# Tomography of the Nucleon at the Electron-Ion Collider

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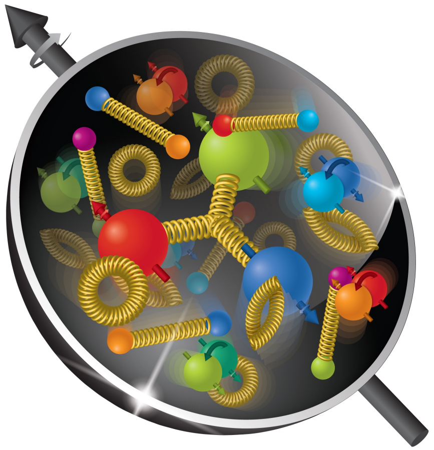
# Upcoming Electron-Ion Collider (EIC)

The EIC to be built at Brookhaven National Lab, USA will collide highly energetic electron beam with proton/heavy ion to take 'snapshots' at high accuracy --tomography of the nucleon

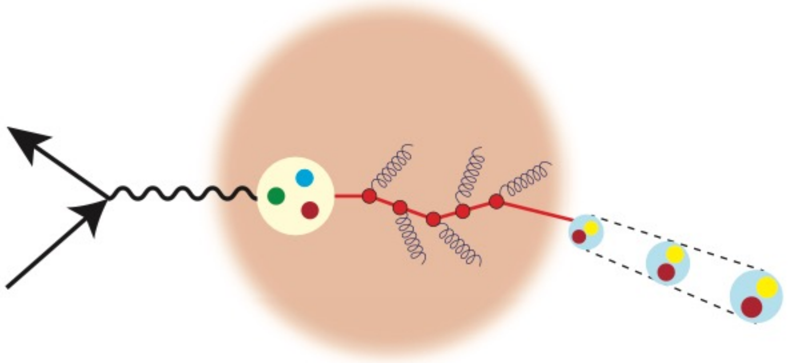


How the quarks and gluons are distributed in space inside the nucleon

How do quarks and gluons bind together and for the nucleon ? What is the Origin of the mass of the nucleon ?

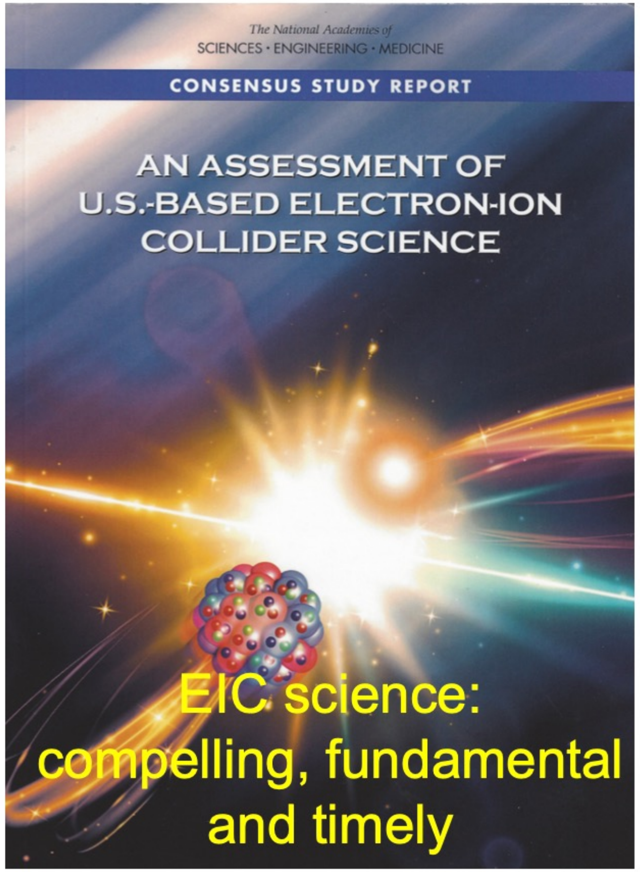
How the spin (1/2) of the proton is made from the spin and orbital angular momentum of the quarks and gluons

Will explore the correlations between spin/OAM and intrinsic transverse momentum

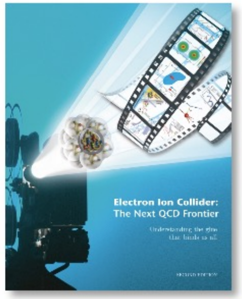


How does a dense nuclear environment affect quarks and gluons and their interactions ?

# National Academy's Assessment




EIC science: compelling, fundamental and timely

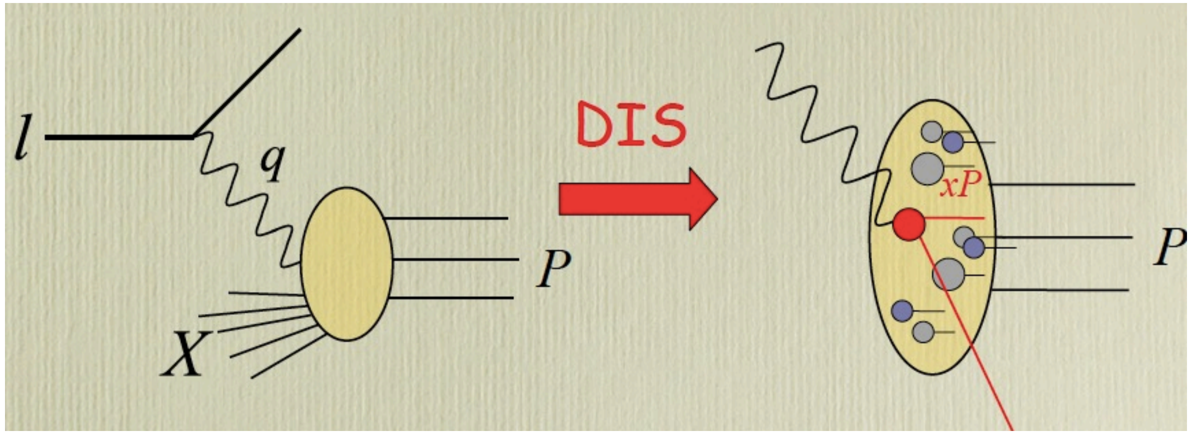



**Machine Design Parameters:**

- High luminosity: up to $10^{33}$-$10^{34}$ $cm^{-2}sec^{-1}$
  - a factor ~100-1000 times HERA
- Broad range in center-of-mass energy: ~20-100 GeV upgradable to 140 GeV
- Polarized beams e-, p, and light ion beams with flexible spin patterns/orientation
- Broad range in hadron species: protons…. Uranium
- Up to two detectors well-integrated detector(s) into the machine lattice

Slide : Abhay Deshpande

# Nucleon structure : probed through electron-proton deep inelastic scattering




$$Q^2 = -q^2 \to \infty$$

$$x = \frac{Q^2}{2P \cdot q} \quad \text{fixed}$$

Virtual photon 'sees ' the partons (quarks) inside the proton

Proton is Lorentz contracted, like a pancake in transverse plane

Target is a collection of partons moving with fraction x of proton momentum, and collinearly with the proton

In the deep inelastic limit, the electron passes target at almost zero time, sees partons frozen in transverse plane.

Electron can interact with the partons only if the impact parameter is less than 1/Q. Electron-parton scattering happens at a much shorter time scale than the hadronization scale of proton remnants

# Factorized form of the cross section

$$\frac{d\sigma}{dx\, dQ^2} = \sum_f \left(\frac{d\hat{\sigma}}{dQ^2}\right)_f e_f^2\, \phi_f(x)$$

Probability density of finding a parton of momentum fraction x inside the proton

Differential scattering cross section

Elastic electron-parton scattering

Incoherent sum over all partons

Parton model : Partons are non-interacting

Factorization of the hard part, that is interaction of electron with the parton, and the soft part, that is the parton distributions in the cross section

In parton model, parton distributions show scaling : they are functions of x only

Hard part can be calculated perturbatively but the parton distributions are non-perturbative. They are also not dependent on the process

Bjorken & Paschos, Phys. Rev D185, 1975, (1969).

# Parton model to QCD

$$\frac{d\sigma}{dx\,dQ^2} = \frac{2\pi\alpha^2}{xQ^4}\,A(y)\,F_2(x)$$

$$F_2(x) = \sum_f e_f^2\,x\,\phi_f(x)$$

$y$ = electron inelasticity $\sim \frac{\nu}{E}$

$A(y) = 1 + (1-y)^2$

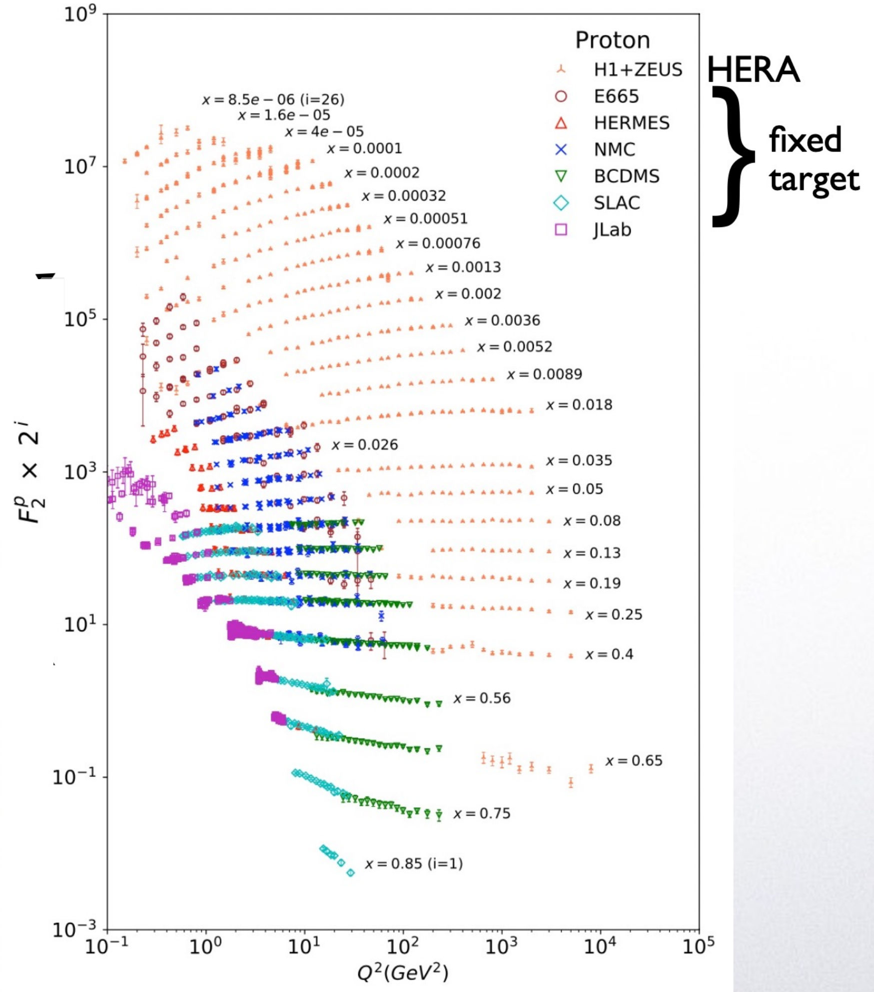
$F_2$ varies also with $Q^2$ : scale evolution

Scale evolution can be calculated using evolution equations

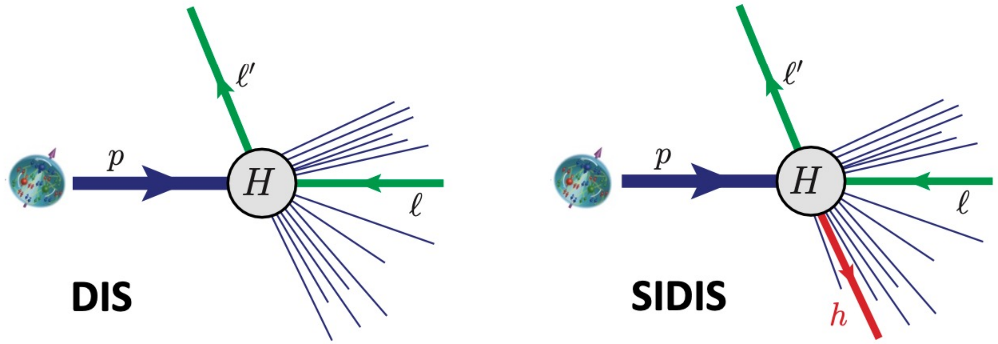
partons, or quarks are not free : they interact through gluons !

Interaction of quarks and gluons are called strong interaction or QCD

$$\frac{d\sigma}{dx\,dQ^2} = \frac{2\pi\alpha^2}{xQ^4}\left[A(y)\,F_2(x,Q^2) - y^2\,F_L(x,Q^2)\right]$$

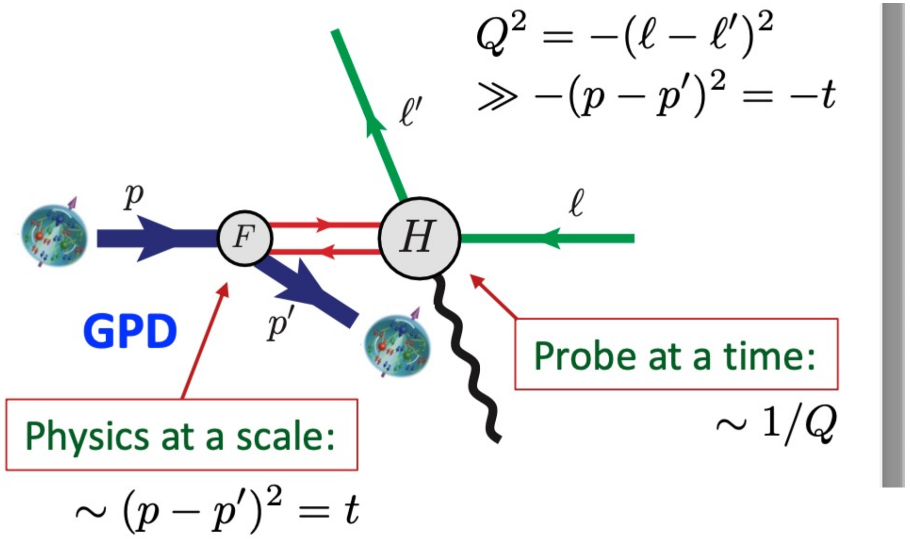


# Exploring partonic structure of the hadron while breaking/not breaking the proton




Deep inelastic scattering (DIS) and semi-inclusive deep inelastic scattering (SIDIS)

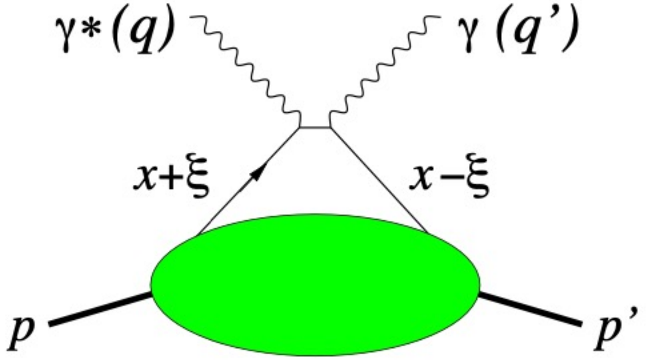
SIDIS probes the transverse momentum dependent parton distributions (TMDs)




Exclusive process; Deeply virtual Compton scattering (DVCS)

The proton remains intact, there is a momentum transfer and a real photon is observed in the final state

# Deeply virtual Compton Scattering and GPDs




$$e + P \to e + \gamma + P'$$

A real photon detected in the final state in addition to the electron and proton remains intact, but there is a momentum transfer

At leading order, the amplitude can be in a factorized form by the handbag diagram

Upper part : perturbative ; lower part non-perturbative: parametrized in terms of GPDs

$$t = (p - p')^2; \quad Q^2 = -q^2$$

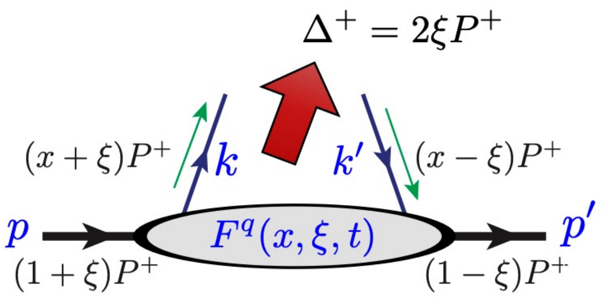
t : momentum transfer of the proton, is much less compared to $Q^2$

$$x_b = \frac{Q^2}{2p \cdot q}$$

$$\xi \approx \frac{x_B}{2 - x_B}$$

Skewness variable

# Generalized parton distributions (GPDs)




Factorization of DVCS amplitude

$$F^q(x,\xi,t) = \int \frac{\mathrm{d}z^-}{4\pi} e^{-ixP^+z^-} \langle p'|\bar{q}(z^-/2)\gamma^+ q(-z^-/2)|p\rangle$$

$$\widetilde{F}^q(x,\xi,t) = \int \frac{\mathrm{d}z^-}{4\pi} e^{-ixP^+z^-} \langle p'|\bar{q}(z^-/2)\gamma^+\gamma_5 q(-z^-/2)|p\rangle$$

- $\frac{x+\xi}{1+\xi}$ and $\frac{x-\xi}{1-\xi}$ are initial and final **light cone momentum fractions** of struck quark.
- $t$ is the invariant momentum transfer **to the target**.
- $Q^2$ is the invariant momentum transfer **from the electron**.
- $t \neq -Q^2$, in contrast to elastic scattering.
- $Q^2$ acts as a **resolution scale**, like in deeply inelastic scattering (DIS).
- $t$ tells us about structure seen from redistribution of momentum kick.
- $t$ is the "form factor variable" rather than $Q^2$.

These are parametrized in terms of GPDs

$$F^q(x,\xi,t) = \int \frac{\mathrm{d}z^-}{4\pi} e^{-ixP^+z^-} \langle p'|\bar{q}(z^-/2)\gamma^+ q(-z^-/2)|p\rangle$$
$$= \frac{1}{2P^+}\left[H^q(x,\xi,t)\,\bar{u}(p')\,\gamma^+ u(p) - E^q(x,\xi,t)\,\bar{u}(p')\,\frac{i\sigma^{+\alpha}\Delta_\alpha}{2m} u(p)\right],$$

$$\widetilde{F}^q(x,\xi,t) = \int \frac{\mathrm{d}z^-}{4\pi} e^{-ixP^+z^-} \langle p'|\bar{q}(z^-/2)\gamma^+\gamma_5 q(-z^-/2)|p\rangle$$
$$= \frac{1}{2P^+}\left[\widetilde{H}^q(x,\xi,t)\,\bar{u}(p')\,\gamma^+\gamma_5 u(p) - \widetilde{E}^q(x,\xi,t)\,\bar{u}(p')\,\frac{\gamma_5\Delta^+}{2m} u(p)\right].$$

Off-forward matrix elements

# Properties of GPDs

As GPDs appear in the amplitude and are expressed as off-forward matrix elements, they do not have a probabilistic interpretation like parton distributions

A Fourier transform wrt momentum transfer in the transverse direction, $\Delta^{\perp}$ gives parton distribution in impact parameter space, which have probabilistic interpretation (more later)

Forward limit of GPDs give parton distributions $H_q(x,0,0) = q(x)$ $\tilde{H}^q(x,0,0) = \Delta q(x)$

$$\int_{-1}^{1} dx\, H^q(x,\xi,t) = F_1^q(t), \qquad \int_{-1}^{1} dx\, E^q(x,\xi,t) = F_2^q(t),$$

$$\int_{-1}^{1} dx\, \tilde{H}^q(x,\xi,t) = g_A^q(t), \qquad \int_{-1}^{1} dx\, \tilde{E}^q(x,\xi,t) = g_P^q(t),$$

Moment of GPDs give Form factors

$$\langle p'|\, \bar{q}(0)\gamma^{\mu} q(0)\, |p\rangle = \bar{u}(p') \left[ F_1^q(t)\, \gamma^{\mu} + F_2^q(t)\, \frac{i\sigma^{\mu\alpha}\Delta_{\alpha}}{2m} \right] u(p),$$

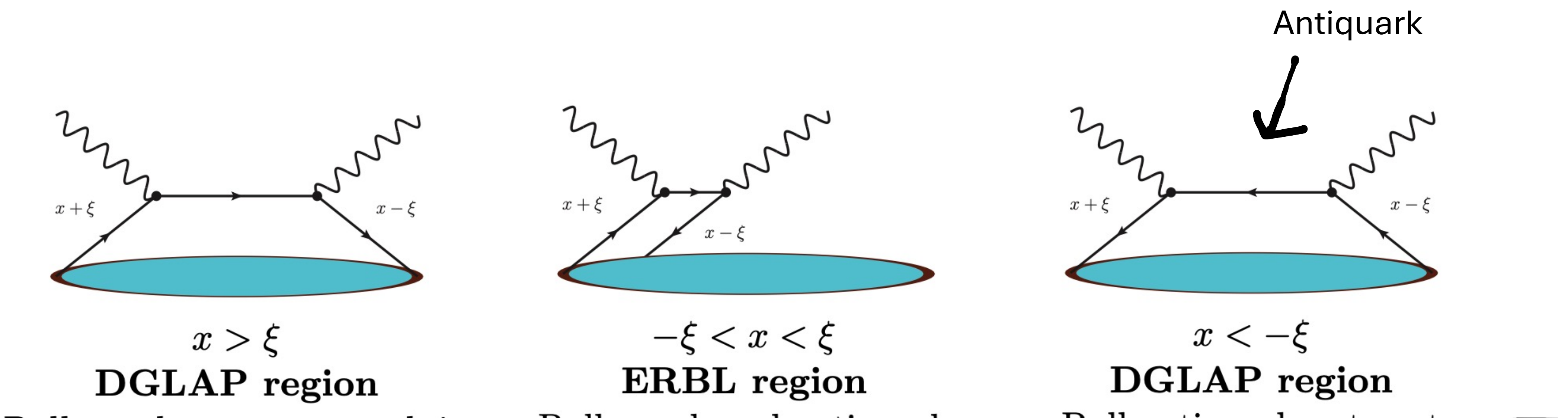
Dirac and Pauli Form factors

$$\langle p'|\, \bar{q}(0)\gamma^{\mu}\gamma_5 q(0)\, |p\rangle = \bar{u}(p') \left[ g_A^q(t)\, \gamma^{\mu}\gamma_5 + g_P^q(t)\, \frac{\gamma_5 \Delta^{\mu}}{2m} \right] u(p),$$

Axial and pseudoscalar form factors

M. Diehl; Phys.Rept.388:41-277,2003

# Evolution of generalized parton distributions




**(Dokshitzer–Gribov–Lipatov–Altarelli–Parisi)**

**(Efremov-Radyushkin-Brodsky-Lepage)**

**ξ** is called skewness, it gives momentum transfer in the light-cone plus or longitudinal direction

GPDs are functions of x, **ξ** and t as well as the scale $Q^2$.

GPDs evolve with the scale, evolution depending on whether skewness is greater or less than x

# Nucleon Spin Puzzle

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + L_q + \Delta g + L_g$$

Proton spin

Quark spin

Quark OAM

Gluon spin

Gluon OAM

EMC (European Muon Collaboration) at CERN in 1989 measured spin asymmetry in polarized muon-proton scattering experiment, and found that the contribution coming from the intrinsic spin of quarks is very small

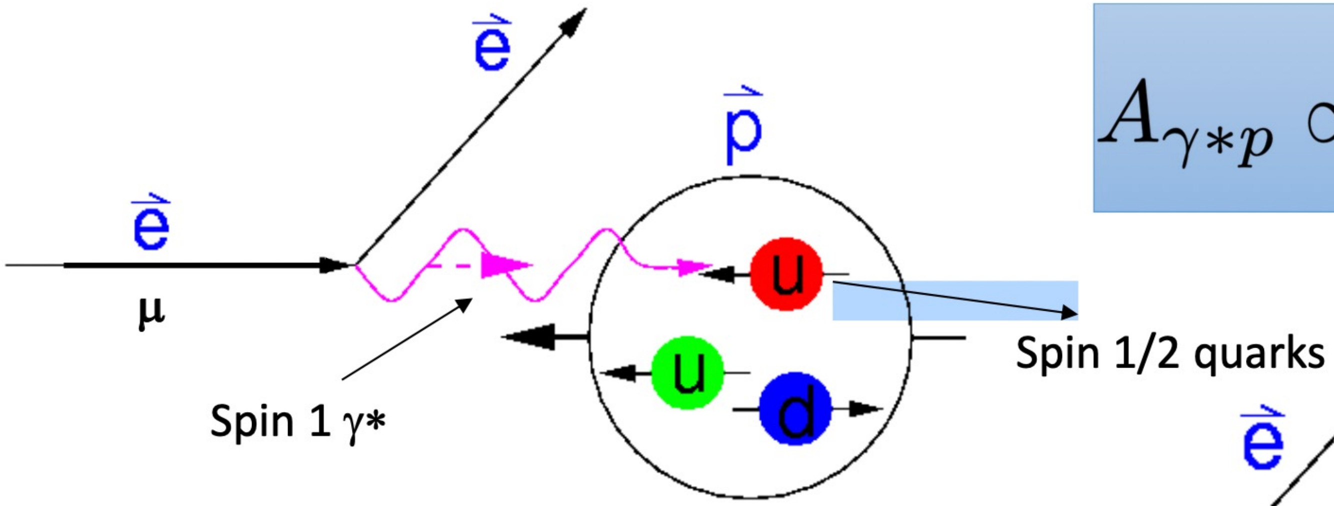
$$\Delta\Sigma/2 = (0.12) \;+/-\; (0.17) \;\text{(EMC, 1989)}$$

Significant contribution comes from gluons as well as the orbital angular momentum of quarks and gluons

How to measure the orbital angular momentum ? Observables ? Can one separate the gluon part intro intrinsic and orbital in a gauge invariant way ?

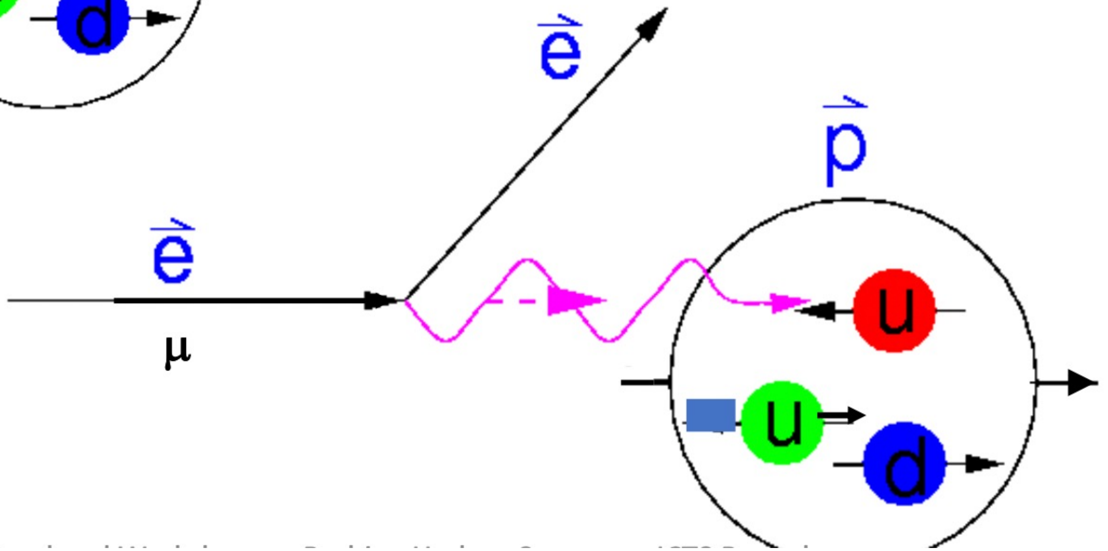
# How to measure quark contribution to the spin ?

Polarized deep inelastic scattering experiment : electron and proton longitudinally polarized




$$A_{\gamma * p} \propto \frac{N^{\uparrow\downarrow} - N^{\uparrow\uparrow}}{N^{\uparrow\downarrow} + N^{\uparrow\uparrow}}$$

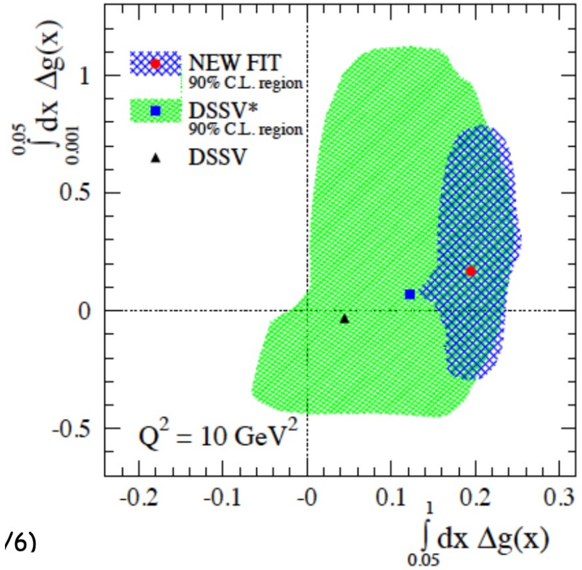
One can measure double longitudinal spin asymmetry : sensitive to the intrinsic spin carried by quarks




Pic : Abhay Deshpande

# Nucleon Spin Puzzle

$\Delta G$ = 0.2 +/- 0.02 +/- 0.5

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D. deFlorian et al., arXiv:1404.4293

RHIC data shows significant contribution from gluon spin.

Several lattice calculations of quark and gluon angular momentum contributions

Total quark angular momentum contribution about 54-57 %, total gluon angular momentum about 38-46 %, quark OAM about 13-18 %

How to measure OAM of quarks and gluon experimentally ? Intrinsic transverse momentum ?

Liu *AAPPS Bulletin* (2022) 32:8
https://doi.org/10.1007/s43673-022-00037-4
AAPPS Bulletin

REVIEW ARTICLE Open Access

Status on lattice calculations of the proton spin decomposition

Check for updates

Keh-Fei Liu

# Ji's sum rule : GPDs

$$\int dx x[H_q(x,0,0)+E_q(x,0,0)] = A_q(0)+B_q(0) = 2J_q$$

Angular momentum of quarks

$$\langle P',S'|T_i^{\mu\nu}(0)|P,S\rangle = \bar{U}(P',S')\Big[-B_i(q^2)\frac{\bar{P}^\mu\bar{P}^\nu}{M} + (A_i(q^2)+B_i(q^2))\frac{1}{2}(\gamma^\mu\bar{P}^\nu+\gamma^\nu\bar{P}^\mu) + C_i(q^2)\frac{q^\mu q^\nu - q^2 g^{\mu\nu}}{M} + \bar{C}_i(q^2)M g^{\mu\nu}\Big]U(P,S),$$

A and B are the form factors of the energy-momentum tensor, called Gravitational form factors

Similarly one can define gluon GPDs and a similar sum rule gives the total angular momentum of the gluons

So GPDs can help us to understand how proton spin is made from quarks and gluons.

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + L_q + \Delta g + L_g$$

Jaffe Manohar Sum rule

Extensive theoretical works to connect the two sum rules .

# Gravitational Form Factors for the Nucleon

$$\langle P', S'|T_i^{\mu\nu}(0)|P,S\rangle = \bar{U}(P',S')\Big[-B_i(q^2)\frac{\bar{P}^\mu \bar{P}^\nu}{M} + (A_i(q^2)+B_i(q^2))\frac{1}{2}(\gamma^\mu \bar{P}^\nu + \gamma^\nu \bar{P}^\mu)$$
$$+ C_i(q^2)\frac{q^\mu q^\nu - q^2 g^{\mu\nu}}{M} + \bar{C}_i(q^2) M g^{\mu\nu}\Big] U(P,S),$$

$$\bar{P}^\mu = \tfrac{1}{2}(P'+P)^\mu,\ q^\mu = (P'-P)^\mu$$

We choose Drell-Yan frame

$$Q^2 = -q^2 = \vec{q}_\perp^{\,2}$$

$$P = (P^+, P_\perp, P^-) = \left(P^+, 0, \frac{M^2}{P^+}\right),$$
$$P' = (P'^+, P'_\perp, P'^-) = \left(P^+, q_\perp, \frac{q_\perp^2 + M^2}{P^+}\right)$$
$$q = P' - P = \left(0, q_\perp, \frac{q_\perp^2}{P^+}\right),$$

GFFs give how matter couples to gravity

$A(Q^2)$ and $B(Q^2)$ are related to the mass and angular momentum of the proton

# Gravitational Form Factors

The GPDs $H_q$ and $E_q$ are related to the GFFs A and B, and to the angular momentum carried by the quarks

$$\int dx x[H_q(x,0,0) + E_q(x,0,0)] = A_q(0) + B_q(0) = 2J_q$$

X. Ji, PRD, 1997

Similarly, for the gluon counterpart.

$$\sum_{a=q,G} A_a(0) = 1, \quad \sum_{a=q,G} B_a(0) = 0, \quad \sum_{a=q,G} \bar{C}_a(t) = 0,$$

Follow from Poincare invariance

$\bar{C}(Q^2)$ arises due to non-conservation of EM tensor separately for quarks and gluons, and must vanish when summed over both

Lorce, Moutarde, Trawinski, EPJC (2019)

However, $C(Q^2)$ , also called the D-term, is not related to any Poincare generator and is unconstrained

# Energy and pressure distributions

GFF C is related to the pressure and shear force distributions inside the nucleon

Polyakov and Schweitzer, IJMPA (2018)

Breit frame (nucleon rest frame **P**=0) is a popular choice for analysis of form factors

3D Fourier transform of the FF can be given as $\mathcal{F}_a(r) = \int \frac{d^3\boldsymbol{\Delta}}{(2\pi)^3}\, e^{-i\boldsymbol{\Delta}\cdot\boldsymbol{r}}\, F_a(t)$ $\quad t = -\boldsymbol{\Delta}^2.$

Energy density

Isotropic pressure

$x^\mu = (0, r)$

EMT can be written as

$$\Theta^{\mu\nu}(\boldsymbol{r}) = [\varepsilon(r) + p(r)]\, u^\mu u^\nu - p(r)\eta^{\mu\nu} + s(r)\left(\chi^\mu\chi^\nu - \frac{1}{3}h^{\mu\nu}\right)$$

Pressure anisotropy

$u^\mu$ $\quad \chi^\mu = x^\mu / r$

Unit timelike and spacelike 4 vectors orthogonal to each other

$$h^{\mu\nu} = u^\mu u^\nu - \eta^{\mu\nu}$$

# Pressure distributions

Isotropic pressure and pressure anisotropy are related to radial and tangential pressure

$$p(r) = \frac{p_r(r) + 2\,p_t(r)}{3}, \quad s(r) = p_r(r) - p_t(r).$$

These can be expressed in terms of the GFFs

Where $$\mathcal{F}_a(r) = \int \frac{d^3\boldsymbol{\Delta}}{(2\pi)^3}\, e^{-i\boldsymbol{\Delta}\cdot\boldsymbol{r}}\, F_a(t)$$

Average squared mass radius is defined as

$$R_M^2 = \frac{1}{M}\int d^3\boldsymbol{r}\, r^2\, \varepsilon(r) = 6\left[\frac{dA(t)}{dt}\bigg|_{t=0} - \frac{1}{M^2}\, C(0)\right].$$

$$\varepsilon_a(r) = M\left\{\mathcal{A}_a(r) + \bar{\mathcal{C}}_a(r) + \frac{1}{4M^2}\frac{1}{r^2} \times \frac{d}{dr}\left(r^2\frac{d}{dr}[\mathcal{B}_a(r) - 4\mathcal{C}_a(r)]\right)\right\},$$

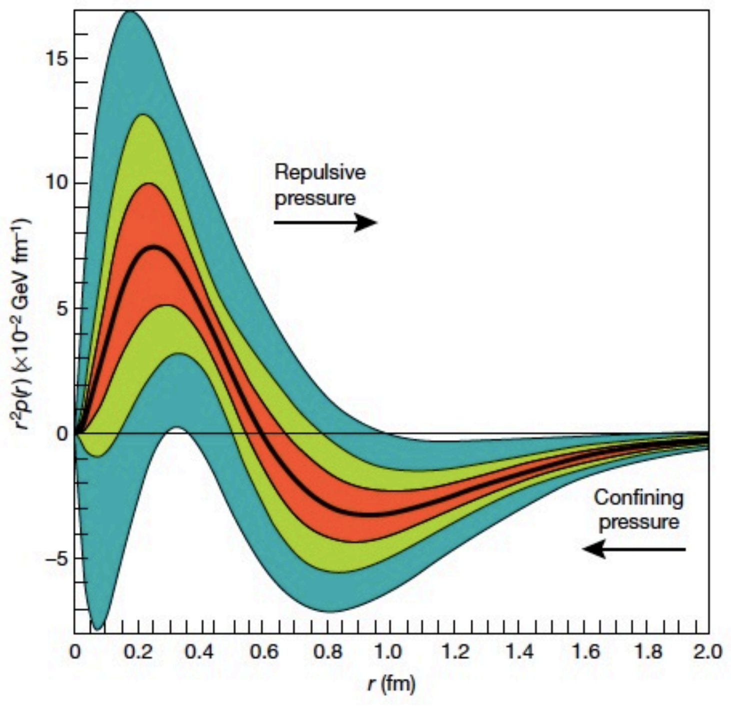
$$p_{r,a}(r) = M\left\{-\bar{\mathcal{C}}_a(r) + \frac{1}{M^2}\frac{2}{r}\frac{d\mathcal{C}_a(r)}{dr}\right\},$$

$$p_{t,a}(r) = M\left\{-\bar{\mathcal{C}}_a(r) + \frac{1}{M^2}\frac{1}{r}\frac{d}{dr}\left(r\frac{d\mathcal{C}_a(r)}{dr}\right)\right\},$$

$$p_a(r) = M\left\{-\bar{\mathcal{C}}_a(r) + \frac{2}{3}\frac{1}{M^2}\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{d\mathcal{C}_a(r)}{dr}\right)\right\},$$

$$s_a(r) = M\left\{-\frac{1}{M^2}\, r\,\frac{d}{dr}\left(\frac{1}{r}\frac{d\mathcal{C}_a(r)}{dr}\right)\right\},$$

# Pressure distribution inside the nucleon




Burkert, Elouadrhiri, Girod, Nature(2018)

Pressure distribution obtained from fits to Jlab data to extract the GPDs, in particular the D-term

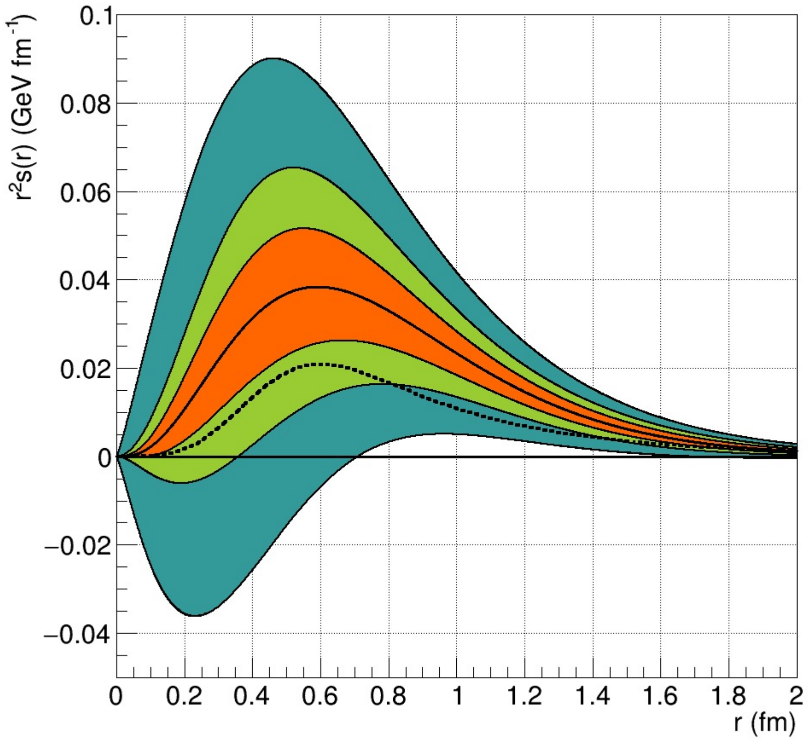
Pressure distribution is repulsive at the center of the nucleon and confining in the outer region

At the core it exceeds the pressure density of the most dense object that is neutron star , average peak pressure $10^{35}$ Pascals

This also connects a set of collider observables (GPDs) to the investigation of the equation of state (EoS) of neutron stars

Rajan, Gorda, Liuti, Yagi (2018)

# Shear Distribution Inside the Nucleon




Shear (tangential) force inside the nucleon from DVCS data at JLab

Maximum shear force at 0.6 fm from the center of the nucleon : confinement may be dominant

Shear forces change direction at r=0.45 fm from the center

Burkert, Elouadrhiri, Girod, 2104.02031[nucl-ex]

# GFFs and Pressure Distribution

Jlab result triggered a lot of interest : theoretical model calculations of the pressure distributions

Polyakov and Schweitzer, IJMPA (2018)

Most calculations are done in the Breit frame and are subject to relativistic corrections

2-D distributions in the infinite momentum frame or light-front formalism introduced in

Lorce, Moutarde, Trawinski, EPJC (2019), Freese and Miller, PRD(2021)

Because of transverse Galilean symmetry on the light-front these are free from relativistic corrections

Connection between 2D and 3D pressure distributions can be established through Abel transformation

Panteleeva and Polyakov, 2021

# Problem with 3 D densities

R. L. Jaffe, PRD 103, 016017 (2021); A Freese and G Miller, PRD 103, 094023 (2021)

Coordinate r in the Fourier transform has to be defined wrt the location of the system

It is thus necessary to construct a localized wave packet whose center is the refrence point wrt which r is measured

The more precisely one tries to localize the system, the higher the momentum components one has to introduce in the wave packet, making the relativistic corrections larger

Thus this picture is not very accurate for a system having a size of the same order as Compton wavelength, for example the nucleon. The relativistic corrections become model dependent of the wave functions as Lorentz Boosts depend on the dynamics of the system.

One can define 2 D light front distributions instead, where the FT are taken wrt the transverse momentum transfer. Such distributions are free from relativistic corrections, as transverse boosts are Galilean or free from dynamics in light-front framework.

M. Burkardt, Int.J.Mod.Phys.A 18 (2003) 173

Lorce, Moutarde, Trawinski, EPJC 79:89 (2019)

# Light-front distributions

Drell-Yan frame $\Delta^+ = 0$

2D Fourier transform of the GFFs

$$\tilde{F}(b_\perp) = \int \frac{d^2\Delta_\perp}{(2\pi)^2} e^{-i\Delta_\perp \cdot b_\perp} F(-\Delta_\perp^2)$$

Momentum transfer is in the transverse direction

Lorce, Moutarde, Trawinski, EPJC 79:89 (2019)

2D Galilean energy density

$$\mu_a(b) = M\left\{\frac{\mathsf{A}_a(b)}{2} + \bar{\mathsf{C}}_a(b) + \frac{1}{4M^2}\frac{1}{b}\frac{\mathrm{d}}{\mathrm{d}b}\left(b\frac{\mathrm{d}}{\mathrm{d}b}\left[\frac{\mathsf{B}_a(b)+\mathsf{D}_a(b)}{2} - 4\mathsf{C}_a(b)\right]\right)\right\},$$

2D pressure distributions

$$\sigma_{r,a}(b) = M\left\{-\bar{\mathsf{C}}_a(b) + \frac{1}{M^2}\frac{1}{b}\frac{\mathrm{d}\mathsf{C}_a(b)}{\mathrm{d}b}\right\},$$

$$\sigma_{t,a}(b) = M\left\{-\bar{\mathsf{C}}_a(b) + \frac{1}{M^2}\frac{\mathrm{d}^2\mathsf{C}_a(b)}{\mathrm{d}b^2}\right\},$$

$$\sigma_a(b) = M\left\{-\bar{\mathsf{C}}_a(b) + \frac{1}{2}\frac{1}{M^2}\frac{1}{b}\frac{\mathrm{d}}{\mathrm{d}b}\left(b\frac{\mathrm{d}\mathsf{C}_a(b)}{\mathrm{d}b}\right)\right\},$$

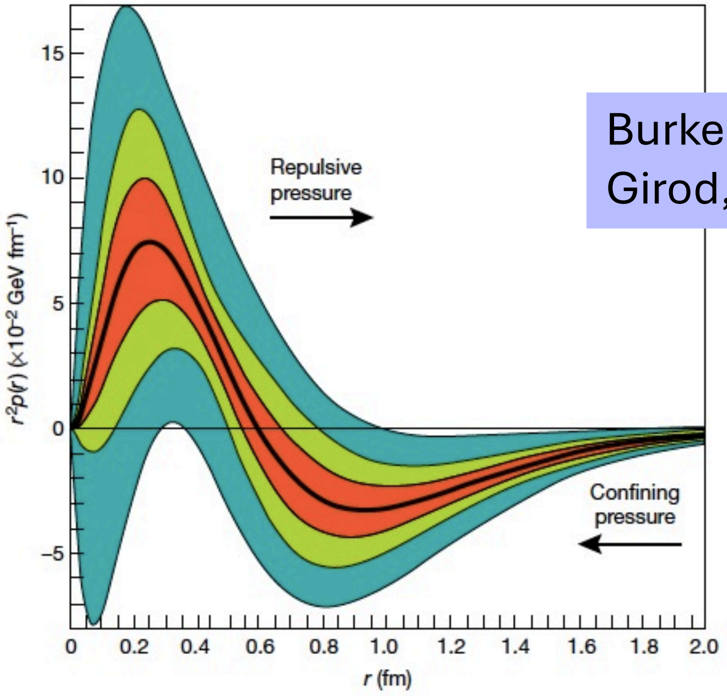
$$\Pi_a(b) = M\left\{-\frac{1}{M^2}b\frac{\mathrm{d}}{\mathrm{d}b}\left(\frac{1}{b}\frac{\mathrm{d}\mathsf{C}_a(b)}{\mathrm{d}b}\right)\right\}.$$

Pressure anisotropy

# Pressure distributions

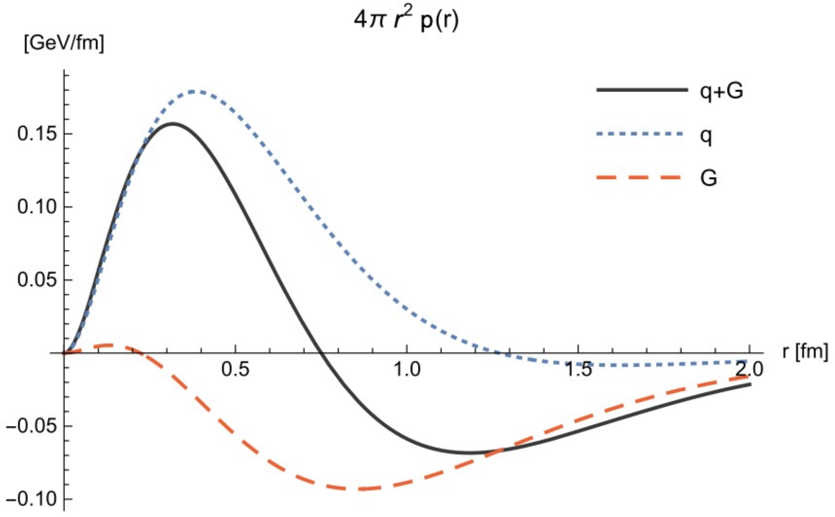
$$\int_0^\infty \mathrm{d}r\, r^2\, p(r) = 0,$$ Von Laue Condition — states that isotropic pressure has to change sign

$$\int_0^\infty \mathrm{d}b\, b\, \sigma(b) = 0,$$ 2 D version of Von Laue condition



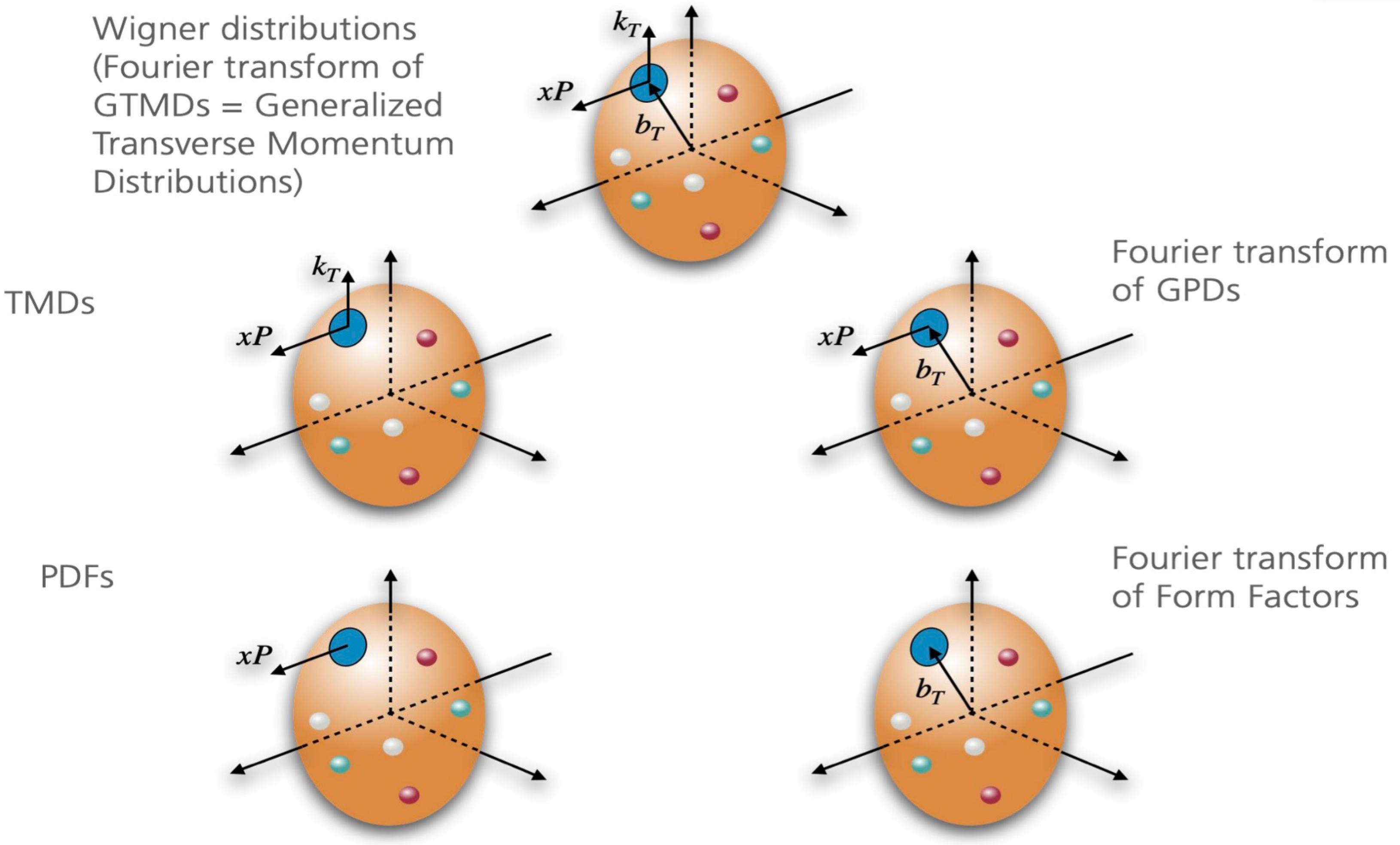

Burkert, Elouadrhiri, Girod, Nature(2018)

Multipole model




Lorce, Moutarde, Trawinski, EPJC 79:89 (2019)

# Hadron Structure in three dimensions




Lorce, Pasquini, Vanderhaeghen, JHEP 1105 (11)

# Generalized parton correlation functions

GPCFs are the most general correlation functions in the hadron-often called the 'mother distributions'

Both GPDs an TMDs can be obtained from the GPCFs-rich in information about the internal structure of the hadron

Integrating the GPCFs over the minus component of momentum, one gets the correlator that is parametrized in terms of GTMDs (generalized transverse momentum dependent parton distributions)

$$W^{[\Gamma]}_{\lambda\lambda'}(P, x, \vec{k}_T, \Delta, N; \eta) = \int dk^- \, W^{[\Gamma]}_{\lambda\lambda'}(P, k, \Delta, N; \eta)$$
$$= \frac{1}{2} \int \frac{dz^- \, d^2\vec{z}_T}{(2\pi)^3} \, e^{ik\cdot z} \, \langle p', \lambda' | \, \bar{\psi}\Big(-\frac{1}{2}z\Big) \, \Gamma \, \mathcal{W}\Big(-\frac{1}{2}z, \frac{1}{2}z \,|\, n\Big) \, \psi\Big(\frac{1}{2}z\Big) \, |p, \lambda\rangle \Big|_{z^+=0} \, .$$

Gauge link

Required for color gauge invariance

Meissner, Metz, Schlegel, JHEP 08, (2009) 056

# Twist Two GTMDs

$$W_{\lambda\lambda'}^{[\gamma^+]} = \frac{1}{2M}\,\bar{u}(p',\lambda')\left[F_{1,1} + \frac{i\sigma^{i+}k_T^i}{P^+}\,F_{1,2} + \frac{i\sigma^{i+}\Delta_T^i}{P^+}\,F_{1,3} + \frac{i\sigma^{ij}k_T^i\Delta_T^j}{M^2}\,F_{1,4}\right]u(p,\lambda)\,,$$

$$W_{\lambda\lambda'}^{[\gamma^+\gamma_5]} = \frac{1}{2M}\,\bar{u}(p',\lambda')\left[-\frac{i\varepsilon_T^{ij}k_T^i\Delta_T^j}{M^2}\,G_{1,1} + \frac{i\sigma^{i+}\gamma_5 k_T^i}{P^+}\,G_{1,2} + \frac{i\sigma^{i+}\gamma_5\Delta_T^i}{P^+}\,G_{1,3} + i\sigma^{+-}\gamma_5\,G_{1,4}\right]u(p,\lambda)\,,$$

$$W_{\lambda\lambda'}^{[i\sigma^{j+}\gamma_5]} = \frac{1}{2M}\,\bar{u}(p',\lambda')\left[-\frac{i\varepsilon_T^{ij}k_T^i}{M}\,H_{1,1} - \frac{i\varepsilon_T^{ij}\Delta_T^i}{M}\,H_{1,2} + \frac{M\,i\sigma^{j+}\gamma_5}{P^+}\,H_{1,3} + \frac{k_T^j\,i\sigma^{k+}\gamma_5 k_T^k}{M\,P^+}\,H_{1,4} + \frac{\Delta_T^j\,i\sigma^{k+}\gamma_5 k_T^k}{M\,P^+}\,H_{1,5} + \frac{\Delta_T^j\,i\sigma^{k+}\gamma_5\Delta_T^k}{M\,P^+}\,H_{1,6} + \frac{k_T^j\,i\sigma^{+-}\gamma_5}{M}\,H_{1,7} + \frac{\Delta_T^j\,i\sigma^{+-}\gamma_5}{M}\,H_{1,8}\right]u(p,\lambda)\,.$$

All GTMDs are functions of

$$(x, \xi, \vec{k}_T^2, \vec{k}_T\cdot\vec{\Delta}_T, \vec{\Delta}_T^2; \eta).$$

gives the direction of the gauge link

After integrating over $k^-,\ k_T$

dependence on η goes out

In the limit of vanishing momentum transfer **Δ** =0 one gets TMDs and integration over $k_T$ gives GPDs.

# Wigner Distributions

A one dimensional quantum system with wave function $\psi(x)$

Wigner distribution is defined as

$$W(x,p)=\int d\eta e^{ip\eta}\psi^*(x-\eta/2)\psi(x+\eta/2),$$

If we integrate over p, we get a positive definite coordinate space density $|\psi(x)|^2$

If we integrate over x, we get a positive definite momentum space density $|\psi(p)|^2$

For arbitrary p and x Wigner distribution is not positive definite and does not have probabilistic interpretation

Can be used to calculate average of observables

$$\langle \hat{O}(x,p)\rangle=\int dxdp W(x,p)O(x,p),$$

Quantum mechanical 'phase space' distribution

E.P. Wigner, Phys. Rev. **40**, 749. 1932.

# Wigner distribution for quarks

Introduce the Wigner operator

$$\hat{\mathcal{W}}_\Gamma(\vec{r},k)=\int d^4\eta e^{ik\cdot\eta}\bar{\Psi}(\vec{r}-\eta/2)\Gamma\Psi(\vec{r}+\eta/2),$$

Quark phase space position

Phase space momentum conjugate to η

Bilocal operator, need gauge link for gauge invariance

$$\Psi(\eta)=\exp\left(-ig\int_0^\infty d\lambda n\cdot A(\lambda n+\eta)\right)\psi(\eta),$$

Define reduced Wigner operator

$$W_\Gamma(\vec{r},\vec{k})=\int\frac{dk^-}{(2\pi)^2}W_\Gamma(\vec{r},k),$$

Cannot be measured experimentally

5 D Wigner operator is defined as

$$\hat{W}^{[\Gamma]}(\vec{b}_\perp,\vec{k}_\perp,x)\equiv\frac{1}{2}\int\frac{dz^-d^2z_\perp}{(2\pi)^3}e^{i(xp^+z^--\vec{k}_\perp\cdot\vec{z}_\perp)}\times\bar{\psi}\left(y-\frac{z}{2}\right)\Gamma\mathcal{W}\psi\left(y+\frac{z}{2}\right)\Big|_{z^+=0}$$

Belitsky, Ji, Yuan, PRD 69,074014 (2004)

# Wigner distribution for quarks

$$\hat{W}^{[\Gamma]}(\vec{b}_\perp, \vec{k}_\perp, x) \equiv \frac{1}{2}\int \frac{dz^- d^2 z_\perp}{(2\pi)^3} e^{i(xp^+ z^- - \vec{k}_\perp \cdot \vec{z}_\perp)} \times \bar{\psi}\left(y - \frac{z}{2}\right)\Gamma \mathcal{W} \psi\left(y + \frac{z}{2}\right)\Big|_{z^+=0}$$

$$y^\mu = [0, 0, \vec{b}_\perp],$$

x is the average fraction of the nucleon momentum carried by active quark

$b_\perp, k_\perp$ Are not Fourier conjugate variables But do not commute, so subject to uncertainty principle

Conjugate to $\Delta_\perp = k_{f\perp} - k_{i\perp}$

Average quark momentum

Wigner distribution is the 2D Fourier transformation of the GTMD correlator introduced earlier

GTMDs are in general complex values functions, however, their 2D FT are real valued

Wigner distributions do not have probabilistic interpretation due to Heisenberg uncertainty principle, and they are not positive definite

One can obtain three dimensional probability densities from Wigner distributions.

Lorce and Pasquini, PRD 84, 014015 (2011)

# Wigner distribution for quarks

$$\rho^{[\Gamma]}(\vec{b}_\perp, \vec{k}_\perp, x, \vec{S}) \equiv \int \frac{d^2\Delta_\perp}{(2\pi)^2} \left\langle p^+, \frac{\vec{\Delta}_\perp}{2}, \vec{S} \middle| \hat{W}^{[\Gamma]}(\vec{b}_\perp, \vec{k}_\perp, x) \middle| p^+, -\frac{\vec{\Delta}_\perp}{2}, \vec{S} \right\rangle.$$

Wigner operator sandwiched between nucleon states.

Integration over $b_\perp$ effectively sets $\Delta_\perp = 0$ and gives the TMD correlator

$$\int d^2 b_\perp \rho^{[\Gamma]}(\vec{b}_\perp, \vec{k}_\perp, x, \vec{S}) = W^{[\Gamma]}(\vec{0}_\perp, \vec{k}_\perp, x, \vec{S}) \equiv \Phi^{[\Gamma]}(\vec{k}_\perp, x, \vec{S}),$$

Integration over $k_\perp$ sets $z_\perp = 0$ and one gets 2 D Fourier transform of GPDs

These are the impact parameter dependent pdfs.

$$\int d^2 k_\perp \rho^{[\Gamma]}(\vec{b}_\perp, \vec{k}_\perp, x, \vec{S}) = \int \frac{d^2\Delta_\perp}{(2\pi)^2} e^{-i\vec{\Delta}_\perp \cdot \vec{b}_\perp} F^{[\Gamma]}(\vec{\Delta}_\perp, x, \vec{S})$$

# Wigner distribution for quarks

Integrating over $b_y$ and $k_x$ effectively sets $\Delta_y = z_x = 0$

Lorce and Pasquini, PRD 84, 014015 (2011)

One gets

$$\int db_y dk_x \rho^{[\Gamma]}(\vec{b}_\perp, \vec{k}_\perp, x, \vec{S}) \equiv \tilde{\rho}^{[\Gamma]}(b_x, k_y, x, \vec{S}).$$

Wigner distribution for quark with longitudinal polarization $\lambda$ in a nucleon with longitudinal polarization $\Lambda$ is given as

$$\rho_{\Lambda\lambda}(\vec{b}_\perp, \vec{k}_\perp, x) \equiv \frac{1}{2}[\rho^{[\gamma^+]}(\vec{b}_\perp, \vec{k}_\perp, x, \Lambda\vec{e}_z) + \lambda\rho^{[\gamma^+\gamma_5]}(\vec{b}_\perp, \vec{k}_\perp, x, \Lambda\vec{e}_z)].$$

Unpolarized quark in unpolarized nucleon

Longitudinally polarized quark in unpolarized nucleon

This can be decomposed as

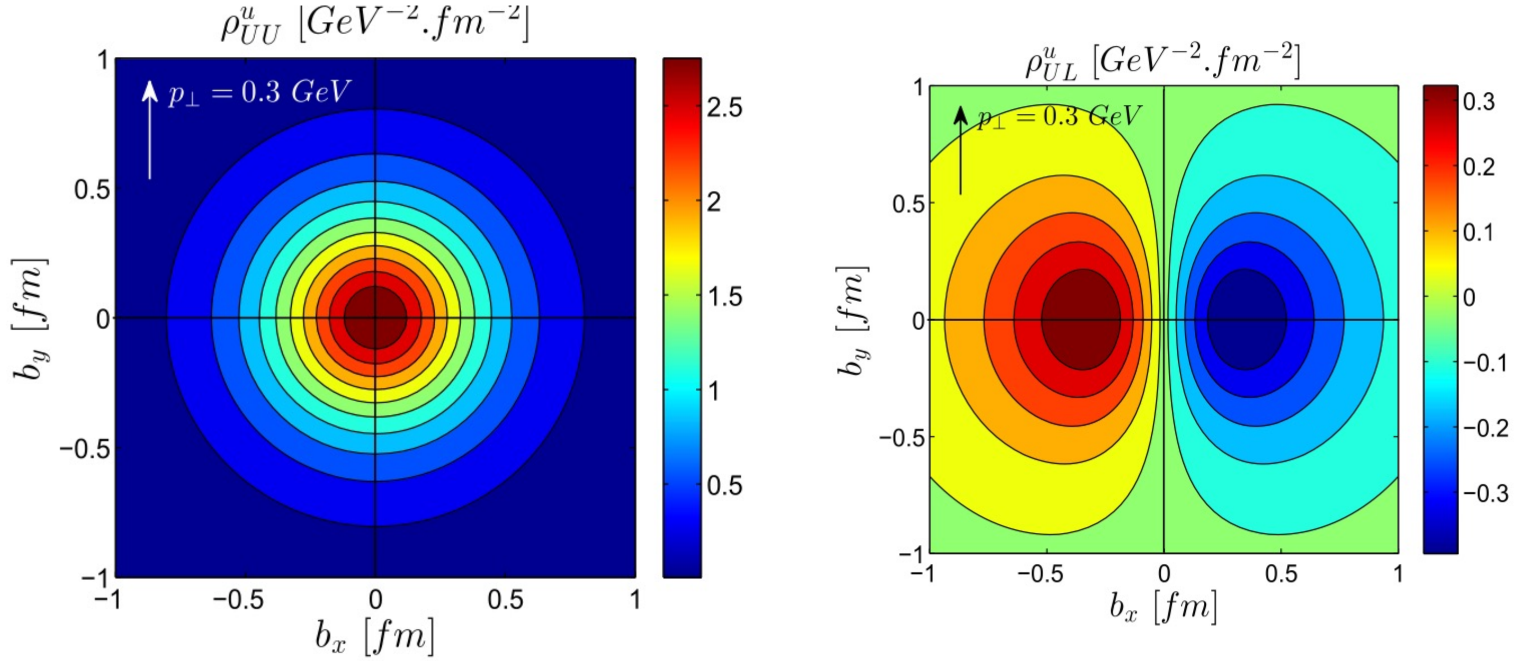
$$\rho_{\Lambda\lambda}(\vec{b}_\perp, \vec{k}_\perp, x) = \frac{1}{2}[\rho_{UU}(\vec{b}_\perp, \vec{k}_\perp, x) + \Lambda\rho_{LU}(\vec{b}_\perp, \vec{k}_\perp, x) + \lambda\rho_{UL}(\vec{b}_\perp, \vec{k}_\perp, x) + \Lambda\lambda\rho_{LL}(\vec{b}_\perp, \vec{k}_\perp, x)],$$

Unpol quark in long pol nucleon

Long pol quark in long pol nucleon

# Model calculations of transverse Wigner distributions

(Wigner distributions integrated over x)




Wigner distributions in a spectator model (ADS-QCD)

Chakrabarti, Maji, Mondal and Mukherjee, PRD 95, 074028 (2017)

# Wigner distributions and GTMDs

In terms of GTMDs we have

$$\rho_{UU}(\vec{b}_\perp, \vec{k}_\perp, x) = \mathcal{F}_{1,1}(x, 0, \vec{k}_\perp^2, \vec{k}_\perp \cdot \vec{b}_\perp, \vec{b}_\perp^2),$$

$$\rho_{LU}(\vec{b}_\perp, \vec{k}_\perp, x) = -\frac{1}{M^2}\epsilon_\perp^{ij} k_\perp^i \frac{\partial}{\partial b_\perp^j} \times \mathcal{F}_{1,4}(x, 0, \vec{k}_\perp^2, \vec{k}_\perp \cdot \vec{b}_\perp, \vec{b}_\perp^2),$$

$$\rho_{UL}(\vec{b}_\perp, \vec{k}_\perp, x) = \frac{1}{M^2}\epsilon_\perp^{ij} k_\perp^i \frac{\partial}{\partial b_\perp^j} \mathcal{G}_{1,1}(x, 0, \vec{k}_\perp^2, \vec{k}_\perp \cdot b_\perp, b_\perp^2),$$

$$\rho_{LL}(\vec{b}_\perp, \vec{k}_\perp, x) = \mathcal{G}_{1,4}(x, 0, \vec{k}_\perp^2, \vec{k}_\perp \cdot \vec{b}_\perp, \vec{b}_\perp^2),$$

FT of GTMDs

$$\mathcal{X}(x, \xi, \vec{k}_\perp^2, \vec{k}_\perp \cdot \vec{b}_\perp, \vec{b}_\perp^2) = \int \frac{d^2\Delta_\perp}{(2\pi)^2} e^{-i\vec{\Delta}_\perp \cdot \vec{b}_\perp} X(x, \xi, \vec{k}_\perp^2, \vec{k}_\perp \cdot \vec{\Delta}_\perp, \vec{\Delta}_\perp^2).$$

Connection with TMD and GPDs

$$f_1(x, \vec{k}_\perp^2) = \int d^2b_\perp \mathcal{F}_{1,1}(x, 0, \vec{k}_\perp^2, \vec{k}_\perp \cdot \vec{b}_\perp, \vec{b}_\perp^2) = F_{1,1}(x, 0, \vec{k}_\perp^2, 0, 0),$$

$$H(x, 0, \vec{\Delta}_\perp^2) = \int d^2k_\perp F_{1,1}(x, 0, \vec{k}_\perp^2, \vec{k}_\perp \cdot \vec{\Delta}_\perp, \vec{\Delta}_\perp^2),$$

$$g_{1L}(x, \vec{k}_\perp^2) = \int d^2b_\perp \mathcal{G}_{1,4}(x, 0, \vec{k}_\perp^2, \vec{k}_\perp \cdot \vec{b}_\perp, \vec{b}_\perp^2) = G_{1,4}(x, 0, \vec{k}_\perp^2, 0, 0),$$

$$\tilde{H}(x, 0, \vec{\Delta}_\perp^2) = \int d^2k_\perp G_{1,4}(x, 0, \vec{k}_\perp^2, \vec{k}_\perp \cdot \vec{\Delta}_\perp, \vec{\Delta}_\perp^2).$$

Lorce and Pasquini, PRD 84, 014015 (2011)

# Quark orbital angular momentum

Ji's sum rule $$J_z^q = \frac{1}{2}\int dx x[H^q(x,0,0) + E^q(x,0,0)].$$

So the OAM is given by $$L_z^q = \frac{1}{2}\int dx\{x[H^q(x,0,0) + E^q(x,0,0)] - \tilde{H}^q(x,0,0)\}.$$

This is called kinetic quark OAM.

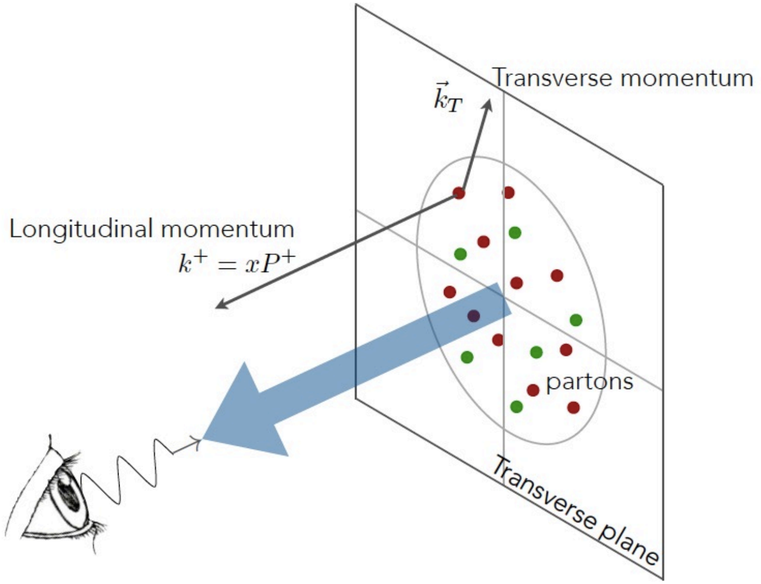
On the other hand, canonical quark OAM can be given by $$\ell_z^q = -\int dx d^2k_\perp \frac{\vec{k}_\perp^2}{M^2} F_{1,4}^q(x,0,\vec{k}_\perp^2,0,0).$$

OAM obtained using these two definitions agree in models with no gluonic degree of freedom

Correlation between quark spin and OAM is given by
$$C_z^q \equiv \int dx d^2k_\perp d^2b_\perp (\vec{b}_\perp \times \vec{k}_\perp)_z \rho_{UL}^q(\vec{b}_\perp, \vec{k}_\perp, x)$$
$$= \int dx d^2k_\perp \frac{\vec{k}_\perp^2}{M^2} G_{1,1}^q(x,0,\vec{k}_\perp^2,0,0),$$

If it is greater than zero, quark spin and OAM are aligned

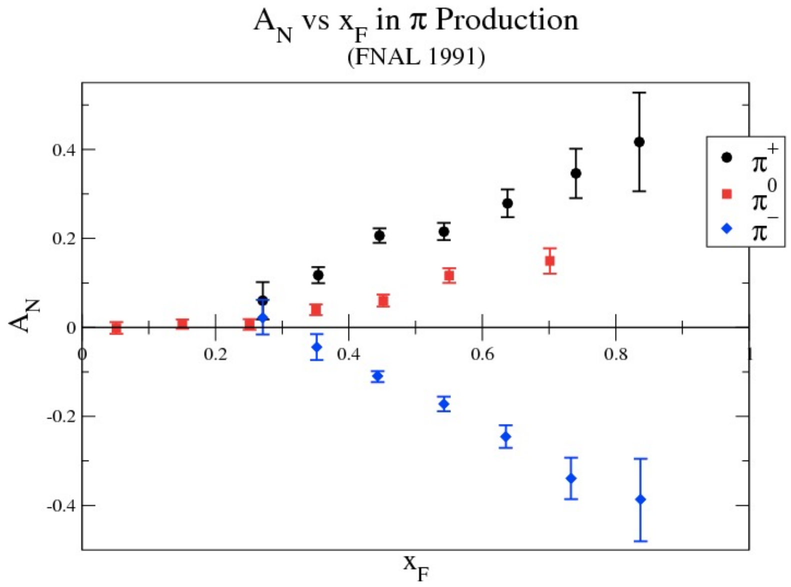
# TRANSVERSE MOMENTUM DEPENDENT PARTON DISTRIBUTIONS (TMDS)




Large (30-40% ) Single transverse spin asymmetries were seen at FermiLab and RHIC experiments

Such large asymmetries cannot be explained in terms of collinear leading twist pdfs : need TMDs, or twist three pdfs

$$A_N = \frac{d\sigma^\uparrow - d\sigma^\downarrow}{d\sigma^\uparrow + d\sigma^\downarrow}$$

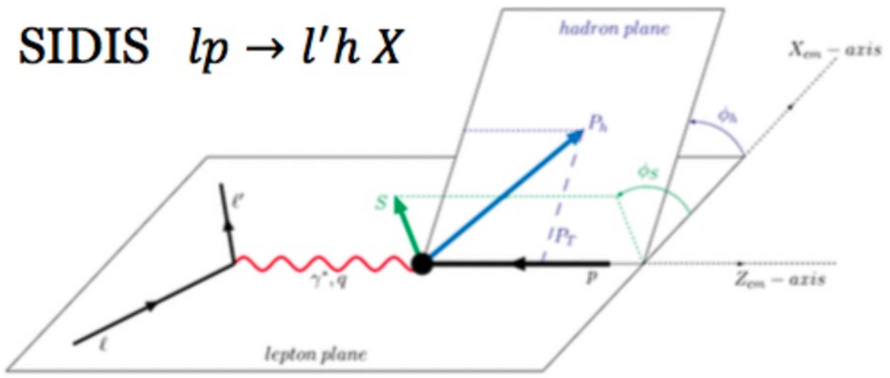


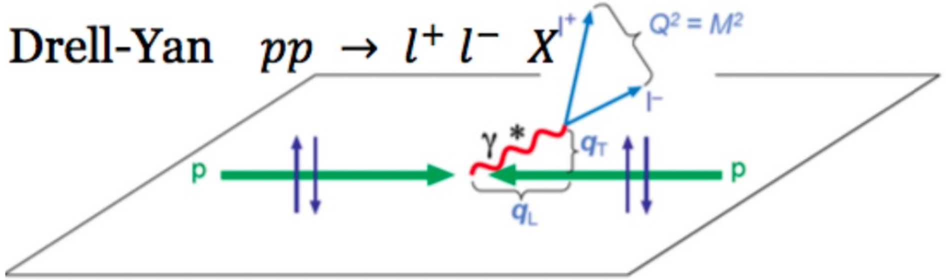

TMDs : functions of x and intrinsic transverse momentum : Gives a 3 D picture of the nucleon in momentum space

Correlations of spin, OAM and $k_T$ : in terms of TMDs

# TRANSVERSE MOMENTUM DEPENDENT PDFS (TMDS)

TMDs play a role in processes where two scales are present $Q^2 >> q_T^2$




For semi-inclusive DIS and Drell-Yan process, TMD factorization is proven to all orders in $\alpha_s$ and leading twist

Collins, Cambridge University Press (2011)
Boussarie et al, TMD handbook 2304.03302

# TRANSVERSE MOMENTUM DEPENDENT PDFS (TMDS)

TMD-PDFs hard scattering TMD-FFs

$$d\sigma^{\ell p \to \ell h X} = \sum_q f_q(x, \boldsymbol{k}_\perp; Q^2) \otimes d\hat{\sigma}^{\ell q \to \ell q}(y, \boldsymbol{k}_\perp; Q^2) \otimes D_q^h(z, \boldsymbol{p}_\perp; Q^2)$$

Fragmentation function for final hadron

TMDs play an important role in single spin and azimuthal asymmetries

Process dependent due to the gauge link or Wilson line in the operator

**Gauge invariant definition of $\Phi$ (not unique)**

$$\Phi^{[\mathcal{U}]} \propto \left\langle P, S \left| \overline{\psi}(0)\, \mathcal{U}^{\mathcal{C}}_{[0,\xi]}\, \psi(\xi) \right| P, S \right\rangle \qquad \mathcal{U}^{\mathcal{C}}_{[0,\xi]} = \mathcal{P}\exp\left(-ig \int_{\mathcal{C}[0,\xi]} ds_\mu A^\mu(s)\right)$$

$\Phi$ : quark correlator, parametrized in terms of TMDs

Gauge link : resummation of initial and/or final state gluon exchanges : process dependent

# QUARK TMDS

Talk by C Pisano

| QUARKS | *unpolarized* | *chiral* | *transverse* |
|---|---|---|---|
| U | $f_1$ | | $h_1^\perp$ |
| L | | $g_{1L}$ | $h_{1L}^\perp$ |
| T | $f_{1T}^\perp$ | $g_{1T}$ | $h_{1T}, h_{1T}^\perp$ |

Angeles-Martinez *et al.*, Acta Phys, Pol. B46 (2015)
Mulders, Rodrigues, PRD 63 (2001)
Meissner, Metz, Goeke, PRD 76 (2007)

There are eight quark TMDs at leading twist

Only three of them survive after transverse momentum integration

Two TMDs, Sivers function and Boer-Mulders function are odd under time reversal

TMDs contribute in different azimuthal angle asymmetries

Quite a lot of advances in extracting the quark TMDs

Also gluon TMDS can be defined.

Pavia 2017, JHEP 06 (2017)
Scimemi, Vladimirov, JHEP 06 (2020)
MAP Collaboration, JHEP (2022)

Bury, Prokudin, Vladimirov, PRL 126 (2021)
Echevarria, Kang, Terry, JHEP 01 (2021)
Bacchetta, Delcarro, Pisano, Radici, CP, PLB 827 (2022)

# Quark TMDs for the nucleon

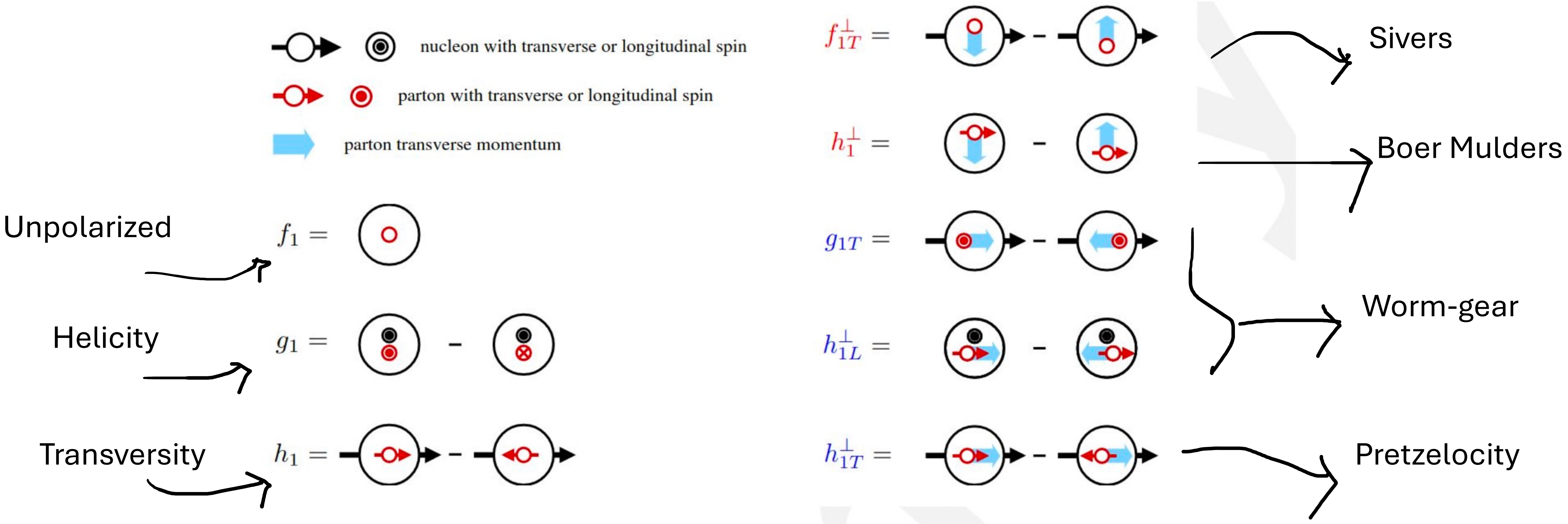



Figure 3.5: Probabilistic interpretation of twist-2 transverse-momentum-dependent distribution functions. To avoid ambiguities, it is necessary to indicate the directions of quark's transverse momemtum, target spin and quark spin, and specify that the proton is moving out of the page, or alternatively the photon is moving into the page.

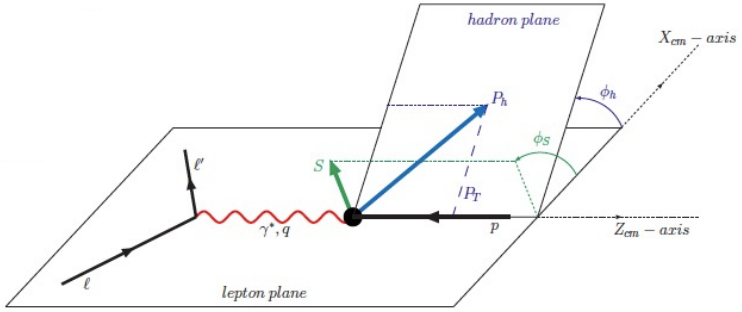
Lecture notes, A. Bacchetta

# SIMPLE EXAMPLE OF PROCESS DEPENDENCE OF TMDS : SIVERS EFFECT

Diff cross section for SIDIS with transversely polarized proton can be written as

$$\frac{d\sigma^{\ell+p(S_T)\to\ell' hX}}{dx_B\, dQ^2\, dz_h\, d^2\boldsymbol{P}_T\, d\phi_S} = \frac{2\,\alpha^2}{Q^4} \times$$

$$\Big\{ \frac{1+(1-y)^2}{2} F_{UU} + (2-y)\sqrt{1-y}\, \cos\phi_h\, F_{UU}^{\cos\phi_h} + (1-y)\, \cos 2\phi_h\, F_{UU}^{\cos 2\phi_h}$$
$$+ \Big[ \frac{1+(1-y)^2}{2} \sin(\phi_h - \phi_S)\, F_{UT}^{\sin(\phi_h-\phi_S)} + (1-y)\, \sin(\phi_h+\phi_S)\, F_{UT}^{\sin(\phi_h+\phi_S)}$$
$$+ \ (1-y)\, \sin(3\phi_h - \phi_S)\, F_{UT}^{\sin(3\phi_h-\phi_S)}$$
$$+ \ (2-y)\sqrt{1-y}\,\Big( \sin\phi_S\, F_{UT}^{\sin\phi_S} + \sin(2\phi_h - \phi_S)\, F_{UT}^{\sin(2\phi_h-\phi_S)} \Big) \Big] \Big\}$$

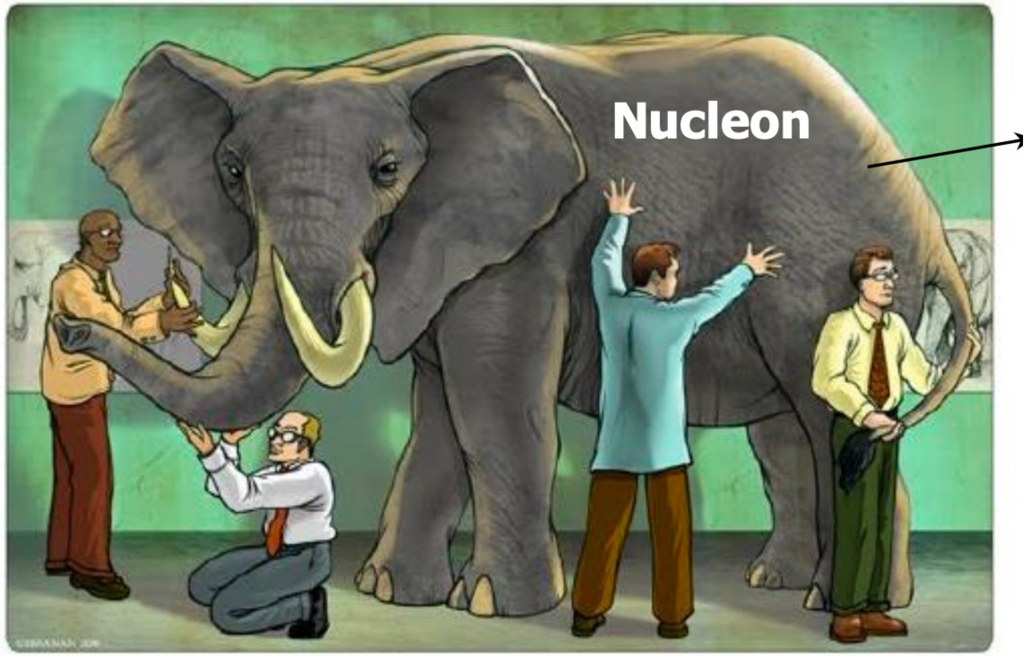



$$F_{UT}^{\sin(\phi-\phi_S)} \sim \sum e_a^2\, f_{1T}^{\perp a} \otimes D_1^a$$

F functions contain different TMDs : each come with a different azimuthal modulation

Sivers Function

# Summary and Outlook




GPDs TMDs FFs PDFs