

Entanglement and Bell test at high energy Colliders

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Hard probes in non-equilibrium QCD matter

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Entanglement and Bell test at high energy Colliders

- Quick review of density matrix, qubits, pure and mixed states, separable and entangled state.
- Non locality of quantum mechanics, Bells inequality and CHSH inequality
- Bell test at high energy collider
- Connecting to GPDs and Decoherence at NLO

Density Matrix

1927 von Neumann

- The density matrix is something that represents a quantum state. Instead of describing a system by a single wave function, the density matrix takes statistical average of different possible quantum states of the associated ensemble,

$$\rho = \sum_i p_i |\psi_i\rangle\langle\psi_i|$$

Wave Function

Only pure state

Density Matrix

Both pure and mixed states

statistical ensemble of pure states

Open Quantum System

- Independent real parameters $N^2 - 1$

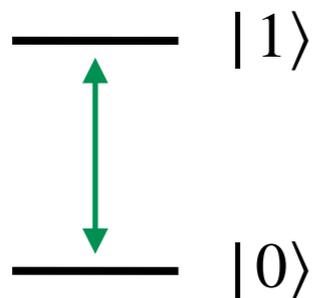
$$\dim(\mathcal{H}) = N$$

- Expectation value of an observable

$$\langle O \rangle = \text{tr}(O\rho)$$

Qubits

- Qubits or quantum bits are two state quantum systems. Unlike the classical bits, qubits are a superposition of both states 0 and 1.



Quantum coding
 B. Schumacher
 Physical Review A 1995

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$$

Examples:

Spin of an electron (up/down),
 photon polarisation (horizontal/vertical),
 trapped ions, and superconducting circuits.

- Density matrix of a qubit

$$\rho = \frac{1}{2} \left[I_2 + \sum_i B_i \sigma^i \right]$$

Two qubit system - Entangled and Separable

- The density matrix for a bipartite two-qubit system can be decomposed as,

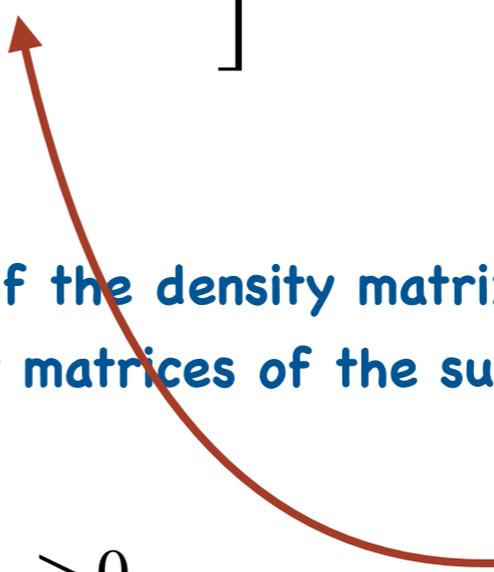
$$\rho = \frac{1}{4} \left[1 + \sum_i (B_i^+ \sigma^i \otimes I_2 + B_i^- I_2 \otimes \sigma^i) + \sum_{i,j} C_{ij} \sigma^i \otimes \sigma^j \right]$$

Hilbert space
 $\mathcal{H} = \mathcal{H}_A \otimes \mathcal{H}_B$

- Now, a quantum state is called separable if and only if the density matrix can be written as the convex sum of products of the density matrices of the subsystems,

$$\rho = \sum_n p_n \rho_n^A \otimes \rho_n^B \quad \text{with} \quad p_n \geq 0.$$

Correlation Matrix



- A state that is not separable is an entangled state.

Entanglement

- Two quantum systems are entangled if the joint state cannot be written as a product of the individual states:

$$|\psi_{AB}\rangle \neq |\psi_A\rangle \otimes |\psi_B\rangle$$

- The system is described by a single non-separable quantum state.

Example: EPR Spin State:

$$|\Psi^-\rangle = \frac{1}{\sqrt{2}} (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$$

● Qubit A

● Qubit B

- Shows two particles show perfect (anti)correlation. Spin measurement on one particle mediately fixes the spin measurement outcome of the other (100% certainly for maximally entangled states, with less certainty for other entangle states).

Example of maximum entanglement

$$\begin{aligned} S_A^z = +\frac{1}{2} &\implies S_B^z = -\frac{1}{2} \\ S_A^z = -\frac{1}{2} &\implies S_B^z = +\frac{1}{2} \end{aligned}$$

Outcome of the spin measurement along Z axis

Entanglement measures

- Entanglement measures quantify how strongly two subsystems are quantum correlated. They are constructed so that they vanish for separable states.

Entanglement entropy

Bennett and Schumacher (1996)

$$S_A = -\text{Tr}(\rho_A \log \rho_A)$$

$$\rho_A = \text{Tr}_B(\rho_{AB})$$

Concurrence

W. K. Wootters (1998)

$$C = \max(0, \lambda_1 - \lambda_2 - \lambda_3 - \lambda_4)$$

$$\sqrt{\rho(\sigma_y \otimes \sigma_y) \rho^*(\sigma_y \otimes \sigma_y)}$$

Widely used
Entanglement measures

Entanglement of Formation

Bennett, DiVincenzo,
Smolin, Wootters (1996)

$$E_F(\rho) = \min_{\{p_i|\psi_i\rangle\}} \sum_i p_i S(\rho_A^i)$$

Negativity

Vidal and Werner (2002) Based on Peres Horodecki
Positive Partial Transpose
criteria

$$\mathcal{N} = (||\rho^{T_B}|| - 1)/2$$

Quantum Mechanics – Probabilistic vs Deterministic

Quantum mechanics

Newtonian physics and relativity

Probabilistic

Deterministic

- Is quantum mechanics incomplete?
 - Perhaps quantum systems are controlled by variables, possibly hidden from us, that determine the outcome of the measurements.
 - If we have direct access to these hidden variables, then the outcome of all measurements performed on a quantum system could be predicted with certainty.
- EPR Paper (1935)**
- The EPR (Einstein, Podolsky, and Rosen) paper was a critique of quantum mechanics.
- If we can predict a physical quantity with certainty without disturbing the system, then there exists an element of reality corresponding to that quantity.

Hidden variable vs Non-locality

Schrödinger (1935)

In the same year, Schrödinger wrote a paper where he first used the term “Entanglement”.

Entanglement is not one but rather the characteristic trait of quantum mechanics.

- So the real question was whether there are hidden variables and whether quantum mechanics is incomplete? Or Quantum mechanics in its core a non-local, is theory inherently probabilistic?

Another way of expressing the peculiar situation is: the best possible knowledge of a *whole* does not necessarily include the best possible knowledge of all its *parts*; even though they may be entirely separated and therefore virtually capable of being “best possibly known”, i.e. of possessing, each of them, a representative of its own. The lack of knowledge is by no means due to the interaction being insufficiently known—at least not in the way that it could possibly be known more completely—it is due to the interaction itself.

Schrödinger (1935)

Bell inequality

John Bell (1964)

Bell's / CHSH inequality



- One may consider a Bell test in the form proposed by Clauser, Horne, Shimony, and Holt (CHSH). The test involves two boxes labelled A and B. Each box accepts a binary input (0 or 1) and subsequently delivers a binary output +1 or -1.
- In each trial of the Bell test, a random input bit is generated on each side and input to the respective box. The random input bits trigger the box to produce an output value that is recorded.
- The test concerns the correlations between the output values generated within the same trial.

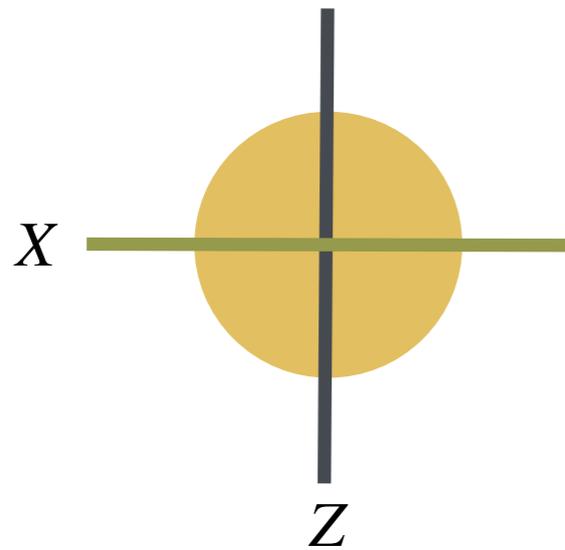
Bell's / CHSH inequality



- The discovery made by Bell is that in any theory of physics that is both local (physical influences do not propagate faster than the speed of light) and realistic (physical properties are defined before and independent of observation) those correlations are bounded more strongly than they are in quantum theory.
- In particular, if the input bits are truly random and boxes are specially separated such that locality prevents communication between the boxes during a trial, then the following inequality holds under local realism

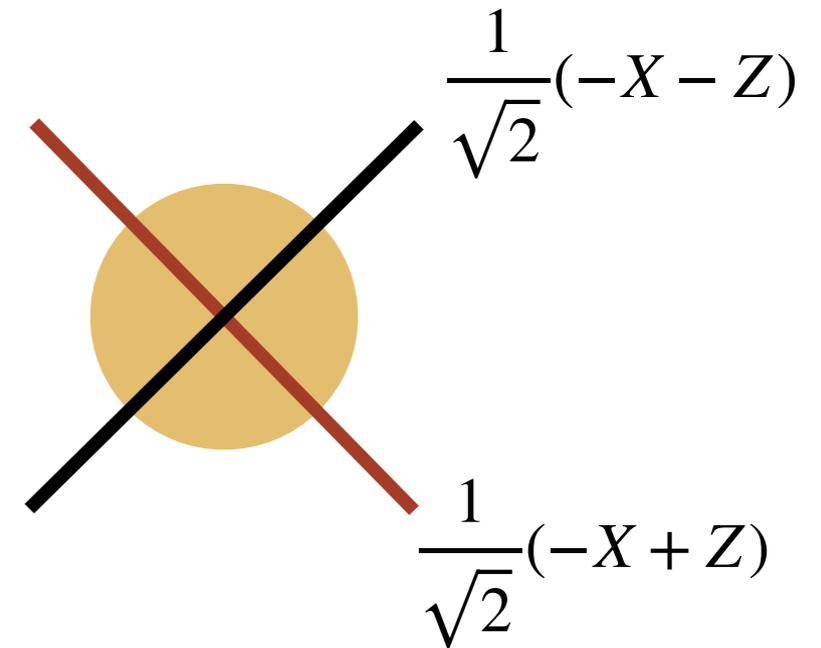
$$S = \left| \langle x \cdot y \rangle_{(0,0)} + \langle x \cdot y \rangle_{(0,1)} + \langle x \cdot y \rangle_{(1,0)} - \langle x \cdot y \rangle_{(1,1)} \right| \leq 2$$

Bell's / CHSH inequality



Qubit A

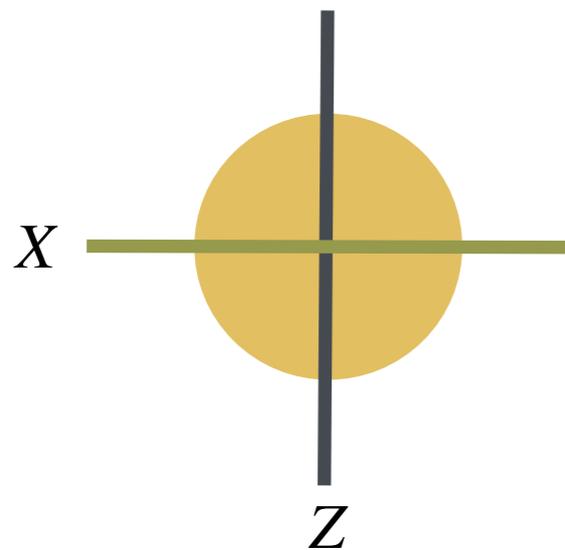
$$|\Phi^+\rangle = \frac{1}{\sqrt{2}} (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$$



Qubit B

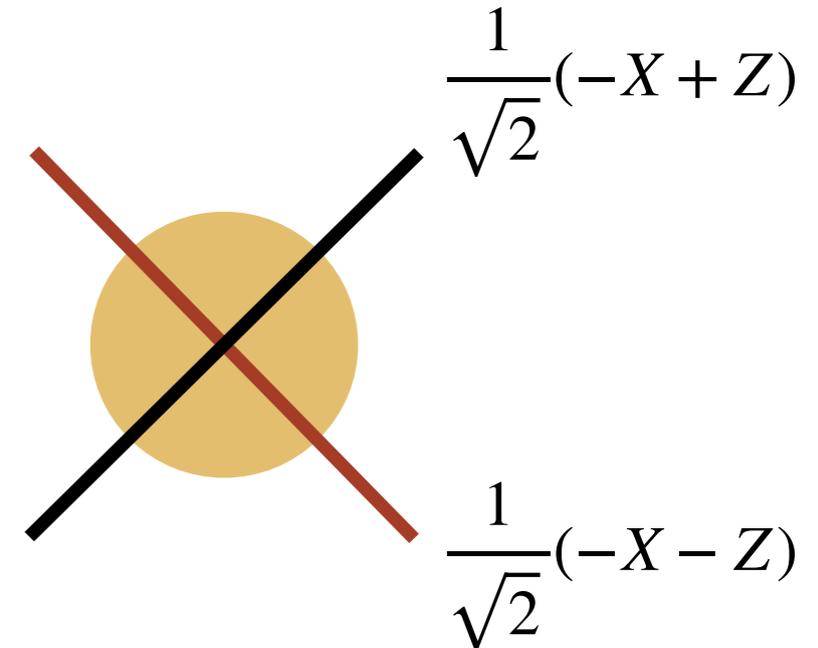
- The spin in A is measured along direction Z (for input bit $a = 0$) or X (for input bit $a = 1$) and the spin in the box B is measured along $(-Z+x)$ (for $b=0$) and $(-Z-x)$ for ($b = 1$).

Bell's / CHSH test with Spin



Qubit A

$$|\Psi^-\rangle = \frac{1}{\sqrt{2}} (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$$



Qubit B

- The quantum theory predicts $S = 2\sqrt{2}$

Bell's / CHSH inequality violation (maximally)

- This shows that the combination of locality and realism is fundamentally incompatible with the predictions of quantum mechanics. Quantum mechanics is fundamentally non-local; entanglement is an essential character of quantum mechanics.

Experiments

First Bell Test

Freedman and Clauser

Phys. Rev. Lett. 28, 938 (1972)

1972

CHSH inequality

J. Clauser, M. A. Horne, A. Shimony, R. A. Holt

Phys. Rev. Lett. 23, 880 (1969)

1969

1964

Bell inequality

John Bell

Physics 1, 195 (1964)

Entanglement

E. Schrödinger

Proc. Cambridge Phil.

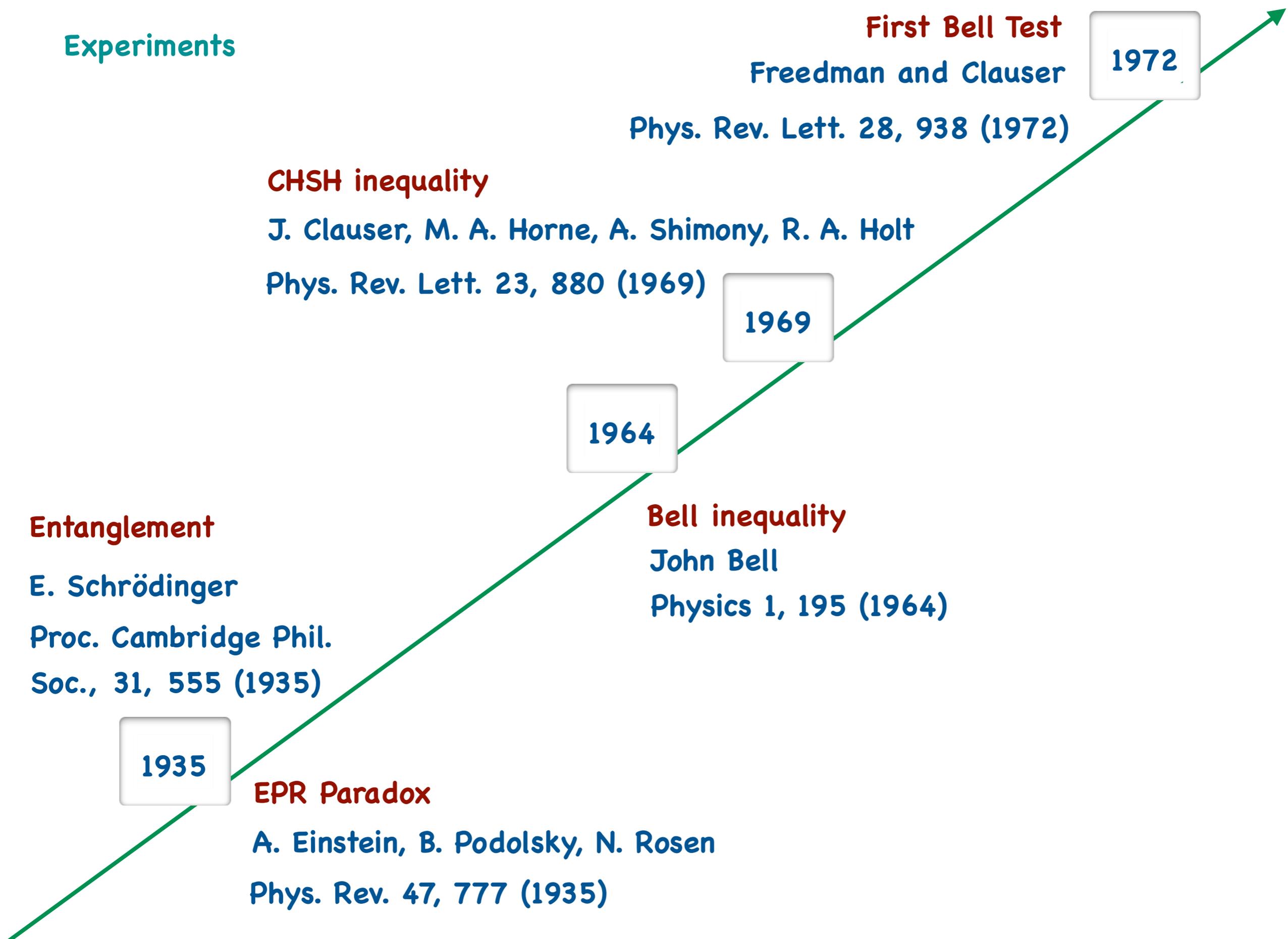
Soc., 31, 555 (1935)

1935

EPR Paradox

A. Einstein, B. Podolsky, N. Rosen

Phys. Rev. 47, 777 (1935)



Experiments

Cosmic Bell Tests

Satellite Bell Tests

First loophole free Test
Nature 526, 682 (2015)
Delft experiment

Trapped Ion Bell Test
David Wineland Group

LHC
First evidence of
entanglement in top -
anti top system

1982

Aspect's experiment
Phys. Rev. Lett. 49, 1804 (1982)

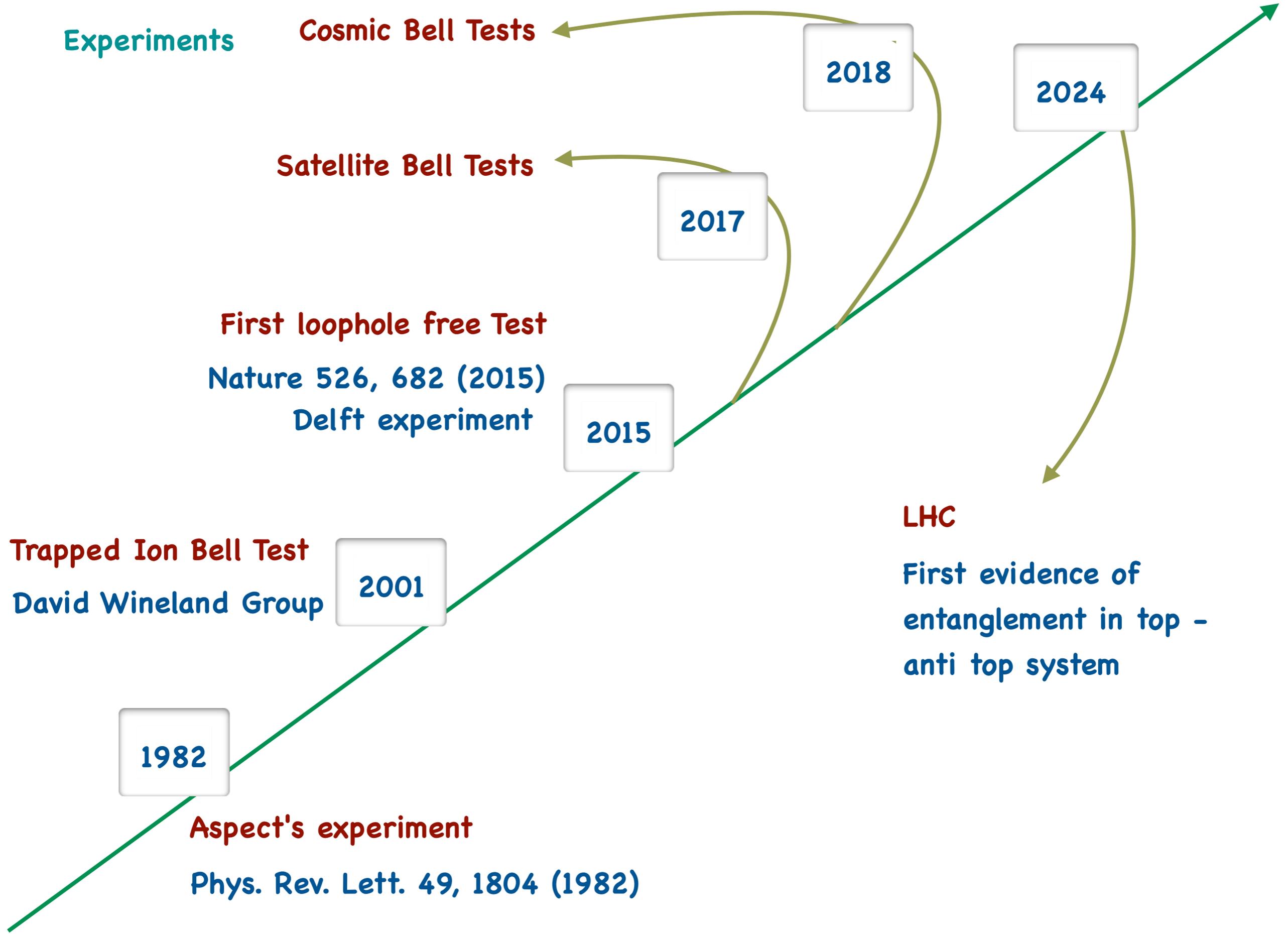
2001

2015

2017

2018

2024



Experiments

Year 2015

Loophole free experiments

- 2015 is widely regarded as the breakthrough year of the 'loophole free' Bell tests
- Before 2015, experiments violated the Bell inequality many times but every test had at least one loophole.
- For 50 years, since the Bell's 1964 paper, critics could argue that experimental violations might be explained by loopholes.
- **Delft experiment (Oct 2015)**
Nature 526, 682 (2015)
Entangled electrons separated by 1.3 km
- **NIST experiments**
PRL 115, 250402 (2015)
- **Vienna experiments (Dec 2015)**
PRL 115, 250401 (2015)
Both used entangled photons

Loopholes

Locality Loophole

- Bell violation would not truly rule out locality unless the two measurement events are space like separated.
- May be there was communication leading to the violation

Detection Loophole

- Bell violation would not truly rule out local hidden variable theory unless the detection efficiency is so high that Bell violation can't be faked by favourable events.
- May be we are looking at only at data that favour violation

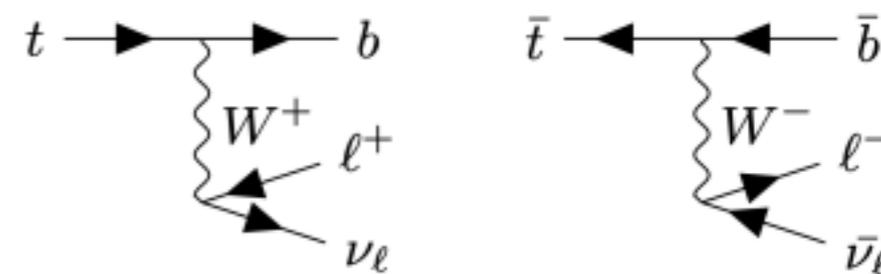
Entanglement and Bell test at LHC

- When a hard scattering produces a quark-antiquark pair, then the spin density matrix is generally not factorizable, and the quark-antiquark pair is entangled.

initial state $\rightarrow Q\bar{Q}$

$$\rho_{Q\bar{Q}} \neq \rho_Q \otimes \rho_{\bar{Q}}$$

At LHC



leptons are finally detected.

Spin density matrix for the quark antiquark pair

$$\rho = \frac{1}{4} \left[1 + \sum_i (B_i^+ \sigma^i \otimes I_2 + B_i^- I_2 \otimes \sigma^i) + \sum_{i,j} C_{ij} \sigma^i \otimes \sigma^j \right]$$

$$\frac{1}{\sigma} \frac{d\sigma}{d\Omega_+ d\Omega_-} = \frac{1}{(4\pi)^2} [1 - B^+ \cdot \hat{q}_- - B^- \cdot \hat{q}_+ - \hat{q}_+ \cdot C \cdot \hat{q}_-]$$

The spin-correlation matrix
can be measured from the decay products.

\hat{q}_\pm anti-lepton (lepton) directions in the rest
frame of their parent top (anti top) quarks

Top quark and anti top quark polarisation
 $B^\pm \sim 0$ CP symmetry of QCD

Entanglement Observables

- From the C matrix, experiments reconstruct the density matrix for the top anti top spin system.

Afik & Nova (2020)

Fabbrichesi, Floreanini & Panizzo (2021)

- The entanglement measures can be computed: concurrence, entanglement entropy, even the Bell's parameter.

$$D = -\frac{1}{3} \sum_i C_{ii} = -3 \langle \cos \phi \rangle \quad \text{If } D > \frac{1}{3} \text{ then there is entanglement,}$$

Average of cos of angles between charged

Lepton directions (after they have been subjected to Lorentz boosting into the top anti top rest frame and then the rest frame of their parent top and the anti top quarks)

which can be measured experimentally in an ensemble data set.

ATLAS Collaboration (2024)

Nature 633, 542–547 (2024)

CMS Collaboration (2024)

Reports on Progress in Physics 87, 117801 (2024)

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ATLAS Collaboration (2024)

- ATLAS measured $D \sim 0.6$ at 5 sigma level.

Nature 633, 542–547 (2024)

$$D = 0.537 \pm 0.002 \pm 0.019$$

Statistically significant
entanglement detected.

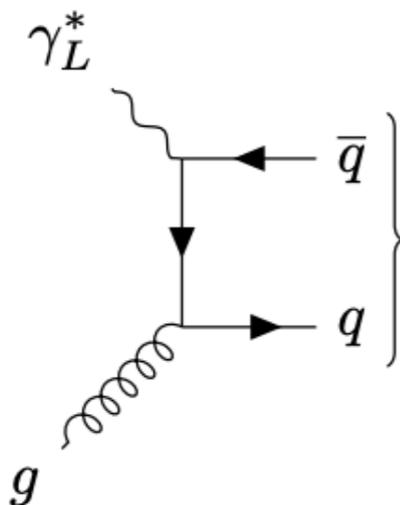
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Spin density matrix at LO in a DIS/EIC set up

- Heavy quark pair production in unpolarised electron-proton (or electron-nucleus) scattering

$$e + p \rightarrow e' + \gamma^* + p \rightarrow e' + p' + q + \bar{q}$$



$$d\sigma^L \propto I_4 + B_i^+ \sigma^i \otimes I_2 + B_j^+ \otimes \sigma^j I_2 + C_{ij} \sigma^i \otimes \sigma^j$$

Spin density matrix

(Can be calculated in the (n, r, k) basis)

Spin unresolved cross section

$$C^L = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \chi_1 & \chi_2 \\ 0 & \chi_2 & -\chi_1 \end{pmatrix}$$

Pure state

$$(C^L)^T C^L = I \quad \text{CHSH optimum reaches} \quad S = 2\sqrt{2}$$

For longitudinally polarise virtual photon, the quark antiquark pair always exhibits maximal entanglement and maximal Bell non-locality both in single gluon or multi gluon (pomeron) exchange.

Maximal entanglement and maximal violation of Bell's inequality

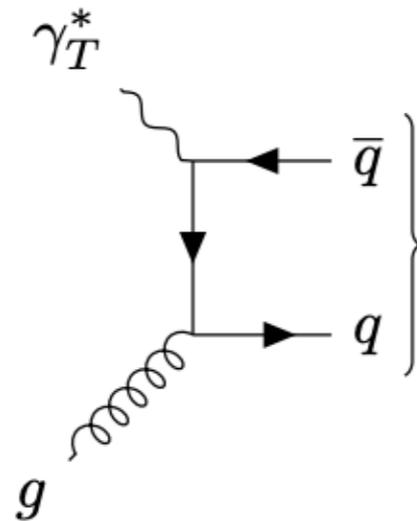
Qi, Guo, Xiao (arXiv: 2506.12889)

Fucilla, Hatta (arXiv: 2509.05267)

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Spin density matrix

(Can be calculated in the (n, r, k) basis)

Spin unresolved cross section

$$C^T = \begin{pmatrix} C_{nn} & 0 & 0 \\ 0 & C_{rr} & C_{kr} \\ 0 & C_{rk} & -C_{kk} \end{pmatrix}$$

Mixed state

For transversely polarise virtual photon, the density matrix is generally in a complicated mixed state.

Qi, Guo, Xiao (arXiv: 2506.12889)

Fucilla, Hatta (arXiv: 2509.05267)

Elements of the correlation matrix are kinematic

Spin density matrix at LO in a DIS/EIC set up

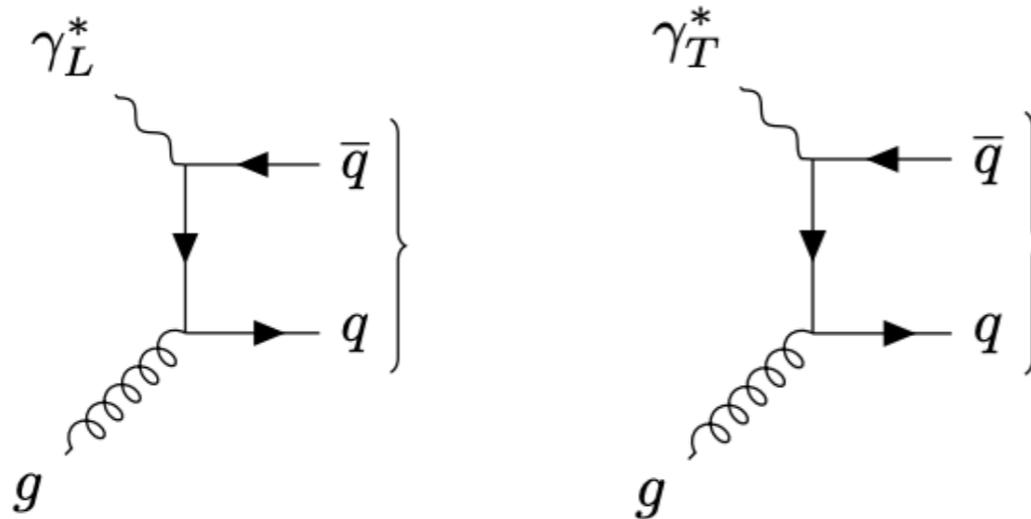
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Schoenleber, Hatta (arXiv: 2511.04537)



Spin density matrix

(Can be calculated in the (n, r, k) basis)

- By measuring the lepton pair in the decay product, one can extract the spin correlation matrix for the heavy quark pair, thereby studying the entanglements and Bell's non locality at EIC

Spin density matrix at LO

- At the leading order (in the longitudinal channel), the observed quark-anti quark system is exact. All information about the final state is exclusively available.
- There are no unobserved gluons in the final state, and the density matrix is therefore pure.
 - What is there in the NLO?
 - Can the entanglement measures and the spin density matrix depends on GPDs and TMDs?

Decoherence at NLO

What is there in the NLO?

- At the next-to-leading order, there is one gluon in the final state. If the gluon was not detected, then we have to integrate over it.
 - The reduced renormalization density matrix

$$\rho_{\alpha\alpha',\beta\beta'} = \mathcal{M}_{\alpha_s\alpha_s}^0 \mathcal{M}_{\beta\beta'}^0 + \alpha_s \mathcal{M}_{V,\alpha\alpha'}^1 \mathcal{M}_{\beta\beta'}^0 + h.c. + \alpha_s \sum_{\lambda} \int d\Phi_g \mathcal{M}_{R,\alpha\alpha';\lambda}^{(1)}(k) \mathcal{M}_{R,\beta\beta';\lambda}^{(1)*}(k)$$

↑
↑
↑

LO
NLO Virtual
NLO real (traced over)

- This is essentially tracing the gluon degrees of freedom. This modifies the pure quark- anti quark system at LO; now, it is a reduced subsystem with a reduced density matrix.

NLO density matrix should be IR safe!

Decoherence at NLO

A white noise mix model of NLO induced decoherence

- Treat the observed sample as a mixture of :
 1. A fraction $1 - \epsilon$ of truly exclusive quark anti quark events.
 2. A fraction ϵ of events where unobserved radiation scramble the two qubit correlations into some noise state, say, white noise.

This makes the density matrix a Werner state build from LO Bell like pure state.

- Maximal CHSH violation $S_{max} = 2\sqrt{2}(1 - \epsilon)$
 - Still violates CHSH inequality $\epsilon < 0.29$
 - Entanglement persists until $\epsilon < 0.66$

NLO + tracing over radiation tends the system from pure and maximally entangled to entangled but not Bell non-local region.

Decoherence at NLO

Lindblad description

- If the gluon is not measured, it acts like an 'environment'. The evolution of the quark anti quark density matrix can be written as a Lindblad equation.

$$\frac{d\rho}{dt} = -i \left[H_{eff}, \rho \right] + \sum_j \gamma_j \left(L_j \rho L_j^\dagger - \frac{1}{2} \left\{ L_j^\dagger L_j, \rho \right\} \right)$$

Where j indicate which spin components are monitored or rephased by the environment.

$$L = \sigma_k \otimes 1 - 1 \otimes \sigma_k$$

- Diagonal element in the chosen basis stay

Off diagonal coherences decay exponentially $\lambda = e^{-\gamma t}$

$$C = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \lambda C_{rr} & \lambda C_{kr} \\ 0 & \lambda C_{rk} & -\lambda C_{kk} \end{pmatrix}$$

$$\lambda(Q) = \left(\frac{\ln(\mu^2/\Lambda^2)}{\ln(Q^2/\Lambda^2)} \right)^{\frac{\Delta\gamma}{2\pi\beta_0}}$$

$\Delta\gamma$
Difference of non-cusp anomalous dimension of two spin channel

Decoherence grows logarithmically with scale

Decoherence at NLO

At small x

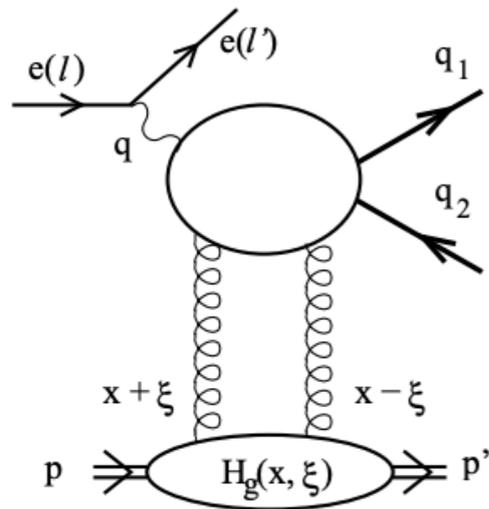
- Radiation is dominated by BFKL type evolution
- The evolution is governed by the BFKL kernel eigenvalue $\omega = \bar{\alpha}_s \chi(\gamma)$
- Spin decoherence scales like $\lambda(x) \sim x^{\Delta\omega}$

Difference of eigenvalues
between spin channels

- The leading BFKL kernel is spin blind -
implies decoherence is weak at small x .

Longitudinal maximum entanglement is
perturbatively stable at small x to leading accuracy

Spin density matrix connects to GPDs and TMDs.



- There are very recent efforts where GPDs are used to probe the entanglement and Bell non locality in EIC.

Schoenleber, Hatta (arXiv: 2511.04537)

- Spin - orbit entanglements of gluon in a proton: the entanglement negativity is directly connected to the gluon helicity PDF.

Agrawal, Abir; Phys. Lett. B 868 (2025) 139802

Outlook

- Hard scattering processes create entangled partonic states
 - As these states interact with QCD radiation or medium, decoherence gradually degrades the entanglement.
 - Studying this evolution may provide a new way to probe thermalization dynamics in QCD.
-
- Is entanglement a generic property of QCD dynamics?
 - How does decoherence appear at NLO and in a QCD medium?
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Thank you