CHIRAL MATRIX MODEL IN $T - \mu_B$ PLANE

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Presenting @ Hard Probes in non-equilibrium QCD matter

(Work in progress!)

INTERNATIONAL CENTRE *for* THEORETICAL SCIENCES

TATA INSTITUTE OF FUNDAMENTAL RESEARCH

The Quark-Gluon Plasma near T_c

From $T = 0$ to $T = 130$ MeV : HRG, Chiral Perturbation theory

$T > 300$ MeV : Resummed perturbation theory

(HTLpt: Haque, Bandopadhyay, Mustafa, Strickland, NanSu, Andersen)

But in Heavy-Ion collision - Interesting region is near T_c - the cross-over temperature

semi-QGP region

(partial deconfinement of color charges)

Need matrix model

WHAT IS MATRIX MODEL?

[Pisarski & Hidaka, arXiv: 0906.1751]

For $SU(3)$ gauge theory without quarks-

Background Field (Diagonal): $A_0^{bg} = \frac{2\pi T}{3g}(q\lambda_3 + r\lambda_8) = \frac{2\pi T}{g}\hat{\mathbf{q}}$

constant in space

Eigen values are: $\rightarrow \frac{2\pi T}{3g}(q + r, -q + r, -2r)$

Thermal Wilson line: $L(A_0) = \mathscr{P}\exp\left(i\,g\int_0^{1/T} A_0\,d\tau\right)$

Eigen-values : $e^{i2\pi(q+r)/3}, e^{i2\pi(-q+r)/3}, e^{-i2\pi(2r)/3}$

Polyakov loop: $\ell_{bg} = \frac{1}{3}\mathrm{Tr}L(A_0^{bg}) = \frac{e^{i2\pi r/3}}{3}(e^{-i2\pi r} + 2\cos(2\pi q/3))$

HOLONOMOUS POTENTIAL

For $SU(3)$, the holonomous potential/free energy:

$$V^{\rm gl}_{pert}(q,r) = \frac{1}{V}\mathrm{Tr}\log(-D^2_{gl}) = \pi^2 T^4\Big(-\frac{8}{45} + \mathscr{V}_4(q,r)\Big)$$

$$\mathscr{V}_4(q,r) = \left|\frac{2q}{3}\right|^2\left(1-\left|\frac{2q}{3}\right|\right)^2 + \left|\frac{q}{3}+r\right|^2\left(1-\left|\frac{q}{3}+r\right|\right)^2 + \left|\frac{q}{3}-r\right|^2\left(1-\left|\frac{q}{3}-r\right|\right)^2$$

$(|x| = |x|_{\mathrm{mod}\,1})$

However, the effective free energy near the transition temperature is denoted by, $\sim T^2T_d^2$. Add a non-perturbative term by Hand →

$$V^{gl}_{\mathrm{npt}} = \frac{4\pi^2}{3}T^2T_d^2\Big(-\frac{1}{3}c_1\mathscr{V}_2(q,r) - c_2\mathscr{V}_4(q,r) + \frac{2}{15}c_3\Big)$$

$$\mathscr{V}_2(q,r) = \left|\frac{2q}{3}\right|\left(1-\left|\frac{2q}{3}\right|\right) + \left|\frac{q}{3}+r\right|\left(1-\left|\frac{q}{3}+r\right|\right) + \left|\frac{q}{3}-r\right|\left(1-\left|\frac{q}{3}-r\right|\right)$$

P.S. Teen field (a dynamical 2D ghost) can produce $V^{\rm gl}_{\mathrm{npt}}$ [arXiv: 2504.20138, 2009.03903]

ADDING QUARKS!

[Pisarski & Skokov, arXiv: 1604.00022]

For 3 Flavors of massless quarks coupled to gauge field:

$$\mathscr{L}^{qk} = \bar{\psi}(\gamma^\alpha D_\alpha + \mu\gamma^0) = \bar{\psi}_L(\gamma^\alpha D_\alpha + \mu\gamma^0)\psi_L + \bar{\psi}_R(\gamma^\alpha D_\alpha + \mu\gamma^0)\psi_R, \qquad \psi_{L,R} = P_{L,R}\psi$$

↓

Completely decoupled left and right part

$$P_{L,R} = \frac{1 \pm \gamma_5}{2}$$

When massless, It follows global flavor symmetry, $G_f = SU(3)_L \times SU(3)_R \times U(1)_A$

$$\psi_L \to e^{-i\alpha/2} U_L \psi_L, \quad \psi_R \to e^{i\alpha/2} U_R \psi_R$$

At $T = 0$, quarks are not massless. Simplest order parameter for chiral symmetry breaking: $\Phi \sim \bar{\psi}_L \psi_R$

$$\Phi \to \mathrm{e}^{+i\alpha} U_R \Phi U_L^\dagger$$

Quantum mechanically, $U(1)_A$ is broken by instantons to $Z(3)_A$ at $T = 0$

$$\det\Phi \to e^{i3\alpha} \det\Phi$$

For three flavor, Φ field can be represented as a complex nonet,

$$\Phi = (\sigma^A + i\pi^A)\lambda^A$$

$\lambda^0 = \mathbf{1}/\sqrt{6}$, and $\lambda^1, \cdots, \lambda^8 \rightarrow$ Gell-Mann matrices

Linear sigma model for Φ,

$$V_\Phi = m^2 \,\mathrm{Tr}(\Phi^\dagger\Phi) - c_A(\det\Phi + \det\Phi^\dagger) + \lambda\,\mathrm{Tr}\,(\Phi^\dagger\Phi)^2$$

[Here, $\sim (\mathrm{Tr}\,\Phi^\dagger\Phi)^2$ has been dropped]

Now, add a symmetry breaking term linear in Φ, causing the generation of mass, $V_H = -\,\mathrm{Tr}\,(H(\Phi^\dagger + \Phi))$

H is proportional current quark mass, $H = \mathrm{diag}\,(h_u, h_u, h_s)$

arXiv:nucl-th/0004006

We can introduce quark coupling to Φ field, which causes a potential for q, r (Polyakov loop eigen values) as well

$$\mathcal{L}^{qk}_\Phi = \bar{\psi}\left(\gamma^\alpha D_\alpha + \mu\gamma^0 + y\left(\Phi P_L + \Phi^\dagger P_R\right)\right)\psi$$

y: Yukawa coupling

covariant derivative: $D_\alpha = \partial_\alpha - igA_\alpha$

QUARK LOOP CONTRIBUTION

At, $T = 0$, from quark loop, due to quark - Φ interaction, we get,

$$\sim \sum_{i=1}^{N_f} \frac{3m_i^4}{16\pi^2}\left(\frac{1}{\epsilon} + \log\left(\frac{\Lambda^2}{m_i^2}\right)\right)$$

assuming chiral symmetry breaking occurs at $m_i = y\phi_i$, $\langle \Phi_{ii} \rangle = \phi_i$

After renormalization, for general form of Φ, this becomes-

$$V_\Phi^{\text{log}} = \frac{3y^4}{16\pi^2}\text{Tr}\left[(\Phi^\dagger\Phi)^2 \log\left(\frac{\Lambda^2}{\Phi^\dagger\Phi}\right)\right]$$

At finite temperature $(T \neq 0)$, the quark loop contribution to the potential

$$V_{pert,T}^{qk} = -\frac{1}{V}\text{Tr}\log(\gamma^\alpha D_\alpha + \mu\gamma^0 + y(\Phi P_L + \Phi^\dagger P_R))$$

[Pisarski & Skokov, arXiv: 1604.00022]

Finite temperature contribution corresponding to V_H will be given by,

$$V_h^T = -\frac{m_{qk}}{V}\left(\mathrm{Tr}\frac{1}{\gamma^\alpha D_\alpha + \mu\gamma^0 + y\Phi_{ii}}\bigg|_{T\neq 0} - (T=0)\right)$$

(m_{qk} is the current quark mass)

Finite temperature contribution for each flavor:

$$V_f^{qk,T}(q,r,\Sigma_f) = -2T\sum_{a=1}^{3}\int\frac{d^3k}{(2\pi)^3}[\ln(1+e^{-(E_f-\mu)/T+i2\pi q_a/3}) + \ln(1+e^{-(E_f+\mu)/T-i2\pi q_a/3})]$$

$$E_f = \sqrt{k^2+m_f^2}, \qquad m_f = y\Sigma_f, \qquad q_a = (q+r,\, -q+r,\, -2r)$$

V_h^T can be rewritten as, $V_h^T = -\sum_{f=u,d,s}\Sigma_f^0\frac{\partial}{\partial\Sigma_f}V_f^{qk,T}$

EFFECTIVE POTENTIAL

$$V_{\text{eff}}(q, r, \Sigma_f) = V^{\text{gl}}(q, r) + V_\Phi^{\text{tot}}(\Sigma_f) + V^{qk}(q, r, \mu, \Sigma_f) + V_h^T(q, r, \mu, \Sigma_f)$$

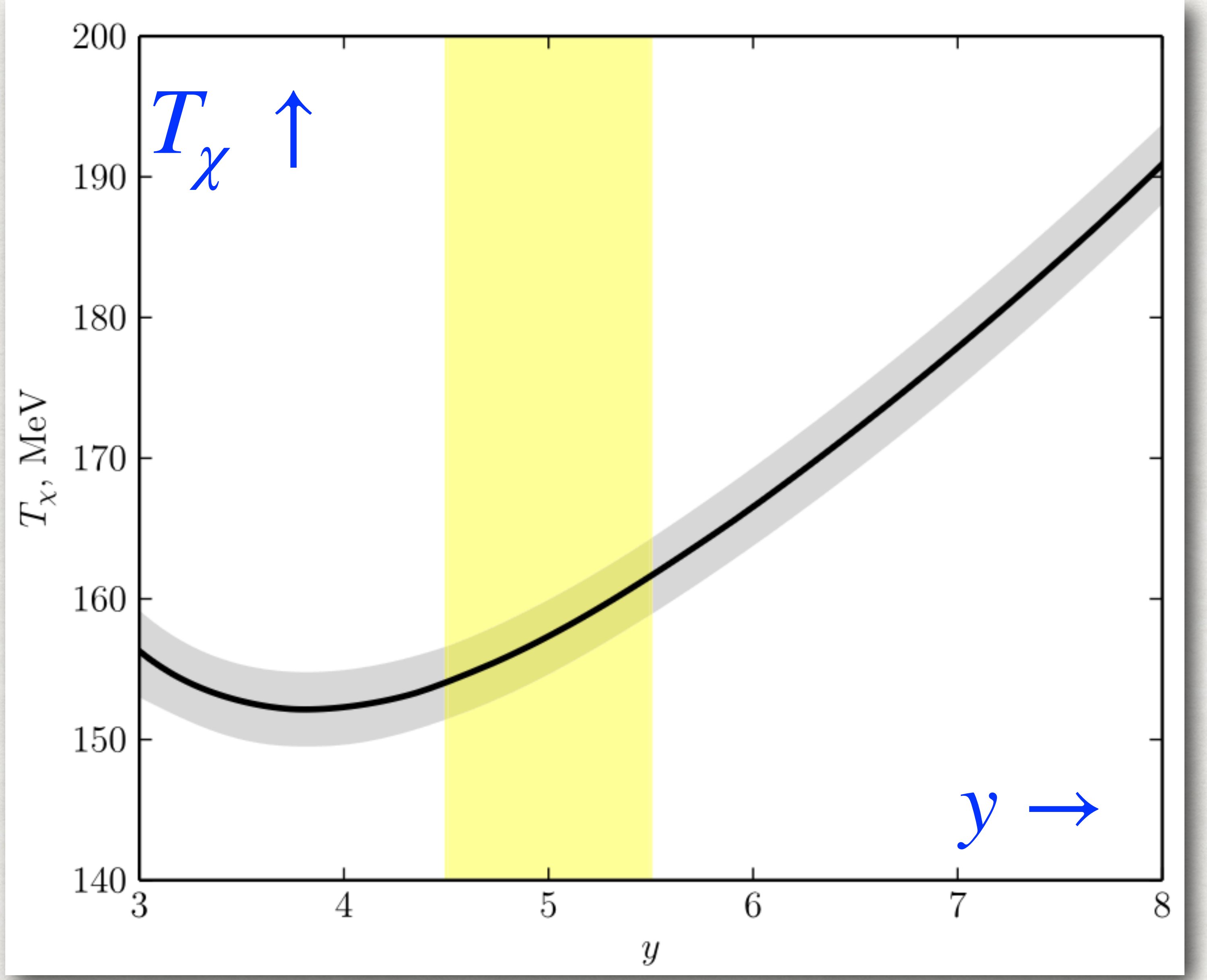
$$V_\Phi^{\text{tot}} = -2h_u\Sigma_u - h_s\Sigma_s + m^2(2\Sigma_u^2 + \Sigma_s^2) - 2c_A\Sigma_u^2\Sigma_s + \lambda(2\Sigma_u^4 + \Sigma_s^4)$$

$$V^{qk}(q, r, \Sigma_f) = \sum_{f=u,d,s} V_f^{qk} = \left(-\frac{3}{16\pi^2} y^4 \Sigma_f^4 \ln\left(\frac{y^2\Sigma_f^2}{\Lambda^2}\right) + V_f^{qk,T}(q, r, \Sigma_f)\right)$$

$$V^{\text{gl}} = V_{pert}^{\text{gl}} + V_{non-pert}^{\text{gl}}$$

$$V_h^T = -\sum_{f=u,d,s} \Sigma_f^0 \frac{\partial}{\partial \Sigma_f} V_f^{qk,T}$$

T_χ defined from the maximum in light quark susceptibility, $d\Sigma_u/dT$

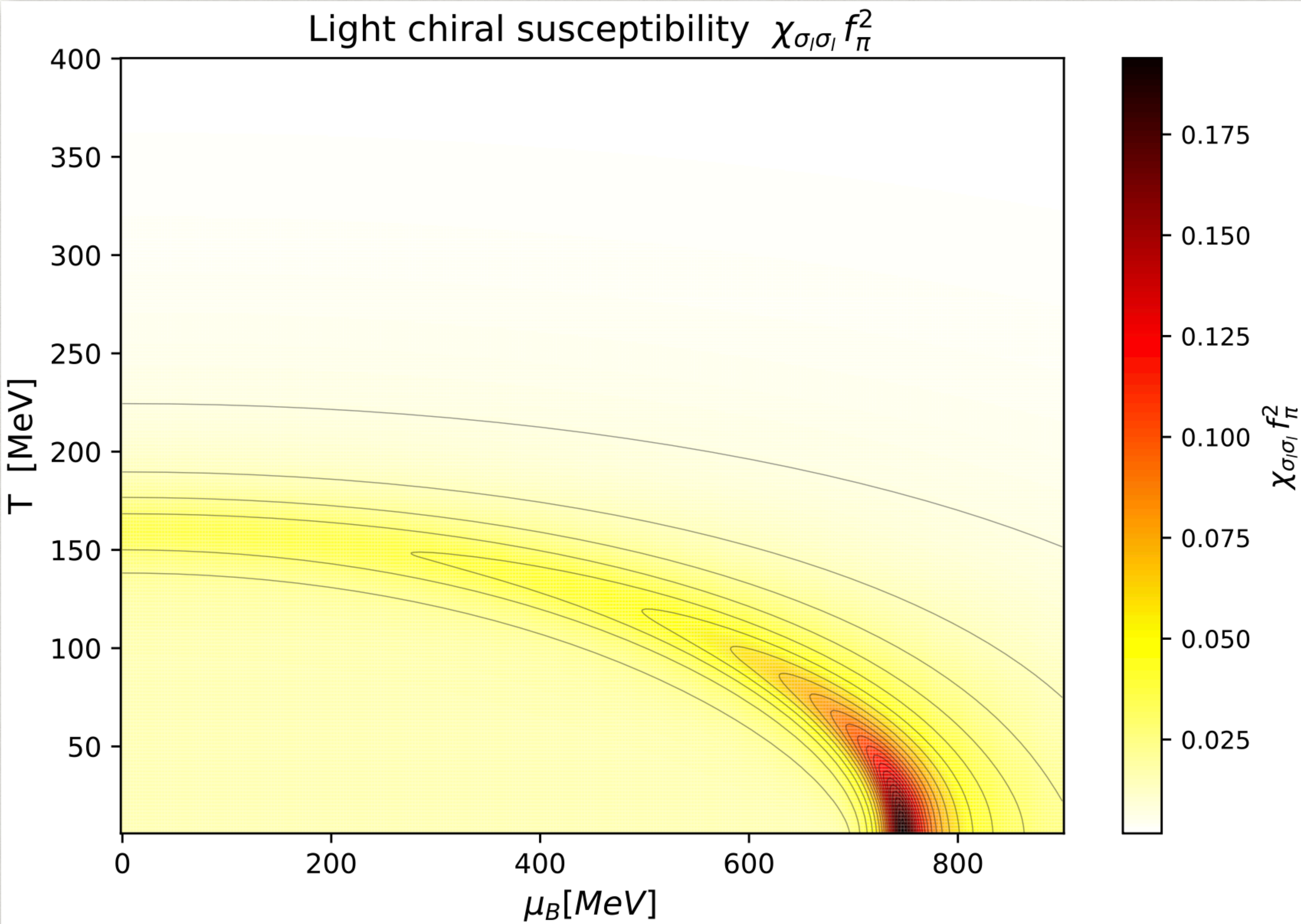


←Grey band: varying T_d from $260 \to 280$

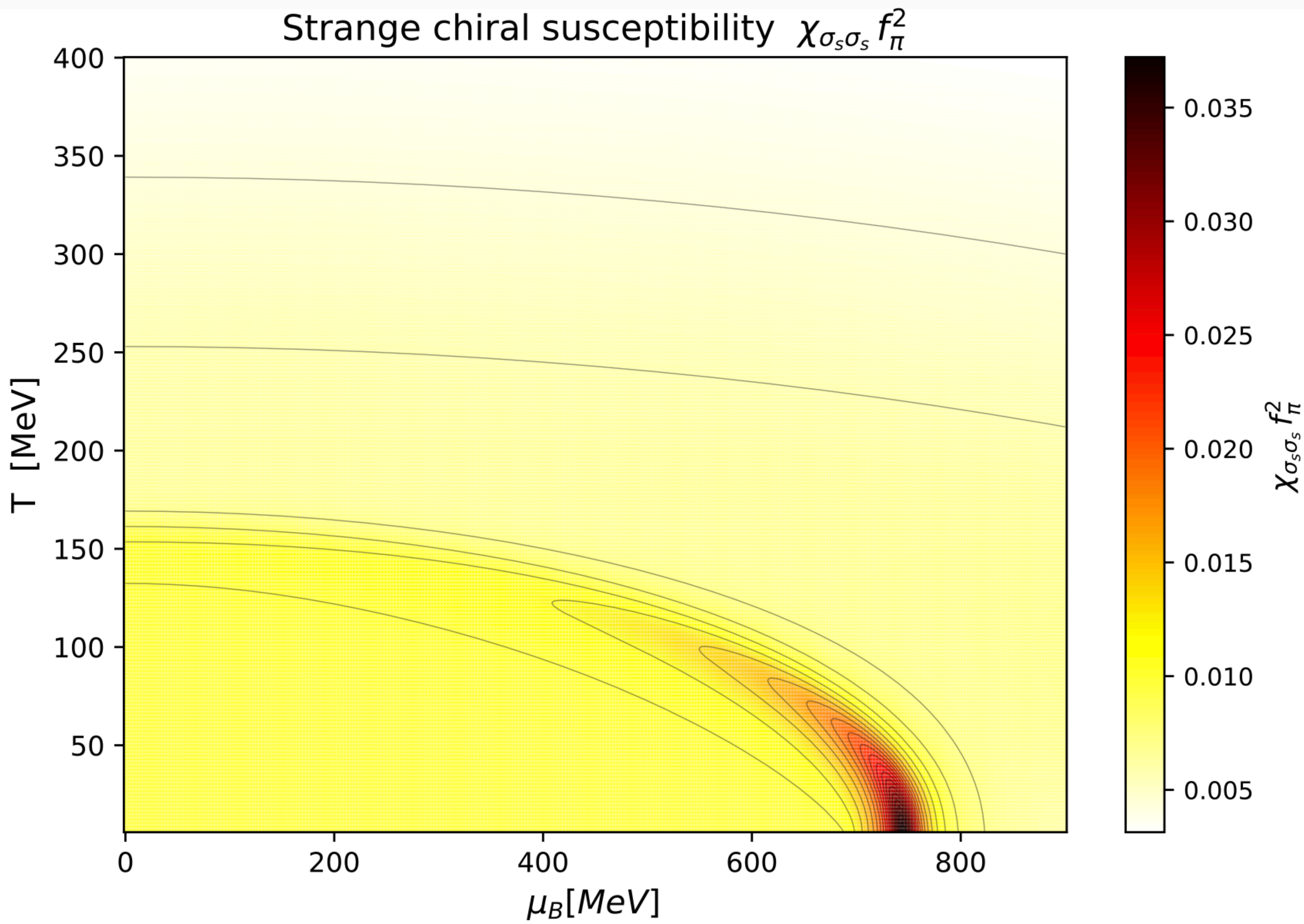
Yellow band: $y = 4.5 \to 5.5$

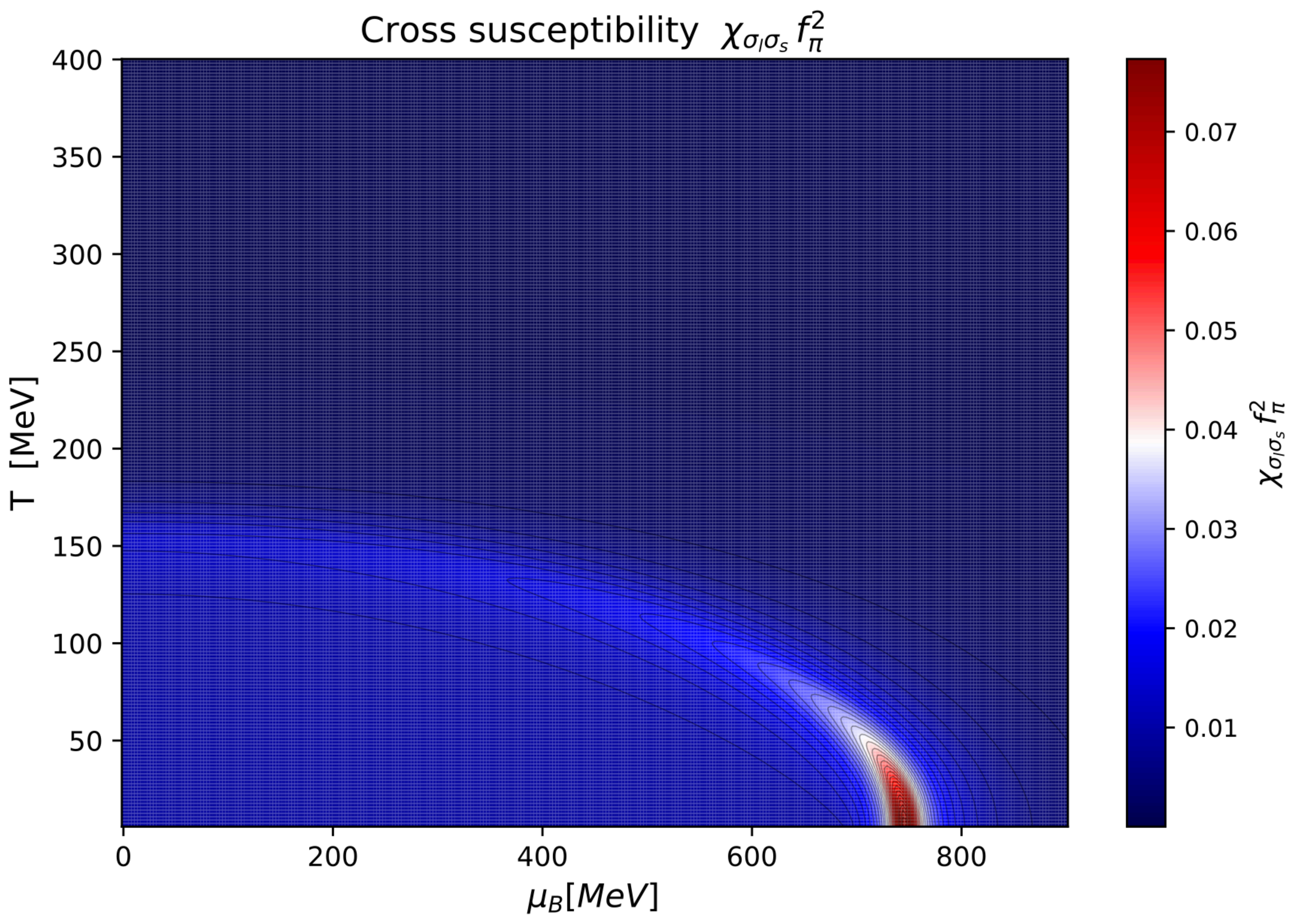
central value: y =5

(Current work: y= 5)

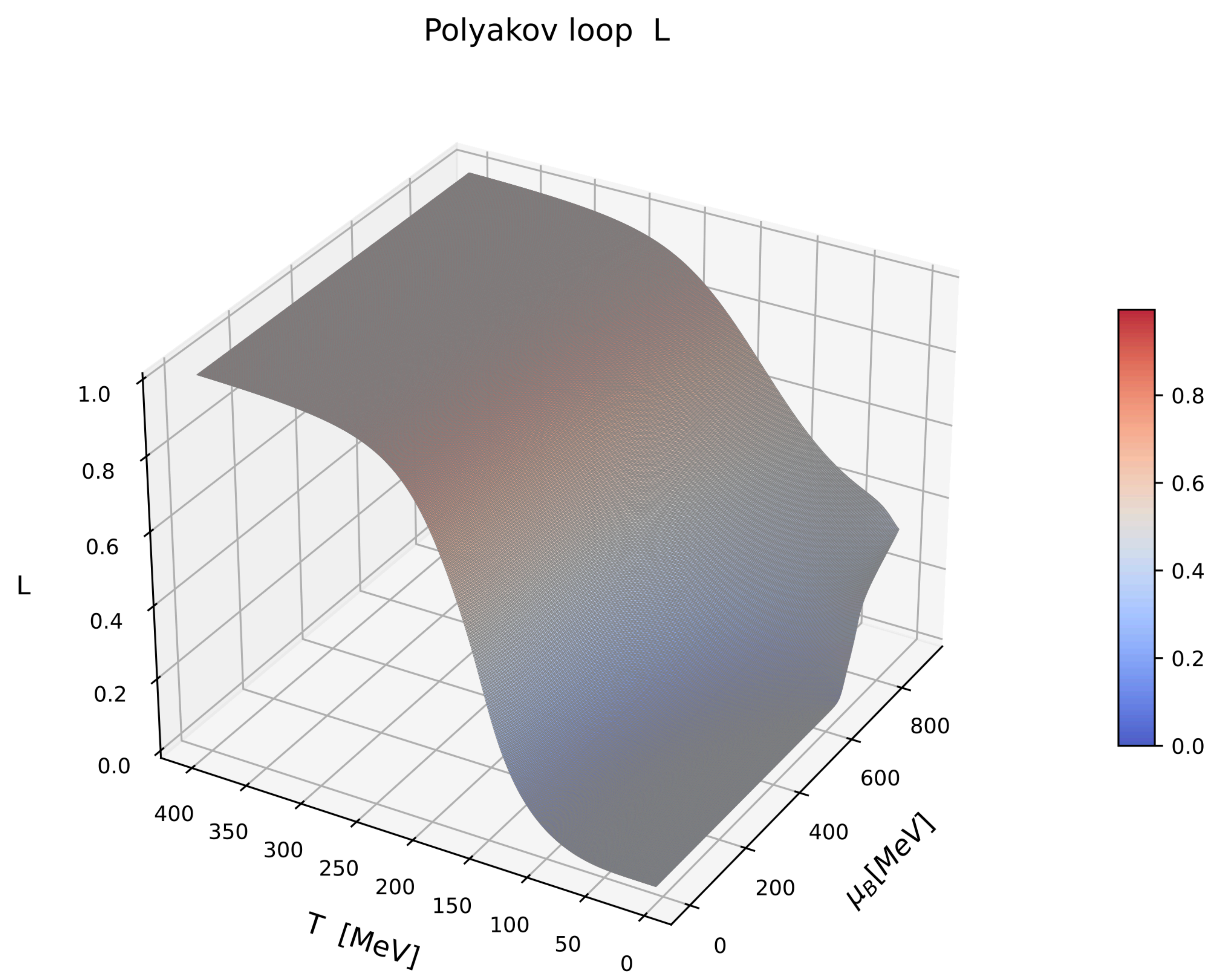


Light chiral susceptibility $\chi_{\sigma_l\sigma_l} f_\pi^2$
T [MeV]
400
350
300
250
200
150
100
50
0
200
400
600
800
$\mu_B [MeV]$
$\chi_{\sigma_l\sigma_l} f_\pi^2$
0.175
0.150
0.125
0.100
0.075
0.050
0.025


Strange chiral susceptibility $\chi_{\sigma_s\sigma_s} f_\pi^2$
T [MeV]
400
350
300
250
200
150
100
50
0
200
400
600
800
$\mu_B[MeV]$
0.035
0.030
0.025
0.020
0.015
0.010
0.005
$\chi_{\sigma_s\sigma_s} f_\pi^2$

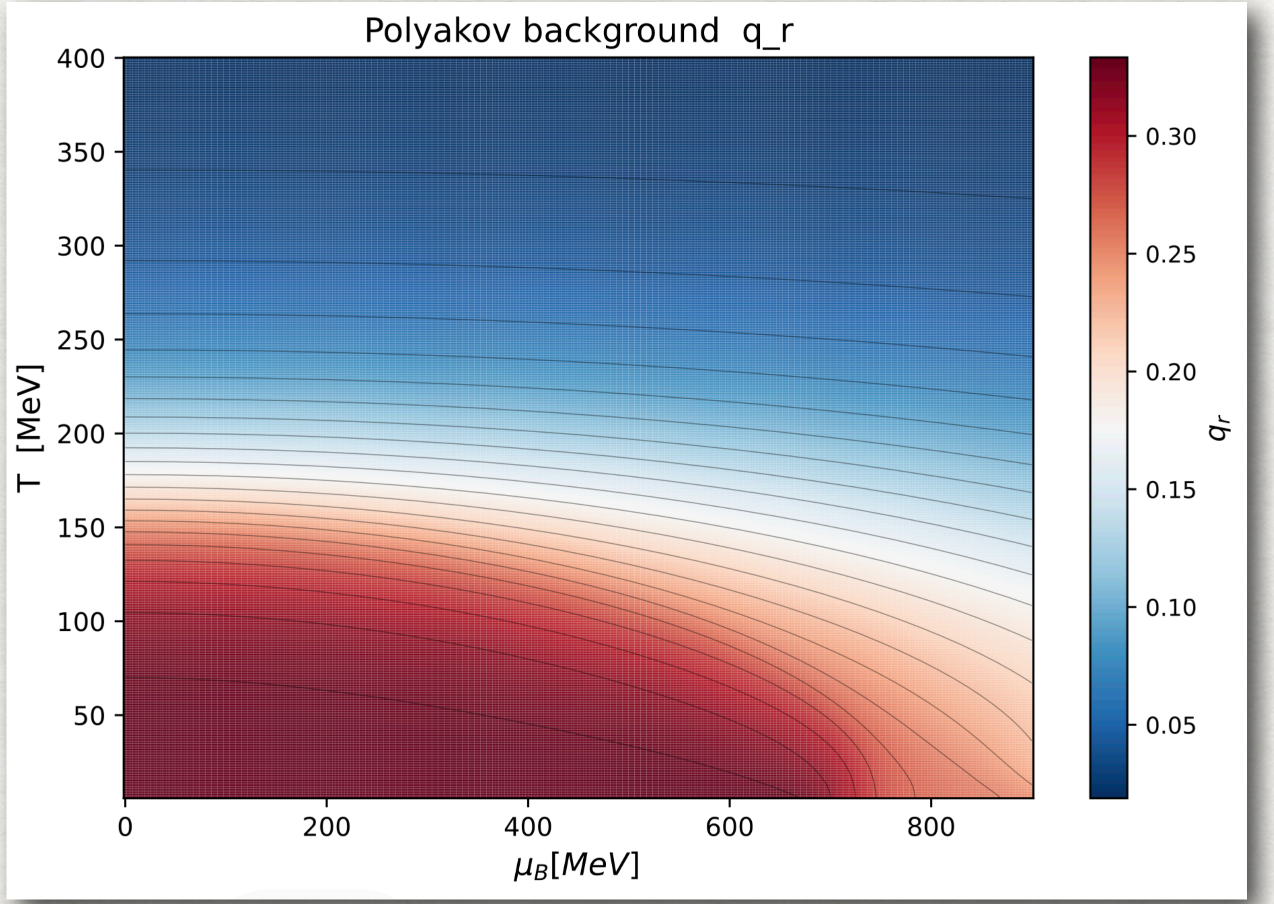


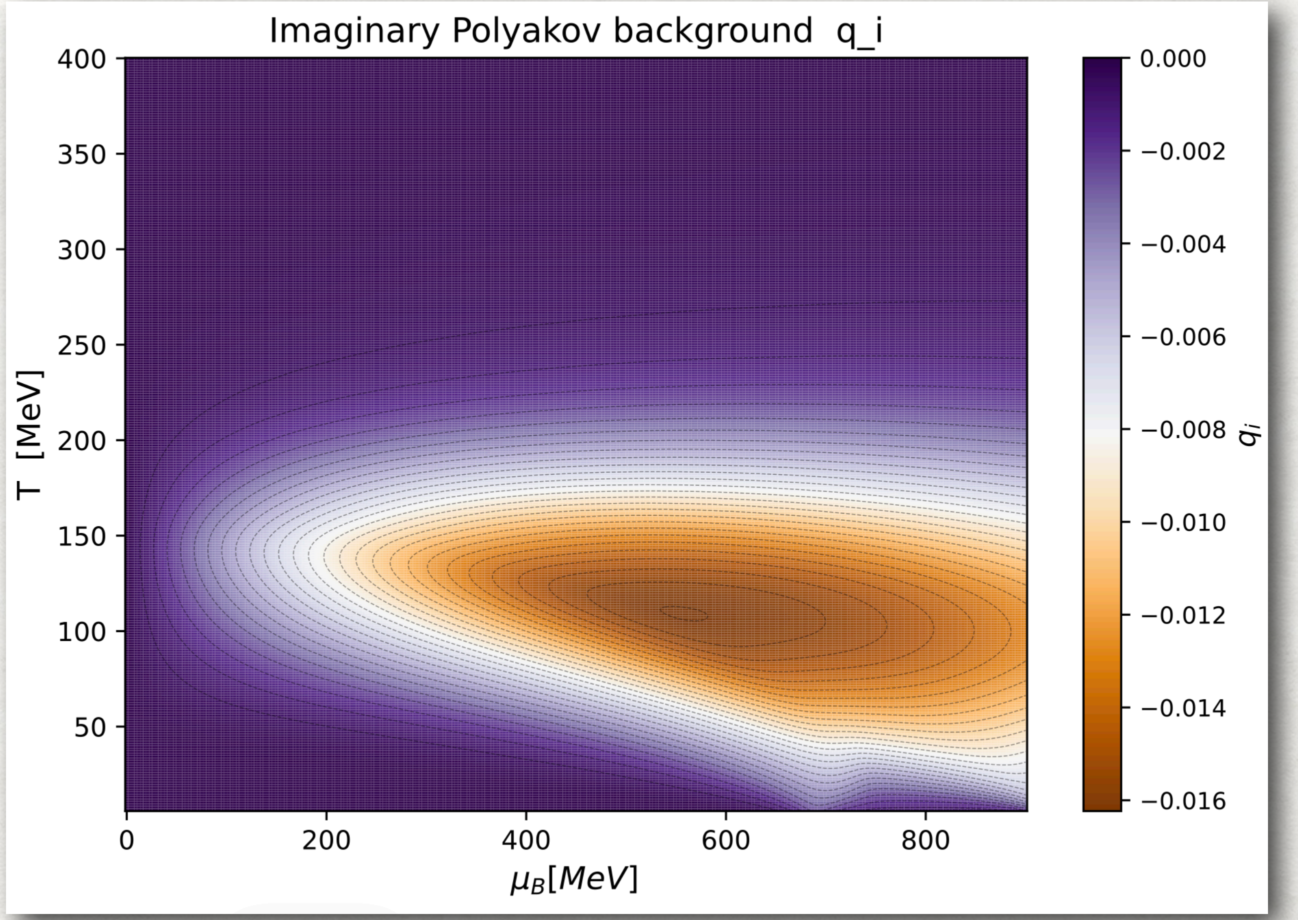
Cross susceptibility $\chi_{\sigma_l \sigma_s} f_\pi^2$
T [MeV]
400
350
300
250
200
150
100
50
0
200
400
600
800
$\mu_B [MeV]$
0.07
0.06
0.05
0.04
0.03
0.02
0.01
$\chi_{\sigma_l \sigma_s} f_\pi^2$

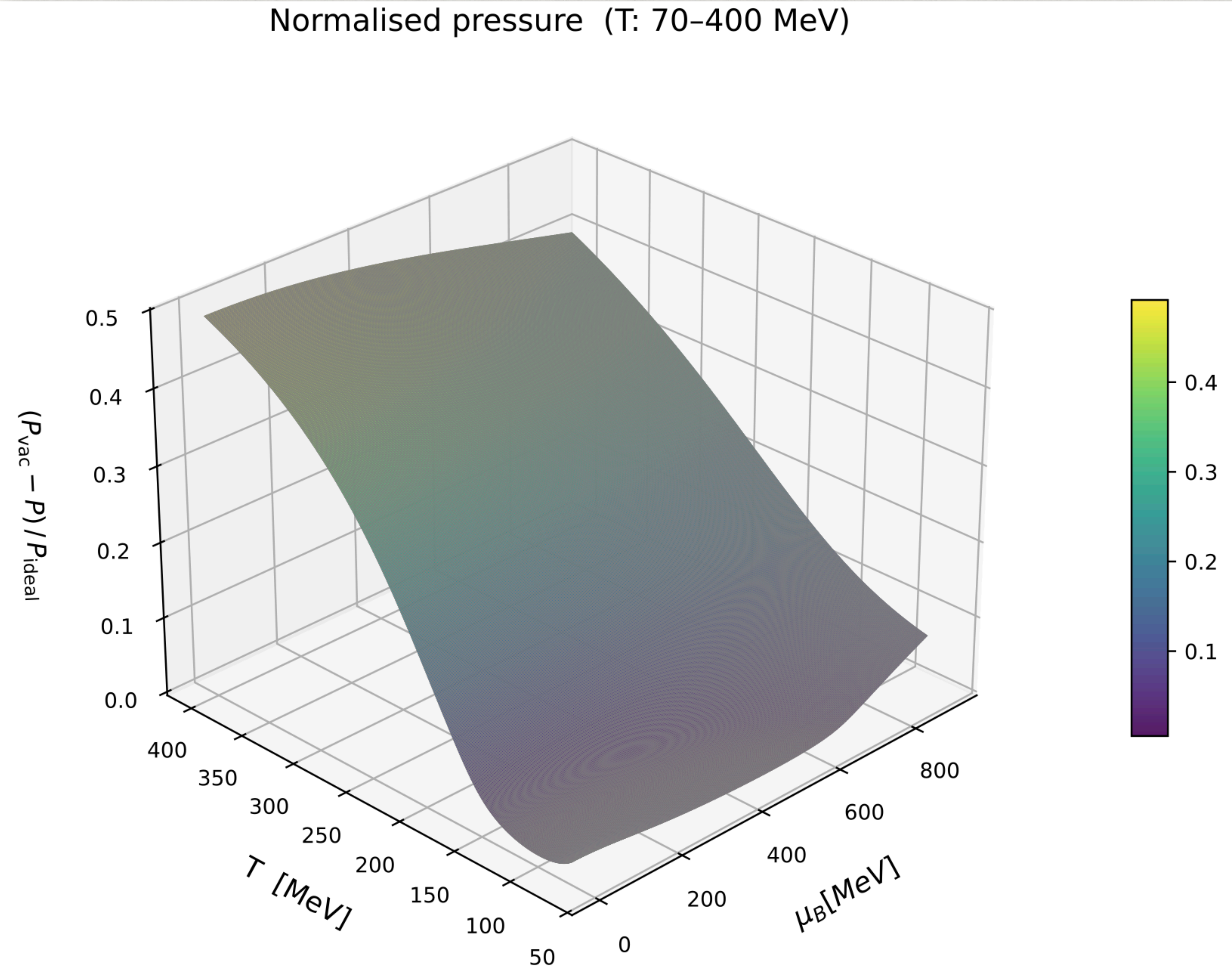


Polyakov loop L
1.0
0.8
0.6
0.4
0.2
0.0
L
400
350
300
250
200
150
100
50
0
T [MeV]
800
600
400
200
0
μB[MeV]
0.8
0.6
0.4
0.2
0.0

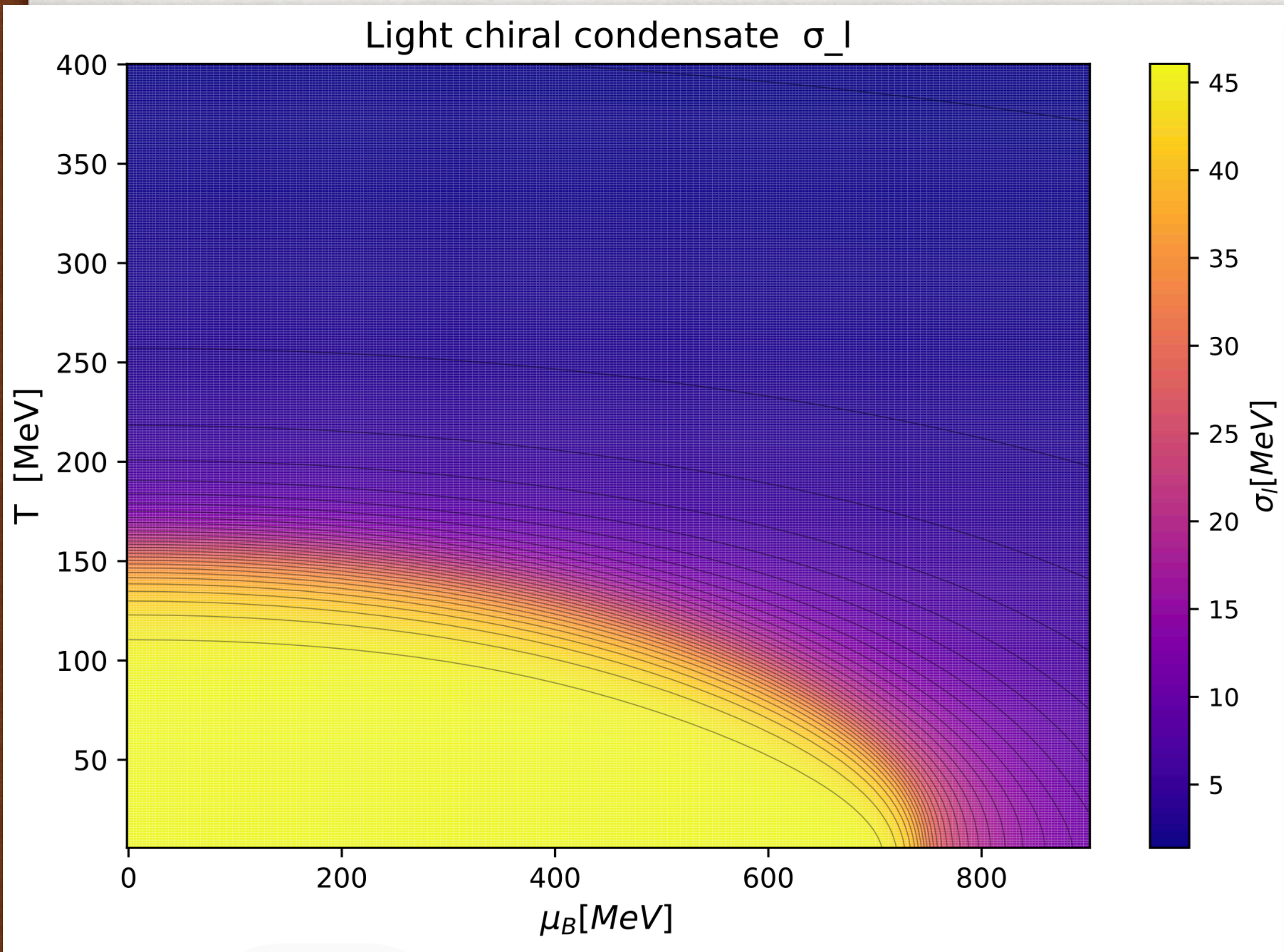
$$\ell_{bg} = \frac{1}{3}\mathrm{Tr}L(A_0^{bg}) = \frac{e^{i2\pi q_i/3}}{3}(e^{-i2\pi q_i} + 2\cos(2\pi q_r/3))$$

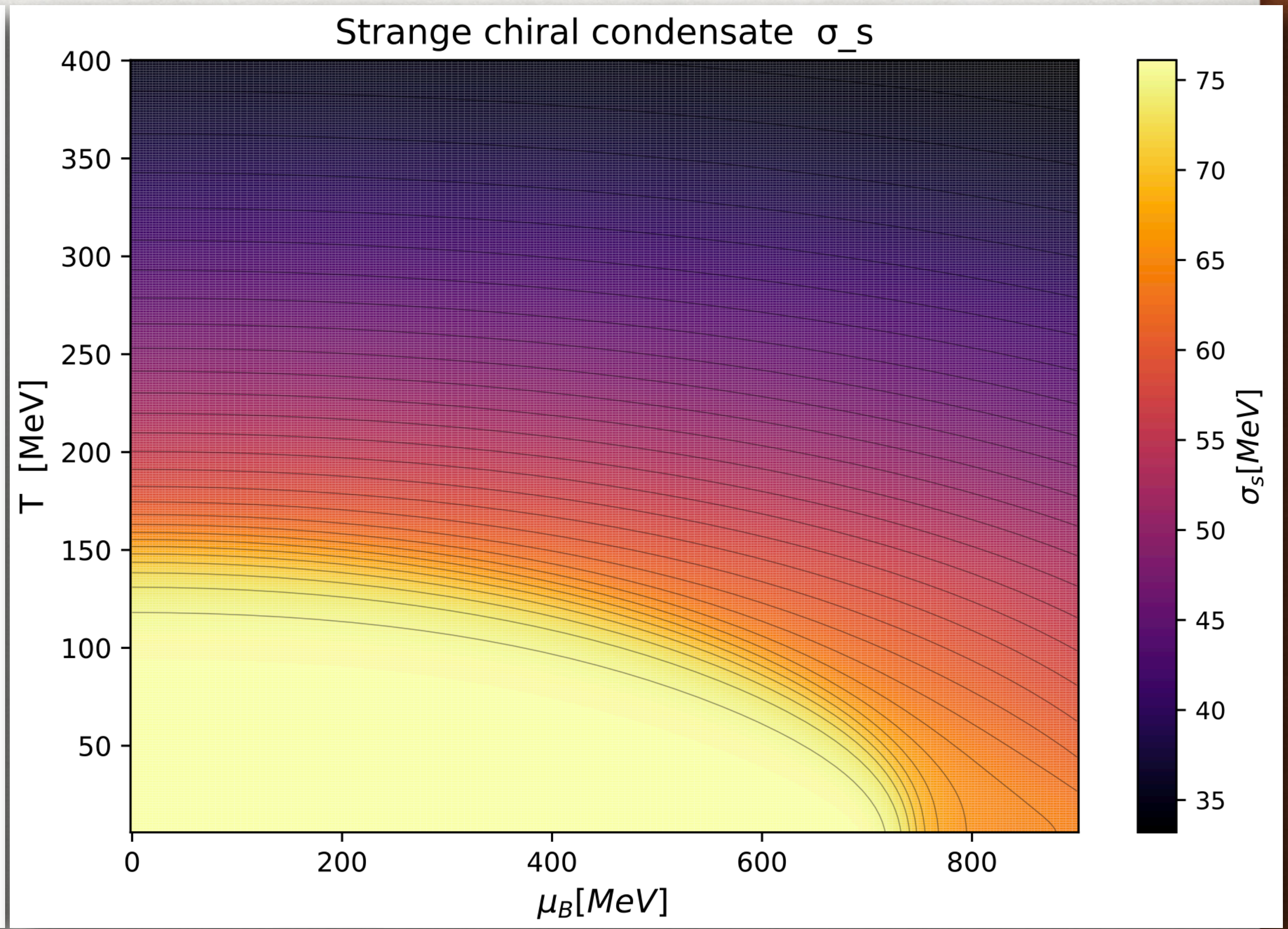






Normalised pressure (T: 70–400 MeV)
$(P_{vac} - P)/P_{ideal}$
0.0
0.1
0.2
0.3
0.4
0.5
T [MeV]
400
350
300
250
200
150
100
50
μ_B[MeV]
0
200
400
600
800
0.1
0.2
0.3
0.4


Light chiral condensate σ_l
T [MeV]
400
350
300
250
200
150
100
50
0
200
400
600
800
μ_B[MeV]
45
40
35
30
25
20
15
10
5
σ_l[MeV]


Strange chiral condensate σ_s
T [MeV]
400
350
300
250
200
150
100
50
0
200
400
600
800
μ_B[MeV]
75
70
65
60
55
50
45
40
35
σ_s[MeV]

Conclusions!

- Chiral matrix model (CMM) keeps the Quarks degrees of freedom alive along with the mesonic fields Φ, unlike other models (NJL, LSM), which integrated out the quarks degrees of freedom.

- CMM used a novel symmetry breaking term $V_h^T \sim -m_{qk}\dfrac{\partial V^{qk,T}}{\partial \Sigma_f}$, ensuring $m_f \to m_{\text{current}}$ as $T \to \infty$.

- Chiral matrix model $\equiv$ Matrix model + Chiral perturbation theory

- Till now no critical point on QCD phase diagram. Work is in progress.

Thank you for your attention