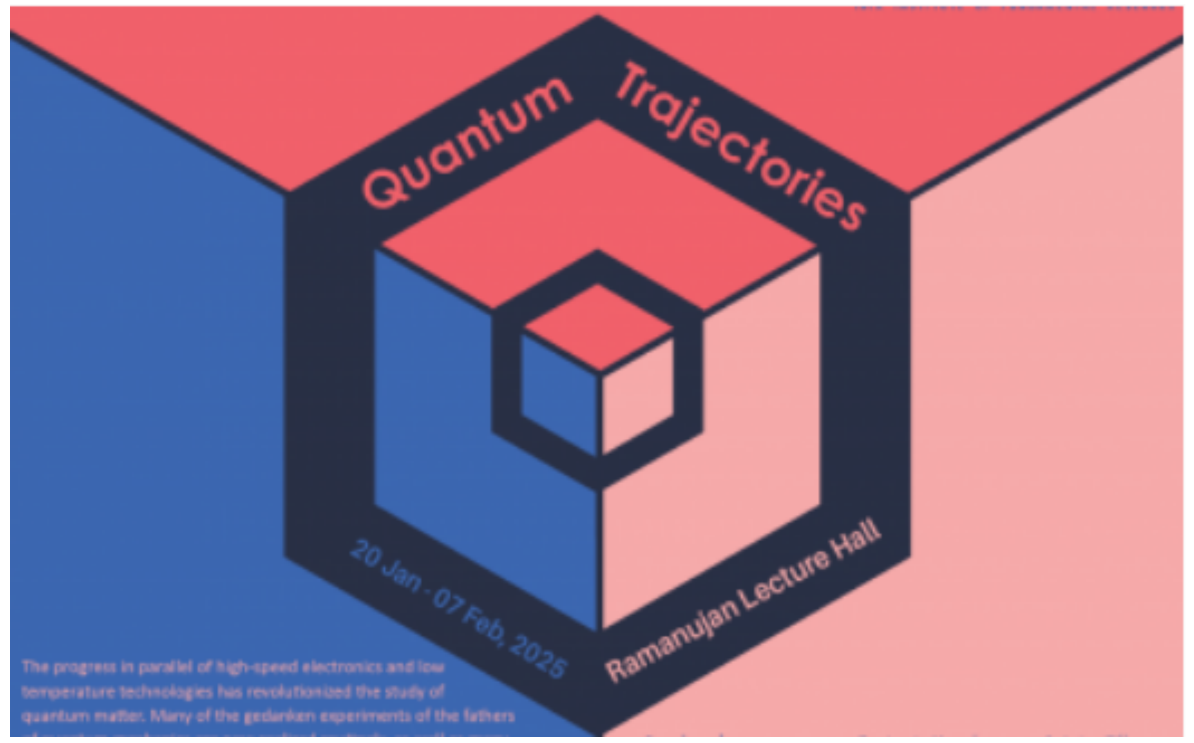
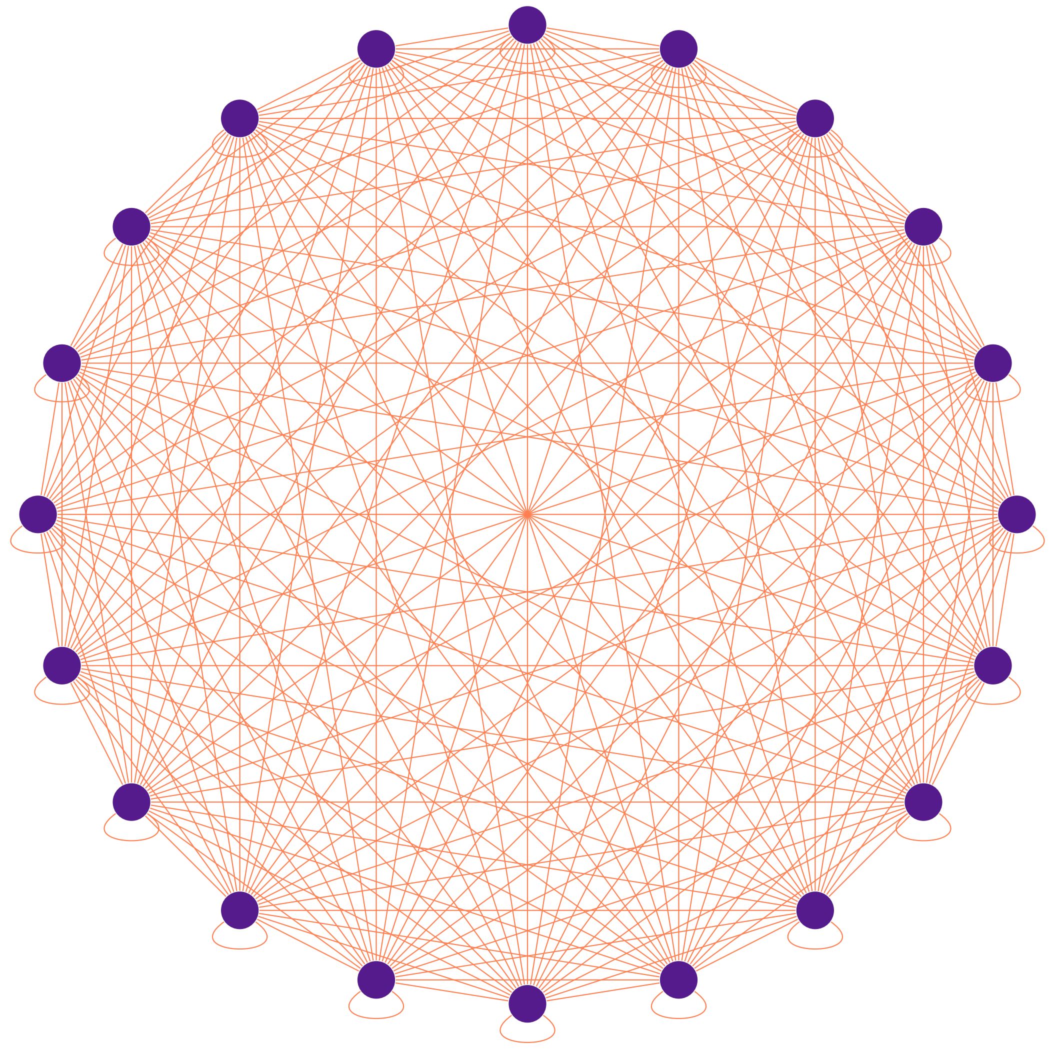
Towards a Solvable many-body model with a chaotic "transition"



Quantum Trajectories, ICTS-TIFR (2025)



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Outline

1. A Solvable Model: What, Who, Why?
2. Why Chaotic?
3. A New Model
4. The Spectrum
5. The Underlying Mechanism
6. Solving the model
7. Conclusions etc...

Collaborators

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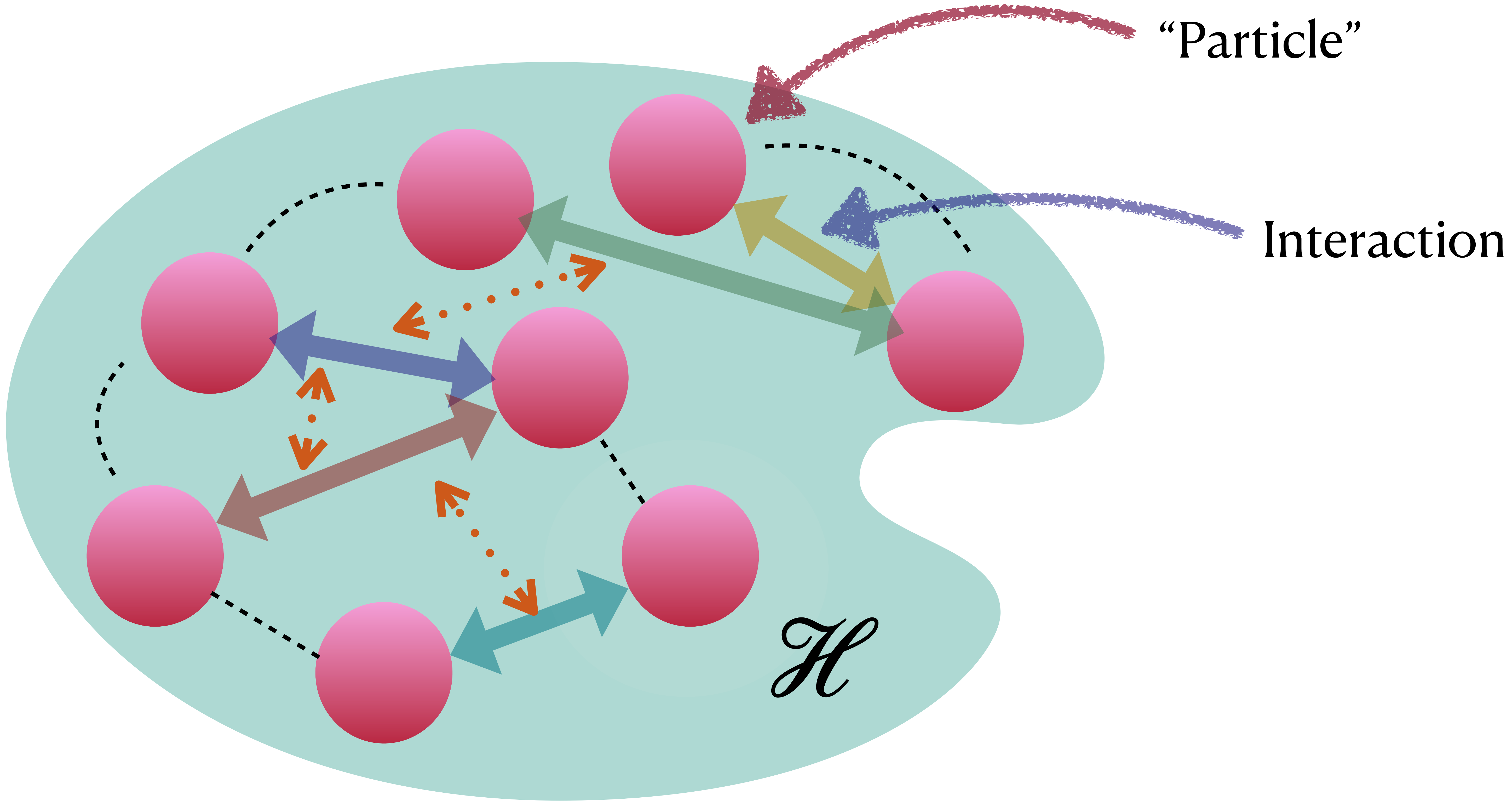
A "Solvable" model : What ?



Number of particles	$= N$
Degrees of freedom/ Hilbert space dimension	$\sim \mathrm{O}\left(e^{N}\right)$

A "Solvable" model : What ?

Number of particles $= N$

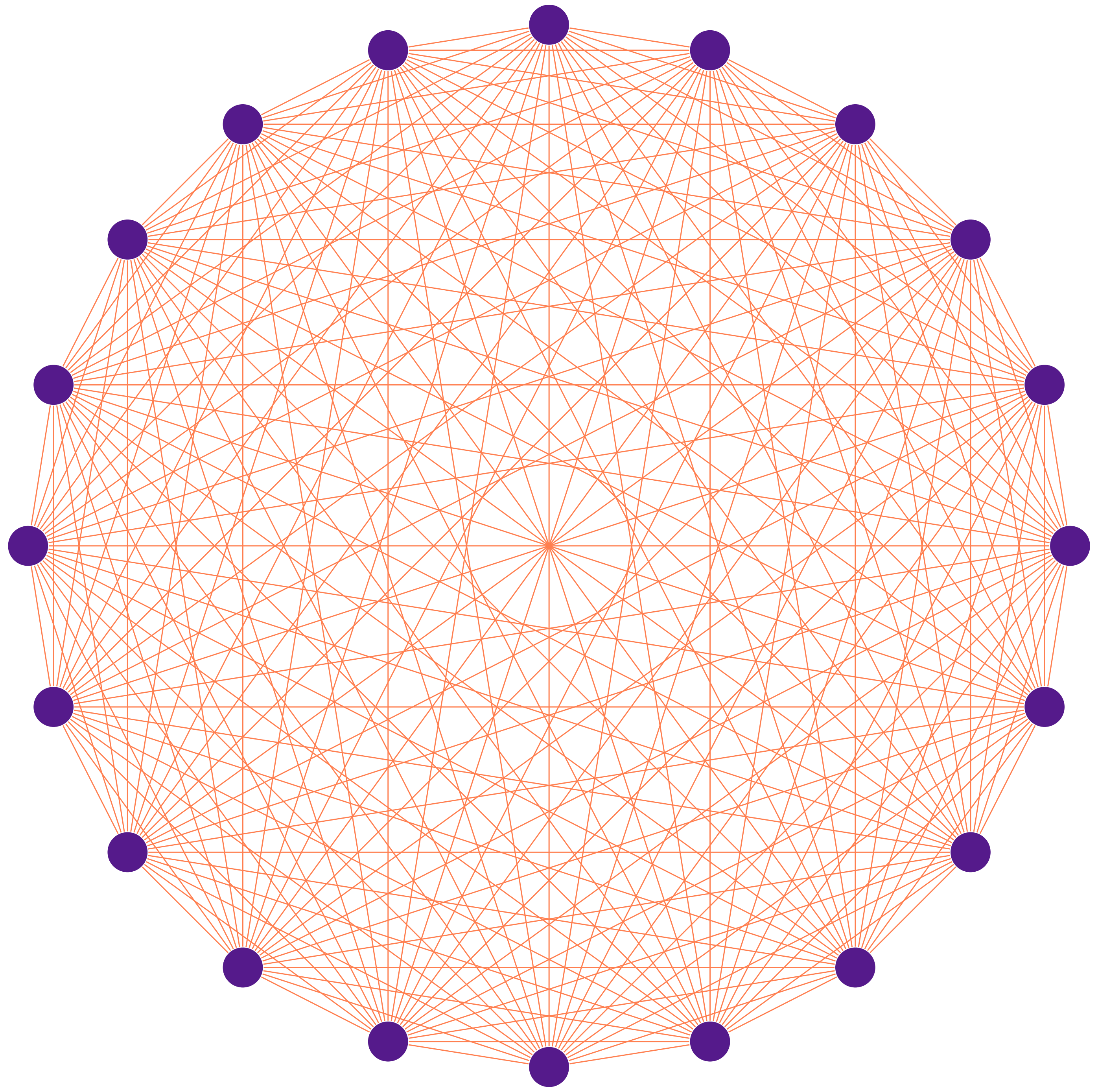
Degrees of freedom/ Hilbert space dimension $\sim O\left(e^N\right)$

Effective coupled (dynamical) fields F_i $\sim O(1)$

Partition function $Z = \mathrm{Tr}(e^{-\beta H}) = Z(F_1, F_2, F_3)$

A "Solvable" model : Who ?

All-to-All Models:



● → *Particle*

▬ → *Interaction*

In general, not solvable!

2 *Solvable* cases:

Sachdev-Ye-Kitaev model
● → *Fermions/bosons*

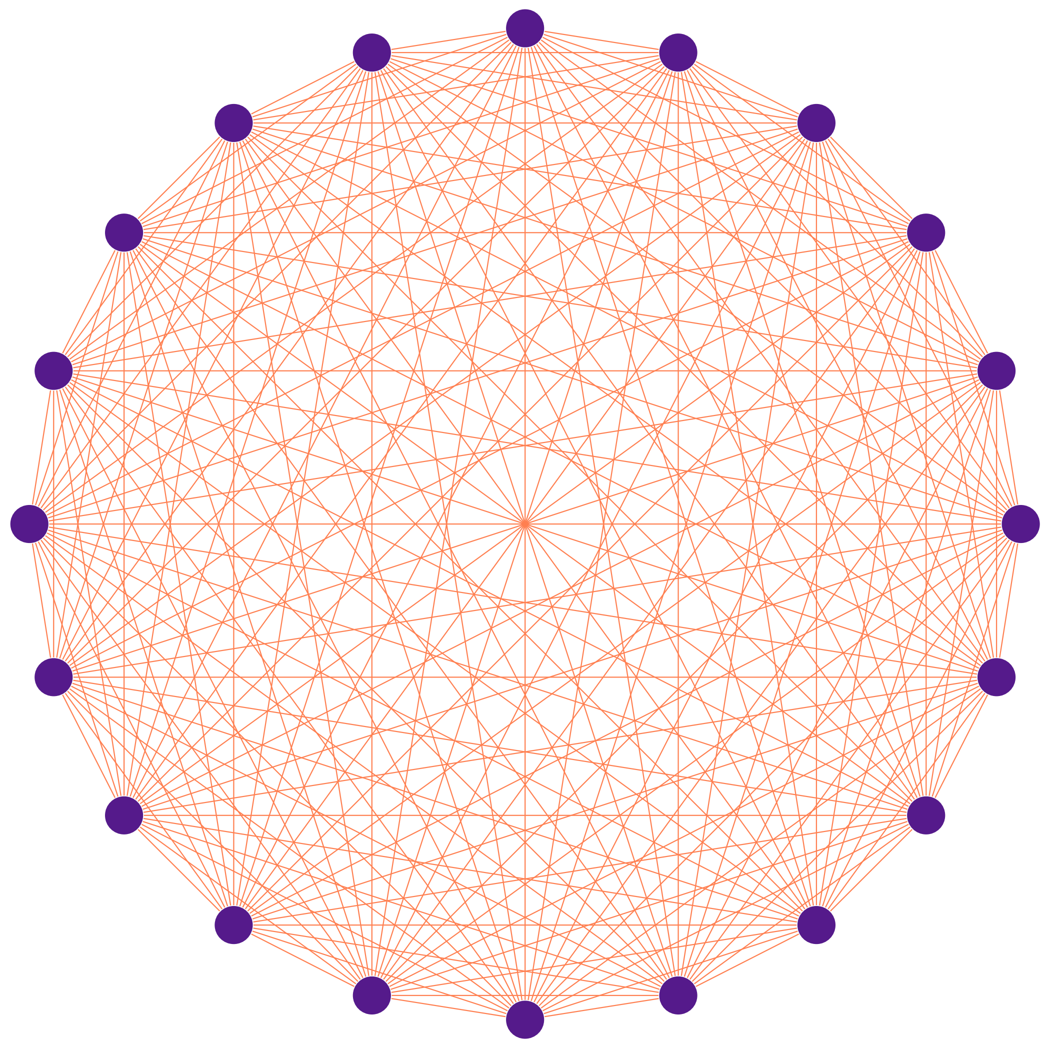
Quantum p− spin glass
● → *Spins*

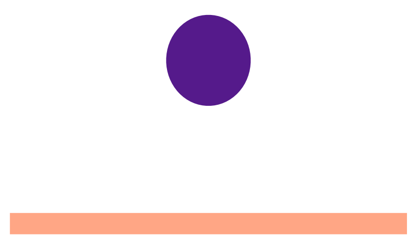
Phys. Rev. D 94, 106002
Maldacena and Stanford (2016)

J. Stat. Mech. (2021) 113101
Anous and Haehl (2021)

A "Solvable" model : Why ?

Sachdev-Ye-Kitaev Model:





→ *Majorana fermion* $\hat{\chi}$

→ *Random Interaction* J_I

$$H = \sum_I J_I \Psi_I = \sum_{i_1, i_2, \ldots, i_q}^{N} J_{i_1, i_2, \ldots, i_q} \hat{\chi}_{i_1} \hat{\chi}_{i_2} \cdots \hat{\chi}_{i_q}$$

$$\{\hat{\chi}_i, \hat{\chi}_j\} = \delta_{ij}$$

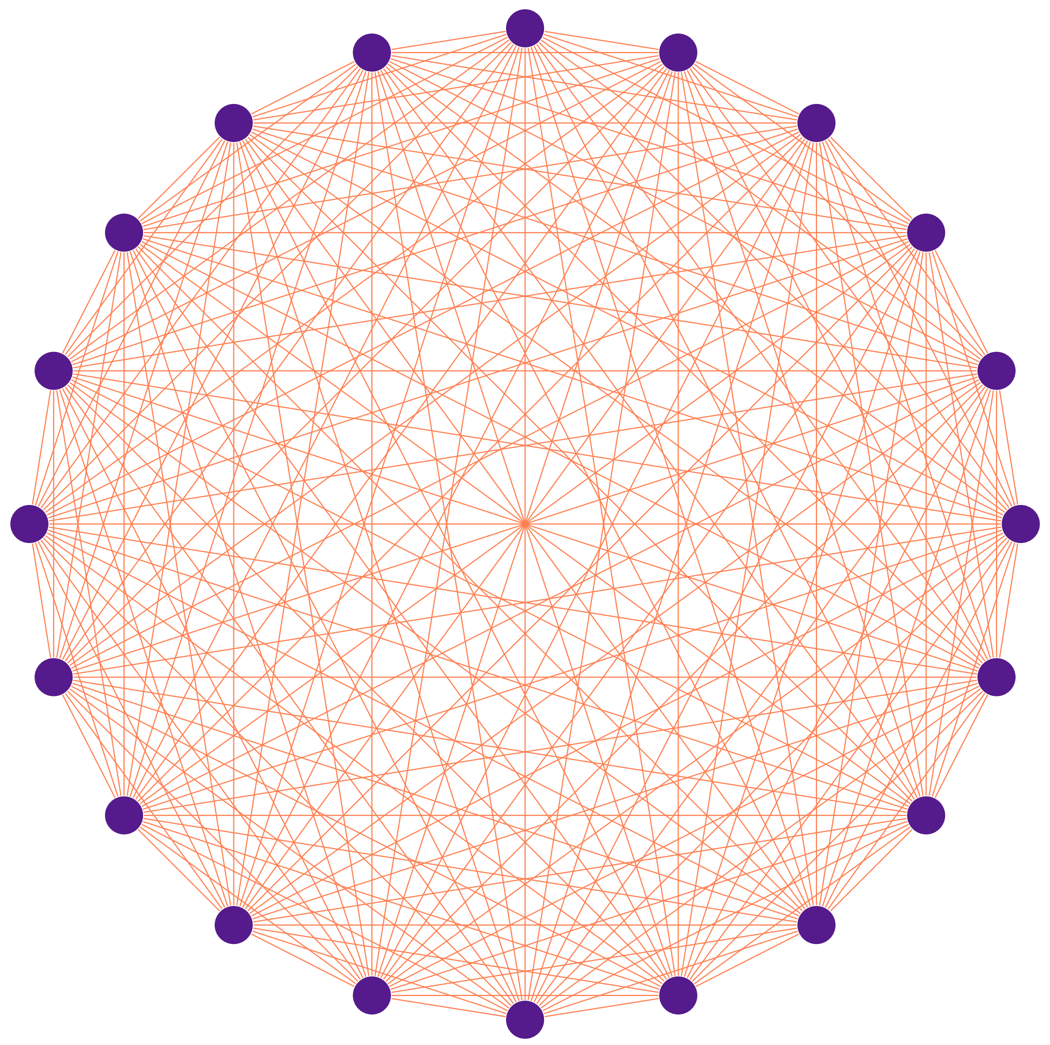
$$J_I \equiv J_{i_1, i_2, i_3, \ldots, i_q}$$

$$\Psi_I \equiv \hat{\chi}_{i_1} \hat{\chi}_{i_2} \hat{\chi}_{i_3}, \ldots, \hat{\chi}_{i_q}$$

$$\langle J_I \rangle = 0 \qquad \langle J_I J_{I'} \rangle = \delta_{I,I'} \Omega^2 = \delta_{I,I'} \frac{(q-1)!}{N^{q-1}} J^2 \qquad i_k \in \{1, \ldots, N\}$$

A "Solvable" model : Why ?

Sachdev-Ye-Kitaev Model:



$$H = \sum_I J_I \Psi_I \qquad N \text{ fermions}$$

A Solvable Model, but what exactly does it mean?

Thermal Partition function $\langle Z \rangle_J = \langle \mathrm{Tr}(e^{-\beta H}) \rangle_J = \langle Z \rangle (G, \Sigma)$

Limit: $N \to \infty$ fermions

Green's function

Self energy

$$G^{-1} = \partial_\tau - \Sigma$$

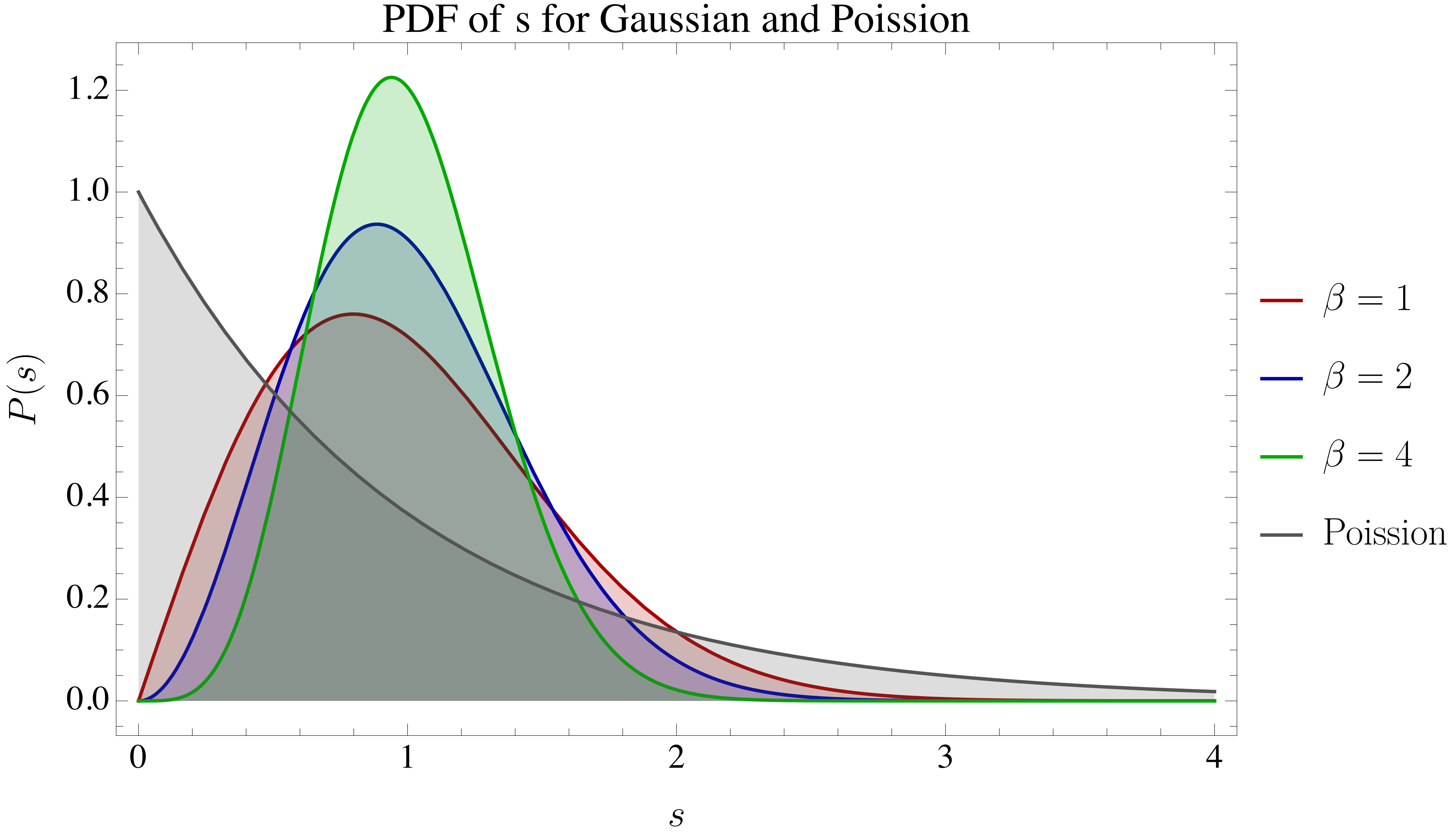
$$\Sigma = J^2 G^{q-1}$$

Why Chaotic?

C.F. TALKS BY:- ARITRA, AURELIA AND ADOLFO

Chaotic $\implies$ looks like a Gaussian Random Matrix

Short-range correlations: $s_k = E_{k+1} - E_k, \quad k = \{1,\ldots,N\}$



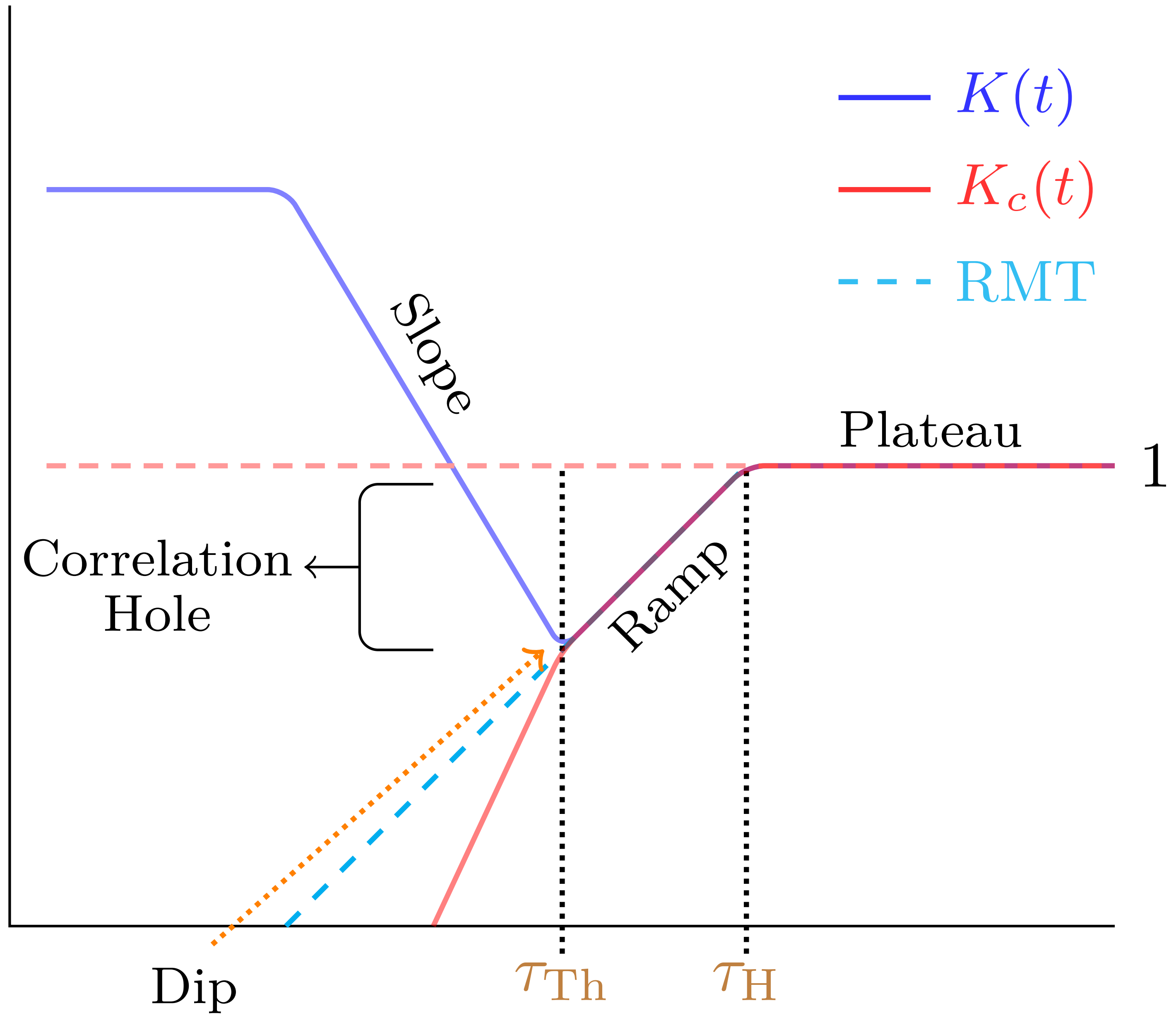
Mean Level-spacing ratio:

$$r = \left\langle \min\left(\frac{s_k}{s_{k-1}}, \frac{s_{k-1}}{s_k}\right)\right\rangle$$

~ 0.53 (GOE)
~ 0.60 (GUE)
~ 0.67 (GSE)
~ 0.35 (Poission)

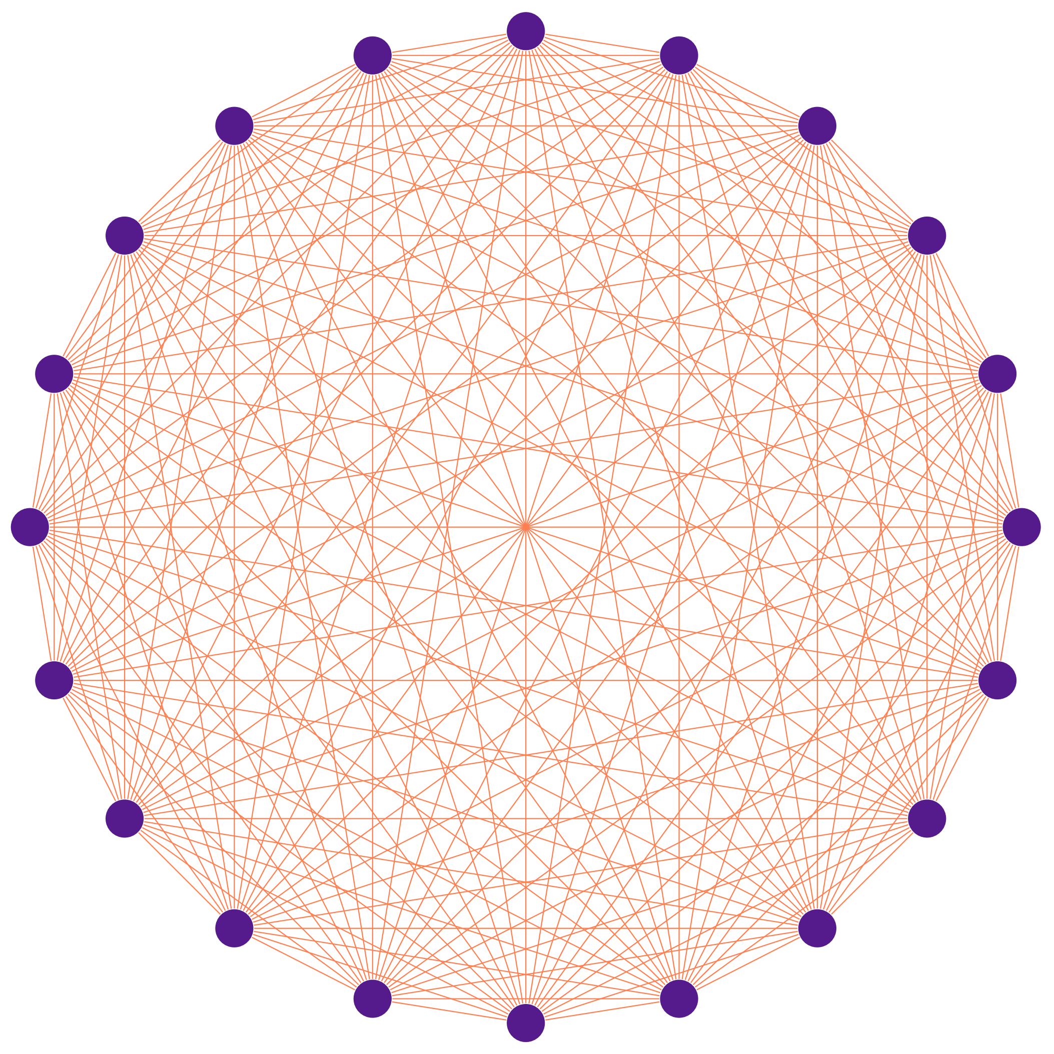
Long-range correlations:

Spectral Form Factor



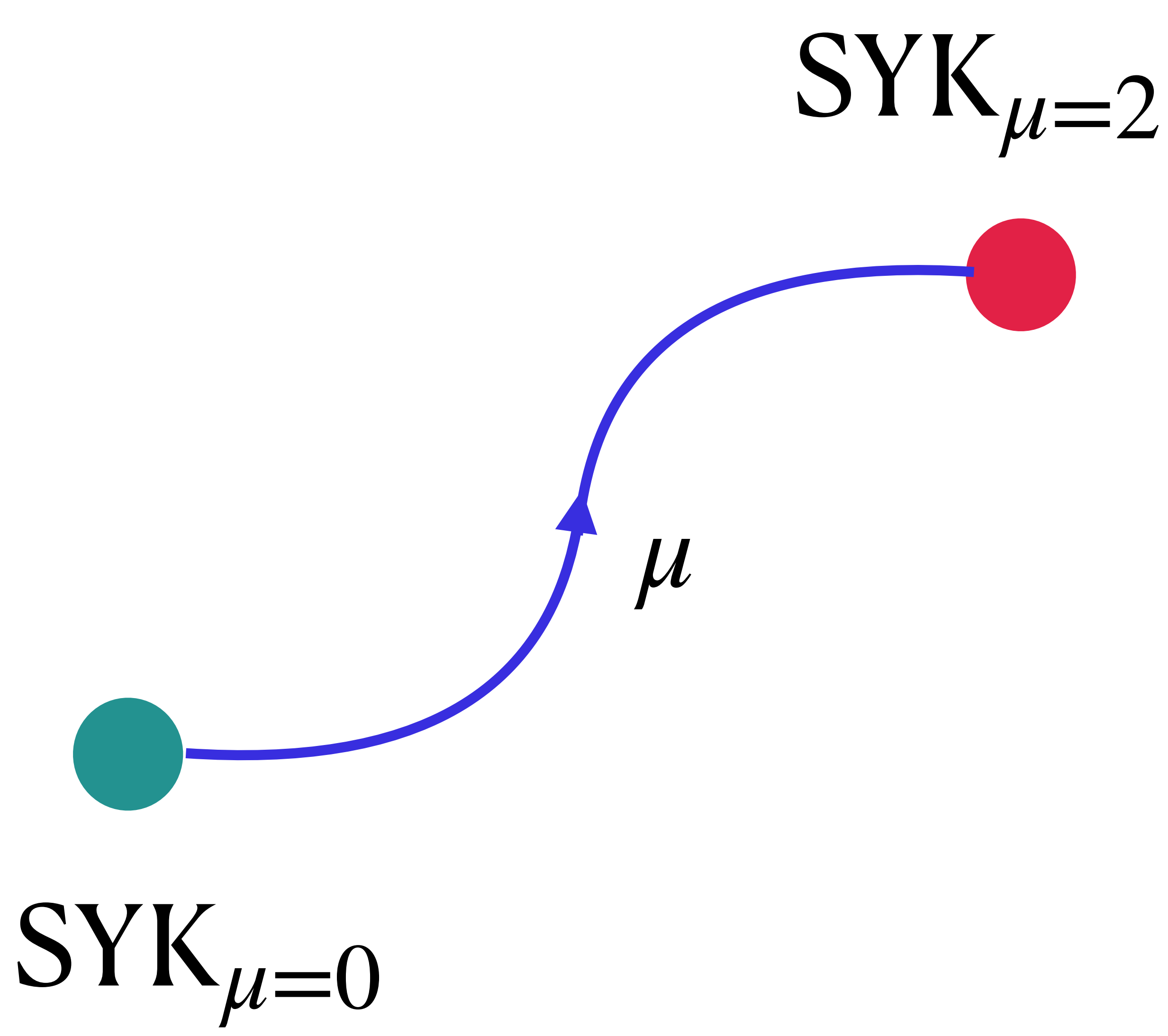
$$K(t) = \left\langle \frac{|Z(t)|^2}{|Z(0)|^2} \right\rangle, \quad Z(t) = \text{Tr}\left(e^{itH}\right)$$

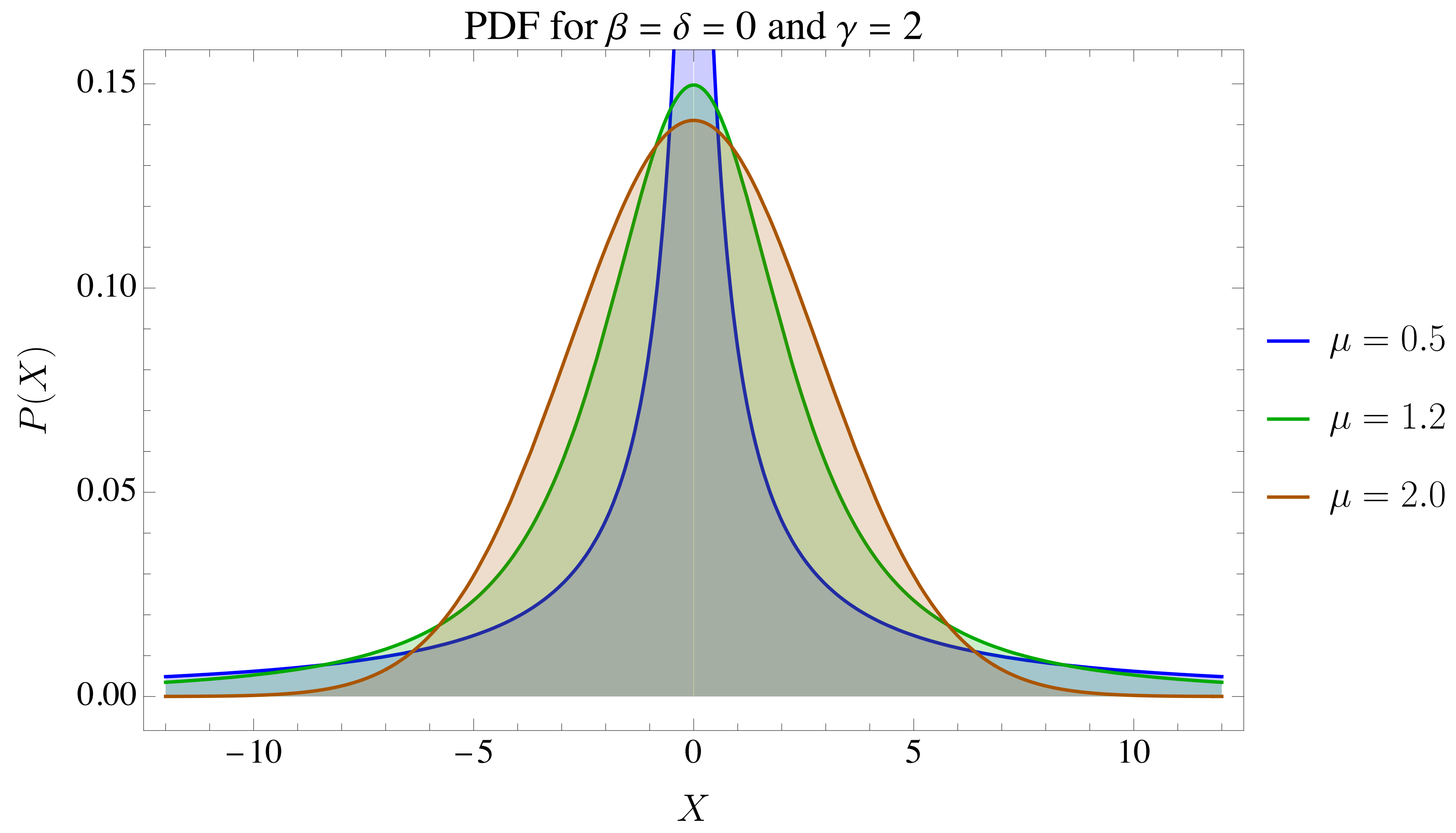
A New Model



The same SYK configuration:

$$H = \sum_{i_1, i_2, \ldots, i_q} J_{i_1, i_2, \ldots, i_q} \hat{\chi}_{i_1} \hat{\chi}_{i_2} \cdots \hat{\chi}_{i_q}$$





Univariate Stable Distribution

John. P. Nolan

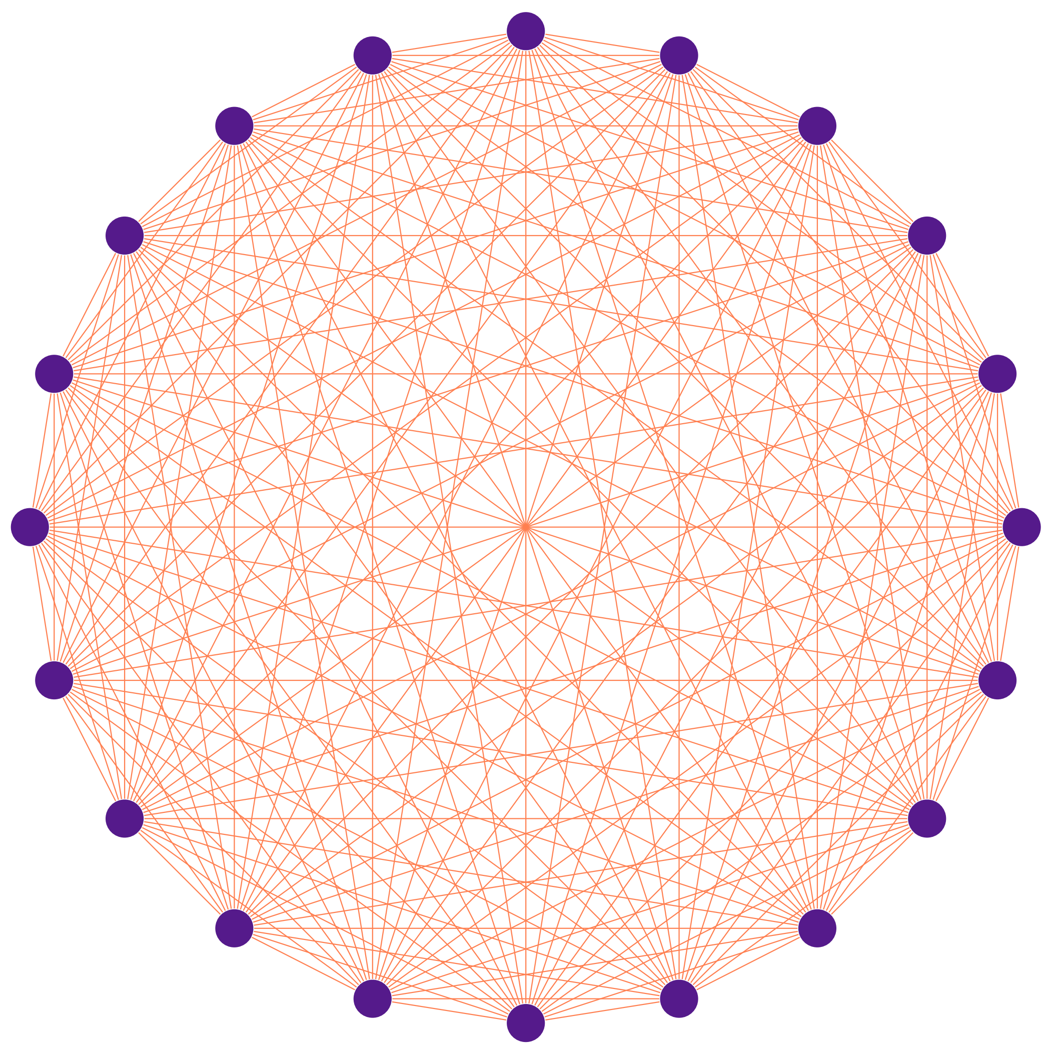
where $J_{i_1, i_2, \ldots, i_q} \sim \mathtt{C}_\mu(\gamma, \beta, \delta)$ → Lévy Dist.

we choose: $\gamma = \mathcal{J}_L \left(\frac{N^2}{2q} \binom{N}{q} \right)^{\frac{-1}{\mu}}$ and $\beta = \delta = 0$

Usual SYK for $\mu = 2$

A New Model

Lévy SYK



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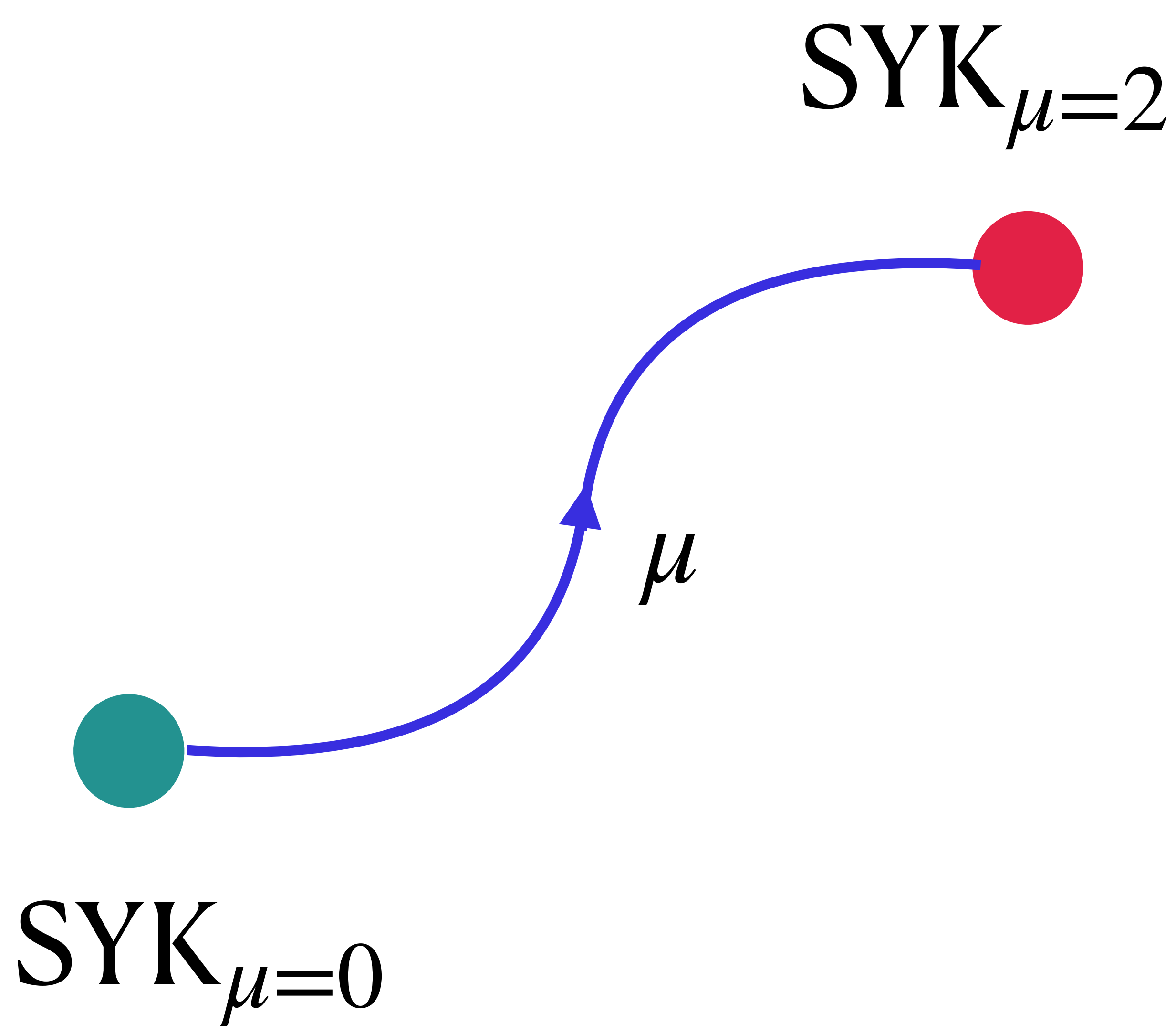

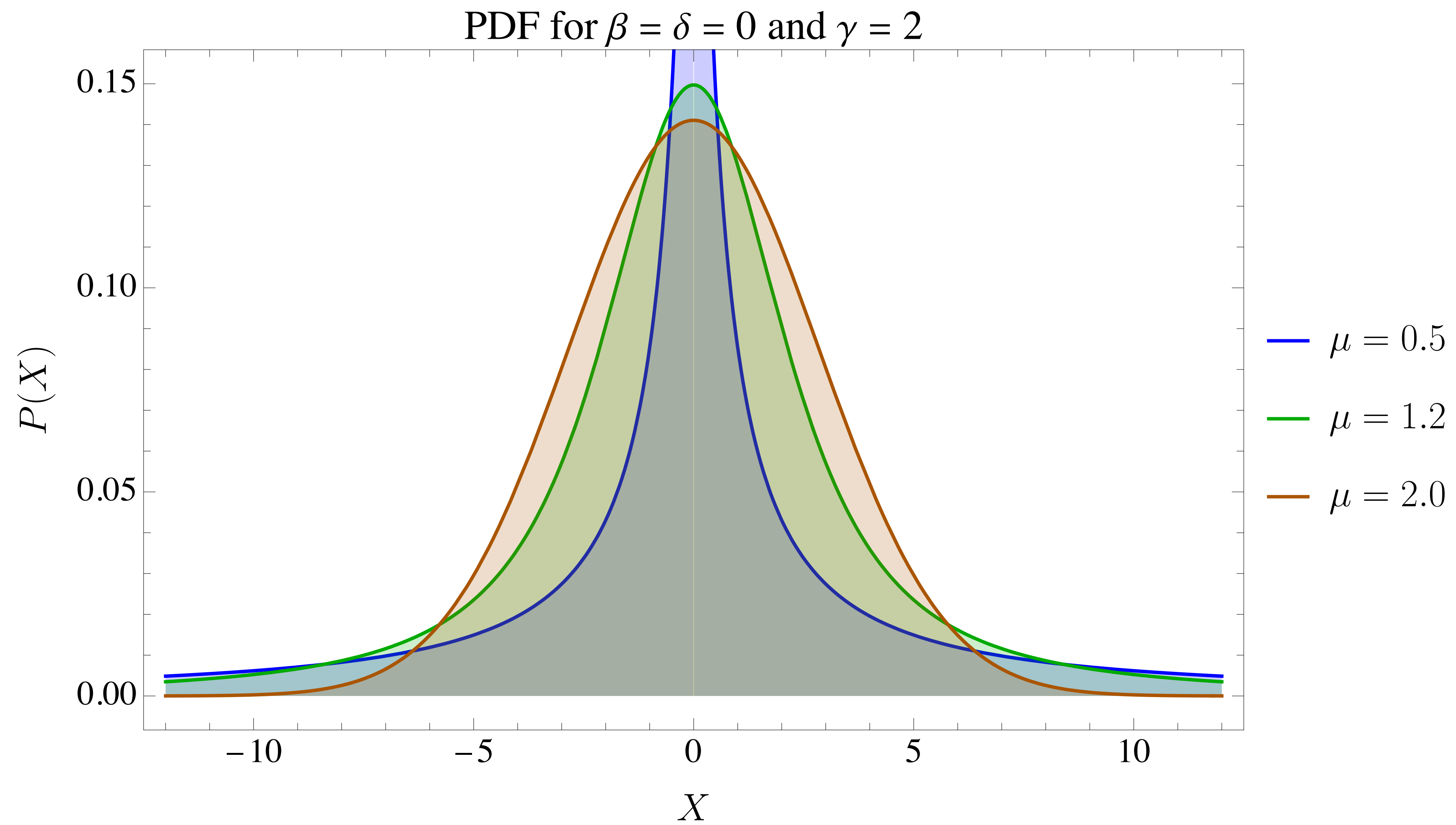

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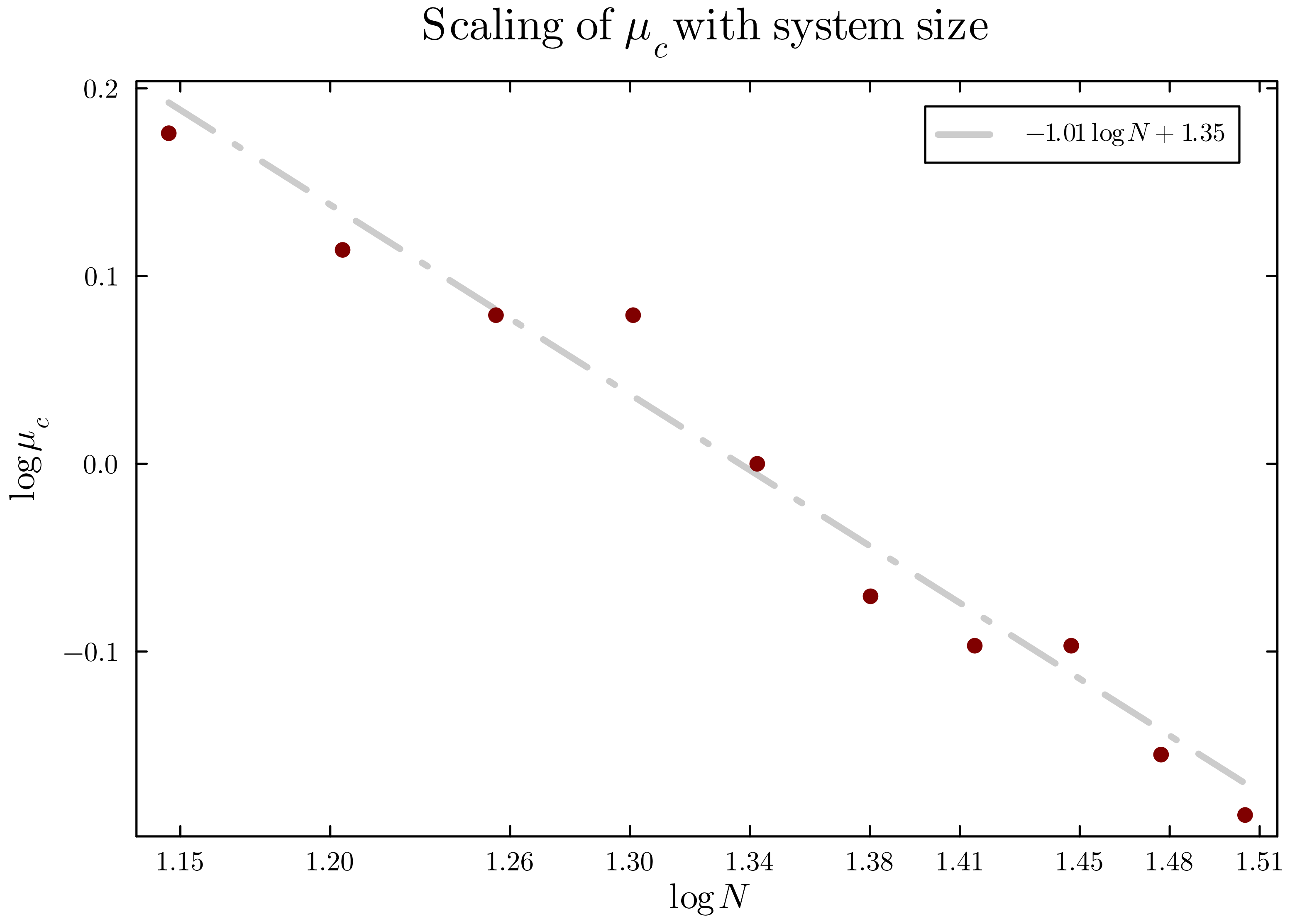
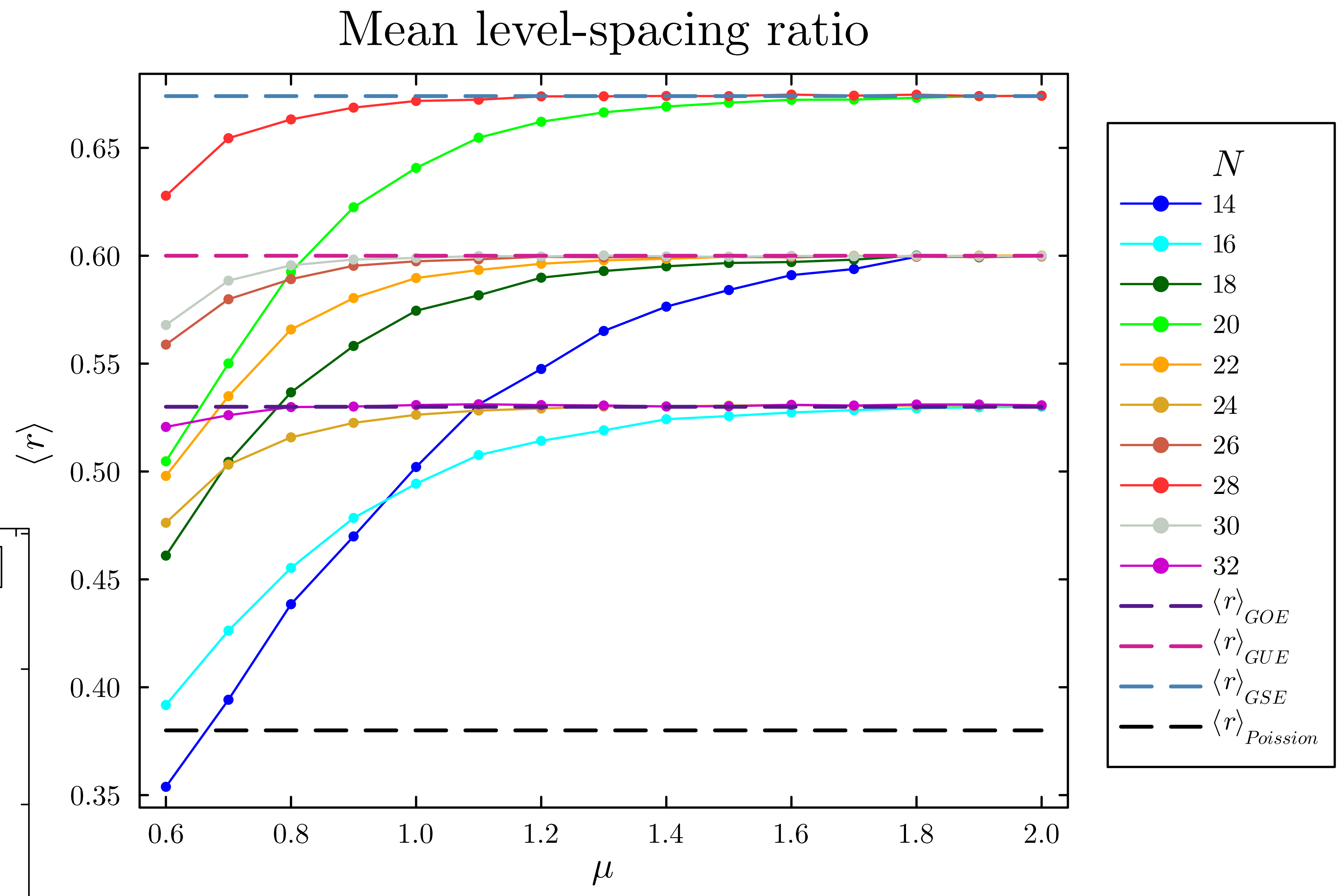
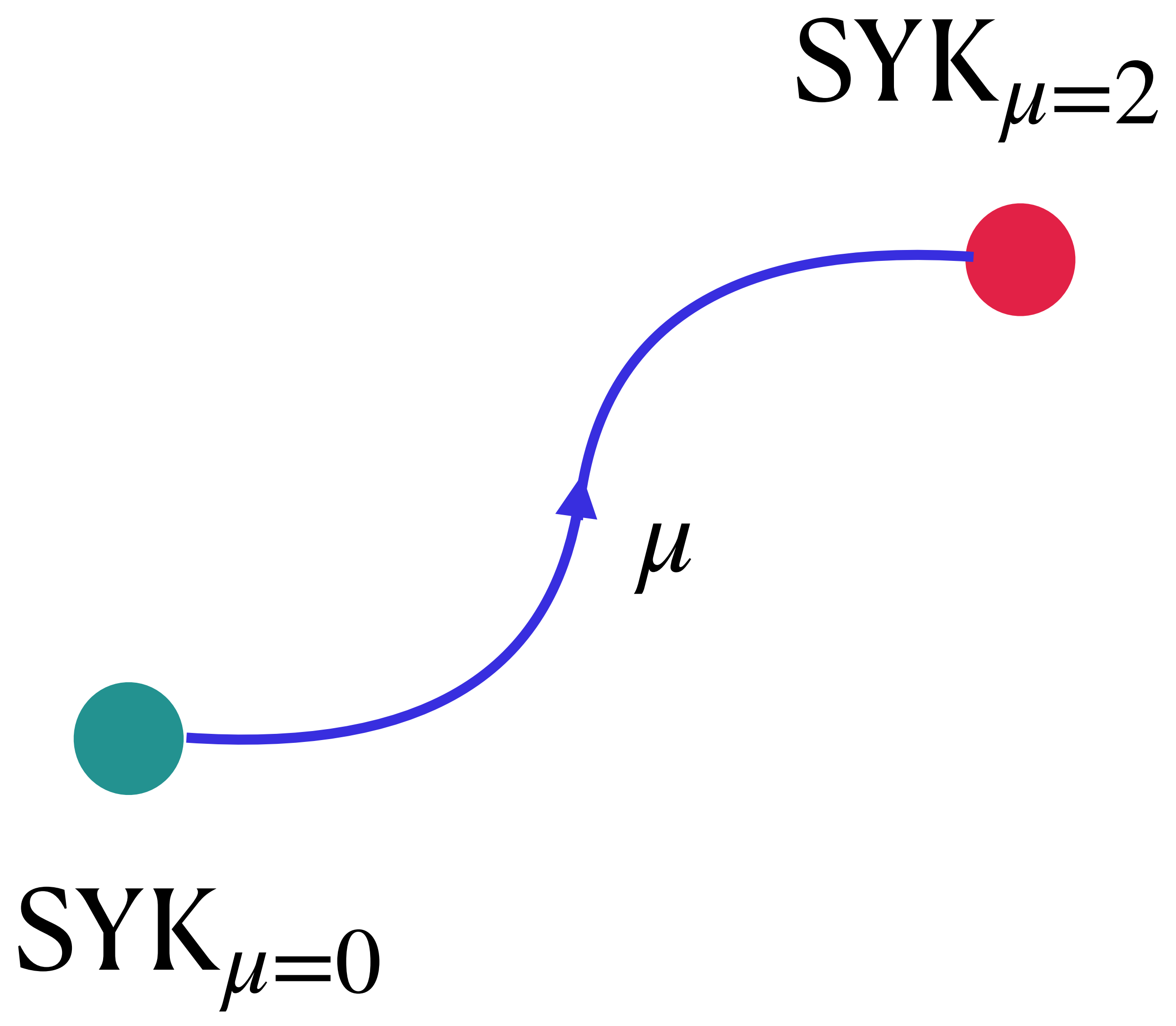
The Spectrum - Short Range

$$r = \left\langle \min\left(\frac{s_k}{s_{k-1}}, \frac{s_{k-1}}{s_k}\right)\right\rangle$$



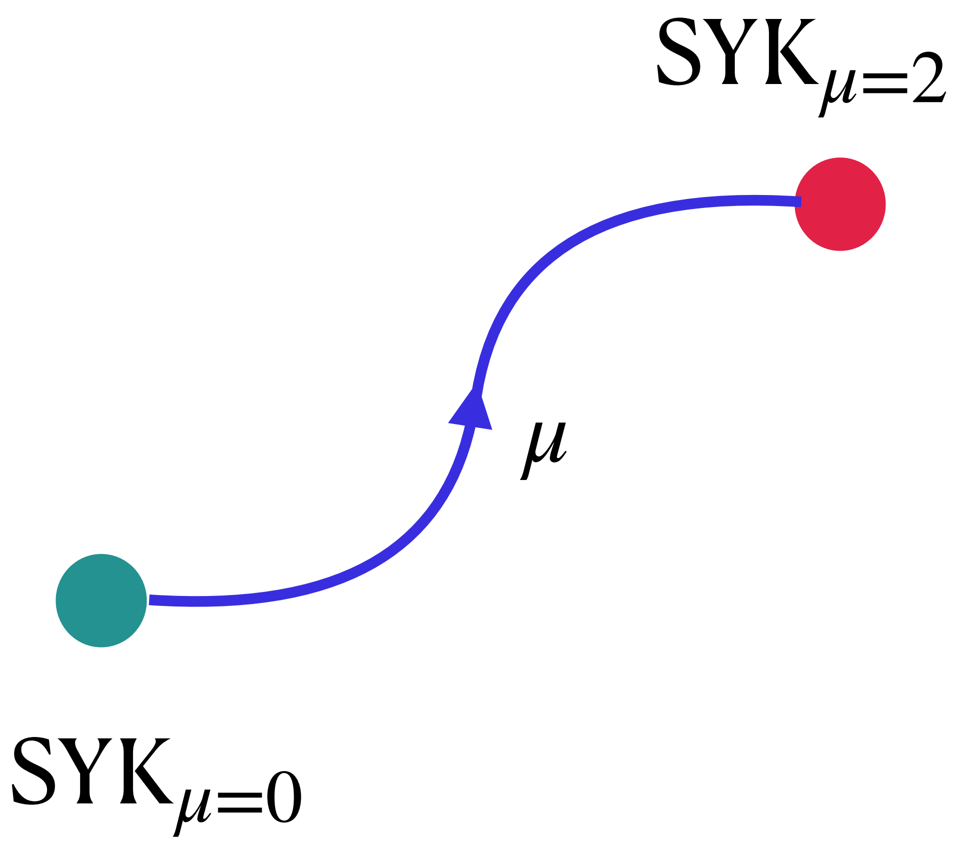


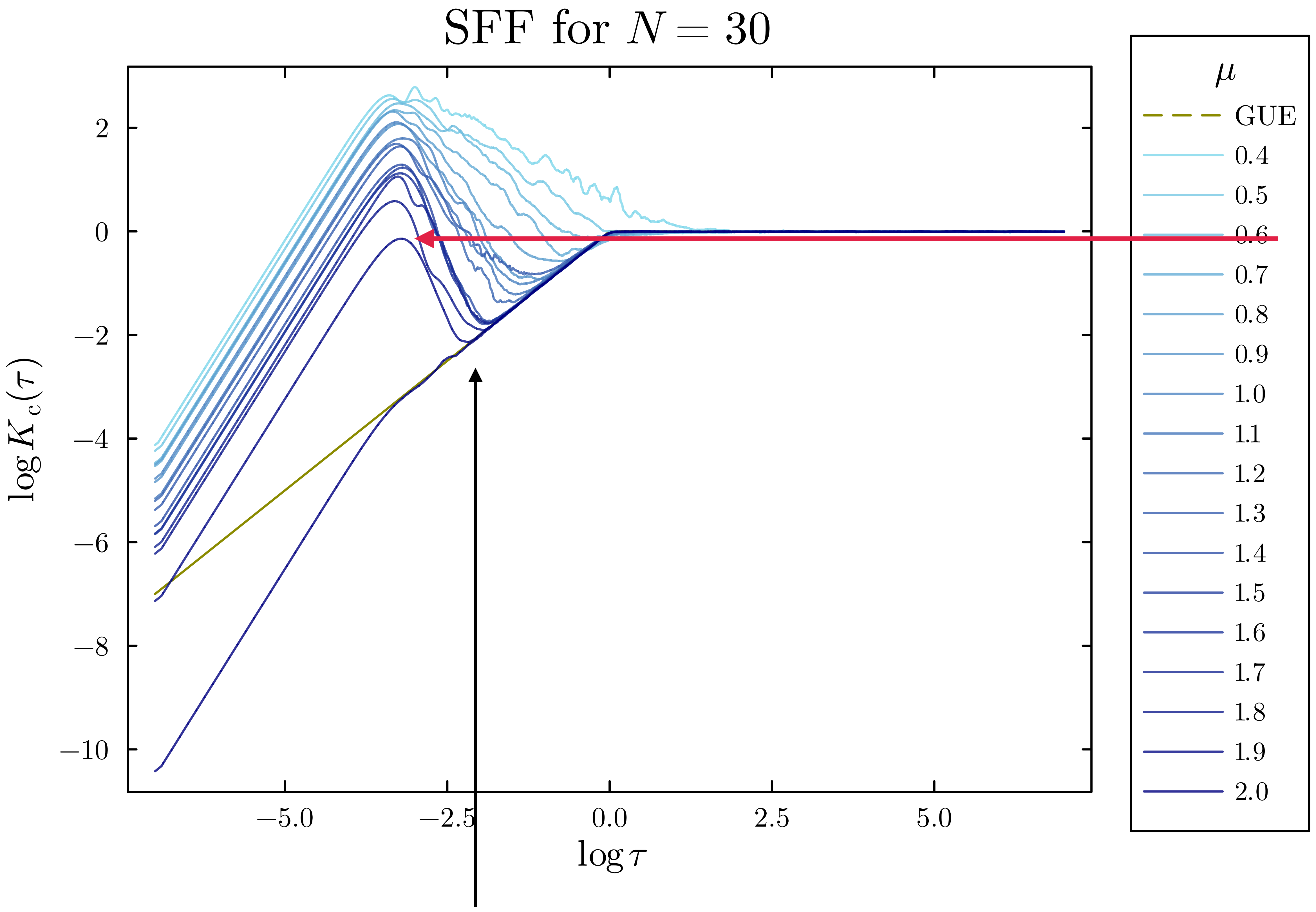


$$\mu_c(N) \sim N^{-1}$$

$\mu_c \rightarrow$ Value of μ where deviation from RMT value is observed.

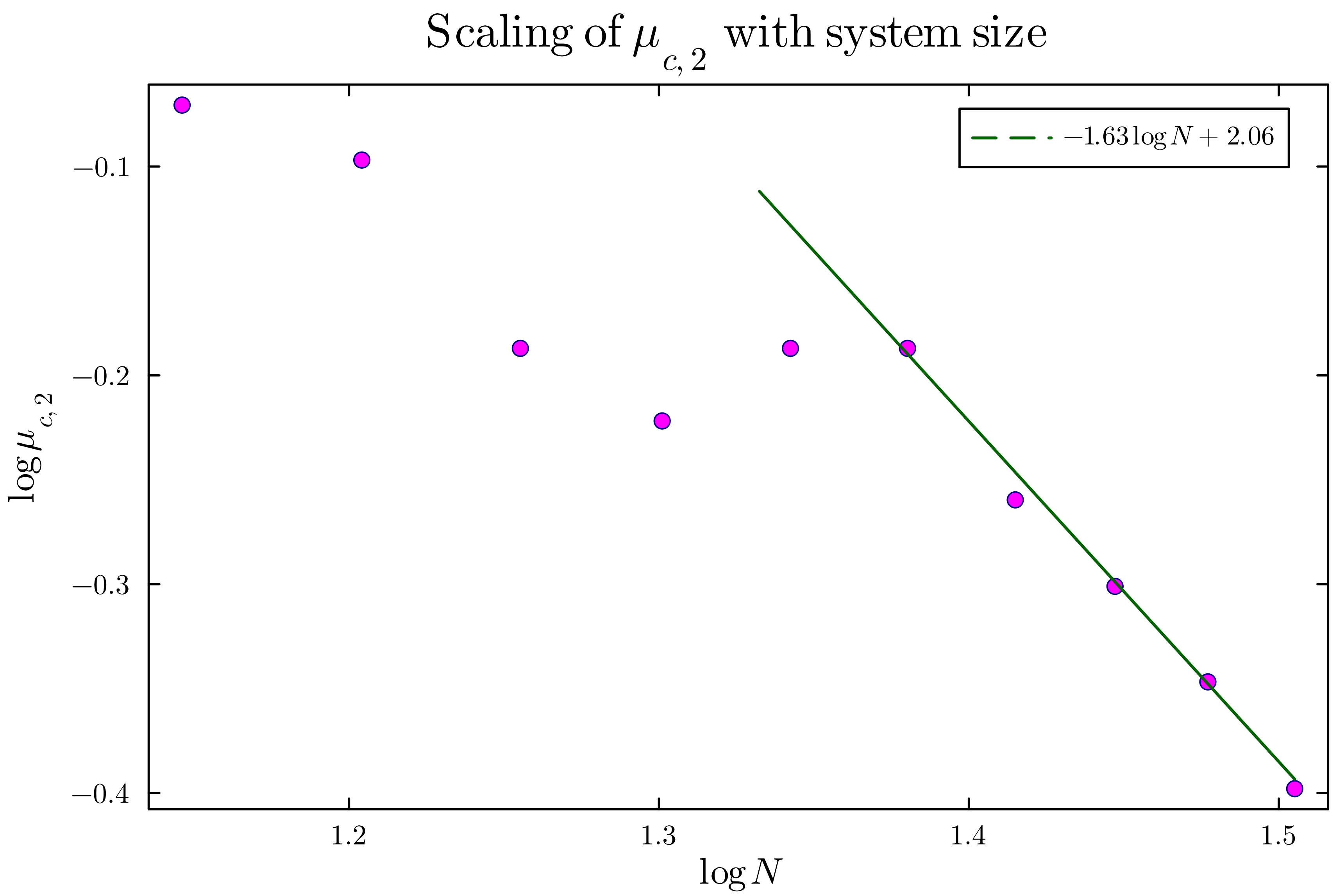
The Spectrum - Long Range





New feature (time-scale) arising from breaking RI (Lévy-ness)

Decrease in Thouless time as μ decreases



Critical $\mu_{c,2} \to$ Value at which $\tau_{\text{Th}} = 1$

$$\mu_{c,2}(N) \sim N^{-1.5}$$

The Underlying Mechanism

All this is fine, but what is *really* leading to chaotic transition?

Chaotic Transition!!

$\therefore \quad \exists$ a *few* $J_I \gg$ other $J_{I' \neq I}$

$$H = \sum_I J_I \Psi_I \approx \sum_{I \in \text{Large}} J_I \Psi_I$$

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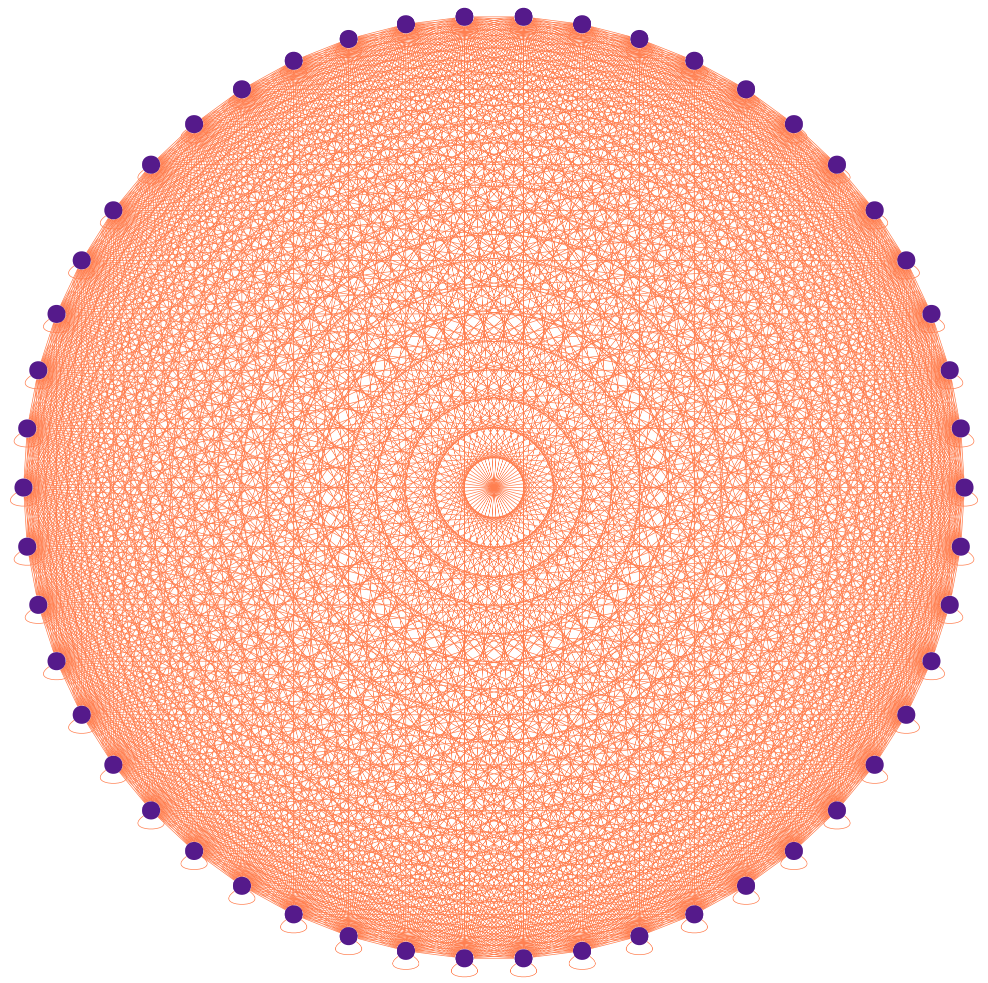
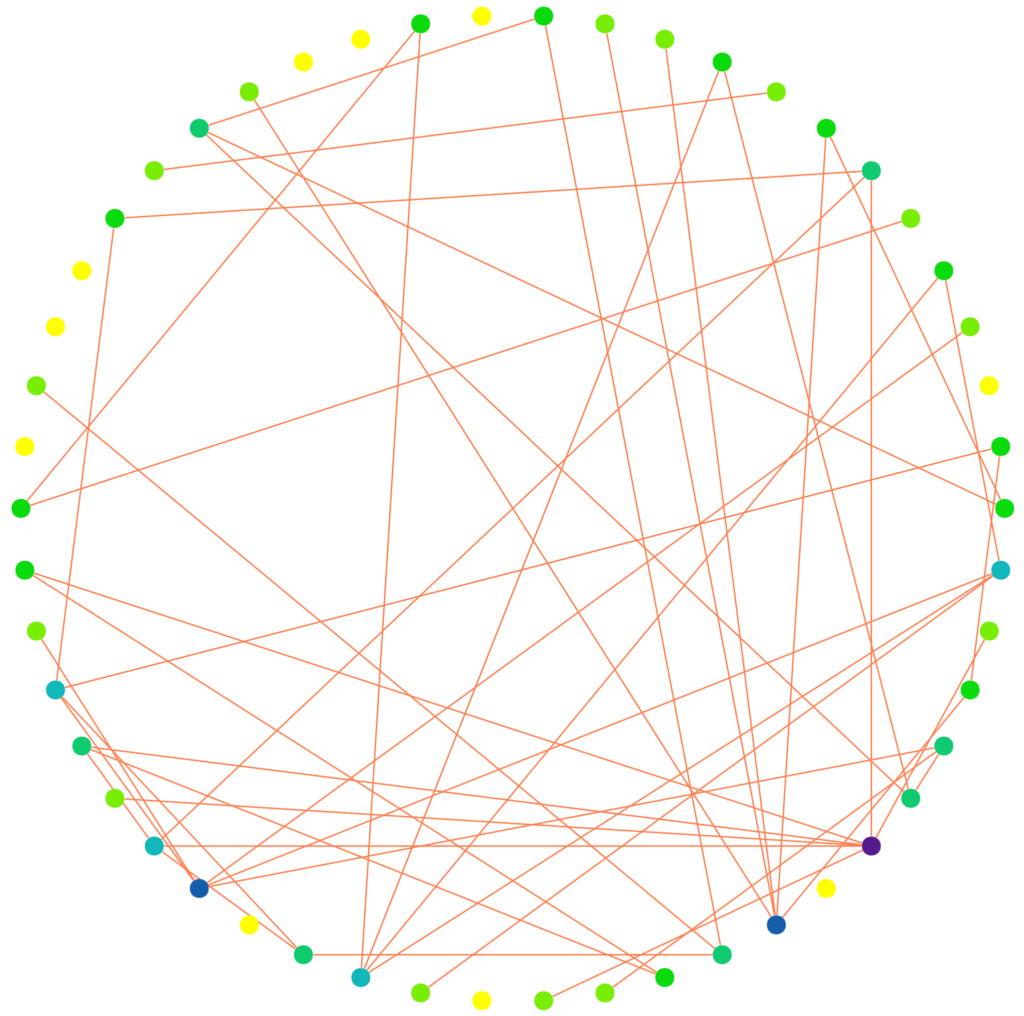
Gaussian

SYK





Lévy

SYK

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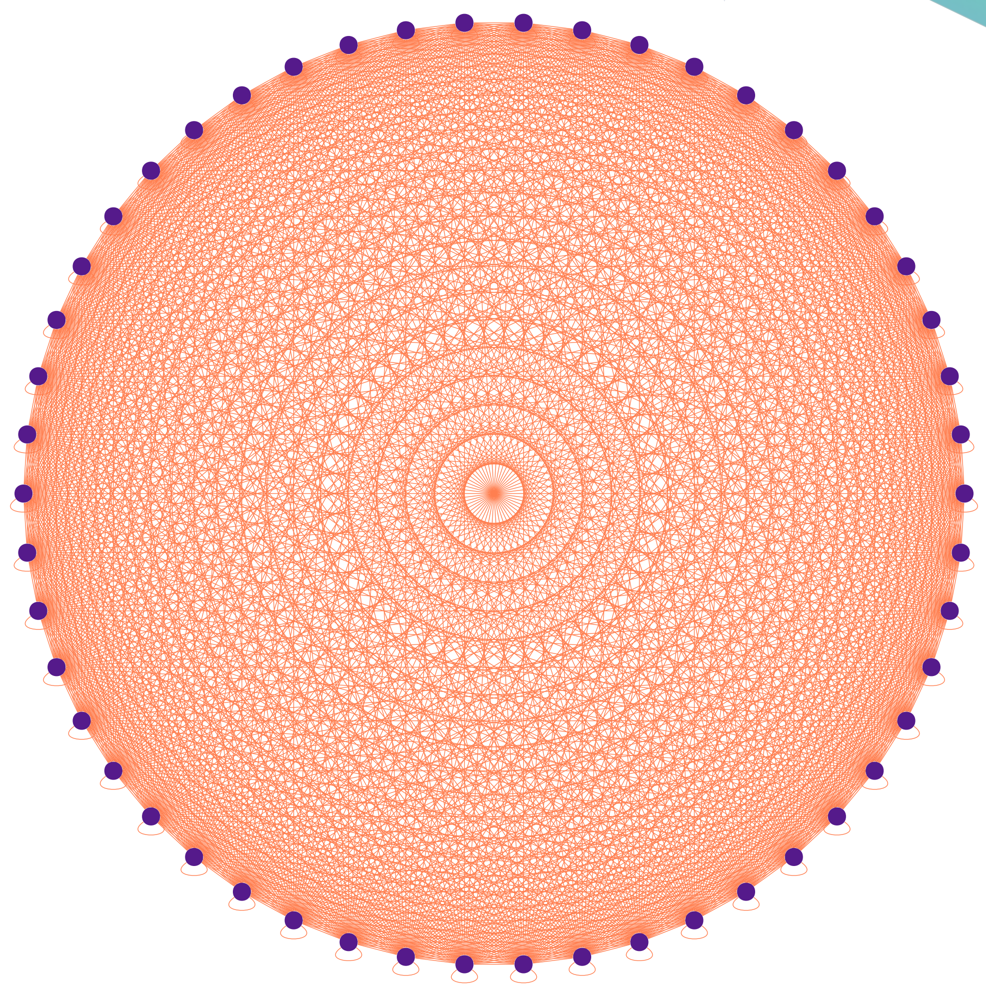
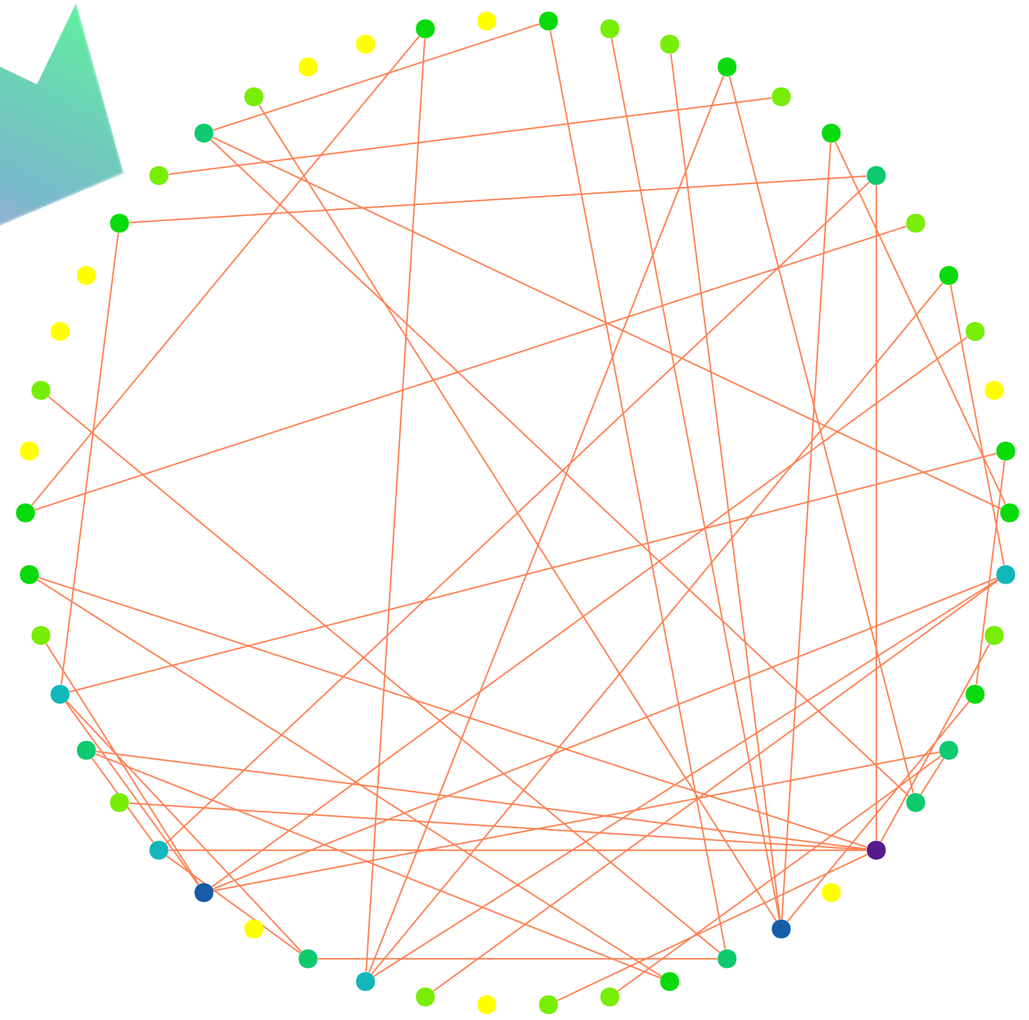
EMERGENT SYMMETRY

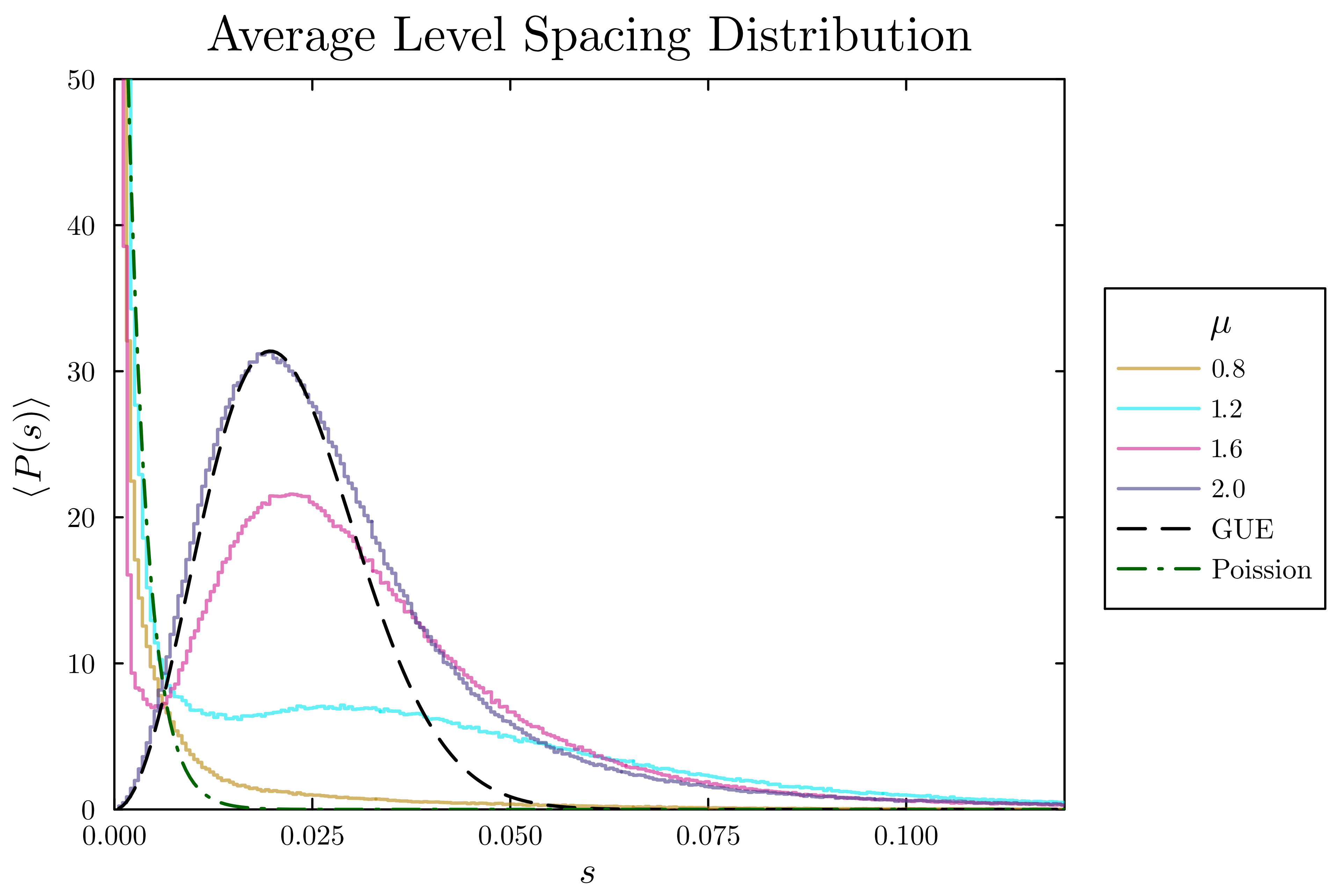
Gaussian SYK





Lévy SYK

ERDÖS-RÉNYI



Solving the model

Univariate Stable Distribution
John. P. Nolan

The main issue: $\langle |J_{i_1,i_2,\ldots,i_q}|^{1,2} \rangle \to \infty$ $\implies$ Usual approach fails outright

The saviour: STOCHASTIC REPRESENTATION OF STABLE DISTRIBUTION

In the SYK model: $H = \sum_I J_I \Psi_I \quad \to \sum_I \sum_{\sigma=1}^{\infty} \Gamma_{I,\sigma}^{-1/\mu} \bar{J}_{I,\sigma} \Psi_I \quad \to \sum_{\sigma=1}^{\infty} \sum_I \Gamma_{I,\sigma}^{-1/\mu} \bar{J}_{I,\sigma} \Psi_I$

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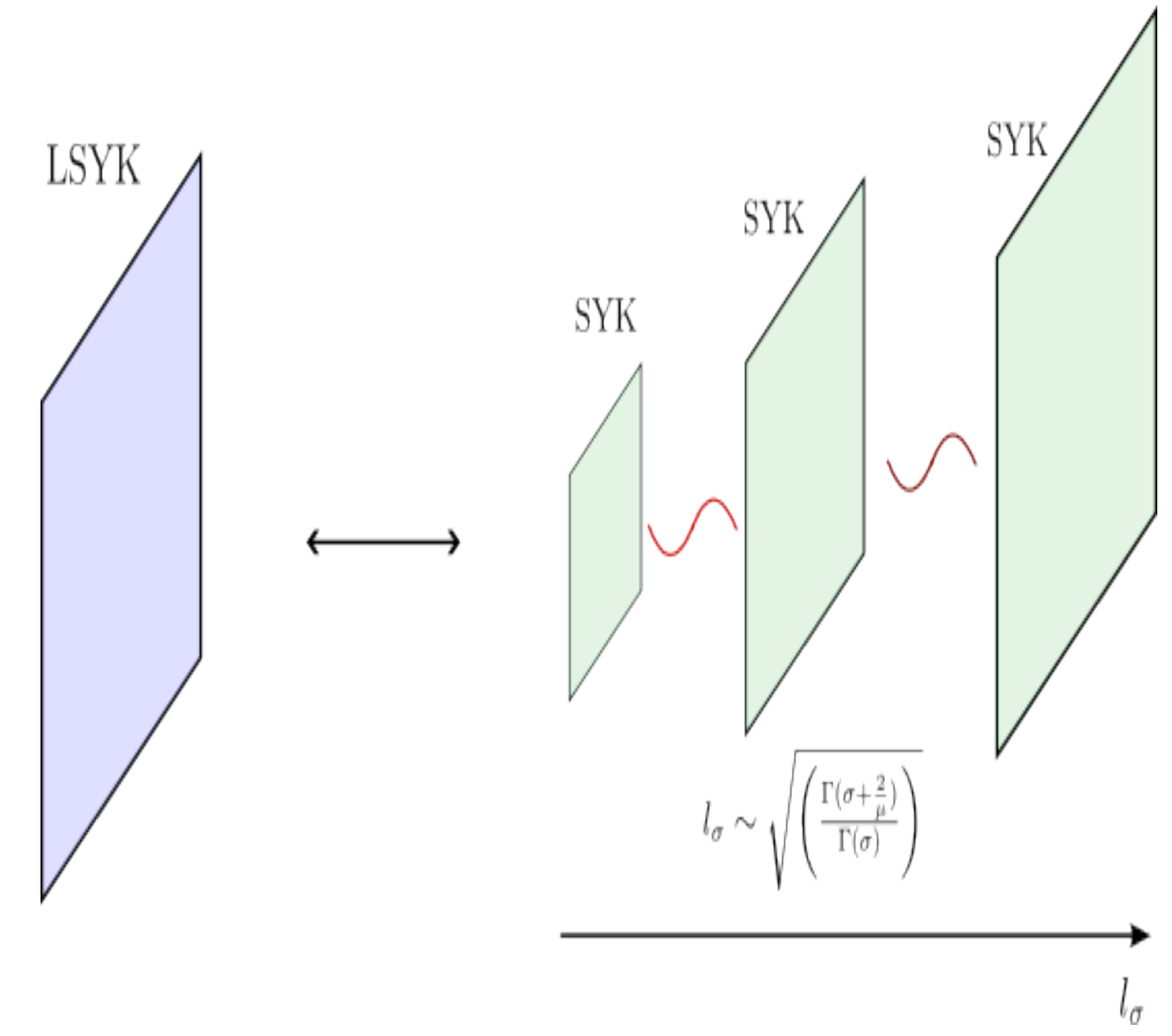
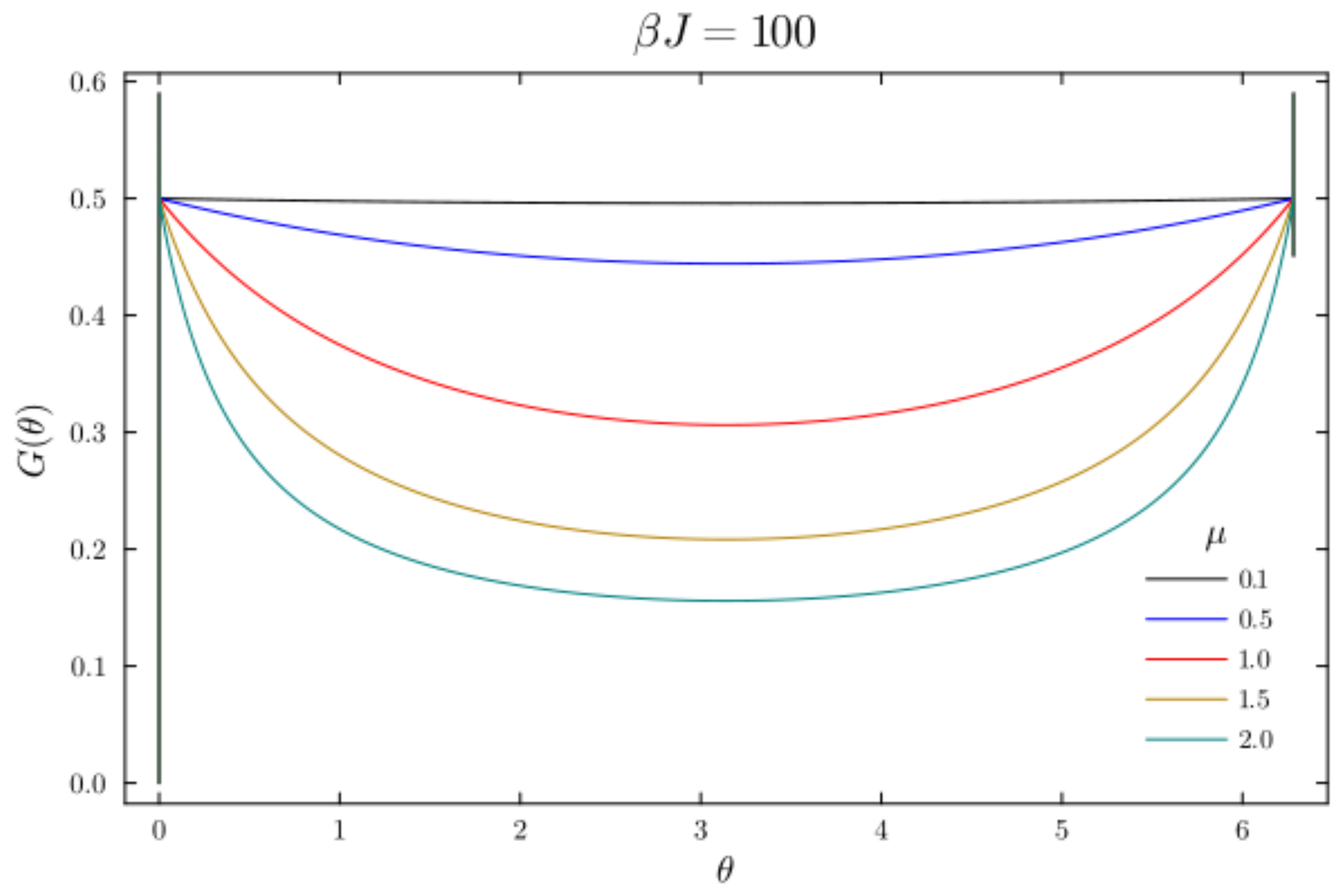
Lévy SYK is an infinite series of correlated Gaussian SYK!



$$G^{-1} = \partial_\tau - \Sigma \qquad \text{(In large}-N\text{ limit)}$$

$$\Sigma(\tau) = \frac{\mu}{2q} \mathcal{J}_L^2 \left(\int_0^\beta \beta \mathcal{J}_L^2 G^q(\tau') d\tau' \right)^{\frac{\mu}{2}-1} G(\tau)^{q-1}$$



Conclusions

We construct a model that has *a chaotic transition* **and** *can be solved!*

Other models such as

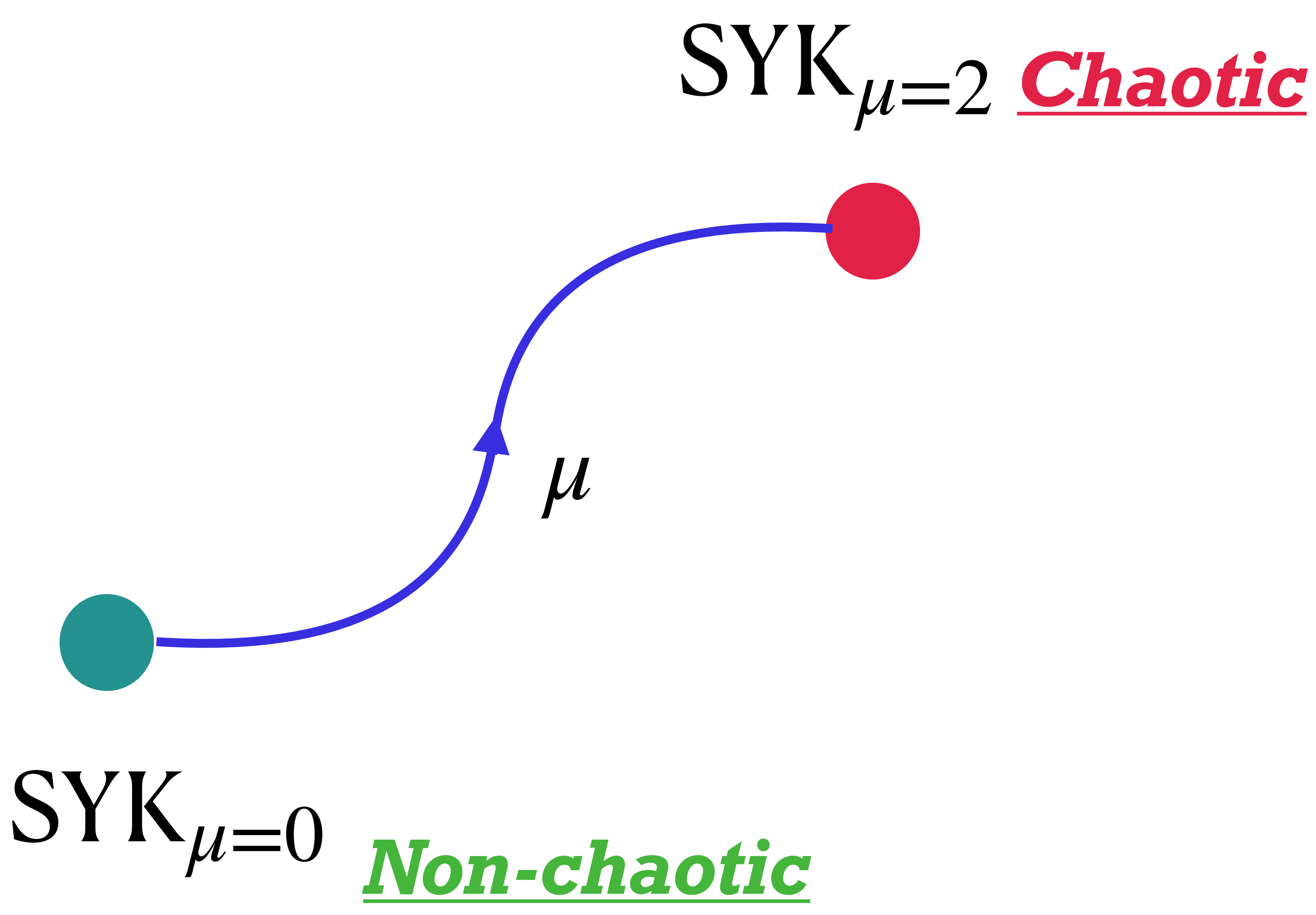
Sparse SYK, *SYK on Graphs* etc.

Not Solvable due to additional structure.

Other models such as

Binary SYK, SUSY *SYK* etc.

Almost equivalent to Gaussian SYK by CLT.



What more can we do?

1. Actually capture the transition *analytically* $\Longrightarrow \dfrac{1}{N}$ correction to SD eqns.

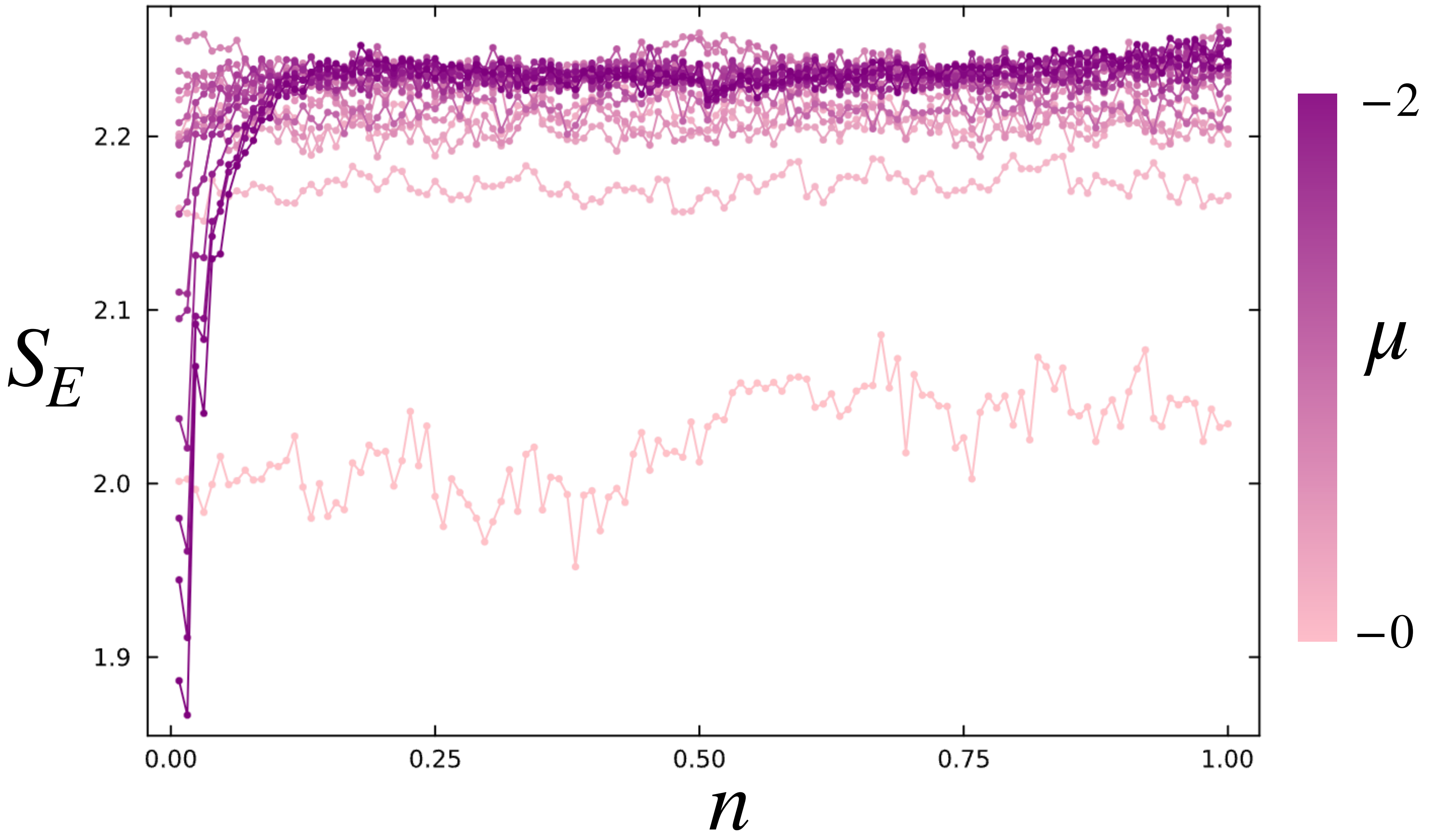
2. Different scaling other than large$-N$? Double-Scaled? *Triple*-scaled ?

3. Dynamics ? Higher-point functions ? Entanglement ? Mobility edge ? Localization ? ....

4. Dissipative Lévy SYK ?

JHEP 01 (2024) 094
BB, Cao, Nandy and Pathak

5. Lévy SYK quantum batteries?



Lots To Do!!!