Magnetogenesis from inflationary perturbation and axion: Its GW signatures

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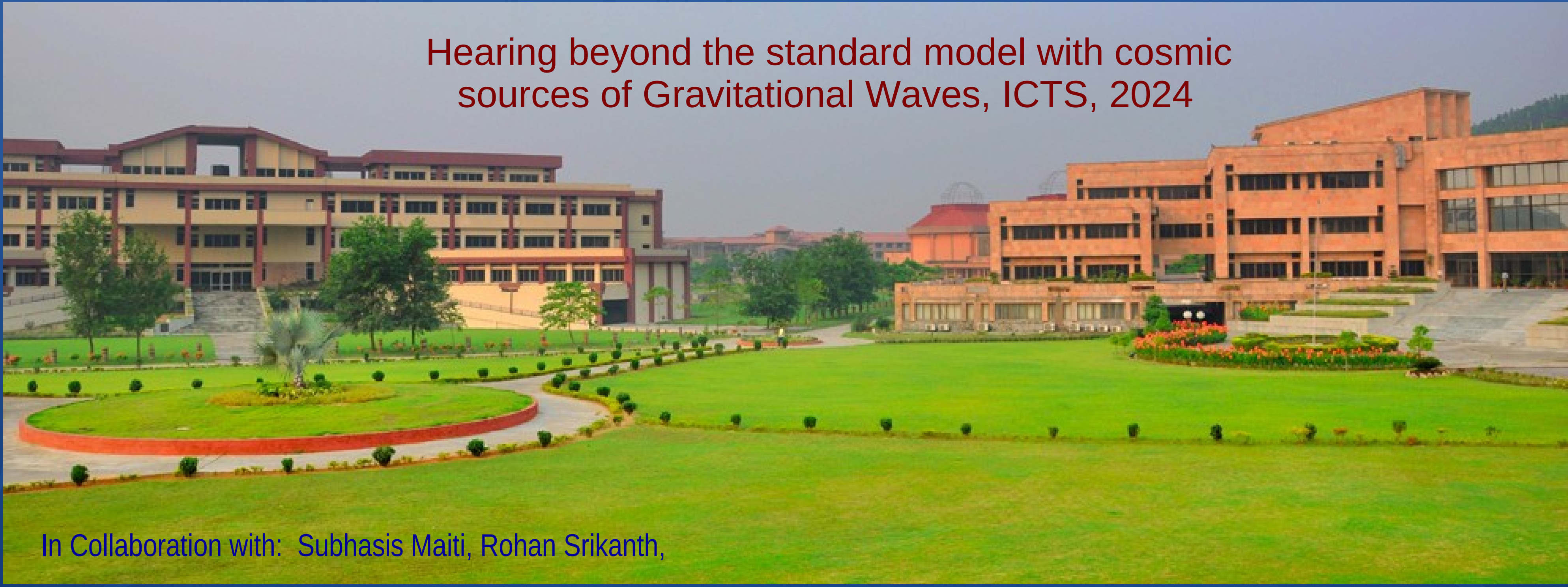

Hearing beyond the standard model with cosmic sources of Gravitational Waves, ICTS, 2024

In Collaboration with: Subhasis Maiti, Rohan Srikanth,

How does our universe become magnetized at all scales?

Introduction

- In observational cosmology we constrain the history of the Universe by different relics:

  i) The best example: CMB of the time of recombination,($\simeq 10^5$ years), and probably also of inflation ($\sim 10^{-35}$ sec)

  ii) Light elements at the time of BBN (100 sec)

  iii) Dark matter at the time of decoupling depending on its nature

  iv) May be Dark energy (do not know)

  v) Gravitational waves at various epochs

  vi) Large scale Magnetic fields

  v) ...

Introduction

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  v) Gravitational waves at various epochs

  vi) Large scale Magnetic fields

  v) ...

Magnetized universe

Magnetic fields have long established their ubiquitous presence in the universe.

1. Contributing as a major component in the interstellar medium, to the total pressure, affecting the gas dynamics, the distribution of cosmic rays and star formation.

3. Most galaxies, including the Milky Way, carry coherent large-scale magnetic fields of μG order strength. (1-10 Kpc)

4. Analogous fields have also been detected in galaxy clusters and in young, high-redshift protogalactic structures. (10-100 Kpc)

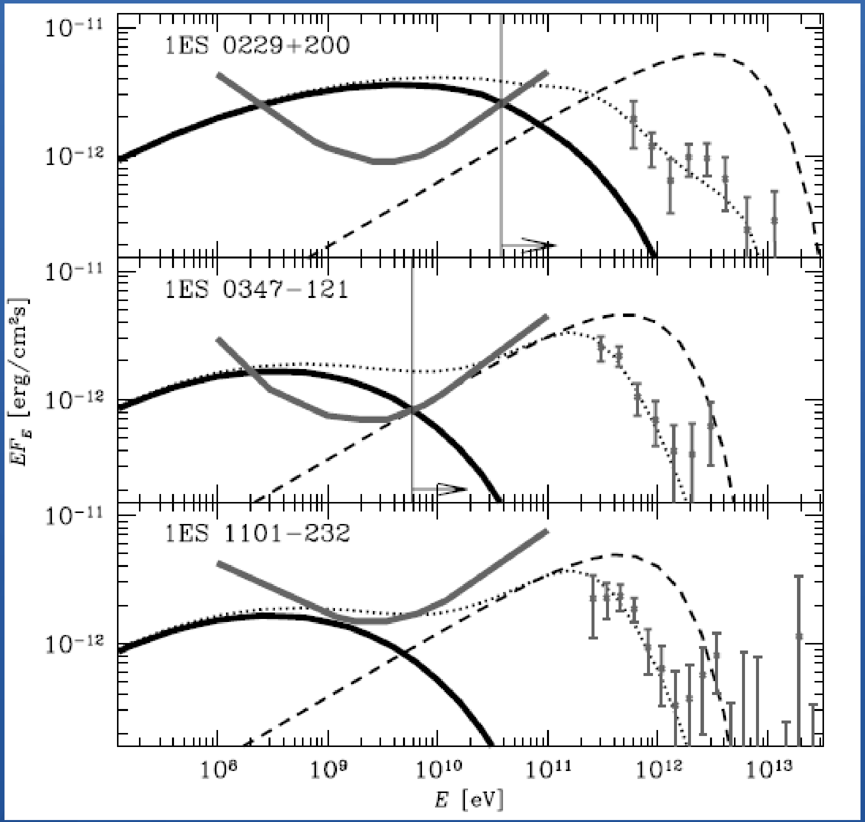
5. The cosmic microwave background (CMB) constrain the magnetic fields at the scale of 1 Mpc to be less than 10^{-9} G

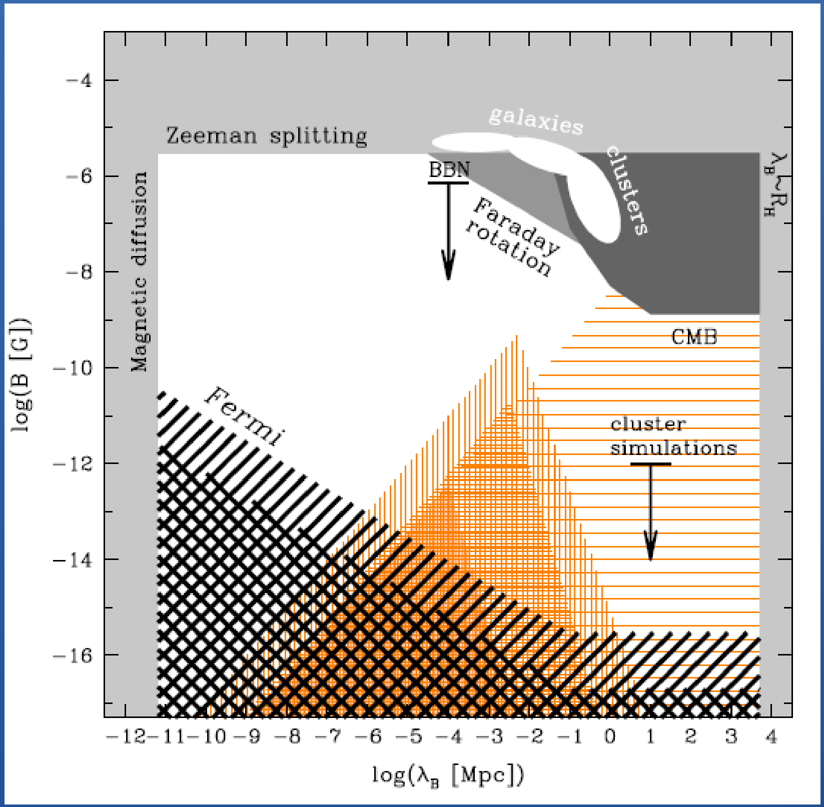
6. In short, the deeper we look for them in the universe, the more widespread we find them to be.

Hinting more towards cosmological rather than astrophysical origin

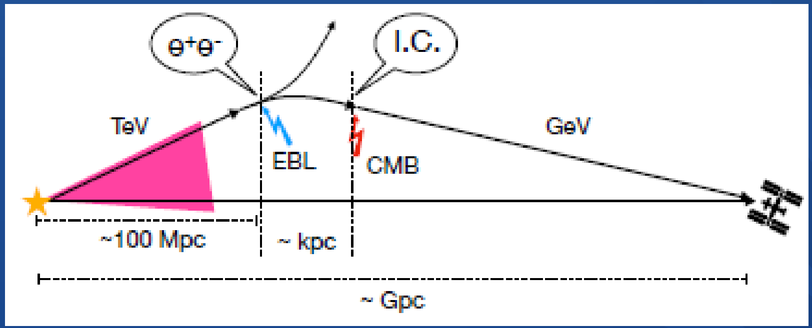
R. Beck, Space Sci. Rev. 99, 243 (2001);
T. E. Clarke, P. P. Kronberg and H. Böhringer, Astrophys. J. 547, L111 (2001);
Planck Collaboration (P. A. R. Ade et al.), Astron. Astrophys. 594, A19 (2016)

Magnetized universe





A. Neronov and I. Vovk, Science 328, 73 (2010).



Tanmay Vachaspati 2021 Rep. Prog. Phys. 84 074901

Magnetized universe
Mechanisms

1. Astrophysical origin

The galactic dynamo paradigm

- nonlinear dynamo action is responsible for amplifying and sustaining the galactic magnetic fields via converting kinetic energy into magnetic energy
- Growth continues until it reaches saturation, which typically occurs when B $\sim 10^{-6}$ G
- The galactic dynamo needs the presence of seed not less than approximately 10^{-22} G, with coherenc length > 100 Kpc :

  Biermann-battery mechanism turns out to be insufficient

A. Brandenburg, K. Subramanian, Phys. Rep. 417 (2005) 1. arXiv:astro-ph/0405052.

Magnetized universe

Mechanisms

2. Primordial origin

- Inflationary magnetogenesis

  Produces magnetic field at all length scales

- Phase transition (Electroweak, QCD)

  Due to its causal process scales involves are within Hubble.

  For exapmle $t_{EW}\,(T_{EW}/T_0) \sim 10^{15}$ cm << 1 Kpc

- Inhomogeneous universe

Magnetized universe

Mechanisms

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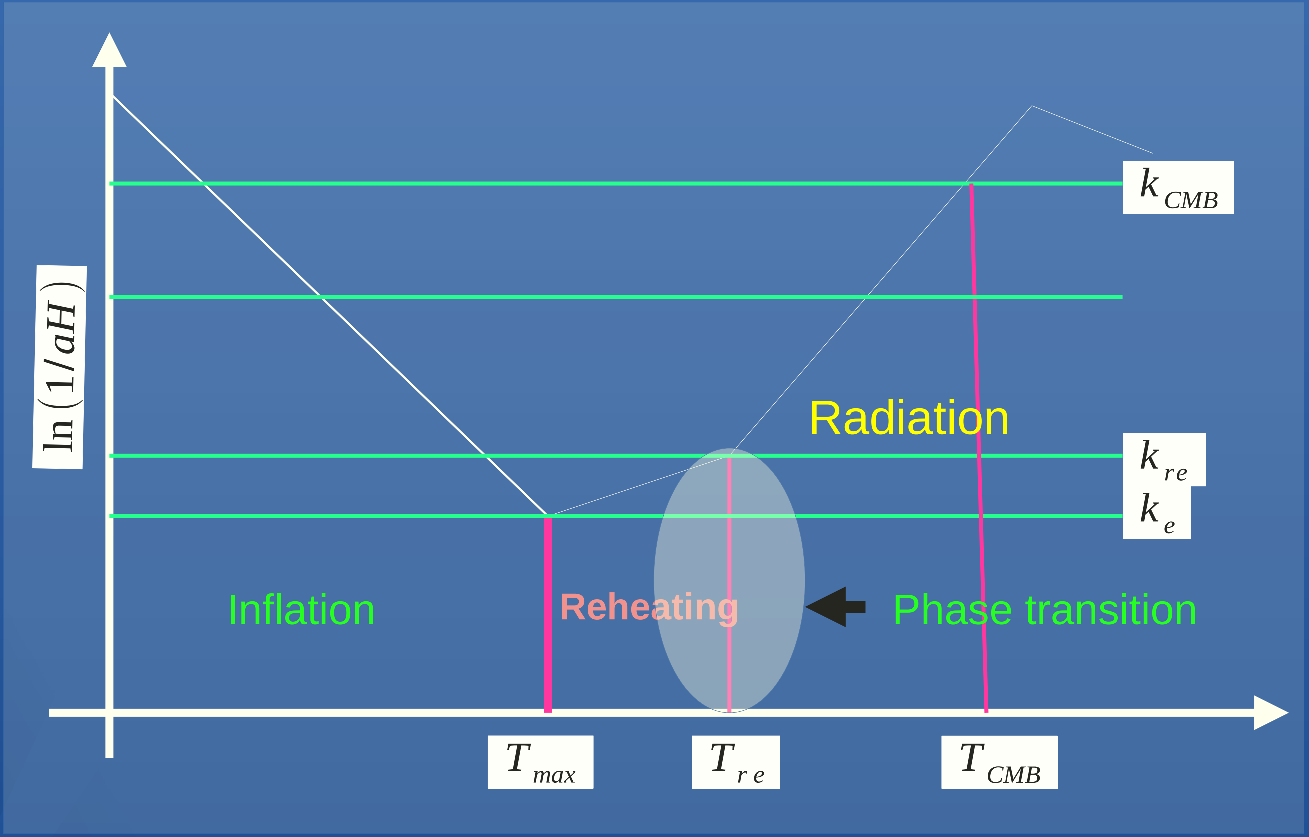
Magnetized universe
Primordial origin

Evolution of scales



- Inflationary magnetogenesis produces magnetic field at all length scales
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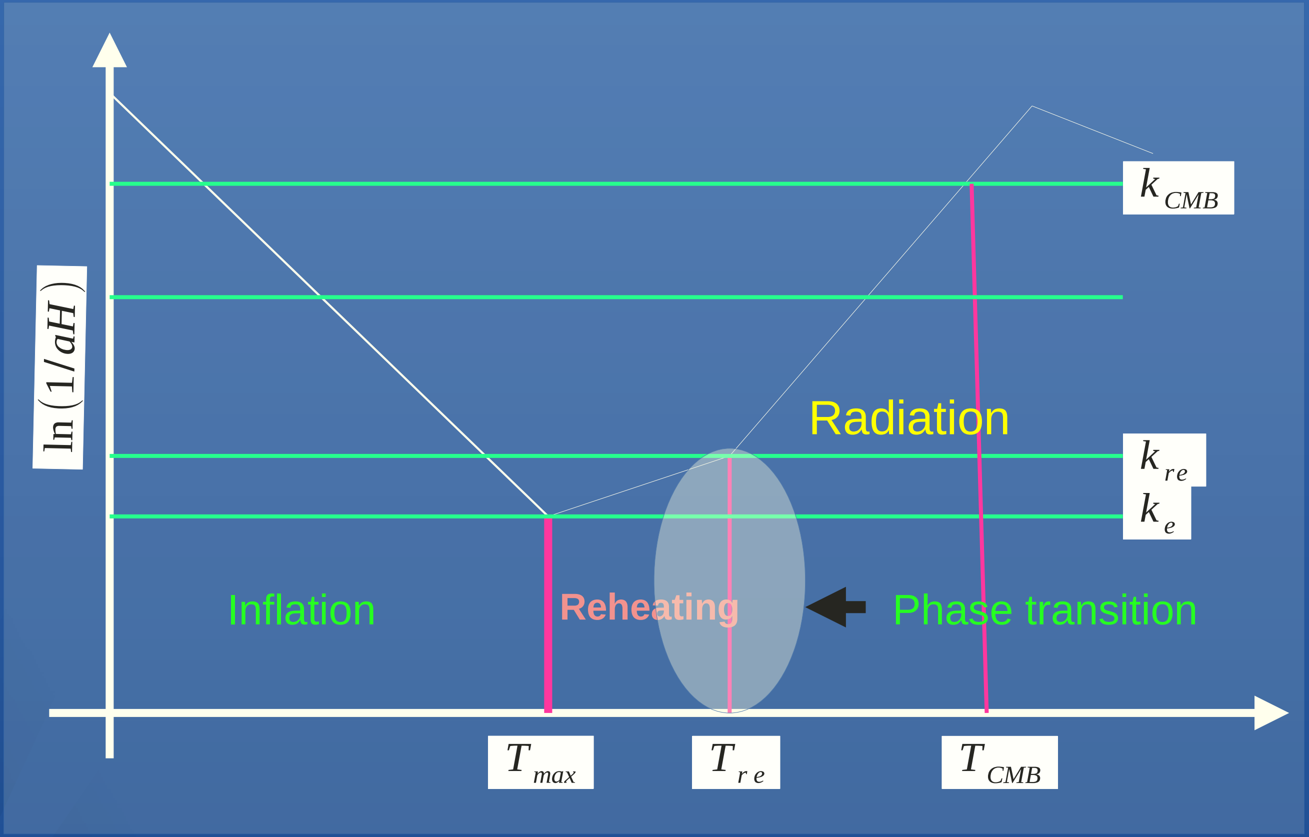
Behavior of the comoving wave number k (horizontal lines) and the comoving Hubble radius $d_H/a = 1/(a H)$ white lines across different epochs

Magnetized universe
Primordial origin

Evolution of scales



- Inflationary magnetogenesis produces magnetic field at all length scales
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  Due to its causal process scales involves are within Hubble.

  For exapmle $t_{EW}\,(T_{EW}/T_0) \sim 10^{15}$ cm << 1 Kpc

GOAL

1. Produce magnetic field at all scales → Inflationary magnetogenesis
2. $10^{-17} – 10^{-19}$ G around 1 Mpc scale

Inflationary Magnetogenesis
Widely studied approach: Example-I

Conventional Ratra model

$$S = \int d^4x \sqrt{-g} \left[R + \frac{1}{2}(\partial\phi)^2 - V(\phi) + \frac{I_1(\phi)}{4} F^2 \right]$$

Asuming the following form of the coupling function

$$I_1(\phi) \sim a^{\gamma} \quad ; \quad ds^2 = a^2(-d\eta^2 + dx^2 + dy^2 + dz^2)$$

Renormalized vector potential satisfies

$$\ddot{\mathcal{A}} + \left(k^2 - \frac{\ddot{I}_1}{I_1} \right) \mathcal{A} = 0$$

Additional scalar/pseudo scalar necessary

Note for I_1=1, conformal case

= 0

Thought of as time dependent mass term, giving rise to amplification at large scale

$$A_k^{\lambda}(\eta \to \infty) \sum_{\lambda'} \sum_q \left(\alpha_{\mathbf{k}}^{\lambda\lambda'}(q,\eta) \frac{\epsilon_{\mathbf{q}}^{\lambda}}{\sqrt{2q}} e^{i(\mathbf{q}\cdot\mathbf{x} - q\eta)} + \beta_{\mathbf{k}}^{\lambda\lambda'}(\mathbf{q},\eta) \frac{\epsilon_{\mathbf{q}}^{\lambda *}}{\sqrt{2q}} e^{-i(\mathbf{q}\cdot\mathbf{x} - q\eta)} \right)$$

Turner MS, Widrow LM , Phys Rev D 37, 2743 (1998), Ratra B, Astrophys J 391:L1 (1992), Subramanian K, Astron Nachr 331:110, (2010), Durrer, R., Neronov, A, Astron Astrophys Rev 21, 62 (2013) (references there in)

Inflationary Magnetogenesis

Widely studied approach: Example-I

Conventional Ratra model

Asumming the following form of the coupling function

$$\rho_B = \left(\frac{k}{a}\right)^4 \sum_{\lambda\lambda'} \int_{k*}^{k_e} |\beta_k^{\lambda\lambda'}|^2$$

Parity violating

=0

Asumming statistically isotropic EM field, the magentic power spectrum:

$$\langle B_i(\eta,\mathbf{k}) B_j^*(\eta,\mathbf{q})\rangle = \frac{(2\pi)^3}{2}\delta(\mathbf{k}-\mathbf{q})\left\{(\delta_{ij}-\hat{k}_i\hat{k}_j)P_S(k,\eta) - i\epsilon_{ijn}\hat{k}_n P_A(k,\eta)\right\}$$

$$k^3 P_S \propto B_+^2 + B_-^2 \quad ; \quad k^3 P_A \propto B_+^2 - B_-^2$$

For Ratra model $P_A = 0$,
The magentic power spectrum:
$P_S = P_B$ (energy density)

$$k^3 P_B \propto k^{n_B} \quad \text{with} \quad n_B = \begin{cases} 4-2\gamma & \text{if } \gamma \geq -1/2 \\ 6+2\gamma & \text{if } \gamma \leq -1/2 \end{cases}$$

Similalry, the electric power spectrum: P_E (energy density)

$$k^3 P_E \propto k^{n_E} \quad \text{with} \quad n_E = \begin{cases} 6-2\gamma & \text{if } \gamma \geq 1/2 \\ 4+2\gamma & \text{if } \gamma \leq 1/2 \end{cases}$$

Turner MS, Widrow LM , Phys Rev D 37, 2743 (1998), Ratra B, Astrophys J 391:L1 (1992), Subramanian K, Astron Nachr 331:110, (2010), Durrer, R., Neronov, A, Astron Astrophys Rev 21, 62 (2013) (references there in)

Take aways

Electric and magnetic power sepctrum

Conventional Ratra model

$$k^3 P_B \propto k^{n_B} \quad \text{with} \quad n_B = \begin{cases} 4-2\gamma & \text{if } \gamma \geq -1/2 \\ 6+2\gamma & \text{if } \gamma \leq -1/2 \end{cases}$$

$$k^3 P_E \propto k^{n_E} \quad \text{with} \quad n_E = \begin{cases} 6-2\gamma & \text{if } \gamma \geq 1/2 \\ 4+2\gamma & \text{if } \gamma \leq 1/2 \end{cases}$$

$I_1(\phi) \sim a^{\gamma}, 0<\gamma<2$ → Can produce 10^{-12} – 10^{-17} G at 1 Mpc

Strong blue tilt P_E → Backreact inflation Need fine tuning of parameters

$I_1(\phi) \sim a^{\gamma}, \gamma>2$ → IR Divergent P_B

$I_1(\phi) \sim a^{\gamma}, \gamma<0$ → Strong coupling problem
Decreasing I_1 , increaing effectice electromagnetic coupling: Non-perturbative → $e \rightarrow e/I_1(\phi)$

Turner MS, Widrow LM , Phys Rev D 37, 2743 (1998), Ratra B, Astrophys J 391:L1 (1992), Subramanian K, Astron Nachr 331:110, (2010), Durrer, R., Neronov, A, Astron Astrophys Rev 21, 62 (2013) (references there in)

Inflationary Magnetogenesis
Widely studied approach: Example-II

Conventional axion-like model

$$S = \int d^4x \sqrt{-g} \left[R + \frac{1}{2}(\partial\phi)^2 - V(\phi) + \frac{1}{4}F^2 + \frac{I_2(\phi)}{4} F \cdot \tilde{F} \right]$$

evade strong coupling problem

Asumming statistically isotropic EM field, the magentic power spectrum:

$$\langle B_i(\eta, \mathbf{k}) B_j^*(\eta, \mathbf{q}) \rangle = \frac{(2\pi)^3}{2} \delta(\mathbf{k} - \mathbf{q}) \left\{ (\delta_{ij} - \hat{k}_i \hat{k}_j) P_S(k, \eta) - i\epsilon_{ijn} \hat{k}_n P_A(k, \eta) \right\}$$

$$k^3 P_S \propto B_+^2 + B_-^2 \quad ; \quad k^3 P_A \propto B_+^2 - B_-^2$$

Non-zero helicity

Electric and magnetic spectrum $k^3 P_E \approx k^3 P_B \propto k^{4-2\gamma}, \gamma > 0$

The model can generate magnetic field of required strength!

Severely constrained by the large non-gaussianity

→ Of course we can construct more complecated models

L Sorbo, JCAP 06 (2011) 003; R Durrer etl, JCAP 03 (2011) 037, N. Barnaby et, JCAP 04 (2011) 009
Some of our paper in this direction: R. Haque etal, Phys.Rev.D 103 (2021) 10, 103540 ; K. Bamba, Phys.Dark Univ. 36 (2022) 101025; DM, Sourav Pal, JCAP 05 (2021) 045 ; Sourval Pal, etal Phys.Rev.D 109 (2024) 8, 083507; S Maiti etal, 2401.01864

Coming back to our original GOAL

1. Produce magnetic field at all scales : not difficult

2. $10^{-17} – 10^{-19}$ G around 1 Mpc scale : Very difficult it seems

Rather than amplifying during inflation we can consider late time enhancement mechanism:
Within minimalistic scenario

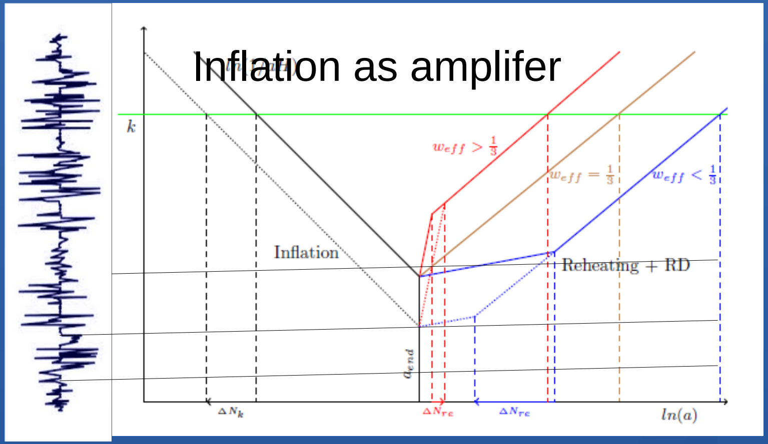
We can indeed produce magnetic field at all scales through inhomogeneous perturbation (No model)

Inflationary Magnetogenesis
Inhomogenous background

- Less studied approach: Conformally non-flat background

$$ds^2 = a^2\left(-(1+2\Phi)\,d\eta^2 + \delta_{ij}(1-2\Phi)\,dx^i dx^j\right)$$

- Canonical EM field FF



$$A_{\mathbf{k}}^{\lambda\prime\prime} + A_{\mathbf{k}}^{\lambda} = \frac{1}{k^2}\mathcal{J}_{\mathbf{k}}^{\lambda}(x),$$

$$\mathcal{J}_{\mathbf{k}}^{\lambda} = -\sqrt{2q}\left[\left(i\Phi'(\mathbf{k}+\mathbf{q},\eta) + \frac{q^2 - \mathbf{k}\cdot\mathbf{q}}{q}\Phi(\mathbf{k}+\mathbf{q},\eta)\right)\epsilon_{\mathbf{q}}^{\lambda} e^{-iq\eta} + (\epsilon_{\mathbf{q}}^{\lambda}\cdot\mathbf{k})\Phi(\mathbf{k}+\mathbf{q},\eta)\frac{q_i}{q}e^{-iq\eta}\right].$$

General Solution: $$A_{\mathbf{k}}^{\lambda} = A_{\mathbf{k}}^{\text{vac}} + \frac{1}{k^2}\int dx_1 G_k(x, x_1)\mathcal{J}_{\mathbf{k}}^{\lambda}(x_1)$$

A. L Maroto, Phys.Rev.D 64 (2001) 083006

Inflationary Magnetogenesis

Inhomogenous background

- Less studied approach: Connformally non-flat background

$$ds^2 = a^2\left(-(1+2\Phi)\,d\eta^2 + (\delta_{ij}(1-2\Phi)+h_{ij})\,dx^i\,dx^j\right)$$

- Canonical EM field FF

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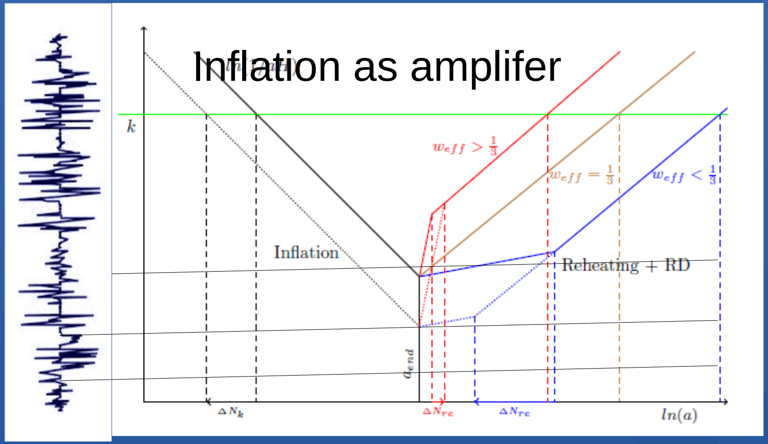


Because of the mode-mode coupling, the aymptotic solutions can be expressed as (unlike conformally flat background)

$$A_k^{\lambda}(\eta\to\infty)\sum_{\lambda'}\sum_{q}\left(\alpha_{\mathbf{k}}^{\lambda\lambda'}(q,\eta)\frac{\epsilon_{\mathbf{q}}^{\lambda}}{\sqrt{2q}}e^{i(\mathbf{q}\cdot\mathbf{x}-q\eta)}+\beta_{\mathbf{k}}^{\lambda\lambda'}(\mathbf{q},\eta)\frac{\epsilon_{\mathbf{q}}^{\lambda *}}{\sqrt{2q}}e^{-i(\mathbf{q}\cdot\mathbf{x}-q\eta)}\right)$$

Quantify the EM production

A. L Maroto, Phys.Rev.D 64 (2001) 083006

Inflationary Magnetogenesis

Single slow roll phase: Standard inflation

- Energy density of the preduced EM field

$$\rho_B=\left(\frac{k}{a}\right)^4 \sum_{\lambda\lambda'} \int_{k*}^{k_e} |\beta_k^{\lambda\lambda'}|^2$$

$$\beta_{\mathbf{k}}^{\lambda\lambda'}(q,\eta) = -\frac{i}{\sqrt{2q}} \int_{\eta_i}^{\eta} \epsilon_{\mathbf{q}}^{\lambda *} \mathcal{J}_{\mathbf{k}}(\mathbf{k}+\mathbf{q},\eta_1) e^{-ik\eta_1} d\eta_1$$

$$= \frac{3\pi^3}{(2\pi)^{3/2}} \frac{16\pi^2}{9} \left(\frac{2+\beta}{3+2\beta}\right)^2 \left[\frac{\mathcal{P}_{\mathcal{R}}(k)}{4}\left(1-\frac{k_*}{k}\right)^4 + \int_1^{k_e/k} du \mathcal{P}_{\mathcal{R}}(uk)\right]$$

$$\mathcal{P}_{\Phi}(k) = \left(\frac{2+\beta}{3+2\beta}\right)^2 \mathcal{P}_{\mathcal{R}}(k)$$

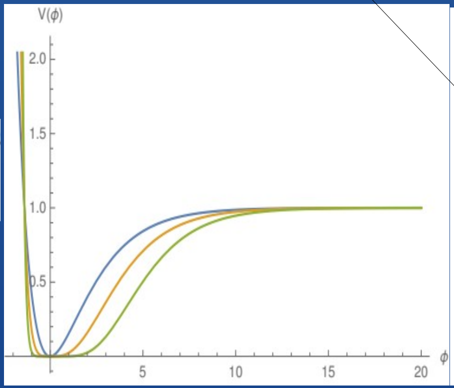
Planck 2018@arXiv :1807.0621

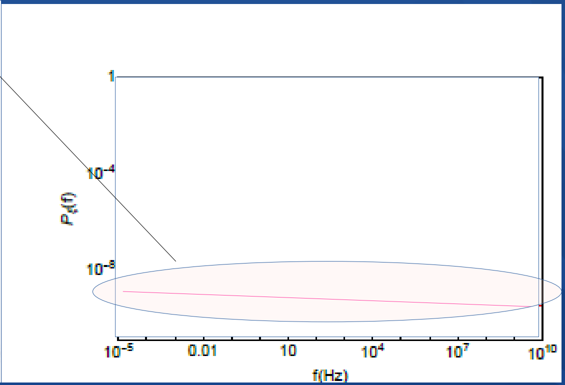
Inflationary Curvature Power spectrum ~:

$$\mathcal{P}_{\mathcal{R}}(k) = A_s \left(\frac{k}{k_*}\right)^{n_s - 1}$$

$n_s = 0.9670 \pm 0.007$
$A_s \sim 2.1 \times 10^{-9}$
$k_* \sim 0.05\ \mathrm{Mpc}^{-1}$

$$V(\phi) = \Lambda^4 \left[1 - e^{-\sqrt{\frac{2}{3\alpha}}\phi/M_p}\right]^{2n}$$





A. L Maroto, Phys.Rev.D 64 (2001) 083006

Inflationary Magnetogenesis

Single slow roll phase: Standard inflation

- Magnetic power spectrum

$$\mathcal{P}_{\rm B}(k,\eta) = \frac{d\rho_B(k,\eta)}{d\ln k} \simeq \left(\frac{k}{a(\eta)}\right)^4 \frac{8\pi^2}{9}\left(\frac{2+\beta}{3+2\beta}\right)^2\left(\frac{k_{\rm e}}{k}\right)\left[\frac{A_s}{n_s}+\right.$$

Present day magnetic field strength:

$$B_0 \sim \sqrt{A_s k_e}\, k^{3/2}$$

~ 10^{-46} G at 1 Mpc

Blue tilted

Can PBH help?

Planck 2018@arXiv :1807.0621

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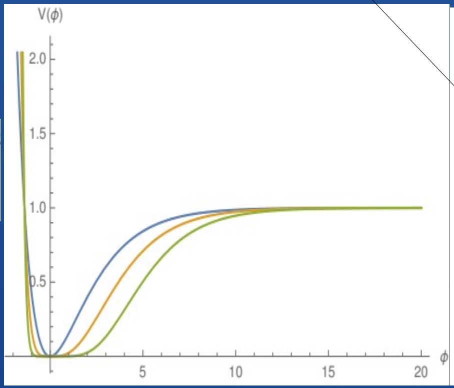
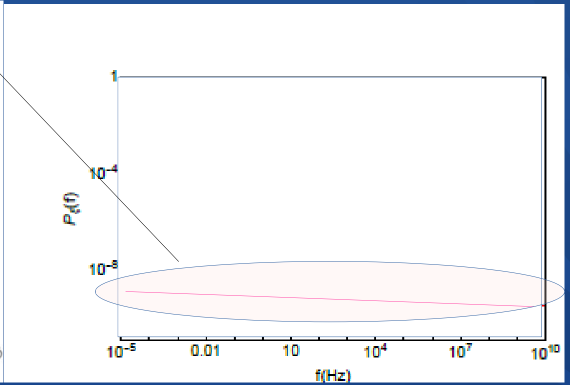


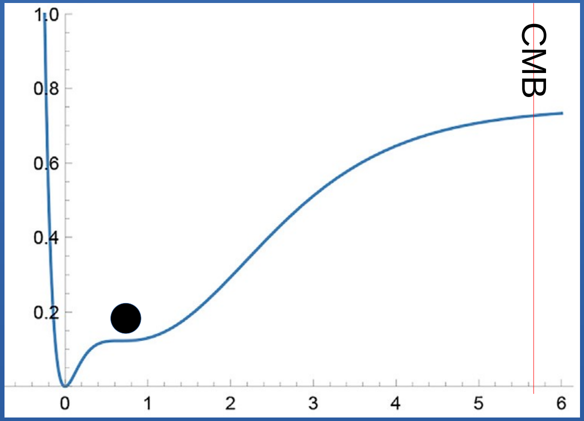


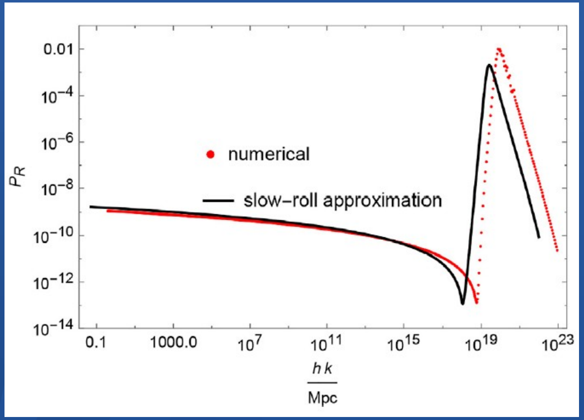
A. L Maroto, Phys.Rev.D 64 (2001) 083006

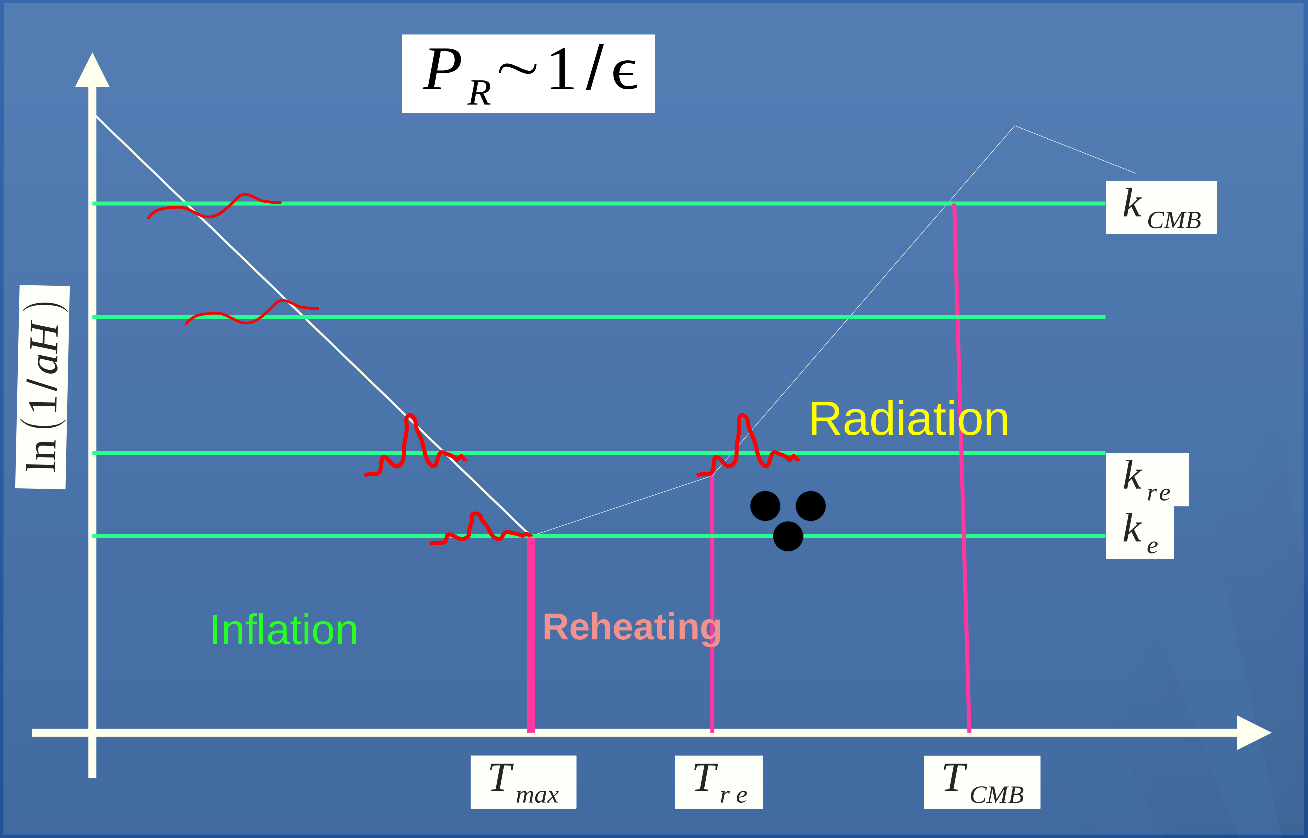
Magnetized universe:

Double slow roll phase: inflation & PBH

Daniel Frolovsk etal, Front. Phys., 04 October 2022







Behavior of the comoving wave number k (horizontal lines) and the comoving Hubble radius $d_H/a = 1/(a H)$ white lines across different epochs

Inflationary Magnetogenesis

PBH Power spectrum

- Energy density of the preduced EM field at

$$\rho_B = \left(\frac{k}{a}\right)^4 \sum_{\lambda\lambda'} \int_{k*}^{k_e} |\beta_k^{\lambda\lambda'}|^2$$

$$\beta_{\mathbf{k}}^{\lambda\lambda'}(q,\eta) = -\frac{i}{\sqrt{2q}} \int_{\eta_i}^{\eta} \epsilon_{\mathbf{q}}^{\lambda *} \mathcal{J}_{\mathbf{k}}(\mathbf{k}+\mathbf{q},\eta_1) e^{-ik\eta_1} d\eta_1$$

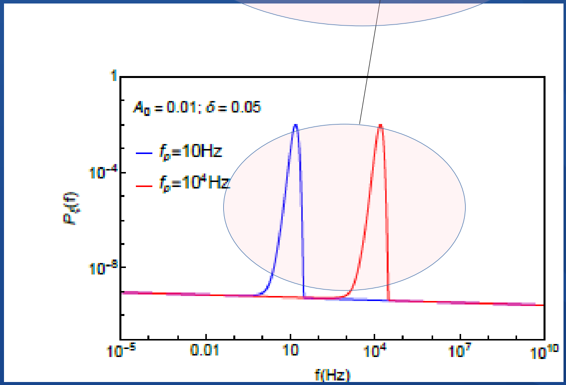
Total number density~:

$$= \frac{3\pi^3}{(2\pi)^{3/2}} \frac{16\pi^2}{9} \left(\frac{2+\beta}{3+2\beta}\right)^2 \left[\frac{\mathcal{P}_{\mathcal{R}}(k)}{4}\left(1-\frac{k_*}{k}\right)^4 + \int_1^{k_e/k} du \mathcal{P}_{\mathcal{R}}(uk)\right]$$

Inflationary Curvature Power spectrum ~:

$$\mathcal{P}_{\mathcal{R}}(k) = A_s \left(\frac{k}{k_*}\right)^{n_s - 1} + A_0 \,\mathrm{Exp}\left[-\frac{(k-k_p)^2}{\delta k_p^2}\right]$$

Two sample PBH curvature power spectrums for two different peak frequencies k_p



Inflationary Magnetogenesis

PBH Power spectrum

Magnetic power spectrum

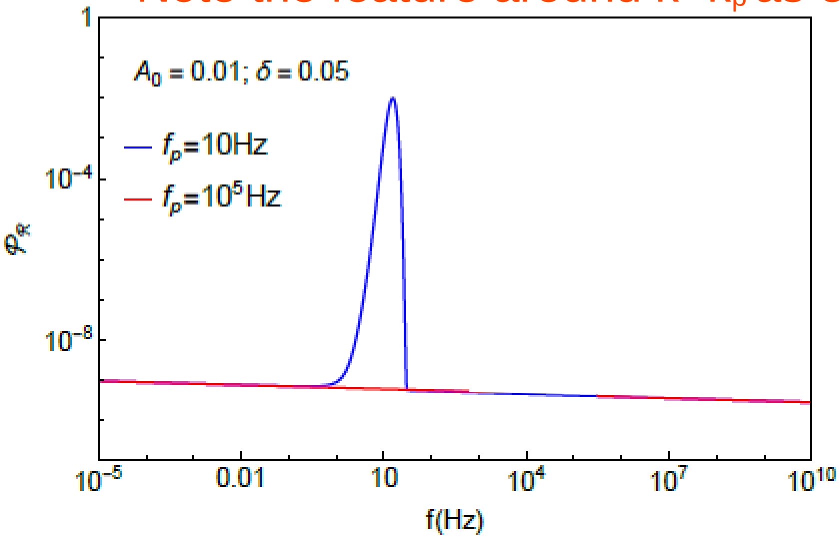
$$\mathcal{P}_{\mathrm{B}}(k,\eta) = \frac{d\rho_B(k,\eta)}{d\ln k} \simeq \left(\frac{k}{a(\eta)}\right)^4 \frac{8\pi^2}{9}\left(\frac{2+\beta}{3+2\beta}\right)^2 \left(\frac{k_{\mathrm{e}}}{k}\right)\left[\frac{A_s}{n_s} + \frac{1}{2}\sqrt{\pi\delta}\,\frac{A_0 k_p}{k_{\mathrm{e}}}\left(1+\mathrm{Erf}\left[\frac{k_p-k}{k_p\sqrt{\delta}}\right]\right)\right]$$

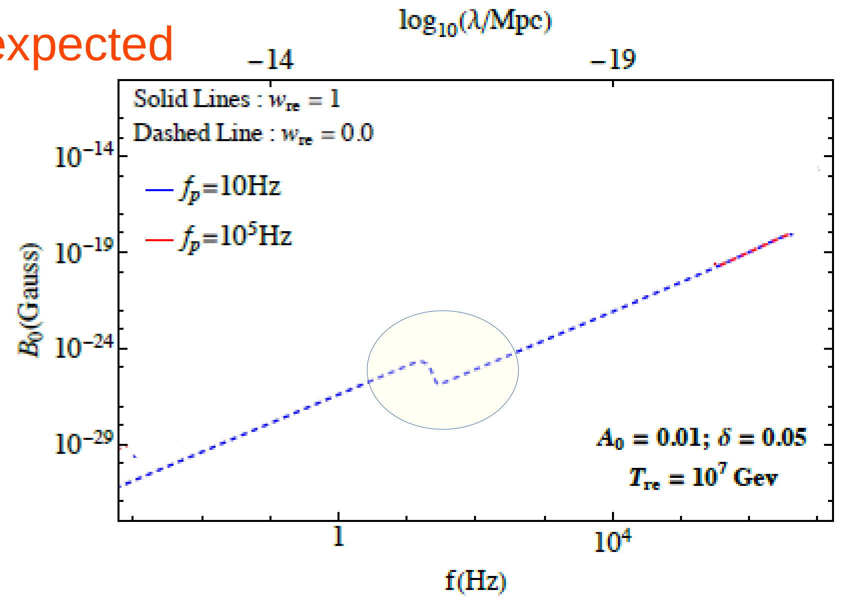
Present day magnetic field strength:

$B_0 \sim \sqrt{A_s k_e}\, k^{3/2}$ for $k > k_p$

$B_0 \sim \sqrt{A_0 k_p}\, k^{3/2}$ for $k < k_p$

Note the feature around $k=k_p$ as expected





Inflationary Magnetogenesis

PBH Power spectrum

Magnetic power spectrum

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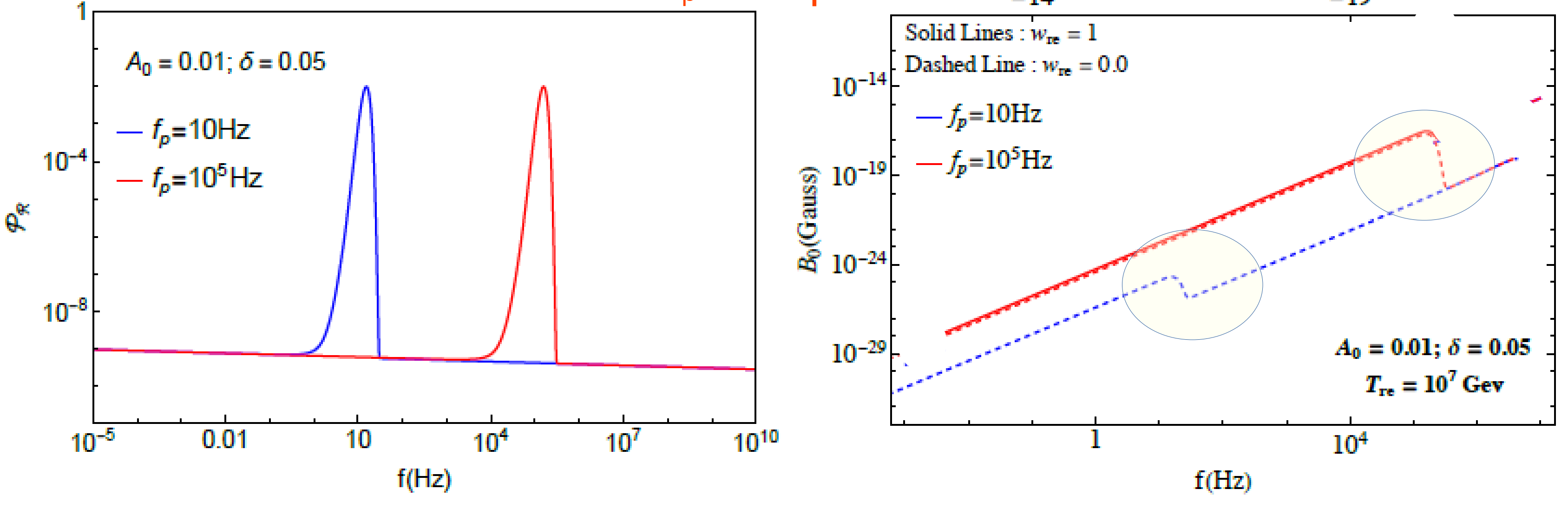

Inflationary Magnetogenesis
PBH Power spectrum

Magnetic power spectrum

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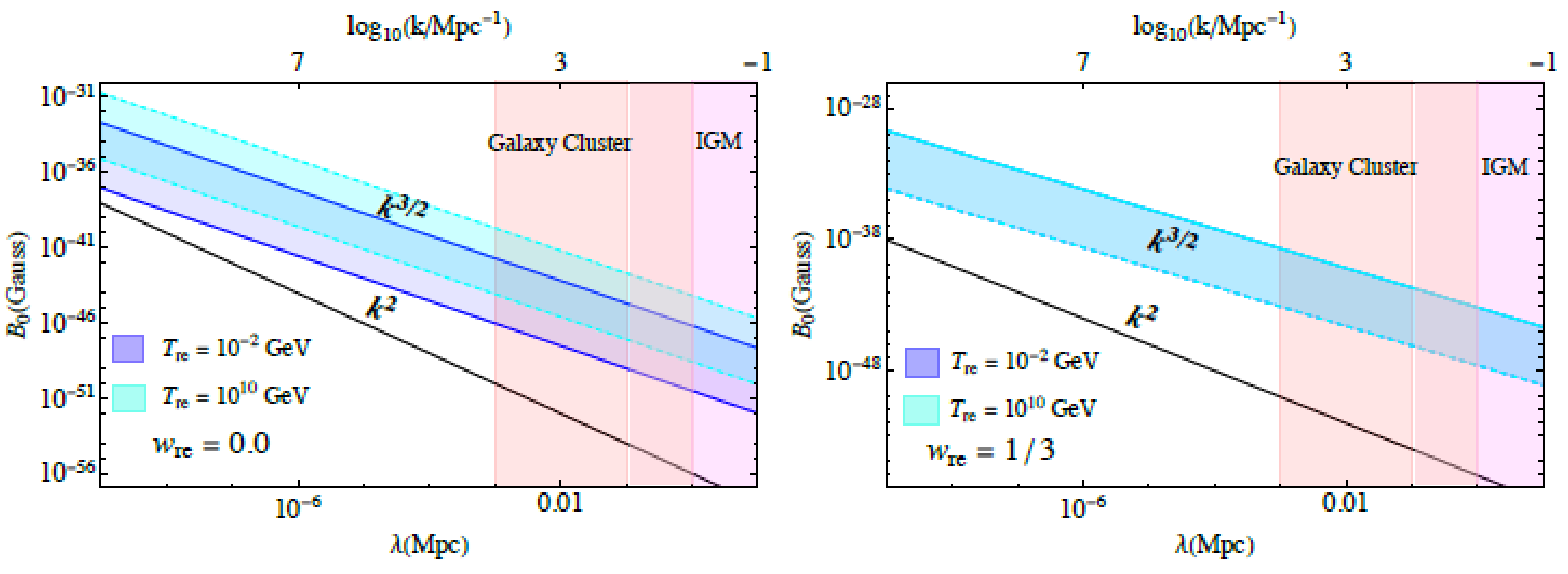
Present day magnetic field strength:

$B_0 \sim \sqrt{A_s k_e}\, k^{3/2}$ ~ 10^{-46} G at 1 Mpc

$B_0 \sim \sqrt{A_0 k_p}\, k^{3/2}$ ~ 10^{-39} G at 1 Mpc



Inflationary Magnetogensis

Important obsevations so far

- Inflationary perturbation generates magnetic field from quantum vacuum
- Without breaking explicit conformal invariance universe can be magnetized at all scales: PBH spectrum can generate ~10^{-39} G at 1 Mpc
- Distinct feature appears in P_B which can give rise to interesting GW signal in addition to well known scalar induced GW

Late time magnetogensis

10^{-39} G at 1 Mpc: too small@inflationary

Can we enhance B-field at late time?

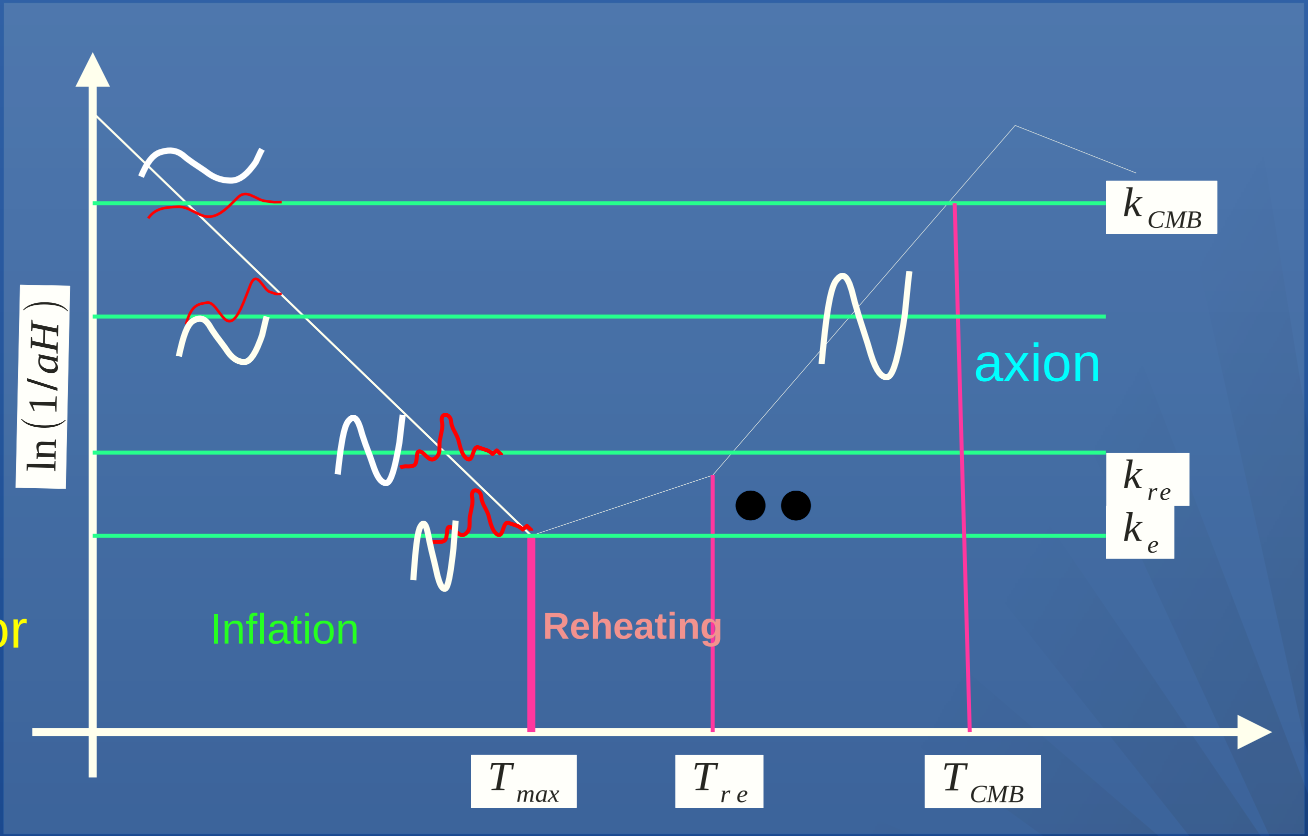
Late time Magnetogensis

~ 10^{-39} G at 1 Mpc: too small

Can we enhance B-field at late time?

YES:
Axion-Photon coupling

$$L \sim \chi F \tilde{F}$$



Axion is assumed as spectator throughout and frozen with constant value, and start to oscillate at late time

Classical Axion background as spectator

Tachyonic growth of EM field

$$\mathcal{L} = -\frac{1}{2}\partial_\mu \chi \partial^\mu \chi - V(\chi) - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{\alpha}{4f_a}\chi F_{\mu\nu}\tilde{F}^{\mu\nu}$$

For QCD axion

$$m_a \simeq 5.7 \times 10^{-6}\ \text{eV}\left(\frac{f_a}{10^{12}\ \text{GeV}}\right)$$

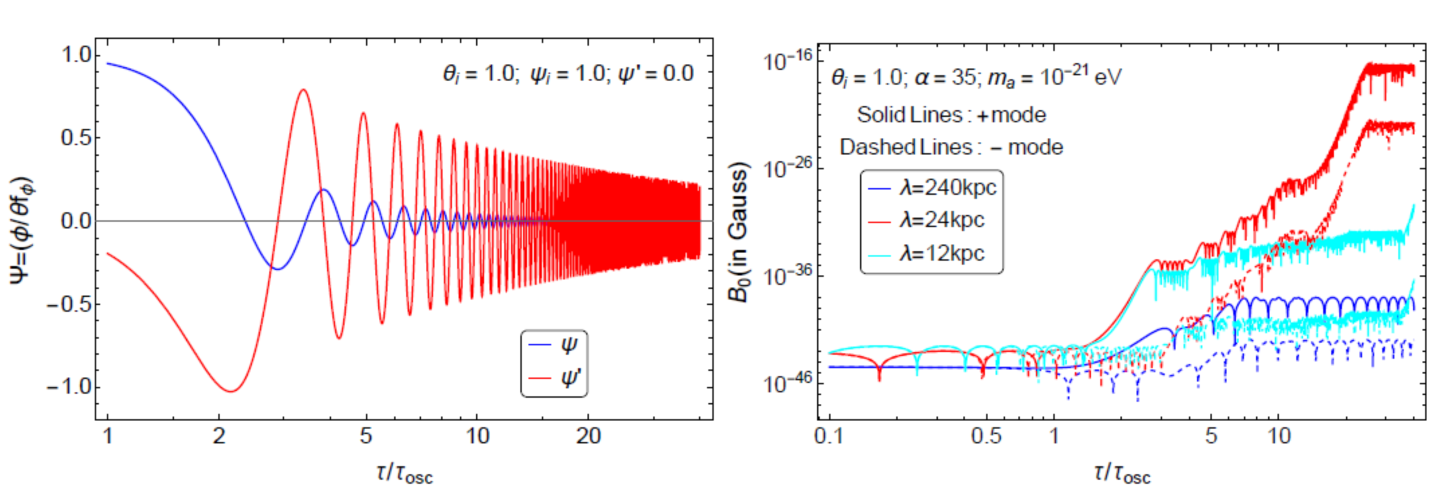
$$V(\chi) = (m_a f_a)^2\left[1 - \cos\left(\frac{\chi}{f_a}\right)\right]$$

$$\chi'' + 2\mathcal{H}\chi' + a^2 V_{,\chi} = \frac{a^2\alpha}{f_a}\langle \mathbf{E}\cdot\mathbf{B}\rangle$$

$$A''_\lambda(k,\tau) + \left(k^2 - \lambda\frac{\alpha k \chi'}{f_a}\right)A_\lambda(k,\tau) = 0$$

Axion is assumed as spectator throughout and frozen with constant value, and start to oscillate at late time: Misalignment

$$T_{\rm osc} \simeq 1.003\ g_*(T_{\rm osc})^{-1/4}(m_a M_{\rm P})^{1/2}$$



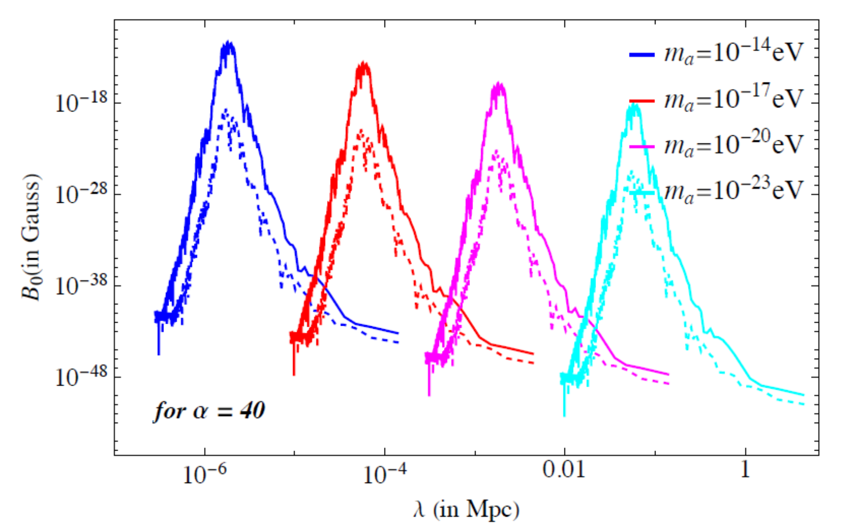
T. Patel et al, JCAP 01 (2020) 043; N. Kitajima et al, JCAP 10 (2018) 008

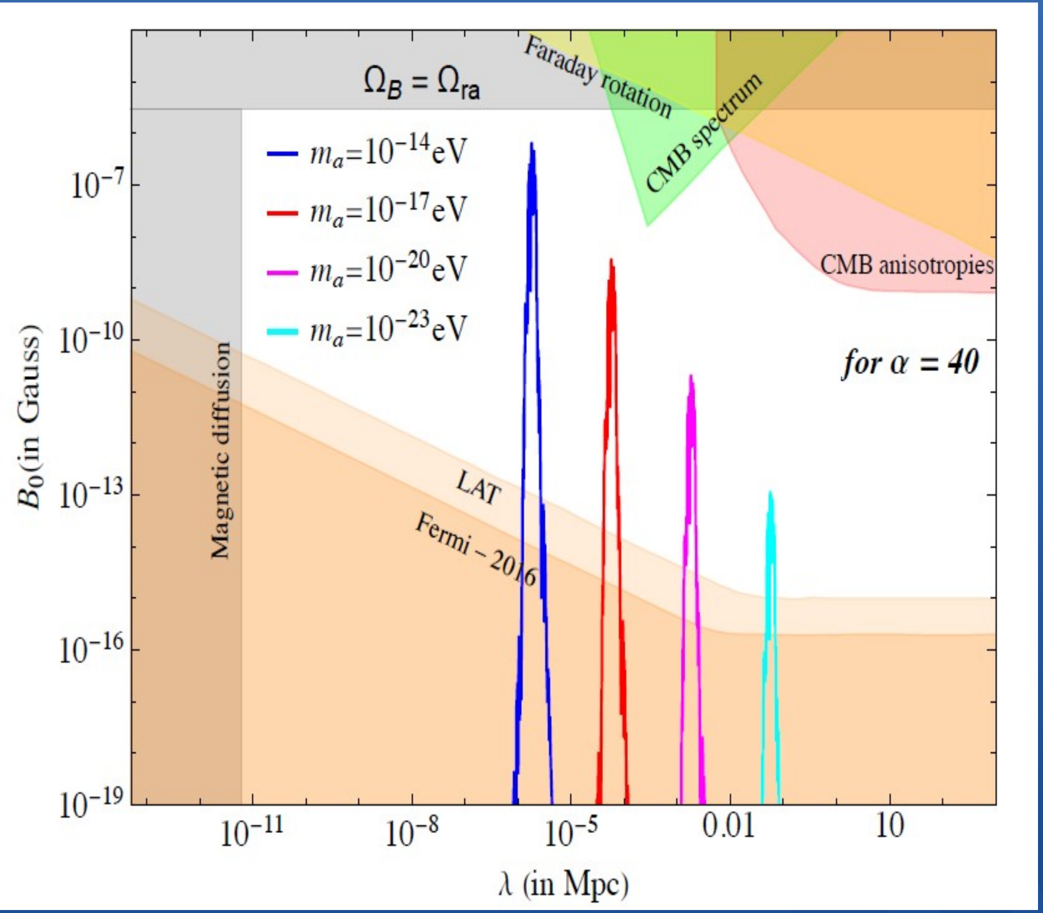
Magnetic field at different scales

$$\chi'' + 2\mathcal{H}\chi' + a^2 V_{,\chi} = \frac{a^2\alpha}{f_\chi}\langle \mathbf{E}\cdot\mathbf{B}\rangle$$

$$A''_\lambda(k,\eta) + \left(k^2 - \lambda\frac{\alpha k\chi'}{f_\chi}\right)A_\lambda(k,\eta) = 0$$

Helical in nature



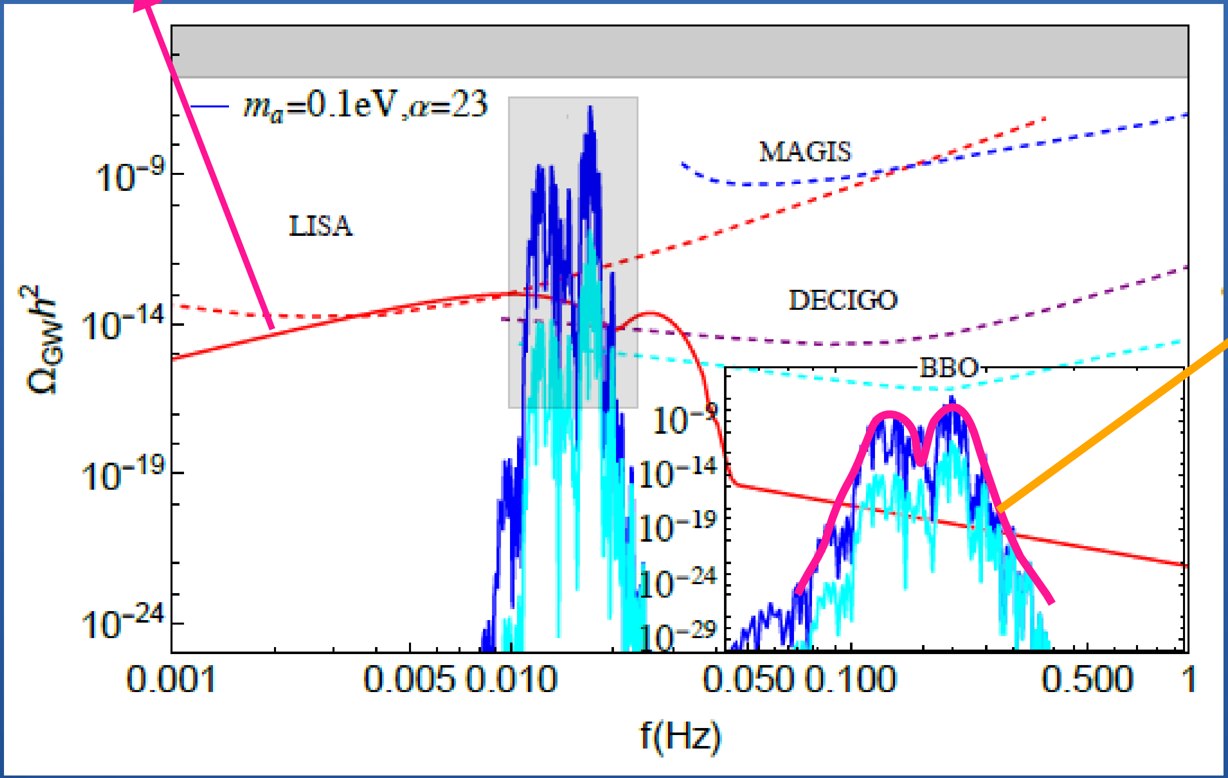


We require axion parameter sapce: $m_a \sim 10^{-20} - 10^{-25}\, GeV \;\; ; \;\; \alpha \sim 35 - 45$

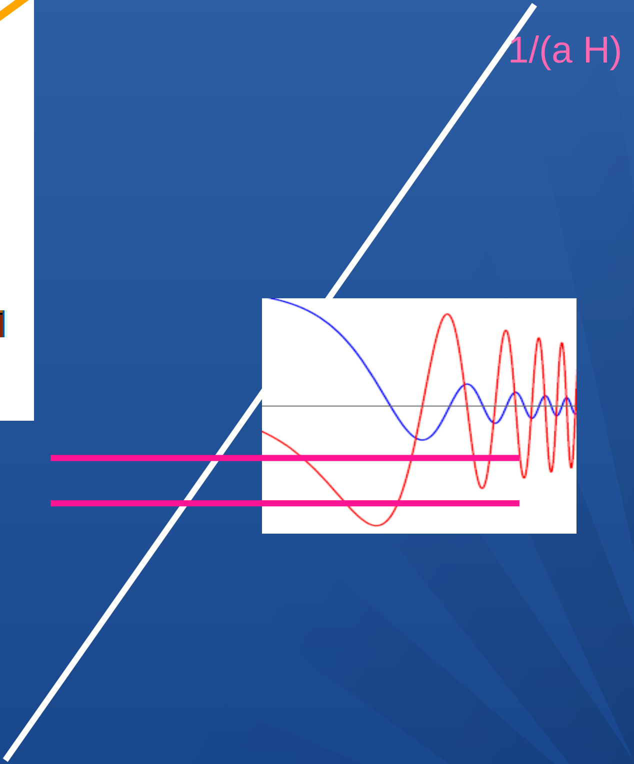
MHD + backreacion of gauge field, however, can change this parameter space

GW signature-I

GW from PBH

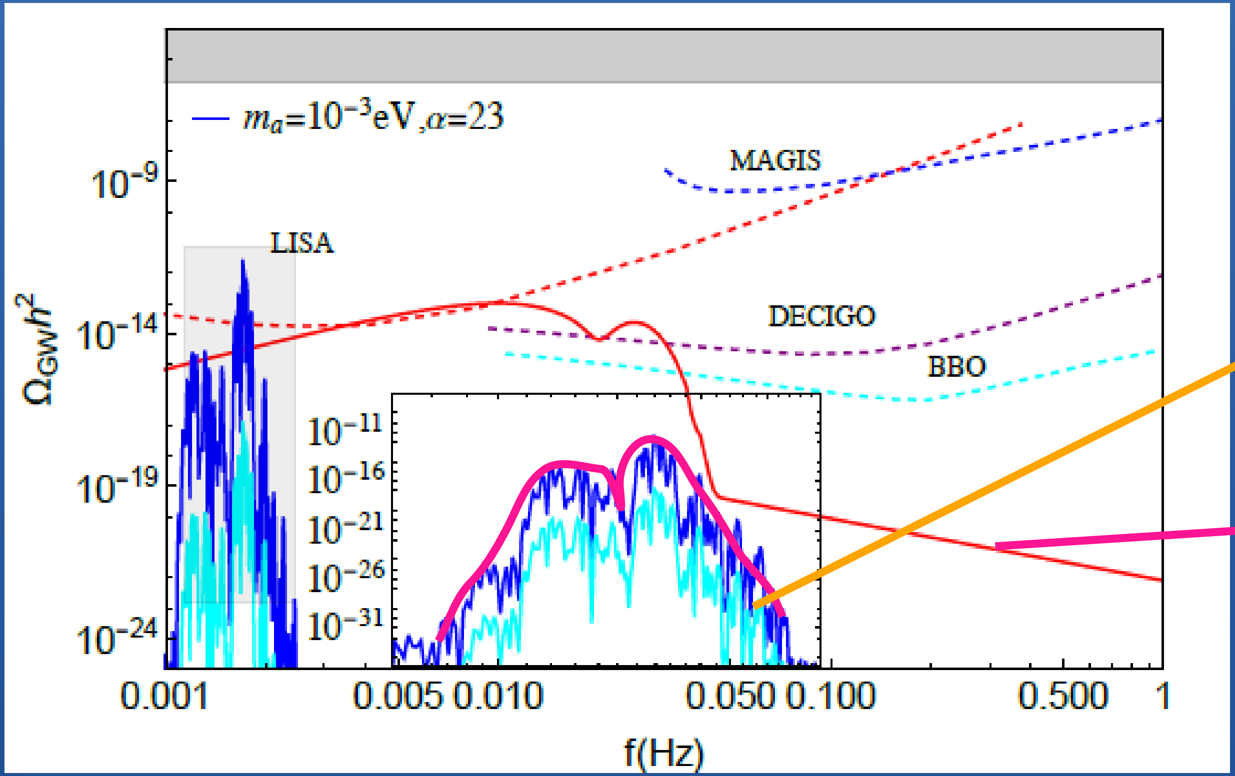


Two distinct peaks appear



For QCD axion $m_a \simeq 5.7 \times 10^{-6}\ \mathrm{eV} \left(\frac{f_a}{10^{12}\ \mathrm{GeV}}\right)$

GW signature-I



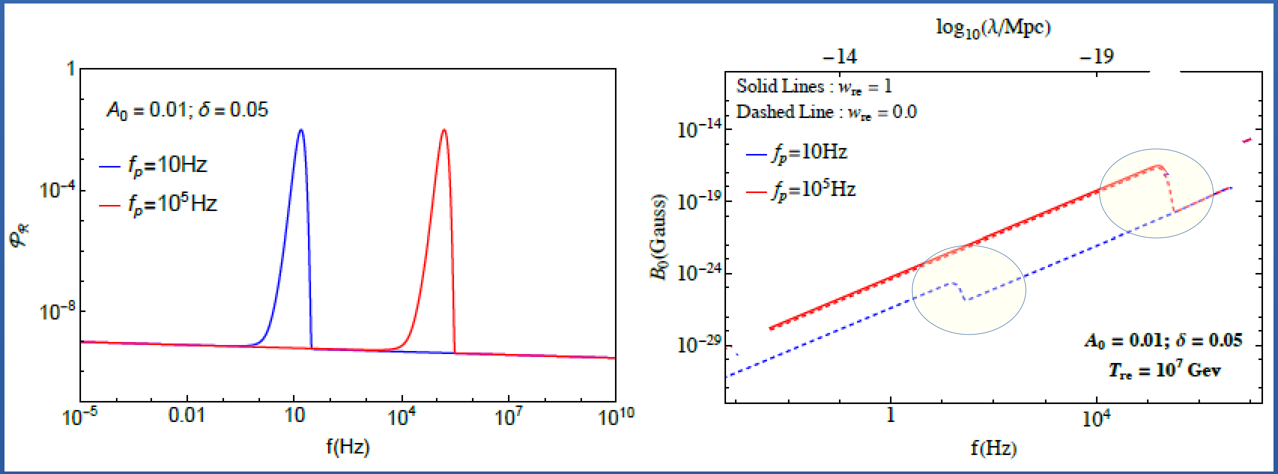
Two distinct peaks appear

GW from PBH

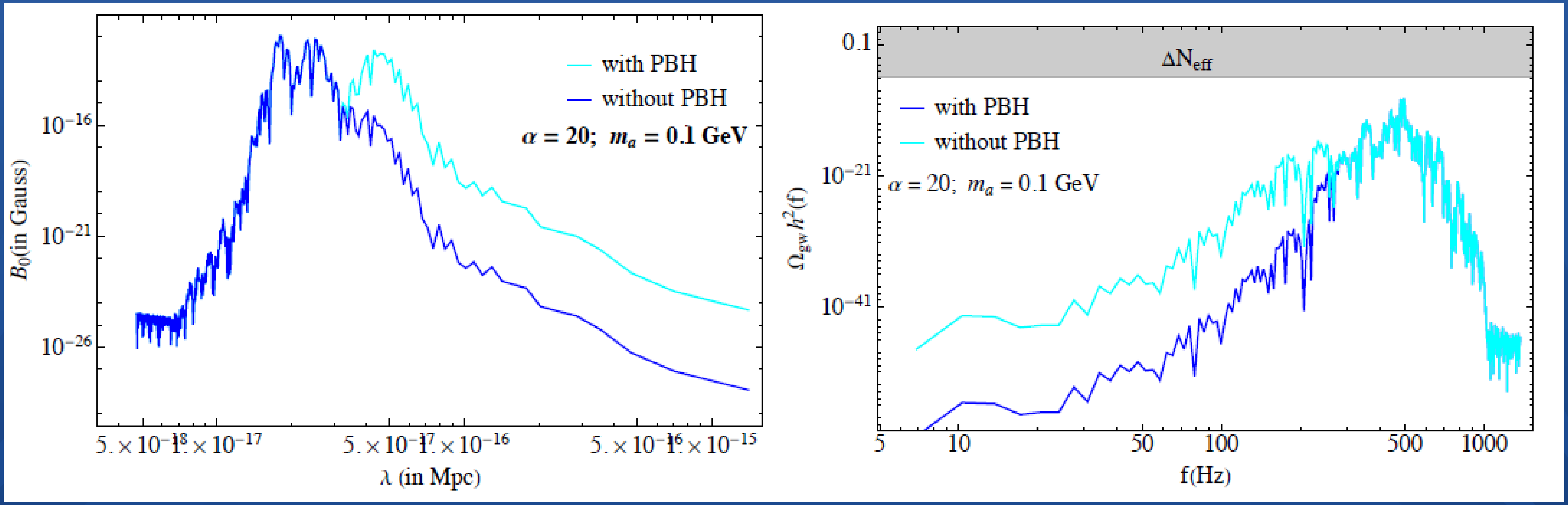
For QCD axion

$$m_a \simeq 5.7 \times 10^{-6}\ \mathrm{eV} \left(\frac{f_a}{10^{12}\ \mathrm{GeV}}\right)$$

GW signature-II



If tachyonic enhnacement occurs at the peak of the PBH spectrum



Conclusions and future directions

- Inflationary scalar perturbation can generate magnetic field at all scales but very weak
- Axion can help enhance such weak field to observable strengh at required length scale
- Depending on the parameter, it predicts distinct GW spectrum with interesting features within detectable range. Could be an interesting probe to look for axion through GW
- Full non-linear analysis needs to be looked into before any conclusive results

Thank you for your patience

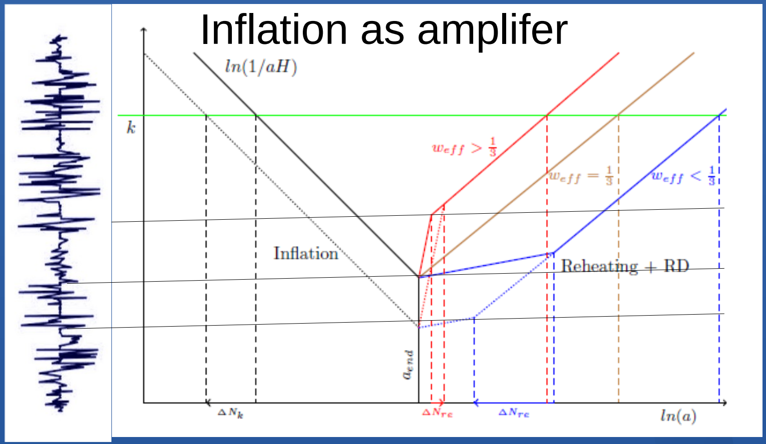
Inflationary Magnetogenesis

- Quantum-mechanically produced magnetic seeds with inflation as an amplifying mechanism
- Conformal invarint EM (no mode coupling)

$$P_B \sim k^4$$

$\sim 10^{-53}$ G at 1Mpc

- Widely studied approach
  Break conformal invariance



$$f(\phi, R) FF, a F\tilde{F}, F R \tilde{R} \ldots$$

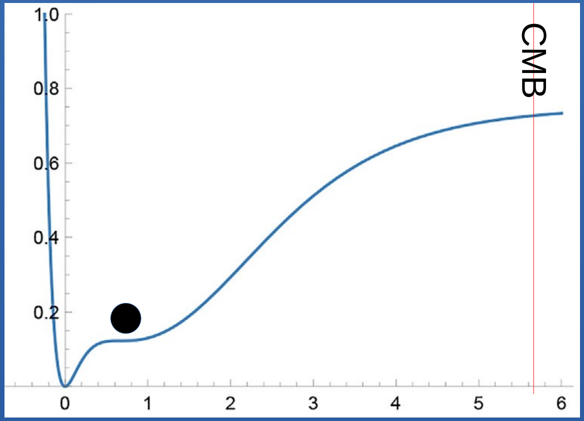
$$ds^2 = a^2(-d\eta^2 + dx^2 + dy^2 + dz^2)$$

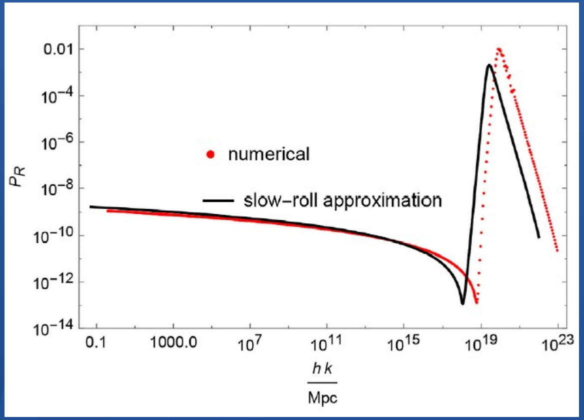
Turner MS, Widrow LM , Phys Rev D 37, 2743 (1998), Ratra B, Astrophys J 391:L1 (1992), Subramanian K, Astron Nachr 331:110, (2010), Durrer, R., Neronov, A, Astron Astrophys Rev 21, 62 (2013) (references there in)

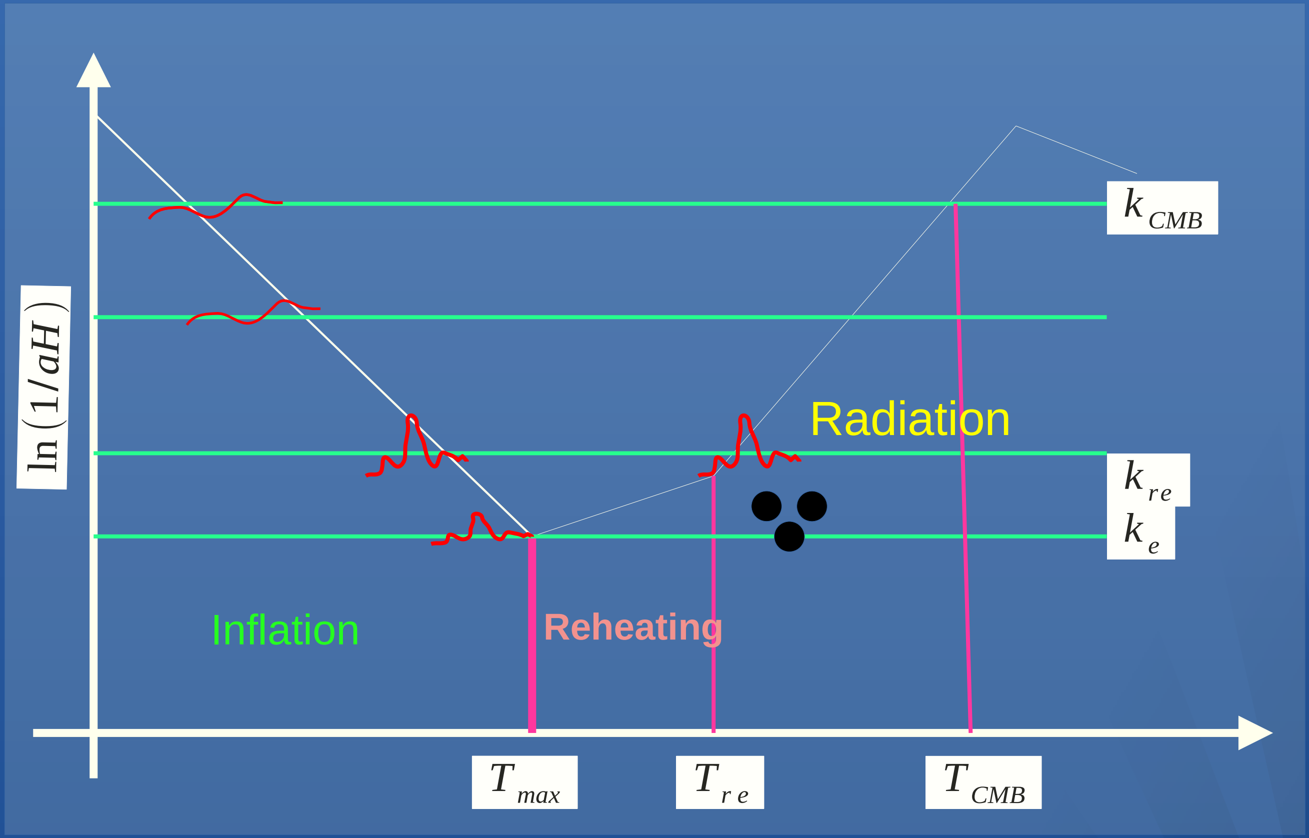
Magnetized universe:

Double slow roll phase: inflation & PBH

Daniel Frolovsk etal Front. Phys., 04 October 2022







Behavior of the comoving wave number k (horizontal lines) and the comoving Hubble radius $d_H/a = 1/(a\,H)$ white lines across different epochs

Inflationary Magnetogenesis
PBH Power spectrum

- Energy density of the preduced EM field at

$$\rho_B = \left(\frac{k}{a}\right)^4 \sum_{\lambda\lambda'} \int_{k*}^{k_e} |\beta_k^{\lambda\lambda'}|^2$$

$$\beta_{\mathbf{k}}^{\lambda\lambda'}(q,\eta) = -\frac{i}{\sqrt{2q}} \int_{\eta_i}^{\eta} \epsilon_{\mathbf{q}}^{\lambda *} \mathcal{J}_{\mathbf{k}}(\mathbf{k}+\mathbf{q}, \eta_1) e^{-ik\eta_1} d\eta_1$$

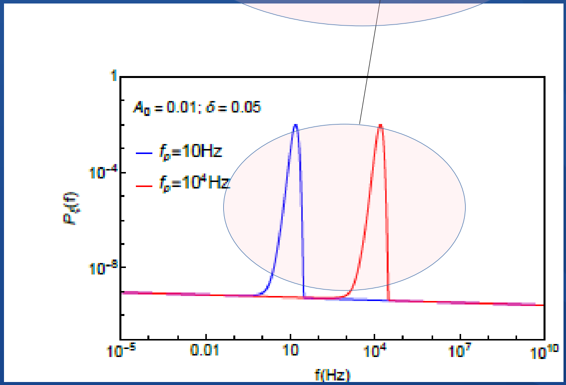
Total number density~:

$$= \frac{3\pi^3}{(2\pi)^{3/2}} \frac{16\pi^2}{9} \left(\frac{2+\beta}{3+2\beta}\right)^2 \left[\frac{\mathcal{P}_{\mathcal{R}}(k)}{4}\left(1-\frac{k_*}{k}\right)^4 + \int_1^{k_e/k} du \mathcal{P}_{\mathcal{R}}(uk)\right]$$

Inflationary Curvature Power spectrum ~:

$$\mathcal{P}_{\mathcal{R}}(k) = A_s \left(\frac{k}{k_*}\right)^{n_s - 1} + A_0 \, \mathrm{Exp}\left[-\frac{(k-k_p)^2}{\delta k_p^2}\right]$$

Two sample PBH curvature power spectrums for two Different peak frequencies k_p



Inflationary Magnetogenesis

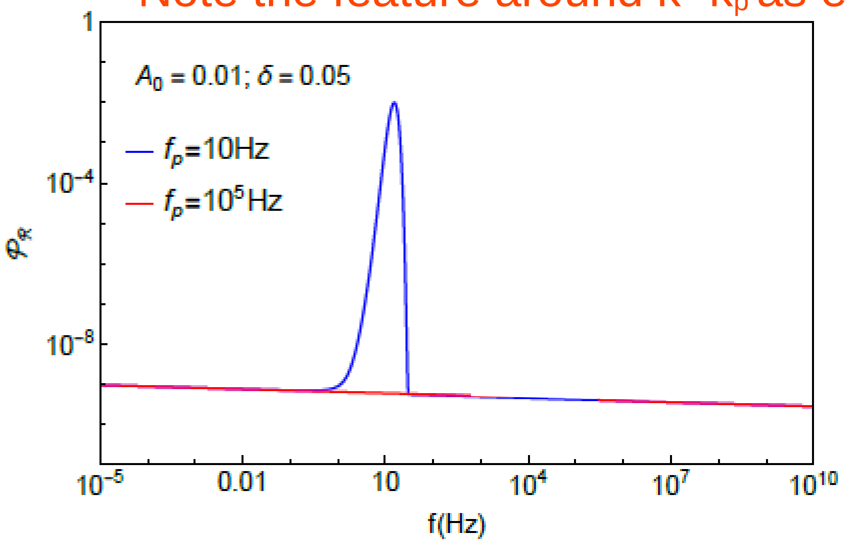
Single slow roll phase: Standard inflation

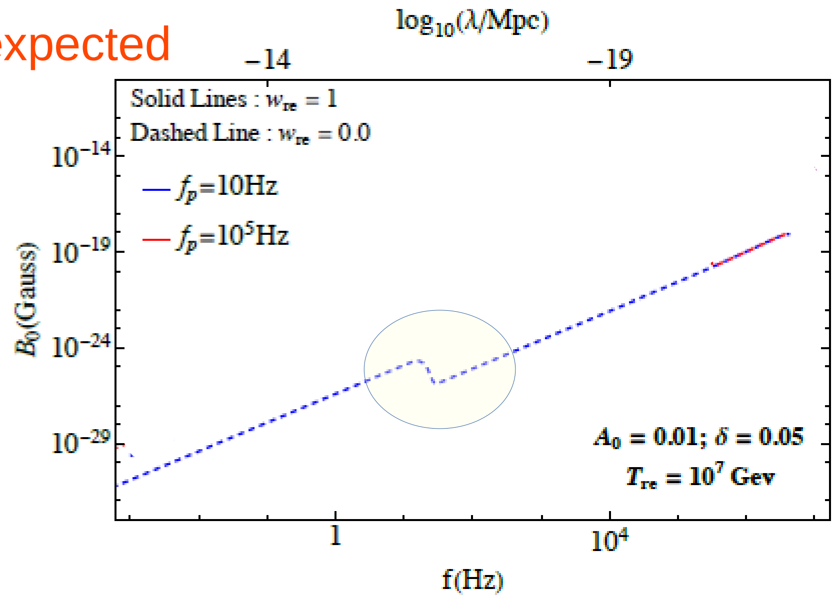
Magnetic power spectrum

$$\mathcal{P}_{\rm B}(k,\eta)=\frac{d\rho_B(k,\eta)}{d\ln k}\simeq\left(\frac{k}{a(\eta)}\right)^4\frac{8\pi^2}{9}\left(\frac{2+\beta}{3+2\beta}\right)^2\left(\frac{k_{\rm e}}{k}\right)\left[\frac{A_s}{n_s}+\frac{1}{2}\sqrt{\pi\delta}\frac{A_0k_p}{k_{\rm e}}\left(1+{\rm Erf}\left[\frac{k_p-k}{k_p\sqrt{\delta}}\right]\right)\right]$$

Present day magnetic field strength: $B_0\sim\sqrt{A_s k_e}k^{3/2}$ ($k>k_p$), $B_0\sim\sqrt{A_0 k_p}k^{3/2}$ ($k<k_p$)

Note the feature around $k=k_p$ as expected





Inflationary Magnetogenesis

Single slow roll phase: Standard inflation

Magnetic power spectrum

$$\mathcal{P}_{\mathrm{B}}(k,\eta) = \frac{d\rho_B(k,\eta)}{d\ln k} \simeq \left(\frac{k}{a(\eta)}\right)^4 \frac{8\pi^2}{9}\left(\frac{2+\beta}{3+2\beta}\right)^2\left(\frac{k_{\mathrm{e}}}{k}\right)\left[\frac{A_s}{n_s} + \frac{1}{2}\sqrt{\pi\delta}\,\frac{A_0 k_p}{k_{\mathrm{e}}}\left(1+\mathrm{Erf}\left[\frac{k_p-k}{k_p\sqrt{\delta}}\right]\right)\right]$$

Present day magnetic field strength: $B_0 \sim \sqrt{A_s k_e}\, k^{3/2}$ for $k > k_p$; $B_0 \sim \sqrt{A_0 k_p}\, k^{3/2}$ for $k < k_p$

Note the feature around $k=k_p$ as expected