Imprints of Early Universe Cosmology on Gravitational waves

arXiv: 2411.09757 [hep-ph]

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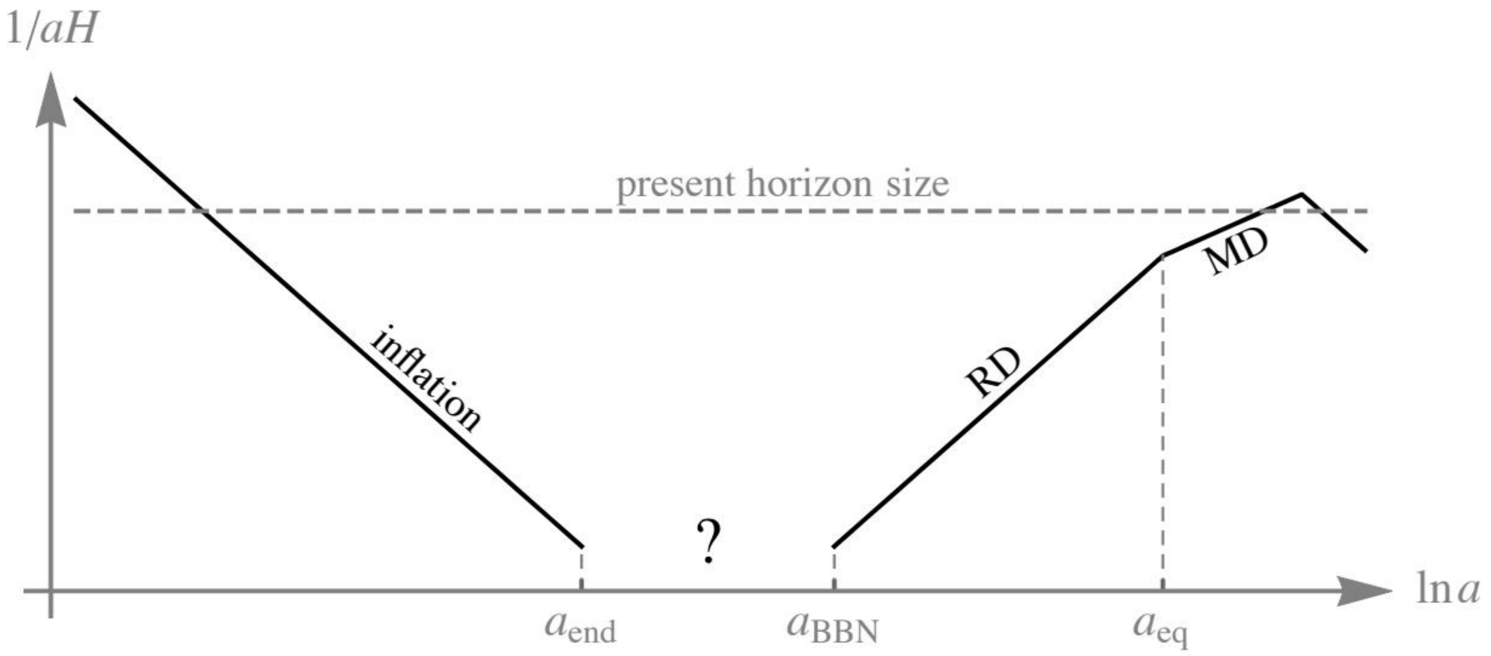
Motivation

- Physics before Big Bang Nucleosynthesis (BBN) ($T{\sim}MeV$) is not well understood due to lack of observational data.
- Gravitational waves can be a natural way to probe this epoch between end of inflation and BBN.



https://arxiv.org/pdf/2006.16182

Cosmological setup

- We consider the scenario where the hidden sector is thermally decoupled to the SM.

- We assume that the SM makes up bulk of the energy density of the universe.

- The ratio of hidden sector temperature and that of SM is given by $\xi = \frac{T_h}{T_{SM}} < 1$ which also implies a hierarchy in the energy densities of the two sectors.

- Net energy density of the universe is given as, $\rho_R(T) = \frac{\pi^2}{30}\left(g_h^*(T) + \frac{g_{SM}^*(T_{SM})}{\xi^4}\right)T^4$

Energy injection

- Energy/entropy injection before BBN has been discussed extensively:
  - Fluctuations generated during inflation and later reentry [Carr & Lidsey, ...]
  - Collapse of domain walls [Cai et al, ...]
  - **PBH reheating** [Bernal et al, ...]
  - Bubble collisions during phase transition [Kodama et al, ...]
  - Temperature increase during reheating [Co et al, ...]
  - **Moduli decay** [Dutta et al...]

- The rate of energy injection can be either be fast where the field remains stuck as the temperature rises or can be slow where the field tracks its T dependent minima.

Impacts of energy injection

- The hierarchy in the two sectors imply that any small change in the energy density of the universe will impact the hidden sector more as compared to SM.
- The energy injection leads to an effective rise in hidden sector temperature

$$T \to \tilde{T} = T(1+\delta)$$

- The energy injection can lead to multiple phase transitions in the hidden sector, and we will show that it can be probed by the associated GW spectrum.

- SM is dominant over the hidden sector implies,

$$\xi(1+\delta) < \left(\frac{g^*_{SM}(T_i/\xi)}{g^*_h(T_i(1+\delta))}\right)^{1/4}$$

Energy injection : Moduli decay

- The amount of energy injection to hidden sector via Moduli decay is given as,

$$\rho_h' = \rho_h + \rho_\chi$$
$$\Longrightarrow T_h' = T_h \left(1 + \frac{30\, m_\chi^2 \chi_i^2}{\pi^2\, g_h^*\, T_h^4}\right)^{1/4}$$

- For hidden sector of $T_h \approx 100$ GeV and small delta,

$$\delta \approx 0.4 \left(\frac{m_\chi}{2.4 \times 10^8\ GeV} \frac{\chi_i}{4 \times 10^{-5}\ GeV}\right)^2$$

- Larger initial field value leads to larger injection,

$$\delta \approx 4 \left(\frac{m_\chi}{2.4 \times 10^8\ GeV} \frac{\chi_i}{4.63 \times 10^{-4}\ GeV}\right)^{\frac{1}{2}} - 1 \approx 3$$

Energy injection : PBH reheating

- Another instance for energy dumping to early universe happens via PBH evaporation.
- Following energy conservation before and after PBH evaporation, we get

$$T'_{SM} = T_{SM}\left(1 + \frac{\eta T_0}{T_{SM}}\right)^{1/4}$$

$$T'_h = T_h\left(1 + \frac{\eta T_0}{T_h}\right)^{1/4}$$

- For hidden sector of $T_h \approx 100$ GeV and small delta,

$$\delta \approx 0.45 \times \frac{\eta}{10^{-11}} \times \frac{0.1}{\xi} \times \left(\frac{M_{BH0}}{5.3 \times 10^4 g}\right)$$

- Larger initial mass fraction leads to larger injection,

$$\delta \approx 4\left(\frac{\eta}{1.39 \times 10^{-9}} \times \frac{0.1}{\xi} \times \left(\frac{M_{BH0}}{5.3 \times 10^4 g}\right)\right)^{1/4} - 1$$

$\eta \equiv \frac{\rho_{BH}}{\rho_R}|_{T_0}$: Initial PBH mass fraction

Model realization

$$V \approx D(T^2 - T_0^2)\phi^2 - ET\phi^3 + \frac{\lambda}{4}\phi^4$$

- Initially, at high T, the field is in symmetric phase and there's just 1 minima at $\phi = 0$
- As universe cools, $T < T_1$, there exist a second minima

$$T_1^2 = \frac{T_0^2}{1 - \frac{9E^2}{8\lambda D}}, \quad \phi_1 = \frac{3ET_1}{2\lambda}$$

- As it further cools, these two minima become equi-potential and we have an onset of phase transition,

$$V(0, T_c) = V(\phi_c, T_c) \qquad T_c^2 = \frac{T_0^2}{1 - \frac{E^2}{\lambda D}}, \quad \phi_c = \frac{2ET_c}{\lambda}$$

- After $T = T_0$, $\phi = 0$ ceases to be a minima and we are left with,

$$\phi_0 = \frac{3ET_0}{\lambda}$$

Criteria for three transitions

- For multiple transitions to occur due to the energy injection, following should be satisfied :

$$T_i < T_c < T_i(1+\delta) < T_1$$

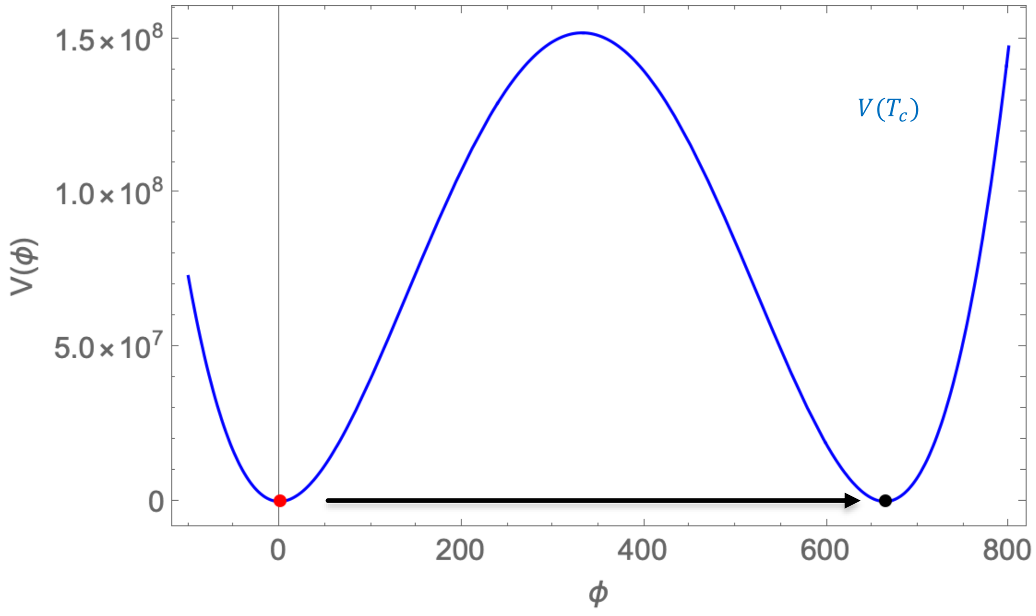
$$V(\phi_{min}(T_i(1+\delta))) > 0$$

$$\phi_{max}(T_i(1+\delta)) < \phi_i(T_i)$$

- In terms of model parameters,

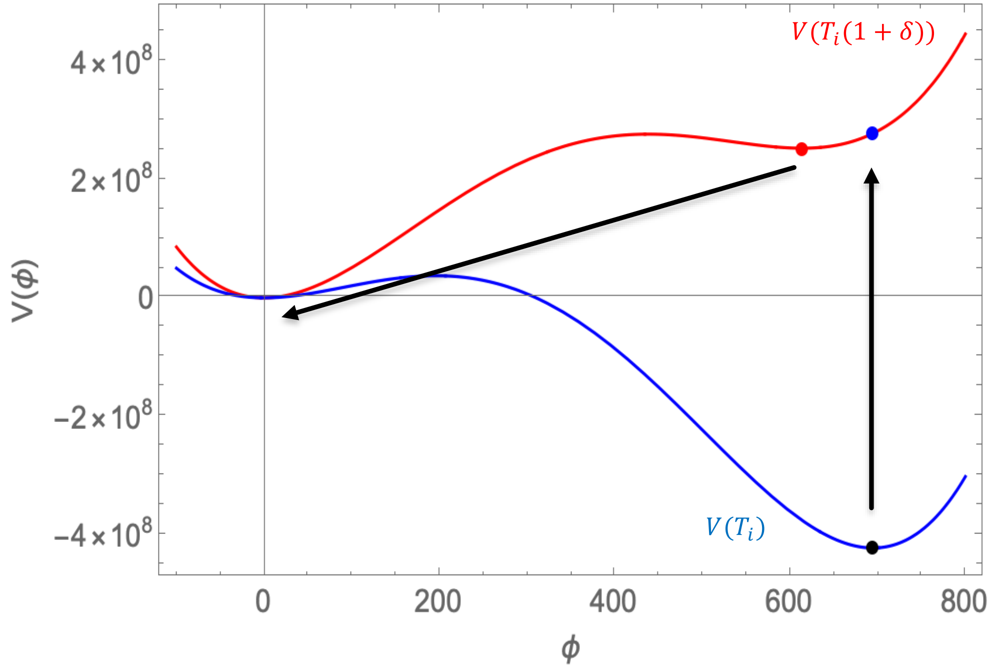
$$\eta := \frac{2\lambda D}{E^2} \in (9/4, 9/2)$$

Phase 1



- This transition corresponds to the standard scenario, where the field undergoes a PT $0 \rightarrow \phi_c$ when the two minimas become equipotential.

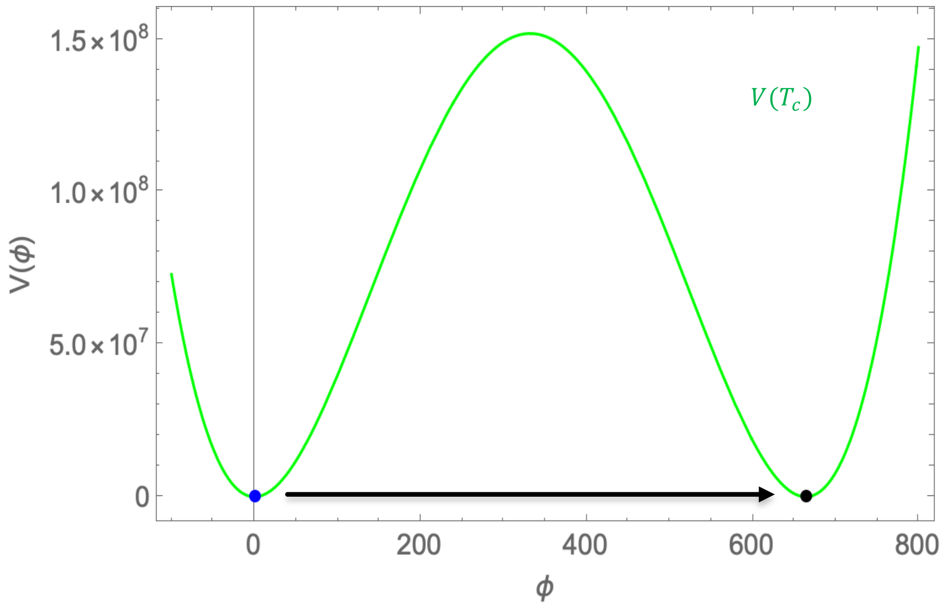
Phase 2



- In the broken phase, an energy injection, say at T_i, ($\mathrm{T_c} > T_i > T_0$) will lead to $T_i \to T_i(1+\delta) > T_c$
- Initially the field remains stuck[1] at $\phi_i(T_i)$ and eventually it rolls down to its T dependent minima $\phi_i(T_i(1+\delta))$, leading to PT from $\phi_i \to 0$ (Phase 2).

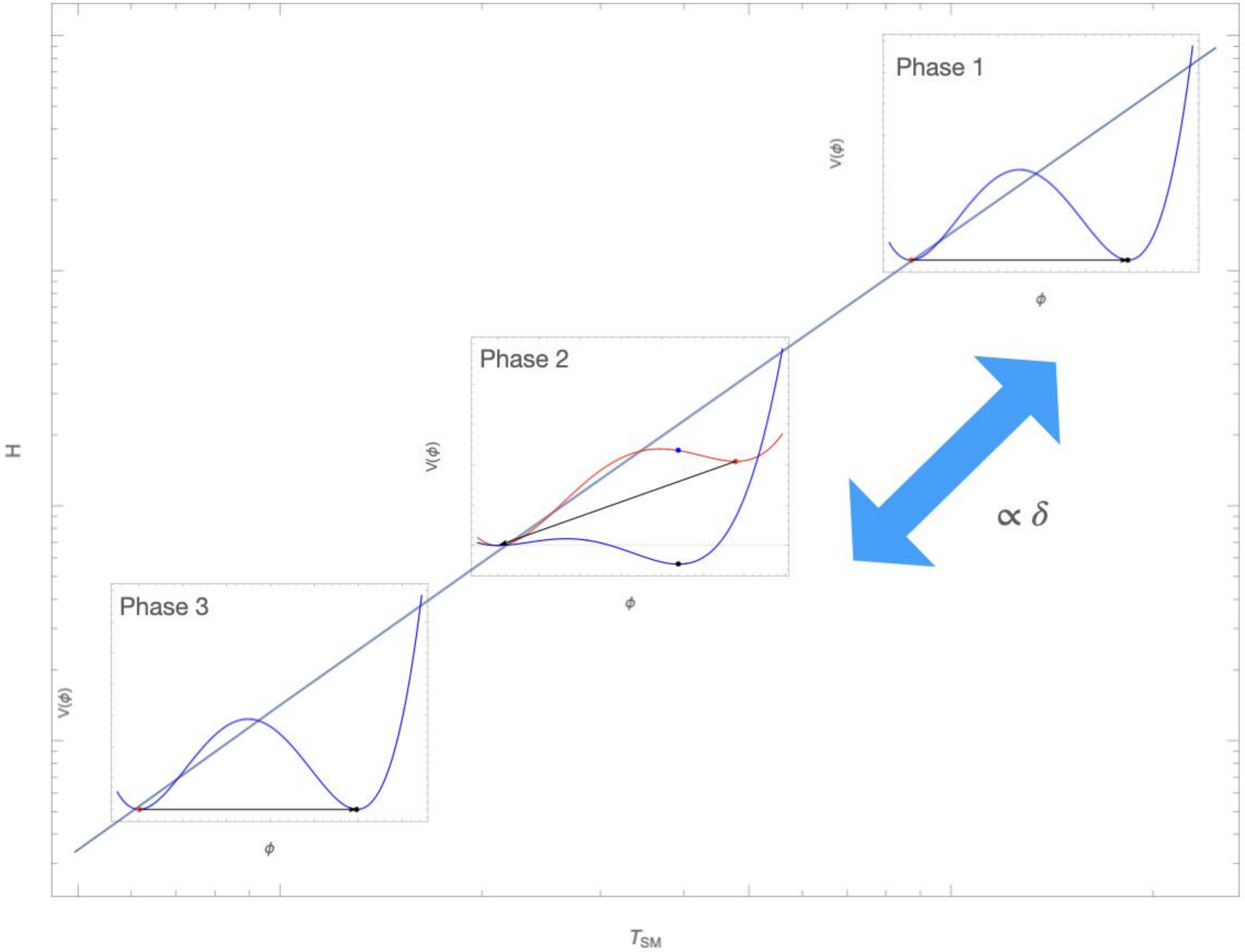
[1] Provided the rate of energy injection is large

Phase 3



- As universe cools down, there's another PT from $0 \to \phi_c$, which is like the standard transition but happens at later redshift (Phase III)
- For scenarios where hidden sector and SM have comparable energies, $\xi > 1$, Phase 1 and Phase 3 will become indistinguishable, similar to resetting the clock (Hubble).

Multiple transitions due to injection



Euclidean Action

- For simple polynomial like potentials, the Euclidean action determining the tunneling rate from a false vacuum state to the true vacuum state [Adams] ,

$$\frac{S_3}{T} \approx \frac{8\,E}{\lambda^{3/2}} f_S(\kappa(T)) \qquad\qquad \kappa = \frac{2\lambda\, D\,(T^2 - T_0^2)}{E^2\,T^2}, \quad 0 \le \kappa \le 2$$

$$f_S(x) = \frac{(8\pi\sqrt{x})\,(0.818x^3 - 5.533x^2 + 8.2938x)}{81(2-x)^2}$$

- For Phase II, we can modify the parameters accordingly as

$$\tilde{D} = D + \frac{3\phi_i(-2\,E\,T + \lambda\phi_i)}{2((T_i(1+\delta))^2 - T_0^2)} = D\left(\frac{9 - 4\kappa(\tilde{T}_i) + 3\sqrt{9 - 4\,\kappa(\tilde{T}_i)}}{2\,\kappa(\tilde{T}_i)}\right)$$

$$\tilde{E} = E - \frac{\lambda\phi_i}{T_i(1+\delta)} = E\left(\frac{1 + \sqrt{9 - 4\,\kappa(\tilde{T}_i)}}{2}\right)$$

$$\tilde{\kappa} = \frac{2\,\lambda\,\tilde{D}}{\tilde{E}^2}\left(1 - \frac{T_0^2}{T^2}\right) = \kappa(T)\left(\frac{9 - 4\,\kappa(\tilde{T}_i) + 3\sqrt{9 - 4\,\kappa(\tilde{T}_i)}}{\kappa(\tilde{T}_i)(5 - 2\,\kappa(\tilde{T}_i) + \sqrt{9 - 4\,\kappa(\tilde{T}_i)})}\right)$$

$$\tilde{T}_i \;=\; T_i(\delta + 1)$$

Nucleation Temperature and PT rate

- Nucleation temperature can be thought of as the temperature where a true vacuum bubble arise within a Hubble volume, i.e,

$$\Gamma/H^4 \approx 1$$

where,

$$\Gamma = T^4 \left(\frac{S_3}{2\pi T}\right)^{3/2} \exp^{-\frac{S_3}{2\pi T}}$$

- This simplifies as,

$$\left(\frac{S_3}{T}\right)_j \approx 173.7 - 2\log g_{*,j} + 8\log \xi_j - 4\log \frac{T_0}{\mathrm{GeV}} + 2\log\left(1 - \frac{\kappa_{N,j}}{\eta}\right)$$

- Rate of the phase transition can be defined in terms of the Euclidean bounce action as,

$$\frac{\beta}{H_N} = T\frac{d(S_3/T)}{dT}|_{T_N}$$

Strength of PT and wall velocity

- Amplitude of GW signal is controlled by strength parameter α :

$$\alpha = \frac{\Delta(V - \frac{1}{4}\partial_T V)}{\rho_R}\bigg|_{T=T_N}$$

where $\Delta X = X_f - X_t$

- For wall velocity, we use analytical approximation[Ellis et al],

$$v_w = \begin{cases} \sqrt{\frac{\Delta V}{\alpha\rho_R}} & \sqrt{\frac{\Delta V}{\alpha\rho_R}} < v_J \\ 1 & \sqrt{\frac{\Delta V}{\alpha\rho_R}} > v_J \end{cases}$$

where

$$v_J = \frac{1}{1+\alpha}\left(\sqrt{\frac{1}{3}} + \sqrt{\alpha\left(\frac{2}{3} + \alpha\right)}\right)$$

Gravitational Waves signal

- Differential GW density parameter characterizes them :

$$\Omega_{GW} = \frac{1}{\rho_c}\frac{d\rho_{GW}}{d\log f}, \qquad \rho_c = 3M_{pl}^2 H^2$$

- Semi-analytical parametrizations can be used to describe them,

$$\Omega_{GW}(f) \simeq \sum_i \mathcal{N}_i\, \Delta_i(v_w) \left(\frac{\kappa_i\, \alpha}{1+\alpha}\right)^{p_i} \left(\frac{H}{\beta}\right)^{q_i} s_i(f/f_{p,i})$$

$$\Longrightarrow \quad \Omega^0_{\rm GW}(f) = \mathcal{R}\, \Omega_{\rm GW}\left(\frac{a_0}{a} f\right)$$

$$\mathcal{R} \equiv \left(\frac{a}{a_0}\right)^4 \left(\frac{H}{H_0}\right)^2 \simeq 2.473 \times 10^{-5}\, h^{-2} \left(\frac{g_s^{\rm EQ}}{g_s}\right)^{4/3} \left(\frac{g_\rho}{2}\right)$$

General features of Hidden sector PT

- $\alpha \propto \xi^4$ implies that for strong GW signals, Hidden sector and SM should have comparable temperature.
- Large β means faster transitions, which results in signals that are weaker and peaking at higher frequencies vs slower transitions.
- Larger v_w implies smaller peak frequency and larger GW amplitude.
- Smaller T_N would lead to lesser redshift suppression.
- Smaller T_N/T_c is an indication of long-lasting phase transitions.

Relationship among phase 1 and 3

- Phase 1 and 3 are similar in nature, differing only due to redshift,

$$\kappa_3 \approx \kappa_1 + \chi, \quad \chi \ll 1$$

- Thus,

$$\left(\frac{S_3}{T}\right)_3 \approx \left(\frac{S_3}{T}\right)_1 + \frac{\chi\,(\beta/H)_1}{2\,(\eta - \kappa_1)}$$

$$\begin{aligned} \implies \kappa_3 &\approx \kappa_1 + \frac{16\,\log(1+\delta)\,(\eta - \kappa_1)}{(\beta/H)_1} \\ &\approx \kappa_1 + \frac{8\,\log(1+\delta)}{(S_3/T)_1\,(\log(f_S(\kappa_1)))'} \end{aligned}$$

- This leads to

$$\frac{f_{p,obs,3}}{f_{p,obs,1}} \approx \sqrt{\frac{\eta - \kappa_{N,1}}{\eta - \kappa_{N,3}}\,\frac{f_\beta(\kappa_3)}{f_\beta(\kappa_1)}} \approx 1 \qquad \alpha_{h,3}/\alpha_{h,1} \approx 1$$

Relationship among phase 2 and 3

- Actions between the two phases be related as

$$\left(\frac{S_3}{T}\right)_2 \approx \left(\frac{S_3}{T}\right)_3 \frac{f_S(\tilde{\kappa}(T,\tilde{T}_i))}{f_S(\kappa(T))}\left(\frac{1+\sqrt{9-4\kappa(\tilde{T}_i)}}{2}\right)$$

where,

$$\frac{f_S(\tilde{\kappa}(T_{N,2},\tilde{T}_i))}{f_S(\kappa_{N,3})} \approx 1 - \frac{2\log\left(\frac{\eta-\kappa_{N,2}}{\eta-\kappa_{N,3}}\right)}{173.7 - 2\log g_{*,3} + 8\log\xi_f - 4\log\frac{T_0}{\mathrm{GeV}} + 2\log\left(1-\frac{\kappa_{N,3}}{\eta}\right)}$$

- The ratio of peak frequencies for the emitted GW is

$$\frac{f_{p,2,0}}{f_{p,3,0}} \approx \sqrt{\frac{\eta-\kappa_{N,3}}{\eta-\kappa_{N,2}}}\,\frac{\sqrt{9-4\,\kappa_i}}{\kappa_i}\left(\frac{\sqrt{9-4\,\kappa_i}+3}{\sqrt{9-4\,\kappa_i}+1}\right)\frac{f_\beta(\tilde{\kappa}(\tilde{T}_i,T_{N,2}))}{f_\beta(\kappa_{N,3})} \qquad f_\beta(x) =: \frac{f_S'(x)}{6\,\pi}$$

Impact of cosmology on GW

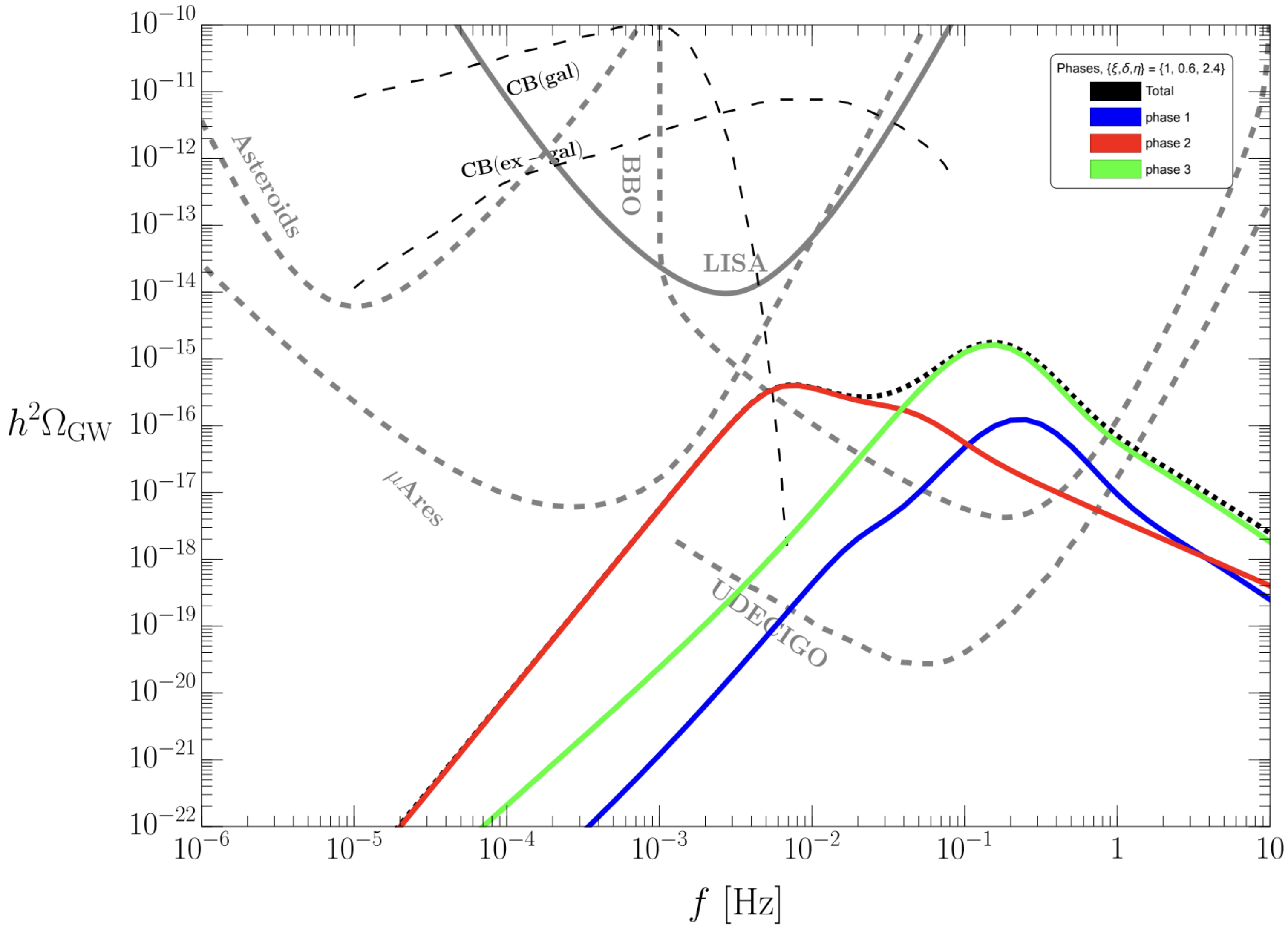
- The amplitude of GW for phase 1 vs phase 3 scales with amount of energy injected,

$$\frac{\Omega^{(p)}_{\mathrm{GW},3,obs}}{\Omega^{(p)}_{\mathrm{GW},1,obs}} \approx (1+\delta)^6 \left(1 - \frac{8\,\log(1+\delta)}{(S_3/T)_1}\, g(\kappa_1,\,\eta)\right) \qquad \frac{8\,\log(1+\delta)}{(S_3/T)_1}\, g(\kappa_1,\,\eta) \ll 1$$

- For phase 2 vs phase 3, we find that the amplitude ratio depends proportionally to the model parameter η in addition to ξ, δ .

- Knowing the peak frequency difference between phase 2 and 3 yields the value of $\xi(1+\delta)$, in terms of model parameters and the scale of the hidden sector.

Results : $\xi = 1, \delta = 0.6, \eta = 2.4, T_0 = 450\ GeV$



Results : $\xi = 0.75, \eta = 2.6, T_0 = 350\ GeV$

$\xi = 0.75$	α	β/H	α_h	T_N^{SM} (GeV)	f_P (Hz)	$h^2\,\Omega_P$	κ
I	0.0009	1070.5	0.055	941.46	0.145	1.1×10^{-17}	1.546
II ($\delta = 0.57$)	0.002	429.8	0.017	839.3	0.09	2.5×10^{-17}	1.61
II ($\delta = 0.93$)	0.003	435.62	0.017	565.1	0.009	1.2×10^{-16}	1.62
III ($\delta = 0.57$)	0.006	1108.8	0.055	565.02	0.1	1.4×10^{-16}	1.55
III ($\delta = 0.93$)	0.01	1126.46	0.055	379.9	0.08	4.0×10^{-16}	1.55

Discussion

- Energy injection δ plays a significant role in shaping GW features.
- Larger energy injection δ leads to stronger GW amplitudes in phases 2 and 3 compared to phase 1, primarily due to smaller redshift suppressions, as shown in the tables.
- The tables verify that the peak frequencies ratios for phases 1 vs 3 are similar, differing from unity by a factor proportional to $\mathrm{Log}(1+\delta)$.
- The tables also verify the ratios for peak amplitude between phase 3 vs phase 1 scaling as $(1+\delta)^6$.
- Smaller ξ leads to smaller peak amplitude due to weaker phase transition strength (α).

Conclusion

Energy injection(χ, PBH)

$T_h \rightarrow T_h(1+\delta)$

3 GW peaks

- Energy injection leads to more than one peak frequencies for GW from FOPT in hidden sector.
- For any reasonable ξ value (small or large), GW spectra have distinctive features due to multiple peaks.
- It is fairly independent w.r.to the mass scale of the hidden sector.
- Hidden sectors with GW can probe a variety of new physics scenario in the pre-BBN era.

THANK YOU!

BACKUP Slides

Phase 1 : Hydrodynamics

- Driving force for the PT in terms of the latent heat is,

$$F_{dr} \approx \alpha\, \rho_R = \Delta(V - \frac{T}{4}\frac{dV}{dT})$$
$$= \frac{T_N^2\, \phi_c^2}{2\lambda}\left(\lambda\, D \frac{T_N}{T_c} - E^2\right) > 0$$

- The friction force is due to particles gaining mass as they go from the false vacua(symmetric phase) to the true vacua (broken phase).
- For runaway transition ($v_w \to c$), $F_{dr} > \Delta p_{LO}^{\gamma_w \to \infty}$ where,

$$\Delta p_{LO}^{\gamma_w \to \infty} \approx \sum_i \frac{c_i\, g_i\, (m_{t,i}^2 - m_{b,i}^2)}{24} > 0$$

- For most of our parameter space, we find that this happens to be a non-runaway transition, i.e, where the bubble wall never reaches the speed of light.

Phase 2 : Hydrodynamics

- The pressure difference due to mass difference is negative, since the particles loose mass as they pass from the false vacua (broken phase) to the true vacua (symmetrical phase).

$$\Delta p_{LO}^{\gamma_w \to \infty} \approx \sum_i \frac{c_i\, g_i\, (m_{t,i}^2 - m_{b,i}^2)}{24} < 0$$

- The force due to latent heat difference is also negative, and it acts as an effective friction.

$$F_{opp} \approx -T_i(1+\delta)\, \eta\, (4\, D\, T_i(1+\delta) - 3\, E\, \phi_i) - 2\, D\, T_0^2 < 0$$

- This transition happens to be runaway[1], with the condition being

$$|\Delta p_{LO}^{\gamma_w \to \infty}| > |F_{opp}|$$

[1] For models without any production of soft vector bosons at the boundary

GW signal efficiency factors

- Turbulence efficiency factor can be estimated from sound wave as,

$$\kappa_{\rm Turb} = \epsilon \kappa_{\rm SW} \,,$$

$$\epsilon = (1 - \min\left(H_* \tau_{\rm sh}, 1\right))^{\frac{2}{3}}$$

- Sound wave efficiency factor differs in run-away vs non-runaway scenarios.
- For runaway scenarios, we have,

$$\kappa_{\rm SW} = (1 - \kappa_{BW}) \frac{\alpha_{eff}}{0.73 + 0.083\sqrt{\alpha_{eff}} + \alpha_{eff}}, \quad \alpha_{eff} = \alpha_h (1 - \kappa_{\rm BW})$$

- Efficiency factor for runaway in Phase 2 is based on numerical analysis in the works of Blasi et al on inverse transitions.

GW signal efficiency factors

- Sounds wave efficiency factor for non-runaway transitions is:

$$\kappa_{\rm SW} = \begin{cases} \frac{c_s^{11/5}\kappa_A\kappa_B}{\left(c_s^{11/5}-v_w^{11/5}\right)\kappa_B+v_w c_s^{6/5}\kappa_A}, & \text{if } v_w < c_s \\ \kappa_B + (v_w - c_s)\,\delta\kappa + \frac{(v_w-c_s)^3}{(v_J-c_s)^3}\left[\kappa_C - \kappa_B - (v_J - c_s)\,\delta\kappa\right], & \text{if } c_s < v_w < v_J \\ \frac{(v_J-1)^3 v_J^{5/2} v_w^{-5/2}\kappa_C\kappa_D}{\left[(v_J-1)^3-(v_w-1)^3\right]v_J^{5/2}\kappa_C+(v_w-1)^3\kappa_D}, & \text{if } v_J < v_w \end{cases}$$

with

$$\kappa_A \simeq v_w^{6/5}\frac{6.9\,\alpha_h}{1.36 - 0.037\sqrt{\alpha_h} + \alpha_h}, \qquad \kappa_B \simeq \frac{\alpha_h^{2/5}}{0.017 + (0.997 + \alpha_h)^{2/5}},$$

$$\kappa_C \simeq \frac{\sqrt{\alpha_h}}{0.135 + \sqrt{0.98 + \alpha_h}}, \qquad \kappa_D \simeq \frac{\alpha_h}{0.73 + 0.083\sqrt{\alpha_h} + \alpha_h},$$

$$\delta\kappa \simeq -0.9\log\frac{\sqrt{\alpha_h}}{1 + \sqrt{\alpha_h}}.$$

Results : $\xi = 1, \eta = 2.4, T_0 = 450\ GeV$

$\xi = 1$	α	β/H	α_h	T_N^{SM} (GeV)	f_P (Hz)	$h^2\,\Omega_P$	κ
I	0.004	885.7	0.074	751.3	0.23	1.2×10^{-16}	1.545
II($\delta = 0.6$)	0.003	256.9	0.013	832.7	0.008	3.5×10^{-16}	1.61
II($\delta = 0.9$)	0.005	257.8	0.013	696.7	0.007	9.8×10^{-16}	1.615
III ($\delta = 0.6$)	0.018	909.9	0.071	477.3	0.15	1.5×10^{-15}	1.55
III ($\delta = 0.9$)	0.03	917.3	0.07	400.75	0.137	3.3×10^{-15}	1.55