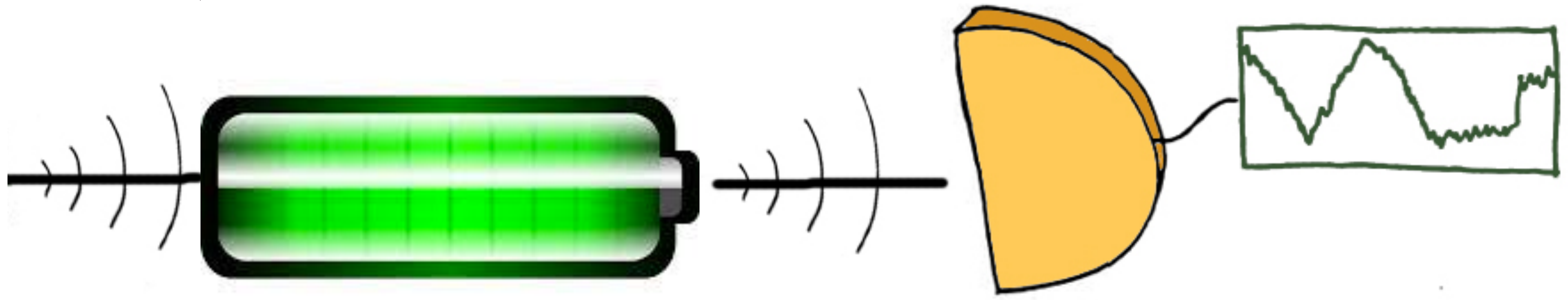
Daemonic work extraction in continuously-monitored open quantum batteries



Marco Genoni

@ Pure and Applied Quantum Mechanics Group
University of Milan

INTERNATIONAL CENTRE for THEORETICAL SCIENCES
TATA INSTITUTE OF FUNDAMENTAL RESEARCH

Quantum Trajectories, ICTS - Bangalore,
3rd February 2025

outline of the talk

- daemonic ergotropy in continuously monitored open quantum batteries


Daniele Morrone
University of Milan (IT)

Matteo A.C. Rossi
Algorithmiq (FI)

D. Morrone, M.A.C. Rossi and MGG, Phys. Rev. Applied **20**, 044073 (2023).

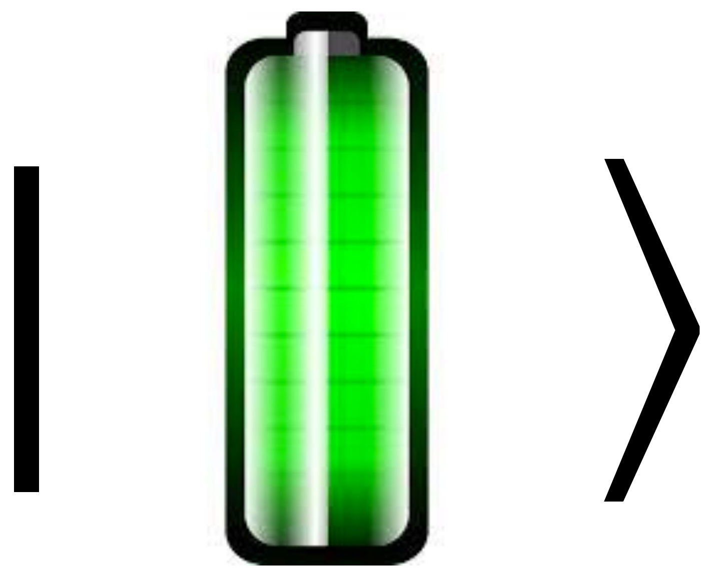
- experimental verification of daemonic work extraction on a digital quantum computer


S. Navid Elyasi
Univ. of Kurdistan (IR)

Matteo A.C. Rossi
Algorithmiq (FI)

S.N. Elyasi, M.A.C. Rossi and MGG, arXiv:2410.16567 (*to appear in Quantum Science & Technology*)

quantum batteries



What is a quantum battery?

It is a temporary energy storing device, whose charging and discharging processes are described by the law of quantum mechanics.

Alicki & Fannes, PRE **87**, 1 (2013)
Campaioli et al., RMP **96**,031001 (2024)

Why is interesting to study quantum batteries?

<u>fundamental reasons</u>: study the fundamental limits on work extraction in the quantum realm

<u>technological reasons</u>: a theoretical quantum advantage has been proven in the charging of quantum batteries thanks to purely quantum/collective effects.

Binder et al., NJP **17**, 075015 (2015)
Campaioli et al., PRL **118**, 150601 (2017)
Andolina et al., PRB **98**, 1 (2018)
Julia-Farré et al., PRResearch **2**, 023113 (2020)

Ferraro et al., PRL **120**, 117702 (2018)
Quach et al., Science Adv. **8**, 3160 (2022)

ergotropy - definition

What is the relevant figure of merit to assess the properties of a quantum battery model?

Let us consider a quantum system

- described by a Hamiltonian $\hat{H}_0 = \sum_k \epsilon_k |\epsilon_k\rangle\langle\epsilon_k|$

- prepared in a given quantum state $\rho = \sum_j r_j |r_j\rangle\langle r_j|$ with energy $E(\rho) = \mathrm{Tr}[\rho \hat{H}_0]$

Work extracted from ρ via a unitary operation $\hat{U}$: $W_{\hat{U}}(\rho) = E(\rho) - E(\hat{U}\rho\hat{U}^\dagger)$

Allahverdyan et al., Europhys. Lett. **125** (2004)

ergotropy - definition

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Work extracted from ρ via a unitary operation $\hat{U}$: $W_{\hat{U}}(\rho) = E(\rho) - E(\hat{U}\rho\hat{U}^\dagger)$

*We define the **ergotropy** $\mathscr{E}(\rho)$ of the state ρ as the maximum amount of work that one can extract from ρ via unitary operations:*

$$\mathscr{E}(\rho) = \max_{\hat{U}} W_{\hat{U}}(\rho) = \ldots = \sum_{j,k} r_j \epsilon_k (|\langle r_j|\epsilon_k\rangle|^2 - \delta_{j,k}|)$$

Allahverdyan et al., Europhys. Lett. **125** (2004)

ergotropy - properties

ergotropy $\mathcal{E}(\rho) = \max_{\hat{U}} W_{\hat{U}}(\rho)$

- the ergotropy is non-negative and upper-bounded by the quantum state energy
  $0 \leq \mathcal{E}(\rho) \leq E(\rho)$

- the ergotropy is zero iff the state is *passive* *(i.e. diagonal in the Hamiltonian basis and with no energy inversion)*
  $\mathcal{E}(\rho) = 0$ $\qquad \rho = \sum_k r_k |\epsilon_k\rangle\langle\epsilon_k|$ with $r_k \geq r_{k+1}$ and $\epsilon_k \leq \epsilon_{k+1}$

  …Gibbs states are passive states, but not all passive states are Gibbs states…

- for pure states, the ergotropy is equal to the energy: $\mathcal{E}(|\psi\rangle\langle\psi|) = E(|\psi\rangle\langle\psi|)$

- the ergotropy is a convex quantity : $\mathcal{E}(\sum_j p_j\rho_j) \leq \sum_j p_j\mathcal{E}(\rho_j)$

Allahverdyan et al., Europhys. Lett. **125** (2004)

daemonic ergotropy

What if I have a correlated bipartite quantum state and I want to extract energy from one of the two parties only?

Francica et al., NPJ Quantum Information **3** (2017)

system S + auxiliary system A

ρ_{SA}

…I want to extract work from the quantum system S

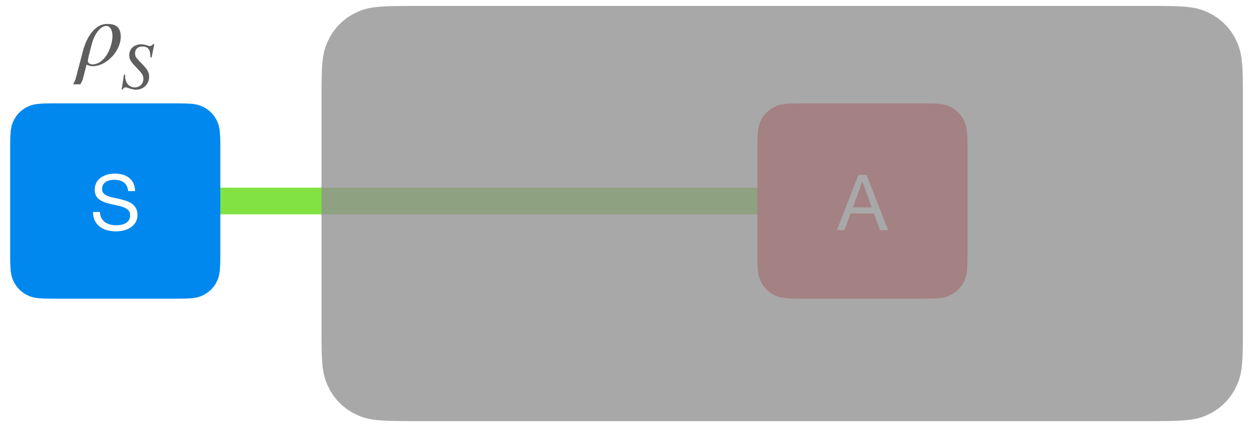
daemonic ergotropy

What if I have a correlated bipartite quantum state and I want to extract energy from one of the two parties only?

Francica et al., NPJ Quantum Information **3** (2017)

- …if I have no access to the auxiliary quantum system A:

reduced state
ρ_S



Maximum extractable work: $\mathcal{E}(\rho_S)$ with $\rho_S = \mathrm{Tr}_A[\rho_{SA}]$

ergotropy of the reduced state

daemonic ergotropy

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Francica et al., NPJ Quantum Information **3** (2017)

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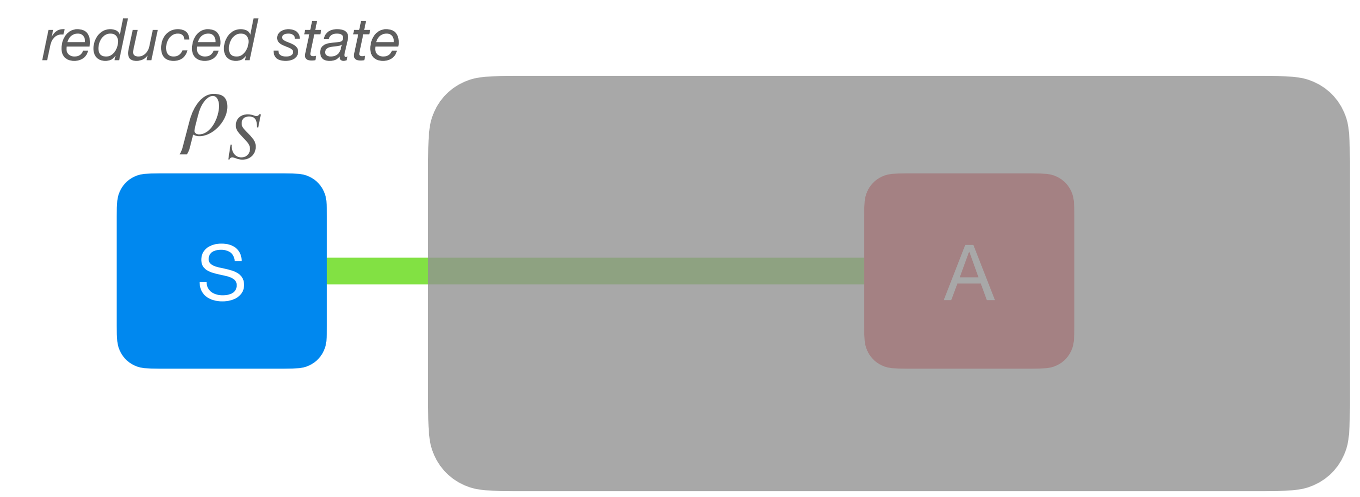
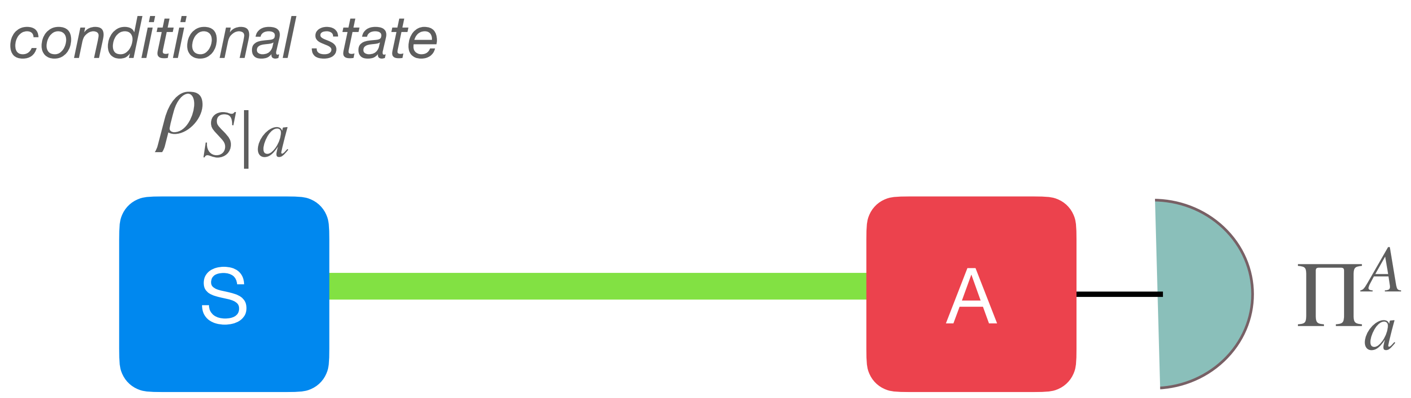


Maximum extractable work: $\mathscr{E}(\rho_S)$ with $\rho_S = \mathrm{Tr}_A[\rho_{SA}]$

ergotropy of the reduced state

- …if I can perform a measurement on the auxiliary system A:



Maximum extractable work: $\overline{\mathscr{E}}_{\Pi_a^A} = \sum_a p_a \mathscr{E}(\rho_{S|a})$

average *ergotropy of the conditional states*

Daemonic ergotropy

daemonic ergotropy

What if I have a correlated bipartite quantum state and I want to extract energy from one of the two parties only?

Francica et al., NPJ Quantum Information **3** (2017)

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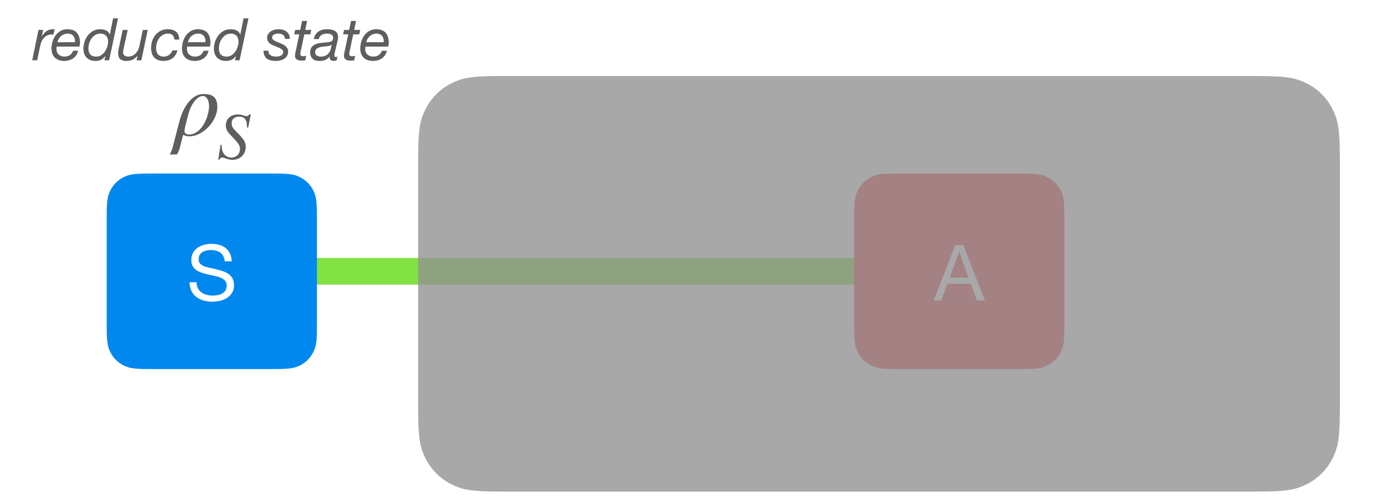

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ergotropy of the reduced state

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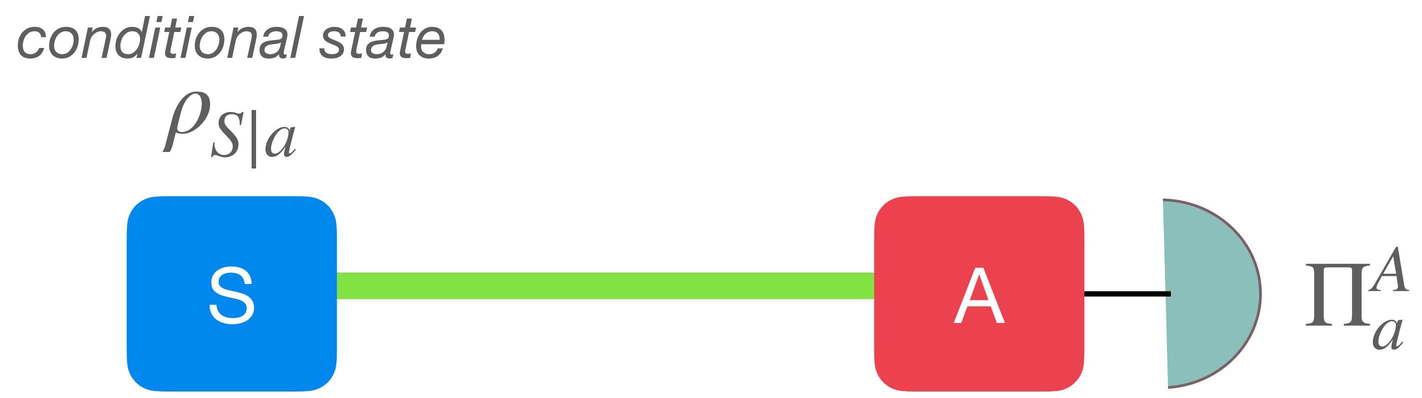


Maximum extractable work: $\overline{\mathcal{E}}_{\Pi_a^A} = \sum_a p_a \mathcal{E}(\rho_{S|a})$

average *ergotropy of the conditional states*

Daemonic ergotropy

…but we know that:

– in quantum mechanics "**tracing out** is equivalent to **measure and forget** (i.e. averaging)" : $\rho_S = \sum_a p_a \rho_{S|a}$

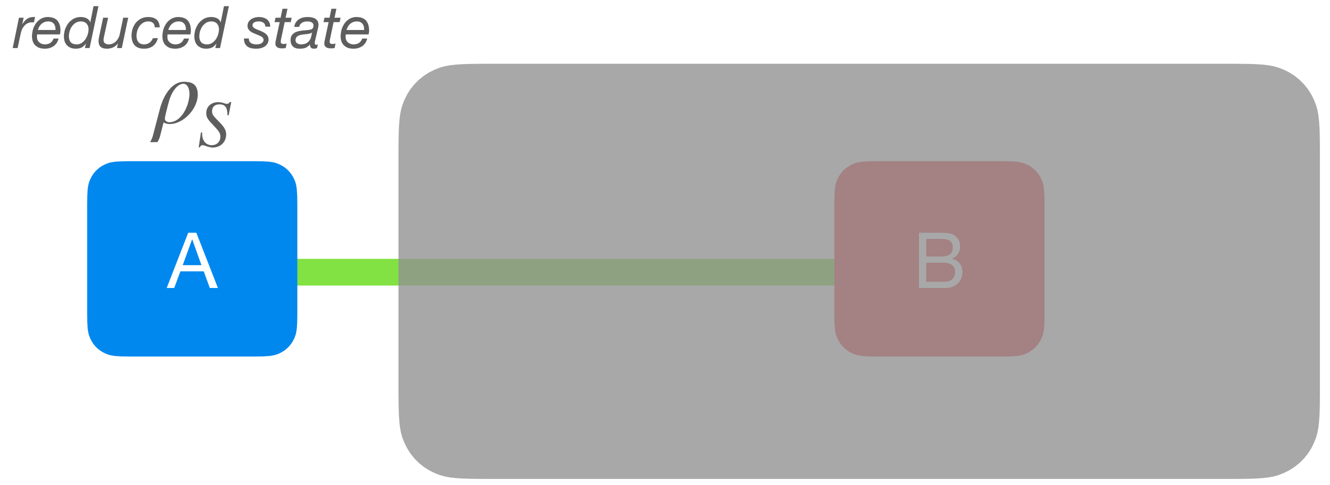
– the quantum **ergotropy** is **convex**: $\sum_k p_k \mathcal{E}(\rho_k) \geq \mathcal{E}(\sum_k p_k \rho_k)$

daemonic ergotropy

What if I have a correlated bipartite quantum state and I want to extract energy from one of the two parties only?

Francica et al., NPJ Quantum Information **3** (2017)

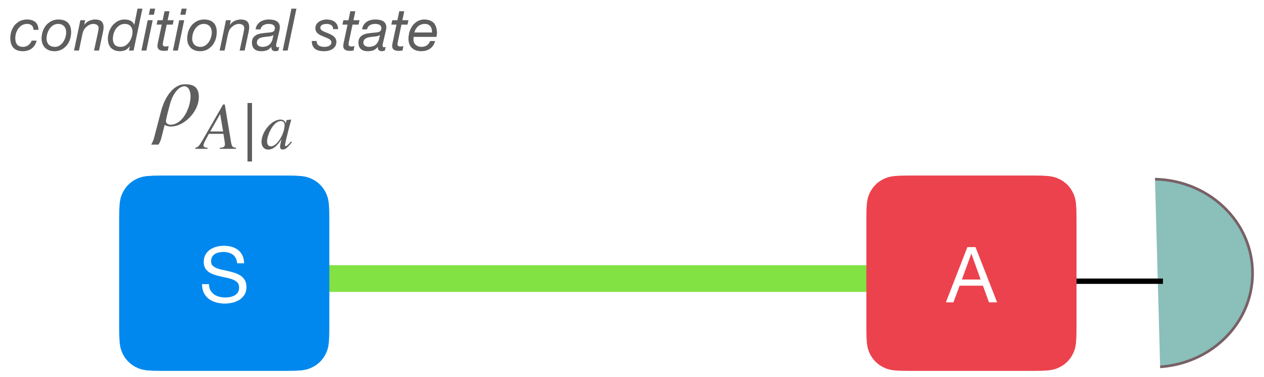
- …if I have no access to the auxiliary quantum system A:



Maximum extractable work: $\mathcal{E}(\rho_S)$ with $\rho_S = \mathrm{Tr}_A[\rho_{SA}]$

ergotropy of the reduced state

- …if I can perform a measurement on the auxiliary system A:



Maximum extractable work: $\overline{\mathcal{E}}_{\Pi_a^A} = \sum_a p_a \mathcal{E}(\rho_{A|a})$

average *ergotropy of the conditional states*

Daemonic ergotropy

Daemonic enhancement $\overline{\mathcal{E}}_{\Pi_a^A} = \sum_a p_a \mathcal{E}(\rho_{A|a}) \geq \mathcal{E}(\rho_S)$

interpretation: *by obtaining information on S via the measurement on A, I can optimize the optimal work-extraction unitary $\hat{U}_a$ differently for each conditional state $\rho_{S|a}$*

daemonic ergotropy

What if I have a correlated bipartite quantum state and I want to extract energy from one of the two parties only?

Francica et al., NPJ Quantum Information **3** (2017)

- …if I have no access to the auxiliary quantum system A:

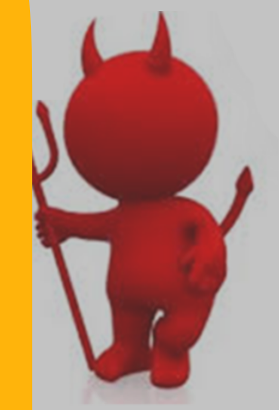
reduced state ρ_S

- …i

*One can **generalize** this framework to the scenario where one has **access** also to an **additional thermal bath** at a certain temperature T and obtain an **equivalent result** for the extractable **daemonic work***

$$\overline{\mathscr{W}^{th}}_{\Pi_a^A} = \sum_a p_a \mathscr{W}^{th}(\rho_{A|a}) \geq \mathscr{W}^{th}(\rho_S)$$

Manzano et al., PRL **121**, 120602 (2018)



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Daemonic enhancement $\overline{\mathscr{E}}_{\Pi_a^A} = \sum_a p_a \mathscr{E}(\rho_{A|a}) \geq \mathscr{E}(\rho_S)$

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daemonic ergotropy

Properties of the daemonic ergotropy

- the *daemonic ergotropy* is lower-bounded by the *reduced-state ergotropy* and upper-bounded by the *reduced-state energy:* $\mathcal{E}(\rho_S) \leq \overline{\mathcal{E}}_{\Pi_a^A} \leq E(\rho_S)$

- If the initial bipartite state is pure $|\Psi\rangle_{SA}$ and one performs a projective (rank-one) measurement $\Pi_a = |a\rangle\langle a|$ on A, then one obtains pure conditional states $|\psi_a\rangle_S$ and thus

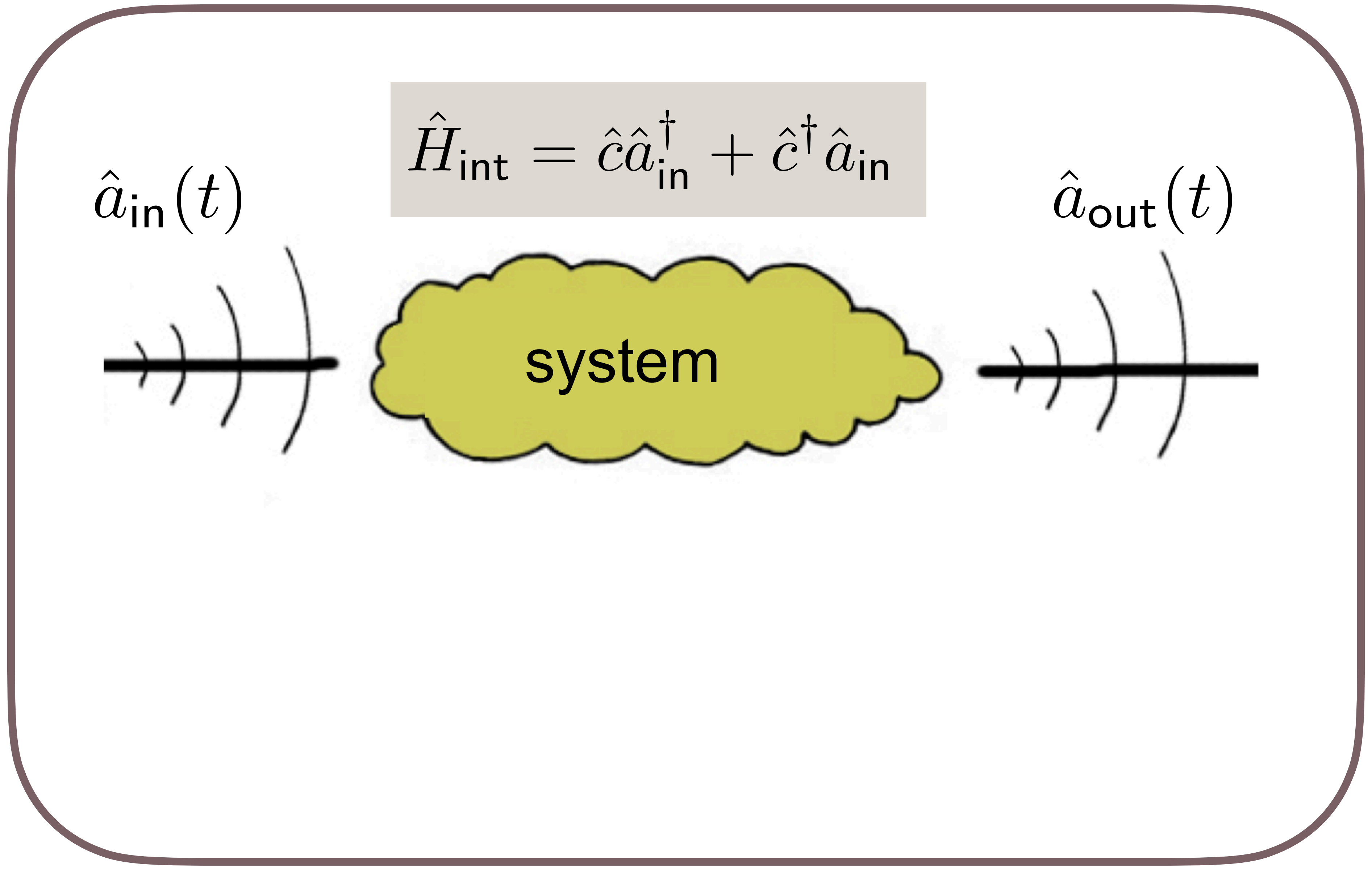
$$\overline{\mathcal{E}}_{\Pi_a^A} = \sum_a p_a \mathcal{E}(|\psi_a\rangle_{SS}\langle\psi_a|) = \sum_a p_a E(|\psi_a\rangle_{SS}\langle\psi_a|) = E(\rho_S)$$

that is, the daemonic ergotropy is equal to the reduced-state energy, **independently on the measurement performed on A**.

- in all the cases where the conditional states are mixed, then the daemonic ergotropy will depend on the particular measurement performed on A !

Francica et al., NPJ Quantum Information **3** (2017)
Morrone, Rossi and MGG, Phys. Rev. Applied **20** (2023).

continuously monitored open quantum systems



Environment : uncorrelated input modes

$$[\hat{a}_{\text{in}}(t), \hat{a}_{\text{in}}^\dagger(t')] = \delta(t - t')$$

Wiseman & Milburn,
Quantum Measurement and Control,
Cambridge University Press (2010)

Albarelli & Genoni, PLA **494**, 129260 (2024)

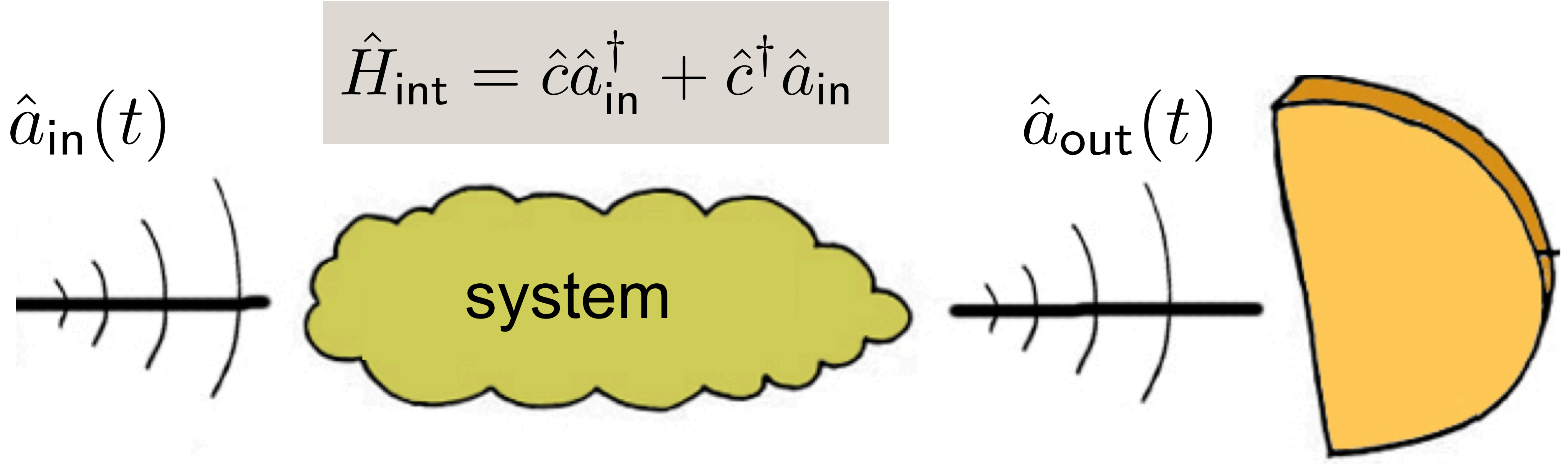
…if the output modes are ***not measured***

Unconditional dynamics : Markovian master equation in the Lindblad form

$$\frac{d\varrho_{\text{unc}}}{dt} = -i\left[\hat{H}_{\text{sys}}, \varrho_{\text{unc}}\right] + \mathcal{D}\left[\hat{c}\right]\varrho_{\text{unc}}$$

where $\mathcal{D}[\hat{A}]\varrho = \hat{A}\varrho\hat{A}^\dagger - \{\hat{A}^\dagger\hat{A}, \varrho\}/2$

continuously monitored quantum systems



…if the output modes are ***continuously monitored*** ?

Environment : uncorrelated input modes

$$[\hat{a}_{\text{in}}(t), \hat{a}^\dagger_{\text{in}}(t')] = \delta(t - t')$$

Wiseman & Milburn,
Quantum Measurement and Control,
Cambridge University Press (2010)

Albarelli & Genoni, PLA **494**, 129260 (2024)

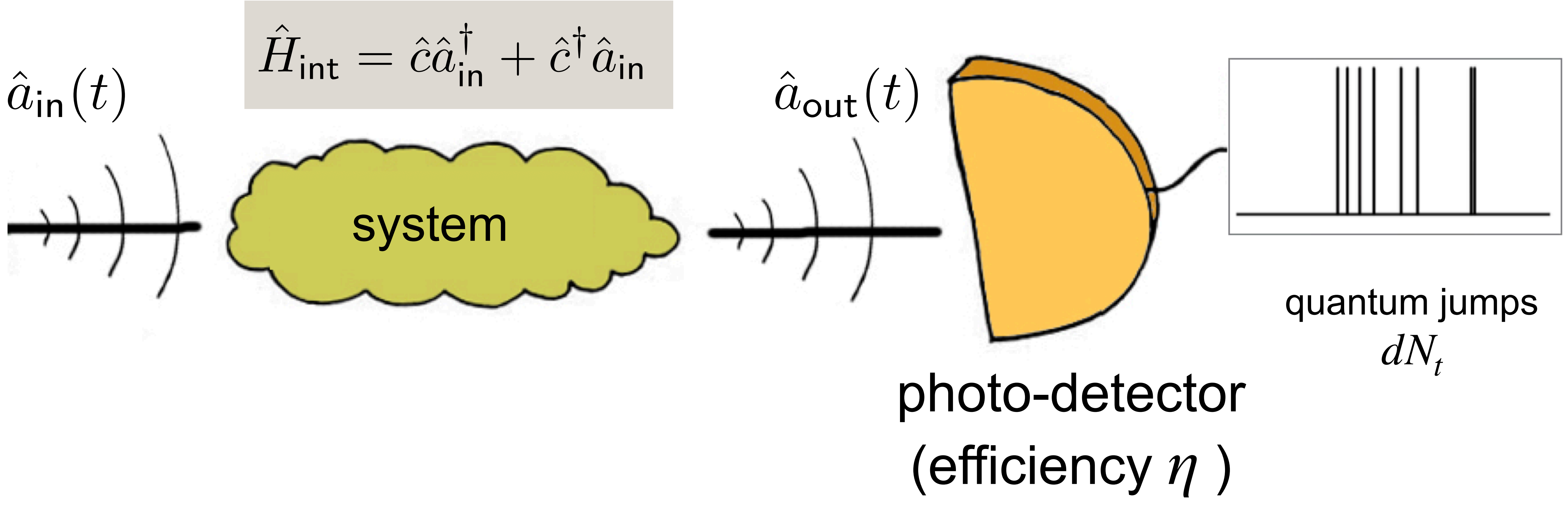
I obtain a **conditional dynamics** described by conditional states ϱ_c , typically called ***quantum trajectories***, *characterized by a certain probability* p_{traj} . In general I have:

$$\varrho_{\text{unc}} = \sum_{\text{traj}} p_{\text{traj}}\, \varrho_c$$

an ***unravelling*** of the unconditional master equation

…I have infinite possible unravellings, depending on the measurement I perform on the output modes…

continuously monitored quantum systems



several experimental demonstrations in cavity QED and circuit QED…

Gleyzes et al., Nature **446** (2007)
Vijay et al., PRL **106** (2011)
…
…

…if the output modes are continuously monitored via ***photodetection***

Conditional dynamics : stochastic master equation for the trajectory

$$d\varrho_c = -i[\hat{H}_{\text{sys}}, \varrho_c]\,dt + (1-\eta)\mathscr{D}[\hat{c}]\varrho_c\,dt - \frac{\eta}{2}(\hat{c}^\dagger\hat{c}\varrho_c + \varrho_c\hat{c}^\dagger\hat{c})\,dt +$$
$$+\eta\,\text{Tr}[\varrho_c\hat{c}^\dagger\hat{c}]\varrho_c\,dt + \left(\frac{\hat{c}\varrho_c\hat{c}^\dagger}{\text{Tr}[\varrho_c\hat{c}^\dagger\hat{c}]} - \varrho_c\right)dN_t$$

Poisson increment

$dN_t = \{0,1\}\quad \mathbb{E}[dN_t] = \eta\,\text{Tr}[\varrho_c\hat{c}^\dagger\hat{c}]\,dt$

…if one averages over the trajectories recovers the Markovian master equation for the unconditional state…

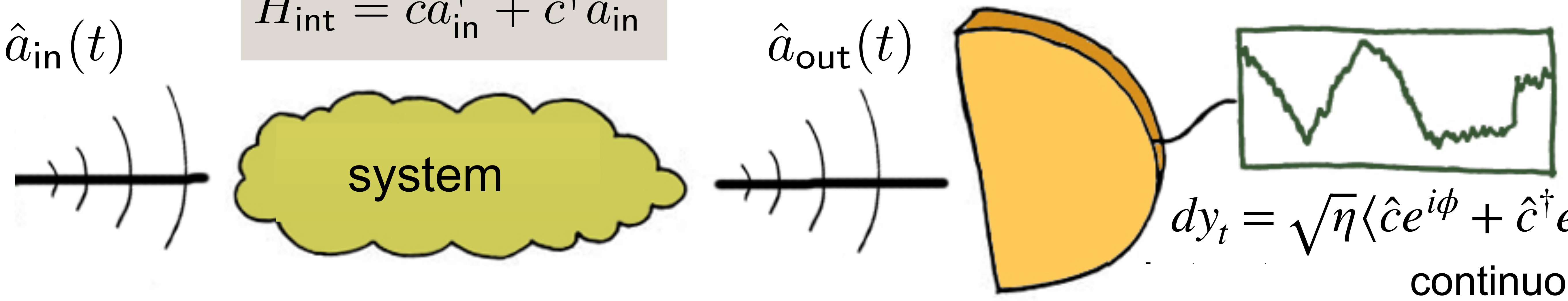
$$\mathbb{E}_{\text{traj}}[d\varrho_c] = \sum_{\text{traj}} p_{\text{traj}}\, d\varrho_c$$
$$= -i[\hat{H}_{\text{sys}}, \varrho_{\text{unc}}]\,dt + \mathscr{D}[\hat{c}]\varrho_{\text{unc}}\,dt$$

continuously monitored quantum systems

$\hat{a}_{\text{in}}(t)$

$\hat{H}_{\text{int}} = \hat{c}\hat{a}^{\dagger}_{\text{in}} + \hat{c}^{\dagger}\hat{a}_{\text{in}}$

$\hat{a}_{\text{out}}(t)$



$dy_t = \sqrt{\eta}\langle \hat{c}e^{i\phi} + \hat{c}^{\dagger}e^{-i\phi}\rangle_t\, dt + dW_t$

continuous
photo-current

homodyne
detector
(efficiency η)

several experimental demonstrations in circuit QED and quantum optomechanics…

Murch et al., Nature **502** (2013)
Campagne-Ibarcq et al., PRX **6** (2016
Naghiloo et al., Nat. Comm. **7** (2016)
Ficheux et al., Nat. Comm. **9** (2018)
…
Rossi et al., PRL **123** (2021)
Magrini et al., Nature **595** (2021)

…if the output modes are continuously monitored via ***homodyne detection***

Conditional dynamics : stochastic master equation for the trajectory

$$d\varrho_c = -i[\hat{H}_{\text{sys}}, \varrho_c]\,dt + \mathscr{D}[\hat{c}]\varrho_c\,dt$$
$$+\sqrt{\eta}\,\mathscr{H}[\hat{c}e^{i\phi}]\varrho_c\,dW_t$$

Wiener increment

$\mathbb{E}[dW_t] = 0, \quad \mathbb{E}[dW_t^2] = dt$

where $\mathscr{H}[\hat{c}]\varrho = \hat{c}\varrho + \varrho\hat{c}^{\dagger} - \text{Tr}[(\hat{c} + \hat{c}^{\dagger})\varrho]\varrho$

…if one averages over the trajectories recovers the Markovian master equation for the unconditional state…

$$\mathbb{E}_{\text{traj}}[d\varrho_c] = \sum_{\text{traj}} p_{\text{traj}}\, d\varrho_c$$
$$= -i[\hat{H}_{\text{sys}}, \varrho_{\text{unc}}]\,dt + \mathscr{D}[\hat{c}]\varrho_{\text{unc}}\,dt$$

quantum metrology ...in continuously monitored systems

- **quantum magnetometry** with atomic ensembles
  - ✓ analytical formula for the QFI in the *large number of atoms and noiseless* regime via continuous quantum non-demolition measurement (**Heisenberg scaling)**

    Albarelli, Rossi, Paris, MGG, NJP **19** (2017)

  - ✓ *(apparent)* **super-classical scaling** for independent atomic dephasing via continuous quantum non-demolition measurement

    Rossi, Albarelli, Tamascelli, MGG, PRL **125** (2022)

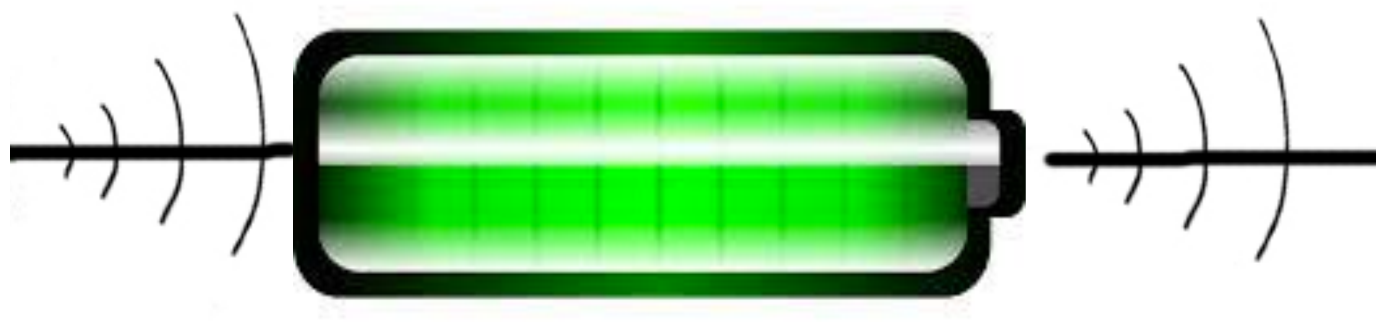
- **Restoring the Heisenberg scaling** in **noisy quantum metrology** by monitoring the environment

  Albarelli, Rossi, Tamascelli, MGG, Quantum **2** (2018)

- Optical **phase-estimation** via **reinforcement-learning optimized continuous feedback** strategies

  Fallani, Rossi, Tamascelli, MGG, PRX Quantum **3** (2022)

continuously monitored open quantum batteries

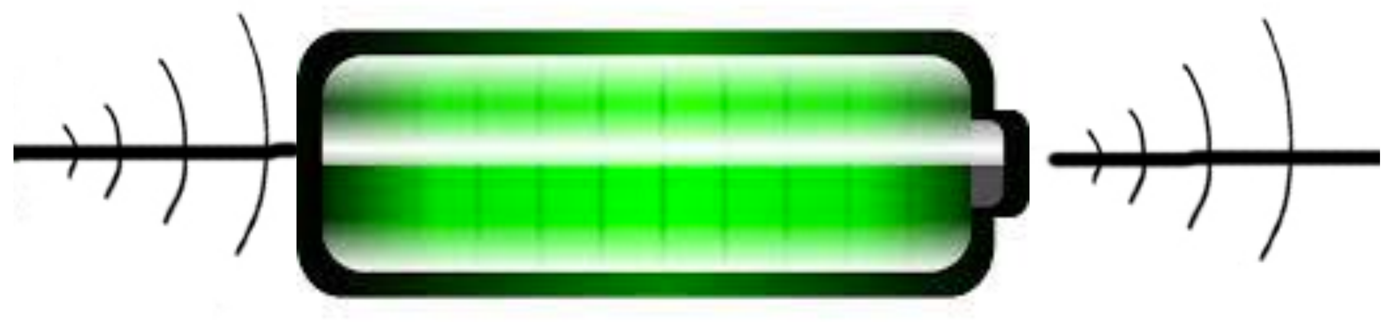


What is an open quantum battery?

a quantum energy-storing device interacting with an environment and thus undergoing decoherence.

Farina et al., PRB **99**, 1 (2019).
Morrone et al., QST **8**, 035007 (2023)

continuously monitored open quantum batteries



What is an open quantum battery?

a quantum energy-storing device interacting with an environment and thus undergoing decoherence.

Farina et al., PRB **99**, 1 (2019).
Morrone et al., QST **8**, 035007 (2023)

What happens if I can monitor the environment?

...I can observe quantum trajectories for the quantum battery...

$$\varrho_{\text{unc}} = \sum_{\text{traj}} p_{\text{traj}}\, \varrho_c$$

Daemonic enhancement via unravelling $\overline{\mathscr{E}}_{\text{unr},\eta} = \sum_{\text{traj}} p_{\text{traj}} \mathscr{E}(\varrho_c) \geq \mathscr{E}(\varrho_{\text{unc}})$

by monitoring the environment, I can optimize the energy extraction unitary on each quantum trajectory!

unravelling daemonic ergotropy

Properties of the "unravelling daemonic ergotropy"

- the *daemonic ergotropy* is lower-bounded by the *unconditional ergotropy* and upper-bounded by the *unconditional energy:* $\mathscr{E}(\varrho_{\text{unc}}) \leq \overline{\mathscr{E}}_{\text{unr},\eta} \leq E(\varrho_{\text{unc}})$

- If I have a conditional dynamics described by
  - an initial pure quantum state
  - a perfect detection of the environment affecting the quantum battery (efficiency $\eta = 1$)

then I have a *pure states unravelling* (each trajectory remains in a pure state): $\varrho_{\text{unc}} = \sum_{\text{traj}} p_{\text{traj}} |\psi_c\rangle\langle\psi_c|$

$$\overline{\mathscr{E}}_{\text{unr},\eta} = E(\varrho_{\text{unc}})$$

daemonic ergotropy is equal to the unconditional energy independently on the measurement performed on the environment!

…any unravelling will give the same identical (and optimal) result in terms of work extractable!

unravelling daemonic ergotropy

Properties of the "unravelling daemonic ergotropy"

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daemonic ergotropy is equal to the unconditional energy independently on the measurement performed on the environment!

…any unravelling will give the same identical (and optimal) result in terms of work extractable!

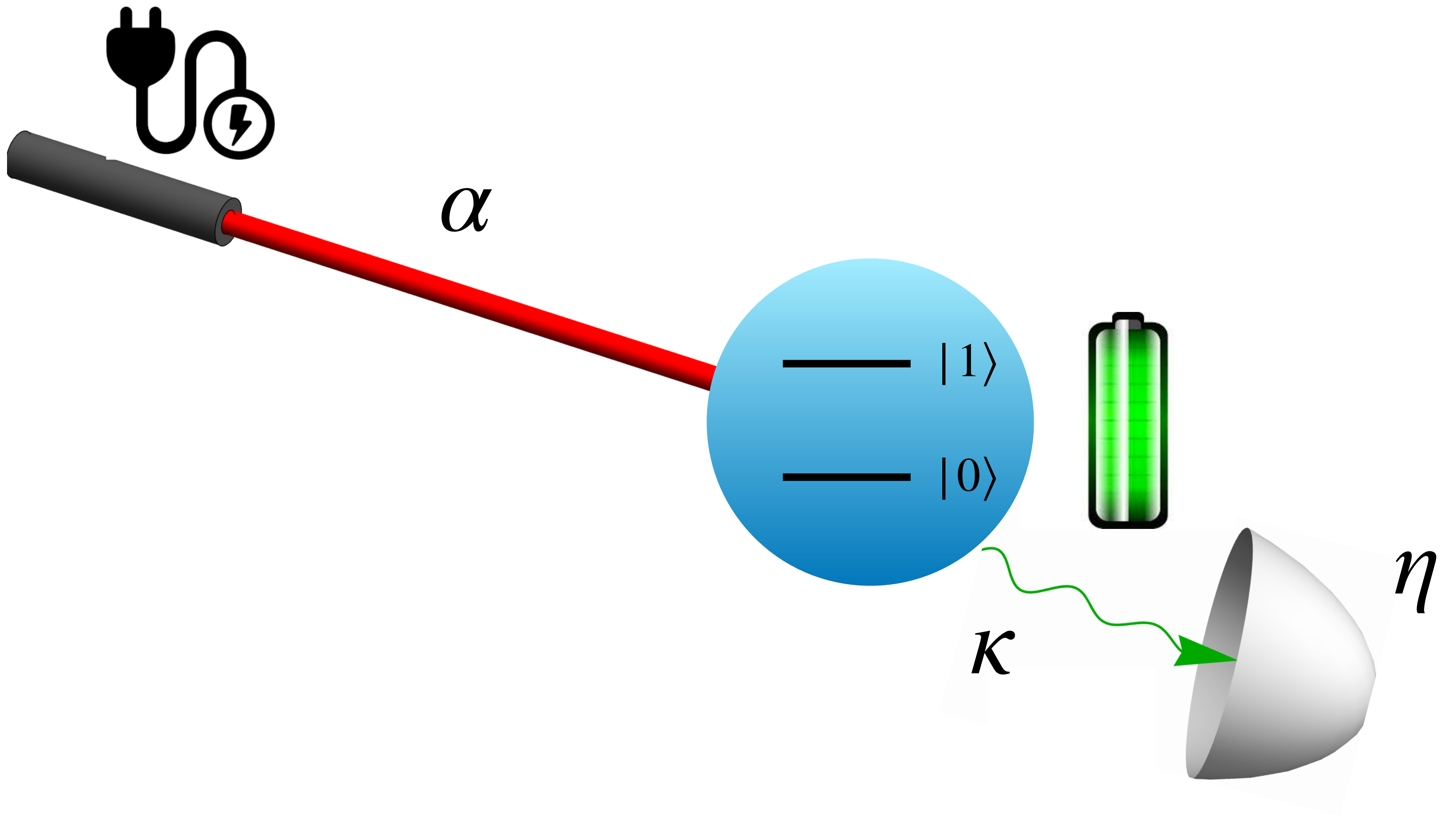
What happens for "mixed states unravelling" (i.e. for non-unit efficiency monitoring or initial mixed states)?

Which unravelling is more efficient for a quantum battery ?

a continuously monitored open quantum battery…

a paradigmatic example…

- *quantum battery*: two-level atom
  $$\hat{H}_0 = \frac{\omega_0}{2}(\sigma_z + 1)$$
- *charger*: classical driving
- *noise*: fluorescence (amplitude damping)



unconditional master equation:

$$\frac{d\varrho_{\text{unc}}}{dt} = -i\alpha[\sigma_x, \varrho_{\text{unc}}] + \kappa\mathscr{D}[\sigma_-]\varrho_{\text{unc}}$$

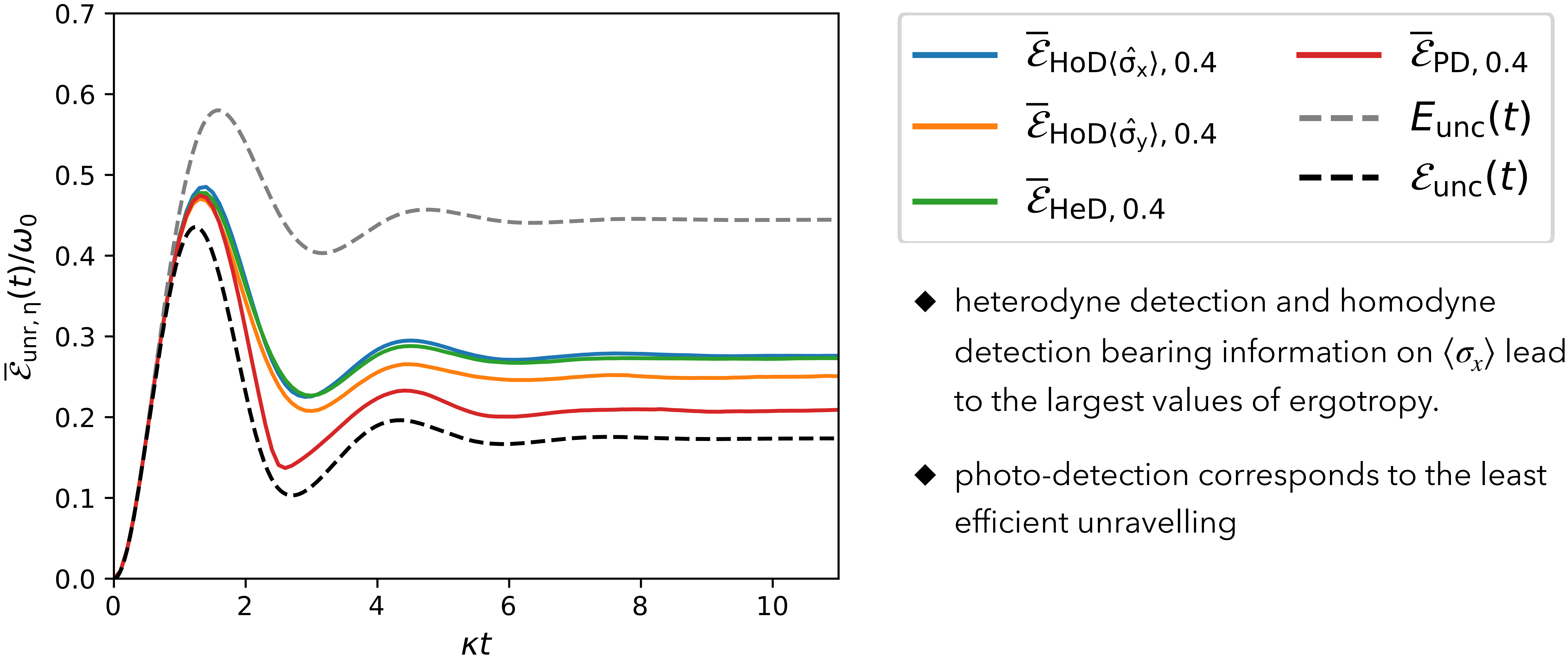
- *possible unravellings/detections*:
  photocounting
  homodyne
  heterodyne
  (*with efficiency* η)

Campagne-Ibarcq et al., PRX 6 (2016)
Naghiloo et al., Nat. Comm. 7 (2016)

a continuously monitored open quantum battery…

Results

Daemonic ergotropy as a function of time for different unravellings with efficiency $\eta = 0.4$ (initial state: ground state $\rho_0 = |0\rangle\langle 0|$).

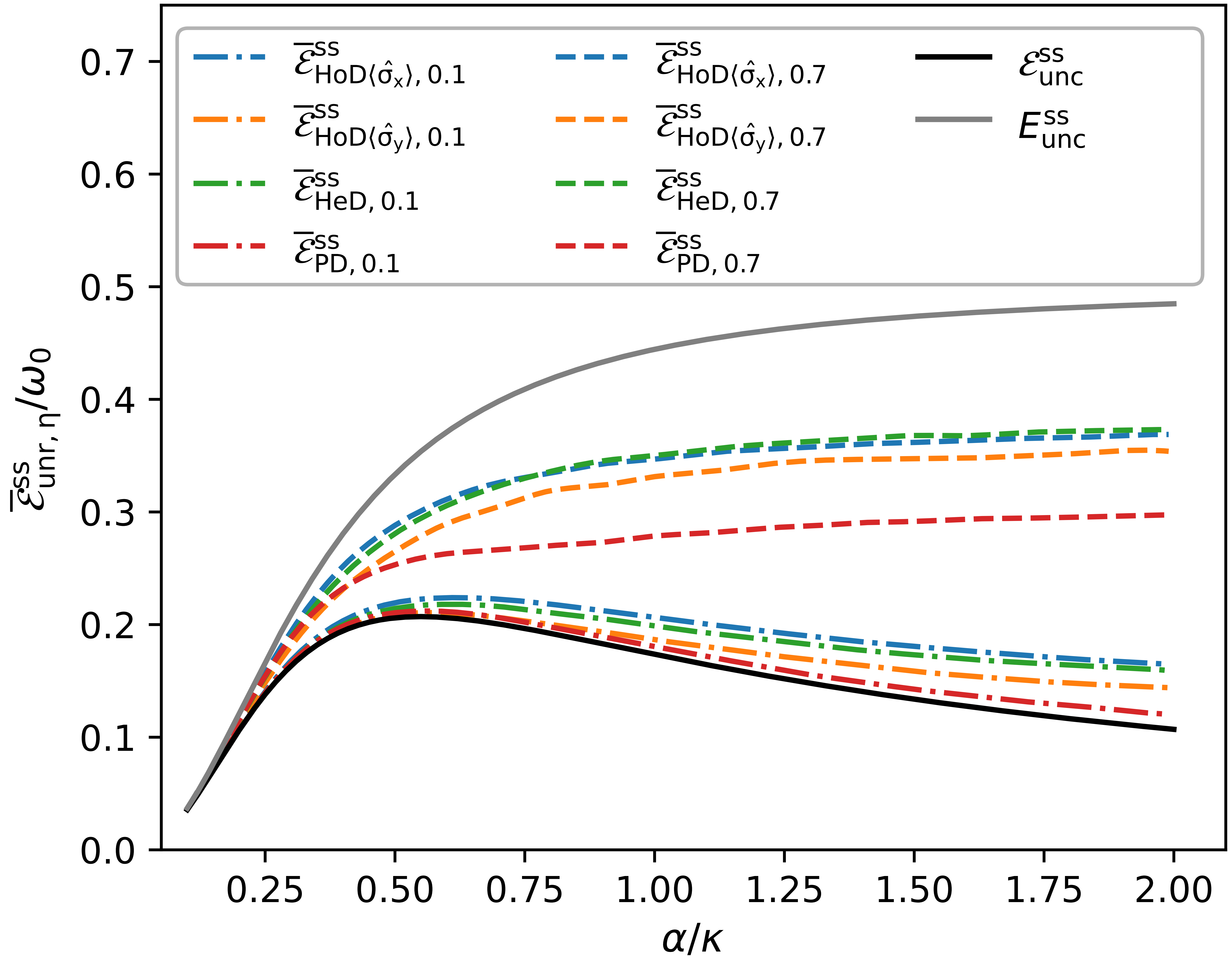


- heterodyne detection and homodyne detection bearing information on $\langle \sigma_x \rangle$ lead to the largest values of ergotropy.
- photo-detection corresponds to the least efficient unravelling

a continuously monitored open quantum battery…

Results

Steady-state daemonic ergotropy as a function of classical driving for $\eta = 0.1$ and $\eta = 0.7$.

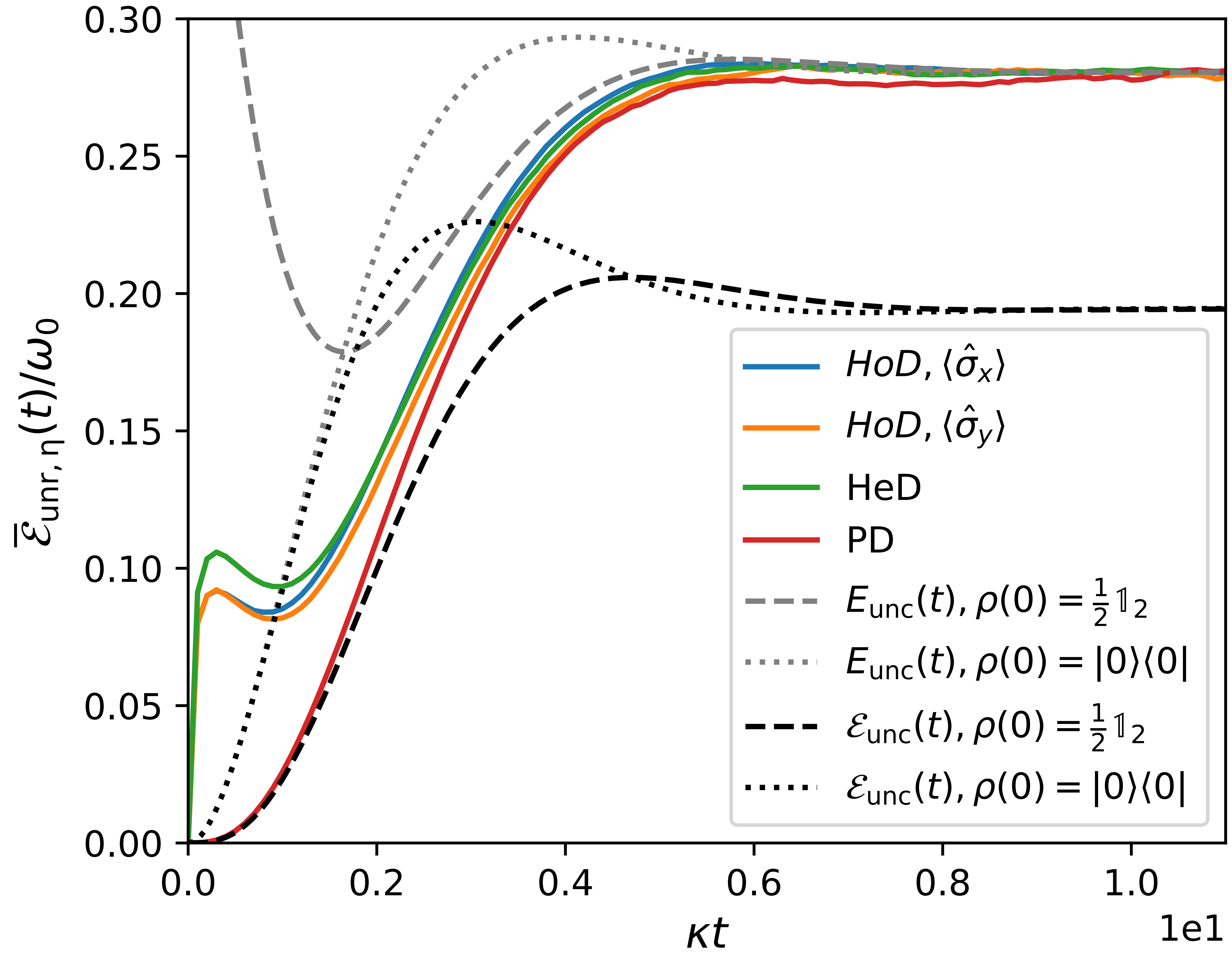


- heterodyne detection and homodyne detection bearing information on $\langle\sigma_x\rangle$ lead to the largest values of ergotropy.
- photo-detection corresponds to the least efficient unravelling
  …also at steady-state….
- for large values of η the steady-state ergotropy seems to saturate by increasing α/κ, similarly to the unconditional energy, while the unconditional ergotropy presents a maximum.

a continuously monitored open quantum battery…

Results

Daemonic ergotropy as a function of time for different unravellings for $\eta = 1$ for an initial mixed state $\varrho_0 = \hat{1}/2$ *(passive state, but with some initial energy…)*



- all unravellings eventually purify all the trajectories, and consequently at steady-state $\overline{\mathscr{E}}_{\mathrm{unr},\eta} = E(\varrho_{\mathrm{unc}})$ independently on the strategy.
- different unravellings lead to a different *purification speed.*
- at small times heterodyne and homodyne detection allow to achieve values of daemonic ergotropy larger than the ones obtainable by starting from the ground state.

at small times in monitoring-enhanced charging protocols, purification is more efficient than pure energy injection

outline of the talk

- daemonic ergotropy in continuously monitored quantum batteries

Daniele Morrone
University of Milan (IT)

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D. Morrone, M.A.C. Rossi and MGG, Phys. Rev. Applied **20**, 044073 (2023).

- experimental verification of daemonic work extraction on a digital quantum computer

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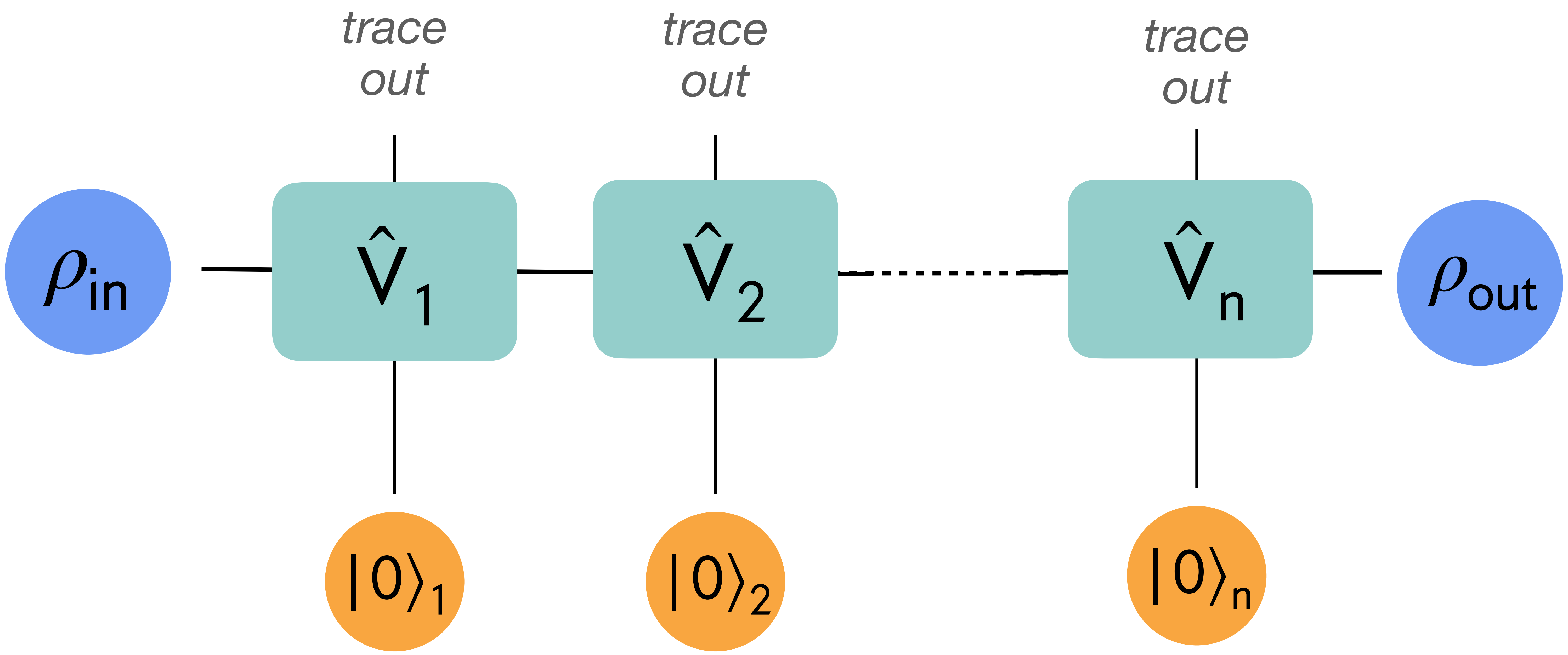
S.N. Elyasi, M.A.C. Rossi and MGG, arXiv:2410.16567 (*to appear in Quantum Science & Technology*)

open quantum systems as collision models

an introduction to quantum (Markovian) collision models

Effective and versatile toolbox to understand the behaviour of open quantum systems :

- discretization of environment
- discretization of time



Markovian assumptions

- auxiliary systems (ASs) do not interact with each other
- ASs are initially uncorrelated
- each AS collides with the system only once .

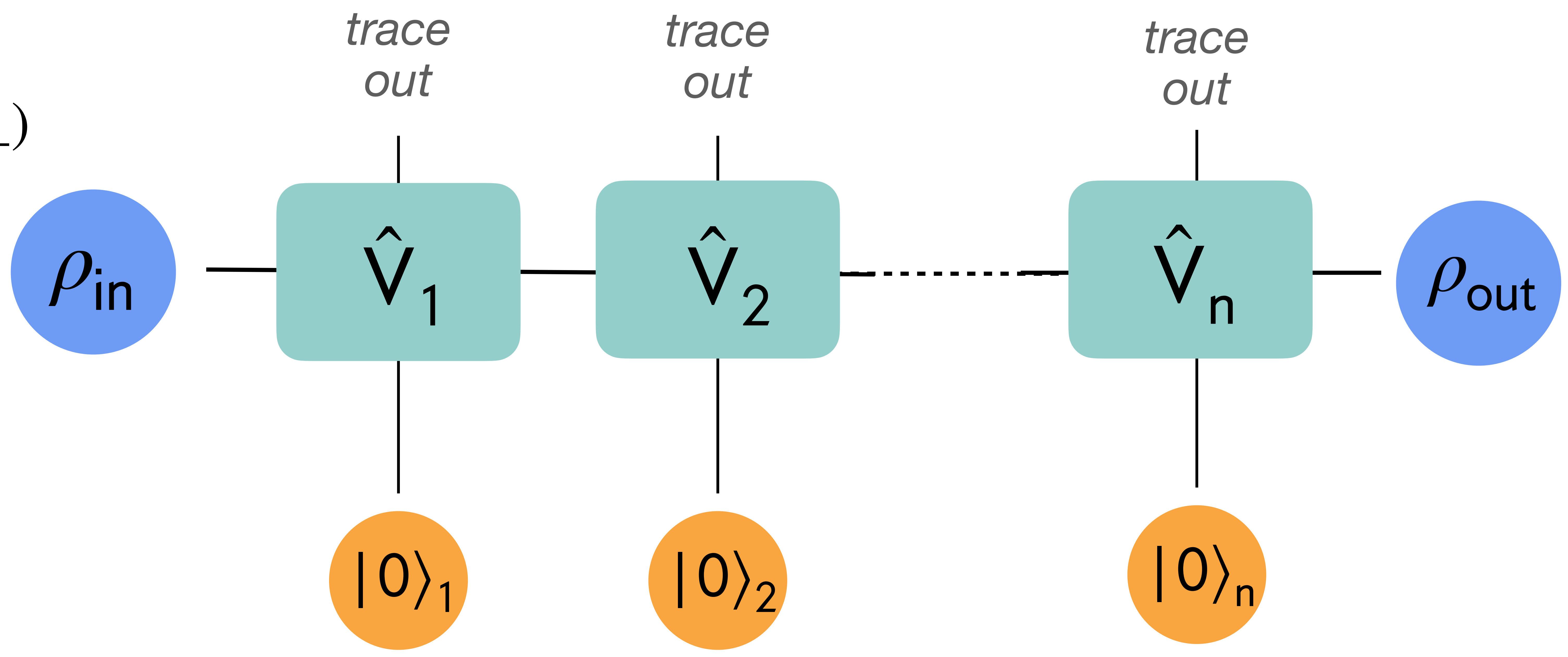
Ciccarello et al., Phys. Rep. **954**, 1 (2022)

open quantum systems as collision models

relationship to continuous time evolution...

if one considers

- $V_j = e^{-iH_{int}}$ with
  $H_{int} = \alpha(\sigma_x \otimes \mathrm{Id}) + \kappa(\sigma_+ \otimes \sigma_- + \sigma_- \otimes \sigma_+)$



then, by introducing a time unit Δt and by choosing $\alpha = \tilde{\alpha}\Delta t$ and $\kappa = \sqrt{\tilde{\kappa}/\Delta t}$, the dynamics described by the collision model converges in the **limit** $\Delta t \to 0$ to the **Markovian master equation**:

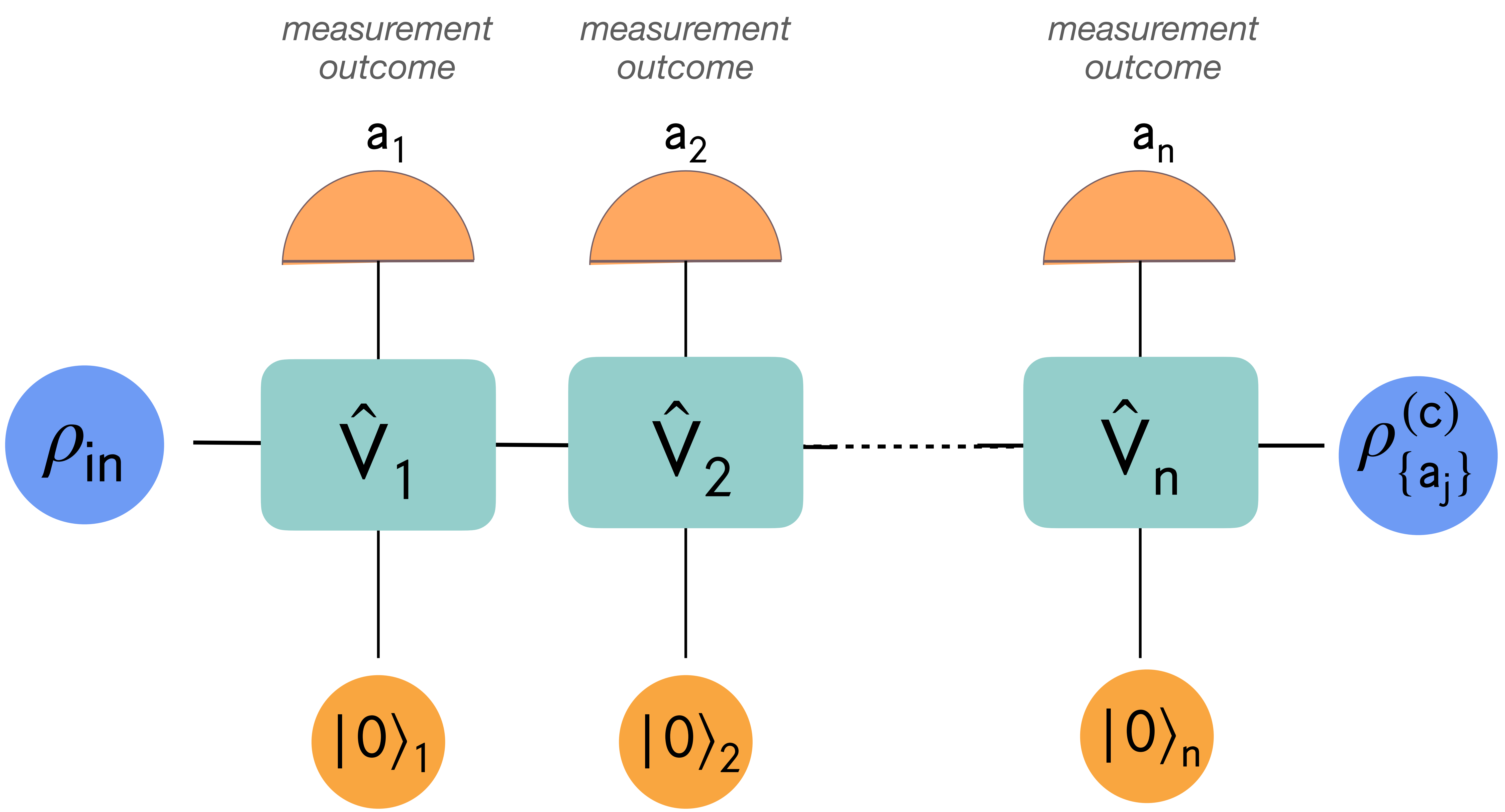
$$\frac{d\rho}{dt} = -i\tilde{\alpha}[\sigma_x, \rho] + \tilde{\kappa}\mathcal{D}[\sigma_-]\rho$$

...the same master equation we considered before

Ciccarello et al., Phys. Rep. **954**, 1 (2022)

continuously monitored collision models (CMCM)

…what if we measure the auxiliary systems?



…we obtain a conditional evolution for the quantum state $\rho^{(c)}_{\{a_j\}}$ depending on the measurement results…

we have an *unravelling* of the previous collision model

G. Landi *et al.*, PRX Quantum **3**, 010303 (2022)

…relationship to continuous time evolution?

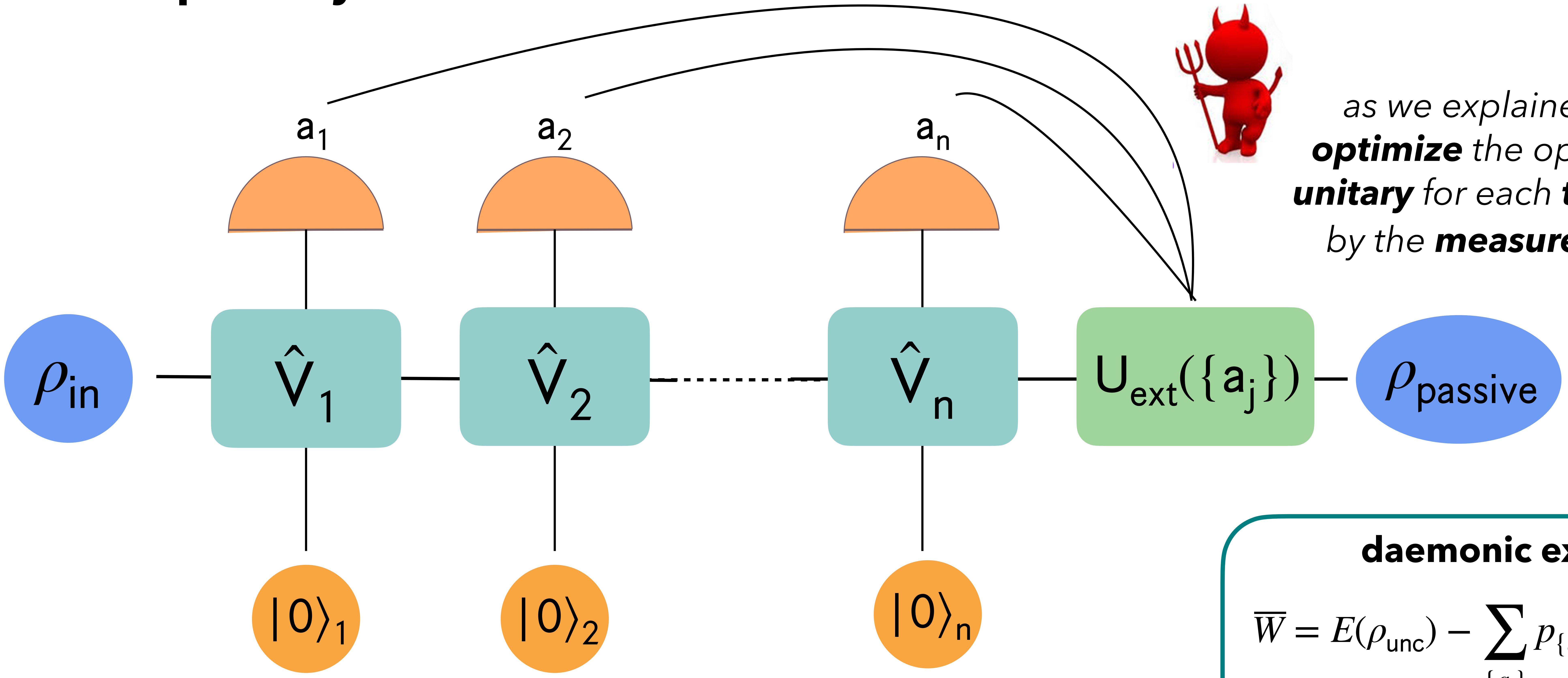
by properly taking the continuous time limit, one can also recover/derive the *usual* stochastic master equations…

Albarelli & Genoni, PLA **494**, 129260 (2024)

Gross et al, Quantum Sci. Technol. **3**, 024005 (2018)

daemonic work extraction in CMCM

how to optimally extract work?



as we explained before, one has to ***optimize*** *the optimal* ***work extraction unitary*** *for each* ***trajectory*** *characterized by the* ***measurement outcomes*** $\{a_j\}$

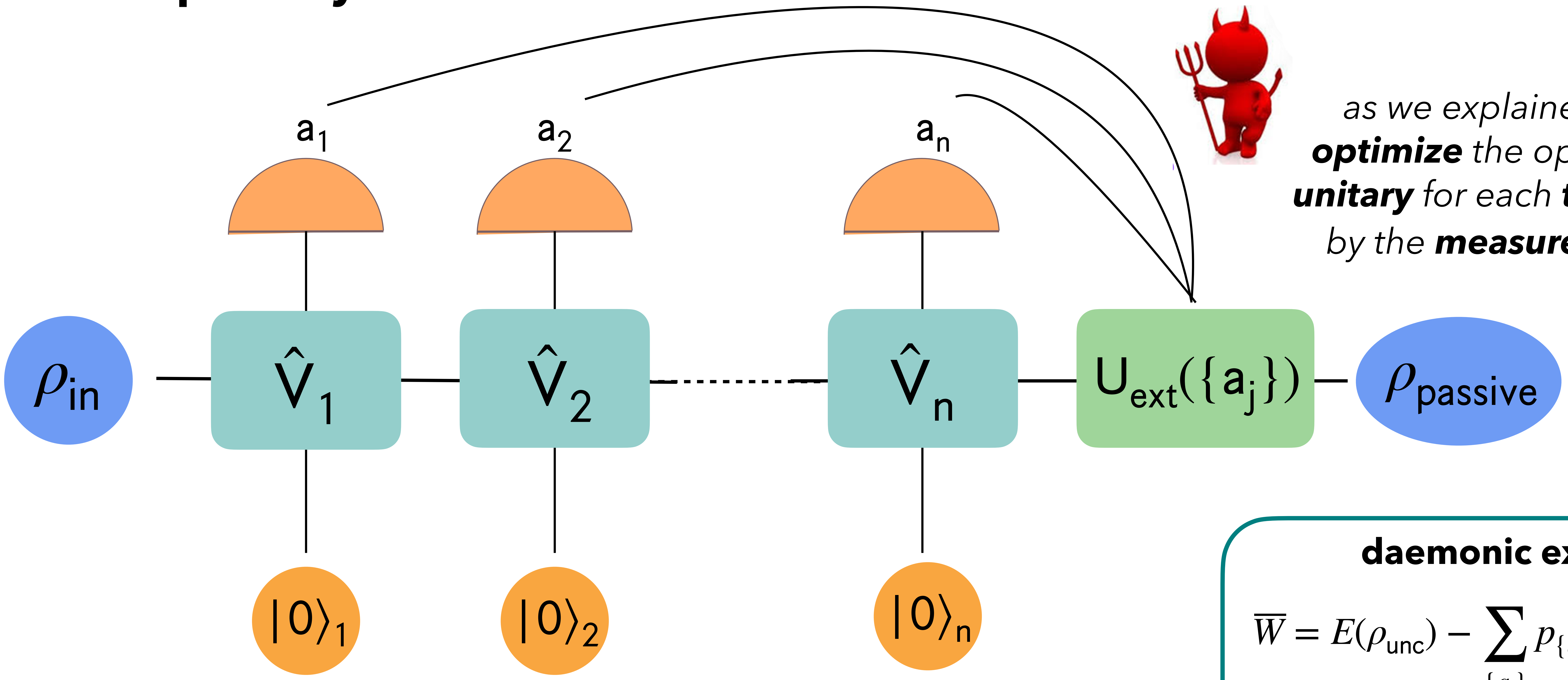
daemonic extracted work

$$\overline{W} = E(\rho_{\text{unc}}) - \sum_{\{a_j\}} p_{\{a_j\}} E(U_{\text{ext}} \rho_{\{a_j\}} U^{\dagger}_{\text{ext}}) \leq \overline{\mathcal{E}}$$

equal to the daemonic ergotropy if one implements the optimal unitary for each trajectory

daemonic work extraction in CMCM

how to optimally extract work?



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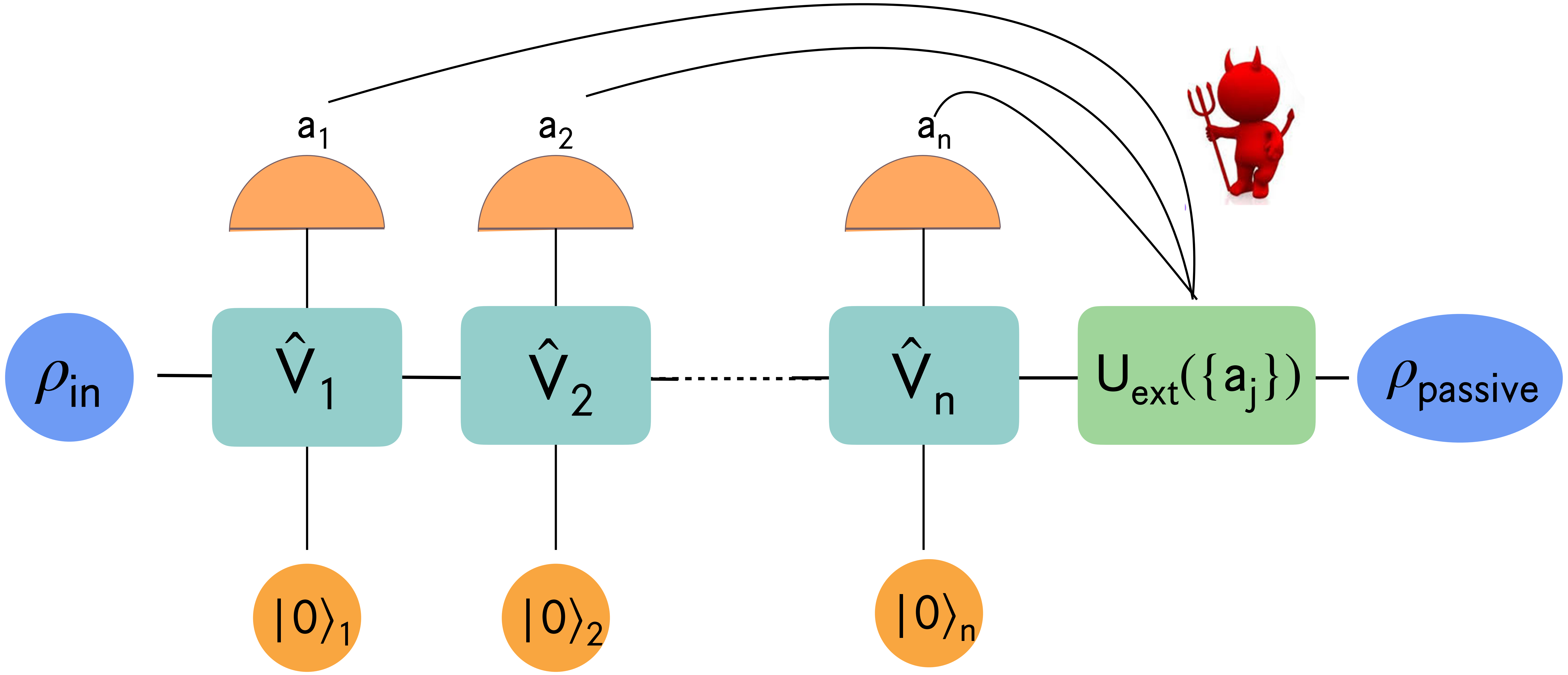
Can we implement a CMCM and demonstrate a daemonic work extraction protocol on a digital quantum computer?

CMCM and daemonic work extraction on an IBM q-computer



- $V_j = e^{-iH_{int}}$ with
  $H_{int} = \alpha(\sigma_x \otimes \mathrm{Id}) + \kappa(\sigma_+ \otimes \sigma_- + \sigma_- \otimes \sigma_+)$
- measurement of σ_x operator

implementable on IBM quantum computer thanks to ***dynamic circuits*** (*mid-circuit measurements and feedback operations*)

Qiskit

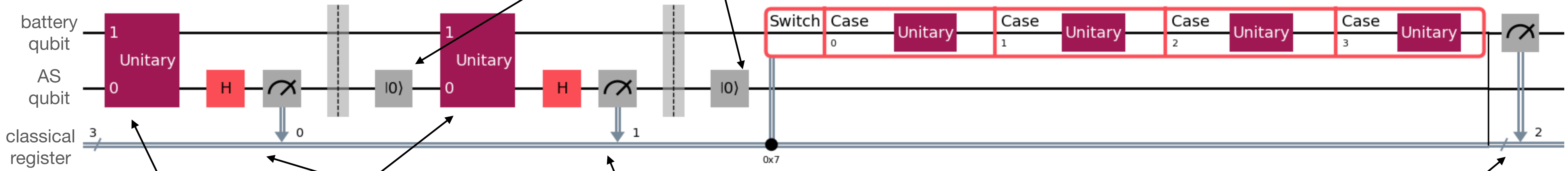
CMCM and daemonic work extraction on an IBM q-computer

example: implementation of a 2-step CMCM with Qiskit

re-inizialization of the AS

conditional and optimal work extraction unitary

(…one different unitary for each one of the 2^n trajectories)



battery energy measurement

unitary charging and battery-environment interaction

$$V = e^{-iH_{int}}$$

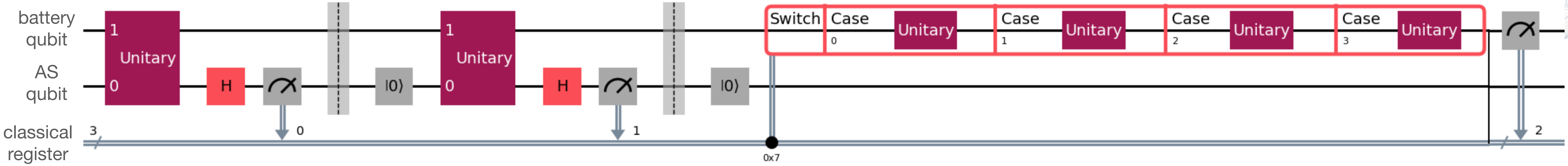
$$H_{int} = \alpha(\sigma_x \otimes \mathrm{Id}) + \kappa(\sigma_+ \otimes \sigma_- + \sigma_- \otimes \sigma_+)$$

measurement of σ_x operator

implementable on IBM quantum computer thanks to ***dynamic circuits*** *(mid-circuit measurements and feedback operations)*

Qiskit

CMCM and daemonic work extraction on an IBM q-computer



pre-processing of optimal work extraction unitaries via numerical simulation

- one first performs a **numerical simulation** of the n-step **collision model** in order to obtain, for each one of the 2^n trajectories, the **optimal conditional unitary** sending the output conditional state towards its corresponding **passive state**.

experimental simulation of q-trajectories to evaluate the daemonic work extracted

- by **running** this circuits on IBM q-computer for a **large number** of times (i.e. by generating a large number of **trajectories**) one gets an **estimate** of the average **energy** of the state **after** the ***work extraction*** unitary:

$$\sum_{\{i_j\}} p_{\{i_j\}}\, E(U_{\text{ext}}\rho_{\{i_j\}}U^\dagger_{\text{ext}})\ .$$

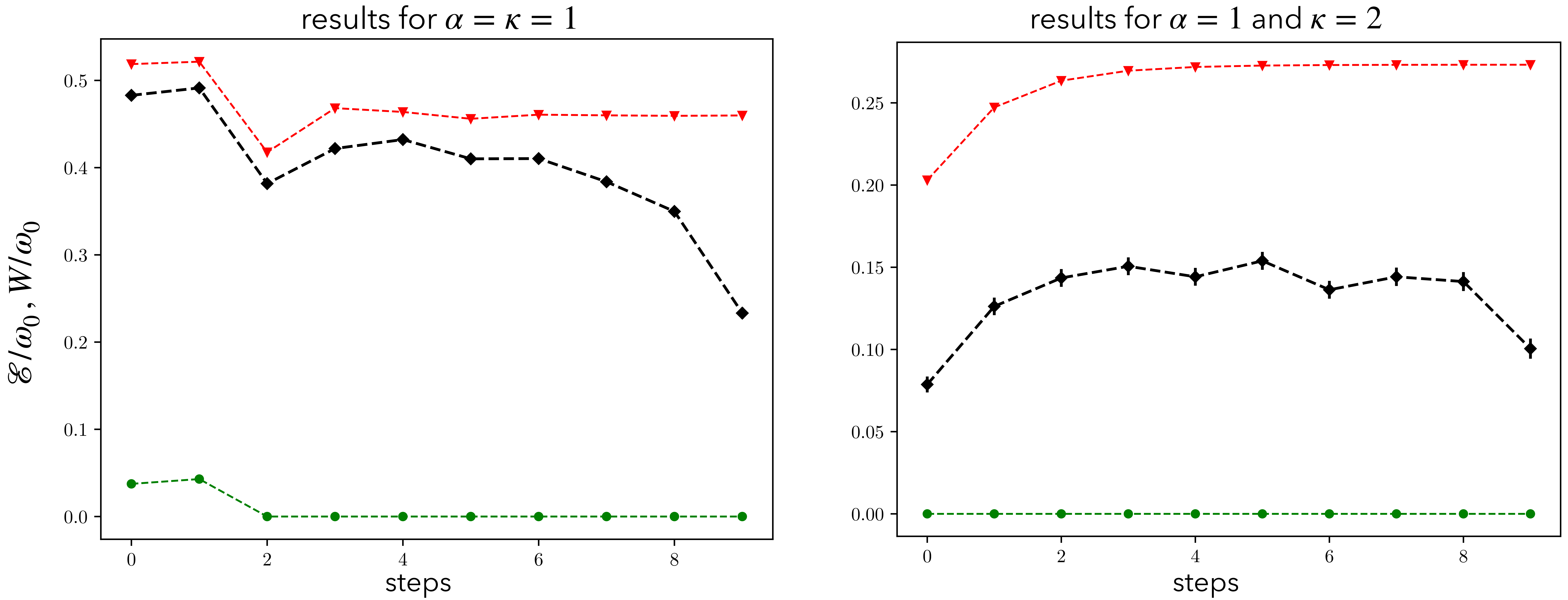
- by **running** a similar circuits, but **without measurements** and work extraction **unitary**, one gets an **estimate** of the **unconditional energy** $E(\rho_{\text{unc}})$.

- by **combining** both **terms**, we obtain the **experimental daemonic extracted work** :

$$\overline{W} = E(\rho_{\text{unc}}) - \sum_{\{i_j\}} p_{\{i_j\}}\, E(U_{\text{ext}}\rho_{\{i_j\}}U^\dagger_{\text{ext}})\ .$$

S.N. Elyasi, M.A.C. Rossi and MGG, arXiv:2410.1656

daemonic work extraction on a IBM quantum computer



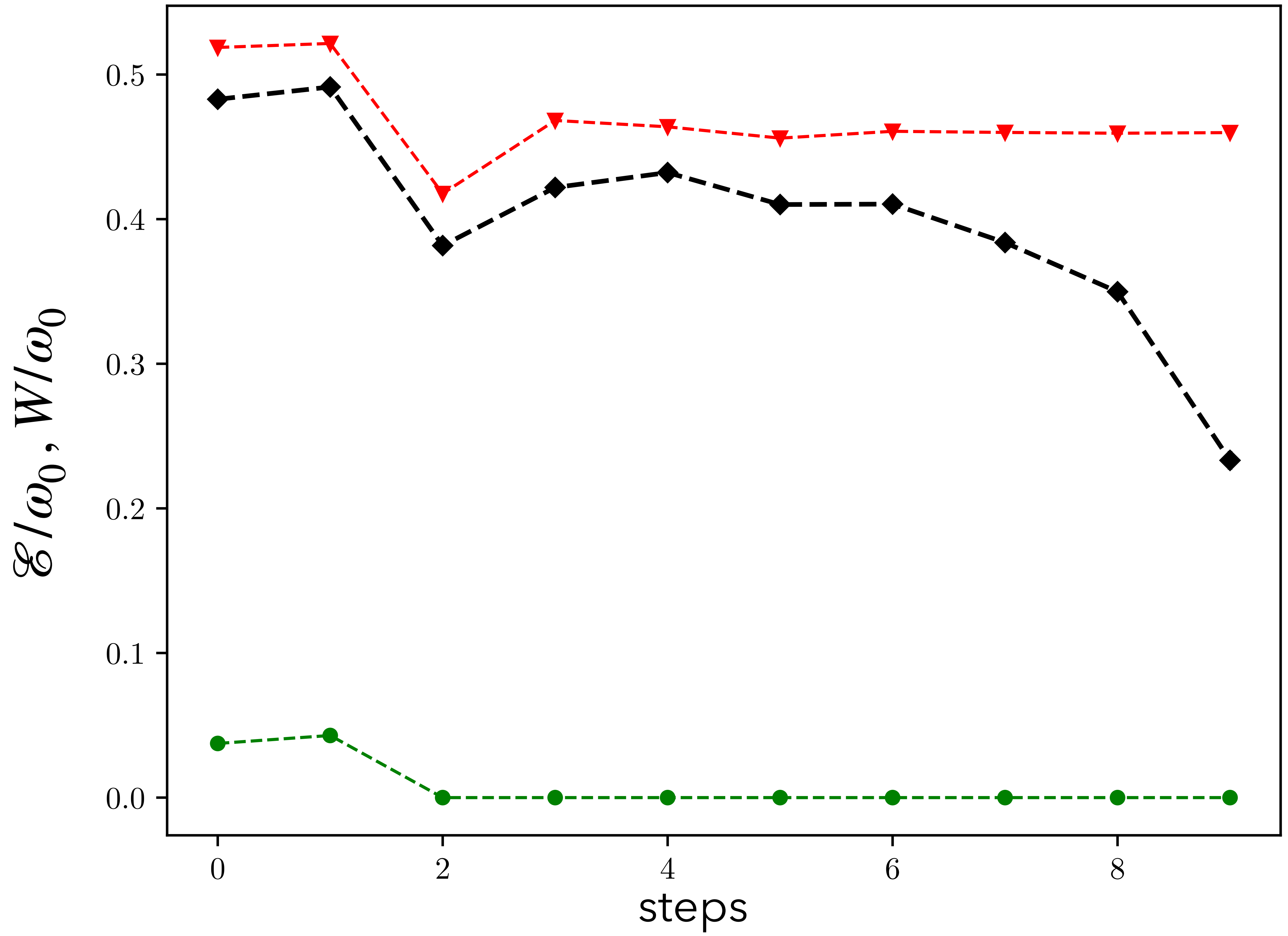
- Daemonic ergotropy - theory (noiseless model)
- Unconditional ergotropy - theory (noiseless model)
- **Daemonic extracted work** - **exp. results** *ibm-osaka* (noiseless model for optimal extraction unitary)

- ***efficient* implementation** of a **CMCM** model with **feedback operation** on a IBM **quantum computer**
- **experimental** proof of principle **demonstration/ simulation** of **daemonic work extraction**: $\overline{W} > \mathscr{E}(\rho_{\text{unc}})$

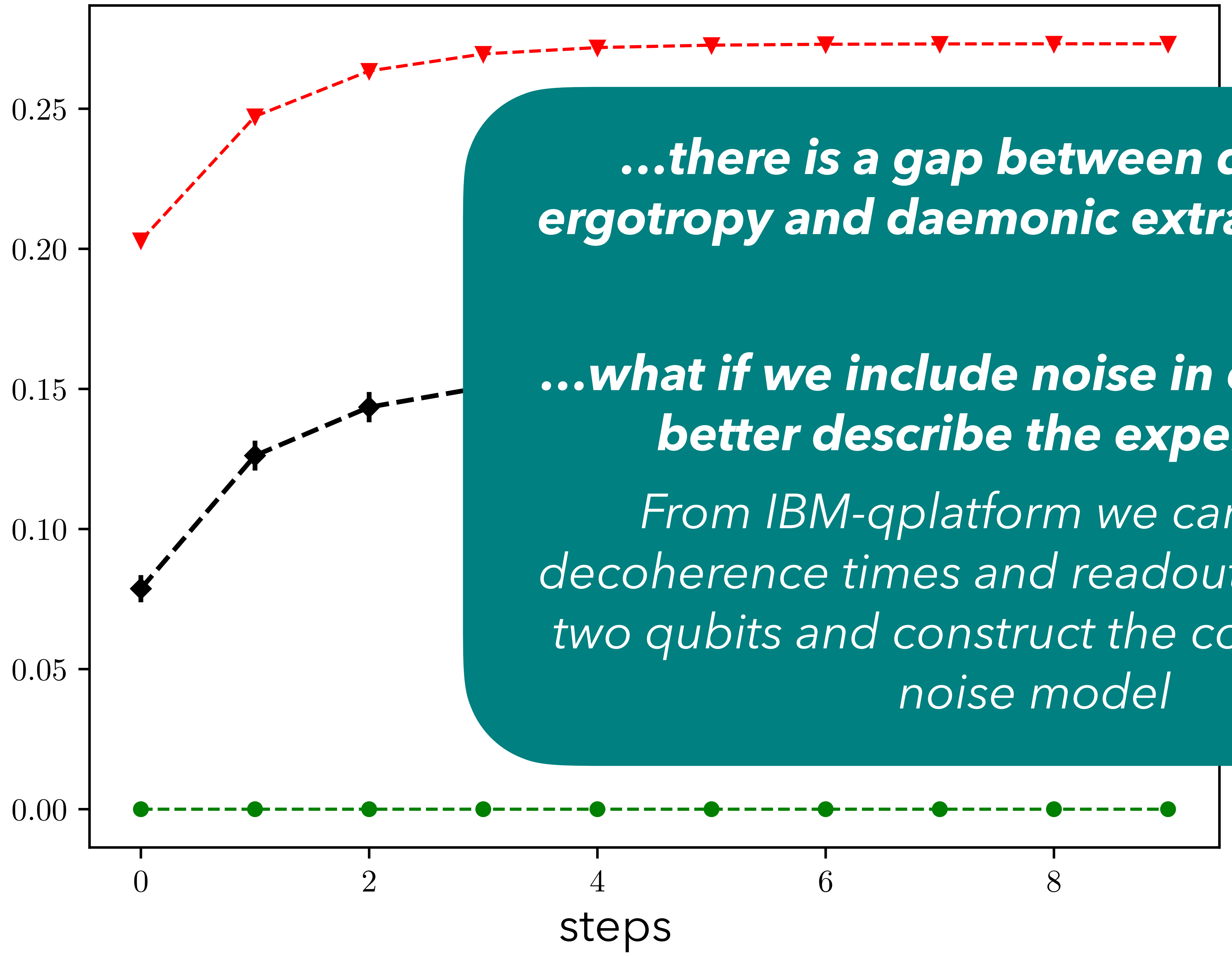
S.N. Elyasi, M.A.C. Rossi and MGG, arXiv:2410.1656

daemonic work extraction on a IBM quantum computer

results for $\alpha = \kappa = 1$



results for $\alpha = 1$ and $\kappa = 2$



...there is a gap between daemonic ergotropy and daemonic extracted work...

...what if we include noise in our model to better describe the experiment?

From IBM-qplatform we can retrieve decoherence times and readout times for the two qubits and construct the corresponding noise model

Daemonic ergotropy - theory (noiseless model)

Unconditional ergotropy - theory (noiseless model)

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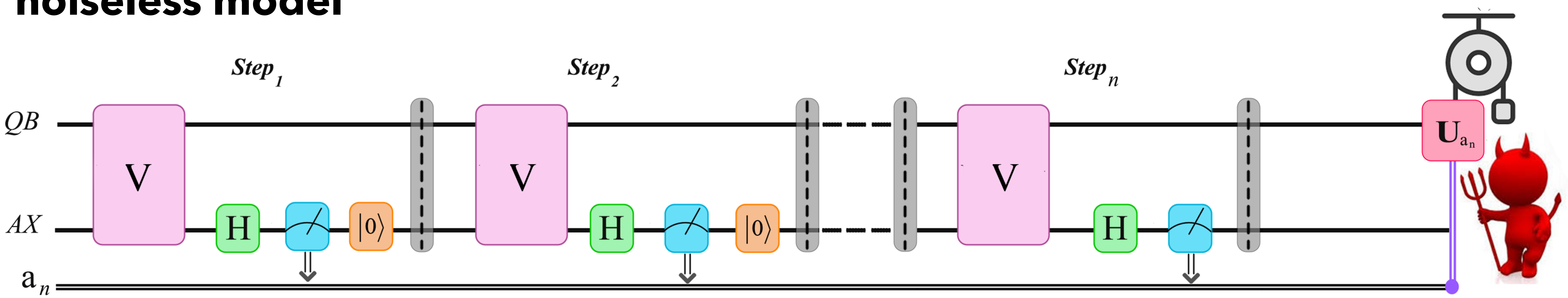
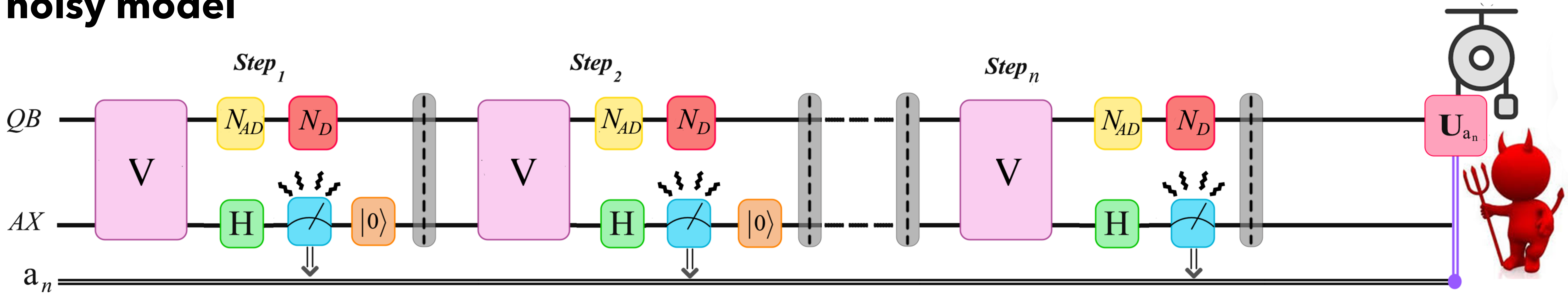
noiseless and noisy CMCM

noiseless model

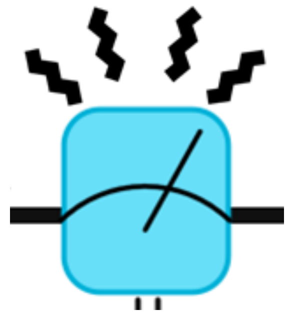


noisy model

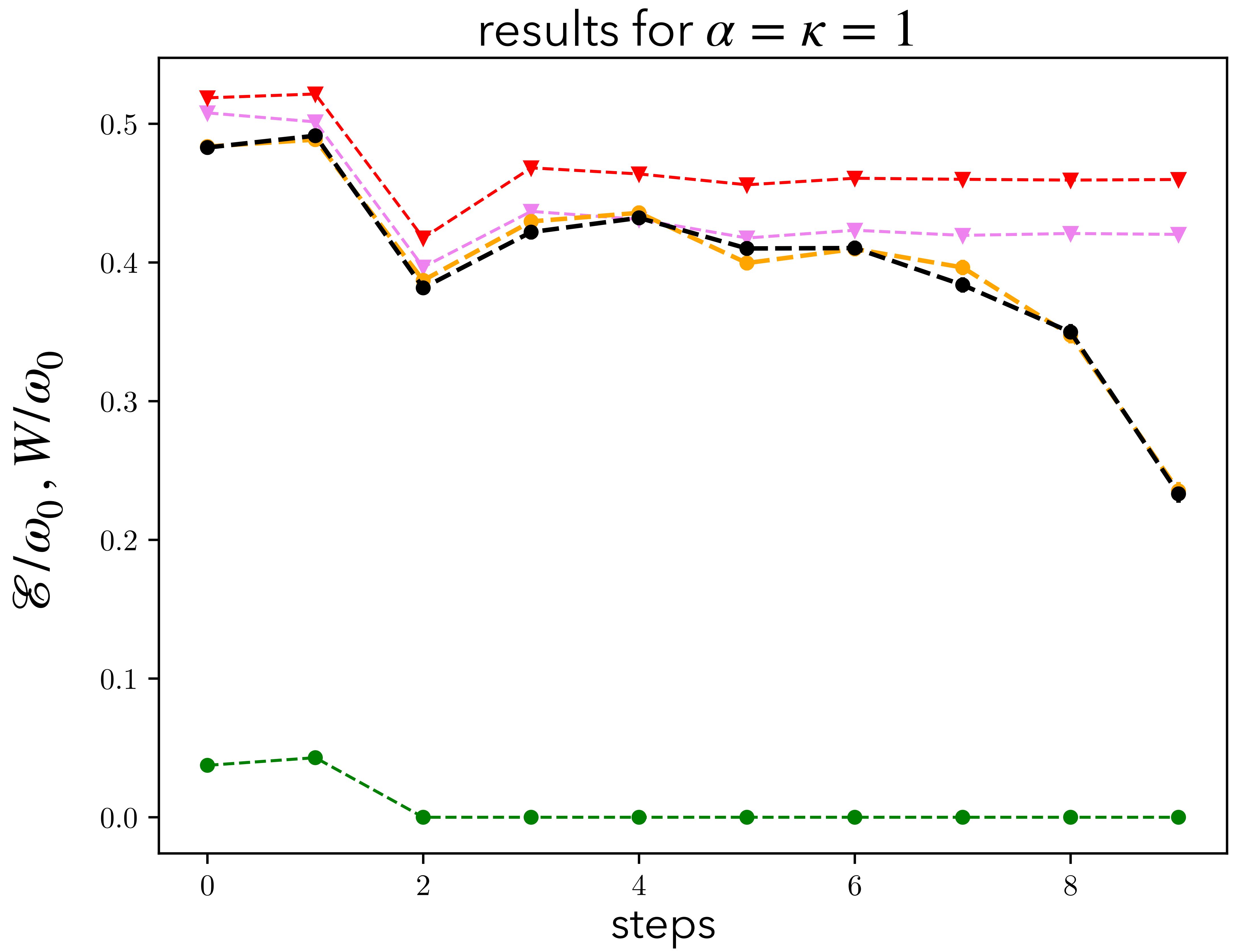


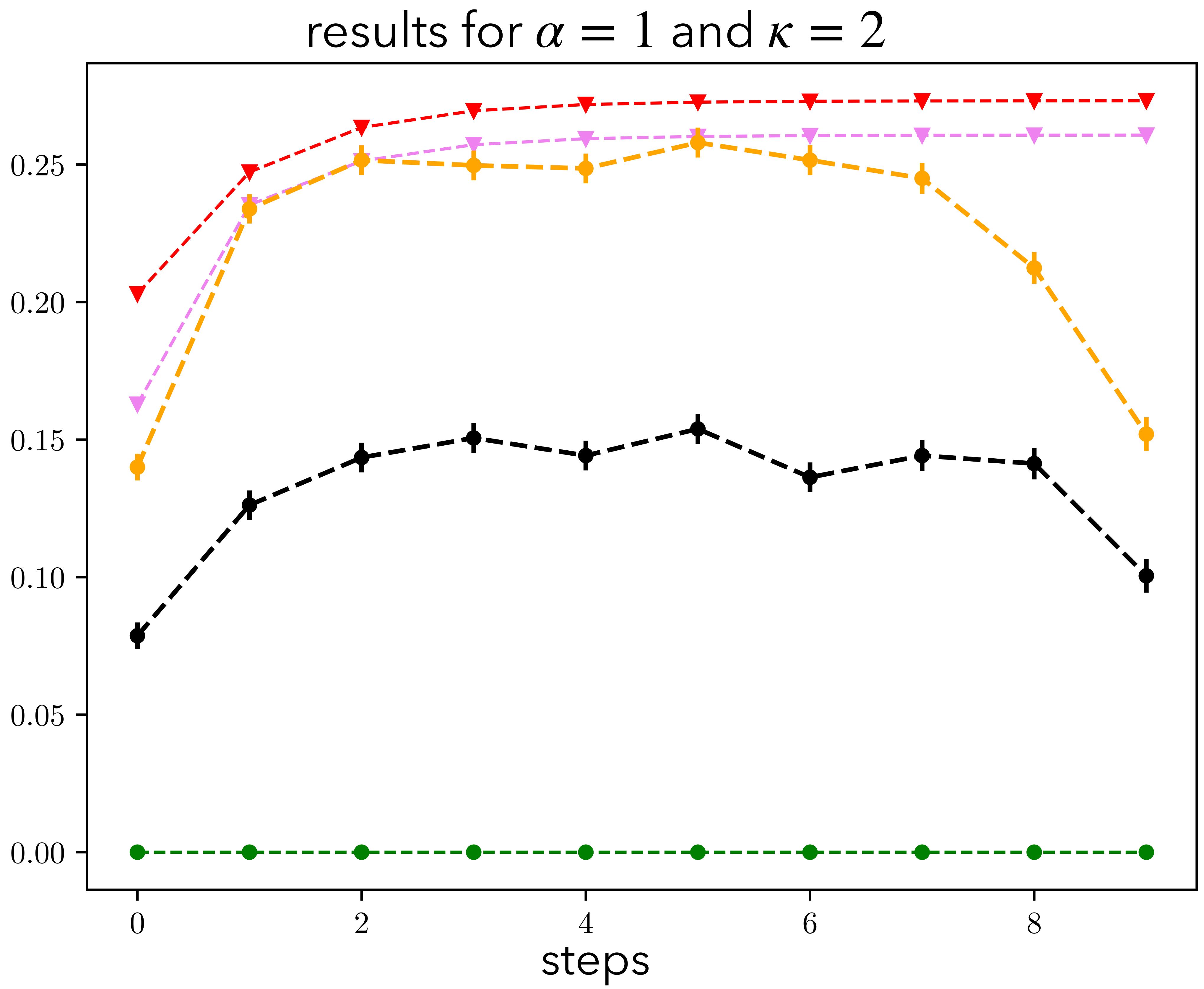
amplitude damping channel

dephasing channel


noisy $\hat{\sigma}_z$ measurement

daemonic work extraction on a IBM quantum computer





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- Daemonic ergotropy - theory (**noisy model**)
- **Daemonic extracted work** - **exp. results** *ibm-osaka* (**noisy model** for optimal extraction unitary)

- ***daemonic extracted work*** *close to* ***theoretical ergotropy*** *of the noisy model*
- ***daemonic work extraction*** *can be* ***improved*** *by properly* ***modelling*** *the* ***noise*** *in the experiment and thus* ***optimizing*** *the work extraction* ***unitary***.

S.N. Elyasi, M.A.C. Rossi and MGG, arXiv:2410.1656

Conclusions, outlooks and acknowledgments

conclusions

- extension of the concept of daemonic ergotropy to open quantum systems.
- simplest example of a *monitoring-enhanced open quantum battery:* hierarchy between unravellings for finite detection efficiency (…*homodyne and heterodyne seem to be the best strategies…*)
- proof of principle exp. demonstration of *daemonic work extraction* on an IBM quantum computer

outlooks

- design of *monitoring*- and *feedback-enhanced* strategies for more complex quantum batteries (e.g. Dicke quantum batteries)
- fundamental relationship between daemonic ergotropy and 2nd law of thermodynamics

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