Aspects of Jet physics in the QGP

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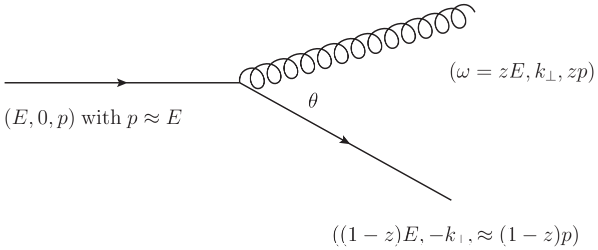
Jets in vacuum

1. Jets originate in showers of energetic partons which hadronize, and carry information about the parton shower
2. The parton shower can be understood perturbatively
3. It is collimated because the (slightly offshell) inital parton radiates energetic particles preferentially at small angles



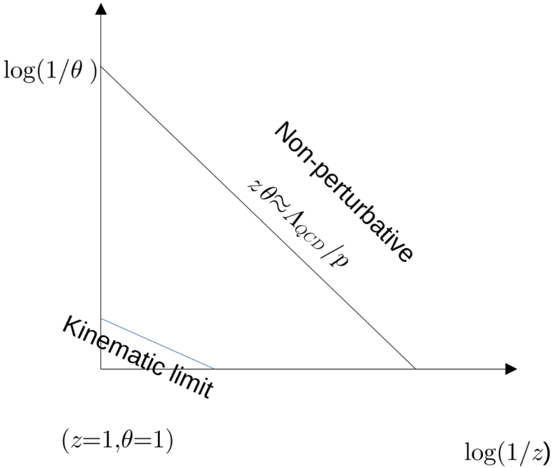
The branching probability

1. For example, the probability of $q \to qg$

$$dP(z, k_\perp) = \frac{\alpha_s}{2\pi} \frac{dk_\perp^2}{k_\perp^2} P_{q\to qg}(x) dz$$

2. $P_{q\to qg}(z) = C_F \frac{1+(1-z)^2}{z}$ is the Alterelli-Parisi splitting function
3. $\log(k_\perp)$ (collinear), $\log(z)$ (soft) divergences

The Lund plane



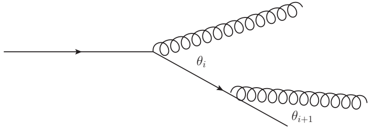
1. With $k_\perp \approx zp\theta$ (small angles)

$$dP(z, k_\perp) \sim d\log(z)d\log(\theta)$$

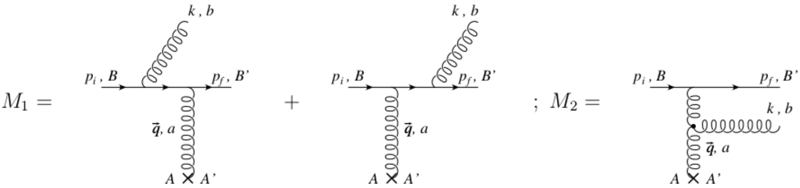
2. Uniform distribution of branching in this plane

Formation time and color flow

1. $t_f \sim \frac{\omega}{k_\perp^2}$, is a measure of the off-shellness of the initial parton
2. Consider the $(i+1)^{\rm th}$ branching with angle θ_{i+1}. Then $t_{f(i+1)} \sim \frac{1}{\omega_{i+1}\theta_{i+1}^2}$
3. The transverse separation between the $i^{\rm th}$ daughters at t_f is $d_i = t_{f(i+1)}\theta_i = \frac{\theta_i}{\omega_{i+1}\theta_{i+1}^2}$
4. The transverse resolution scale of the $(i+1)^{\rm th}$ gluon is $l_{i+1} = 1/k_{\perp(i+1)} = \frac{1}{\omega_{i+1}\theta_{i+1}}$
5. Requiring $l_{i+1} < d_i$ imposes $\theta_{i+1} < \theta_i$: Angular ordering



Jets in the medium: Medium induced Bremsstrahlung



1. *[BDMPS (1993, 1994, 1996, 1998), Zakharov (1996, 1997)]*
2. Transverse scattering from the medium and resulting induced gluon emission from an energetic parton
3. Typical scattering momentum m_D. Mean free path between scatterings λ. Rate of of transverse momentum gain $\hat{q} \sim m_D^2/\lambda$. (Assume transverse scatterings are independent, $m_D < 1/\lambda$.)
4. The formation time of the emitted gluon $l_{coh} = t_f \sim \frac{\omega}{k_\perp^2}$ governs whether multiple transverse scatterings independently give rise to emission or coherently give rise to emission

Coherent emission

1. For independent gluon emission (Bethe-Heitler)

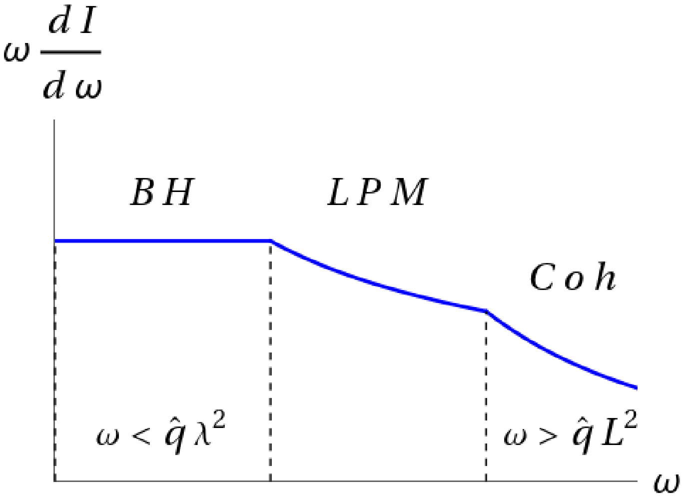
$$\omega \frac{dI}{dLd\omega} \sim \frac{\alpha_s}{\pi} N_c (\frac{1}{\lambda})$$

2. When $L > t_f > \lambda$ emission contributions from multiple scatterings add coherently. Effectively only one emission for a coherence length
3. The net transverse momentum transferred during this period $k_\perp^2 \sim l_{coh} m_D^2/\lambda$ ($\hat{q} \sim m_D^2/\lambda$). Thus $l_{coh} = \sqrt{\frac{m_D^2}{\lambda\, \omega}}$
4. Only a single emission per coherence length

$$\omega \frac{dI}{dLd\omega} \sim \frac{\alpha_s}{\pi} N_c (\frac{1}{l_{coh}}) = \frac{\alpha_s}{\pi} N_c \sqrt{\frac{m_D^2}{\lambda\, \omega}}$$

Emission suppressed by $\frac{1}{\sqrt{\omega}}$ (LPM)

Coherence regimes



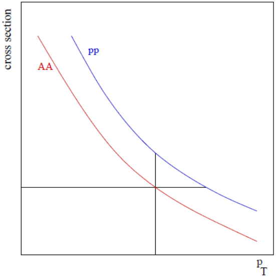
Energy loss in a weakly coupled medium

1. Integrating over ω to get the net energy loss

$$\frac{dE}{dL} \sim -\frac{\alpha_s}{\pi} N_c \sqrt{\frac{m_D^2 \omega_{max}}{\lambda}} = -\frac{\alpha_s}{\pi} N_c \frac{m_D^2}{\lambda} L = -\frac{\alpha_s}{\pi} N_c \langle k_\perp^2 \rangle \; .$$

   (Coherence length can't be larger than L)
2. $\Delta p_T = \kappa L^2 \log(\frac{p_T}{\Omega^2 L})$ (*Zakharov (2000)*)
3. Ω is related to medium scales

Energy loss leads to suppression



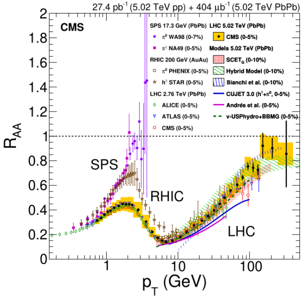
1. Energy loss or equivalently transverse momentum loss (Δp_T)

$$\left.\frac{d\sigma_{AA}}{dp_T dy}\right|_{p_T} = \left.\frac{d\sigma_{pp}}{dp_T dy}\right|_{p_T+\Delta p_T}$$

2. Due to a falling spectrun, leads to a suppression R_{AA}

$$R_{AA} = \frac{d\sigma_{AA}}{dp_T dy} \Big/ \frac{d\sigma_{pp}}{dp_T dy}, \text{ where } \frac{d\sigma_{AA}}{dp_T dy} = \frac{1}{N_{\text{evt}}\, T_{AA}} \frac{dN_{\text{jet}}}{dp_T dy}.$$

R_{AA} and more differential observables



1. This basic idea “works”.
2. Prompts us to learn more from substructures of jets.

Jet Substructure, Angularities

Jet substructure

1. Jet substructure observables measure how energy and (longitudinal and transverse to the jet axis) momentum is distributed in the jet radius and it is natural to ask how these are different in *pp* and *AA*
2. Less sensitive to the difference between the initial state parton distribution functions in *AA*, *pp*
3.
   - Jet fragmentation functions
   - Jet shapes
   - Jet angularities
   - Energy-energy correlators
   - . . .

Jet angularities

1. Consider a jet (start with *pp* collisions) formed by a collection of particles of momenta $\{p_i\}$
2. Define,
$$\tau_a = \sum_{i \in jet} z_i \Delta R_{i,jet}^{2-a}$$
3. $$z_i = \frac{p_{T,i}}{p_{T,jet}}, \quad \Delta R_{i,jet} = \sqrt{\Delta\phi_{i,jet}^2 + \Delta\eta_{i,jet}^2}$$
   [Berger et. al. (2003); Hornig et. al (2009); Thaler et. al (2014)]
4. For infrared safety need $a < 2$. To avoid technical complications, further take $a < 1$.
5. Wider jets have larger τ_a. Can change the relative distribution by changing a
6. Eventually, hope to learn about the QGP by looking at jets in *AA* collisions

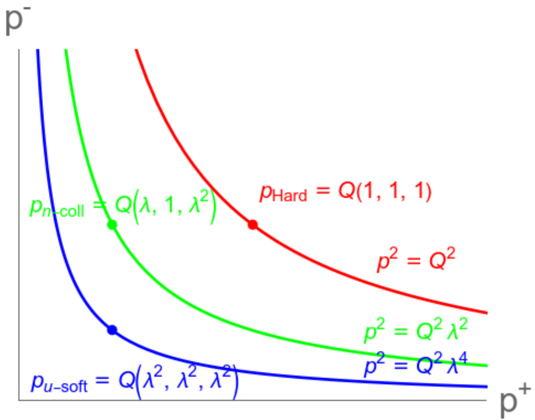
Illustrative example: Thrust

1. $a = 0$ well known as the thrust *[Farhi (1977)]*

$$\tau_0 = \sum_{i \in jet} z_i \Delta R^2_{i,jet}$$

2. Let the jet direction be $\hat{n}$. We are interested in small angle deviations, i.e. $\tau \ll 1$
3. Substantial contribution comes from energetic partons ($z_i \sim 1$) that are collinear with the jet
4. These have momenta $p^\mu \sim p_{T,jet}(1, \hat{n}, 0_\perp)$. In light cone coordinates $p_{n-coll} \sim p_{T,jet}(0, 1, 0_\perp)$. More typically, $p_{n-coll} = p_{T,jet}(\lambda^2, 1, \lambda)$ where $\lambda^2 \sim \tau_0$ ($\lambda \sim \Delta R_{i,jet}$)
5. Ultra-soft partons have much smaller energy ($z_i \ll 1$). But they also give a similar contribution to τ as ΔR is larger $p_{u-soft} = p_{T,jet}(\lambda^2, \lambda^2, \lambda^2)$

Modes (pp)

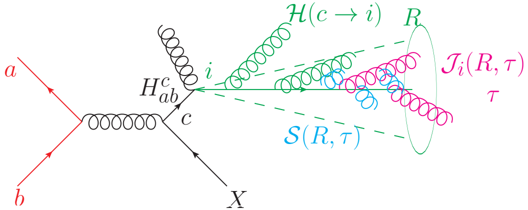


1. $Q \sim p_{T,jet}$
2. Since there is a hierarchy of virtuality of modes, [(ultra-)soft and collinear modes] SCET can be used to factorize the problem *[Manohar, Stewart, Pirjol, Bauer,.. .. Beneke ..]*
3. For general a, $p_{u-soft} = p_{T,jet}(\lambda^{2-a}, \lambda^{2-a}, \lambda^{2-a})$ and $\tau \sim \lambda^{2-a}$

Factorization formula

1. For $a < 1$ angularities and $\tau_a^{\frac{1}{2-a}} \ll R$,

$$\frac{d\sigma^{AA\to(jet[R,\tau_a])X}}{d\tau_a dp_T d\eta} = \sum_{abc}\sum_i f_a(x_a,\mu)\otimes f_b(x_b,\mu)$$
$$\otimes H_{ab}^c(x_a,x_b,\eta,p_T/z,\mu)\times \mathcal{H}_{c\to i}(z,p_T R,\mu)$$
$$\otimes \mathcal{J}(\tau_a^c,p_T,R,\mu)\otimes \mathcal{S}(\tau_a^s,p_T,R,\mu)$$



Factorization II

1. $\mathcal{J}_i(\tau_a^c, p_T, \mu)$ gives the collinear contribution to τ_a
2. $\mathcal{S}_i(\tau_a^s, p_T, R, \mu)$ gives the soft contribution to τ_a
3. These functions and their running with scale can be computed using SCET

SCET: basic structure

1. A lagrangian of soft and collinear fields. Eg. $A^\mu = A^\mu_s + A^\mu_n$
2. The fermion spinor has large components in the helicity bases and the small components can be integrated out
3. The lagrangian for the fermionic field collinear with direction n has the following form

$$\mathcal{L}_{n\xi} = e^{-ix\cdot\mathcal{P}}\bar{\xi}_n[in\cdot D + i\not{D}_{n,\perp}\frac{1}{i\bar{n}\cdot D_n}\not{D}_{n,\perp}]\frac{\not{\bar{n}}}{2}\xi_n$$

4. $\mathcal{P}$ is a momentum projection operator that selects the large component of the momentum $[Q(0,1,\lambda)]$. The collinear derivatives are
   4.1 $iD^\mu_{n\perp} = \mathcal{P}^\mu_\perp + gA^\mu_{n\perp}$
   4.2 $i\bar{n}\cdot D_n = \bar{\mathcal{P}} + g\bar{n}\cdot A_n$
5. Only coupling between collinear quarks and soft gluons is from the $in\cdot D$ term. This can be removed by gauge transformations (soft Wilson lines appear in currents). This is why the collinear ($\mathcal{J}_i$) and the soft factors ($\mathcal{S}_i$) appear separately

SCET: Ingredients

1. Gauge invariant collinear quark and gluon fields are built out of the SCET collinear fields (ξ_n,A_n) and the collinear Wilson line (U_n)

$$\chi_n(x) = U_n^\dagger(x)\xi_n(x)$$
$$\mathcal{B}^\mu_{n,\perp}(x) = \frac{1}{g} U_n^\dagger(x) iD^\mu_{n,\perp} U_n(x) \,.$$

2.

$$U_n(x) = \mathcal{P}\exp\left(-ig\int_0^\infty dy\, \bar{n}\cdot A_n(x+\bar{n}y)\right) \,,$$

3. Y_n, $Y_{\bar{n}}$ defined analogously with collinear fields A_n replaced by soft fields A_s

Angularity operators

1. Collinear function for angularity

$$\mathcal{J}_q(\tau_a^c, p_T, \mu) = \frac{1}{2N_c} Tr\left[\frac{\bar{\not{n}}}{2}\langle 0|\delta(\omega - \bar{n}\cdot\mathcal{P})\delta(\tau_a^c - \hat{\tau}_a^c)\chi_n(0)|JX\rangle \right.$$
$$\left. \langle JX|\bar{\chi}_n(0)|0\rangle\right]$$

   $\omega \sim p_T$ is the large component of the initiating parton i.

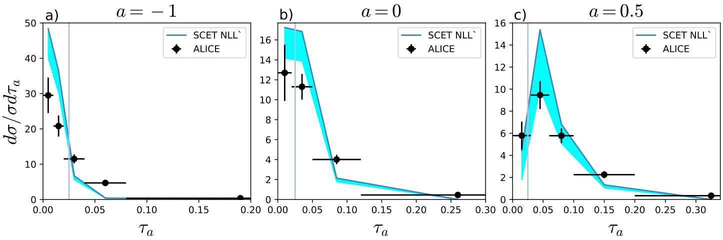
2. Soft function for angularity

$$\mathcal{S}_q(\tau_a^s, p_T, R, \mu) = \frac{1}{N_c}\langle 0|\bar{Y}_n\delta(\tau_a^s - \hat{\tau}_a^s)Y_{\bar{n}}|X\rangle\langle X|\bar{Y}_{\bar{n}}Y_n|0\rangle\,,$$

Resummation

1. H, $\mathcal{H}$, $\mathcal{J}$, $\mathcal{S}$ have to be computed at the same scale μ. But NLO expressions have large logs of μ/p_T, $\mu/(p_T\mathcal{R})$, μ/τ_a respectively. Resum using RG
2. $$\mu\frac{d}{d\mu}\mathcal{J}_i^{\rm vac}(\tau_a, p_T, R, \mu) = \int d\tau_a'\, \gamma_{\mathcal{J}_i}(\tau_a-\tau_a', p_T, R, \mu)\, \mathcal{J}_i^{\rm vac}(\tau_a', p_T, R, \mu)\,.$$
   The collinear anomalous dimensions are given by
   $$\gamma_{\mathcal{J}_i}(\tau_a, p_T, R, \mu) = \frac{\alpha_s(\mu)}{\pi}\left\{\delta(\tau_a)\left(2b_i+\frac{(2-a)}{(1-a)}\ln\frac{\mu^2}{p_T^2}\,C_i\right)-\frac{2}{1-a}C_i\left[\frac{1}{\tau_a}\right]_+\right\}.$$
3. $$\mu\frac{d}{d\mu}\mathcal{S}_i^{\rm vac}(\tau_a, p_T, R, \mu) = \int d\tau_a'\, \gamma_{\mathcal{S}_i}(\tau_a-\tau_a', p_T, R, \mu)\, \mathcal{S}_i^{\rm vac}(\tau_a', p_T, R, \mu)\,,$$
   The soft anomalous dimensions are given as
   $$\gamma_{\mathcal{S}_i}(\tau_a, p_T, R, \mu) = \frac{2\alpha_s(\mu)\,C_i}{\pi\,(1-a)}\left\{\left[\frac{1}{\tau_a}\right]_+ - \ln\frac{\mu\,R^{1-a}}{p_T}\,\delta(\tau_a)\right\}.$$
4. NLL formalism *[Kang, Lee, Ringer (2018)]*

Comparison with data (pp)



1. Ungroomed jet angularity distribution τ_a in pp collisions from *[ALICE (2021)]* for $R = 0.4$
2. $80 < p_T < 100$GeV
3. Compared *[Budhraja, RS, Singh (2023)]* with ALICE data. $\mu = [1 - 2]\, p_T$
4. Now we consider AA collisions

Medium effect on jet functions

1. $\mathcal{J} = \mathcal{J}^{vac} + \mathcal{J}^{med}$
2. $$\mathcal{J}^{med}_{i\to jk}(..) \sim \sum_{j,k} \int dx \frac{d^2 k_\perp}{(2\pi)^2} P^{med}_{i\to jk}(k_\perp, x)\delta(\tau_a - \hat{\tau}_a)$$
3. The physics of $P^{med}_{i\to jk}$ is medium induced bremmstrahlung. *BDMPS-Z, GLV, Wiedemann, Gyulassy, Wang, Majumder...*

Medium effect on angularities

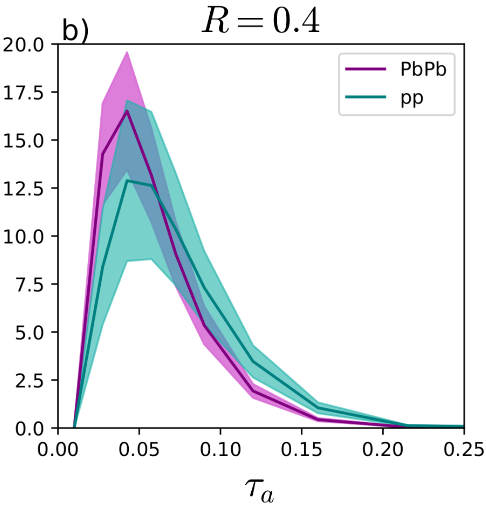
1. Medium splitting kernels derived using $SCET_G$ in *[Vitev, Ovanesyan (2011, 2012, 2013)]*. For eg, in the small x limit,

$$x\frac{dN^{med}_{q\to qg}}{dxd^2k_\perp} = \alpha_s \int_0^L d\Delta z d^2 q_\perp \frac{1}{\sigma}\frac{d^2\sigma}{dq_\perp^2}\frac{2k_\perp q_\perp}{k_\perp^2(q_\perp - k_\perp)^2} \times \left[1-\cos\left(\frac{(q_\perp - k_\perp)^2\Delta z}{x\omega}\right)\right]$$

2. *[Gyulassy, Wang (1994)]*

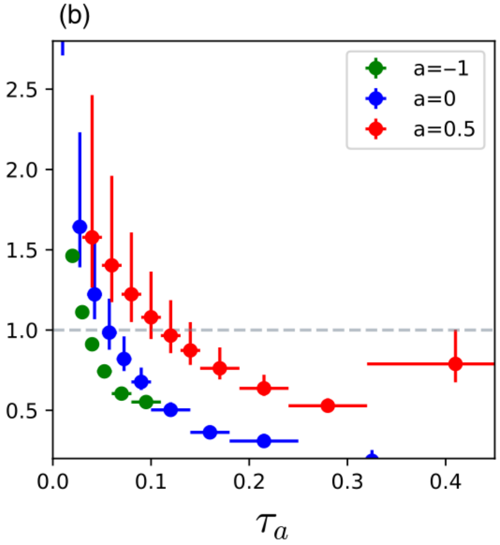
$$\frac{1}{\sigma}\frac{d^2\sigma}{dq_\perp^2} = \frac{m_D^2}{\pi(q_\perp^2 + m_D^2)^2}$$

pp vs PbPb



1. Normalized τ_a distributions for $p_T \in [40, 60]$GeV, $a = 0$, $R = 0.4$, $0 - 10\%$ centrality. Interpretation: (a) more medium induced emissions at small angle (b) the jets in *PbPb* came from "would be" higher energy, narrower jets in *pp*, and have smaller τ_a

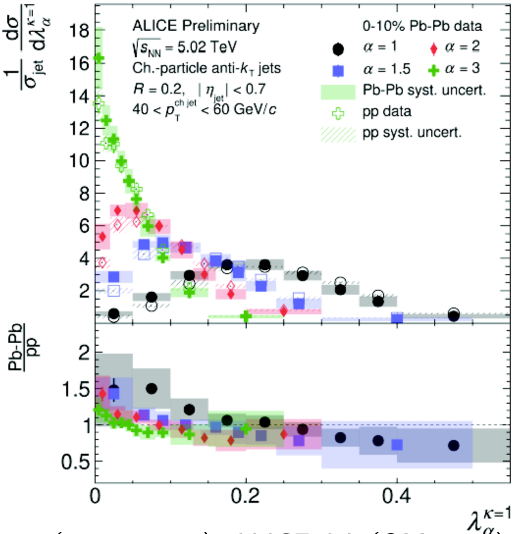
Ratio



1. $p_T \in [40, 60]$GeV, $R = 0.4$, $0 - 10\%$ centrality *[Budharaja, Sharma, Singh [2305.10237]]*

Measured an ' ·· ·

Ungroomed



1. *[ALICE pp (2107.11303); ALICE AA (QM 2022).]*
   Calculations using Monte-Carlo studies (JEWEL, JETSCAPE, H-T, Hybrid) also exist ($\alpha = 2 - a$)

Energy-energy correlators

Energy-energy correlators

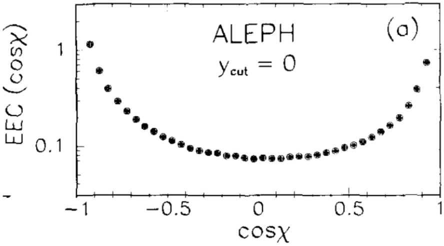
1. Angular correlation between energy flow *[Ellis, Gaillard, Ross (1976)]*

$$EEC_n(\theta) = \sum_{i,j} (E_i)^n (E_j)^n \delta(\cos\theta - \hat{n}_i \cdot \hat{n}_j)$$

2. Related to the angular correlation of the energy flow

$$EEC_n(\theta) = \int d\hat{n}_{1,2} \frac{\langle \mathcal{E}(n_1)^n \mathcal{E}(n_2)^n}{Q^{2n}} \delta(\cos\theta - \hat{n}_2 \cdot \hat{n}_1)$$

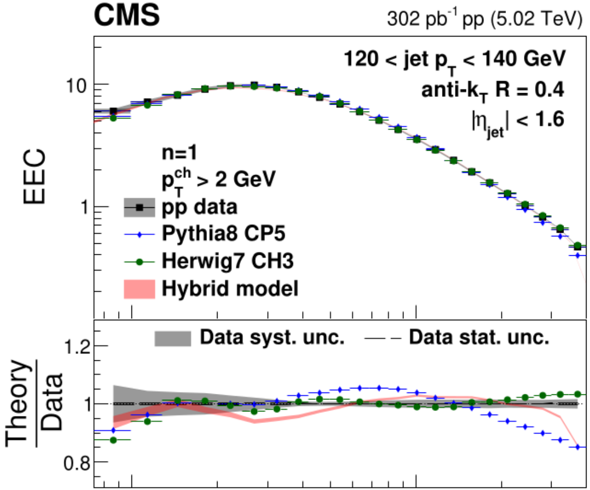
3. For small angles $dEEC(\theta) \sim \frac{d\theta}{\theta}$. In e^+e^- see a clear correlation



Energy-energy correlators in *pp*

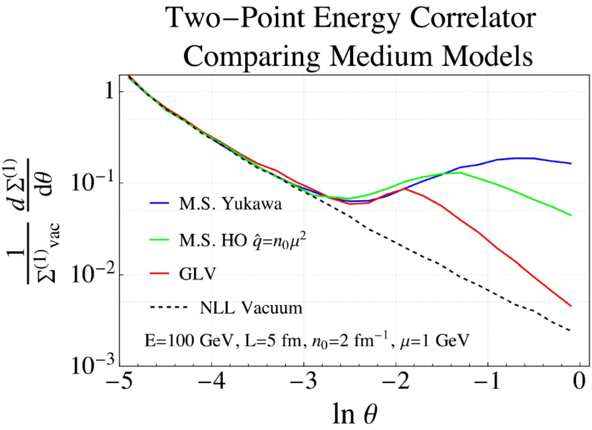
1. CMS *(2024, 2025)* has measured it in *pp*

$$EEC(\Delta R) = \sum_{i,j} E_i E_j \delta(\Delta R - \Delta R_{ij})$$



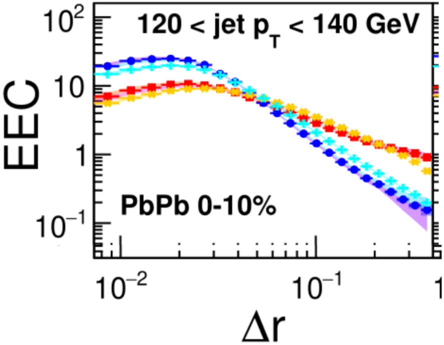
Energy-energy correlators in *PbPb*

1. Theory estimates suggest ability to distinguish models of energy loss *[Andres et. al.(2023, 2024)]*



Energy-energy correlators in *PbPb*

1. Recent results from CMS *(2025)* seem fairly accurate, and this is a promising observable



Summary

1. Jet substructure is sensitive to medium properties and can help us understand finer properties of jet dynamics in the medium
2. Medium effects on angularity can be incorporated using the medium modified splitting functions
3. Angularity distributions show a relative increase at low τ_a and a decrease at high τ_a due to enhanced radiation

Backup slides

Non-perturbative effects in the shape function

1. Non-perturbative dynamics can be appropriately included in a shape function, $\mathcal{S}_{\rm np}$, which can be convolved with the resummed perturbative distribution
$$\frac{d\sigma}{d\eta dp_T d\tau_a} = \int dk \, \frac{d\sigma^{\rm pert}}{d\eta dp_T d\tau_a}\left(\tau_a - \frac{k}{p_T R}\right) \mathcal{S}_{\rm np}(k) \,.$$
2. Single parameter parameterization, Ω_a *[Aschenauer et. al (2019)]*
$$\mathcal{S}_{\rm np}(k) = \frac{4k}{\Omega_a^2} \exp\left(-\frac{2k}{\Omega_a}\right),$$
3. The 'a' dependence factors out as *[Lee, Sterman (2007)]*
$$\Omega_a = \frac{\Omega_{a=0}}{1-a}\,,$$
4. Can be estimated from a global fit to the jet angularity data for different choices of 'a'
5. $\Omega_0 \approx 0.35$GeV for a jet with $80 < p_T < 100$GeV and $R = 0.4.$, $\Omega_0 \approx 0.8$GeV for $40 < p_T < 60$GeV.
6. Not modified between pp and $PbPb$

Resumming the jet function using splitting

1. Jet functions in SCET in some cases can be directly computed from the spin-averaged QCD splitting functions. For eg. see *[Ritzmann et. al. (2014), Cal et. al (2019)]*
2. For angularity jet functions,

$$\mathcal{J}_{i\to jk}(\tau_a, p_T, R, \mu) = \frac{\alpha_s(\mu)}{\pi}\frac{e^{\epsilon\gamma_E}\mu^{2\epsilon}}{\Gamma(1-\epsilon)}\sum_{j,k}\int dx\frac{dk_\perp}{k_\perp^{2\epsilon-1}}\mathcal{P}_{i\to jk}(x,k_\perp)\delta(\tau_a-\hat{\tau}_a)$$

   x is the momentum fraction carried by the final parton, $k_\perp$ is its transverse momentum.

3. $$\hat{\tau}_a = p_T^{a-2}k_\perp^{2-a}(x^{a-1}+(1-x)^{a-1})\,.$$
4. In vacuum,

$$\mathcal{P}_{i\to jk}(x,k_\perp) = \frac{1}{k_\perp^2}\mathcal{P}_{i\to jk}(x)\,,$$

   with $\mathcal{P}_{i\to jk}(x)$ being the usual Altarelli-Parisi QCD splitting functions