Numerical Weather Prediction – Methods

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Overview

- Introduction: Numerical Weather Prediction (NWP)
- NWP: A Historical Perspective
- Method of Finite Difference Method including time differencing
- Linear Stability: CFL (Courant Friedrichs Lewy) condition
- Aliasing and Nonlinear Computational Instability
- Semi-Lagrangian and Semi-Implicit Methods
- Galerkin and Spectral Methods
- Need for Data Assimilation

Introduction: Numerical Weather Prediction (NWP)

- Edmond Halley (1686) explained dynamics of tropical wind system as a result of solar heating
- George Hadley (1735) included Earth's rotation
- William Ferrel (1858) applied Coriolis force to atmosphere
- Leonhard Euler (1755) gave governing inviscid equations
  Claude L. Navier (1822) extended equations to viscous fluids
- George G. Stokes (1845) gave Navier-Stokes equations.
- Osborne Reynolds (1900) showed transition to turbulent flow
- Vilhelm Bjerknes' (1904) presented his rational version of forecasting based on the laws of mechanics and physics of the atmosphere

Atmosphere as a dynamical system (Bjerknes 1904)

- Any system which evolves/ changes with time may be called as a dynamical system. We are interested to know the future state of such dynamical system.
- Prediction of a dynamical system requires a set of rule(s)/law(s) governing the time evolution of the dynamical system and complete information about the present state of the system.
- Atmosphere is a dynamical system; hence its prediction requires complete information about the current/present state of the atmosphere and the set of physical rule(s)/law(s) governing the time evolution of the atmosphere.

History of NWP

- In 1904, V. Bjerknes first realized that problem of weather forecasting is a mathematical initial value problem.
- In 1922, L.F. Richardson attempted to solve the governing equations for predicting pressure tendency using a desk computer. However, he failed as the computed 24 hours pressure change was a several order of magnitude greater than the observed 24 hours pressure change.
- The failure of Richardson's numerical treatment, where he obtained 145 hPa pressure change in 6 hrs, was, at that time, attributed to poor initial data available, especially the absence of upper air data

History of NWP

- Later, it was discovered that the atmospheric equations in its complete form, so called ‘primitive’ form, admit solutions corresponding to not only the slow-moving atmospheric waves (Rossby Waves) but also fast-moving sound and gravity waves.
- These high-speed waves amplify spuriously with the time and mask the solutions relating to atmospheric waves if not properly controlled.
- Later, in 1948, Charney showed that by making use of hydrostatic and geostrophic assumptions the high-speed sound and gravity waves can be effectively ‘filtered’.

History of NWP

- In 1950, using the first electronic calculator "Electronic Numerical Integrator and Calculator" (ENIAC) and the filtered Barotropic/Equivalent barotropic model, Charney, Fjortoft and Von Neumann, produced the first successful numerical prediction.
- Since then, there has been a rapid progress in all aspects of NWP.
- These improvements are mainly due to considerable increase in the quantity of meteorological data, advances in telecommunication system, tremendous progress in computer technology and development of much better and sophisticated numerical models.

Richardson's efforts at NWP in his own words

If the time-step were 3 hours, then 32 individuals could just compute two points so as to keep pace with the weather. If the co-ordinate chequer were 200 km square in plan, there would be 3200 columns on the complete map of the globe. In the tropics the weather is often foreknown, so that we may say 2000 active columns. So that 32 x 2000 = 64,000 computers would be needed to race the weather for the whole globe. That is a staggering figure. But in any case, the organization indicated is a central forecast-factory for the whole globe, or for portions extending to boundaries where the weather is steady, with individual computers specializing on the separate equations.

Richardson’s efforts at NWP in his own words

Imagine a large hall like a theatre, except that the circles and galleries go right round through the space usually occupied by the stage. The walls of this chamber are painted to form a map of the globe. The ceiling represents the north polar regions, England is in the gallery, the tropics in the upper circle, Australia on the dress circle and the antarctic in the pit.

A myriad computers are at work upon the weather of the part of the map where each sits, but each computer attends only to one equation or part of an equation. The work of each region is coordinated by an official of higher rank. Numerous little "night signs" display the instantaneous values so that neighbouring computers can read them

Richardson's efforts at NWP in his own words

Each number is thus displayed in three adjacent zones so as to maintain communication to the North and South on the map. From the floor of the pit a tall pillar rises to half the height of the hall. It carries a large pulpit on its top. In this sits the man in charge of the whole theatre; he is surrounded by several assistants and messengers. One of his duties is to maintain a uniform speed of progress in all parts of the globe. In this respect he is like the conductor of an orchestra in which the instruments are slide- rules and calculating machines. But instead of waving a baton he turns a beam of rosy light upon any region that is running ahead of the rest, and a beam of blue light upon those who are behindhand.

Richardson's efforts at NWP in his own words

Four senior clerks in the central pulpit are collecting the future weather as fast as it is being computed, and despatching it by pneumatic carrier to a quiet room. There it will be coded and telephoned to the radio transmitting station.

Messengers carry piles of used computing forms down to a storehouse in the cellar. In a neighbouring building there is a research department, where they invent improvements

In another building are all the usual financial, correspondence and administrative offices. Outside are playing fields, houses, mountains and lakes, for it was thought that those who compute the weather should breathe of it freely.

This is a remarkable vision, in which 'computer' means only one thing: a human calculator.

Progress in the years between 1920 and 1950

Atmospheric motion has multiple temporal and spatial scales. Scale analysis method is used to simplify the NWP equations based on the scale of motion in which one desires in making a weather forecast.

The Courant-Friedrichs-Lewy (CFL) criterion sets a bottom-line of requirements between sizes of grid spaces and time steps in order to retain computational stability.

Better understanding of nonlinear computational stability also helps in designing schemes that can be used to solve PDEs accurately and efficiently with numerical methods

The invention of radiosonde made it possible to probe conditions in the upper atmosphere. Invention of electronic computers enhanced the efficiency of scientific calculation tremendously.

NWP governing equations

There is a complete set of seven equations with seven unknowns that governs the evolution of the atmosphere: Newton's second law or conservation of momentum (three equations for the three velocity components), the continuity equation or conservation of mass, the equation of state for ideal gases, the first law of thermodynamics or conservation of energy, and a conservation equation for water mass.

$u = r\cos$

In spherical coordinates, assume that λ and φ are longitude and latitude and r is the radius of the Earth, one has

$$u = r\cos\varphi\frac{d\lambda}{dt};\ v = r\frac{d\varphi}{dt};\ w = \frac{dr}{dt};\ r=a+z;\ \ a>>z;\ \ dr\approx dz;\ \frac{d}{dr} = \frac{d}{dz};$$

NWP governing equations (7 equations in 7 variables)

$$\frac{\partial u}{\partial t} = -u\frac{\partial u}{\partial x} - v\frac{\partial u}{\partial y} - w\frac{\partial u}{\partial z} + \frac{\mathrm{uv}\tan\phi}{a} - \frac{\mathrm{uw}}{a} - \frac{1}{\rho}\frac{\partial p}{\partial x} - 2\Omega(w\cos\phi - v\sin\phi) + \mathrm{Fr}_x$$

$$\frac{\partial v}{\partial t} = -u\frac{\partial v}{\partial x} - v\frac{\partial v}{\partial y} - w\frac{\partial v}{\partial z} - \frac{u^2\tan\phi}{a} - \frac{\mathrm{uw}}{a} - \frac{1}{\rho}\frac{\partial p}{\partial y} - 2\Omega u\sin\phi + \mathrm{Fr}_y$$

$$\frac{\partial w}{\partial t} = -u\frac{\partial w}{\partial x} - v\frac{\partial w}{\partial y} - w\frac{\partial v}{\partial z} - \frac{u^2 + v^2}{a} - \frac{1}{\rho}\frac{\partial p}{\partial z} + 2\Omega u\cos\phi - g + \mathrm{Fr}_z$$

$$\frac{\partial T}{\partial t} = -u\frac{\partial T}{\partial x} - v\frac{\partial T}{\partial y} + (\gamma - \gamma_d)w + \frac{1}{c_p}\frac{\mathrm{dH}}{\mathrm{dt}}$$

$$\frac{\partial \rho}{\partial t} = -u\frac{\partial \rho}{\partial x} + -v\frac{\partial \rho}{\partial y} - w\frac{\partial \rho}{\partial z} - \rho\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right)$$

$$\frac{\partial q_v}{\partial t} = -u\frac{\partial q_v}{\partial x} - v\frac{\partial q_v}{\partial y} - w\frac{\partial q_v}{\partial z} + Q_v$$

$$p\alpha = \mathrm{RT}$$

Finite Difference Equations

The equations are a coupled set of nonlinear partial differential equations and is solved using the discrete form with numerical method, say finite difference methods. Example: Advection equation

$$\frac{\partial u}{\partial t} + c\frac{\partial u}{\partial x} = 0$$

We take discrete values for x and t: $x_j = j\Delta x$ and $t_n = n\Delta t$, where Δx is the grid space and Δt is the time step of integration. The solution of the FDE is defined at the discrete points $(x_j, t_n) = (j\Delta x, n\Delta t)$:

$$U_j^n = U(j\Delta x, n\Delta t) = U(x_j, t_n)$$

u indicates solution of PDE & U is solution of FDE

$$\frac{U_j^{n+1} - U_j^n}{\Delta t} + c\frac{U_j^n - U_{j-1}^n}{\Delta x} = 0$$

Since we employ an FDE to approximate a PDE, two fundamental conditions should be satisfied:

(i) FDE must be consistent with PDE

(ii) For any time t> 0, solution of FDE should converge to solution of PDE as $\Delta x \to 0$ and $\Delta t \to 0$

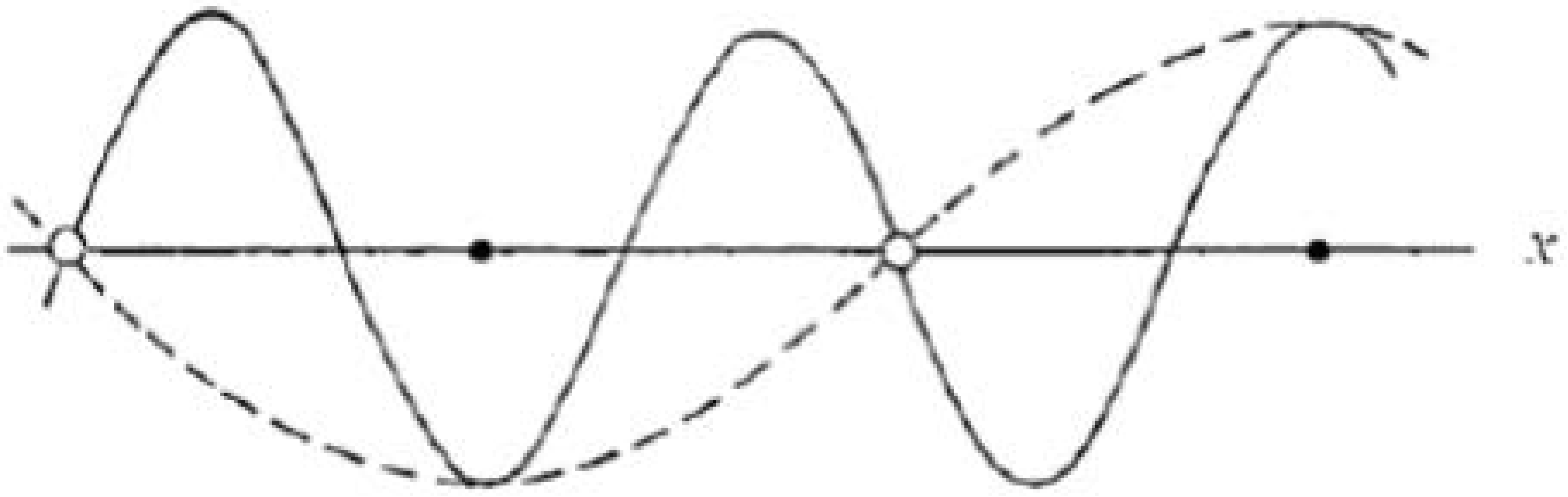
Consistency and Stability of Finite Difference Equations

In order to fulfill these two requirements, the numerical schemes used in the FDE must be as accurate as possible. We say that the FDE is consistent with the PDE if, in the limit $\Delta x \rightarrow 0$ and $\Delta t \rightarrow 0$, the FDE coincides with the PDE. This requires that the solutions of the FDE be consistent approximations of the solutions of the PDE. The difference between the PDE and FDE is the discretization error.

In addition, it is very important to keep computational stability during the integration (prediction) process. Commonly, the Courant-Friedrichs-Lewy, or CFL, condition must be satisfied when specifying the time step Δt for a given grid size Δx: $0 \leq \frac{c\Delta t}{\Delta x} \leq 1$, where $\frac{c\Delta t}{\Delta x}$ is Courant number and c is phase speed.

CFL condition, however, is only a necessary condition that ensures that an FDE is computationally stable so that the solution of the FDE at a fixed time t remains bounded as $\Delta t \rightarrow 0$. Generally Δt is taken smaller than CFL condition

Aliasing and Nonlinear Computational Instability



A wave (continuous line) of wavelength less than 2Δx, say, (4/3)Δx is misrepresented as a resolvable wave (dashed line) of wavelength 4Δx by the finite difference grid.

Aliasing error appears when one solves nonlinear pde's since the nonlinear terms in such equations can produce small waves that have wavelengths that are smaller than the smallest wavelength that can be resolved by the grid system.

Consider two modes 'A' and 'B' having wavelengths k and l and amplitudes A_o and B_o, respectively, in a one-dimensional grid. Product of A & B, gives rise to wave number k+l

$A(x_j) = A_o e^{ikj\Delta x}$ and $B(x_j) = B_o e^{ilj\Delta x}$ $AB = A_o B_o e^{i(k+l)j\Delta x}$

k+l can be higher than highest wave number π/Δx

Aliasing and Nonlinear Computational Instability

Let the maximum value of the wave number for a given grid system be represented by k_{max}, then, $k_{max} > \pi/\Delta x$. What happens if a new wave of wave number k is generated due to nonlinear interactions such that $k > k_{max}$. Since $2k_{max}\Delta x = 2\pi$, one can assume that 2kmax > k > kmax. Writing the expression for sin(k jΔx), one has

$$\sin(k j\Delta x) = \sin[(2k_{max} - 2k_{max} + k) j\Delta x] = \sin[2\pi j - (2k_{max} - k) j\Delta x] = \sin[-(2k_{max} - k) j\Delta x] = \sin(k^* j\Delta x)$$

where $k^* = -(2k_{max} - k)$ Similarly it can be showr $\cos(k j\Delta x) = \cos(k^* j\Delta x)$

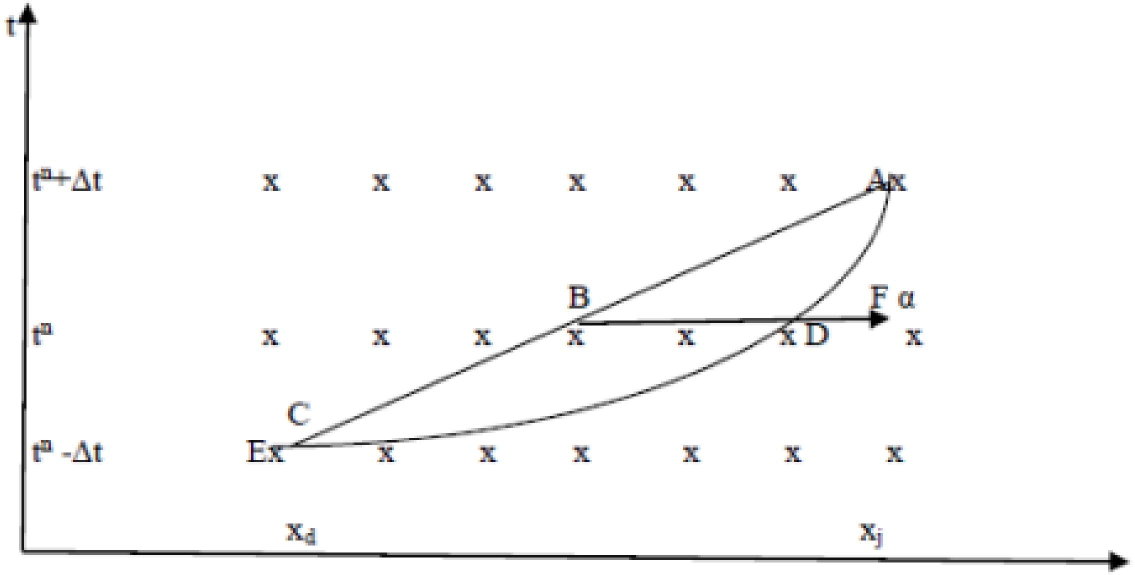
Knowing only the values at the grid points, one cannot distinguish the wavenumbers k from the wavenumbers 2kmax −k. Above shows that wave of wave number k > kmax is misrepresented by the grid system as a wave of wave number $k * = -(2k_{max} - k)$.

Aliasing or misrepresentation of the wavenumber $k > k_{max}$. In previous figure

Semi Lagrangian scheme to solve 1-D advection equation

$$\frac{du}{dt} = \frac{\partial u}{\partial t} + \frac{dx}{dt}\frac{\partial u}{\partial x} = 0$$

Three time level scheme for semi-Lagrangian method for solving 1-D advection equation



Solve the 'trajectory equation' to find out $x(t^n - \Delta t)$, the location of the departure point x_d at the previous time step before the present time $(t^n + \Delta t)$ for fluid particle at the grid point x_j

$$\frac{dx}{dt} = a(x,t)$$ subject to the condition $x(t^{n+1}) = x_j$, in order to find $x(t^n - \Delta t)$.

Using central differences in time and using mid-point rule $$\frac{x_j - x_d}{2\Delta t} = a(x_m, t^n)$$ where

$$x_m = \frac{x_j + x_d}{2}$$ Defining the displacement along the trajectory as 2α where $2\alpha = xj - xd$,

Semi Lagrangian scheme to solve 1-D advection equation

$$\alpha^{(r+1)} = \Delta t\, a(x_j - \alpha^{(r)}, t^n)$$

Solve for the displacement α iteratively until convergence is achieved, r is iteration number

Following sequence of steps defines the semi-Lagrangian method for solving 1-D advection equation

(i) Solve above equation iteratively for the half-displacement α for all grid points x_j at time $t^n+\Delta t$ using some initial guess (the value of α at the previous time step can be utilized as a first guess value) with employing appropriate interpolation formula in space for the velocity

(ii) Evaluate u at upstream location $x_j - 2\alpha$, and at time $t^n - \Delta t$ using an appropriate interpolation formula.

(iii) Evaluate u at arrival location x_j , and at time $t^n - \Delta t$ using

$$\frac{u(x_j, t^n + \Delta t) - u(x_j - 2\alpha, t^n - \Delta t)}{2\Delta t} = 0$$

Semi implicit method to solve shallow water equations

In semi-implicit method, the terms that give rise to high frequency gravity waves are integrated implicitly, while the other terms are treated explicitly. Thus, the equation

$$\frac{\partial u}{\partial t} = F(u) = F_1(u) + F_2(u)$$ is discretised as $$\frac{u^{n+1} - u^{n-1}}{2\Delta t} = F_1(u) + F_2\left(\frac{u^{n-1} + u^{n+1}}{2}\right)$$

Consider the shallow water equations

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \underline{-\frac{\partial \Phi}{\partial x}} + fv$$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = \underline{-\frac{\partial \Phi}{\partial y}} - fu$$

$$\frac{\partial \Phi}{\partial t} + u\frac{\partial \Phi}{\partial x} + v\frac{\partial \Phi}{\partial y} = \underline{-\bar{\Phi}\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right)} - (\Phi - \bar{\Phi})\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right)$$

Terms that are underlined are those that lead to fast gravity waves and are treated implicitly while all other terms are treated explicitly

Semi implicit method to solve shallow water equations

R_u, R_v and R_ϕ terms are nonlinear & Coriolis terms, δ is divergence

$$\frac{\partial u}{\partial t} = -\frac{\partial \Phi}{\partial x} + R_u$$

$$\frac{\partial v}{\partial t} = -\frac{\partial \Phi}{\partial y} + R_v$$

$$\frac{\partial \Phi}{\partial t} = -\bar{\Phi}\delta + R_\Phi$$

$$\left(\frac{u^{n+1} - u^{n-1}}{2\Delta t}\right) = -\left(\frac{\Phi_x^{n+1} - \Phi_x^{n-1}}{2}\right) + R_u^n$$

$$\left(\frac{v^{n+1} - v^{n-1}}{2\Delta t}\right) = -\left(\frac{\Phi_y^{n+1} - \Phi_y^{n-1}}{2}\right) + R_v^n$$

$$\left(\frac{\Phi^{n+1} - \Phi^{n-1}}{2\Delta t}\right) = -\Phi\left(\frac{\delta^{n+1} - \delta^{n-1}}{2}\right) + R_\Phi^n$$

where ϕ_x and ϕ_y are partial derivatives

Now solve for u^{n+1} and v^{n+1}

$$u^{n+1} = -\Delta t \Phi_x^{n+1} + S_u$$

$$v^{n+1} = -\Delta t \Phi_y^{n+1} + S_v$$

Then, the divergence at time $(n+1)\Delta t$ is

$$\delta^{n+1} = \left(u_x^{n+1} + v_y^{n+1}\right) = -\Delta t\left(\Phi_{xx}^{n+1} + \Phi_{yy}^{n+1}\right) + R_\delta$$

$$= -\Delta t \nabla^2 \Phi^{n+1} + R_\delta$$

Semi implicit method to solve shallow water equations

Substituting the expression for divergence in the continuity equation to get

$$\left(\frac{\Phi^{n+1}-\Phi^{n-1}}{2\Delta t}\right)=\frac{1}{2}\Phi\Delta t\nabla^2\Phi^{n+1}+S_\Phi$$

The above equation can be written as a Helmholtz Equation for Φ^{n+1} as follows

$$\left[\nabla^2-\left(\frac{1}{\overline{\Phi}\Delta t^2}\right)\right]\Phi^{n+1}=F_\Phi$$

Solving the above Helmholtz equation using SOR method, for Φ^{n+1}

the velocity components in the zonal and meridional directions are obtained from

$$u^{n+1}=-\Delta t\Phi_x^{n+1}+S_u$$

$$v^{n+1}=-\Delta t\Phi_y^{n+1}+S_v$$

All the variables are now known at time $(n+1)\Delta t$ and φ, u and v at the next time-step can be computed

Spectral method: An Example of Galerkin method

Galerkin method is a method that provides for a more accurate solution to the governing equations of atmospheric motions since it calculates derivatives exactly. Spectral method is an example of Galerkin method. In Galerkin method, we approximate functions as a linear combination of prescribed expansion functions, $\phi_j(x)$, the latter known as 'basis functions' , such as

$$u(x) = \sum_{j=1}^{N} u_j \phi_j(x)$$

where $\phi_j(x)$, j = 1,2,.. N, are the basis functions that each satisfy any boundary conditions on f(x). The coefficients u_j are the unknown coefficients that form a vector of N numbers

Let £(u) = f(x) for $a \le x \le b$ be an ordinary differential equation (ODE) with £ being an ordinary differential operator. Substituting expansion of u in the ODE, we get for the residual

$$\varepsilon(x) = £(u) - f(x) = £\{\sum_{j=1}^{N} u_j \phi_j(x)\} - f(x)$$

The Galerkin method requires that the residual ε(x) be orthogonal to each of the basis functions, i.e.

Spectral method: An Example of Galerkin method

$$\int \phi_j \varepsilon(x) dx = 0$$ for all j =1,2,.....N

Spectral methods are one of the examples of the series expansion method, for which the basis functions form an orthogonal set.

$$\int \phi_i \phi_j dx = 0$$ for $i \neq j$

The error in satisfying the ODE with the N terms of the series sum is given as

$$\varepsilon_N = \pounds\{\sum_{j=1}^{N} u_j \phi_j(x)\} - f(x)$$

The Galerkin method requires that the error ε_N be orthogonal to each basis function $\phi_j(x)$ in the following sense,

$$\int_a^b \varepsilon_N \phi_i dx = 0$$ i=1,2,...N

Substituting the expression for residual in the orthogonal condition, one obtains

$$\int_a^b \phi_i \pounds\{\sum_{j=1}^{N} u_j \phi_j dx\} - \int_a^b \phi_i f(x) dx = 0$$ i=1,2,…N

The system of N algebraic equations involving the unknown coefficients u_j can be solved to obtain u(x), which is the solution of the ODE

Staggered grids in horizontal and vertical directions

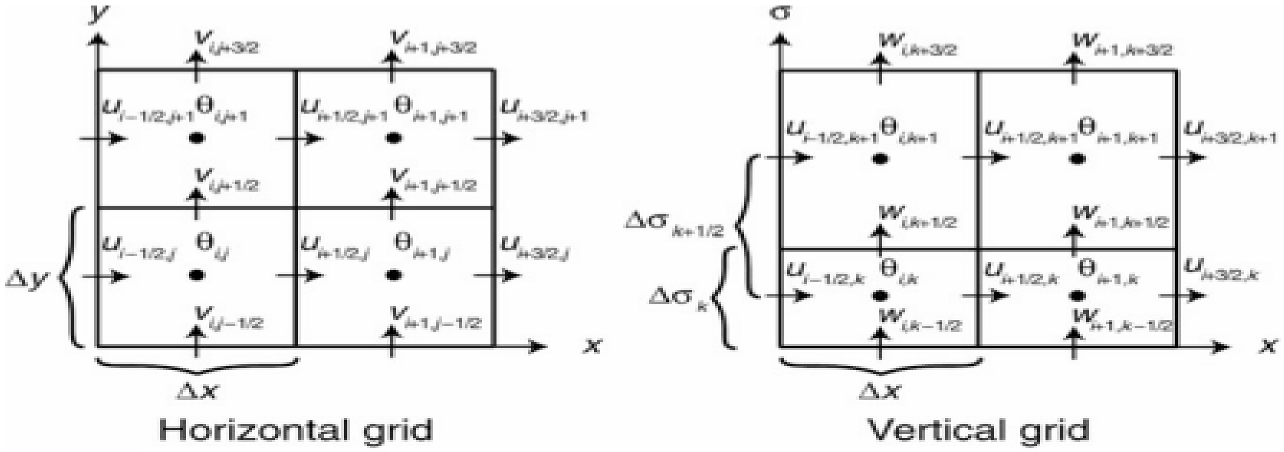


Fig. 2 The Arakawa C staggered grid method (Adapted from Skamarock et al. 2008)

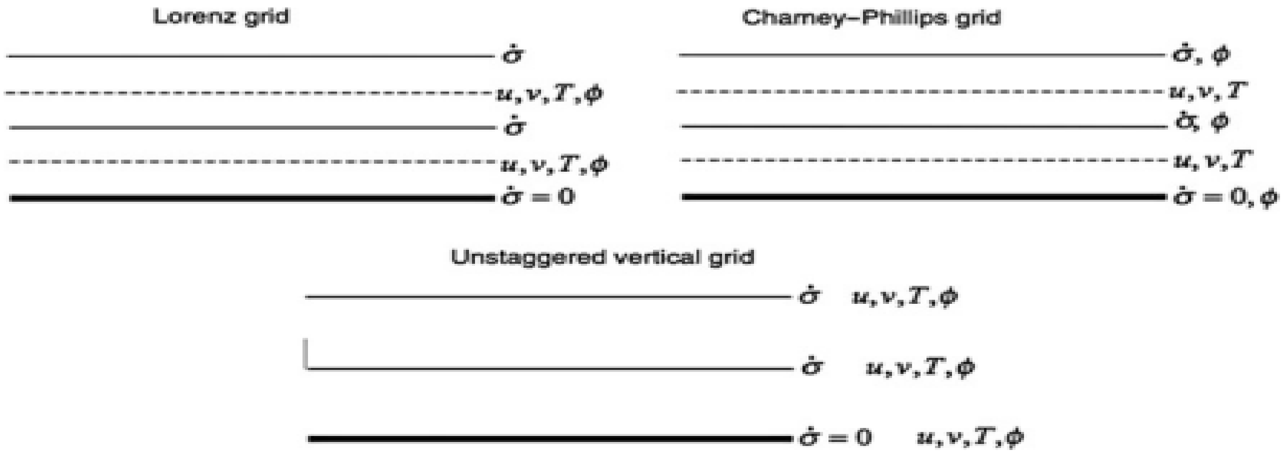


Fig. 3 Staggering in vertical grids (After Arakawa and Konor 1996)

Arakawa C grid is a staggered horizontal grid very commonly employed to provide geostrophic adjustment

Two types of vertical grids are used for atmospheric models: the Lorenz grid (L grid) and the Charney–Phillips grid (CP grid, with CP grid providing more accurate results

Upper and Lower boundary conditions

There are many ways to represent the upper boundary. For instance, a rigid lid can cap the model at some specified altitude so that energy reaching this lid is reflected downward. A free-surface method treats the model atmosphere and higher altitude as two distinct, non-mixing fluids and also reflects energy downward. Since the key issue of representing upper boundary conditions is how to handle the transfer of energy by gravity waves upward and out of the domain, an absorption/damping layer is incorporated with both the rigid-lid and free- surface methods. A radiative boundary condition is also used in some models to mimic the effects of wave energy propagating upward and out of the domain at the top of the model.

The bottom boundary conditions of NWP models are very complicated, as surface characteristics vary significantly. Therefore, the bottom conditions of NWP models are commonly parameterized or represented by a thermal diffusion surface model, land surface and ocean model, or surface drag schemes

Physical Parameterizations: Basic Principles

The basic equations of an NWP model include terms for friction (eddy fluxes of momentum), heating source(radiative heating and cooling, sensible heat fluxes), evaporation and condensation processes as well as moisture flux. These physical processes in numerical models represent their contribution. Thus, the model should include surface and planetary boundary layer processes, radiative transfer, and cloud microphysics in order to represent their contributions

Atmospheric motion includes a broad spectrum of temporal and spatial scales. The timescale spans from 1 to 10^6 s and beyond, and spatial scale ranges from 1 cm to 10,000 km, including the turbulent microscale, convective scale, mesoscale, and large scale.

Due to the use of numerical discretization methods to solve PDEs, the grid resolution of the atmospheric model is always limited. Hence, any processes that occur on a scale smaller than the grid space cannot be explicitly represented in the numerical model, even though their contribution cannot be ignored.

Physical Parameterizations: Apply Reynolds' average to momentum equation

Assume that any variable (e.g., u, v, w, T, p) can be separated into resolvable and unresolvable components, i.e., one can split all dependent variables into mean and turbulent parts. The mean is defined as an average over a grid cell. For example,

$$u = \bar{u} + u',$$
$$T = \bar{T} + T',$$
$$p = \bar{p} + p'.$$

$$u\frac{\partial u}{\partial x} = (\bar{u} + u')\frac{\partial}{\partial x}(\bar{u} + u') = \bar{u}\frac{\partial \bar{u}}{\partial x} + \bar{u}\frac{\partial u'}{\partial x} + u'\frac{\partial \bar{u}}{\partial x} + u'\frac{\partial u'}{\partial x}.$$

$$\overline{u\frac{\partial u}{\partial x}} = \overline{\bar{u}\frac{\partial \bar{u}}{\partial x}} + \overline{\bar{u}\frac{\partial u'}{\partial x}} + \overline{u'\frac{\partial \bar{u}}{\partial x}} + \overline{u'\frac{\partial u'}{\partial x}}.$$

since

$$\overline{a'} = 0,$$
$$\bar{\bar{a}} = \bar{a} \text{ and } \overline{\bar{a}b} = \bar{a}\bar{b} = \bar{a}\bar{b}, \text{ and}$$
$$\overline{\bar{a}b'} = \bar{a}\overline{b'} = \bar{a}\overline{b'} = 0.$$

Hence

$$\overline{u\frac{\partial u}{\partial x}} = \bar{u}\frac{\partial \bar{u}}{\partial x} + \cancelto{0}{\bar{u}\overline{\frac{\partial u'}{\partial x}}} + \cancelto{0}{\overline{u'}\frac{\partial \bar{u}}{\partial x}} + \overline{u'\frac{\partial u'}{\partial x}} = \bar{u}\frac{\partial \bar{u}}{\partial x} + \overline{u'\frac{\partial u'}{\partial x}}.$$

$$\frac{\partial u}{\partial t} = -u\frac{\partial u}{\partial x} - v\frac{\partial u}{\partial y} - w\frac{\partial u}{\partial z} - \frac{1}{\rho}\frac{\partial p}{\partial x} + fv + \frac{1}{\rho}\left(\frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} + \frac{\partial \tau_{zx}}{\partial z}\right)$$

where

$$\tau_{zx} = \mu\frac{\partial u}{\partial z},$$

Physical Parameterizations: Apply Reynolds' average to momentum equation

$$\frac{\partial \bar{u}}{\partial t} = -\bar{u}\frac{\partial \bar{u}}{\partial x} - \bar{v}\frac{\partial \bar{u}}{\partial y} - \bar{w}\frac{\partial \bar{u}}{\partial z} - \frac{1}{\bar{\bar{\rho}}}\frac{\partial \bar{p}}{\partial x} + f\bar{v} - u'\frac{\partial u'}{\partial x} - v'\frac{\partial u'}{\partial y} - w'\frac{\partial u'}{\partial z} + \frac{1}{\bar{\bar{\rho}}}\left(\frac{\partial \bar{\tau}_{xx}}{\partial x} + \frac{\partial \bar{\tau}_{yx}}{\partial y} + \frac{\partial \bar{\tau}_{zx}}{\partial z}\right).$$

$$\frac{\partial u'}{\partial x} + \frac{\partial v'}{\partial y} + \frac{\partial w'}{\partial z} = 0.$$

$$\frac{\partial \bar{u}}{\partial t} = -\bar{u}\frac{\partial \bar{u}}{\partial x} - \bar{v}\frac{\partial \bar{u}}{\partial y} - \bar{w}\frac{\partial \bar{u}}{\partial z} - \frac{1}{\bar{\bar{\rho}}}\frac{\partial \bar{p}}{\partial x} + f\bar{v} - \frac{\overline{\partial u'u'}}{\partial x} - \frac{\overline{\partial u'v'}}{\partial y} - \frac{\overline{\partial u'w'}}{\partial z} + \frac{1}{\bar{\bar{\rho}}}\left(\frac{\partial \bar{\tau}_{xx}}{\partial x} + \frac{\partial \bar{\tau}_{yx}}{\partial y} + \frac{\partial \bar{\tau}_{zx}}{\partial z}\right).$$

$$T_{xx} = -\bar{\rho}u'u',$$
$$T_{yx} = -\bar{\rho}u'v',$$
$$T_{zx} = -\bar{\rho}\,\overline{u'w'}.$$

$$\frac{\partial \bar{u}}{\partial t} = -\bar{u}\frac{\partial \bar{u}}{\partial x} - \bar{v}\frac{\partial \bar{u}}{\partial y} - \bar{w}\frac{\partial \bar{u}}{\partial z} - \frac{1}{\bar{\bar{\rho}}}\frac{\partial \bar{p}}{\partial x} + f\bar{v} + \frac{1}{\bar{\bar{\rho}}}\left(\frac{\partial}{\partial x}(\tau_{xx} + T_{xx}) + \frac{\partial}{\partial y}(\tau_{yx} + T_{yx}) + \frac{\partial}{\partial z}(\tau_{zx} + T_{zx})\right)$$

In the above equation, the first five terms in RHS can be explicitly represented by model grid values. The second component of the three terms inside the parentheses cannot be explicitly resolved at model grid points, but their contributions cannot be ignored, since these subgrid-scale processes depend on and in turn affect the large-scale fields and processes that are explicitly resolved by numerical models.

Overview of Physical Parameterizations

Hence, parameterization schemes are then necessary in order to properly describe the impact of these subgrid-scale mechanisms on the large-scale flow of the atmosphere. **In other words, the ensemble effect of the subgrid-scale processes has to be formulated in terms of the resolved grid-scale variables**. The "dynamics of the model" indicates schematically the resolved processes and the "model physics," the processes that must be parameterized.

Physical parameterization schemes in a numerical model should be designed to (i) represent the physical processes that interact with the dynamics; and (ii) explicitly calculate the contributions from the subgrid-scale processes parameterized as a function of the large-scale, resolved scales.

Physical parameterization schemes are commonly applied for the following physical processes, (i) radiation, (ii) convection, (iii) cloud microphysics and precipitation, (iv) soil/surface, (v) turbulent diffusion and planetary boundary layer

Need for Data Assimilation

In the early NWP experiments, Richardson (1922)and Charney et al. (1950) performed hand interpolations of the available observations to grid points, and these fields of initial conditions were manually digitized, which was a very time consuming procedure. The need for an automatic "objective analysis" quickly became apparent, and interpolation methods fitting data to grids were developed. However for operational primitive equation models, it is not enough to perform spatial interpolation of observations into regular grids, because not enough data are available to initialize current models.

Modern primitive NWP models have a number of degrees of freedom of the order of 10^7. For example, a latitude–longitude model with a typical resolution of 1° and 50 vertical levels would have $360 \times 180 \times 50 = 3.24 \times 10^6$ grid points. At each grid point, we have to seven prognostic variables, giving over 22.68 million variables that need to be given an initial value. Also, the total number of conventional observations of the variables used in the atmospheric models (e.g., from rawinsondes) is of the order of 10^4.

Need for Data Assimilation

There are many new types of data currently available, including remotely sensed data such as satellite and radar observations, which however do not measure directly the variables used in the models (wind, temperature, moisture, and surface pressure). While satellites provide satellite spectral radiances, Doppler radars give reflectivity and radial velocity. Also, the distribution of atmospheric observations in space and time is very non-uniform, with regions like North America and Eurasia that are data-rich, while others that are much more poorly observed.

For this reason, it became clear rather early in the history of NWP that, in addition to the observations, it was necessary to have a complete ***first guess*** estimate of the state of the atmosphere at all the grid points in order to generate the initial conditions for the forecast. The first guess (also known as background field or prior information) should be our best estimate of the state of the atmosphere prior to the use of the observations. Initially climatology, or a combination of climatology and a short forecast were used as a first guess. As forecasts became better, the use of short-range forecasts as a first guess was universally adopted in operational systems in what is called an “analysis cycle”

Need for Data Assimilation

The analysis cycle is an intermittent data assimilation system that continues to be used in most global operational systems, which typically use a 6-h cycle performed four times a day. The model forecast plays a very important role.

Over data-rich regions, the analysis is dominated by the information contained in the observations. In data-poor regions, the forecast benefits from the information upstream. For example, 6-h forecasts over the North Atlantic Ocean are very good, even in the absence of satellite data, because of the information coming from North America.

The forecast is thus able to transport information from data-rich to data-poor areas, and for this reason, data assimilation using a short-range forecast as a first guess has become known as four-dimensional data assimilation (4DDA)

Data Assimilation

The essential pre-requisite for prediction is a best estimate of the initial state of the system, encoded as numbers on a spatial grid, which may have little resemblance to the spatial pattern of available observations. For chaotic systems such as atmosphere whose evolution exhibits sensitive dependence on initial conditions, the forecast can depend critically on how well the grid-based initial state is estimated from disparate observational data. In the case of satellite data, the problem is compounded by the fact that the quantity measured – spectral radiance at different frequencies – is related only indirectly to parameters represented in the model, such as winds, waves and temperatures.

The term 'assimilation' should not be taken to imply a vague process of absorbing data into some computer program, but a carefully constructed procedure that brings to bear all our knowledge of the measurement process, the known errors in the measurements, the governing equations of the system, and the expected errors in those equations as approximated on a computer

Data Assimilation

Data assimilation is the technique whereby observational data are combined with data from forecasts by a numerical model to produce an optimal estimate of the evolving state of the system. The model brings consistency to the observational data, and interpolates or extrapolates data into data-devoid regions in space and time. The observational data correct the trajectory of the imperfect model through state space, keeping it ‘on the road’ in a forecast – observe – correct feedback loop.

The word ‘optimal’ in the above definition indicates the statistical basis of most advanced implementations of the method. The state of the system is estimated essentially as a weighted combination of observations and numerical forecasts, the weights being determined from the (supposedly known) errors in the observations and in the numerical forecasts.

THANKS