Non-Perturbative Approach to Heavy-flavor transport in Nuclear Collisions

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Primary reference. *T. Krishna et al. Phys. Lett. B* ***871***,*(2025)(139999)*

Hard probes in non-equilibrium QCD matter, ICTS-2026

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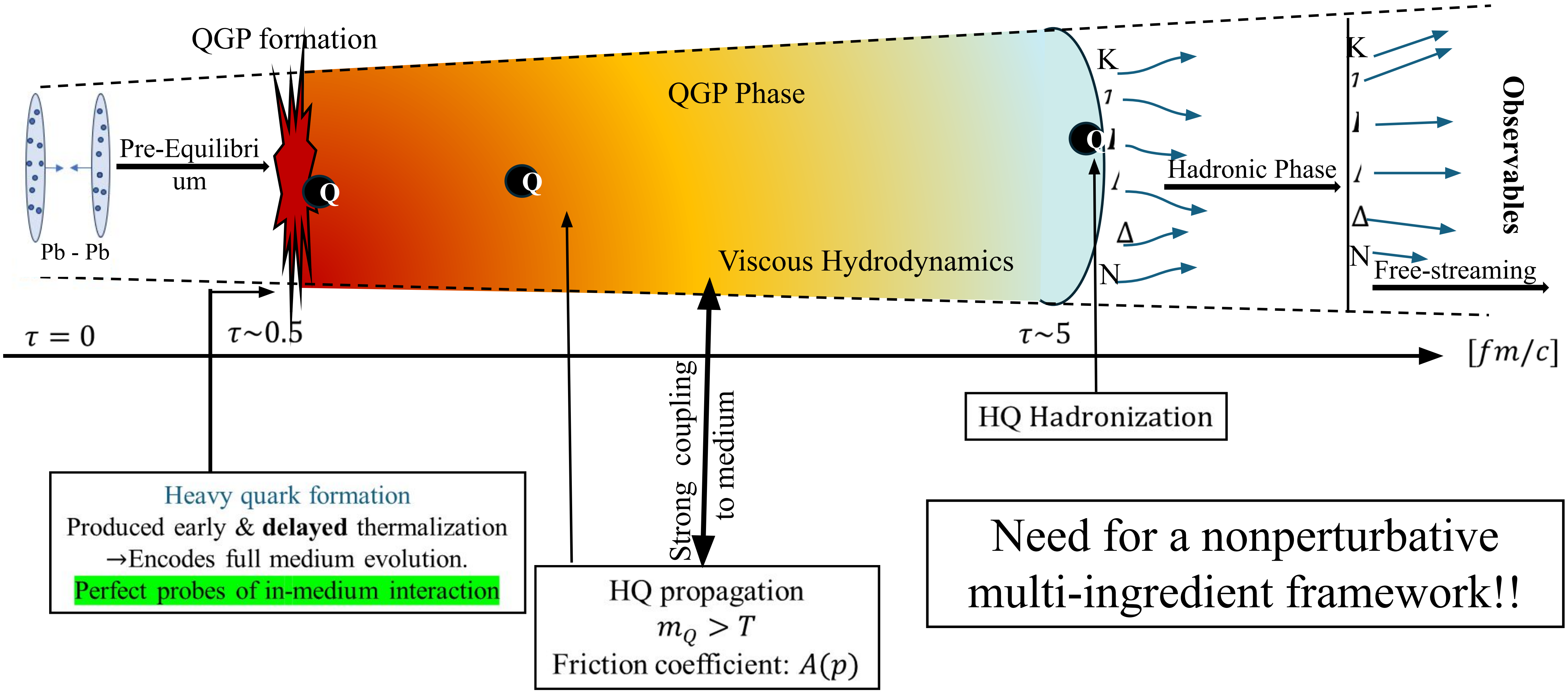
Outline

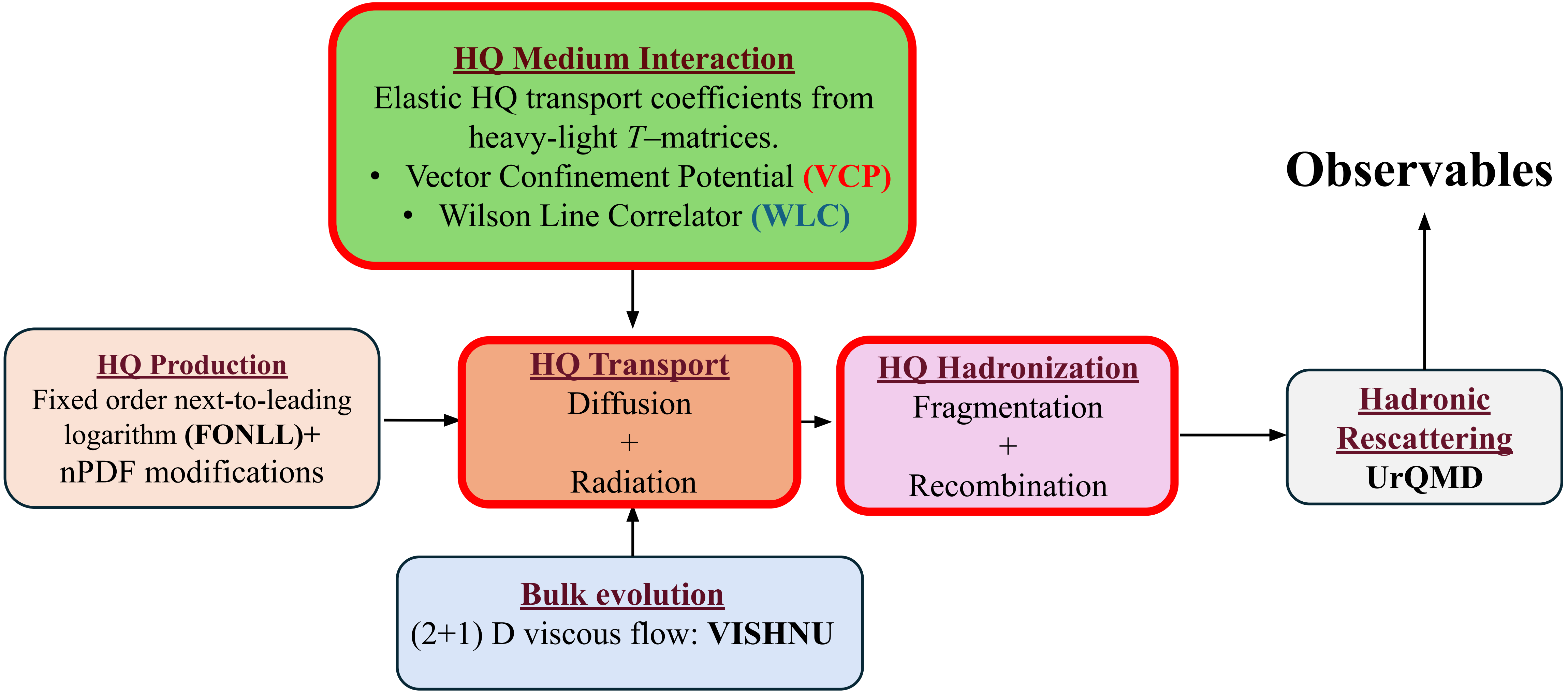
- Motivation
- Overview of our nonperturbative transport approach
- Transport Coefficients
  - *T*-matrix formalism
  - Predictions and comparison to lattice-Quantum ChromoDynamics (lQCD) data
- Heavy-Quark(HQ) transport
  - Diffusion + radiation
- HQ hadronization
  - Recombination + Fragmentation
- Data-theory comparison to ALICE and CMS data for Pb-Pb data and STAR data for Au-Au

Motivation



Need for a nonperturbative multi-ingredient framework!!

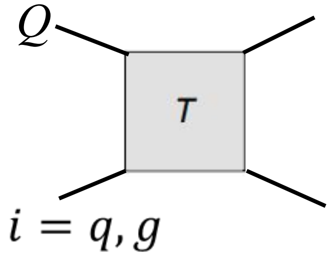
Ingredients for Transport

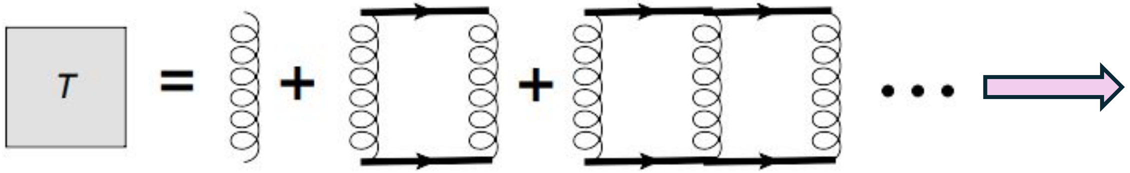


TRANSPORT COEFFCIENTS

Transport Coefficients: *T*-matrix formalism

Based on ***T*-matrix formalism**: nonperturbative scattering framework for strongly coupled QGP;





Infinite-Ladder Resummation

$$T_{ij} = V_{ij} + \int V_{ij} G_i G_j T_{ij}$$

4D Bethe-Salpter Equation

$M_Q \gg T \Rightarrow \frac{q^2}{m_Q}$: Small Energy Transfer limit ->3D Bethe-Salpter eqn

Kernel $\boldsymbol{V_{ij}} \rightarrow \boldsymbol{V(p-p')}$: Input potential

$$G_i = \frac{1}{\omega - \omega_k - \Sigma_i}$$

Quark propagator

$$\boldsymbol{V(p-p')} = \boldsymbol{V^{vec}} + \boldsymbol{V^{sca}} + (\text{spin dependent terms..})$$

$$\Sigma_i = \int_m T_{im} G_m$$

Self-Energy

$$A(p) = \int d^4p' d^4q d^4q' |T|^2 \left(1 - \frac{(\boldsymbol{p.p'})}{p^2}\right) \rho_q \rho_{q'} \rho_{p'} f_q$$

Quantum effects encoded in self-consistently evaluated spectral functions

Self-Consistent Dyson-Schwinger like calculation

T-matrix: [Riek+Rapp PRC 82(2010),Liu+Rapp PRC 97(2018)]

Transport Coefficients: lQCD Constraints

Input potential

[Color-Coulomb (V_c) + Confining term(V_s)]

Vacuum: $V(r) = -\frac{4}{3}\frac{\alpha_s}{r} + \sigma\, r$

α_s: Coupling constant
σ: String tension

→ Determined by vacuum quarkonium spectroscopy

In-medium: $V(r) = -\frac{4}{3}\alpha_s\left(\frac{e^{-m_d r}}{r} + m_d\right) - \frac{\sigma}{m_s}\left(e^{-m_s r - (c_b m_s r)^2} - 1\right)$

$m_{d/s}$: Debye screening masses
c_b : Mimics string breaking

Constraints from thermal lQCD

Vector Confinement Potential (VCP)

Confining potential: scalar + vector admixture

$$V^{vec} = (1-\chi)V_s + V_c$$
$$V^{sca} = \chi V_s$$

$\chi = 0.6$

40% 'vector' confining potential

(Improved Vacuum spin-splittings in Quarkonia)

+

Constrained with **HQ free energies** in-medium

Wilson Line Correlators (WLC)

Same vacuum structure as VCP but with $(\chi = 0.8)$

+

Constrained with **Wilson line correlator** from lQCD computations

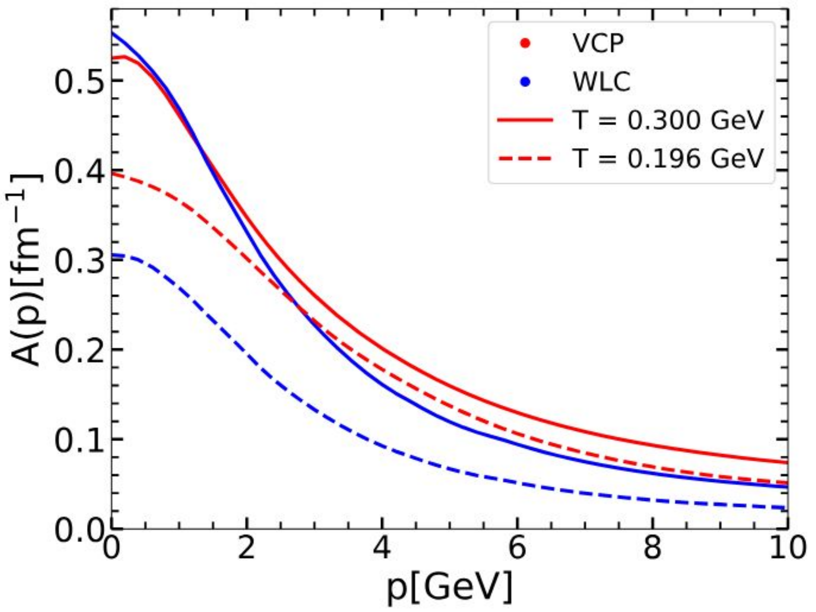
$$W(r,\tau,T) = \int dE e^{-E\tau}\rho_{\bar{Q}Q}(E,r,T)$$

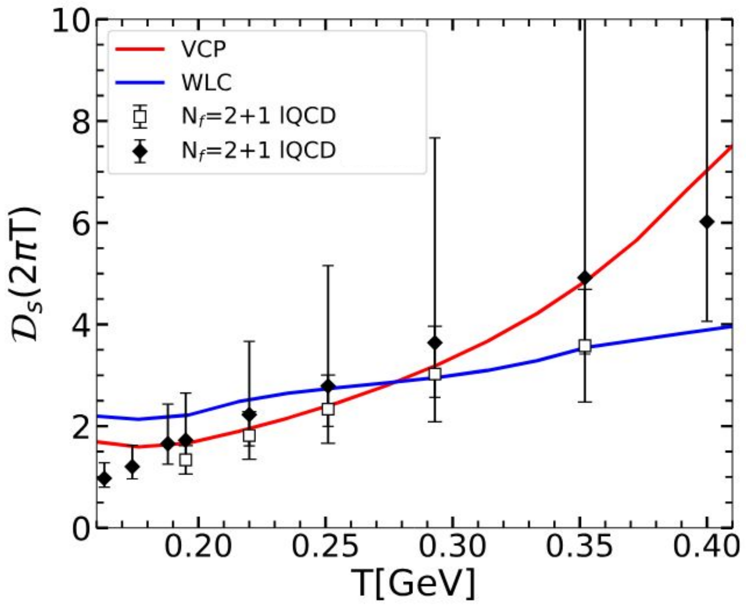
$\rho_{\bar{Q}Q}$:Spectral Function evaluated from in-medium T-matrix

VCP: Tang +Rapp PRC 108(2023) ; WLC: Tang et al EPJA60(2024)

Transport Coefficients: Predictions

VCP vs **WLC**





$$\mathcal{D}_s = \frac{T}{MA(p=0)}$$

Spatial diffusion coefficient

Vector component in confining potential leads to

- Harder momentum dependence of *A(p)*
- Smaller $\mathcal{D}_s$

Wilson Line Correlators leads to

- Softer momentum dependence(due to less vector component)
- Weaker T dependence in $\mathcal{D}_s$: Due to lower screening

VCP tends to agree better with lQCD data for spatial diffusion coefficient

lQCD data: HotQCD, PRL132 (5) (2024), HotQCD, arxiv: 2506.11958

HQ TRANSPORT

HQ Transport: Diffusion

$$\frac{df_Q}{dt} = \mathcal{D}[f_Q] + C^{1\to 2}[f_Q]$$

Diffusion operator: $\mathcal{D} = \frac{\partial}{\partial p_i}\left(A(p)p_i + \frac{\partial}{\partial p^i}B(p)\right)$

Encodes soft momentum kicks from the medium.

Fokker-Planck for HQ diffusion ⬄ Langevin dynamics

$A(p)$: Friction Coefficient
$B(p)$: Drag Coefficient

Equilibrium limit
↓
Enforce Einstein Relation
$B = TEA$

$$\Delta\vec{p} = -\eta_D \vec{p}\Delta t + \vec{\xi}(t)\Delta t \quad \langle \xi_i(t)\xi_j'(t')\rangle = \kappa\delta_{ij}\delta(t-t') \quad (\kappa = 2TE\eta_D)$$

Heavy quark treated as quasi-particle + quantum effects encoded in transport coefficients

[Weiyao Ke, *et al,* PRC 98]

HQ Transport: Radiation

$$\frac{df_Q}{dt} = \mathcal{D}[f_Q] + \mathcal{C}^{1\to 2}[f_Q]$$

$\mathcal{C}^{1\to 2}[f_Q]$: Diffusion-induced radiation

Incoherent medium-induced gluon radiation

Collision rate given by

$$R_{q\to q+g} \propto \alpha_s \, \hat{q}_s P(x)/(k_\perp^2 + m_\infty^2)$$

No interference between successive gluon emissions

$$\tau_f \quad < \quad \lambda$$

(Gluon formation time < mean-free path)

Bethe-Heitler regime

+

LPM effect

Incoherent gluons generated

Accepted with a probability

$$\frac{\lambda}{\tau_f}\sqrt{\ln\frac{\tau}{\lambda}}$$

suppresses emissions for

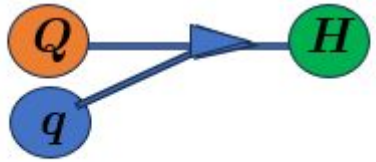
$$\tau_f \quad >> \quad \lambda$$

"Destructive interference"

between successive gluon emissions

[Weiyao Ke, *et al,* PRC 100]

HQ Hadronization



Recombination

Fragmentation: $D(z,r)$

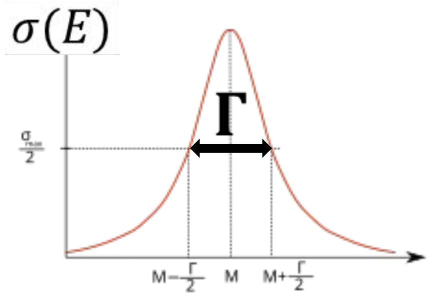
Hadronization: Recombination

Recombination: Resonance Recombination Model(RRM)

[Ravagli+Rapp,2007]

Relativistic Breit-Wigner cross section + Boltzmann Equation

$$\frac{\partial f_M}{\partial t} = -\frac{\Gamma}{\gamma_p} f_M + \beta$$



$\frac{-\Gamma}{\gamma_p} f_M$:**Loss term** through $M \rightarrow q + \bar{q}$

β: **Gain term** $q + \bar{q} \rightarrow M$; Resonance formation from quark distributions

Equilibrium limit->LHS=0 i.e. $f_M = \gamma_p \beta / \Gamma$

Extension to baryons: Formation of an intermediate di-quark

$$q + q \rightarrow dq$$

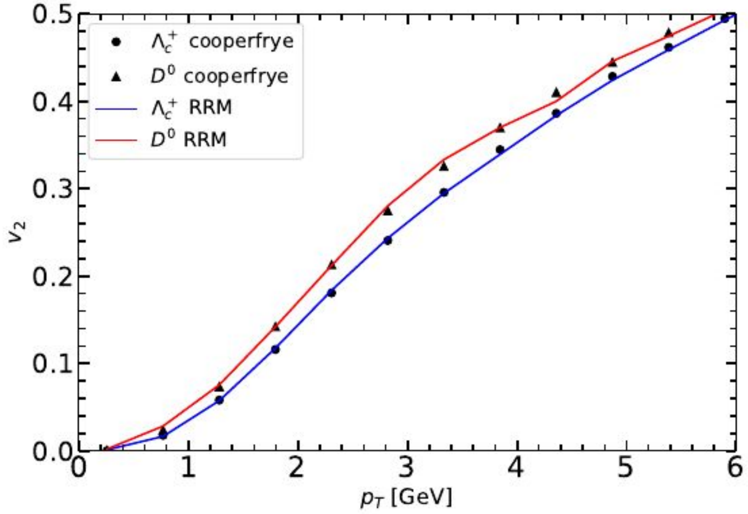
$$dq + Q \rightarrow B$$

$$f_M = \frac{\gamma_M}{\Gamma_M} \int d\Pi \delta^3 \, f_q f_{\bar{q}} \sigma(s) v_{rel}$$

Meson Phase Space Distribution

$$f_B = \frac{\gamma_B}{\Gamma_B} \int d\Pi \delta^3 f_1 f_2 f_3 \sigma_{dq}(s_{dq}) \sigma_B(s_B) v_{rel}$$

Baryon Phase Space Distribution



Equilibrium PSDs for quarks → Equilibrium limit for Hadrons!

For a flowing medium

Hadronization: Fragmentation

Fragmentation: Statistical hadronization approach ; constrained using *p-p* data [He+Rapp, PLB 2019]

Input charm quark p_T spectrum
p_T-dependence from **(FONLL)** – [Cacciarri et al, JHEP 1998]

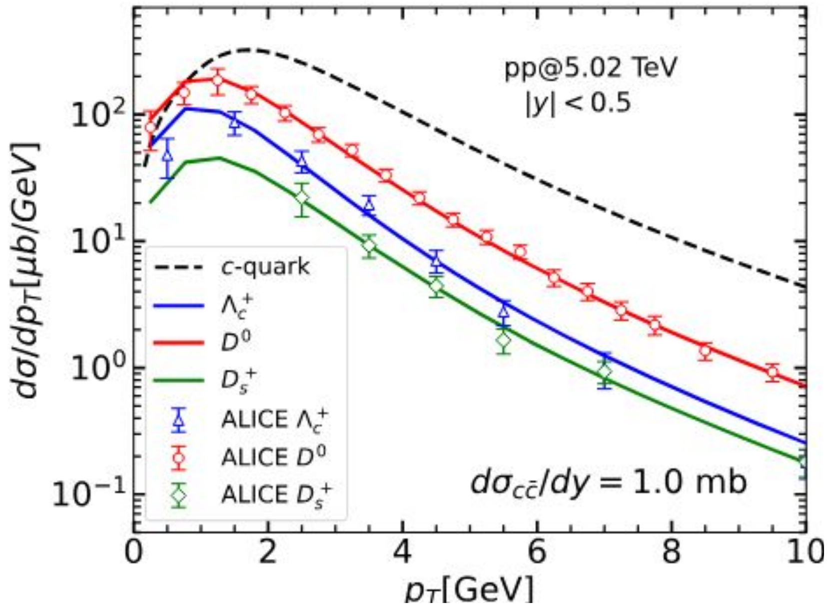
Open Charm-Hadron spectra

- D-mesons(D, D^*, D_S) : PDG.
- Charm Baryons: PDG+RQM*-predicted states,

addnl $18\ \Lambda_c, 42\ \Sigma_c, 62\ \Xi_c, 34\ \Omega_c$ [Ebert et al, PRD 2011]

Fragmentation function

- $D(z,r)$ from Heavy-Quark Effective Theory
- $r_M \propto r_{D^0}(m_M - m_c)$ [Braaten et al, PRD 1995]



- Populate all hadrons with relative thermal weights.

$$w_i \propto \gamma_s d_i m_i^2 T_H K_2\left(\frac{m_i}{T_H}\right)$$

strangeness suppression γ_s(=0.7 for LHC)

- Decay to ground states with appropriate branching ratios.

*RQM: Relativistic Quark Model

lQCD predicts more baryon states [Madanagopalan et al, PoS LATTICE 2014]

[ALICE: EPJC 79 (2019)]; [ALICE: PRC 107 (2023)]

Hadronization: Recombination + Fragmentation

HQs marked for hadronization at $T \sim 160$ MeV

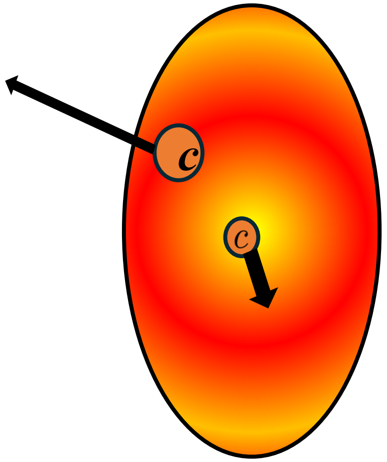
Calculate a recombination probability P_{rec} for every heavy quark

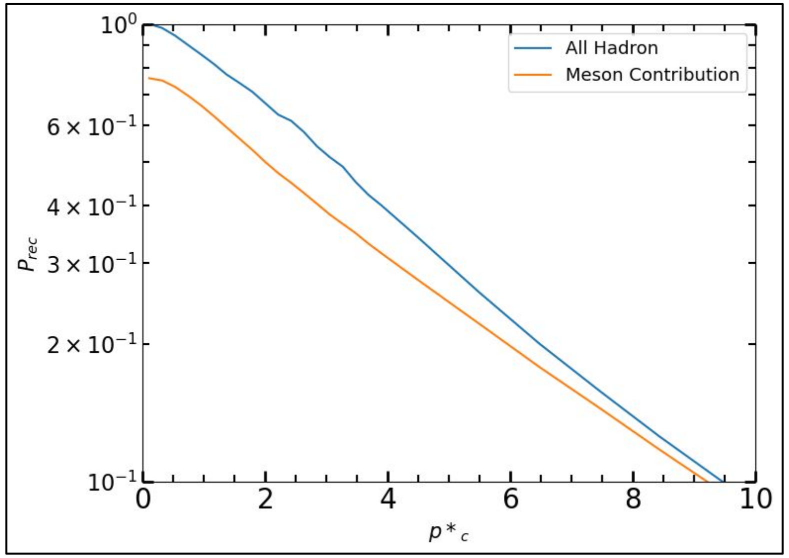
'Populate' heavy quark to one hadron.

$$dN_H = \sum_Q [dN_{RRM} + (1 - P_{rec}) D_H(z)]_Q$$

Do this for all hadrons in spectrum
+
perform strong feed-downs into ground state as in *p*-*p*

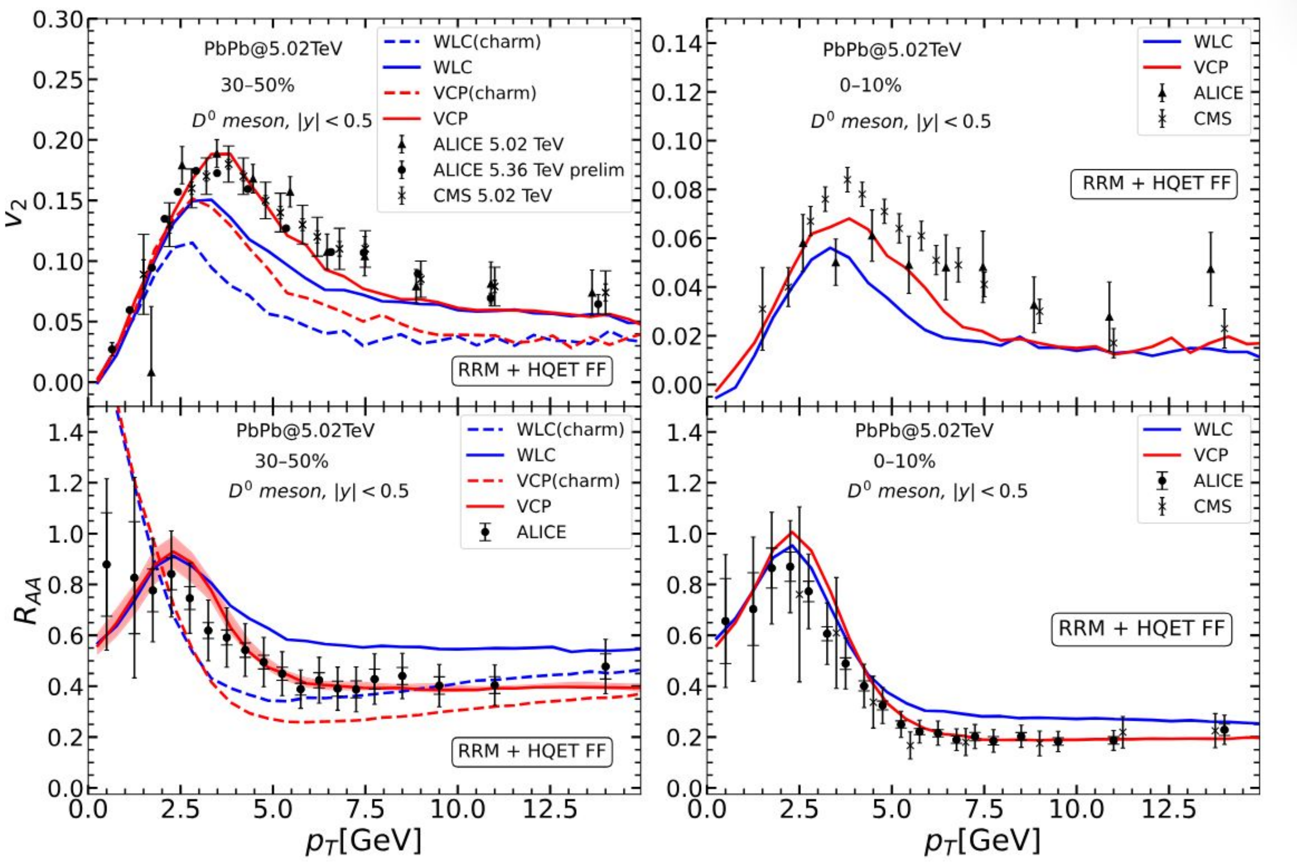
Preserves space-momentum correlations





D^0 at LHC: Pb-Pb @$\sqrt{s_{NN}} = 5.02$ TeV

Semi-Central | Central



Framework has good sensitivity to in-medium QCD force

Observables

$$v_2(p_T) = \frac{\int \frac{dN}{dpT\, d\phi} \cos 2\phi}{\int \frac{dN}{dpT\, d\phi}}$$

Elliptic flow coefficient

$$R_{AA}(p_T) = \frac{1}{N_{col}} \frac{\left(\frac{dN_{AA}}{dp_T}\right)}{\frac{dN_{pp}}{dp_T}}$$

Nuclear Modification Factor

Hadronization

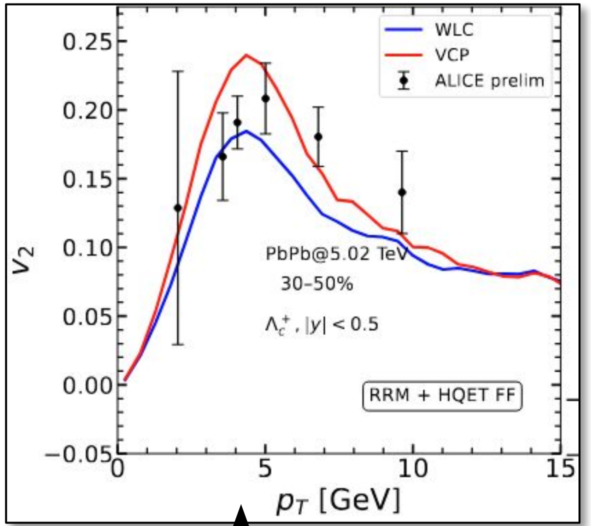
- Augments v_2
- Changes shape of R_{AA} (flow bump)

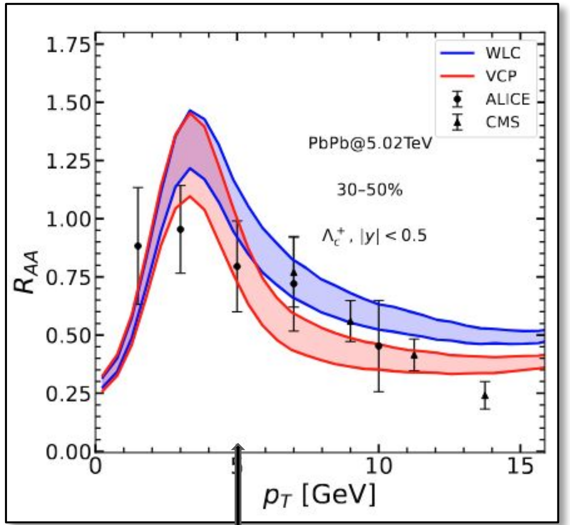
VCP constraints aligns better with lQCD data

Fair agreement with ALICE and CMS data

[ALICE: PLB 813(2021)][CMS: PLB 816(2021)]

Λ_c^+ and Hadro-chemistry

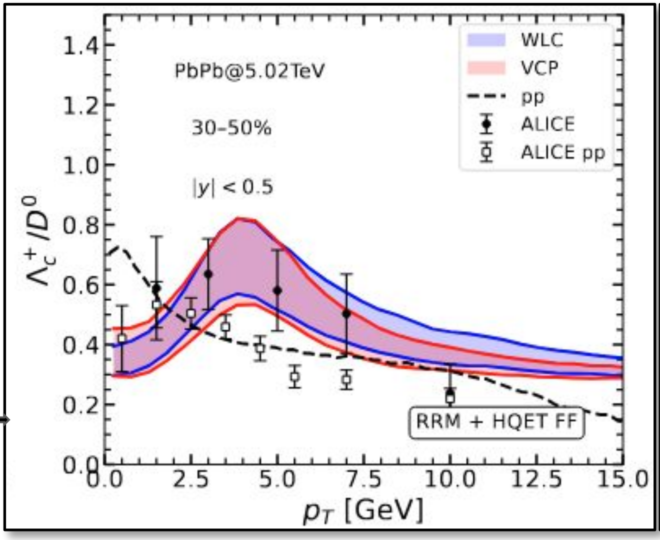


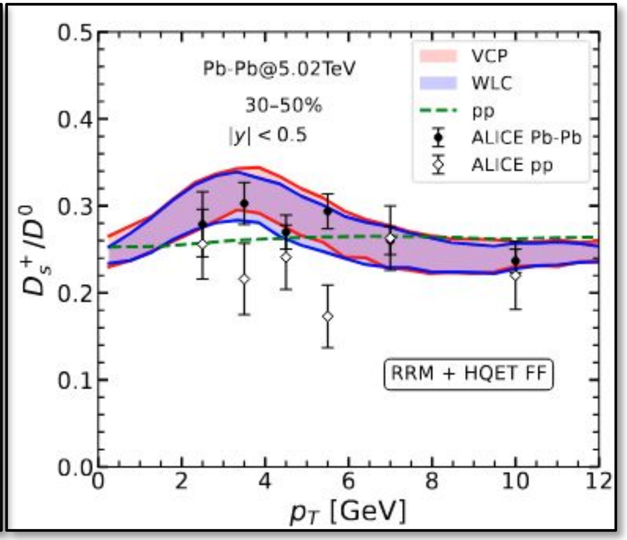


- Enhancement of D_s^+ going from pp to AA.
- Λ_c^+ picks up more flow relative to D^0:Flow bump

Recombination with 2 light quarks.
$v_2(\Lambda_c^+) \gtrsim v_2\ (D^0)$

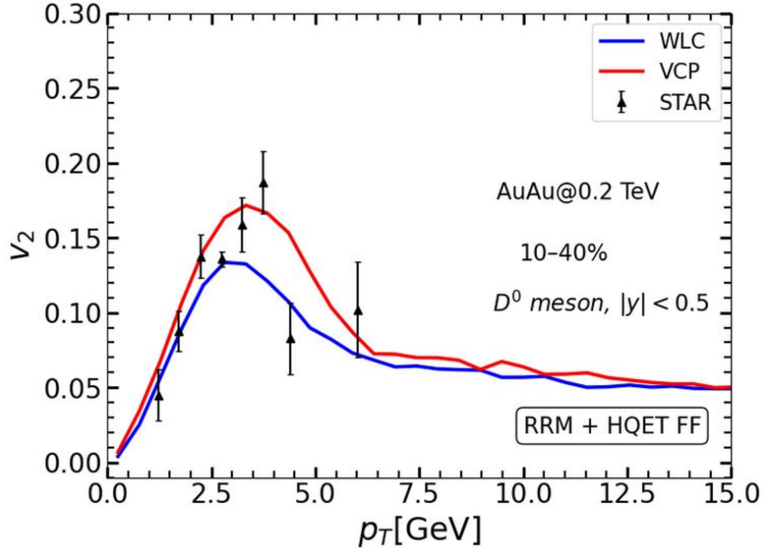
Bands: uncertainty in branching ratios of $\Lambda_c^* \to D + N$ vs $\Lambda_c^* \to \Lambda_c^+ + \pi$

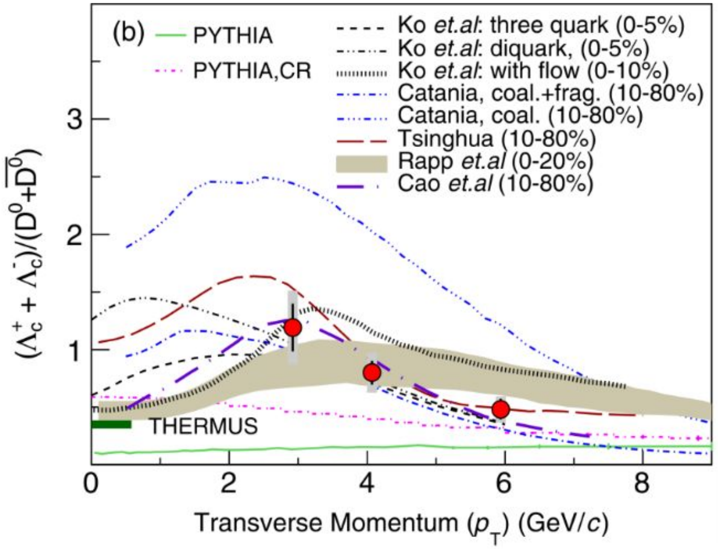


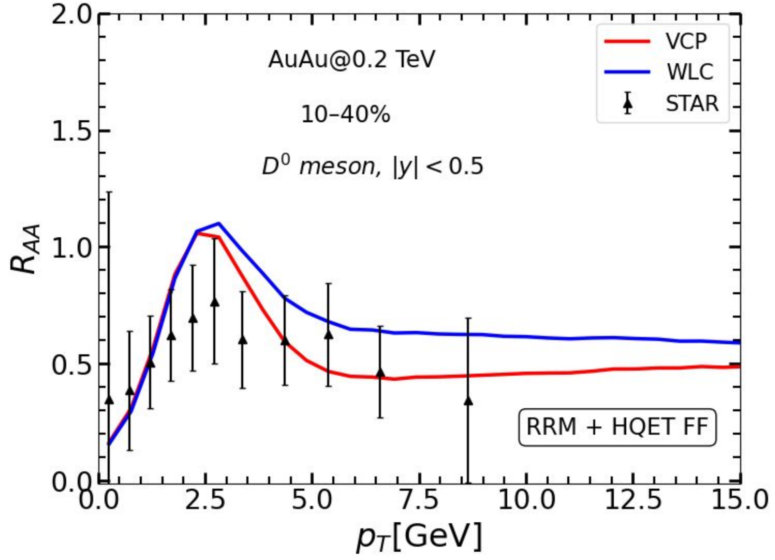


Hadro-chemistry

RHIC: Au-Au @$\sqrt{s_{NN}} = 0.2$ TeV







For RAA, we need shadowing.
But no Λ_c^+ measurement down to low p_T at RHIC

$\frac{\Lambda_c^+}{D^0} \sim 0.5$ in Au-Au + measured D^0 cross section in Au-Au

↓

~30% shadowing for semi-central

←

VCP constraints: fair agreement with STAR data

Conclusion

- We have developed a new framework for heavy-flavor transport in URHICs combining state-of-the-art ingredients into a fully nonperturbative and **lattice-QCD–constrained transport coefficients, without any parameter tuning towards experimental data.**
- Effects such as

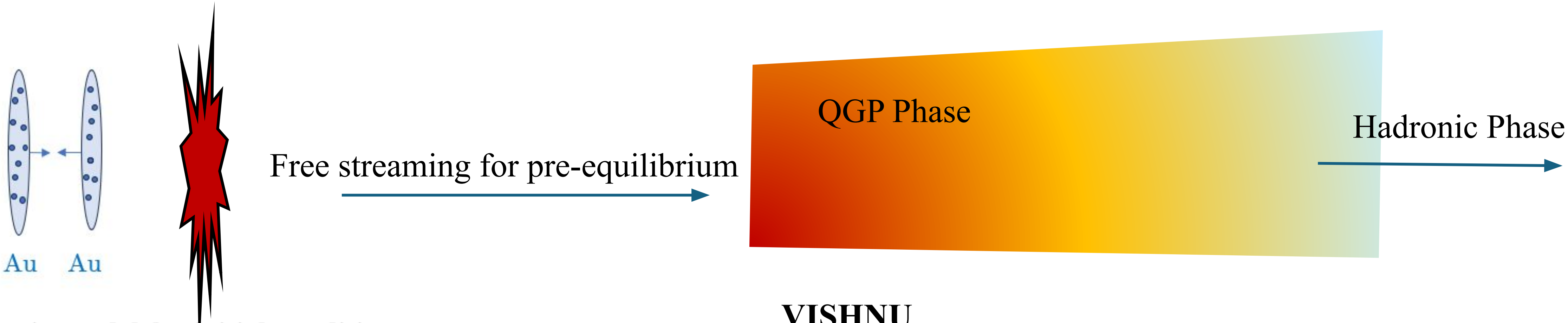
1. Pre-Equilibrium Effects
2. Event-by-event fluctuations in the hydro evolution
3. Hadronic diffusion

are expected to be sub-leading and incorporation of this is underway.

The presentation here is based on work done in
T. Krishna, R. Rapp, Y. Fu, S. A. Bass, and W. Ke, Phys. Lett. B ***871****,(2025)(139999)*

Thank
You

TRENTO and VISHNU



Parametric model for initial conditions

TRENTO used to generate the **initial entropy (or energy) density distribution**

-

Based on nuclear thickness functions

$$T_A(x,y), T_B(x,y)$$

projected nucleon density of A&B in the transverse plane.

The produced entropy density: **generalized mean** of the two thickness functions:

$$s(x,y) \propto \left((T_A(x,y), + T_B(x,y)) \frac{1}{2} \right)^{1/p}$$

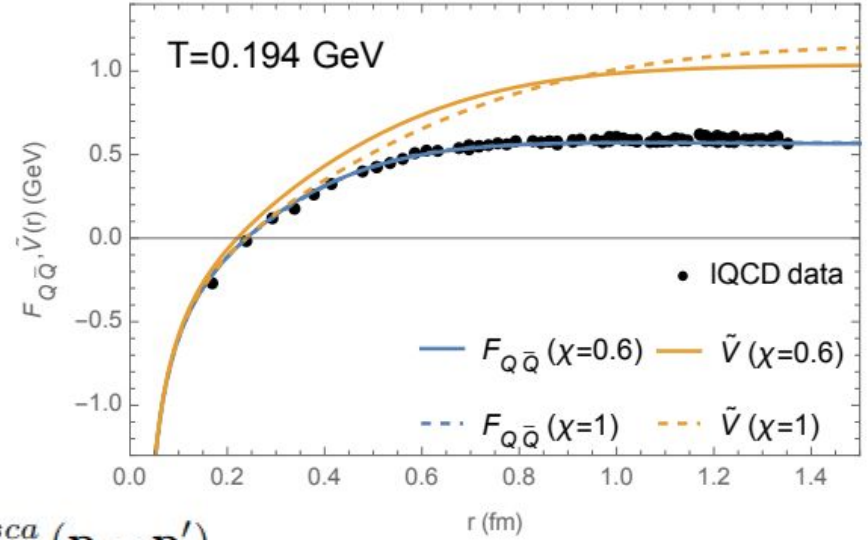
VISHNU

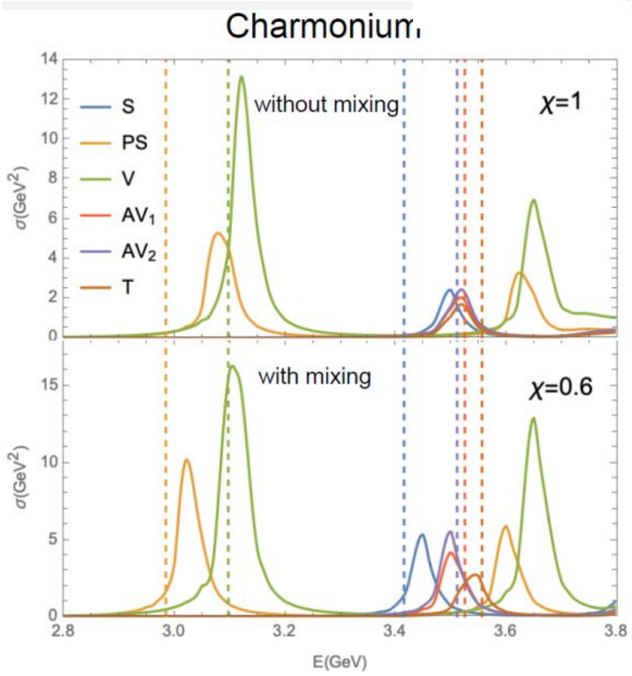
- **2+1D relativistic viscous hydrodynamics solver** that includes shear viscosity.
- Calibrated to produce light hadron observables.

T-matrix: Back-up

VCP:

$$F_{Q\bar{Q}}(r,\beta) = \frac{-1}{\beta}\ln\left[\int_{-\infty}^{\infty} dE\, e^{-\beta E} \times \frac{-1}{\pi}\mathrm{Im}\left[\frac{1}{E+i\epsilon-\tilde{V}(r)-\Sigma_{Q\bar{Q}}(E+i\epsilon,r)}\right]\right]$$





$$V_{ij}(\mathbf{p},\mathbf{p}') = \mathcal{R}_{ij}V^{vec}(\mathbf{p}-\mathbf{p}') + V^{sca}(\mathbf{p}-\mathbf{p}')$$

spin-orbit: $V^{LS} = \frac{1}{2M_Q^2 r}\langle \boldsymbol{L}\cdot\boldsymbol{S}\rangle\left(3\frac{d}{dr}V^{vec} - \frac{d}{dr}V^{sca}\right)$ spin-spin: $V^{SS} = \frac{3}{3M_Q^2}\langle \boldsymbol{S_1}\cdot\boldsymbol{S_2}\rangle \,\Delta\, V^{vec}$

tensor: $V^{T} = \frac{1}{12M_Q^2}S_{12}\left(\frac{1}{r}\frac{d}{dr}V^{vec} - \frac{d^2}{dr^2}V^{vec}\right)$

$$V^{vec} = V_{Coul} + (1-\chi)V_{conf}\,,\quad V^{sca} = \chi V_{conf}$$

Improved hyper/fine splittings

T-matrix: Back-up

WLC:

$$W(r,\tau,T)=\int_{-\infty}^{\infty} d\omega e^{-\omega\tau}\rho_{Q\bar{Q}}(\omega,r,T)$$

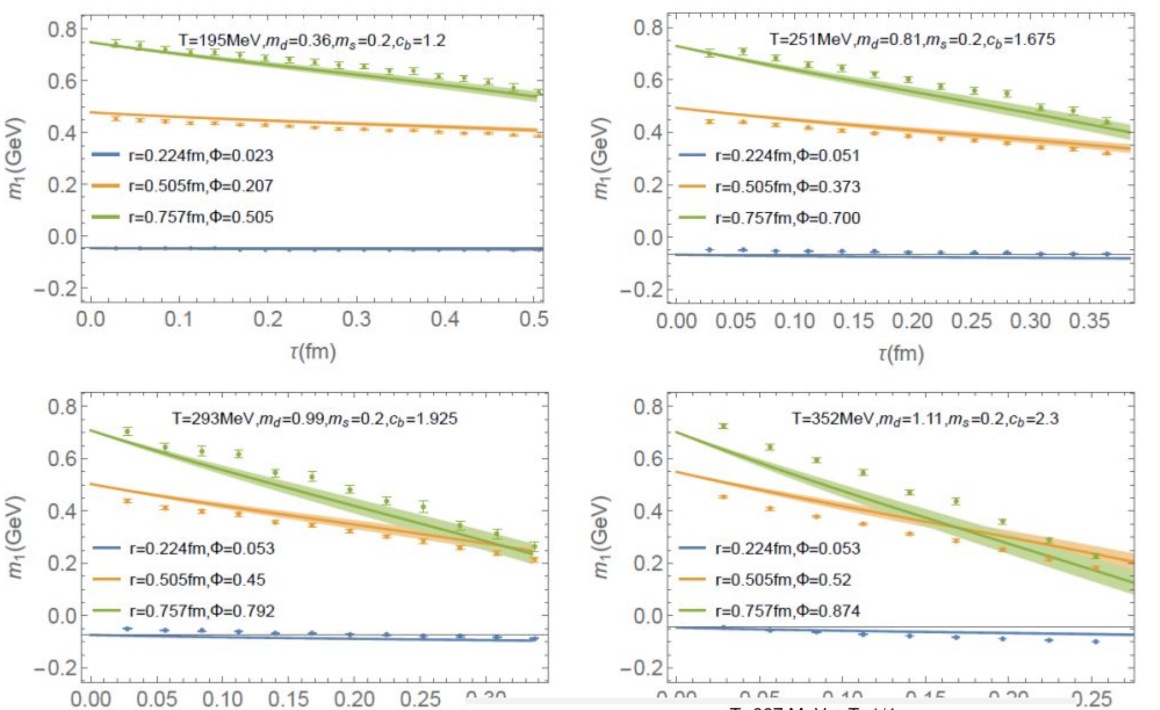
$$\rho_{Q\bar{Q}}(\omega,r,T)=\frac{-1}{\pi}\mathrm{Im}\left[\frac{1}{\omega-V(r,T)-\Phi(r,T)\Sigma_{Q\bar{Q}}(\omega,T)}\right.$$

Quarkonium spectral function

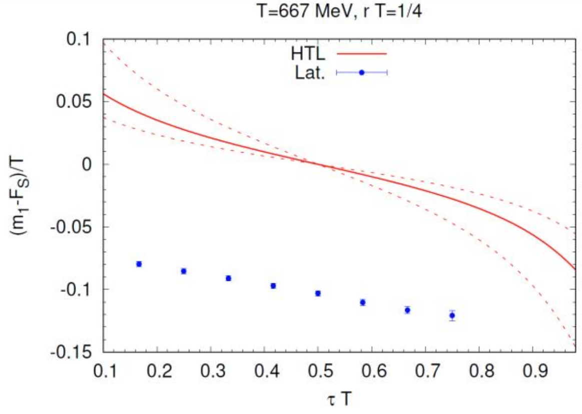
$$m_1(r,\tau,T)=-\partial_\tau \ln W(r,\tau,T)$$

Need smaller screening masses, even less potential screening

$$F=-T\ln\left(\int dE\, e^{-\beta E}\rho(E)\right)$$



Predictions from Perturbative Hard-Thermal-Loop Approach



Implementation of Recombination probability

To effectively partition charm quarks into fragmentation or recombination, we calculate a self consistently derived recombination probability.

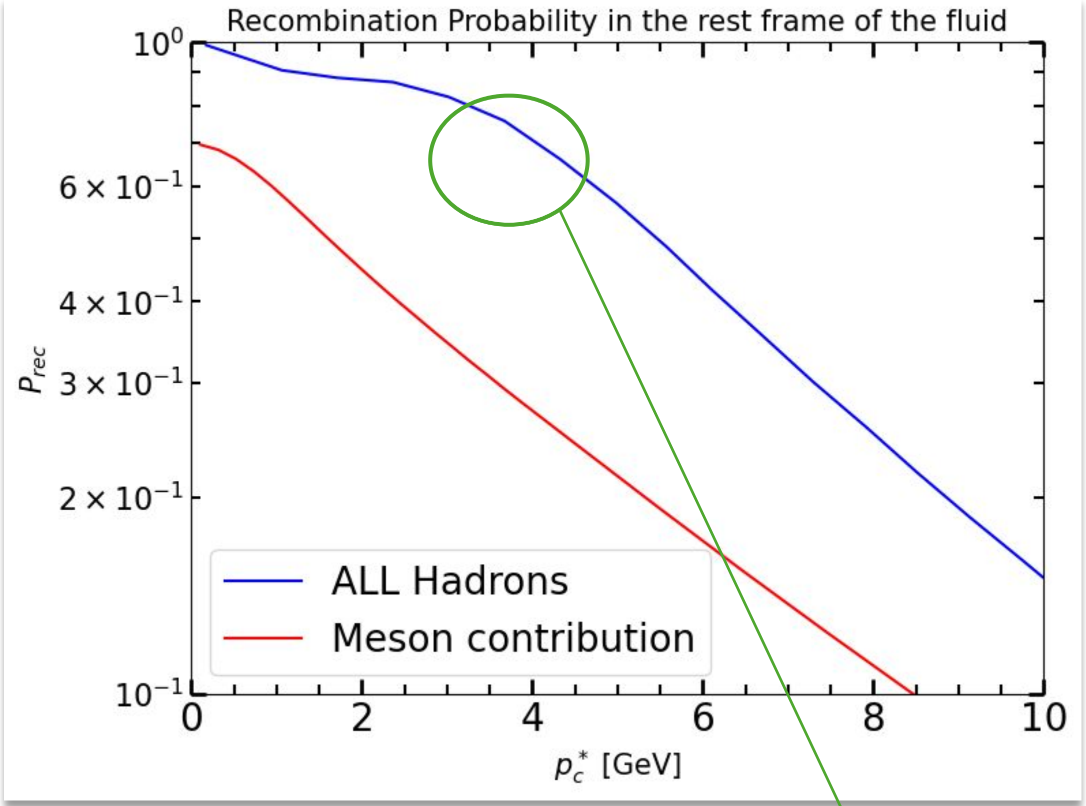
$$P_M(p_c^*) = P_0 \int \frac{d^3p_q}{2\pi^3} f_q(p_q) \frac{\gamma_M}{\Gamma_M} \sigma(s) v_{rel}$$

$$P_B(p_c^*) = P_0 \int \frac{d^3\vec{p}_1^* d^3\vec{p}_2^*}{(2\pi)^6} g_1 e^{-E(\vec{p}_1^*)/T_H} g_2 e^{-E(\vec{p}_2^*)/T_H} \times \frac{\gamma_B}{\Gamma_B} \frac{\gamma_{dq}}{\Gamma_{dq}} \sigma(s_{12}) v_{\mathrm{rel}}^{12} \sigma(s_{d3}) v_{\mathrm{rel}}^{dq3}.$$

Charm quark momentum in fluid-rest frame

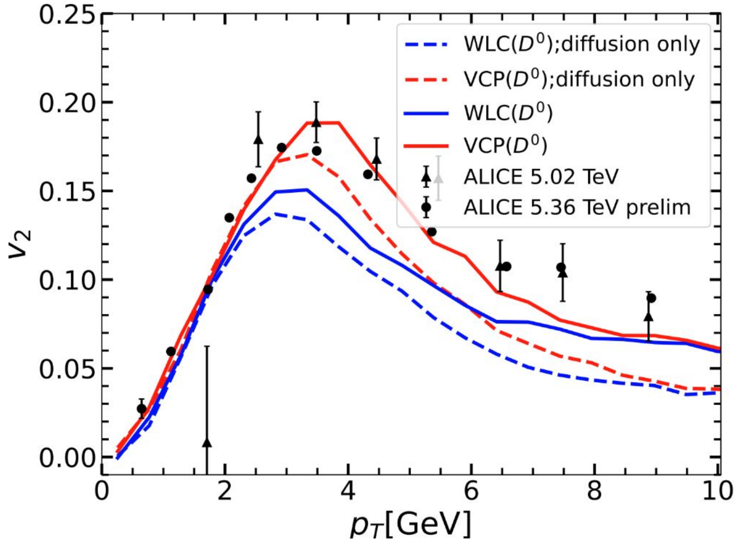
$$P_{\mathrm{tot}}(p_c^* = 0) = \sum_M P_M(0) + \sum_B P_B(0) = 1$$

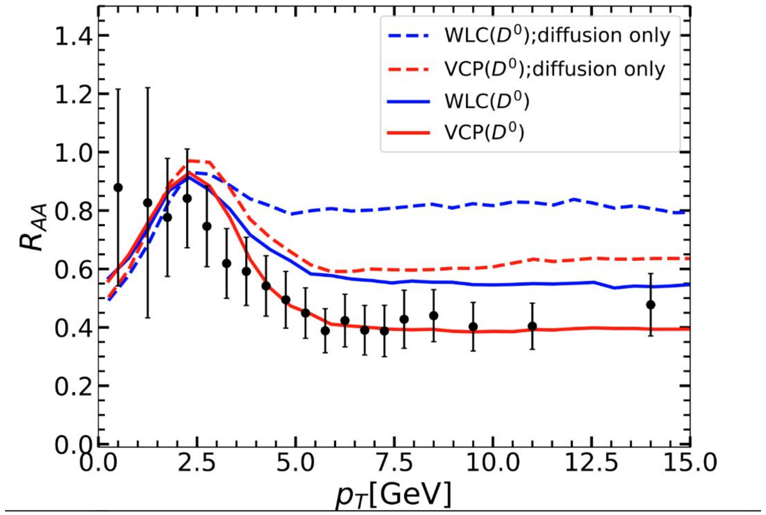
P_{rec} equal to 1 for a charm quark at rest in the thermal frame.



Comes from baryons

HQ Transport: Diffusion vs Radiation





Partons considered independent until

$$t - t_0 = \tau_f \equiv \frac{2x(1-x)E}{k_\perp^2(t, t_0) + x^2 M^2},$$

→ Accept with a probability $\frac{\lambda}{\tau_f}\sqrt{\ln\frac{\tau}{\lambda}}$

$k_\perp$: Transverse momentum broadening.

RQM and RRM info for Baryons

- Similar to RRM. Form qq-> dq and dp+Q->B. With two separate potentials.

qq-> dq potential

$$V(\mathbf{p},\mathbf{q};M)=\bar{u}_1(p)\bar{u}_2(-p)\mathcal{V}(\mathbf{p},\mathbf{q};M)u_1(q)u_2(-q), \quad (5)$$

with

$$\mathcal{V}(\mathbf{p},\mathbf{q};M)=\frac{1}{2}\Big[\frac{4}{3}\alpha_s D_{\mu\nu}(\mathbf{k})\gamma_1^{\mu}\gamma_2^{\nu} + V_{\rm conf}^{V}(\mathbf{k})\Gamma_1^{\mu}(\mathbf{k})\Gamma_{2;\mu}(-\mathbf{k}) + V_{\rm conf}^{S}(\mathbf{k})\Big],$$

Qdq-> B potential

$$V(\mathbf{p},\mathbf{q};M) = \frac{\langle d(P)|J_\mu|d(Q)\rangle}{2\sqrt{E_d(p)E_d(q)}}\bar{u}_Q(p)\frac{4}{3}\alpha_s D_{\mu\nu}(\mathbf{k})\gamma^{\nu}u_Q(q) + \psi_d^*(P)\bar{u}_Q(p)J_{d;\mu}\Gamma_Q^{\mu}(\mathbf{k})V_{\rm conf}^{V}(\mathbf{k})u_Q(q)\psi_d(Q) + \psi_d^*(P)\bar{u}_Q(p)V_{\rm conf}^{S}(\mathbf{k})u_Q(q)\psi_d(Q), \quad (6)$$

Quark content	Diquark type	M (MeV)
$[u, d]$	S	710
$\{u, d\}$	A	909
$[u, s]$	S	948
$\{u, s\}$	A	1069
$\{s, s\}$	A	1203

Width for mesons = 0.1 GeV; width for baryons near threshold = 0.1 GeV, away from thereshold = 0.3 GeV

Recombination Probability

To effectively partition charm quarks into fragmentation or recombination, we calculate a self consistently derived recombination probability.

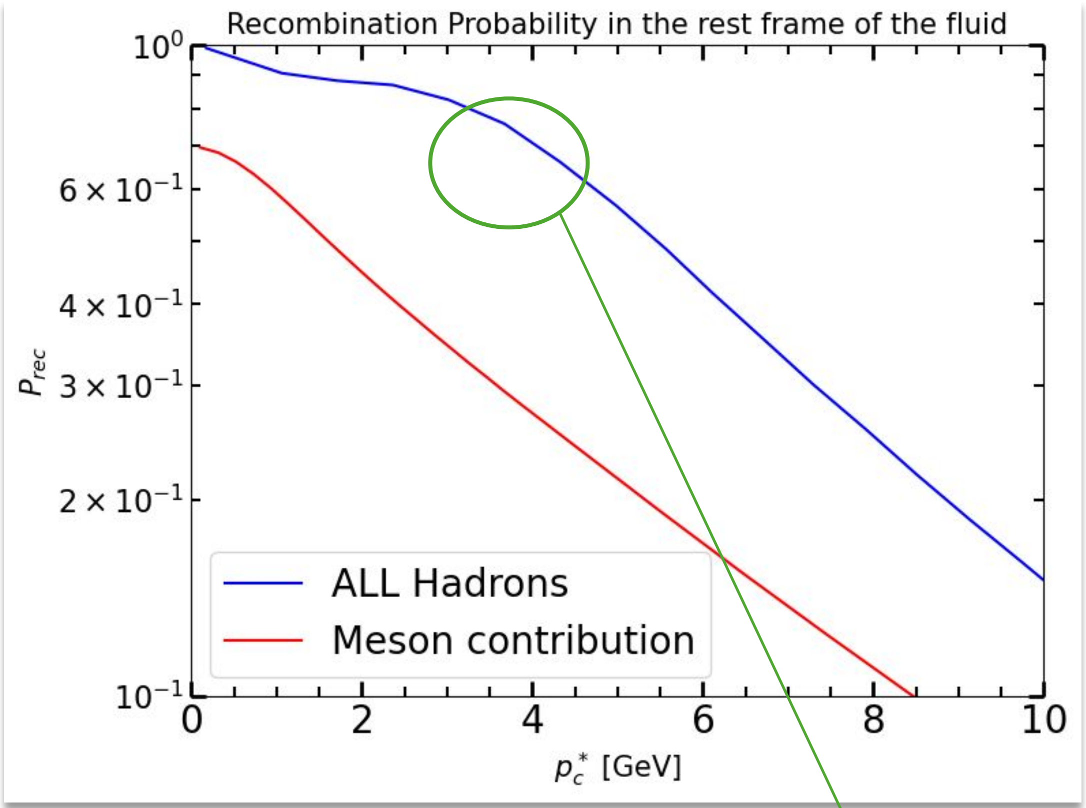
$$P_M(p_c^*) = P_0 \int \frac{d^3p_q}{2\pi^3} f_q(p_q) \frac{\gamma_M}{\Gamma_M} \sigma(s) v_{rel}$$

$$P_B\left(p_c^*\right) = P_0 \int \frac{d^3\vec{p}_1^{\,*} d^3\vec{p}_2^{\,*}}{(2\pi)^6} g_1 e^{-E(\vec{p}_1^{\,*})/T_H} g_2 e^{-E(\vec{p}_2^{\,*})/T_H} \times \frac{\gamma_B}{\Gamma_B} \frac{\gamma_{dq}}{\Gamma_{dq}} \sigma\left(s_{12}\right) v_{\rm rel}^{12} \sigma\left(s_{d3}\right) v_{\rm rel}^{dq3}.$$

Charm quark momentum in fluid-rest frame

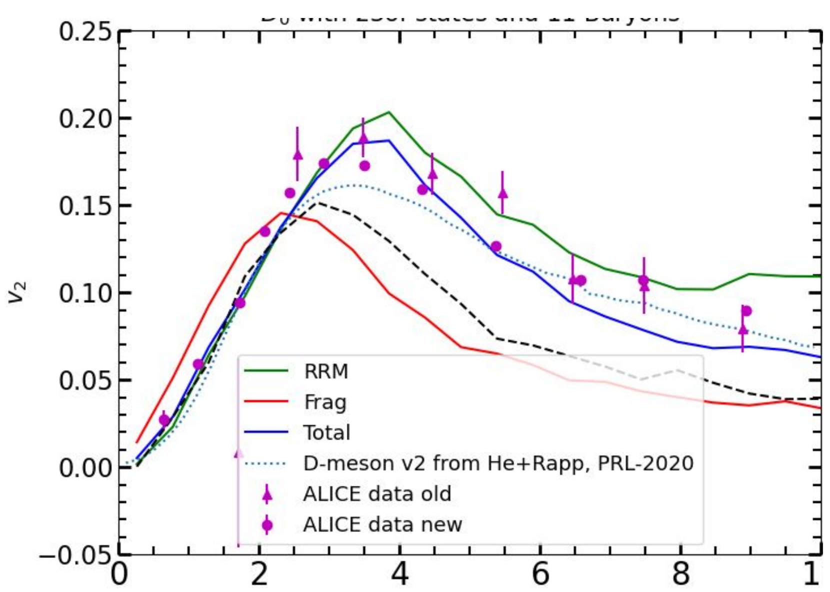
$$P_{\rm tot}\ (p_c^* = 0) = \sum_M P_M(0) + \sum_B P_B(0) = 1$$

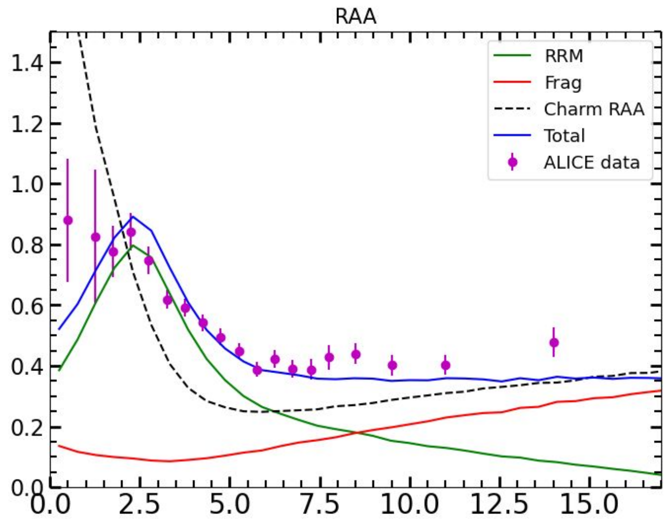
$P_{\rm rec}$ equal to 1 for a charm quark at rest in the thermal frame.



Comes from baryons

RRM vs Fragmentation





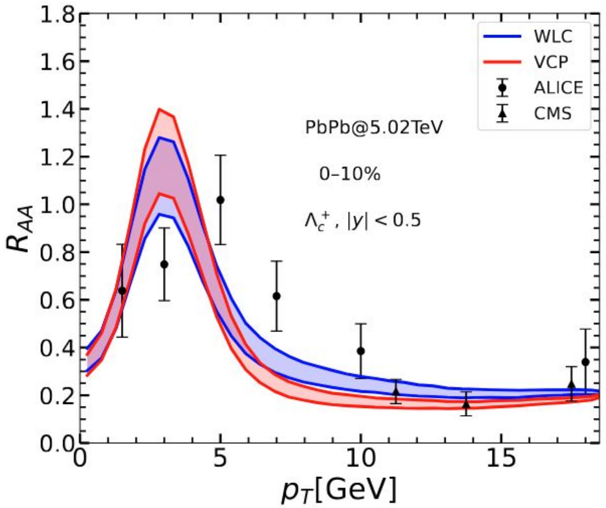
Σ_c^* or $\Lambda_c^* \rightarrow \Lambda_c + \pi$ (100%) *

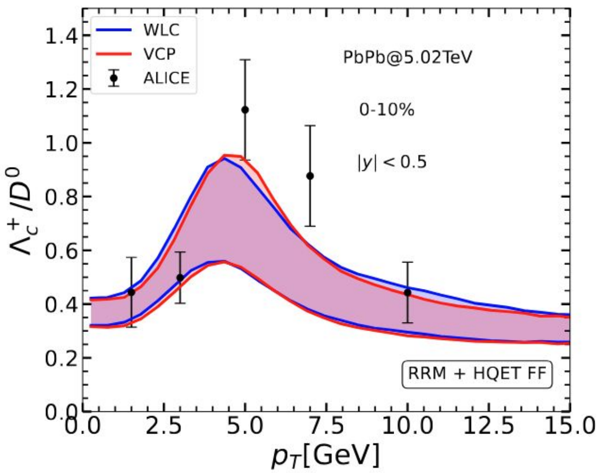
$\Xi_c^* \rightarrow \Xi_c + \pi$ (50%)

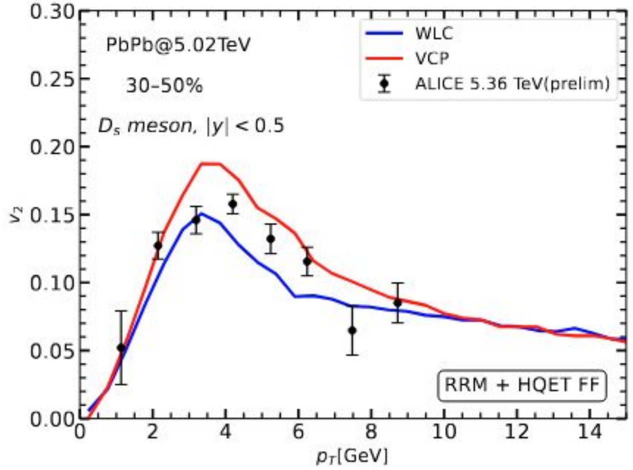
$\Xi_c^* \rightarrow \Lambda_c + K$ (50%)

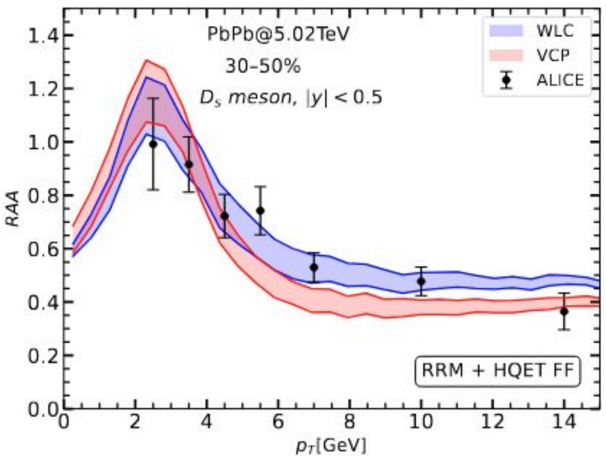
$\Omega_c^* \rightarrow \Xi_c + K$ (100%)

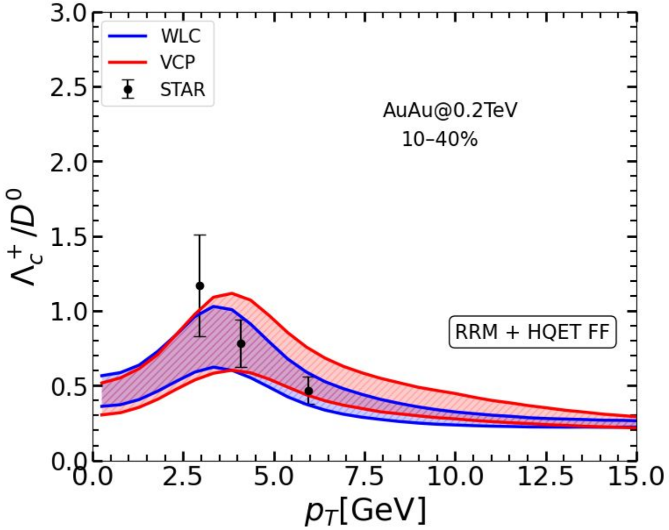
More Theory to Data comparison-I





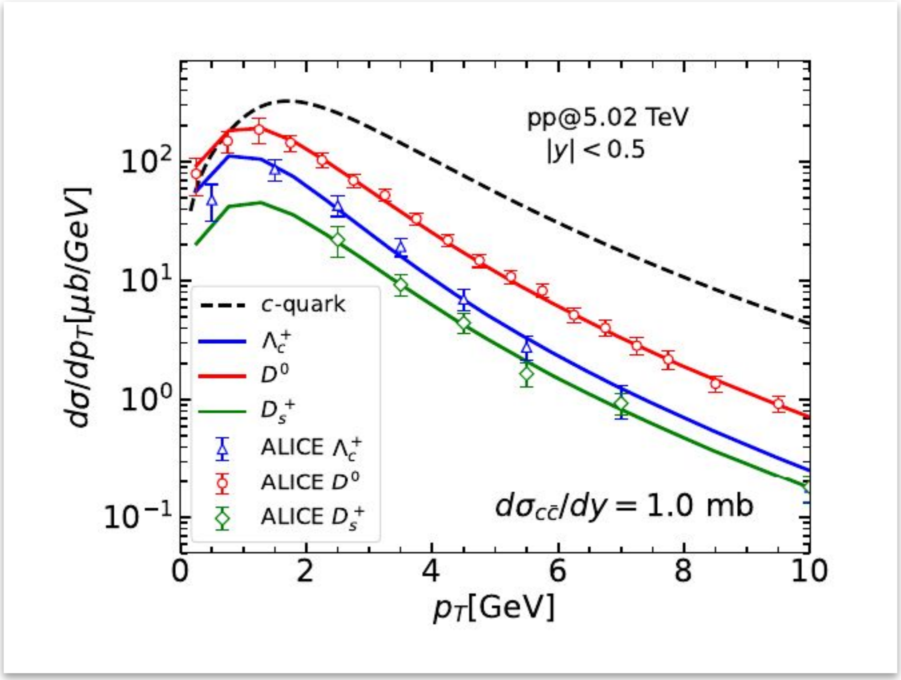


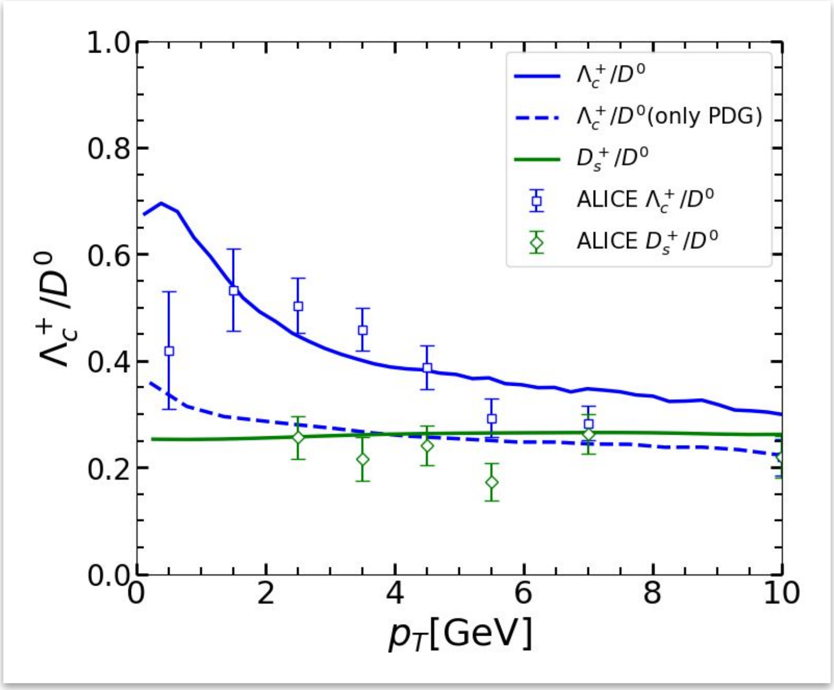




More Theory to Data comparison-II

$p - p \ @ \sqrt{s} = \ 5.02 \,\text{TeV}$





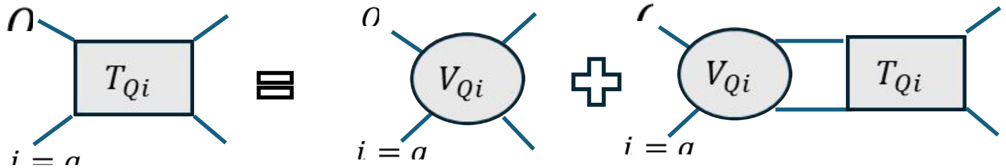
Hadron	Cross Section (mb)
	0.437
	0.178
	0.115
	0.236
	0.026
	0.003
	1.0

- Additional RQM baryons help explain large Λ_c^+/D^0 ratio

[ALICE: EPJC 79 (2019)]; [CMS: PAS-HIN-18-017]; [CMS: JHEP 01 (2024) 128]; [ALICE: PRC 107 (2023)]

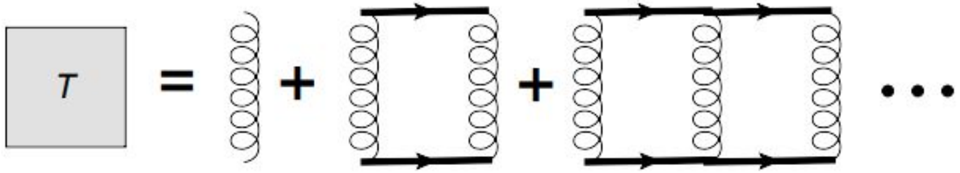
Transport Coefficients: *T*-matrix formalism

Based on ***T*-matrix formalism**: nonperturbative scattering framework for strongly coupled QGP;



$$T_{ij} = V_{ij} + \int V_{ij} G_i G_j T_{ij}$$

4D Bethe-Salpeter Equation



Infinite-Ladder Resummation

$$G_i = \frac{1}{\omega - \omega_k - \Sigma_i}$$

Quark propagator

$M_Q \gg T \Rightarrow \frac{q^2}{m_Q}$: Small Energy Transfer limit ->3D Bethe-Salpeter eqn

Kernel $\boldsymbol{V_{ij}} \rightarrow \boldsymbol{V(p - p')}$: Input potential

$$\Sigma_i = \int_m T_{im} G_m$$

Self-Energy

$$A(p) = \int d^4p' d^4q d^4q' |T|^2 \left(1 - \frac{(\boldsymbol{p}.\boldsymbol{p}')}{p^2}\right) \rho_q \rho_{q'} \rho_{p'} f_q$$

Quantum effects encoded in self-consistently evaluated spectral functions

Self-Consistent Dyson-Schwinger like calculation

T-matrix: [Riek+Rapp PRC 82(2010),Liu+Rapp PRC 97(2018)]