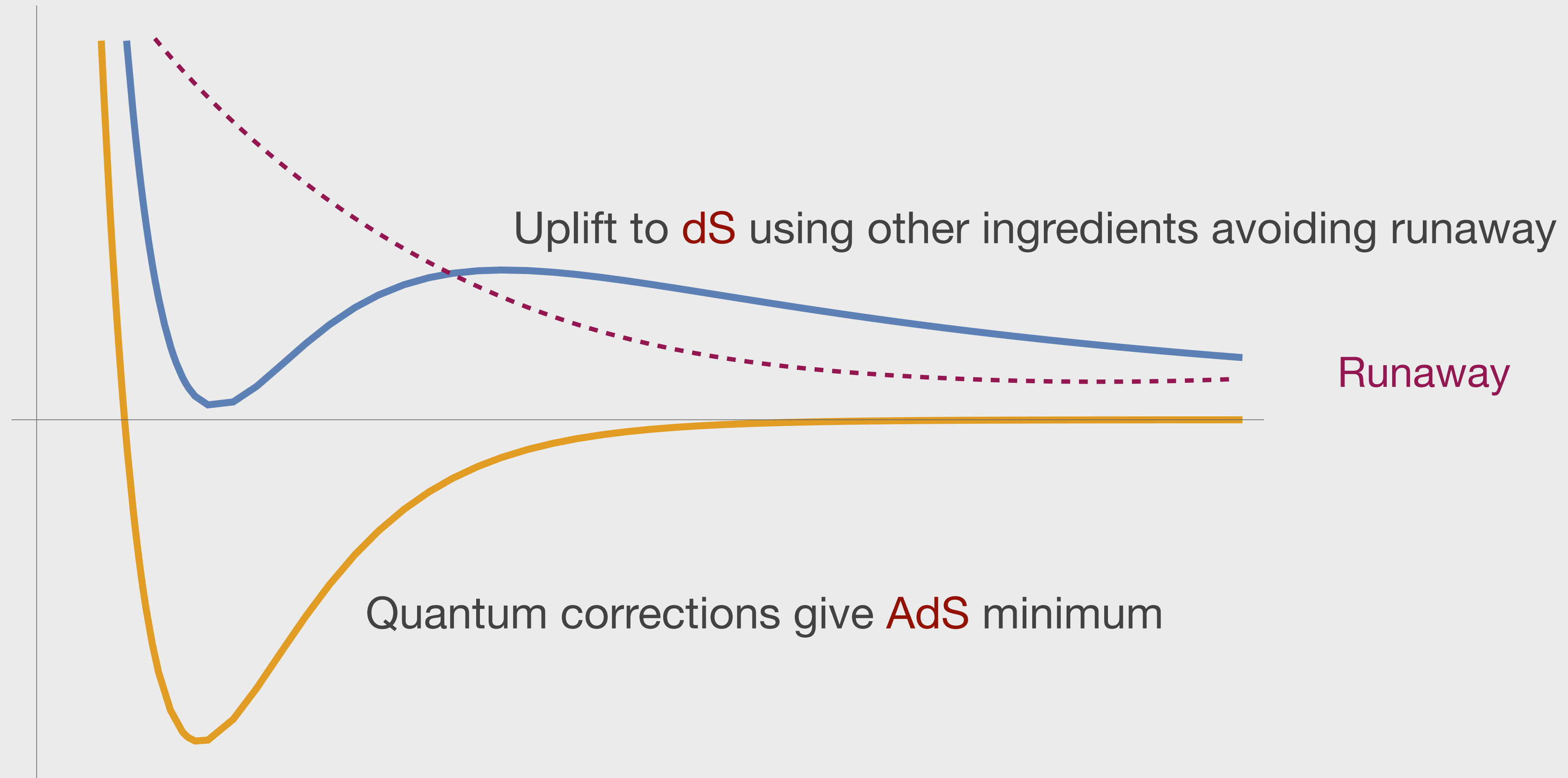
Two-step moduli stabilisation

- Stabilise the complex structure and axio-dilaton by the GVW superpotential.
- Engineering the appropriate geometric setup and calculate Kahler moduli-dependent **quantum corrections**.

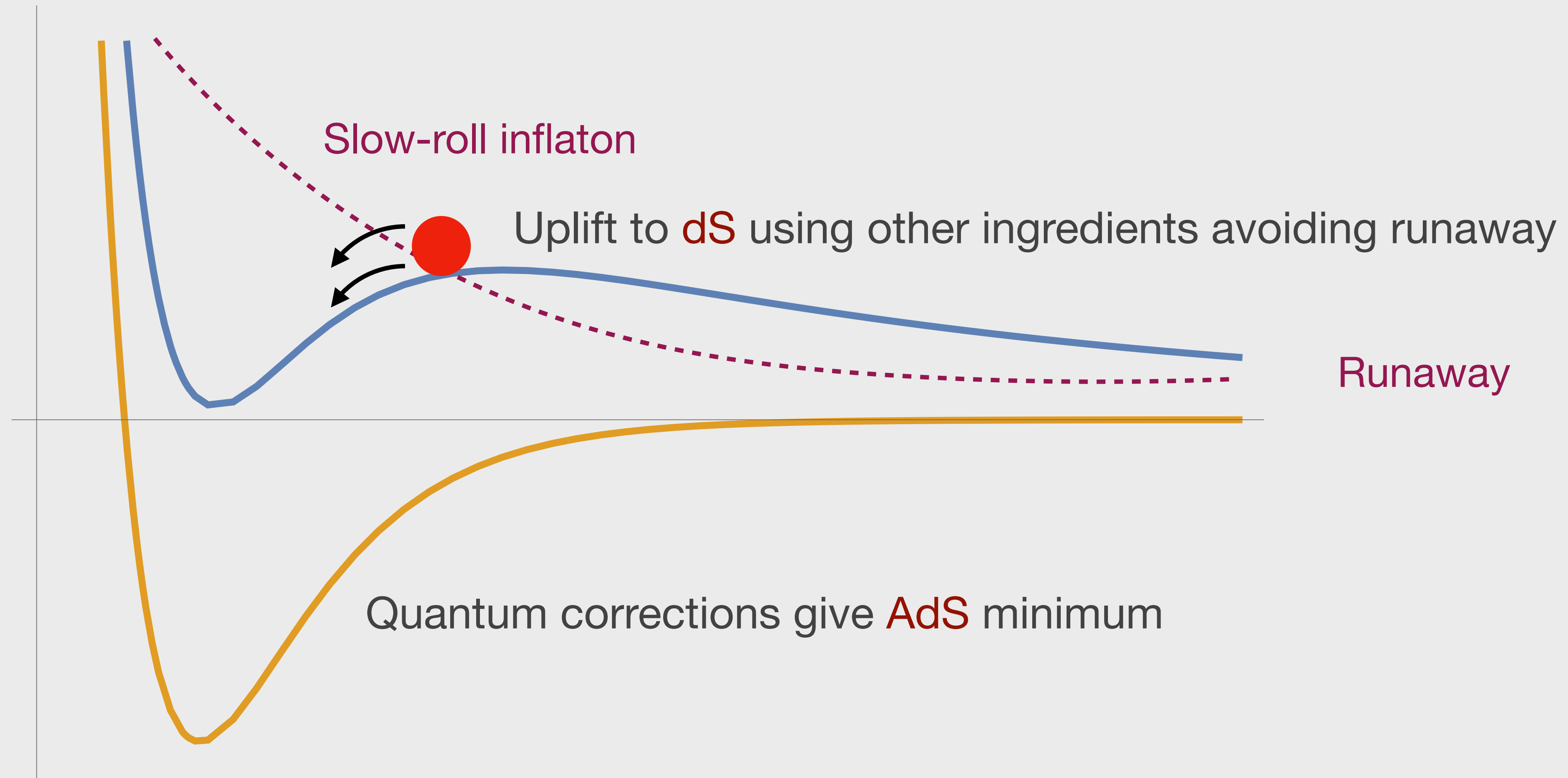
Generate a non-zero vev for Kahler moduli

$$V_{F_T} \neq 0$$

Goal - add quantum corrections



Goal - add quantum corrections and inflate with the lifted direction



The **Classification** of Quantum Corrections

Perturbative
corrections

□ In inverse string tension
 $\propto \alpha'/R^2$

α'^3 – corr

log – corr

higher derivative corr

□ In string coupling constant
 $\propto g_s$

KK 1-loop open string

winding 1-loop open string

Non-perturbative
corrections

□ D3-branes instantons

□ Gaugino condensation

Present focus

Addressing the stabilisation of KM in absence of NP effects

List of Corrections

α'^3 –corrections^a

$$K \propto -2 \log \left(\mathcal{V} + \frac{\xi(S + \bar{S})^{3/2}}{2} \right) = -2 \log \left(\mathcal{V} + \frac{\xi}{2g_s^{3/2}} \right)$$

Log-loop corrections^{b,c,d}

$$K = -2 \log(\mathcal{V} + \eta \log \mathcal{V})$$

Higher derivative corrections^{e,f,g}

$$V_{F_4} = -\frac{-\lambda k^2 |W_0|^4}{g_s^{3/2} \mathcal{V}^4} \Pi_\alpha t^\alpha, \quad \text{where } \Pi_\alpha = \int_X c_2 \wedge D_\alpha$$

String loops^{h,i,k}

$$V_{g_s}^W = -2k \frac{|W_0|^2}{\mathcal{V}^3} \sum_\alpha \frac{C_\alpha^W}{t_\cap^\alpha}$$

(a) Becker, Becker, Haack, Louis' 02

(b) Green, Vanhove'97 (c) Antoniadis, Ferrara, Minasian, Narain' 97 (d) Kiritsis, Pioline'97

(e) Ciupke, Louis, Westphal, (f) Green, Mayer, Weissenbacher, (g) Cicoli, Ciupke, de Alois, Muia

(h) Ciupke, Louis, Westphal, (i) Green, Mayer, Weissenbacher, (k) Cicoli, Ciupke, de Alois, Muia

Large Volume Scenario

Key ingredients: inclusion of α'^3 corrections in the Kahler potential

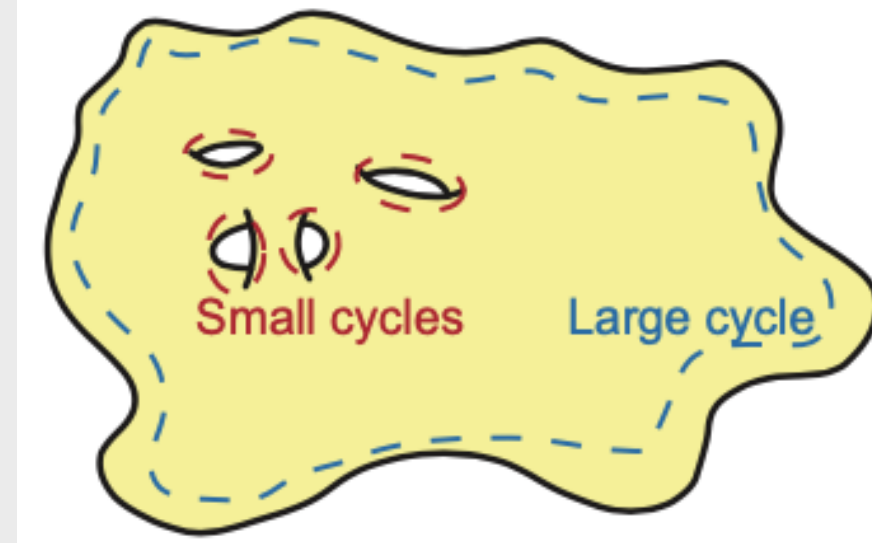
Non-perturbative **instanton corrections** in the superpotential

$$K = -2 \log \left(\mathcal{V} + \frac{\xi}{2g_s^{3/2}} \right) - \log \left(i \int \Omega_3 \wedge \bar{\Omega}_3 \right) - \log(S + \bar{S})$$

$$W = \int G_3 \wedge \Omega_3 + \sum_i A_i e^{-a_i T_i}$$

Non-perturbative
effect

$$\mathcal{V} = \tau_L^{3/2} - \sum_{i=1}^{N_{\text{small}}=h^{1,1}-1} \tau_s^{3/2}$$



$$V_{\text{LVS}} \simeq \frac{\alpha_1}{\mathcal{V}^3} - \frac{\alpha_2 \tau_s}{\mathcal{V}^2} e^{-a_s \tau_s} + \frac{\alpha_3 \sqrt{\tau_s}}{\mathcal{V}} e^{-2a_s \tau_s} \quad \langle \mathcal{V} \rangle \simeq \frac{\alpha_2 \sqrt{\langle \tau_s \rangle}}{2\alpha_3} e^{a_s \langle \tau_s \rangle}, \quad \langle \tau_s \rangle \simeq \hat{\xi}^{2/3} \left(\frac{9 k_{SSS}}{8} \right)^{1/3}.$$

Large Volume Scenario



Global embedding

Perturbative Large Volume Scenario ^(a,b,c)

Key ingredients: inclusion of α'^3 corrections + higher orders in the Kahler potential
NO Non-perturbative instanton corrections in the superpotential

$$K = -2 \log \left(\mathcal{V} + \frac{\xi}{2g_s^{3/2}} \right) - \log \left(i \int \Omega_3 \wedge \bar{\Omega}_3 \right) - \log(S + \bar{S})$$

$$W = \int G_3 \wedge \Omega_3 + \sum_i A_i e^{-\alpha T_i}$$

$$\mathcal{V} = \tau_L^{3/2} - \sum_{i=1}^{N_{\text{small}}=h^{1,1}-1} \tau_s^{3/2}$$

$$\mathcal{V} = n_0 t^1 t^2 t^3 = \frac{1}{\sqrt{n_0}} \sqrt{\tau_1 \tau_2 \tau_3}, \quad \tau_\alpha = \partial_{t^\alpha} \mathcal{V}, \quad n_0 = 2$$

Perturbative Large Volume Scenario

In the large volume limit with **BBHL corrections plus log-loop effects**

$$K = -2 \left(\mathcal{V} + \frac{\xi}{2g_s^{3/2}} + g_s^{1/2} \frac{\zeta(2)}{\zeta(3)} \xi (\ln \mathcal{V} - 1) \right) \quad V_f \approx C_1 \frac{\xi - 4\eta + 2\eta \log(\mathcal{V})}{\mathcal{V}^3}$$

Features:

■ Minimum exists for $\eta < 0$, $C_1 \propto W_0^2$

■ Stabilisation at large volume:

$$\mathcal{V}_{min} = e^{\frac{7}{3} + \frac{\xi}{2|\eta|}} \sim e^{\frac{1}{g_s^2}}$$

$g_s = 0.2 \Rightarrow \langle \mathcal{V} \rangle = 95593.3$
$g_s = 0.1 \Rightarrow \langle \mathcal{V} \rangle = 7 \cdot 10^{16}$

■ For F-term potential, AdS-minimum

$$(V_F)_{min} \propto \frac{\eta}{\mathcal{V}^3} < 0$$

Perturbative Large Volume Scenario

In the large volume limit with **BBHL corrections plus log-loop effects+D-terms**

$$K = -2 \left(\mathcal{V} + \frac{\xi}{2g_s^{3/2}} + g_s^{1/2} \frac{\zeta(2)}{\zeta(3)} \xi (\ln \mathcal{V} - 1) \right) \quad V_f \approx \mathcal{C}_1 \frac{\xi - 4\eta + 2\eta \log(\mathcal{V})}{\mathcal{V}^3} + \sum_{\alpha=1}^3 \frac{d_\alpha}{\tau_\alpha^3}$$

In terms of canonical normalised fields,

$$\varphi^\alpha = \frac{1}{\sqrt{2}} \ln \tau_\alpha, \quad \phi^1 = \frac{1}{\sqrt{3}} (\varphi^1 + \varphi^2 + \varphi^3) = \sqrt{\frac{2}{3}} \ln(\sqrt{n_0} \mathcal{V}),$$
$$\phi^2 = \frac{1}{\sqrt{2}} (\varphi^1 - \varphi^2) \quad \phi^3 = \frac{1}{\sqrt{6}} (\varphi^1 + \varphi^2 - 2\varphi^3)$$

Perturbative Large Volume Scenario - a dS with D-terms and no NP effects

Extremisation conditions yield the following relations

$$a_1 = e^{-\sqrt{\frac{3}{2}}\langle\phi\rangle} \left(\sqrt{\frac{3}{2}}\langle\phi^1\rangle - a_2 \right), \quad \langle\phi^2\rangle = \frac{1}{6} \left(\frac{d_1}{d_2} \right), \quad \langle\phi^3\rangle = \frac{1}{6\sqrt{3}} \left(\frac{d_1 d_2}{d_3^2} \right)$$

$$a_1 \equiv -\frac{(d_1 d_2 d_3)^{1/3}}{n_0^{3/2} \eta \mathcal{C}_1} \geq 0, \quad a_2 = -\frac{\xi}{2\eta} + \frac{7}{3} + \frac{1}{2} \ln n_0 > 0$$

To ensure single-field inflation and a dS minimum imposes the following constraint

$$\mathcal{R}_{\text{hierarchy}} \equiv \frac{m_{\phi^1}^2}{m_{\phi^\alpha}^2} = \frac{\left(1 + a_2 - \sqrt{\frac{3}{2}}\langle\phi^1\rangle\right)}{2 \left(\sqrt{\frac{3}{2}}\langle\phi^1\rangle - a_2\right)} \ll 1, \quad \alpha = \{2, 3\}$$

$$\frac{2}{3} + a_2 \leq \sqrt{\frac{3}{2}}\langle\phi^1\rangle < 1 + a_2$$

$$\langle V_0 \rangle = -\eta n_0^{3/2} \mathcal{C}_1 e^{-3\sqrt{\frac{3}{2}}\langle\phi^1\rangle} \left(\sqrt{\frac{3}{2}}\langle\phi^1\rangle - a_2 - \frac{2}{3} \right)$$

$$M_1 = \frac{1}{2} M_2 = \frac{1}{2} M_3$$

Perturbative Large Volume Scenario - a dS with D-terms and no NP effects

Re-defining some parameters — x takes care of the uplifting now on

$$a_1 \equiv e^{-a_2 - 1 - x}, \quad \sqrt{\frac{3}{2}}\phi^1 - a_2 - 1 = \sqrt{\frac{3}{2}}\phi$$

$$V_{\text{inf}} = -\tilde{\mathcal{B}}e^{-3\sqrt{\frac{3}{2}}\phi} \left(\sqrt{\frac{3}{2}}\phi - \frac{3}{2}e^{-x + \sqrt{\frac{3}{2}}\phi} + \frac{4}{3} \right), \quad \tilde{\mathcal{B}} = \tilde{\mathcal{B}}(|W_0|, g_s) > 0$$

The potential features several extremas:

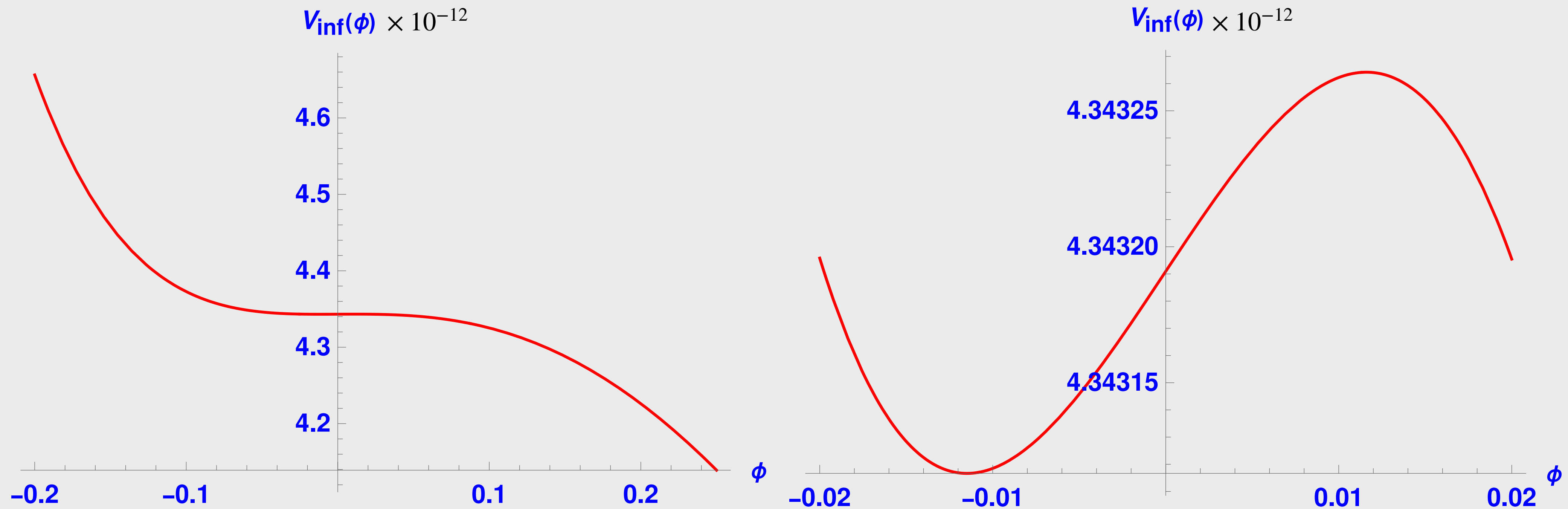
$$\phi_{\text{min}} = -\sqrt{\frac{2}{3}} (1 + \mathcal{W}_0[-e^{-1-x}]), \quad \phi_{\text{max}} = -\sqrt{\frac{2}{3}} (1 + \mathcal{W}_{-1}[-e^{-1-x}]),$$

$$\phi_{\text{inflec1}} = -\sqrt{\frac{2}{3}} \left(\frac{2}{3} + \mathcal{W}_0 \left[-\frac{2}{3} e^{-\frac{2}{3}-x} \right] \right), \quad \phi_{\text{inflec2}} = -\sqrt{\frac{2}{3}} \left(\frac{2}{3} + \mathcal{W}_{-1} \left[-\frac{2}{3} e^{-\frac{2}{3}-x} \right] \right).$$

A benchmark model

The inflationary observables are the following:

$$V_{\text{inf}} = -\tilde{\mathcal{B}}e^{-3\sqrt{\frac{3}{2}}\phi} \left(\sqrt{\frac{3}{2}}\phi - \frac{3}{2}e^{-x+\sqrt{\frac{3}{2}}\phi} + \frac{4}{3} \right), \quad \tilde{\mathcal{B}} = \tilde{\mathcal{B}}(|W_0|, g_s) > 0$$



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$$\begin{aligned} x &= 0.0001, & a_2 &= 6, & \tilde{\mathcal{B}} &= 7.56 \times 10^{-12}, \\ -\chi(\text{CY})|W_0|^2 &\simeq 1.23, & d &= (d_1 d_2 d_3)^{1/3} = 2.2735 \times 10^{-6} n_0, \\ \langle \phi^1 \rangle &= 5.70398, & g_s &= 0.316, & \langle \tau_\alpha \rangle &= 105.349, \\ \langle \mathcal{V} \rangle &\simeq 1081.31, & \frac{m_{\phi^1}}{m_{\phi^2}} &= 0.0844882 = \frac{m_{\phi^1}}{m_{\phi^3}}. \end{aligned}$$

$$\begin{aligned} \epsilon_V^* &\simeq 2.42 \times 10^{-6}, & r &= \epsilon_V^* = 3.88 \times 10^{-5}, \\ \eta_V^* &= -0.02, & n_s^* - 1 &= -0.04 \end{aligned}$$

Robustness of Perturbative Large Volume Scenario

In the presence of **BBHL corrections plus log-loop effects plus string loop and higher derivative corrections** — after integrating out the two moduli, the potential becomes

$$V_{\text{inf}}(\phi) = -\tilde{\mathcal{B}}e^{-3\sqrt{\frac{3}{2}}\phi} \left(\sqrt{\frac{3}{2}}\phi - \frac{3}{2}e^{-x+\sqrt{\frac{3}{2}}\phi} + \frac{4}{3} \right) + \tilde{\mathcal{C}}_2e^{-5\sqrt{\frac{2}{3}}\phi} + \tilde{\mathcal{C}}_3e^{-\frac{11}{\sqrt{6}}\phi}$$

$$\tilde{\mathcal{B}} \equiv \tilde{\mathcal{B}}(|W_0|, g_s) = -\kappa \frac{\chi(\text{CY}) \sqrt{g_s} |W_0|^2 e^{-10 - \frac{9\zeta[3]}{g_s^2 \pi^2}}}{64\pi} > 0,$$

$$\tilde{\mathcal{C}}_2 = \frac{15}{4} \kappa \mathcal{C}_w |W_0|^2 n_0^{1/3} e^{-\frac{100}{9} - \frac{10\zeta[3]}{g_s^2 \pi^2}}, \quad \tilde{\mathcal{C}}_3 = -\frac{72 \kappa^2 \lambda |W_0|^4}{g_s^{3/2} n_0^{1/3}} e^{-\frac{110}{9} - \frac{11\zeta[3]}{g_s^2 \pi^2}},$$

Robustness of Perturbative Large Volume Scenario

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$$\chi(\text{CY}) = -224, \quad n_0 = 2, \quad g_s = \frac{1}{3}, \quad x = 10^{-4},$$

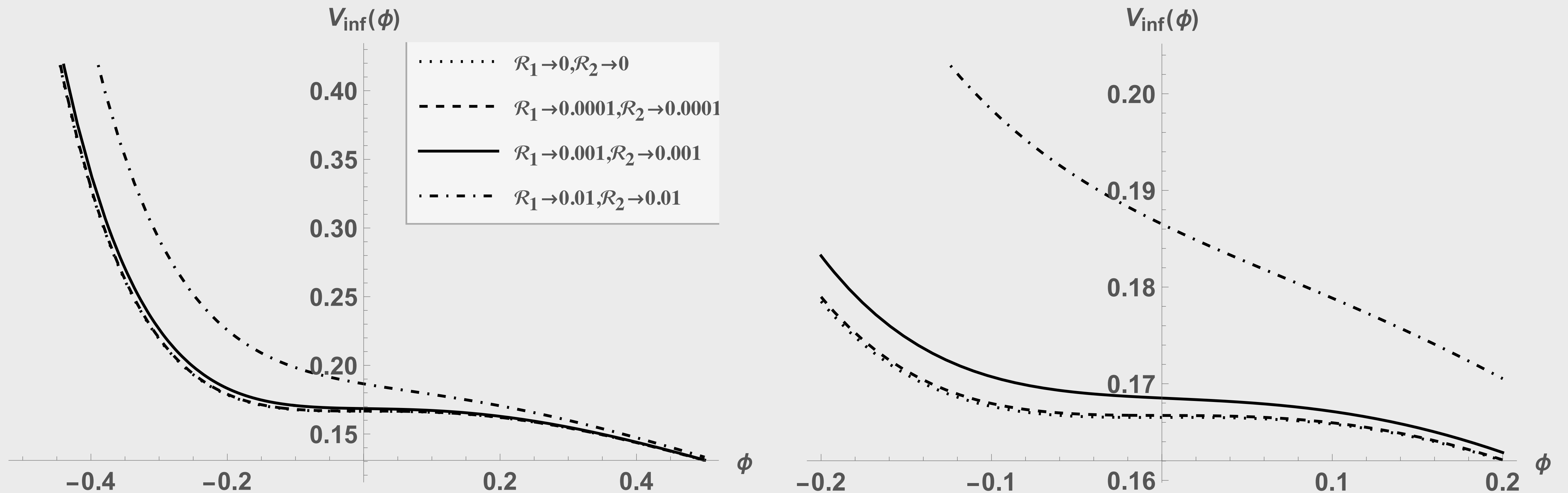
$$\tilde{\mathcal{B}} = 1.51694 \times 10^{-9} |W_0|^2, \quad \tilde{\mathcal{C}}_2 = 1.22570 \times 10^{-9} \mathcal{C}_w |W_0|^2,$$

$$\tilde{\mathcal{C}}_3 = -8.47389 \times 10^{-9} \lambda |W_0|^4.$$

Robustness of Perturbative Large Volume Scenario

$$V_{\text{inf}}(\phi) = -\tilde{\mathcal{B}}e^{-3\sqrt{\frac{3}{2}}\phi} \left(\sqrt{\frac{3}{2}}\phi - \frac{3}{2}e^{-x+\sqrt{\frac{3}{2}}\phi} + \frac{4}{3} \right) + \tilde{\mathcal{C}}_2e^{-5\sqrt{\frac{2}{3}}\phi} + \tilde{\mathcal{C}}_3e^{-\frac{11}{\sqrt{6}}\phi}$$

$$\mathcal{R}_1 = \frac{\tilde{\mathcal{C}}_2}{\tilde{\mathcal{B}}} = 0.80801 \mathcal{C}_w, \quad \mathcal{R}_2 = \frac{\tilde{\mathcal{C}}_3}{\tilde{\mathcal{B}}} = -5.58619 |W_0|^2 \lambda.$$



Robustness of Perturbative Large Volume Scenario

$$V_{\text{inf}}(\phi) = -\tilde{\mathcal{B}}e^{-3\sqrt{\frac{3}{2}}\phi} \left(\sqrt{\frac{3}{2}}\phi - \frac{3}{2}e^{-x+\sqrt{\frac{3}{2}}\phi} + \frac{4}{3} \right) + \tilde{\mathcal{C}}_2e^{-5\sqrt{\frac{2}{3}}\phi} + \tilde{\mathcal{C}}_3e^{-\frac{11}{\sqrt{6}}\phi}$$

$$\mathcal{R}_1 = \frac{\tilde{\mathcal{C}}_2}{\tilde{\mathcal{B}}} = 0.80801 \mathcal{C}_w, \quad \mathcal{R}_2 = \frac{\tilde{\mathcal{C}}_3}{\tilde{\mathcal{B}}} = -5.58619 |W_0|^2 \lambda.$$

$$W_0 = 0.038, \quad \mathcal{C}_w = 5 \cdot 10^{-5}, \quad \lambda = -10^{-4},$$

$$\tilde{\mathcal{B}} = 2.19046 \times 10^{-12}, \quad \tilde{\mathcal{C}}_2 = 8.84958 \times 10^{-17}, \quad \tilde{\mathcal{C}}_3 = 1.76692 \times 10^{-18},$$

$$\langle \phi \rangle = -0.00841545, \quad \langle \tau_\alpha \rangle = 103.409, \quad \langle \mathcal{V} \rangle = 743.568, \quad \langle V \rangle = 3.64835 \times 10^{-13},$$

$$m_\phi^2 = 0.015697 m_{\phi_\alpha}^2, \quad m_{\phi_\alpha}^2 = 6.70767 \times 10^{-12} \quad \text{for } \alpha \in \{2, 3\},$$

$$\phi^* = 0.000567702, \quad \epsilon_V^* = 7.05464 \times 10^{-7}, \quad \eta_V^* = -0.0199979, \quad N_e \simeq 97,$$

$$P_s = 2.1 \times 10^{-9}, \quad n_s = 0.96, \quad r = 1.13 \times 10^{-5}$$

Global Embedding of Fibre Inflation

The Model: $h^{1,1} = 3$ and a toroidal like volume form $\mathcal{V} = \sqrt{\frac{\tau_1 \tau_2 \tau_3}{k_{123}}}$

The ansatz: $\tau_1 = q\tau_2, \quad q = 1$

$$\tau_3 = \tau_f = e^{2\varphi/\sqrt{3}}, \quad \varphi = \langle \varphi \rangle + \phi$$

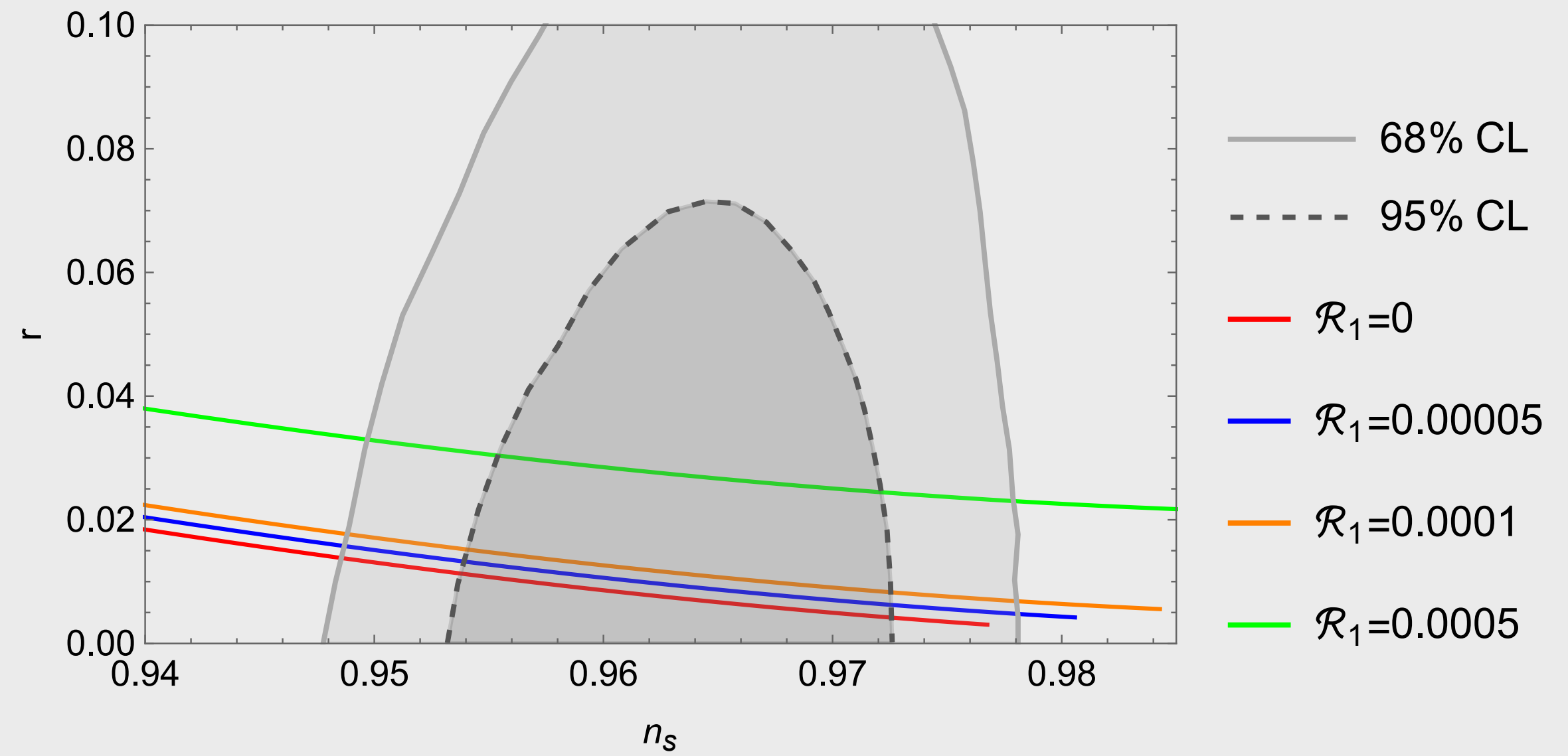
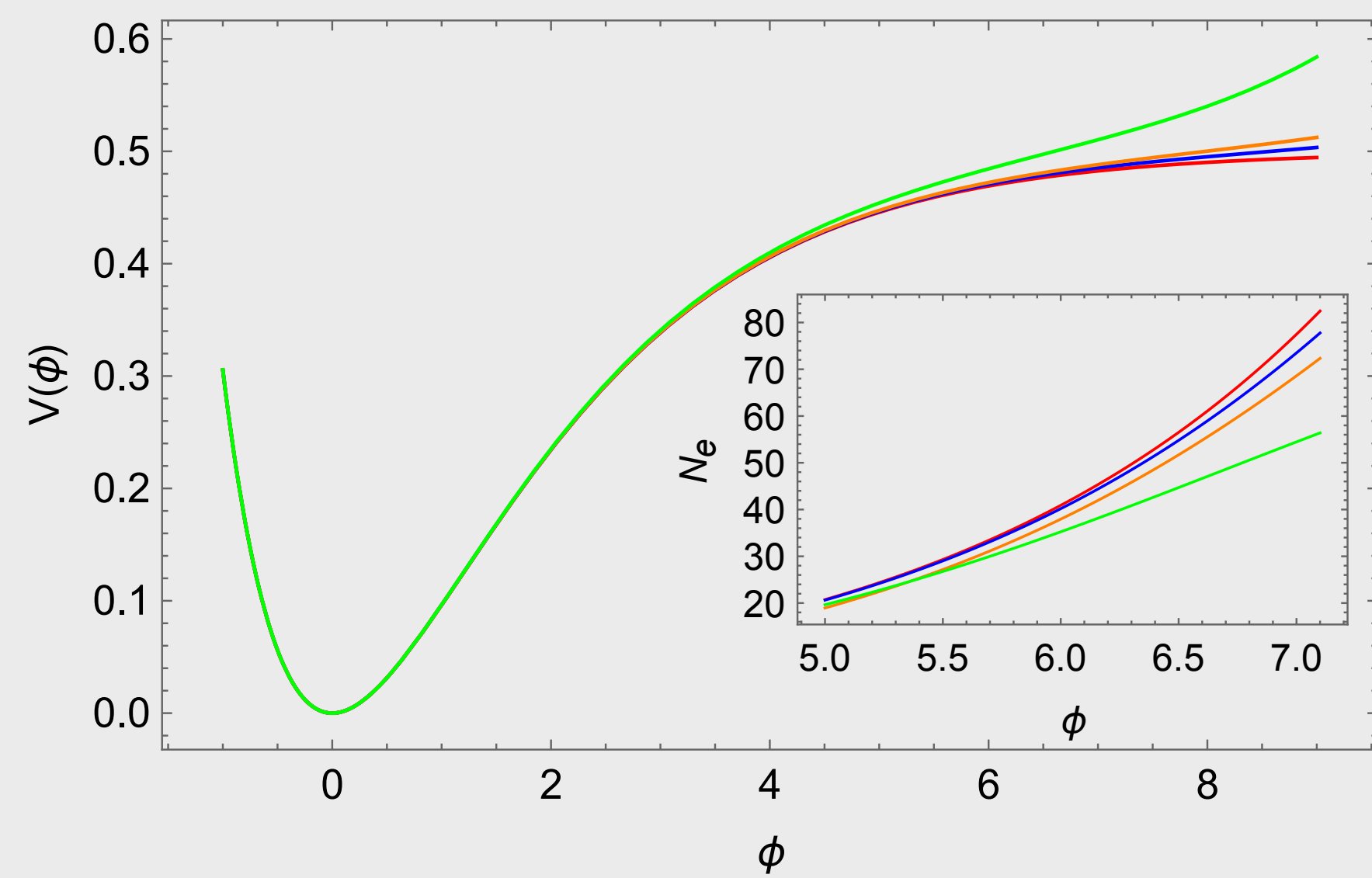
The Potential: $V = \mathcal{C}_0 \left(\mathcal{C}_{\text{up}} + \mathcal{R}_0 e^{-\gamma\phi} - e^{-\frac{\gamma}{2}\phi} + \mathcal{R}_1 e^{\frac{\gamma}{2}\phi} + \mathcal{R}_2 e^{\gamma\phi} \right),$

The parameters: $\mathcal{C}_{\text{up}} = 1 - \mathcal{R}_0 - \mathcal{R}_1 - \mathcal{R}_2, \quad \mathcal{C}_0 = \frac{\sqrt{2}\mathcal{C}_2\tilde{\mathcal{C}}_w}{\langle \mathcal{V} \rangle^3 e^{\frac{\gamma}{2}\langle \varphi \rangle}}, \quad \mathcal{R}_0 = \frac{\mathcal{C}_3 e^{-\frac{\gamma}{2}\langle \varphi \rangle}}{\sqrt{2}\mathcal{C}_2\tilde{\mathcal{C}}_w}, \quad \frac{\mathcal{R}_1}{\mathcal{R}_0} = \frac{\sqrt{2}e^{\sqrt{3}\langle \varphi \rangle}}{\langle \mathcal{V} \rangle},$

$$\gamma = 2/\sqrt{3}, \quad \frac{\mathcal{R}_2}{\mathcal{R}_0} = \frac{\mathcal{C}_2\mathcal{C}_w\mathcal{C}_3 e^{2\gamma\langle \varphi \rangle}}{\mathcal{C}_3 \langle \mathcal{V} \rangle} \left[1 + \hat{\mathcal{C}}_w \left(1 + \frac{e^{\sqrt{3}(\phi + \langle \varphi \rangle)}}{\langle \mathcal{V} \rangle \sqrt{2}} \right)^{-1} \right],$$

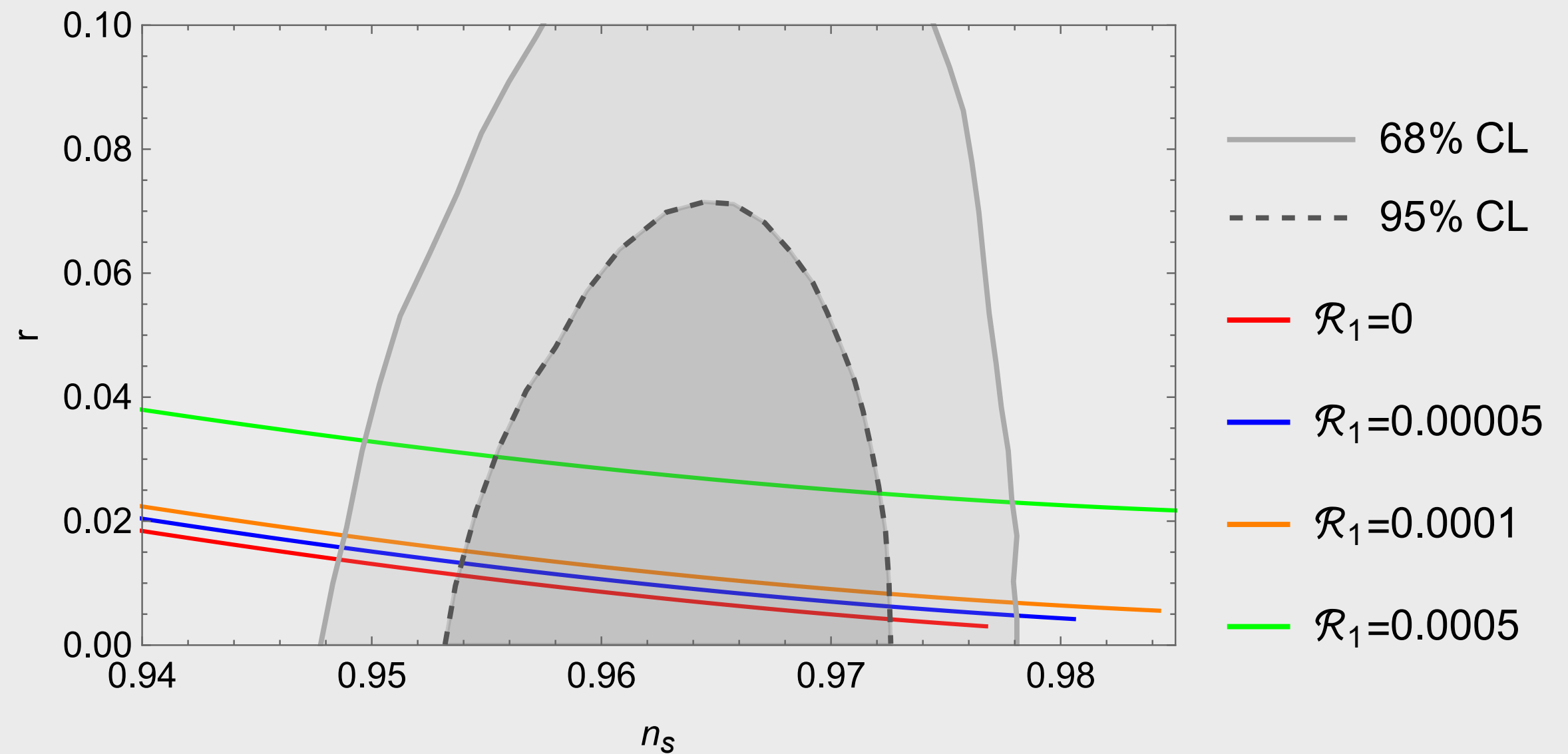
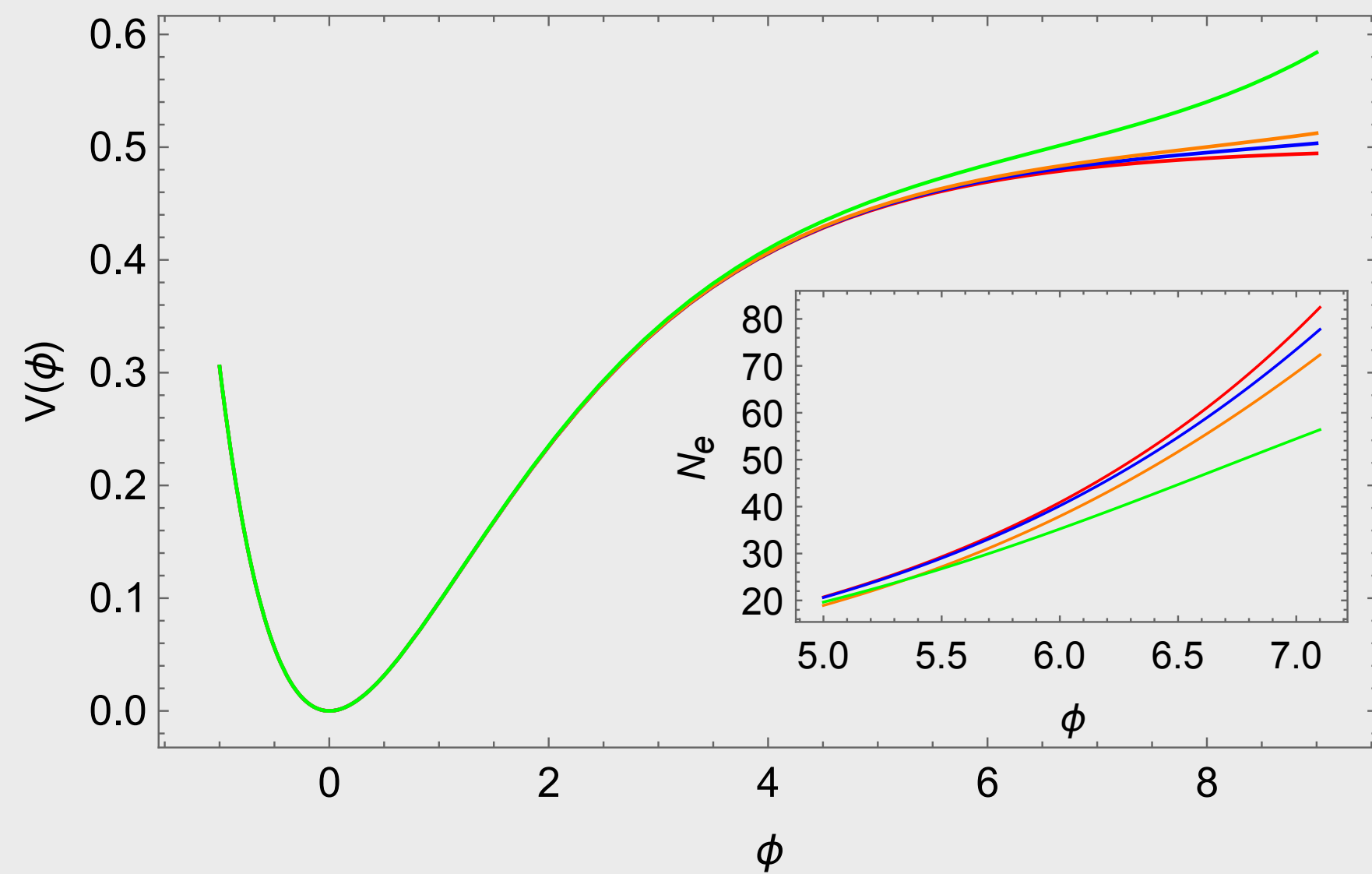
Global Embedding of Fibre Inflation

The Potential:
$$V = C_0 \left(C_{\text{up}} + \mathcal{R}_0 e^{-\gamma\phi} - e^{-\frac{\gamma}{2}\phi} + \mathcal{R}_1 e^{\frac{\gamma}{2}\phi} + \mathcal{R}_2 e^{\gamma\phi} \right),$$



Global Embedding of Fibre Inflation

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$$e^{\frac{1}{2}K_{\text{cs}}} |W_0| = 145, \quad \tilde{C}_w = 5.3, \quad C_{w3} = 10^{-2}, \quad \hat{C}_w = 0, \quad |\lambda| = 10^{-4}, \quad \langle \varphi \rangle = 0.1$$

$$g_s = 0.3, \quad \langle \mathcal{V} \rangle \simeq 15000, \quad C_0 \simeq 5.26 \cdot 10^{-10}, \quad \mathcal{R}_0 \simeq 0.462, \quad \mathcal{R}_1 \simeq 5.18 \cdot 10^{-5}, \quad \mathcal{R}_2 \simeq 1.06 \cdot 10^{-7},$$

$$N_e^* \simeq 50, \quad P_s^* \simeq 2.1 \cdot 10^{-9}, \quad n_s^* \simeq 0.966, \quad r^* \simeq 8 \cdot 10^{-3}, \quad \phi^* \simeq 6.22, \quad \phi_{\text{end}} \simeq 0.92.$$

Conclusion

- ☑ Presented the **global embedding** of the inflationary model in the context of **perturbative LVS** in a K3 fibred CY — global toroidal like structure.

Moduli stabilisation is addressed in presence of only perturbative - **BBHL+ log-loop** type corrections

Robustness of inflationary model is checked — **volume inflation** is studied — **small field inflation** — **satisfying distance conjecture**.

Improvement over previous constructions — is it again robust against another type of log-loop effects? — what are the implications of water fall fields?

- ☑ Global embedding of **fibre inflation** — in absence of exceptional del-pezzo divisor — devoid of non-perturbative effects.
- ☑ Viable inflation **without** a geometric **Kahler cone bound on the field excursion**.

Thank you!

Fibre Inflation revisited

Features:

- ◆ The CYs are “**weak**” **Swiss-cheese** which have $h^{1,1} = 3$

$$\mathcal{V} = \lambda_f \tau_b \sqrt{\tau_f} - \lambda_s \tau_s^{3/2}$$

- ◆ Overall volume and τ_s are fixed by standard LVS, leaving τ_f as a **flat direction** — serves as inflaton.
- ◆ Inclusion **string-loop corrections lifts** the τ_f direction — giving the leading order scalar potential as

$$V(\tau_f) = V_{up} + \frac{|W_0|^2}{\mathcal{V}^2} \left(\frac{B_1}{\tau_f^2} - \frac{B_2}{\mathcal{V} \sqrt{\tau_f}} + \frac{B_3 \tau_f}{\mathcal{V}^2} \right),$$

V_{up} depends on the uplifting and B_i s depend on CS moduli.

Global Embedding of Fibre Inflation

The Model: $h^{1,1} = 4$ with a dP and a toroidal like volume form $\mathcal{V} = \sqrt{\frac{\tau_1 \tau_2 \tau_3}{k_{123}}} - \lambda_s \tau_s^{3/2}$

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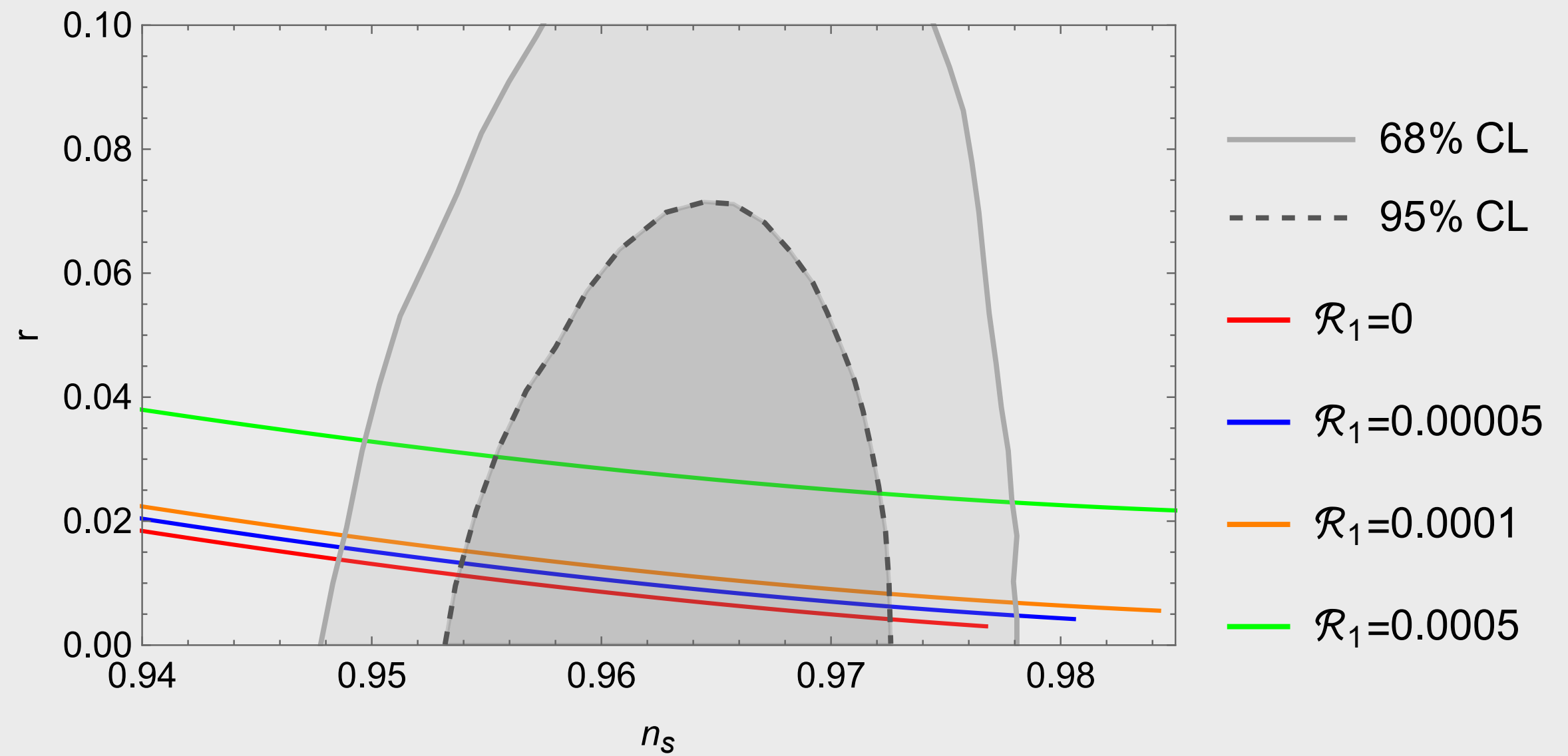
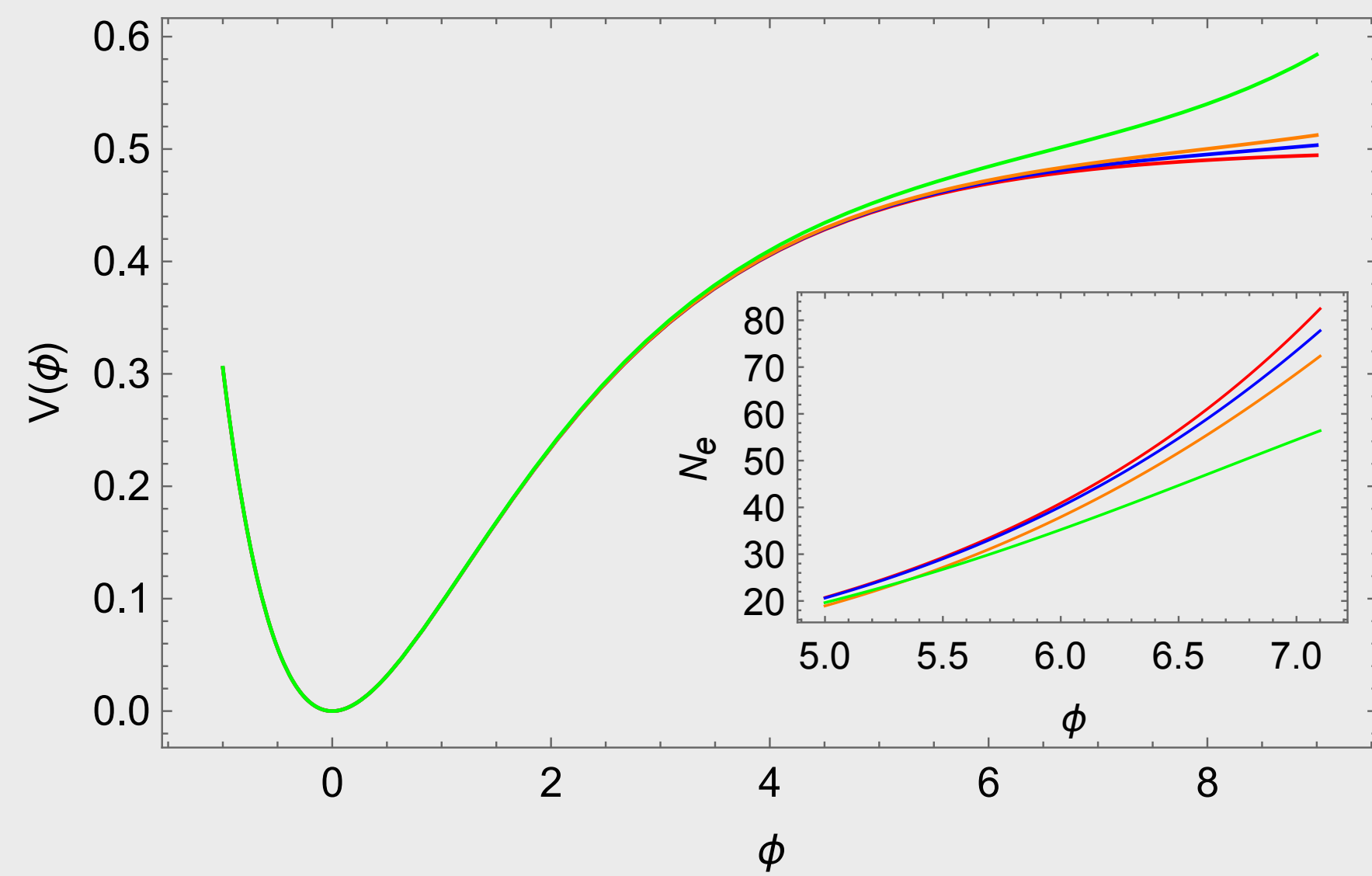
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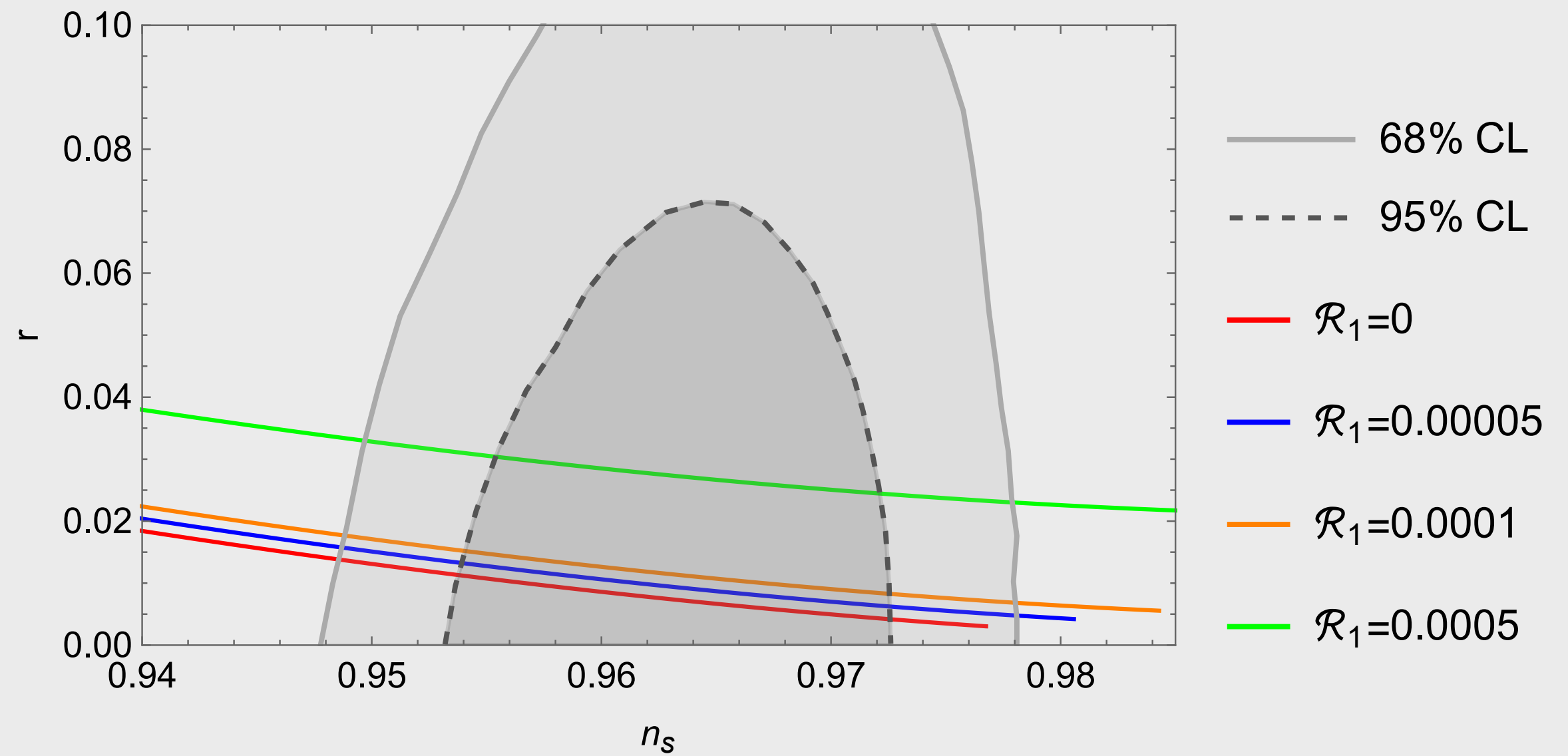
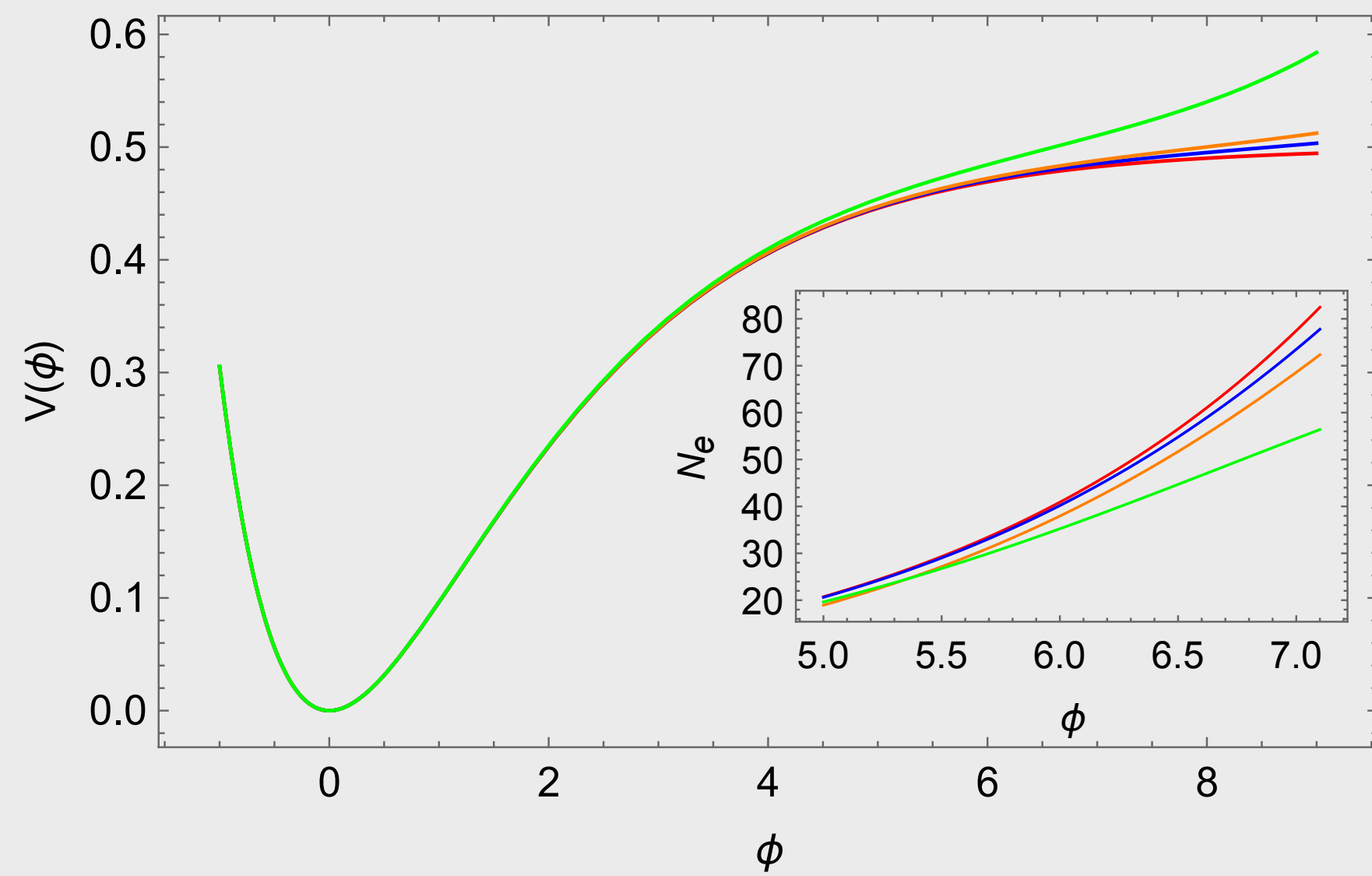
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The **Source** of Quantum Corrections (perturbative)

The **tree-level 10D** action of type IIB supergravity action is given by

$$S_{IIB} = \frac{1}{k_{10}^2} \int d^{10}x \sqrt{-g} [\mathcal{L}_{NSNS} + \mathcal{L}_{RR} + \mathcal{L}_{CS}] + S_{\text{loc}}$$

The corrections from **higher derivatives** operators,

$$S_{IIB} = S_{0,\text{tree}} + (\alpha')^3 S_{0,(3)} + \dots + (\alpha')^n S_{0,(n)} + S_{CS,\text{tree}} + \\ + S_{\text{loc},\text{tree}} + (\alpha')^2 S_{\text{loc},(2)} + \dots + ((\alpha')^n) S_{\text{loc},(n)}$$

$$S_{CS,\text{tree}} = \frac{1}{2k_{10}^2} \int d^{10}x \sqrt{-g} \mathcal{L}_{CS}$$

$$(\alpha')^3 S_{0,(3)} \sim \frac{1}{k_{10}^2} \int d^{10}x \sqrt{-g} [\mathcal{R}^4 + \mathcal{R}^3 (G_3 G_3 + G_3 \bar{G}_3 + \bar{G}_3 \bar{G}_3 + F_5^2 + (\nabla \tau)^2 + \\ + \mathcal{R}^2 (G_3^4 + G_3^2 + \bar{G}_3^2 + \dots + (\nabla G_3)^2 + (\nabla F_5)^2 + \dots) + \\ + \mathcal{R} (G_3^6 + \dots + G_3^2 (\nabla G_3)^2 + \dots) + G_3^8 + \dots]$$

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How does the potential arise?

A 5-d scalar field compactified on $\mathcal{M} = \mathbb{R}^4 \times S^1$ with $x^5 \in [0, 2\pi R)$ splits into an infinite number of scalar fields.

The action:
$$S_5 = \int d^5 X \frac{1}{2} (\partial_M \Phi) (\partial^M \Phi) \quad X \equiv (x^\mu, y)$$

The EOM:
$$\partial_M \partial^M \Phi = \partial_\mu \partial^\mu \phi(x) + \partial_5 \partial^5 \phi(y) = 0$$

Since the 5d scalar is compactified, we can expand the scalar field into Fourier series:

$$\Phi(X) = \sum_{n=0}^{\infty} \phi_n(x) e^{\frac{iny}{R}}$$

The action:

$$S = 2\pi R \int d^4 x \left[\frac{1}{2} (\partial \phi(x))^2 + V(\phi(x)) + \frac{1}{2} \sum_{n=1}^{\infty} (\partial \phi_n(y))^2 + m_n^2 \phi_n(y)^2 \right], \quad m_n = n/R$$

α'^3 -corrections^a

$$S \propto \int d^{10}x \sqrt{-g} e^{-2\phi} (R + 4(\partial\phi)^2 + \alpha'^3 c_1 J_0) \propto f(\mathcal{R}^4)$$

↓
↓
↘

Dilation Chern class

This gives rise to a correction in the Kahler potential

$$K \propto -2 \log \left(\mathcal{V} + \frac{\xi(S + \bar{S})^{3/2}}{2} \right) = -2 \log \left(\mathcal{V} + \frac{\xi}{2g_s^{3/2}} \right)$$

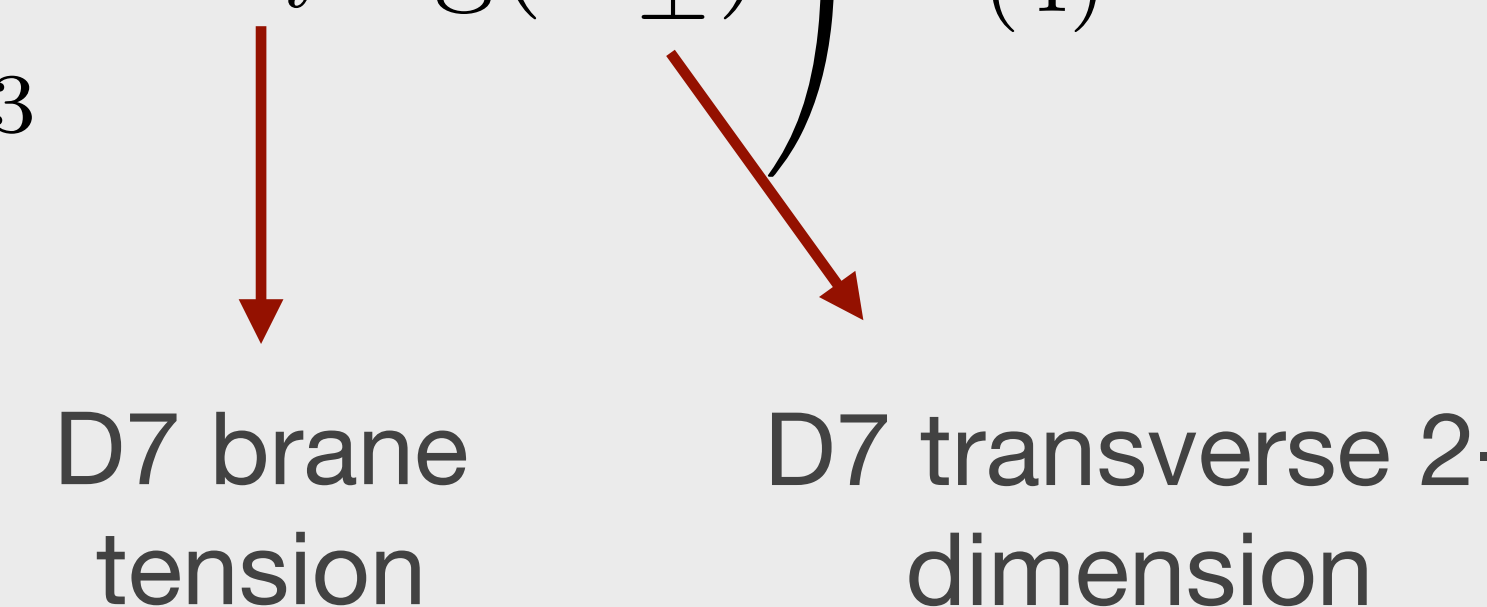
6D volume: $\mathcal{V} = \frac{1}{3!} \int_X J \wedge J \wedge J = \frac{1}{6} k_{ijk} t^i t^j t^k$, where t^i : volume of 2-cycles

$$\xi = -\frac{\zeta(3)\chi(X)}{4(2\pi)^3} \text{ where } \chi(X) : \text{Euler characteristics} \Rightarrow \chi(X) = 2(h^{1,1} - h^{2,1})$$

Log-loop corrections

Correction sourced via fourth power of curvature and KK-exchange induce correction to the Einstein-Hilbert term ^{a,b,c}

$$\propto \zeta(2)\chi(X) \int_{M_4} \left(1 + \sum_{i=1,2,3} e^{2\phi} \tau_i \log(R_{\perp}^i) \right) R_{(4)}$$



D7 brane tension D7 transverse 2-dimension

This again corrects the Kahler potential

$$K = -2 \log(\mathcal{V} + \eta \log \mathcal{V})$$

η depends on the Euler characteristics

Higher derivative corrections

$\alpha'^3 F^4$ terms from 10D $R^2 G_3^4$ term sources the following potential ^a

$$V_{F_4} = -\frac{-\lambda k^2 |W_0|^4}{g_s^{3/2} \mathcal{V}^4} \Pi_\alpha t^\alpha, \quad \text{where } \Pi_\alpha = \int_X c_2 \wedge D_\alpha$$

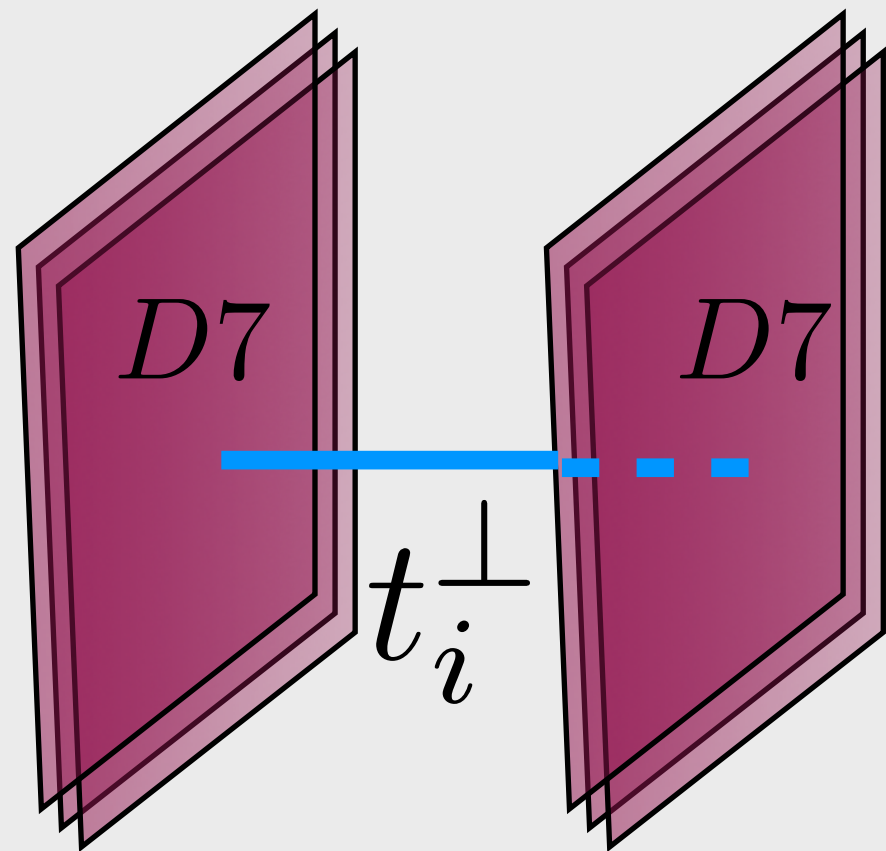
$|\lambda| \sim \mathcal{O}(10^{-4})$: From dimensional reduction ^b

$\lambda < 0$: Fix all LVS flat directions for arbitrary CY ^c

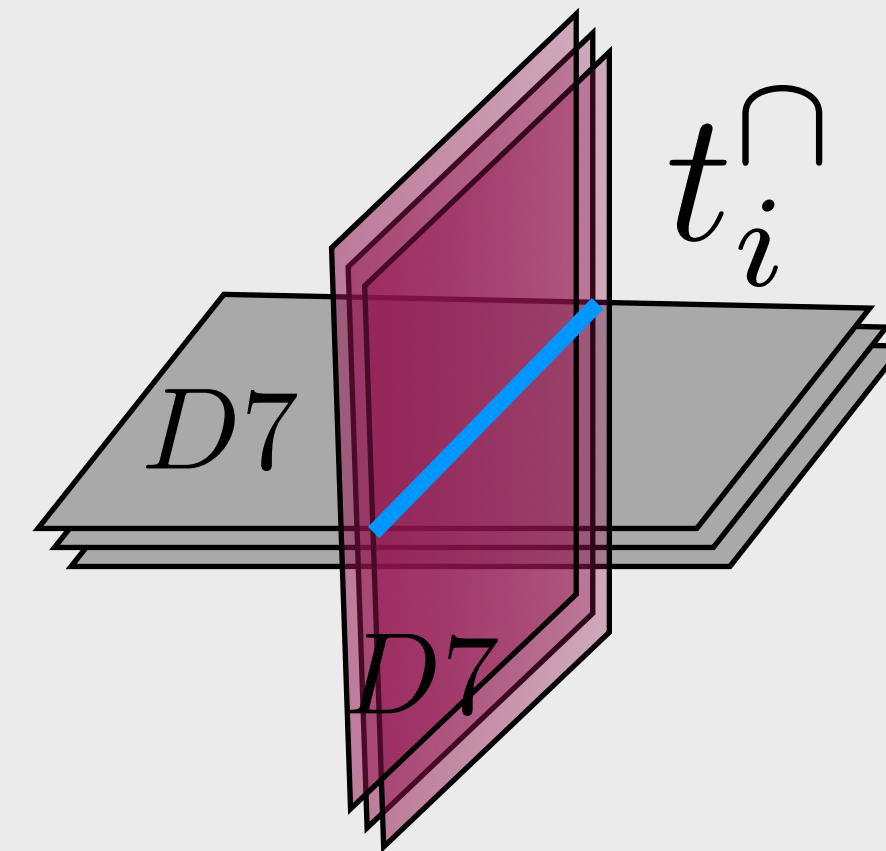
String loops

KK+ winding 1-loop open string corrections^{a,b,c} in terms of 2-cycle volume

$$K_{g_s}^{KKK} = g_s \sum_i \frac{C_i^{KKK} t_i^\perp}{\mathcal{V}}$$



$$K_{g_s}^W = \sum_i \frac{C_i^W}{\mathcal{V} t_i^\cap}$$



$$V_{g_s}^{KKK} = k g_s \frac{|W_0|^2}{16 \mathcal{V}^4} \sum_{\alpha, \beta} C_\alpha C_\beta (2 t_\alpha^\cap t_\beta^\cap - 4 \mathcal{V} k^{\alpha\beta})$$

$$V_{g_s}^W = -2k \frac{|W_0|^2}{\mathcal{V}^3} \sum_\alpha \frac{C_\alpha^W}{t_\alpha^\cap}$$

The Inflationary Potentials - a quick trailer

Global model — master formula — in terms of Kahler moduli

$$V_{\text{tot}} = \left[\frac{d_1}{\tau_1} \left(\frac{q_{12}}{\tau_2} + \frac{q_{13}}{\tau_3} \right)^2 + \frac{d_2}{\tau_2} \left(\frac{q_{21}}{\tau_1} + \frac{q_{23}}{\tau_3} \right)^2 + \frac{d_3}{\tau_3} \left(\frac{q_{31}}{\tau_1} + \frac{q_{32}}{\tau_2} \right)^2 \right] + \mathcal{C}_1 \left(\frac{\hat{\xi} - 4\hat{\eta} + 2\hat{\eta} \ln \mathcal{V}}{\mathcal{V}^3} \right) \\ + \frac{\mathcal{C}_2}{\mathcal{V}^4} \left(\tau_1 + \tau_2 + \tau_3 + \frac{\tau_1\tau_2}{2(\tau_1 + \tau_2)} + \frac{\tau_2\tau_3}{2(\tau_2 + \tau_3)} + \frac{\tau_3\tau_1}{2(\tau_3 + \tau_1)} \right) + \frac{\mathcal{C}_3}{\mathcal{V}^3} \left(\frac{1}{\tau_1} + \frac{1}{\tau_2} + \frac{1}{\tau_3} \right) \\ + 6\mathcal{C}_1 \left(\frac{3\hat{\eta}\hat{\xi} + 4\hat{\eta}^2 + \hat{\xi}^2 - 2\hat{\eta}\hat{\xi} \ln \mathcal{V} - 2\hat{\eta}^2 \ln \mathcal{V}}{\mathcal{V}^4} \right) + \mathcal{O}(\mathcal{V}^{-n}) + \dots, \quad n > 4$$

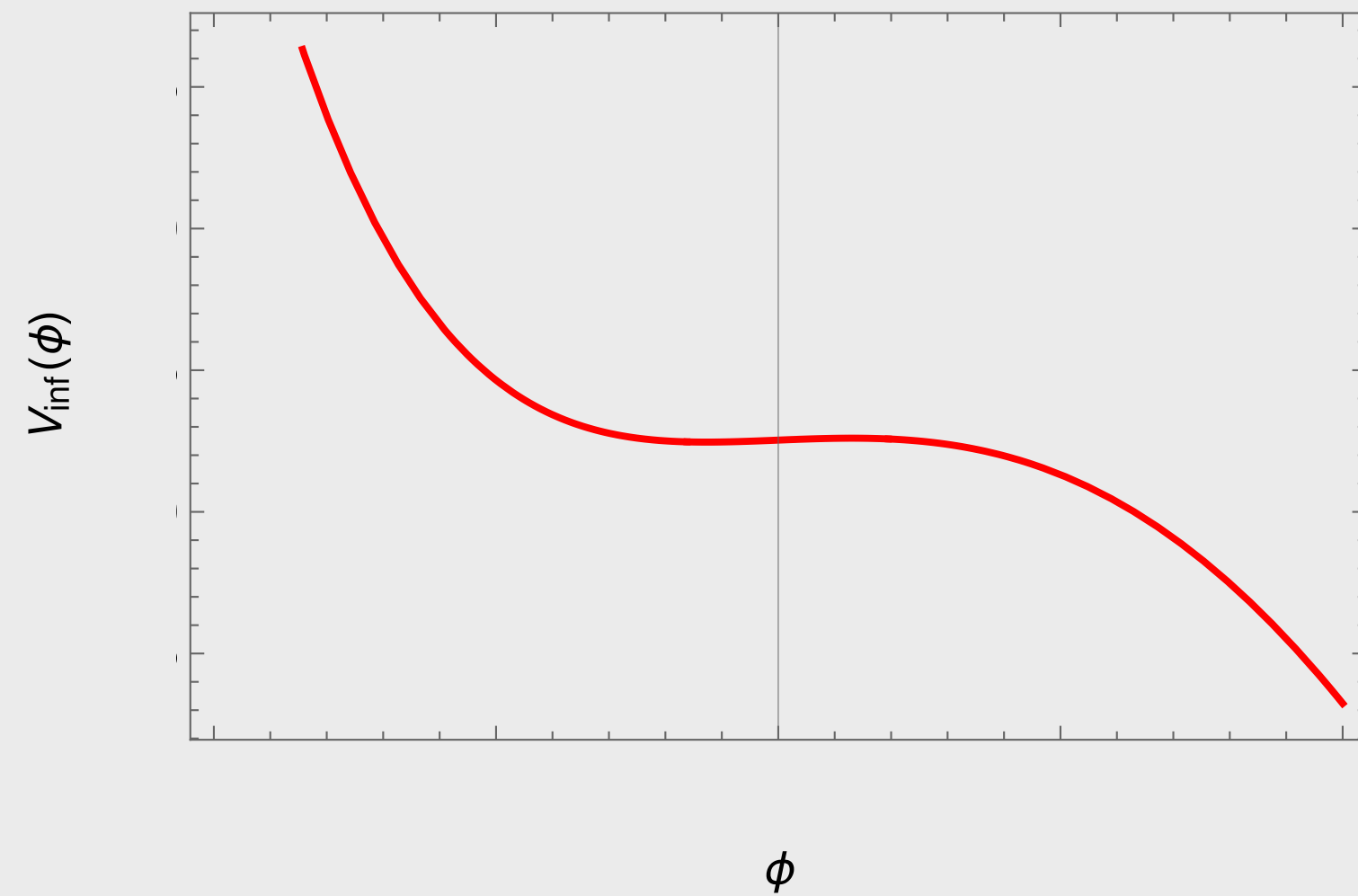
τ_i 's : scalar fields
Parameters: Stringy
details

The Inflationary Potentials - a quick trailer

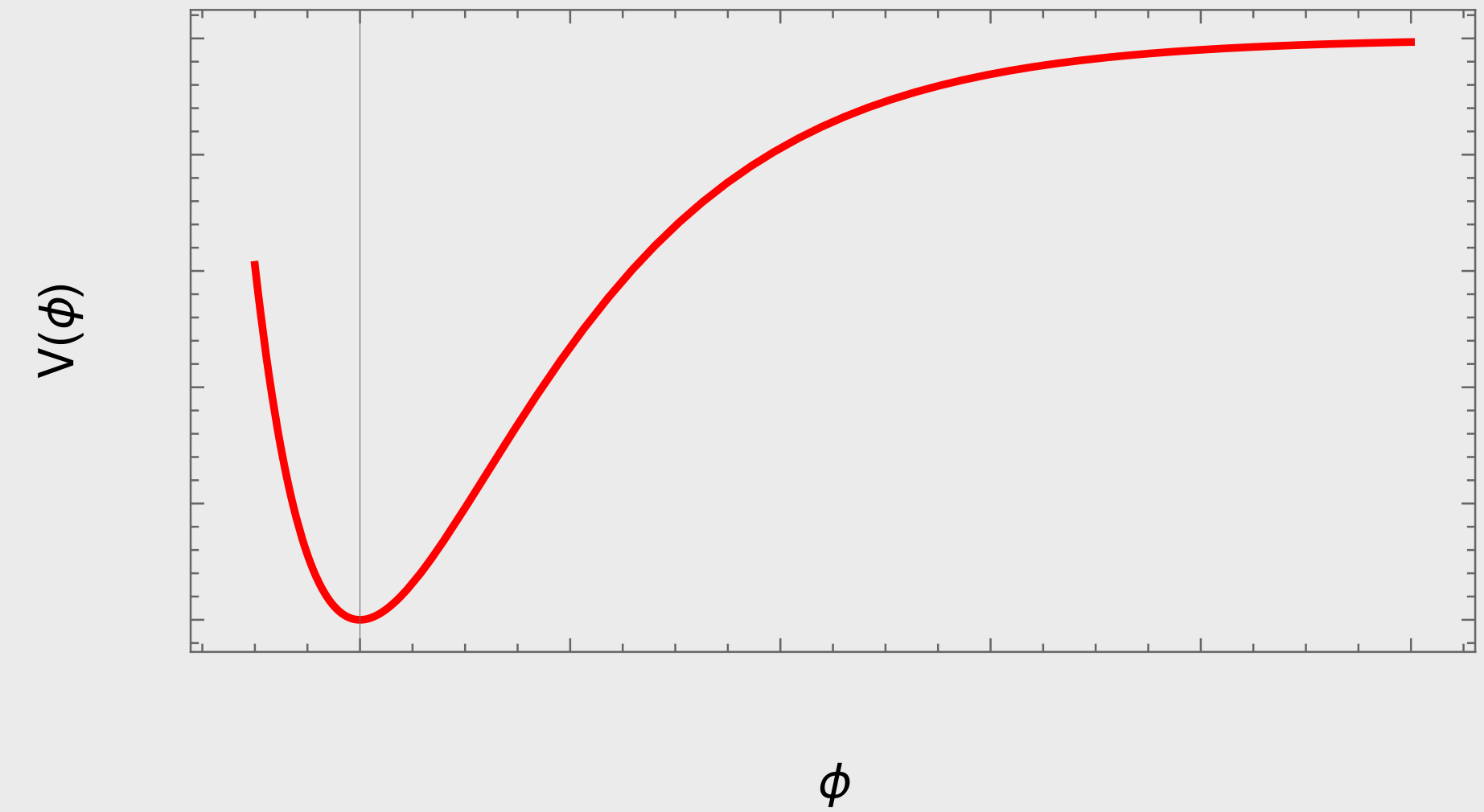
After transforming to canonical coordinates

$$V_{\text{inf}}(\phi) = -\tilde{\mathcal{B}} e^{-3\sqrt{\frac{3}{2}}\phi} \left(\sqrt{\frac{3}{2}}\phi - \frac{3}{2} e^{-x + \sqrt{\frac{3}{2}}\phi} + \frac{4}{3} \right) + \tilde{\mathcal{C}}_2 e^{-5\sqrt{\frac{2}{3}}\phi} + \tilde{\mathcal{C}}_3 e^{-\frac{11}{\sqrt{6}}\phi}$$

$$V(\phi) = \mathcal{C}_0 \left(1 - e^{-\frac{2\phi}{\sqrt{3}}} - \mathcal{R}_0 \left(1 - e^{-\frac{2\phi}{\sqrt{3}}} \right) - \mathcal{R}_1 \left(1 - e^{\frac{\phi}{\sqrt{3}}} \right) - \mathcal{R}_2 \left(1 - e^{\frac{2\phi}{\sqrt{3}}} \right) \right)$$



Volume Inflation



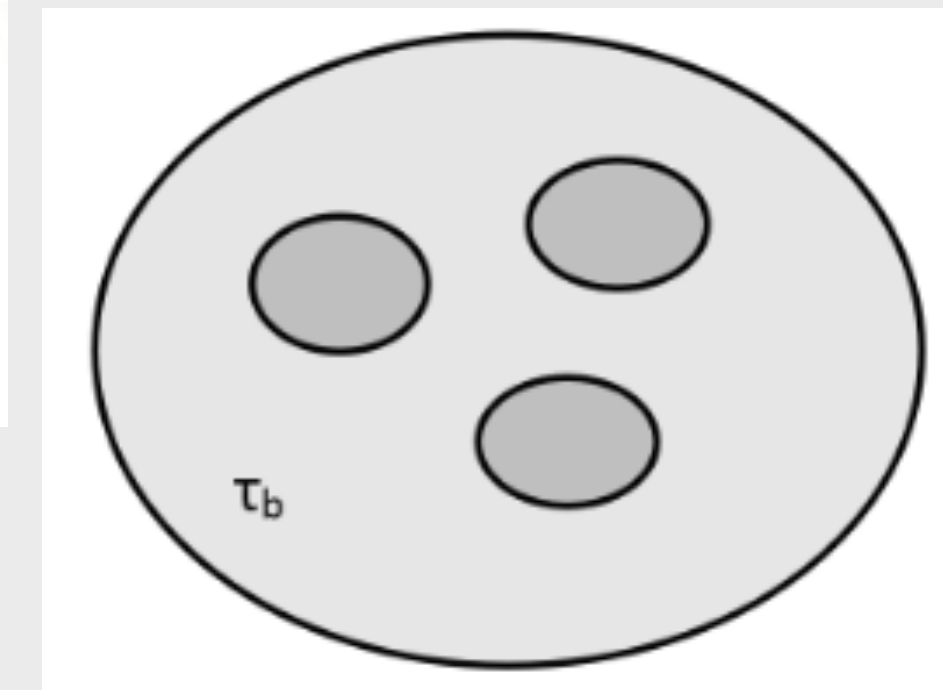
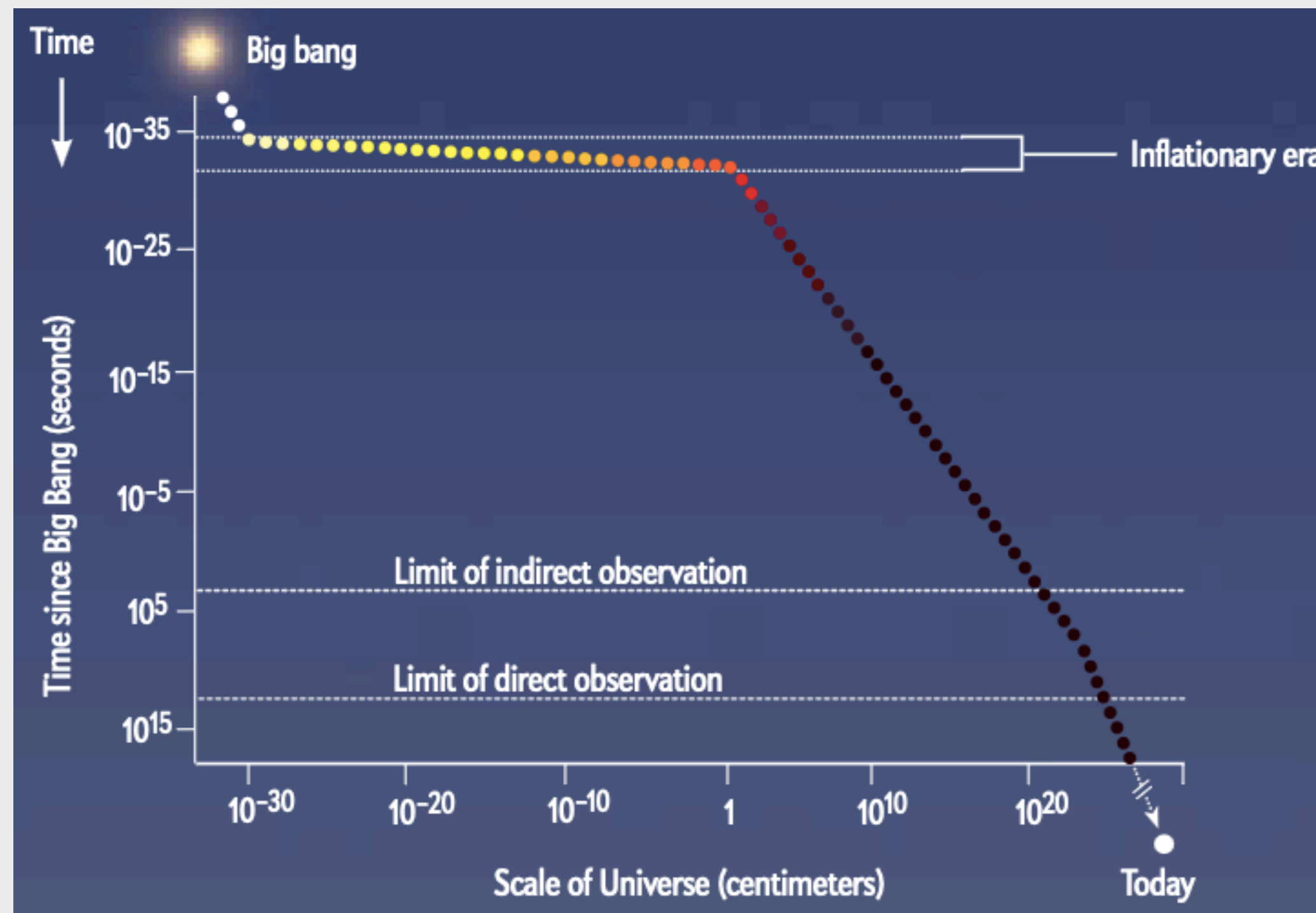
Fibre Inflation

ϕ : scalar fields
Parameters: Stringy details

Keywords

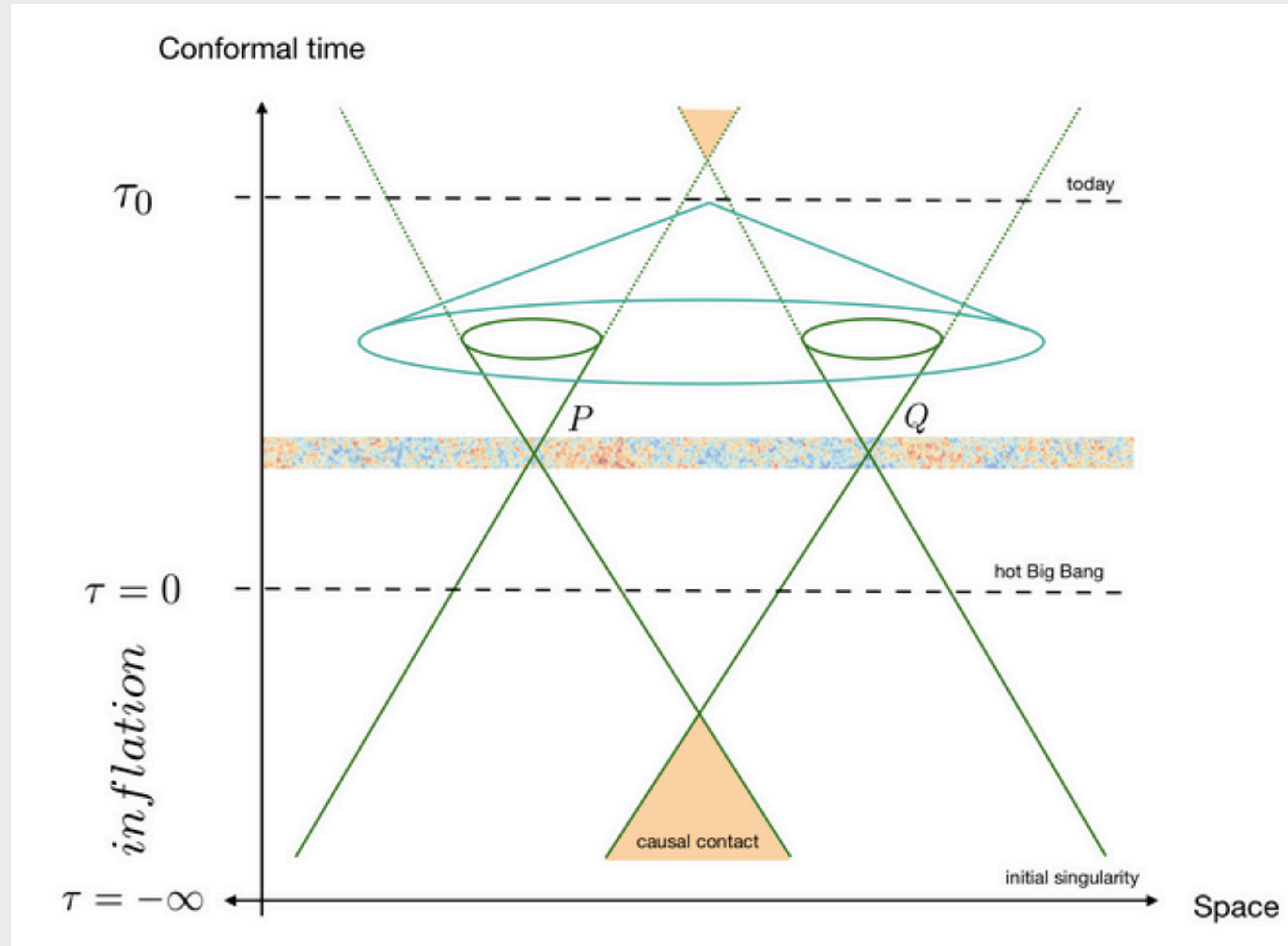
📌 String Theory Model building 📌 Moduli stabilisation

📌 Inflation → **Volume Inflation** **Fibre Inflation**



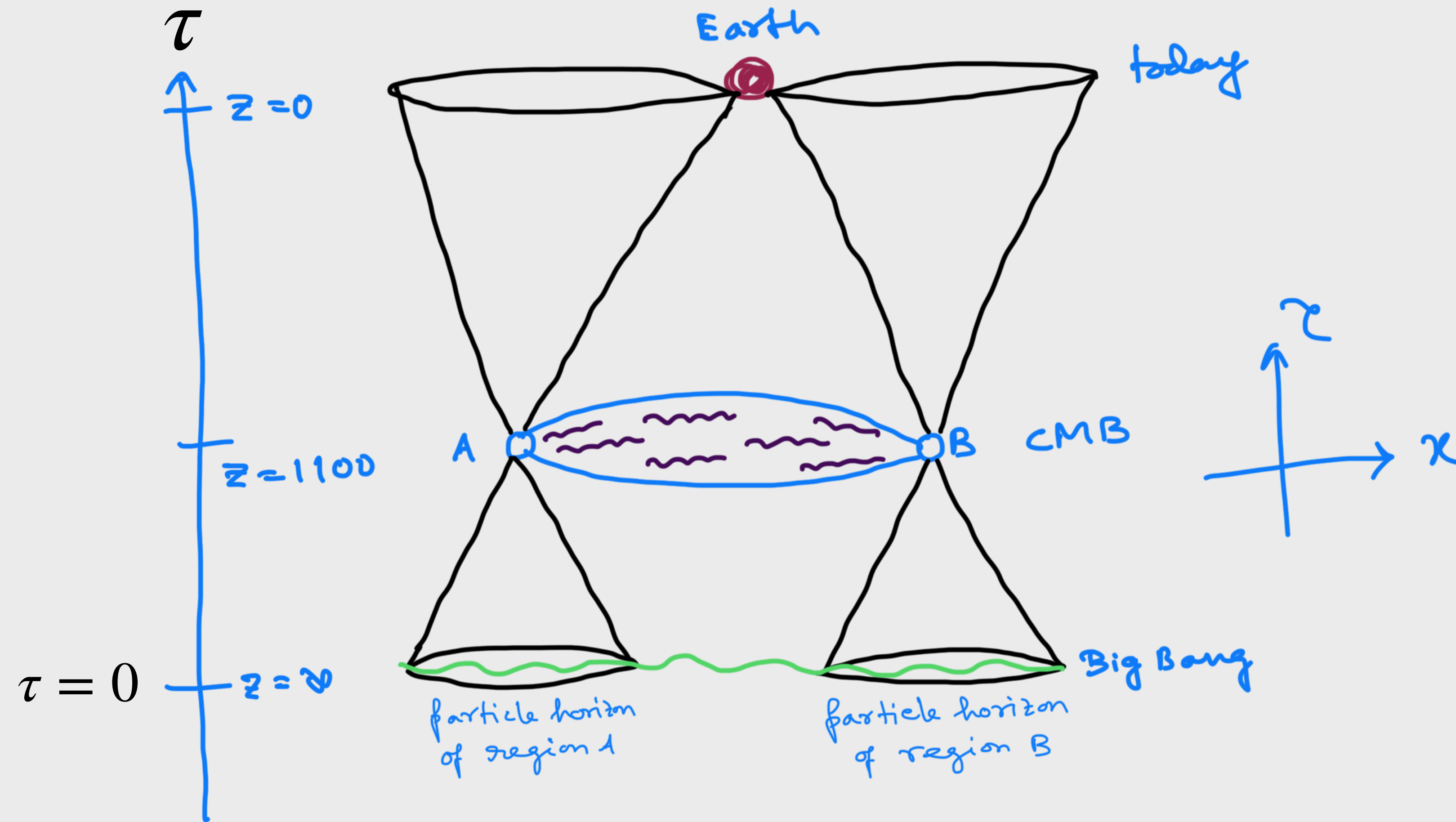
Extra dimensional topology

Why inflation



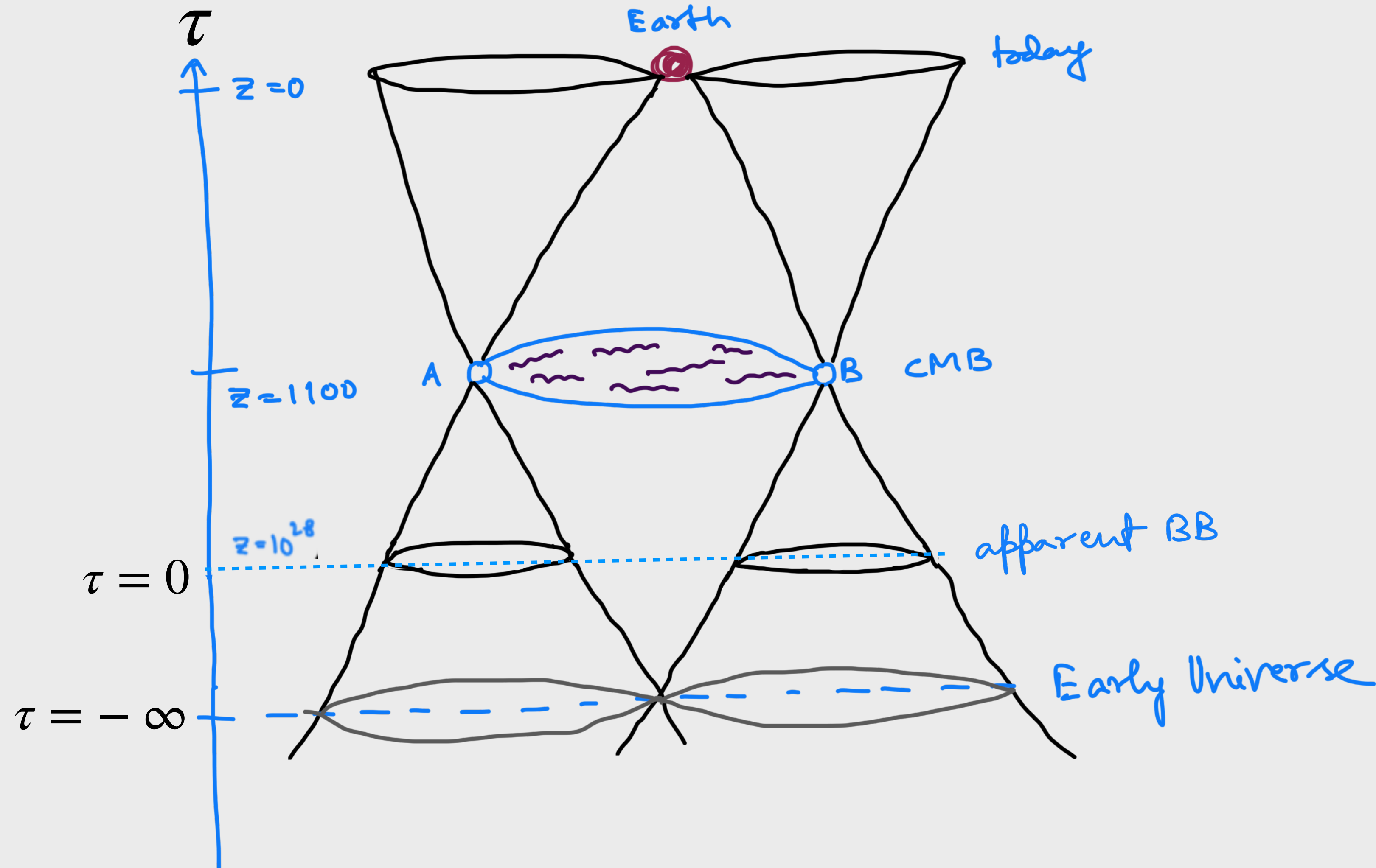
The resolution

The horizon Problem



Lights from A and B met on Earth today were not in contact at the time of Big Bang.

Why inflation



Inflation taking place between $\tau = -\infty \rightarrow \tau = 0$ can make the forward light cone and backward light cone of same size, **solving the horizon problem.**