

Universal Biology in Adaptation and Evolution: Multilevel Consistency, Dimension Reduction, and Fluctuation-Response

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Universal Biology

Life system as a universality class in nature



Phenomenological theory (a la thermodynamics)

→ general characteristics, universal laws

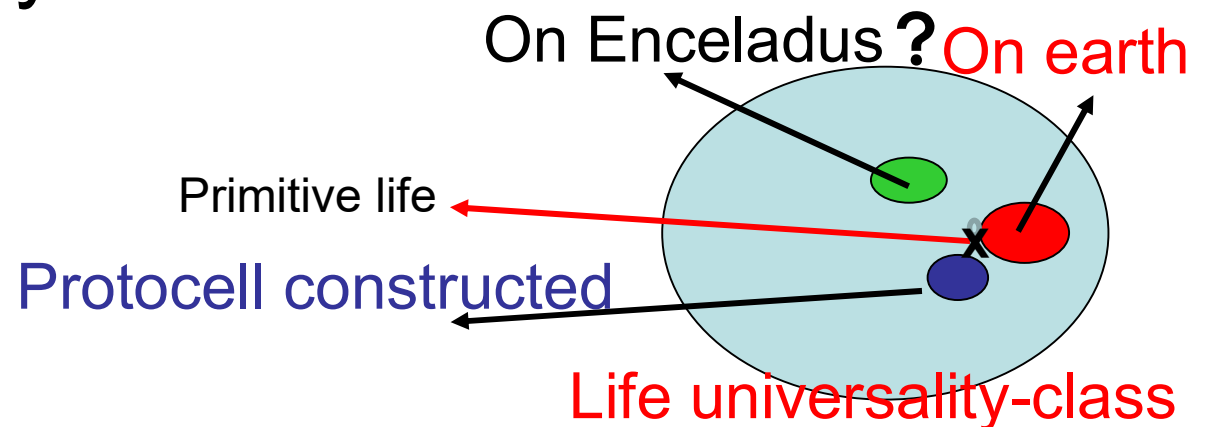
* Biology not restricted to those that happen to be evolved on earth

(coined originally by SF writer

小松左京

(Komatsu Sakyō)

at 1972



Universal Biology – proposed? 1968 by
Sakyo Komatsu (SF novelist)

- *Universal biology – science to explore universal patterns and possible variations of living organisms in this cosmos. It started to expand the end of last century (* i.e. 20c). Since then, characterization of life in terms of topological geometry (* dynamical systems?) has developed, and now, grand theory comparable to relativity is anticipated... ---*

Universal Biology Institute (in real world) launched
2016, Univ Tokyo

- Life ~ System that consists of diverse components and that maintains itself and can continue to produce itself --consequence→
- Guiding Principle--Micro-macro Consistency:
 - micro – many components (**high-dimensional**)
 - macro – unit to sustain/ reproduce as a whole (*low-dimensional description?*)

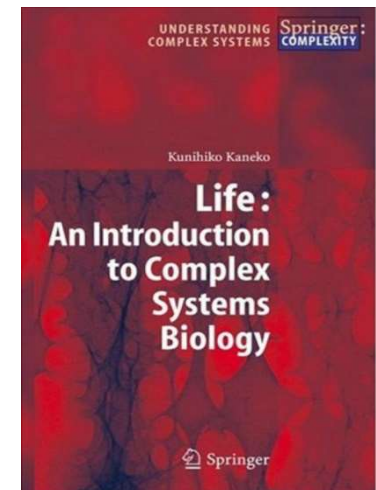
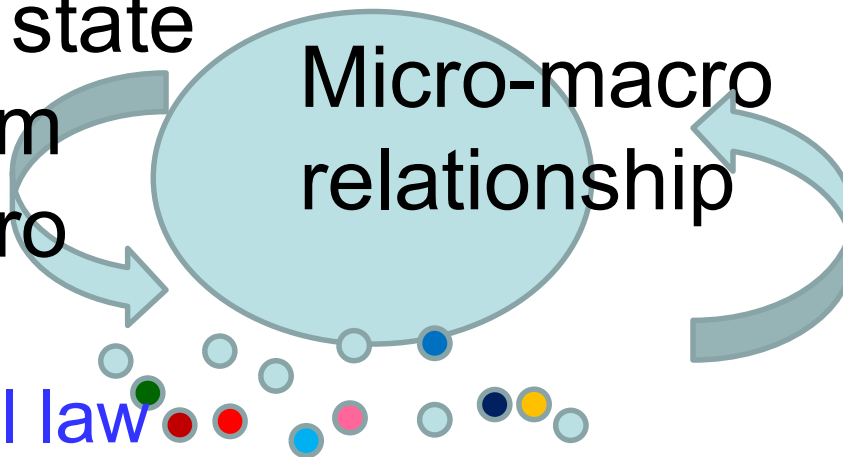
molecule – cell, cell-tissue etc.

Steady (growth) state

Constraint from macro to micro

Micro-macro relationship

Universal statistical law



Complex-systems Biology

Consistency between dynamics of different levels

(1) Cell reproduction vs molecule replication →

universal statistical laws in gene expression

(Furusawa et al, PRL 2003,2012, Biophysics 2006, KK et al, PRX2015)

(2) Adaptation → universal adaptation laws (Kashiwagi et al Plos One2005; Furusawa, KK Phys RevE2018)

(3) Differentiation: Cell vs multicellularity →

Oscillatory dynamics \Rightarrow pluripotency + cell-cell interaction → differentiation, loss of pluripotency

(KK&Yomo 1997, Furusawa&KK,1998,Science 2012)

(4) Genetic vs phenotypic changes →

Isogenic Phenotypic Variance by noise \propto variance by genetic change $V_g \propto$ Evolution Speed (plasticity)

Robustness to noise \sim to robustness to genetic

change, (PNAS03, PLoSOne07, Furusawa, KK, Interface2015, PRE 2018)

Part I: Consistency (with robustness) between molecule and cell levels :

→ Dimension Reduction in phenotypic dynamics by adaptation & evolution

→ Law in Adaptation and Evolution

Response Theory

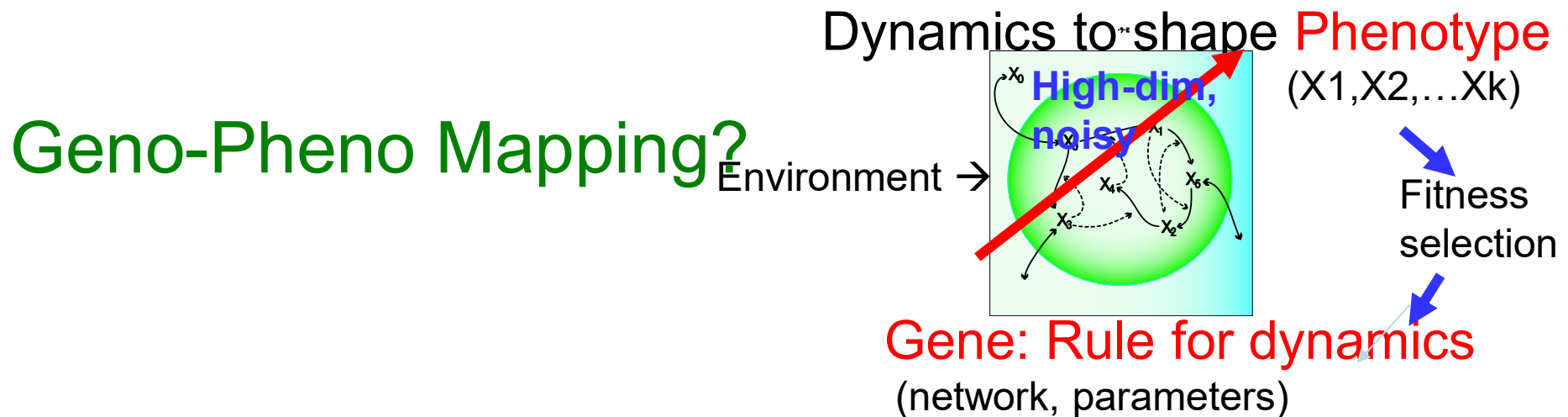
Part II: Evolutionary Fluctuation-Response Relationship

→ Pheno Variance by noise \propto that by mutation
 \propto evolution speed

Phenotypic Evolution is directed, constrained even before genetic change

- Basic Setup (Exp/Theory/Model)
- **Phenotype**=Abundances of each component (e.g., protein/mRNA) (~5000 dimensions)

Genotype- DNA seq, or rule for dynamics:



- * **Experiment:** transcription analysis of E Coli
- * **Model:** (i) catalytic reaction network for growth
(ii) Gene regulation net: (**high-dim dynamics**):
- * **Theory:** Low-dim constraint in high-dim states

Trivial(?) Law in Adaptation: Focus on steady-growth cells → universal constraint

all the components have to be roughly doubled within a cell division time)

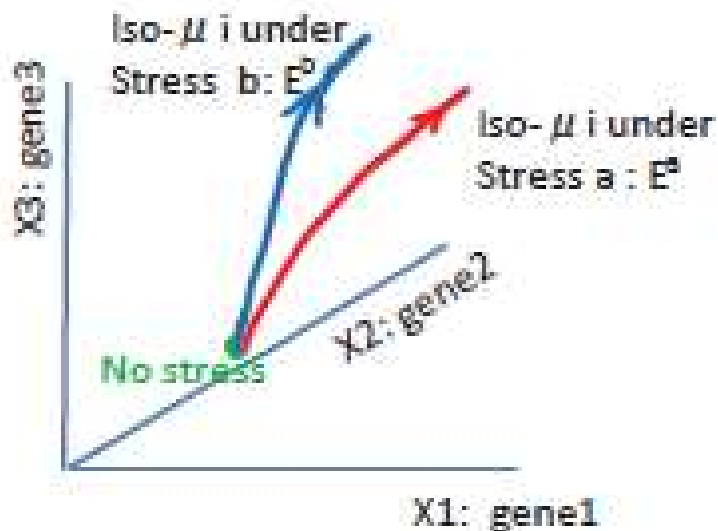
$N_i (i=1, \dots, M)$

$dN_i/dt = \mu_i N_i \rightarrow \exp(\mu_i t)$; all μ_i are equal;

→ (M-1) conditions → 1-dimensional line

M (e.g. proteins) $\sim (10^3 \sim 10^4)$
measurable by microarray

Adaptation/evolution progresses on an iso- μ_i -line ('quasi-static process') in an M-dimensional state space



Concentration $x_i = N_i/V$: $(dV/dt)/V = \mu$ (volume V)

Temporal change of concentration x (Any reaction dynamics)

$$dx_i/dt = f_i(\{x_j\}) - \mu x_i \text{ dilution}$$

Now, the stationary state is given by a fixed point condition

$$x_i^* = f_i(\{x_j^*\})/\mu$$

for all i .

As a convenience, denote $X = \log x$, and $f_i = x_i F_i$. Then,

$$dX_i/dt = F_i(\{X_j\}) - \mu$$

Response under different stress strength E

$$F_i(\{X_j^*(E)\}, E) = \mu(E).$$

Trivial so far

Linearization w.r.t $X(=\log x)$

KK, Furusawa, Yomo,
Phys Rev X(2015)

$$\sum_j J_{ij} \delta X_j(E) + \gamma_i \delta E = \delta \mu(E)$$

Jacobi matrix J_{ij} .

with $\gamma_i \equiv \frac{\partial F_i}{\partial E}$. ← Susceptibility to stress

In the linear regime $\delta \mu = \alpha \delta E$.

Trivial
+ linearization

$$\delta X_j(E) = \delta \mu(E) \times \sum_i L_{ji} (1 - \gamma_i / \alpha) \quad L = J^{-1}.$$

No evolution yet

$$\frac{\delta X_j(E)}{\delta X_j(E')} = \frac{\delta \mu(E)}{\delta \mu(E')} = \text{indep't of } j$$

Common proportionality for log-expression change δX_j for all components j

← Steady-growth sustaining all components + Linear

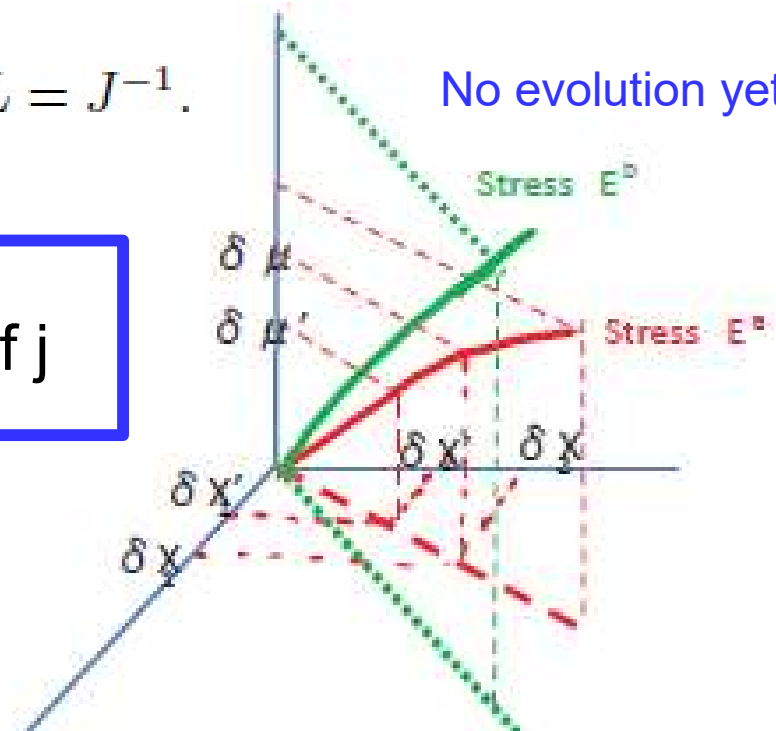
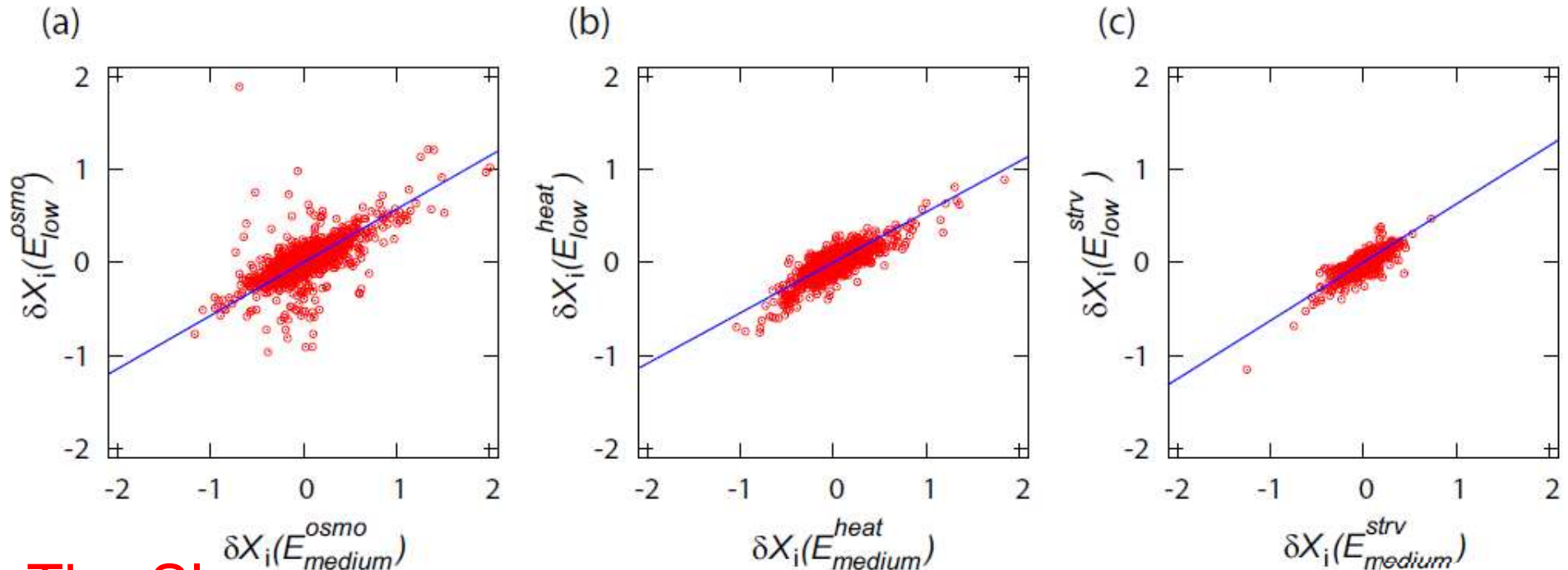


Fig. 2b

Put E Coli under different strength of stress conditions; Measure gene expressions

$$\log(x_i(E)/x_i^O) \text{ and } \log(x_i(E')/x_i^O)$$

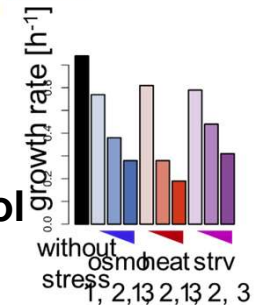


The Slope agrees with the growth rate change $\delta\mu'/\delta\mu$

A: low vs medium osmo
 B: low vs medium heat
 C: low vs medium starvation

$\delta X^E, \delta X^{E'}$
 over few thousand genes

Data from Matsumoto et al
 BMC Evol Biol 12013



KK, Furusawa, Yomo,
 Phys Rev X (2015)

Linearization works for too(?) broad regime

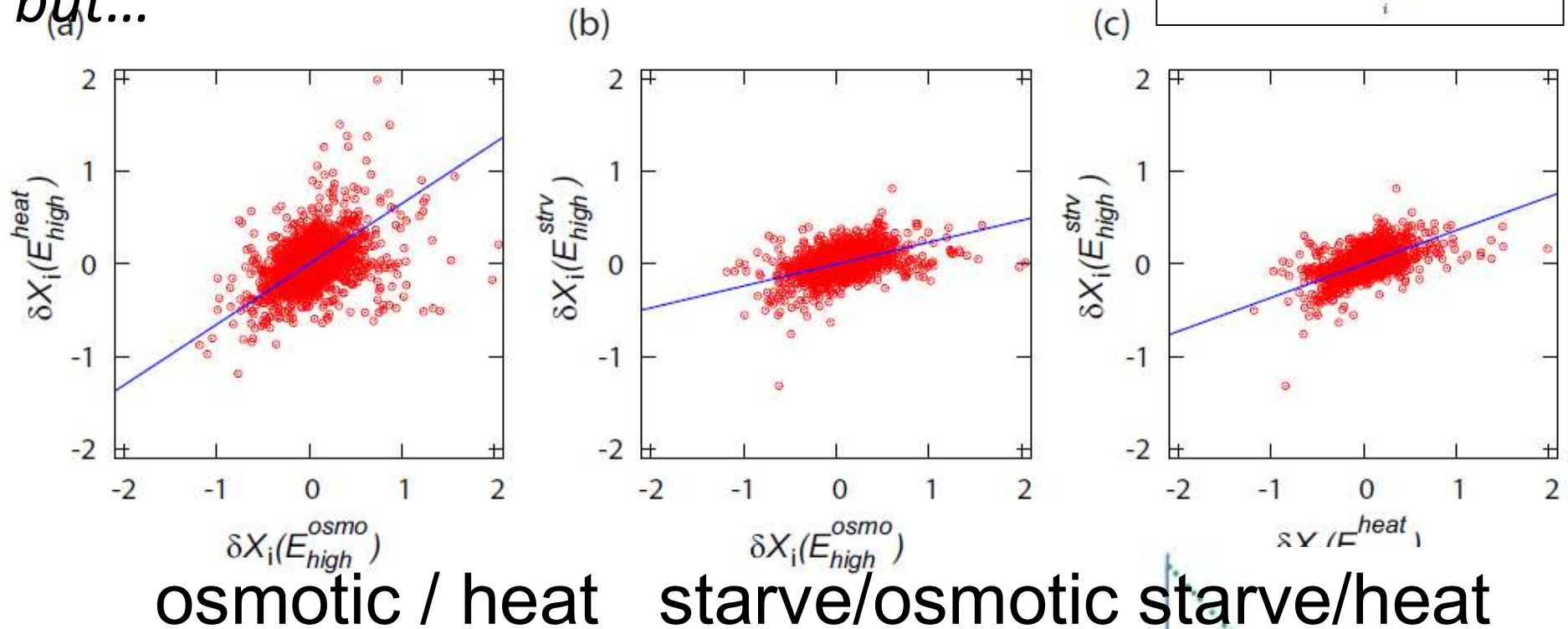
Across Different types of stresses:

$$\gamma_i \equiv \frac{\partial F_i}{\partial E}$$

$\gamma_i(a)$ depends on type a so correlation not expected,

but...

$$\delta X_j(E) = \delta\mu(E) \times \sum_i L_{ji}(1 - \gamma_i/\alpha)$$



Still highly correlated

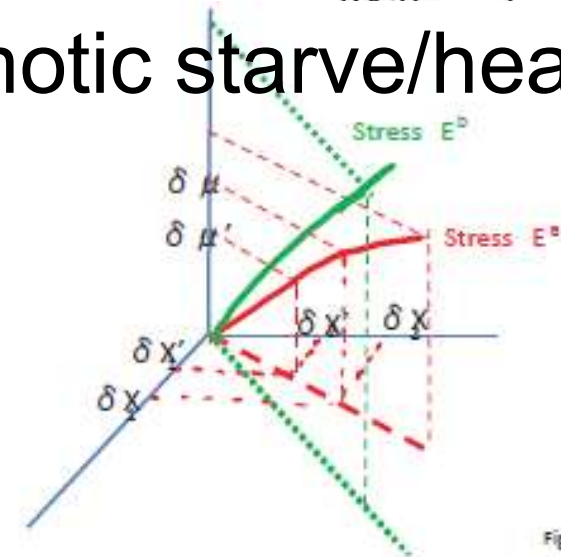
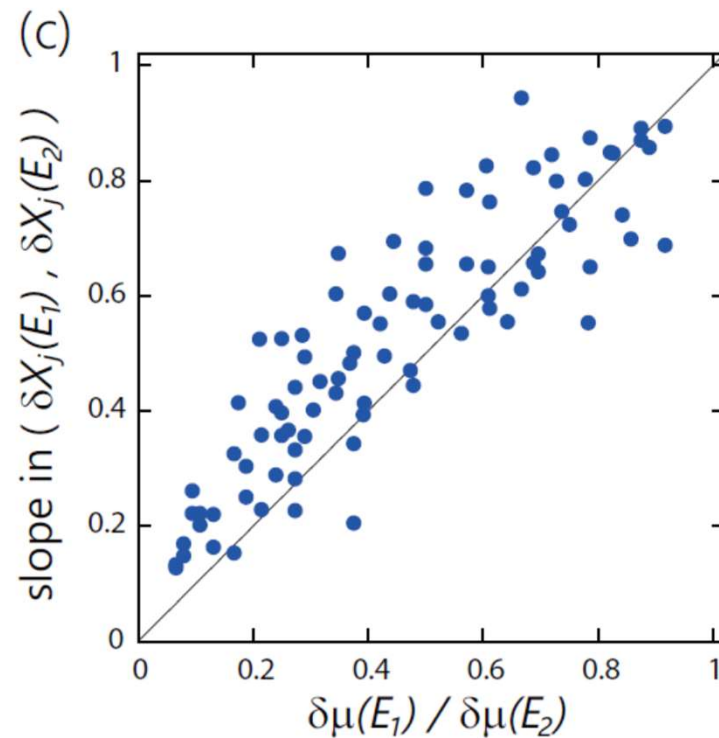
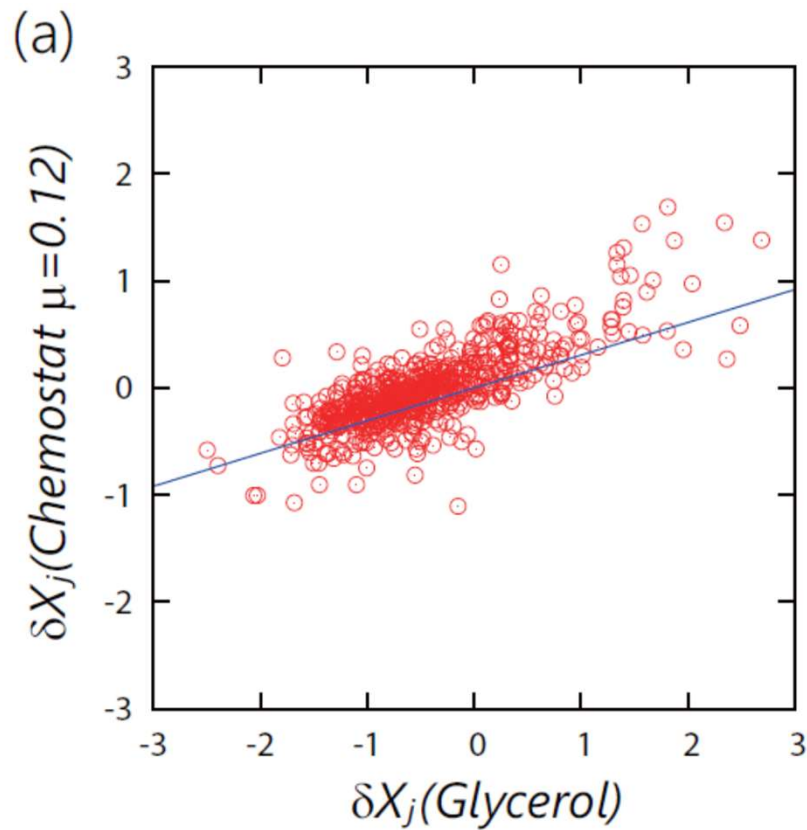


Fig. 2b

Better(?) confirmed in protein expression changes across different environmental conditions (based on the data by Heinemann) 20 different conditions on E Coli



Non-trivial point: Emergent macroscopic Linearity

- (1) Large Linear Regime?
- (2) Validity across different environmental condition?

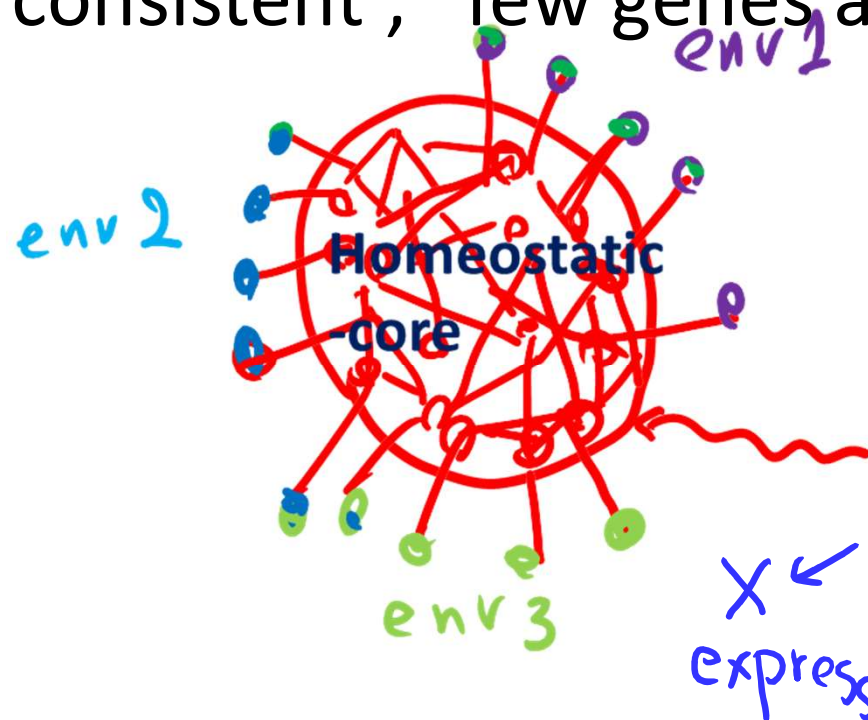
achieved in an evolved system ?

Check by simulations of toy models with high-dim dynamical systems

- High-dimensional adaptation system (diversity) is important for **expanded liner regime** and applicability for **diverse environmental changes**

* emergence of 'collective' slow variable (Image)

homeostatic core (major parts) mutually stabilize; growth-rate as 'mean-field'; self-consistent ; few genes absorb environmental stresses



Relevant for robustness of a high-dimensional state

Core part
(no direct

$$d\mu = C dX + \sum_{\alpha} \delta_{\alpha} dE_{\alpha}$$

α : Environment

Examine by Toy Cell Model with Catalytic Reaction Network

(Cf. Furusawa, KK, PRL 2003, 2012)

■ **k species of chemicals** , $X_0 \cdots X_{k-1}$

number --- $n_0, n_1 \dots n_{k-1}$

■ random catalytic reaction network

with the path rate p

for the reaction $X_i + X_j \xrightarrow{p} X_k + X_l$

□ **Resource chemicals (<- environment) are transported with the aid of a given catalyst, transporter**

■ resource chemicals are thus transformed into impenetrable chemicals, leading to the growth.

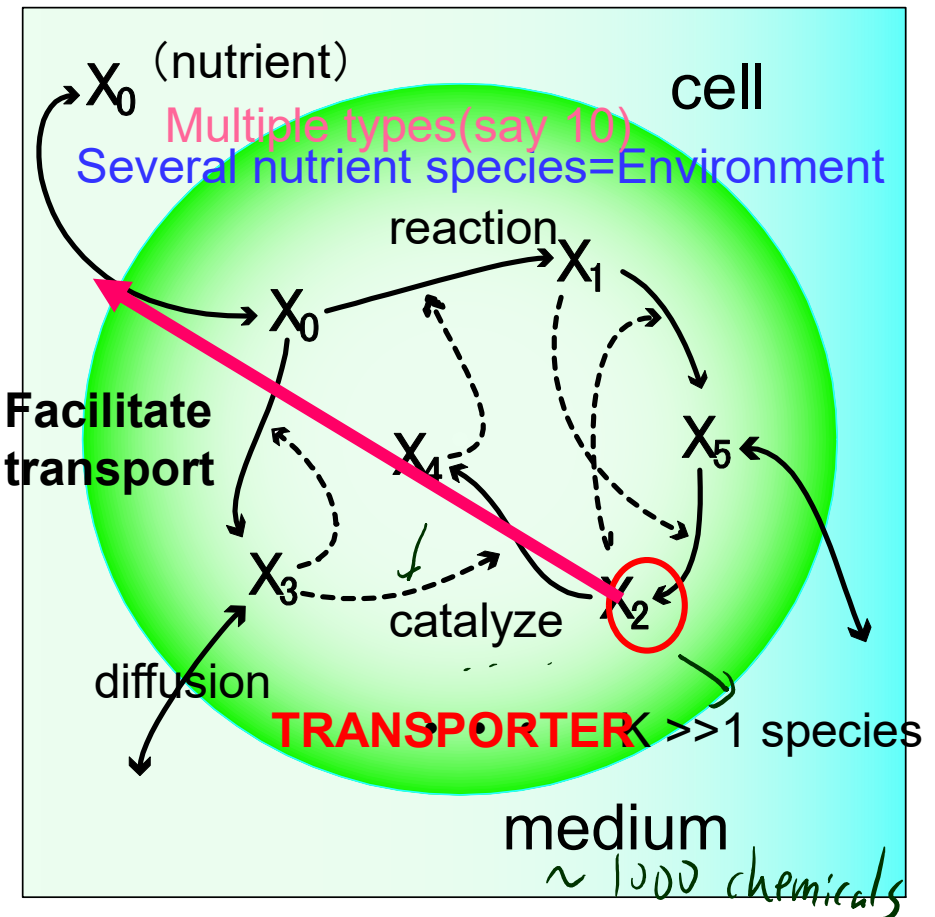
■ $N = \sum n_i$ exceeds N_{\max} (model 1)

■ **Genotype: Network;**

■ **Fitness: e.g., abundances of given component**

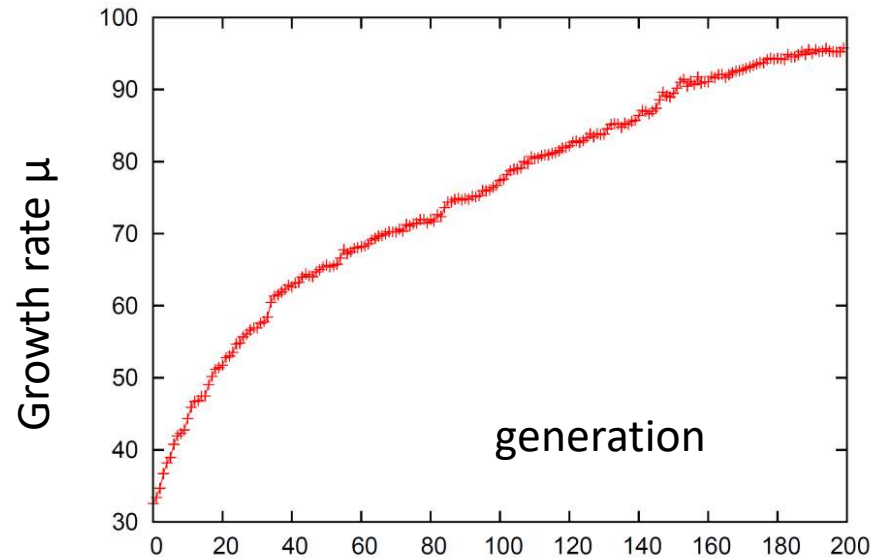
■ **Evolution: Mutate reaction paths, and select those with higher fitness**

model



$dX_1/dt \propto X_0 X_4$; rate equation;
Stochastic model here

Evolve Network to increase the growth rate under given resource condition



evolution under the resource environment
with concentrations $i=1,2,\dots,10$ e.g., (e_0, e_0, \dots, e_0)

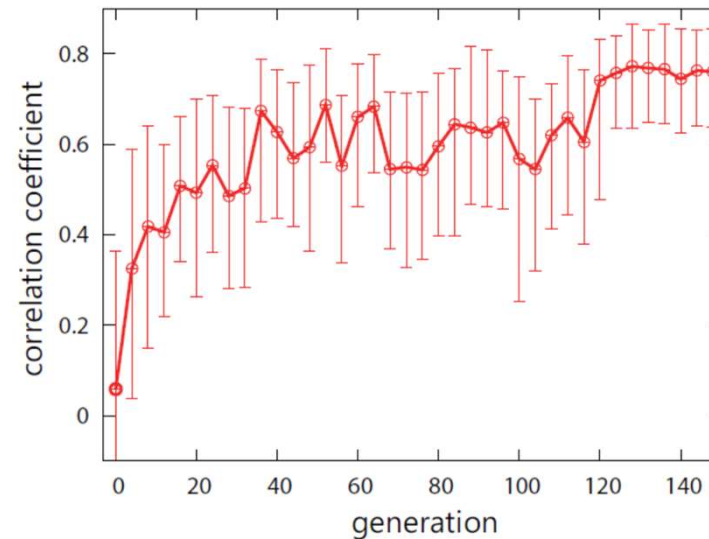
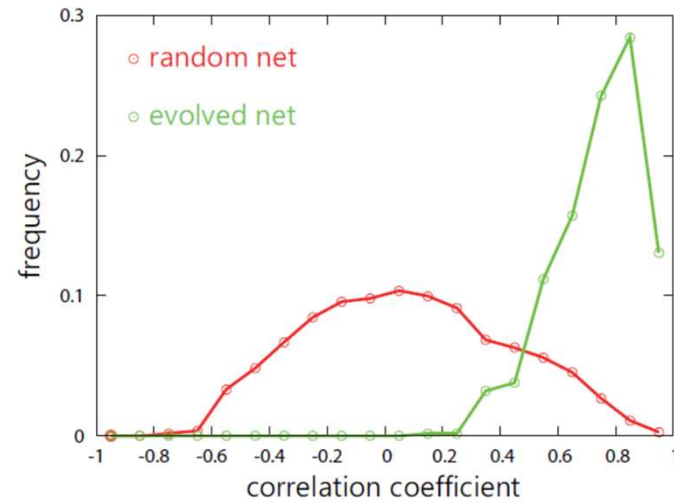
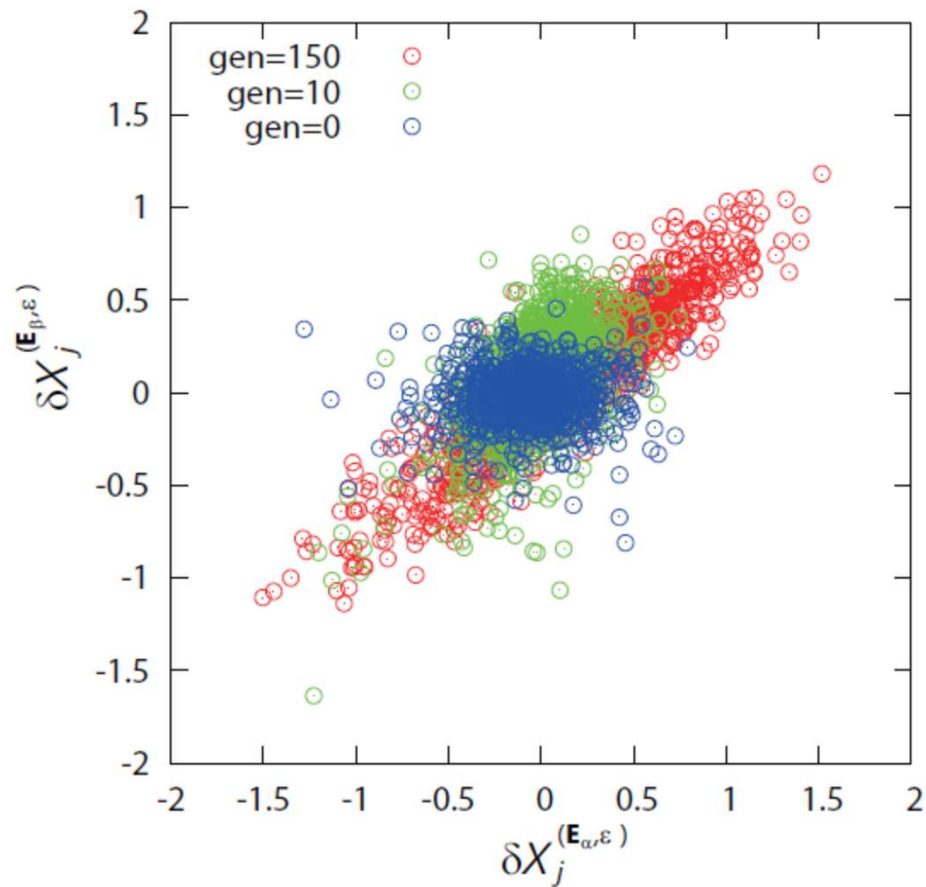
Then put an environment

$$\text{Env} = \lambda (e_1, e_2, e_3, \dots, e_{10}) + (1-\lambda) (e_0, e_0, \dots, e_0)$$

$-1 < e_1, e_2, \dots < 1$ (randomly chosen)

Check the change in concentrations and growth rates against λ

Evolution shapes Global Proportionality across different environmental conditions

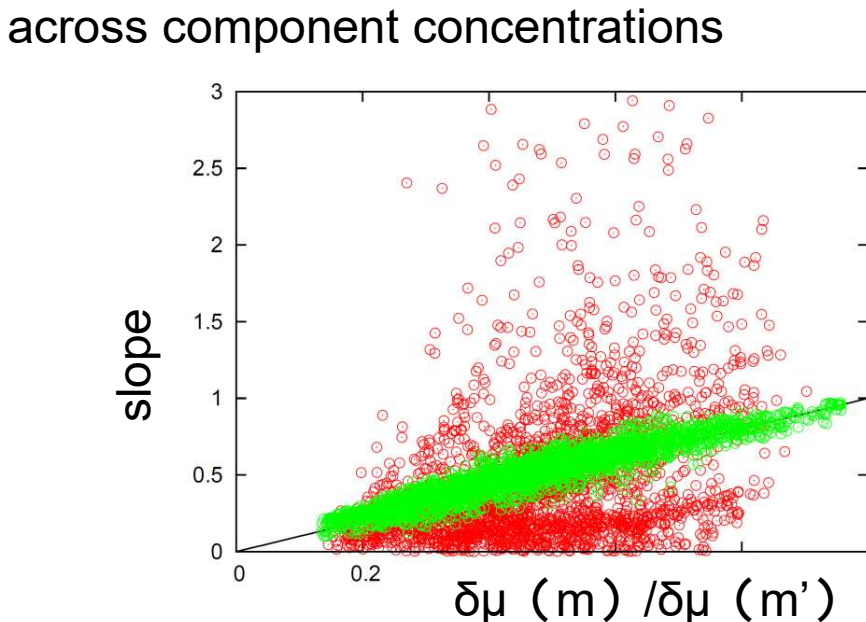
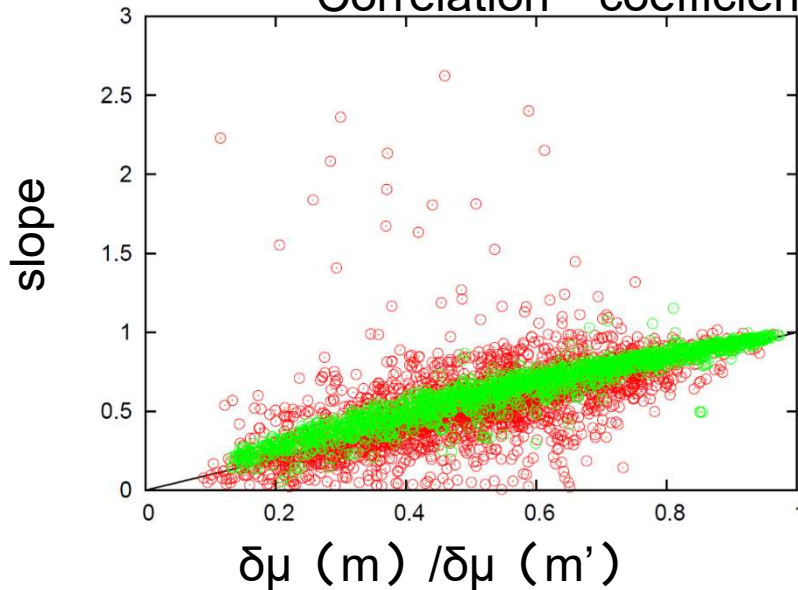
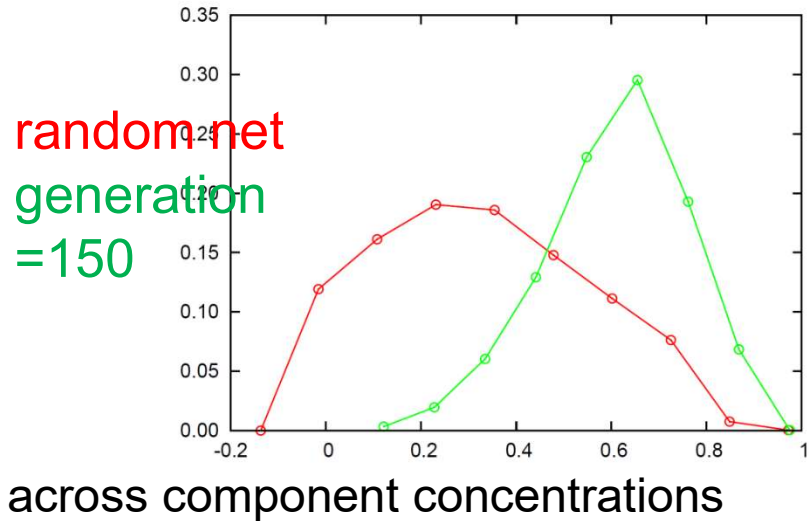
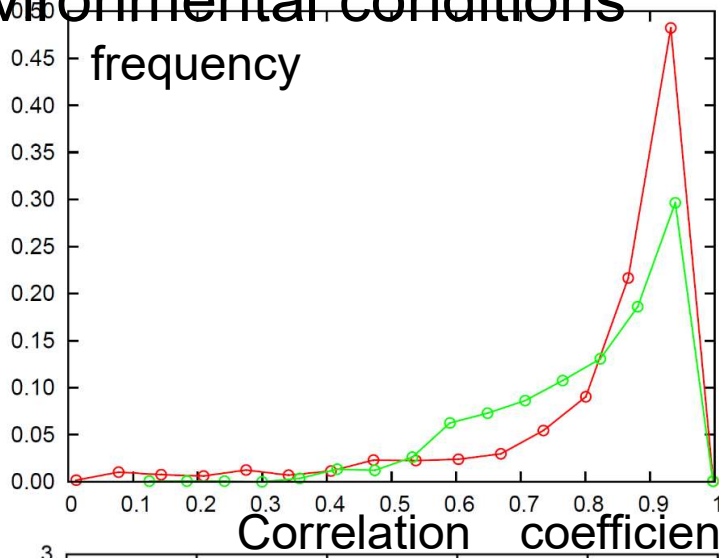


Furusawa, KK, Phys.Rev E 2018
KK, Furusawa, Ann Rev Biophys 2018

After evolution, correlation across different env cond. is increased, and slope-growth-rate linearity is enhanced

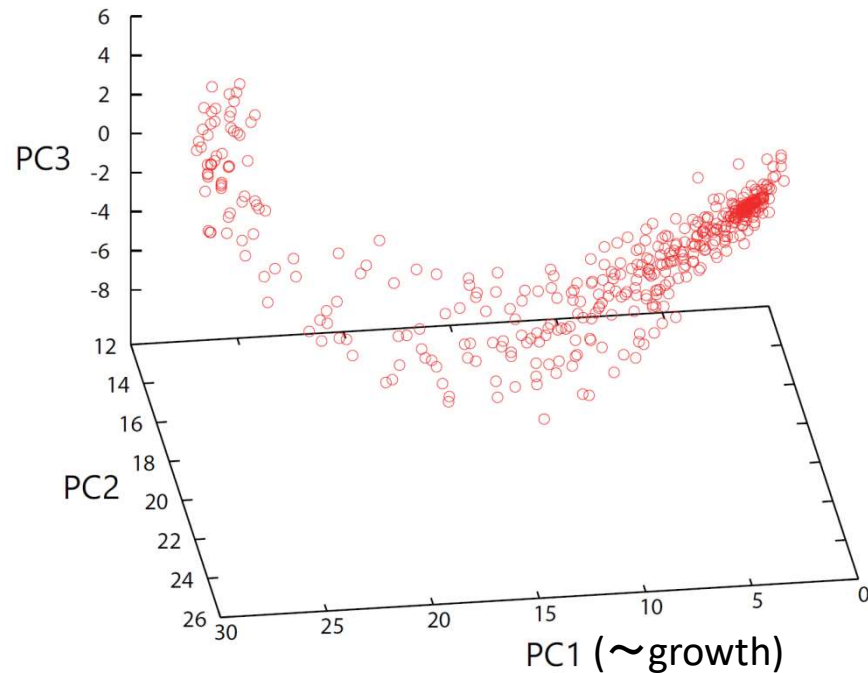
Between same environmental conditions

Across different env conditions

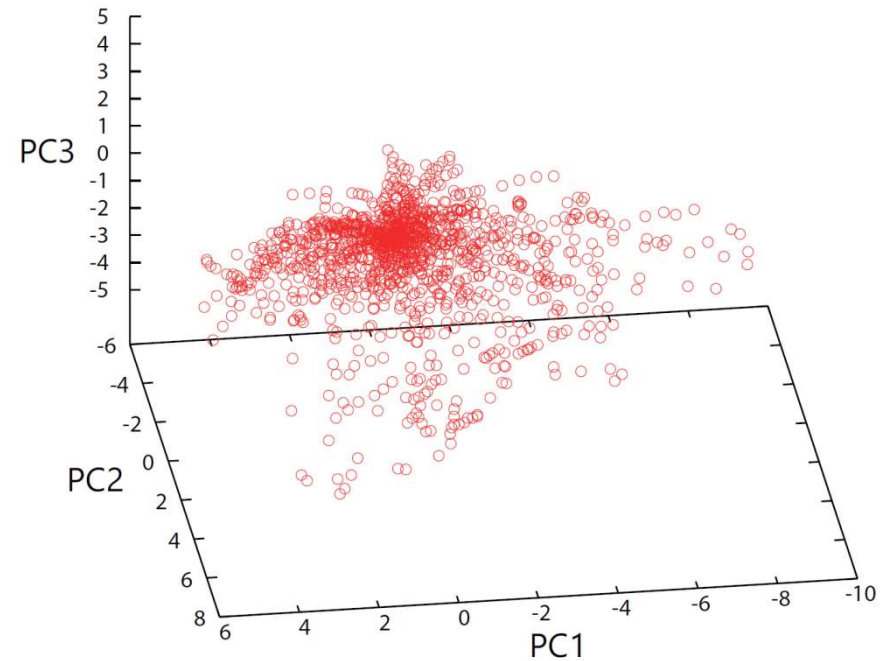


Phenotypic constraint on a low-dimensional space

After evolution

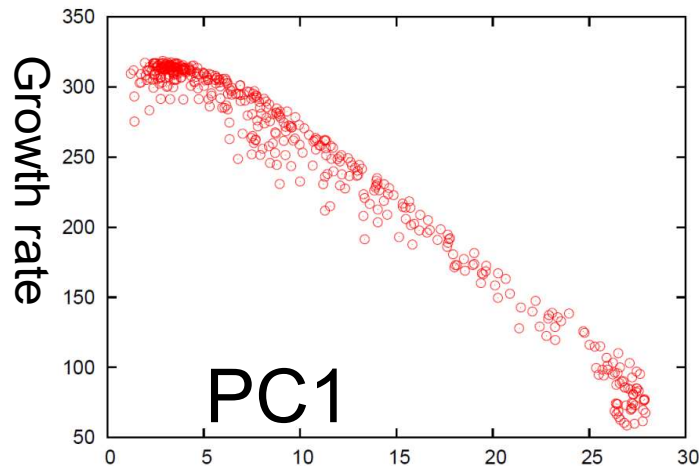
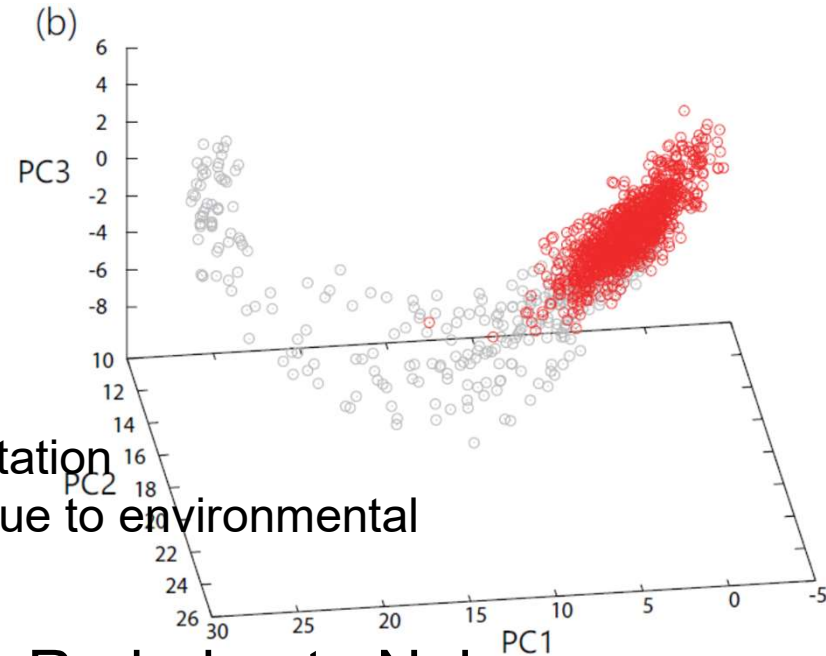
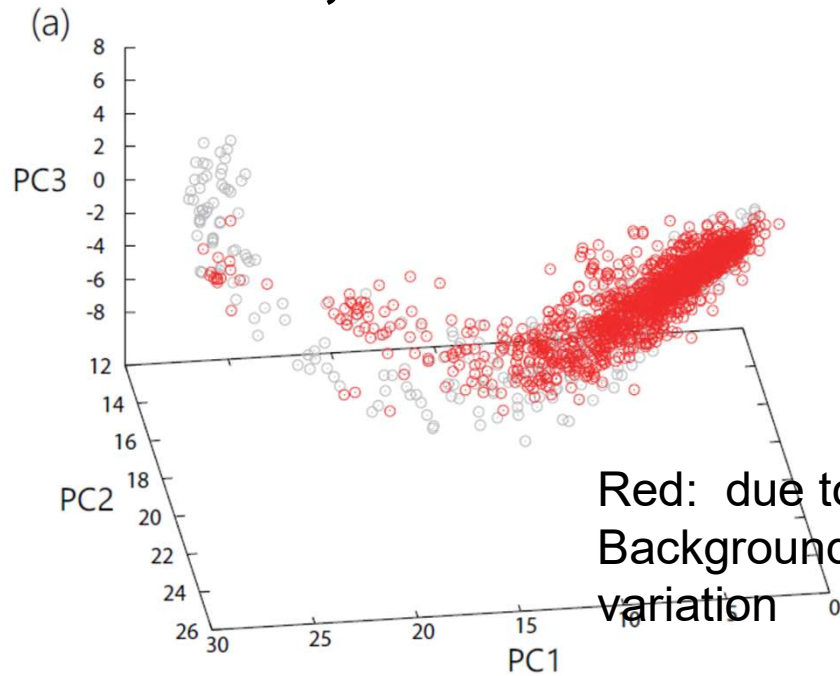


Random network



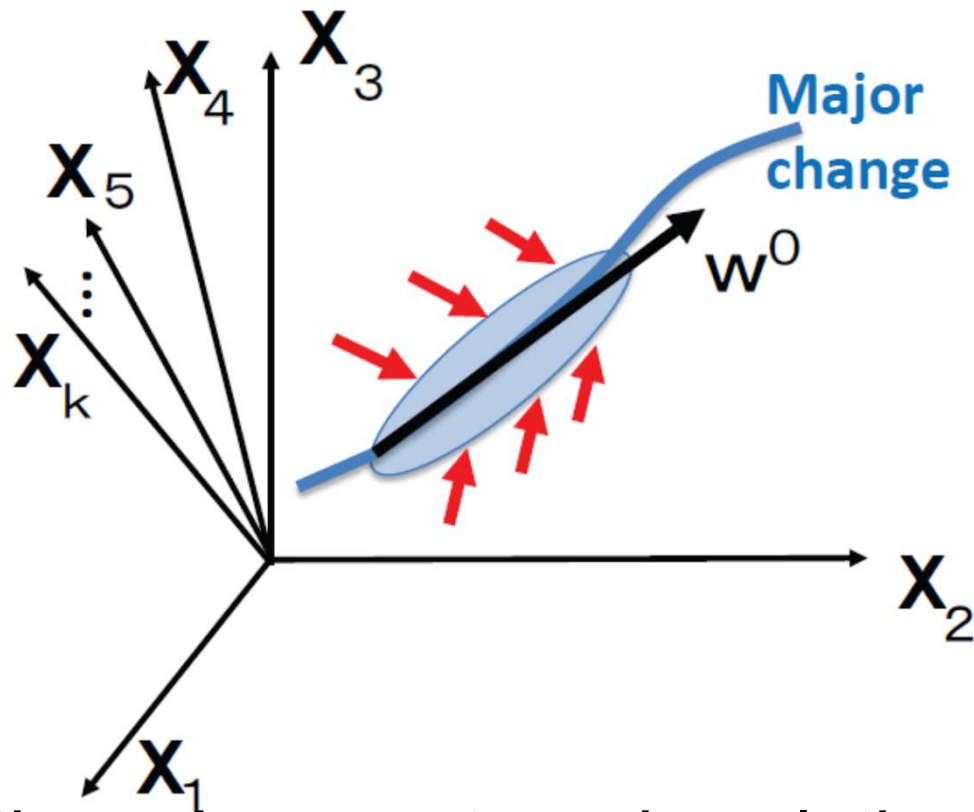
After evolution, the environmental response is constrained on a low-dimensional phenotype space.

Phenotypic change due to environmental variation, mutation, noise are constrained along a major axis



?Phenotypic change occurs along a common slow-manifold

Formation of Dominant Mode Along Major Axis



Robust to perturbations – strong attraction from most directions
except one direction along which evolution progresses

(Both environment- and evolution- induced) changes in high-dimensional phenotype space are constrained along low-dimensional slow-manifold

Formulation and Consequence of Hypthesis

Recall
$$\sum_j J_{ij} \delta X_j(E) + \gamma_i \delta E = \delta \mu(E)$$

with $\gamma_i \equiv \frac{\partial F_i}{\partial E}$.
$$\delta \mathbf{X} = \mathbf{L}(\delta \mu \mathbf{I} - \gamma \delta E)$$

- $\gamma(\mathbf{E})$: susceptibility to environment change

Slow manifold Hypothesis – Only the smallest eigenvalue in \mathbf{J} (or largest in $\mathbf{L}=1/\mathbf{J}$) contributes
Most changes occur along such slow manifold

$$\delta \mathbf{X} = \lambda^0 \mathbf{w}_0 (\delta \mu (\mathbf{v}_0 \cdot \mathbf{I}) - (\mathbf{v}_0 \cdot \gamma) \delta E).$$

Projection to this manifold

\mathbf{w}^0 (\mathbf{v}^0) right(left) eigenvector for the smallest eigenvalue, i.e., Projection to this slow manifold

$$\frac{\delta \mathbf{X}(\mathbf{E})}{\delta \mathbf{X}(\mathbf{E}')} = \frac{\delta \mu(E) - (\mathbf{v}_0 \cdot \gamma(\mathbf{E})) \delta E / (\mathbf{v}_0 \cdot \mathbf{I})}{\delta \mu(E') - (\mathbf{v}_0 \cdot \gamma(\mathbf{E}')) \delta E' / (\mathbf{v}_0 \cdot \mathbf{I})}$$

$\mathbf{v}_0 \cdot \mathbf{v}_0$
small

Consequence of Slow-Manifold Hypothesis (cont'd)

→ Slow manifold is roughly orthogonal to $\boldsymbol{\gamma}$

$$\boldsymbol{\gamma} \cdot \mathbf{v}_0 \sim 0$$

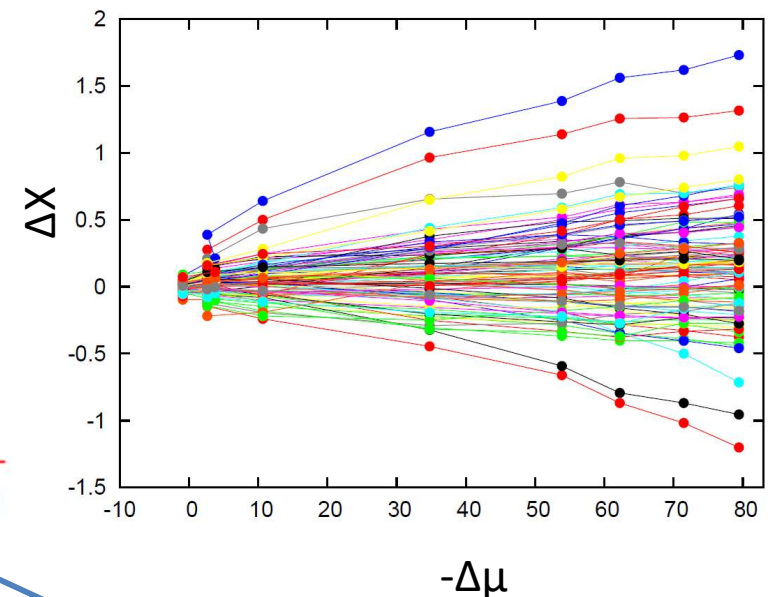
$$\rightarrow \delta \hat{X} = \lambda^0 \delta \mu \mathbf{w}^0$$

Or, from the linear approximation

$$\delta E = \delta \mu / \alpha(E)$$

$$\frac{\delta X(E)}{\delta X(E')} = \frac{\delta \mu(E)}{\delta \mu(E')} \frac{(1 - (\mathbf{v}_0 \cdot \boldsymbol{\gamma}(E)) / (\alpha \mathbf{v}_0 \cdot \mathbf{I}))}{(1 - (\mathbf{v}_0 \cdot \boldsymbol{\gamma}(E')) / (\alpha \mathbf{v}_0 \cdot \mathbf{I}))}$$

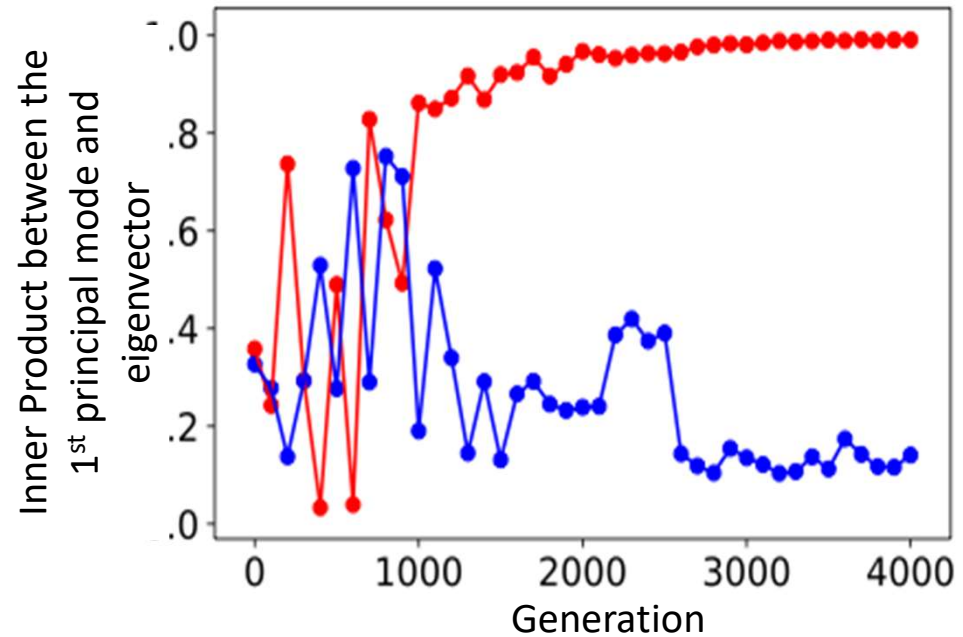
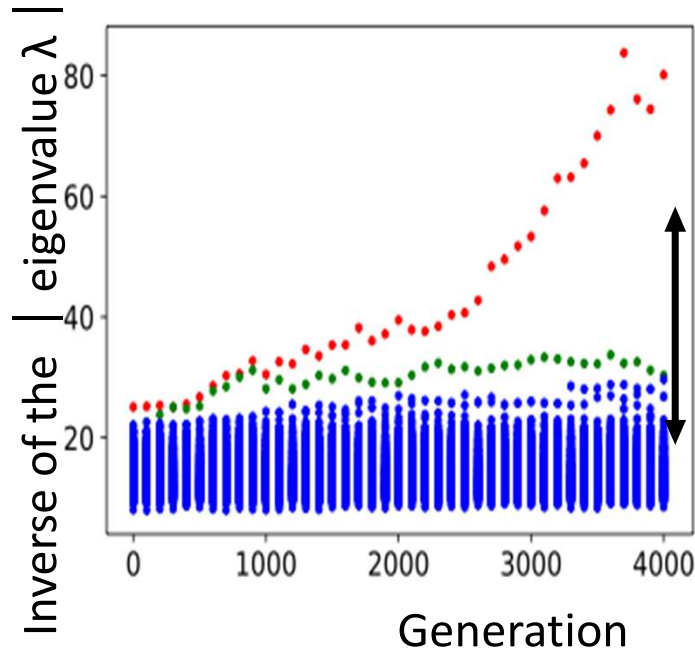
Correction in proportion coefficient



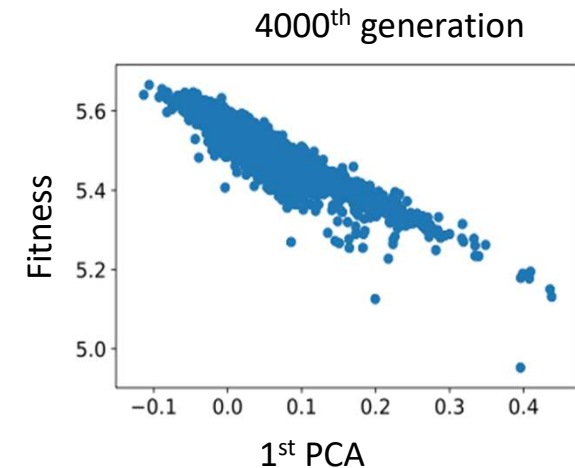
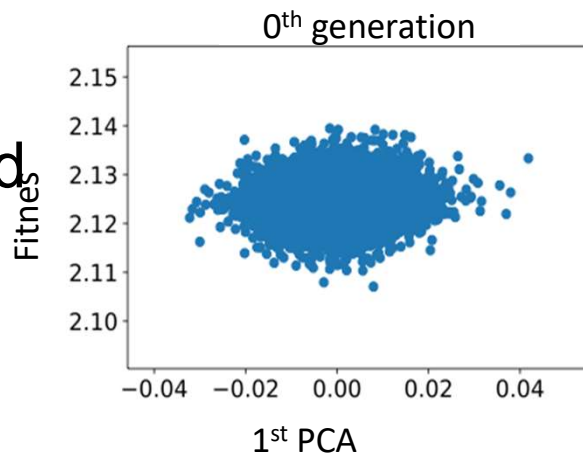
Separation of slowest mode in catalytic reaction net model

Eigenvalues of $J_{ii} = (\partial \dot{X}_i / \partial X_j)_{\mathbf{X}_i = \mathbf{X}_i^*}$

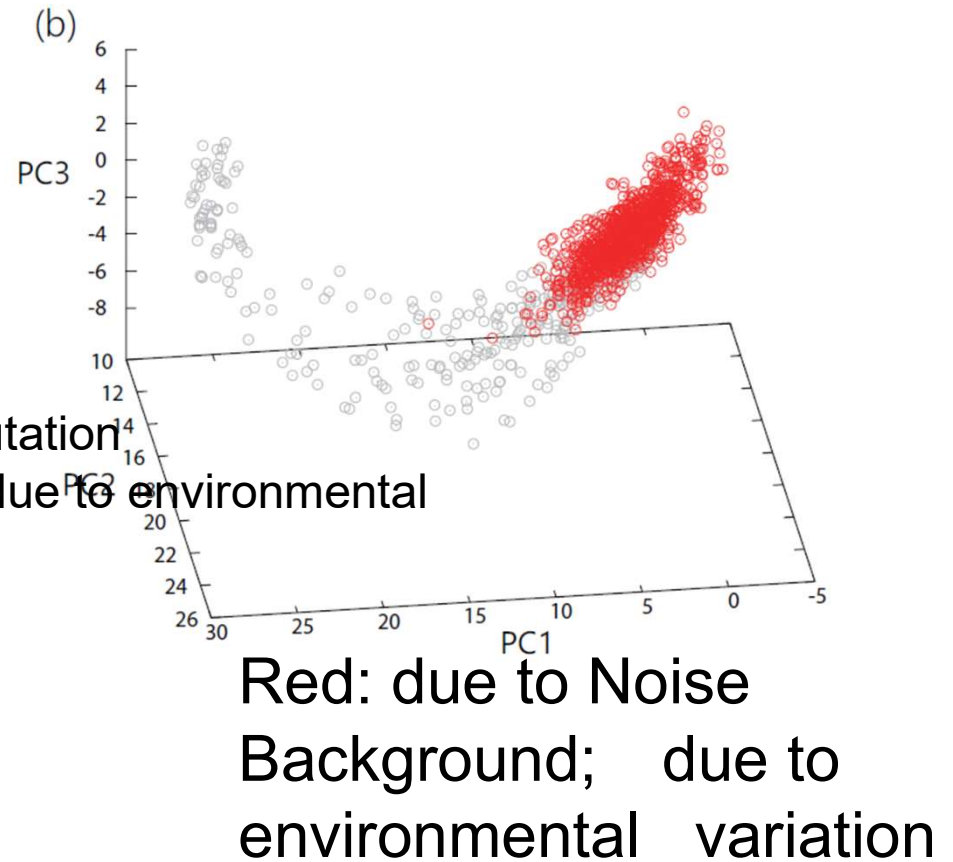
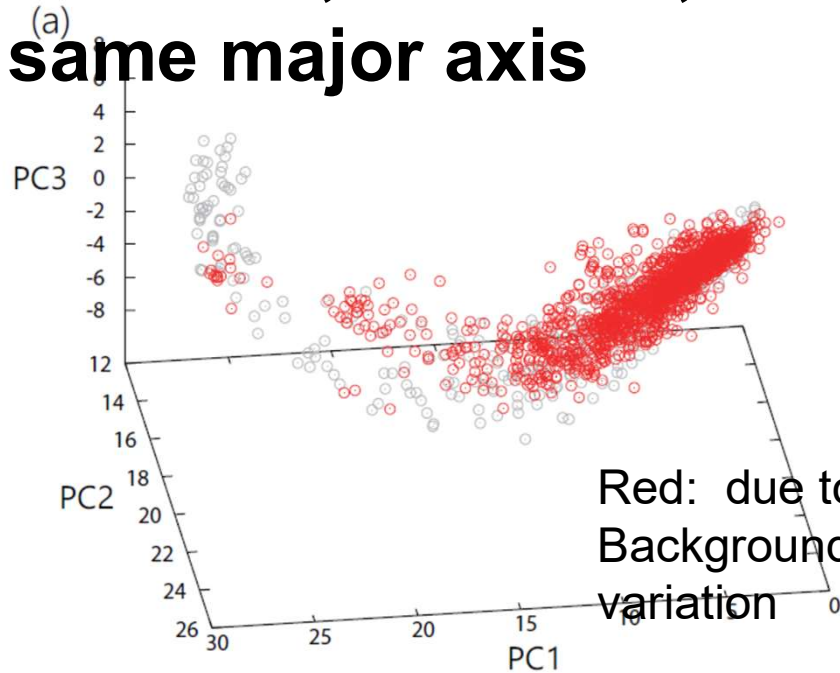
Sato, KK PhysRevR 2020



The directions of slowest mode and the fitness are aligned after evolution



Recall: Phenotypic change due to environmental variation, mutation, noise are constrained along the same major axis



?Phenotypic changes by evolution and environmental changes are along a common slow-manifold

Consequence of Hypothesis → Correlation between Environment vs Evolutionary Changes

Evolution : $\mathbf{J}\delta\mathbf{X} + \gamma(\mathbf{E})\delta E + \gamma(\mathbf{G})\delta G = \delta\mu(E).$

Again, assume that

most changes occur along such slow manifold

Project to this slow manifold →

$$\delta X_i(\mathbf{G})/\delta X_i(\mathbf{E}) = \delta\mu(\mathbf{G})/\delta\mu(\mathbf{E})$$

using $\gamma \cdot v_0 \sim 0$

(Genetic) evolution under the environmental condition

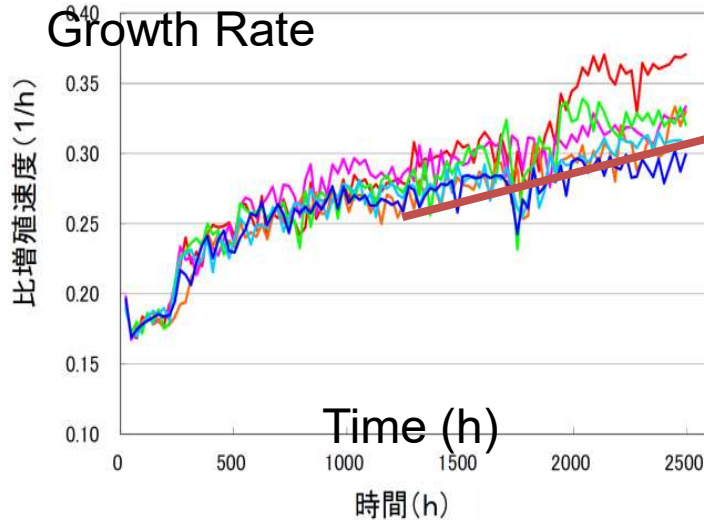
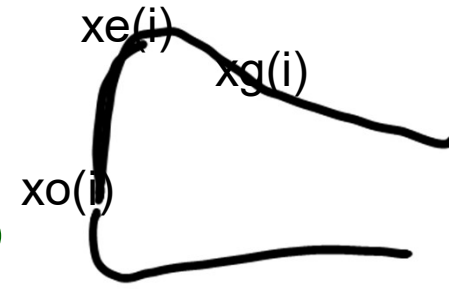
→ recover growth-- $|\delta\mu(\mathbf{E})| < |\delta\mu(\mathbf{G})|$

$$\delta X_i(\mathbf{G})/\delta X_i(\mathbf{E}) = \delta\mu(\mathbf{G})/\delta\mu(\mathbf{E}) < 1$$

→ All the expression levels tend to return the original level by evolution **Le Chatelier Principle?**

Evolution Experiment of E Coli to adapt in stressed (ethanol) condition

Furusawa's Group

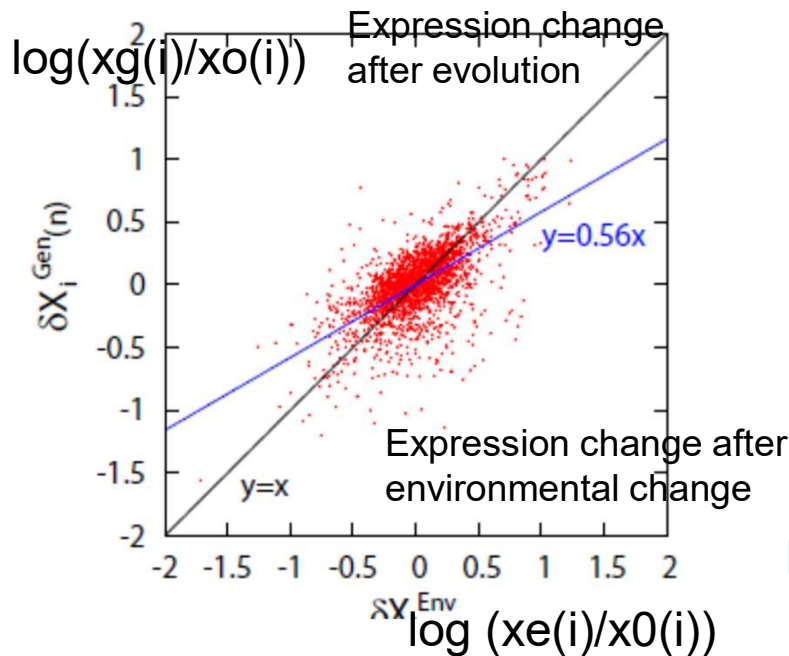
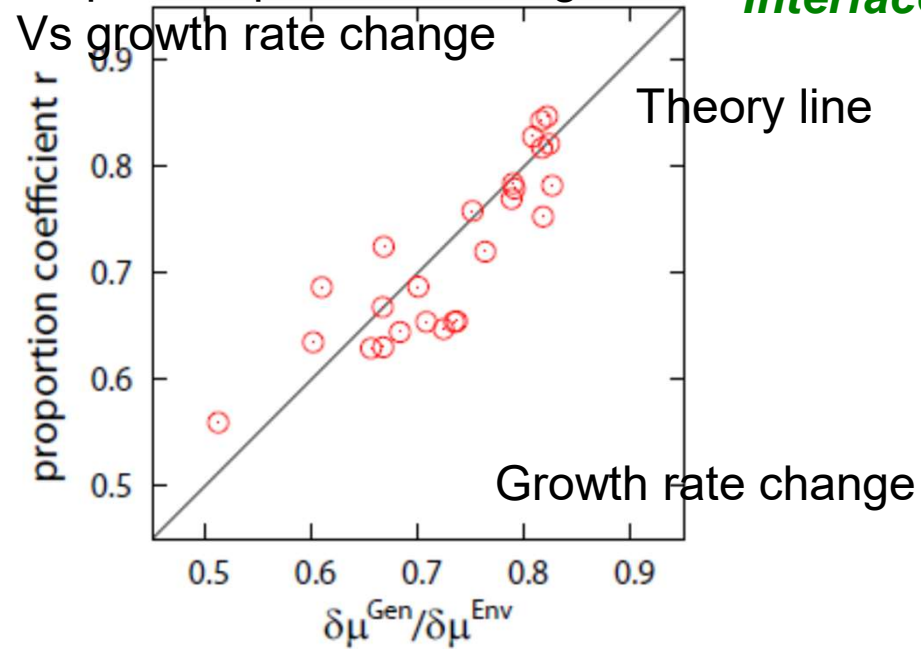


~ 1000 generations



Furusawa, KK Interface, 2015

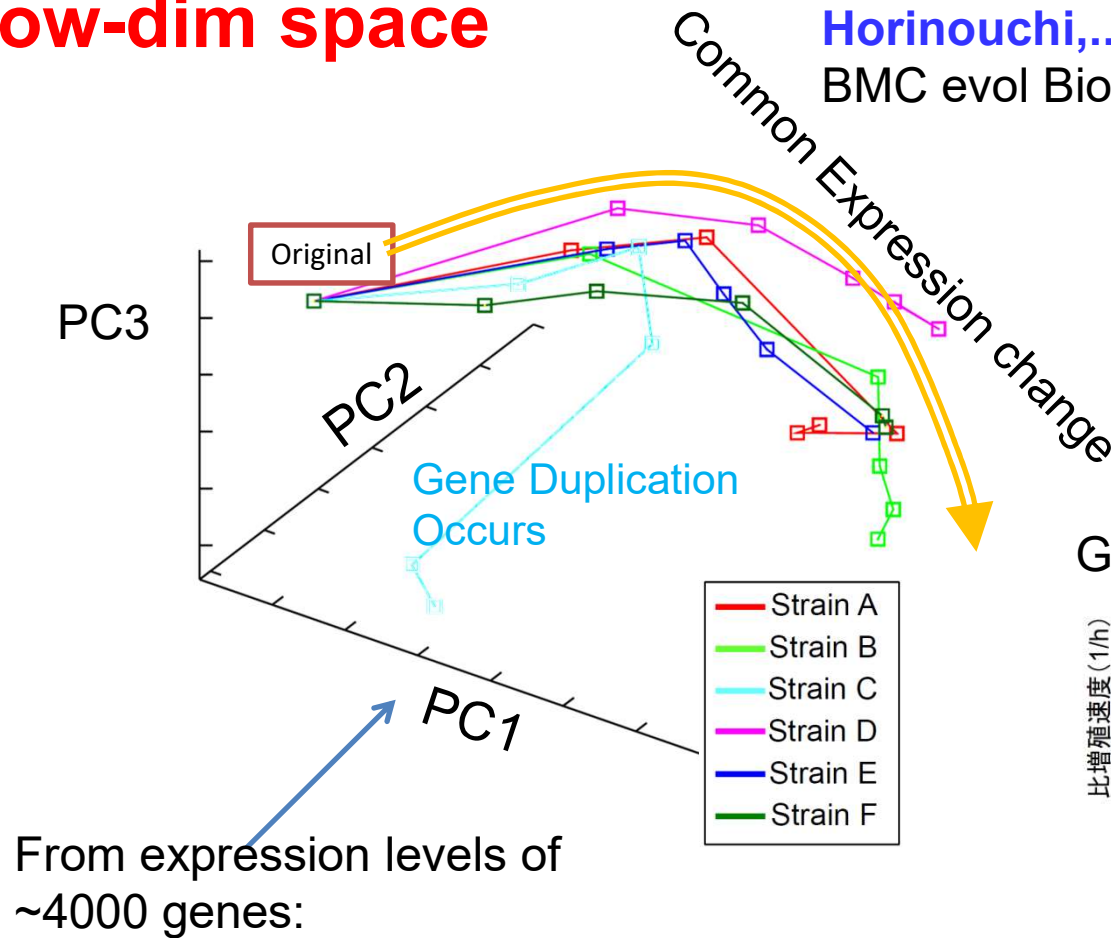
Slope in expression change Vs growth rate change



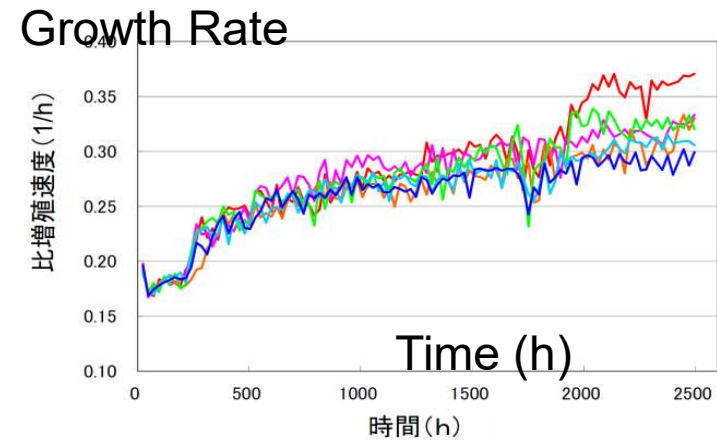
$0 < \delta X_i (E,G) / \delta X_i (E) < 1$
 return to original expression pattern
 (Le Chatelier principle)

Deterministic phenotypic evolution constrained in low-dim space

Horinouchi,...,Furusawa,
BMC evol Biol 2015



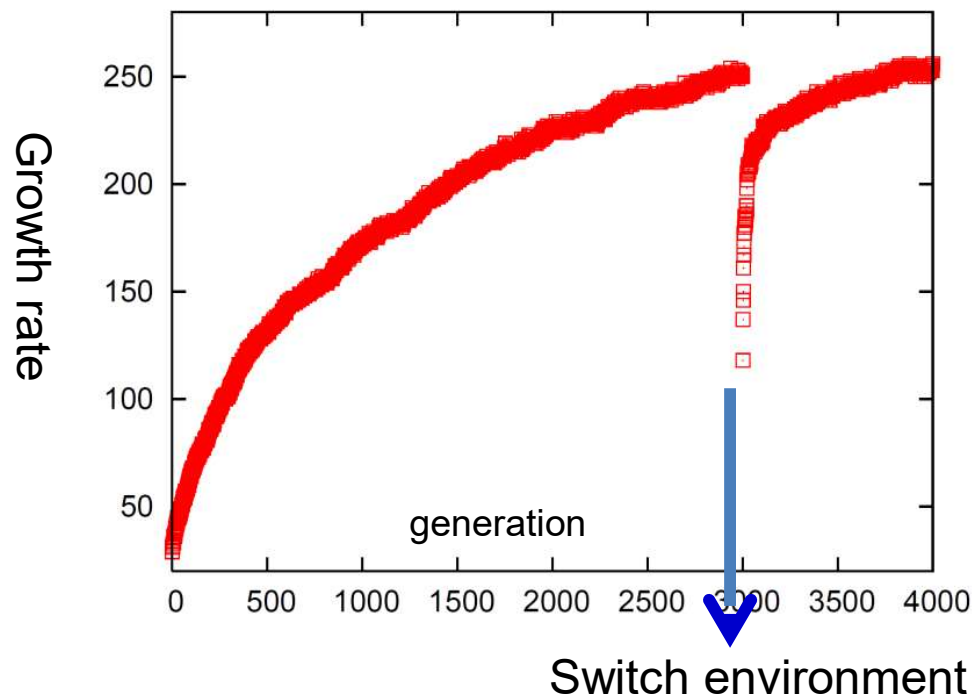
Replaying the tape of evolution, same phenotypic path (not genetic) arises!



Mutation sites are different by strains. But..
Common trends in phenotypic space (low-dim structure)
PC1 is highly correlated with the growth rate

Evolution of Catalytic reaction net model by switching environment (nutrient concentration) and check evol-env response

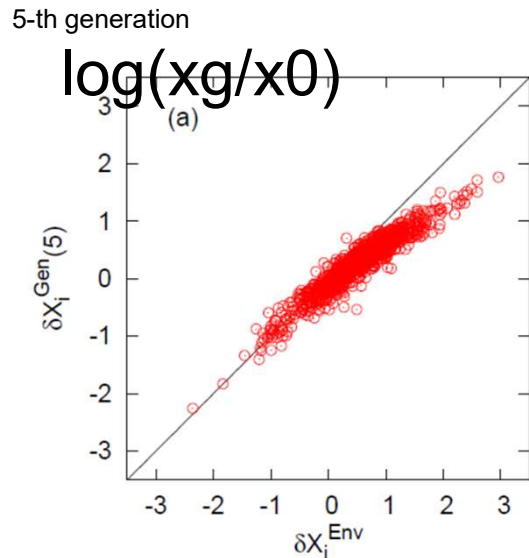
Mutate network and select those with higher growth
—evo



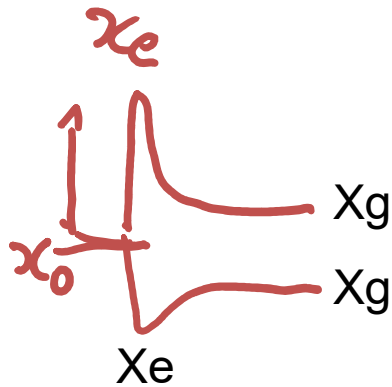
Recovery of growth rate by adaptive evolution to new environment

$\delta X_i(G)/\delta X_i(E) = \delta\mu(G)/\delta\mu(E) < 1$ (Across all components)

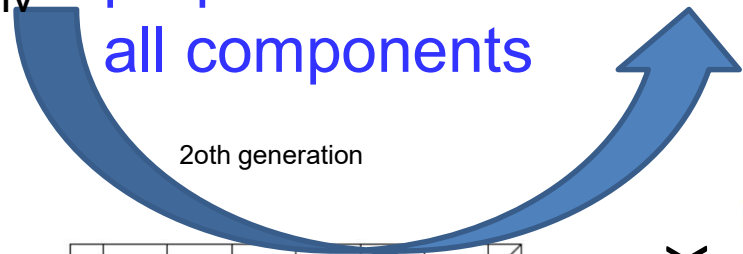
Expression Change by evolution



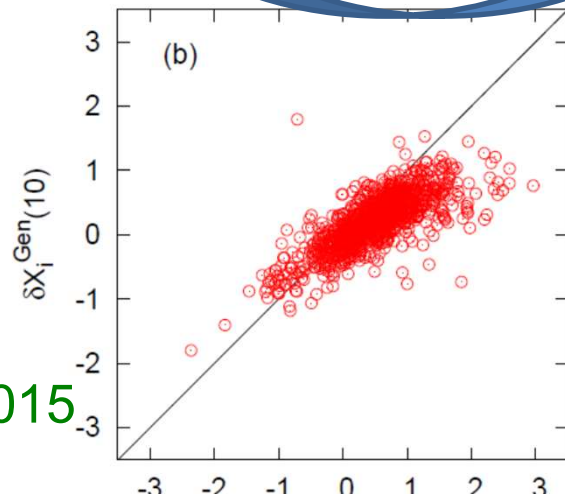
Expression change by env
 $\log(xe/x0)$



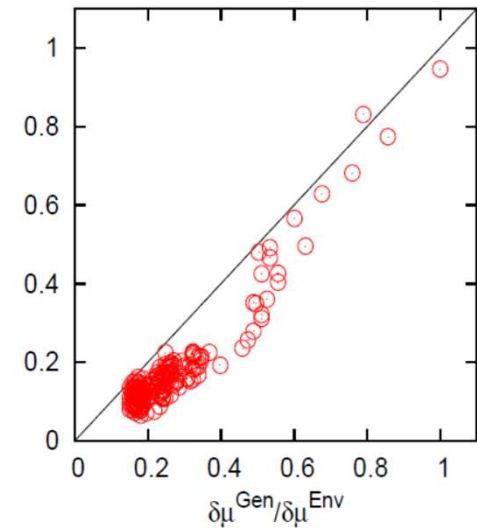
(1) Response by genetic change tends to cancel the change by environment
 (2) The two responses are proportional over all components



20th generation



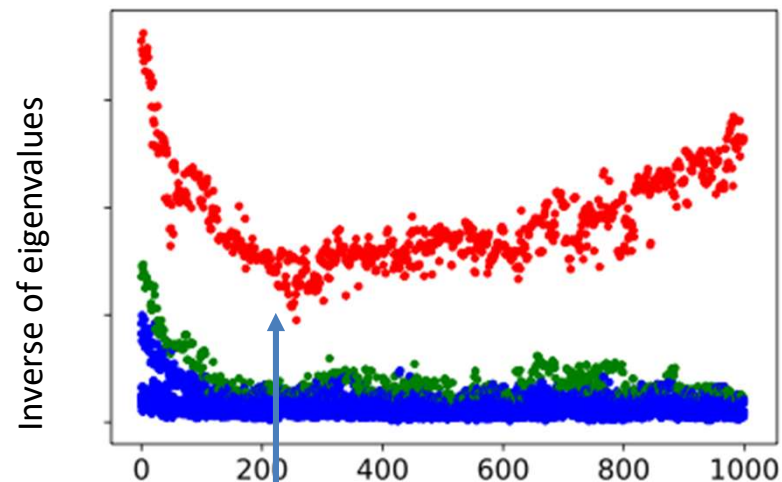
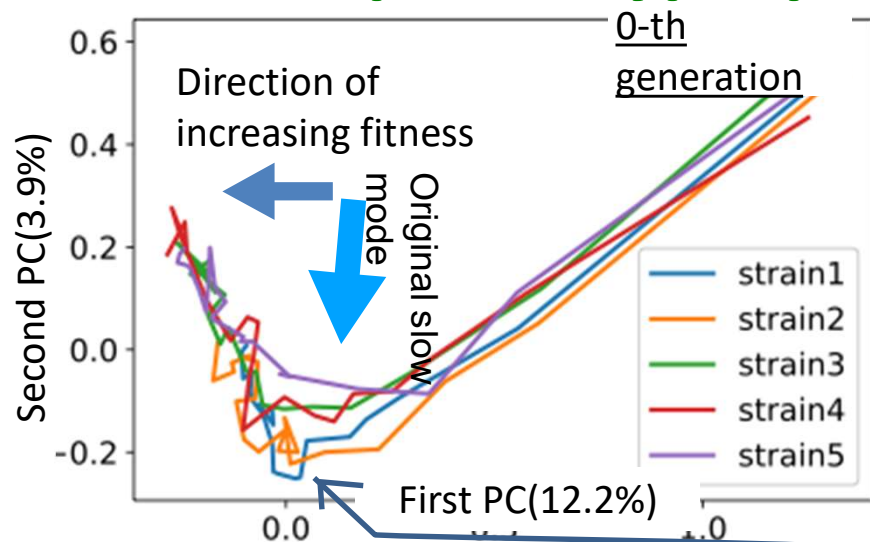
Slope in δX



$-\Delta\mu$ bo by env to by evol

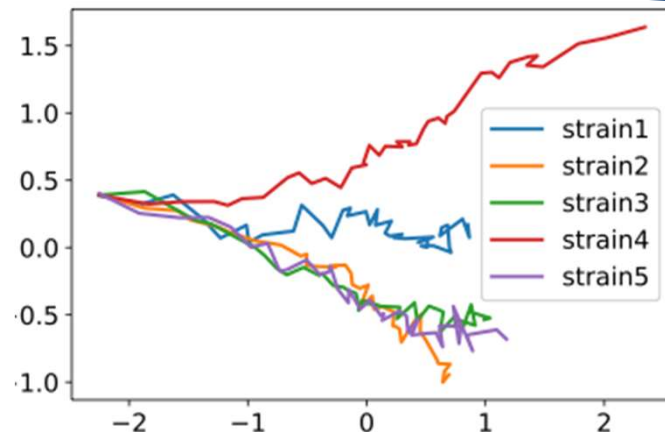
- In evolution to novel environment after adaptation to previous environment, **the already evolved slow mode is adopted** to adapt to new environment

→ **Same phenotypic path when replayed the tape.**



Slow mode is adjusted to novel fitness

Cf. When started from non-adapted case (same random network)

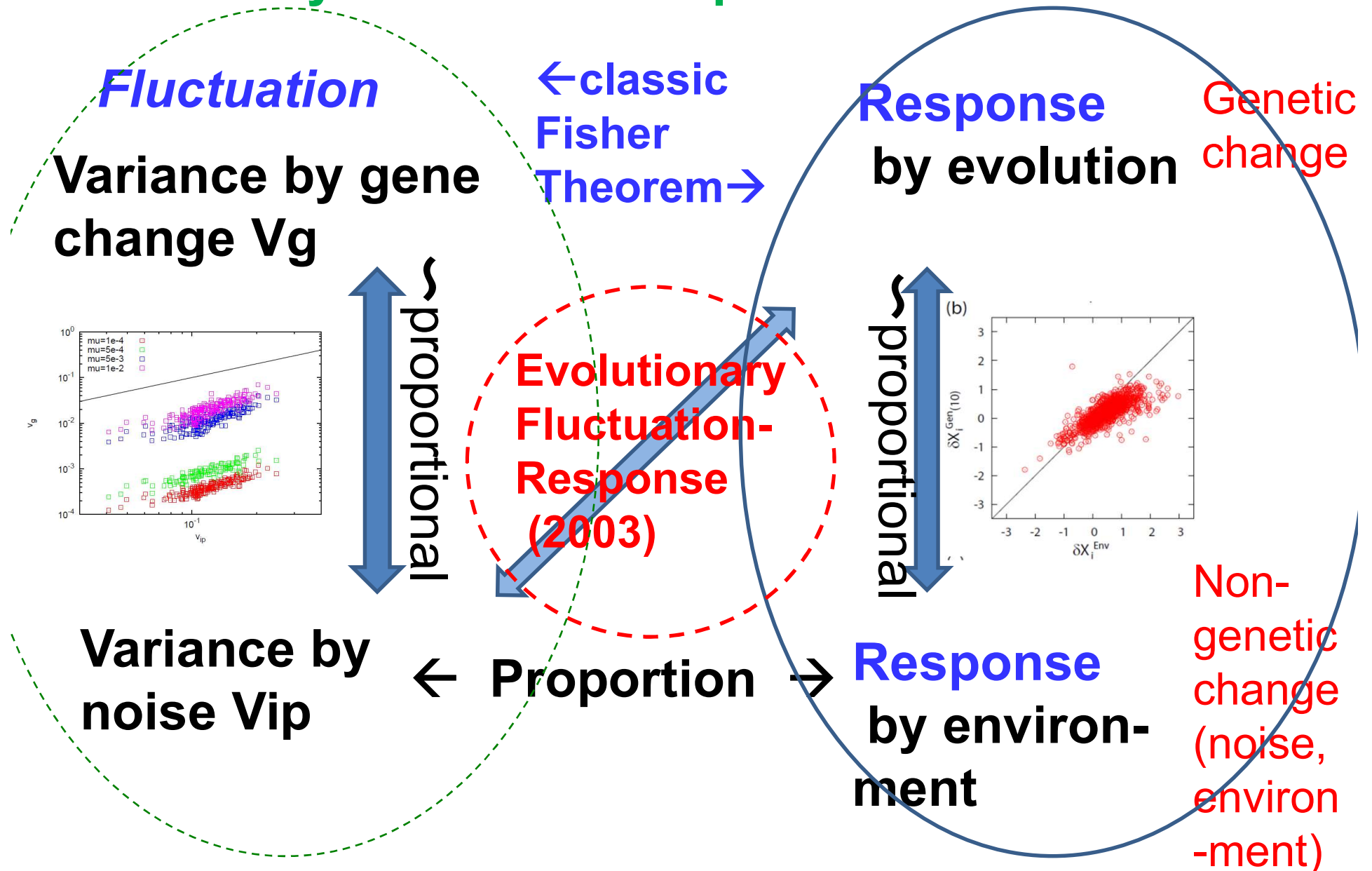


Sato, KK, PhysRevRes2020

Messages

- (Cellular) Phenotypes are high-dimensional, but their adaptive changes are drastically restricted in a low-dimensional space
- ← Result of steady-growth and evolutionary robustness (to noise and to genetic changes)
- Phenotypic evolution is rather deterministic even though genetic changes can be stochastic (replaying the tape, phenotypically same path)
- ← Phenotypic evolvability correlated by fluctuations

So far response relationship: but earlier we showed evolutionary fluctuation response



Part II

1) Evolutionary Fluctuation-Response Relationship

(Sato et al PNAS 2003, Furusawa, KK 2006)

2) Proportionality between Fluctuation by noise and by mutation (robustness relationships)

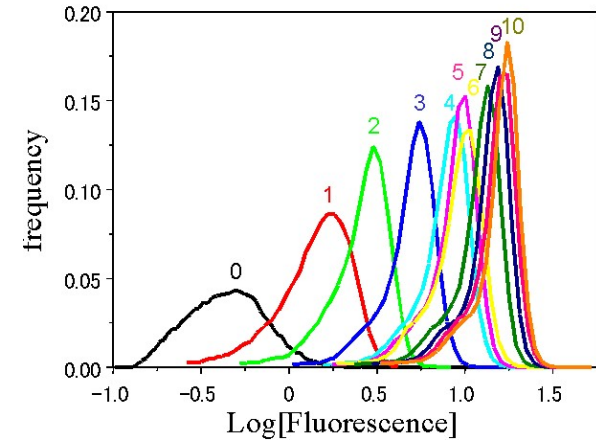
(KK, Plos One 2007, Furusawa, KK Interface 2015)

(i) evolutionary fluctuation-response relationship:

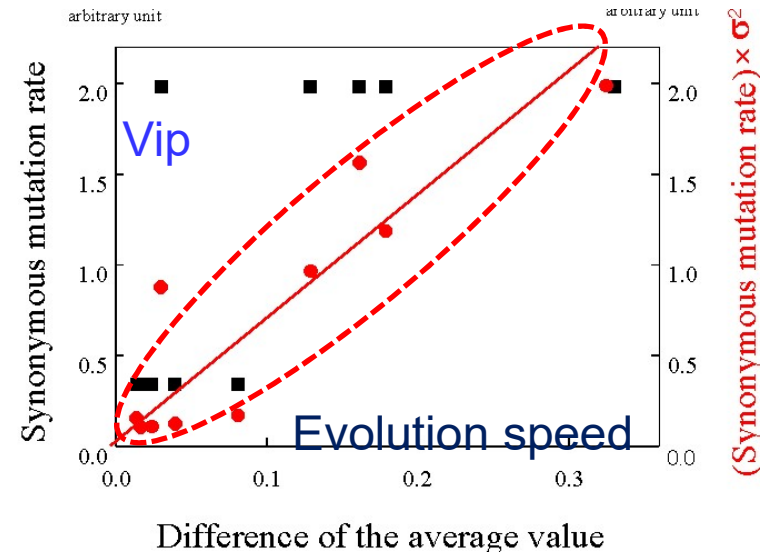
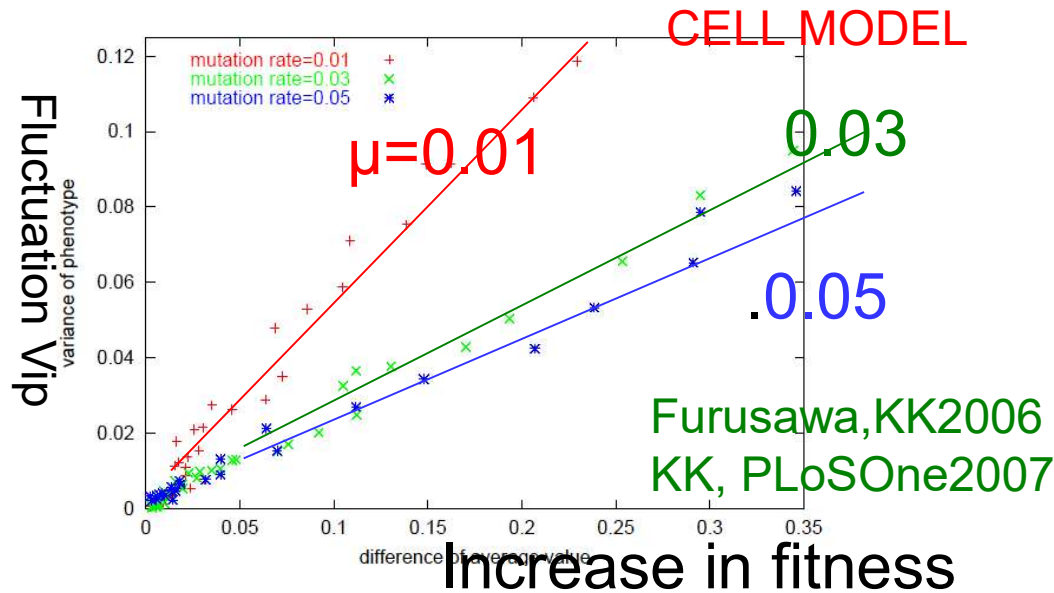
* **Vip** variance of phenotype (fitness) over **isogenic** individuals (Ve, Vnoise)

Vip \propto **evolution speed**
through evolution course

bacteria evolution experiment
+ models (cell, gene-regulation-net),
+ Phenomenological Theory



EXPERIMENT



Sato Ito Yomo KK; PNAS 2003,

Analogy with fluctuation-response relationship

Force to change a variable x ;

response ratio = (shift of x) / force

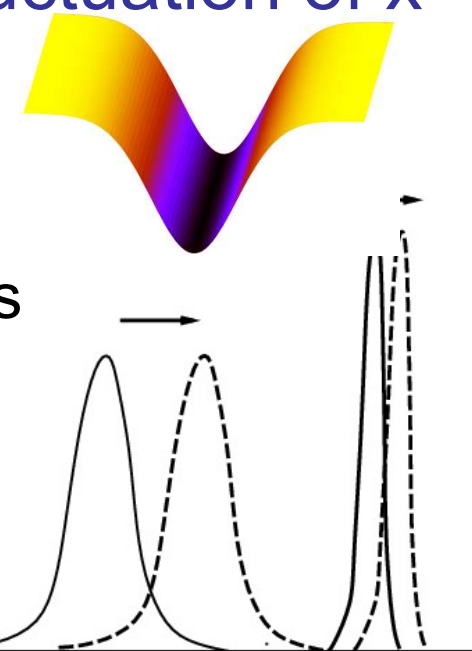
fluctuation of x (without force)

response ratio proportional to **fluctuation**

Generalize by distribution function

response ratio of some variable x against change of parameter a versus fluctuation of x

$P(x;a)$ x variable, a : control parameter
change of the parameter $a \rightarrow$
peak of $P(x;a)$ (i.e., $\langle x \rangle$ average) shifts



$$\frac{\langle x \rangle_{a+\Delta a} - \langle x \rangle_a}{\Delta a} \propto \langle (\delta x)^2 \rangle_a = \langle (x - \langle x \rangle)^2 \rangle$$

-- "Response against mutation+selection" -- Fluctuation

Phenomenological Distribution argument

Gaussian distribution of x ; under the parameter a

$$P(x; a_0) = N_0 \exp\left(-\frac{(x - X_0)^2}{2\alpha_0}\right), \quad \text{at } a=a_0$$

Change the parameter from a_0 to a

$$P(x; a) = N \exp\left(-\frac{(x - X_0)^2}{2\alpha(a)} + v(x, a)\right) \quad v(a, x) = C(a - a_0)(x - X_0) + \dots, \text{ with } C \text{ as a constant,}$$

$$P(x, a_0 + \Delta a) = N' \exp\left(-\frac{(x - X_0 - C\Delta a\alpha(a_0 + \Delta a))^2}{2\alpha(a_0 + \Delta a)}\right)$$

Hence, we get

$$\frac{\langle x \rangle_{a=a_0+\Delta a} - \langle x \rangle_{a=a_0}}{\Delta a} = C\alpha(a_0 + \Delta a),$$

Noting that $\alpha = \langle (\delta x)^2 \rangle$

$$\frac{\langle x \rangle_{a=a_0+\Delta a} - \langle x \rangle_{a=a_0}}{\Delta a} = C \langle (\delta x)^2 \rangle,$$

(1) Assumption of representation by $P(x;a)$ **x** : phenotype
a : gene
 (2) The coupling form Cxa is also assumption

→ Not derivation, but need to check experimentally

(ii) Geno-Pheno relationship on variances

*but $V_g \propto$ evolution speed (Fisher)

* V_{ip} variance of fitness over **isogenic** individuals

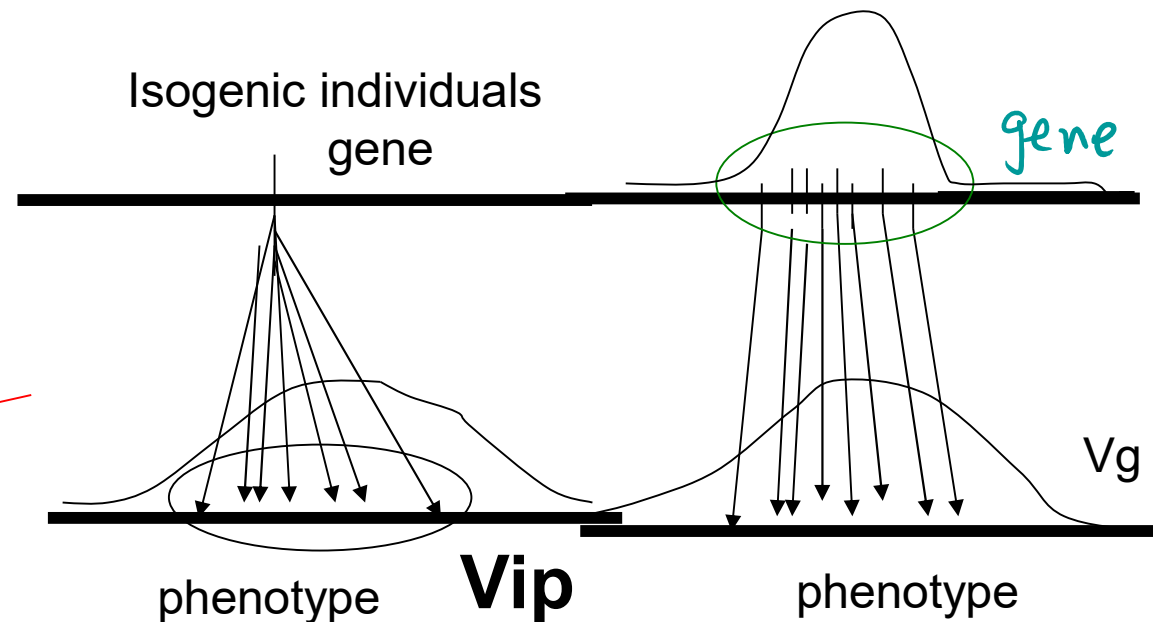
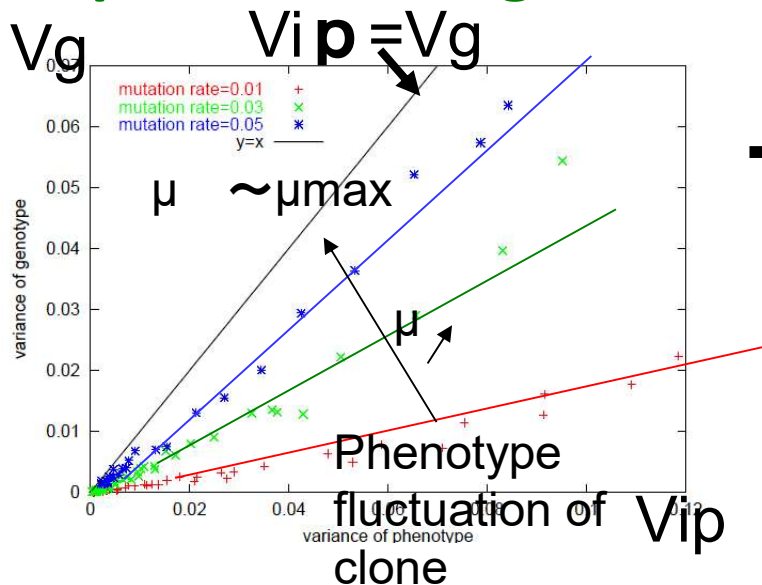
* V_g variance of average fitness over **heterogenic** pop

$V_{ip} \propto V_g \propto$ evolution speed through evolution course
confirmed; experiment, theory, models

WHY?? → result of robust evolution + distribution theory

Robustness to noise ↑ → Robustness to Mutation ↑

$V_{ip} \downarrow \rightarrow V_g \downarrow$

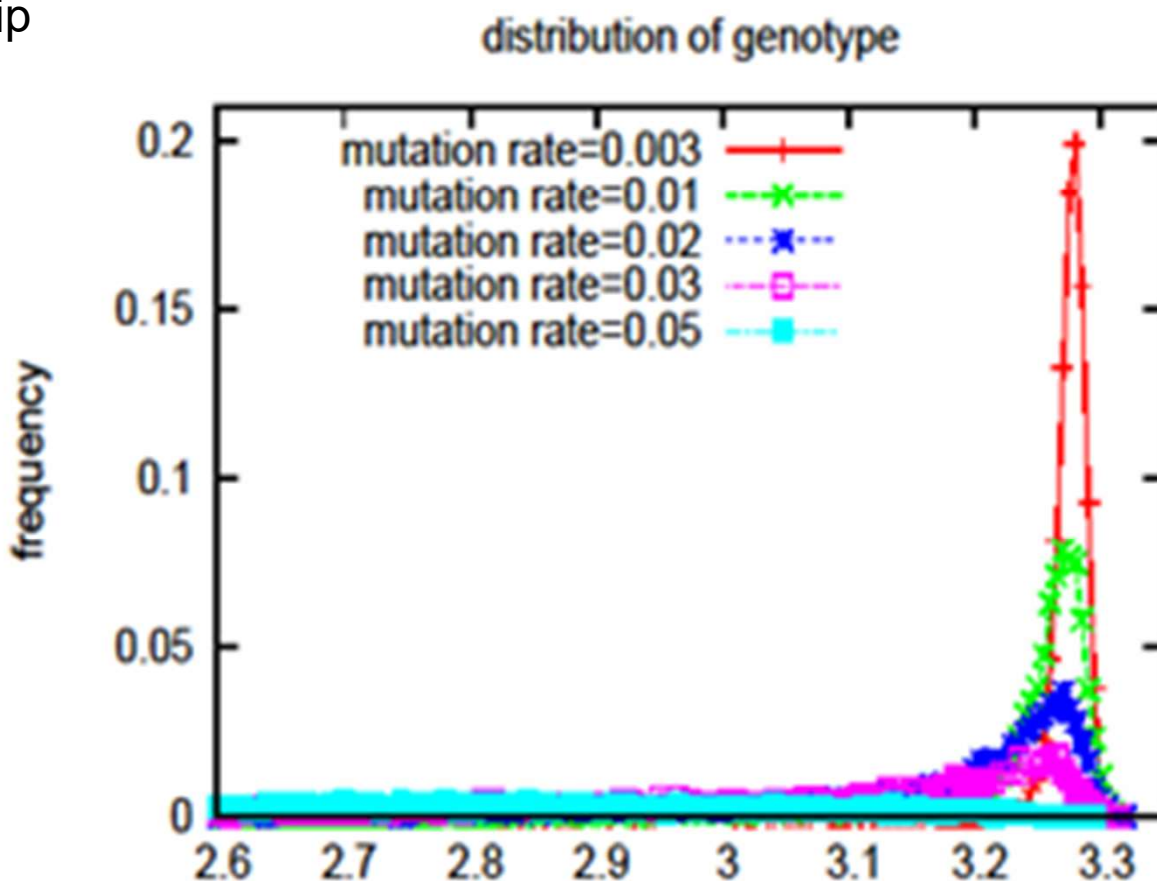


- As μ (mutation rate) increases to μ_{max} ,
- (1) the distribution collapses (error catastrophe)
 - (2) evolution no longer progresses beyond μ_{max}
evolution speed is maximal at $\mu \sim \mu_{max}$
 - (3) V_g approaches V_p

V_p

As μ is increased,
The distribution
'collapses'

Error catastrophe



WHY? (Phenomenological theory assuming evolutionary robustness)

Consider 2-variable distrib

$$P(x=\text{phenotype}, a=\text{genotype}) = \exp(-V(x, a))$$

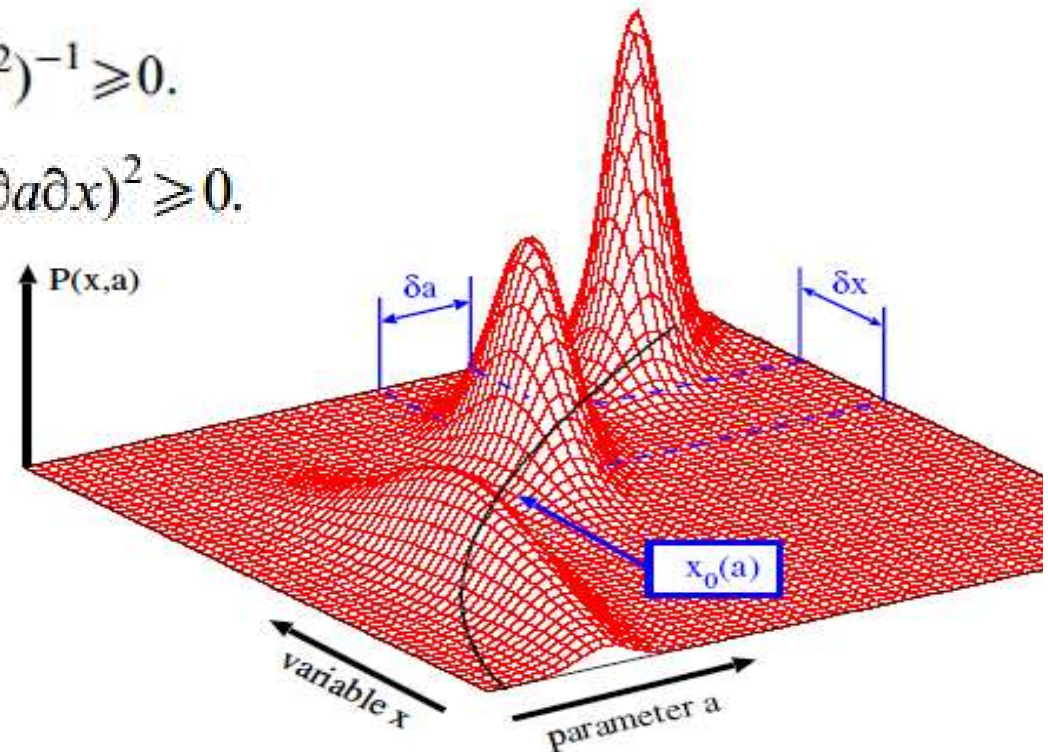
Keep a single-peak (stability condition).

$$(\partial^2 V / \partial a^2)^{-1} \geq 0; \quad (\partial^2 V / \partial x^2)^{-1} \geq 0.$$

$$(\partial^2 V / \partial x^2)(\partial^2 V / \partial a^2) - (\partial^2 V / \partial a \partial x)^2 \geq 0.$$

Hessian condition

Leads to relationship between V_{ip} and V_g



$$P(x, a) = \widehat{N} \exp\left[-\frac{(x - X_0)^2}{2\alpha(a)} + \frac{C(a - a_0)(x - X_0)}{\alpha} - \frac{1}{2\mu}(a - a_0)^2\right].$$



$$P(x, a) = \widehat{N} \exp\left[-\frac{(x - X_0 - C(a - a_0))^2}{2\alpha(a)} + \left(\frac{C^2}{2\alpha(a)} - \frac{1}{2\mu}\right)(a - a_0)^2\right].$$

$$\mu \leq \frac{\alpha}{C^2} \equiv \mu_{max}.$$

$$\bar{x}_a \equiv \int xP(x, a)dx = X_0 + C(a - a_0).$$

$$V_g = \frac{\mu C^2}{1 - \mu C^2/\alpha} \sim V_{ig} = \mu C^2 \quad V_{ip} = \alpha$$

If mutation rate μ is small, $V_g < V_{ip}$,

$$V_g \sim (\mu/\mu_{max}) V_{ip} \propto V_{ip}$$

Consistency between pheno & geno

- (i) $V_{ip} \geq V_g$? (for stability?) (**)
 - (ii) error catastrophe at $V_{ip} \sim V_g$ (**)
 - (where the evolution does not progress)
 - (iii) $V_g \sim (\mu/\mu_{max}) V_{ip} \propto \mu V_{ip}$
 - (\propto evolution speed) at least for small μ

* * Consistent with the experiments, but,,,,,

Assumptions on $P(x,a)$ and Robust Evolution??

Why higher developmental noise leads to robust evolution?

(**) under selection of given trait. if μ is small:

to be precisely V_g , variance those from a given phenotype x : but $V_{ig} \sim V_g$
if μ is small

$V_g/(V_{ip}+V_g)$ is known as heritability (smaller for important trait)

Gene expression dynamics model: Relevance of Noise to evolution?

Simple Model: Gene-net (dynamics of stochastic gene expression) \rightarrow on/off state

X_i – expression of gene i : on off

$$dx_i/dt = F\left[\sum_j^M J_{ij}x_j - \theta_i\right] - x_i + I_i(n) + (\sigma\eta_k(t))$$

(on) $x > \theta_i$

(off) $x < \theta_i$

off  *on*

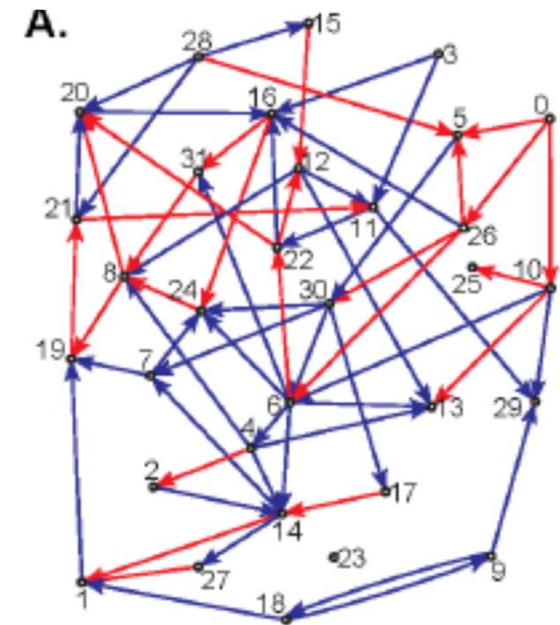
$$F(X) = 1 / (\exp(-\beta X) + 1)$$

$$\langle \eta_i(t) \eta_j(t') \rangle = \delta(t - t') \delta_{ij}$$

Gaussian white noise

M ; total number of genes, k : output genes

Noise strength σ



Activation
Repression
" " = 1, -1, 0

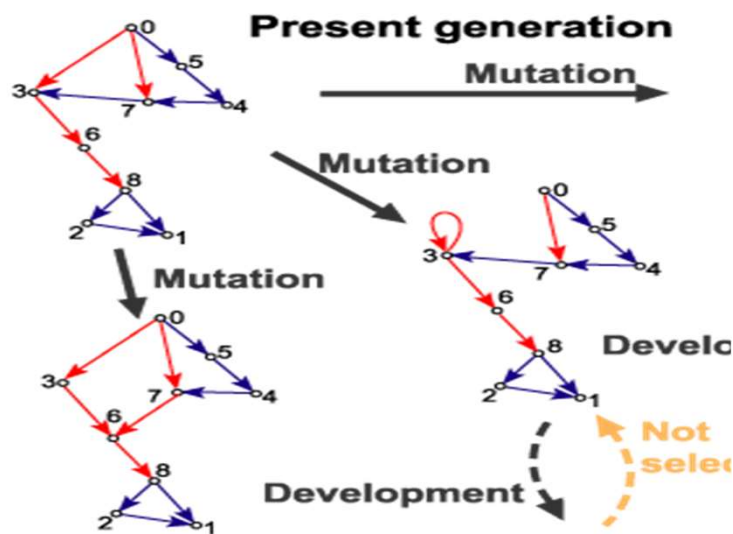
- Fitness: Starting from off of all genes, after development genes x_i $i=1, 2, \dots, k$ should be on (Target Gene Pattern)

Fitness $F = - (\text{Number of off } X_i)$

Genetic Algorithm

Population of N different genotypes(networks). Select those with higher $\langle F \rangle$ and mutate with rate μ

Keep N networks

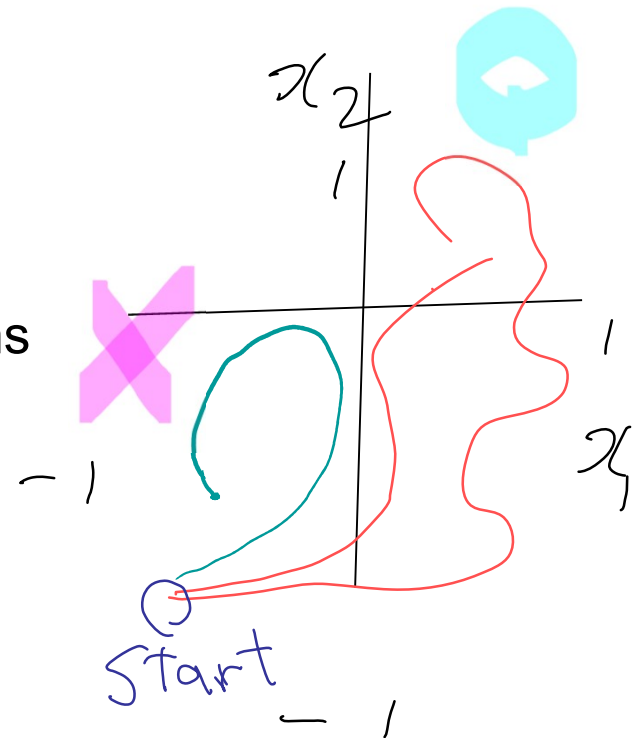


If $M=k=2 \rightarrow$

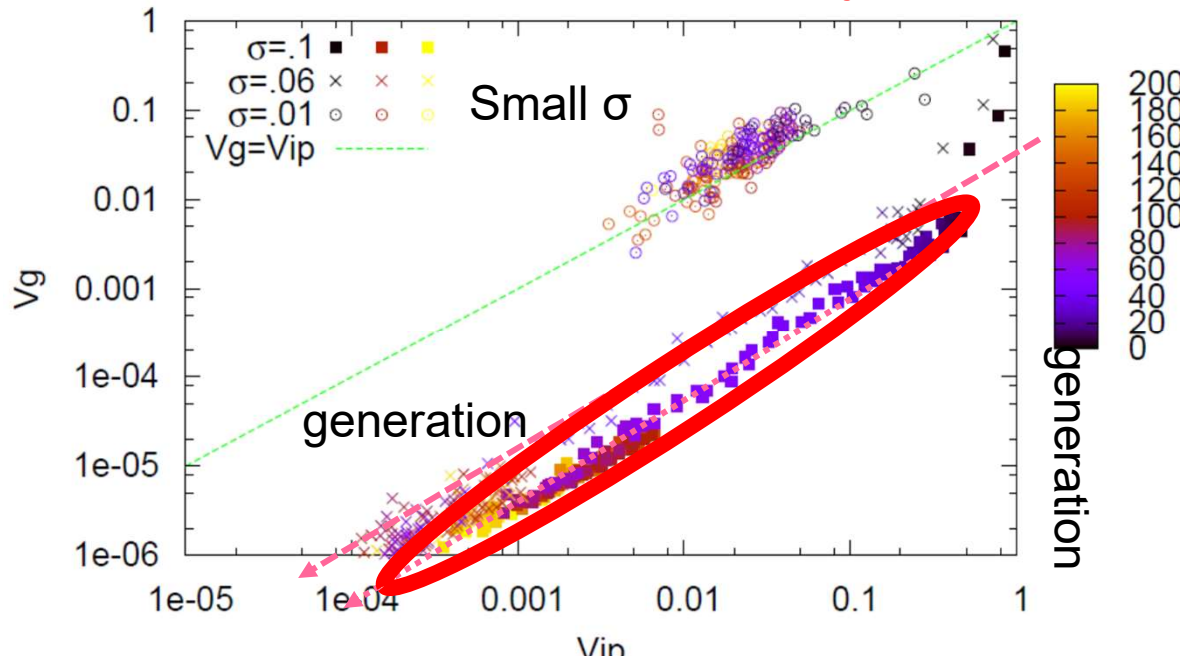
Most simulations

$M=64$

$K=8$



“Robustness transition by increasing noise”



(1) $V_{ip} \geq V_g$ for $\sigma \geq \sigma_c$

(2) $V_g \rightarrow V_{ip}$ as $\sigma \rightarrow \sigma_c$

(4) $V_{ip} \propto V_g$ through evolution course

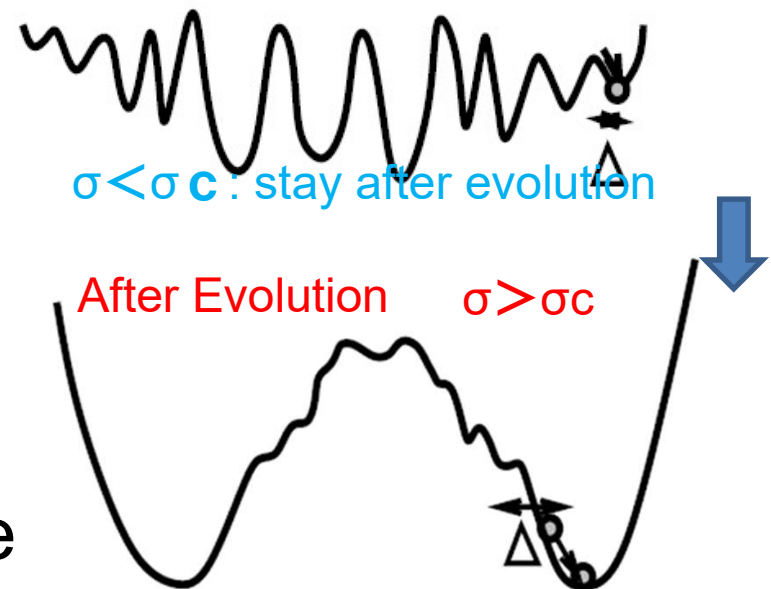
KK, PLoSOne, 2007

Initial (most probable networks, Random)

$\sigma < \sigma_c \rightarrow$ only tiny basin around target orbit

$\sigma > \sigma_c \rightarrow$ robustness evolves proportional decrease in V_{ip} & V_g
Large basin for target attractor

Smooth developmental landscape



Difference in basin structure

Evolution of Robustness

If developmental dynamics (gene expression) are under sufficient noise level, robustness to noise leads to robustness to mutation, through the evolution.

Robustness ----- Insensitivity of Fitness (Phenotype) to system's change –

“Inverse” of phenotypic variances

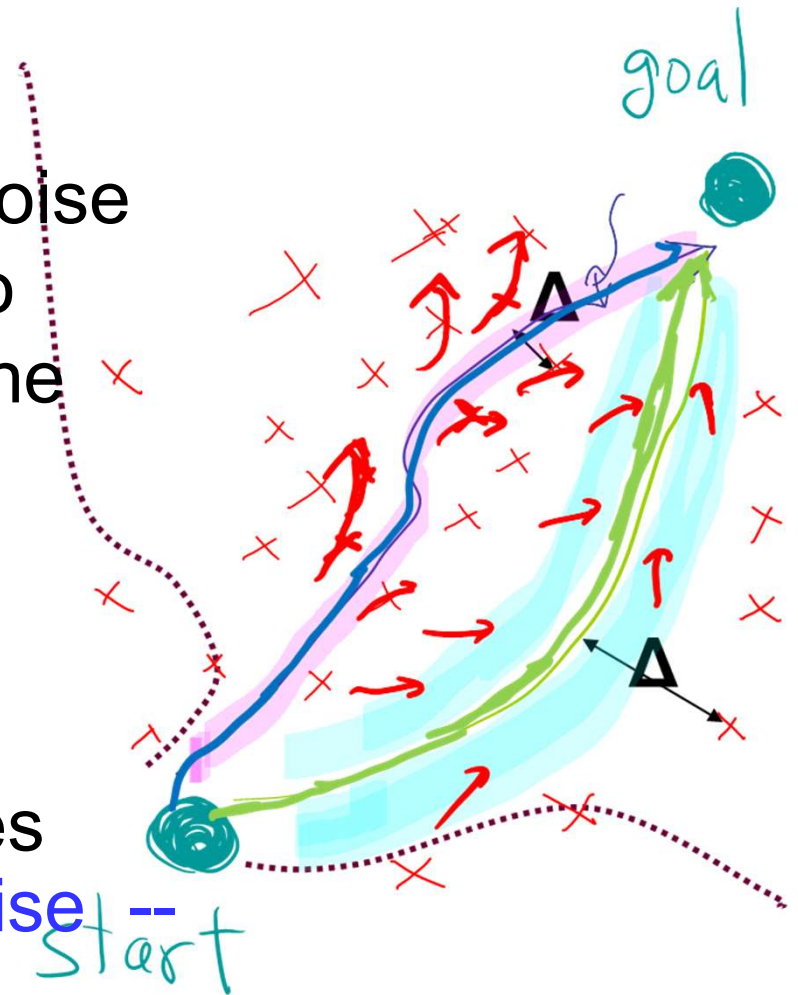
Developmental Robustness to noise --

-- V_p

Robustness to mutation in evolution --

-- V_g

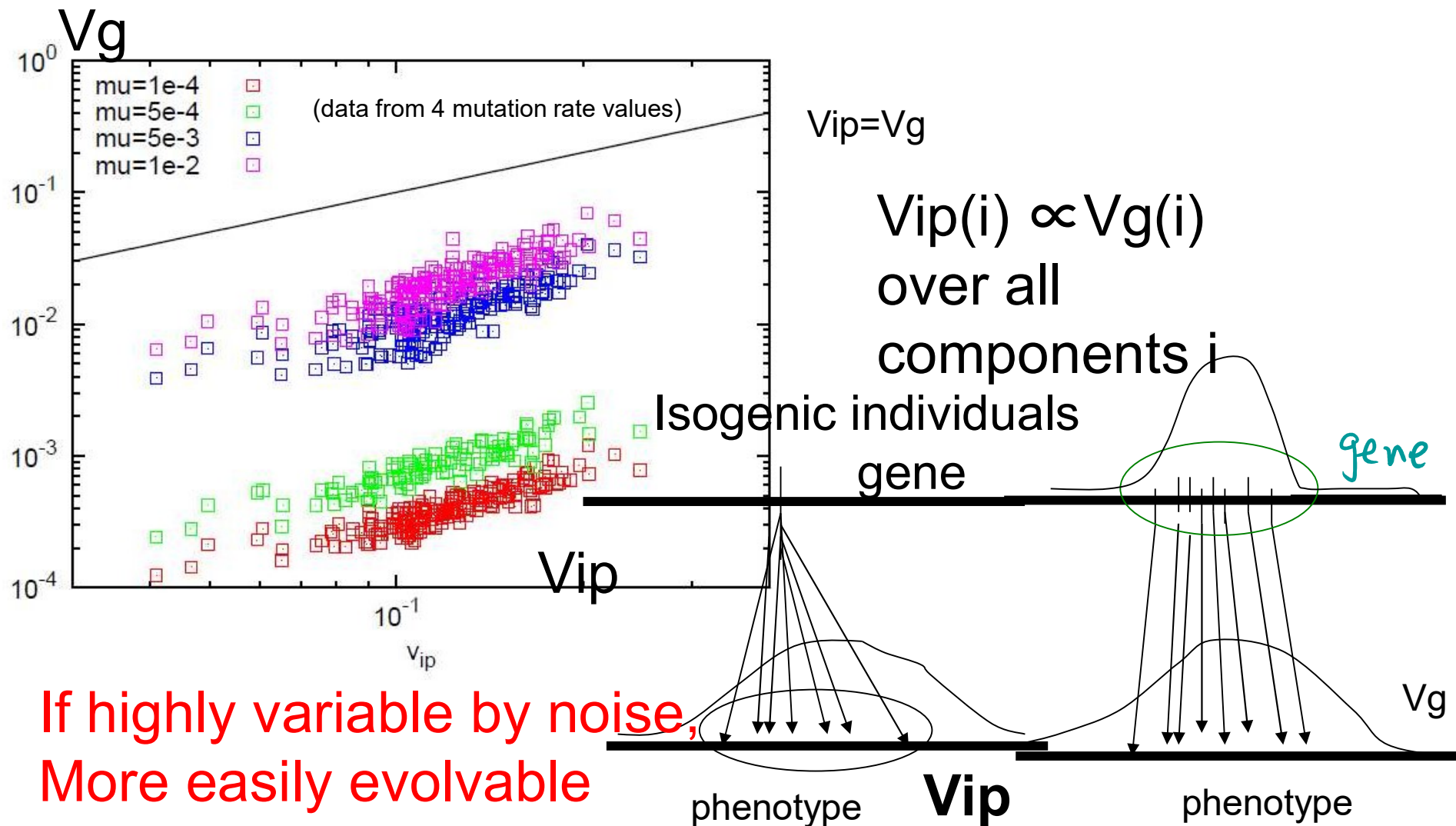
$V_p \propto V_g \rightarrow$ Developmental robustness is embedded into genetic (evolutionary) robustness for $\sigma > \sigma_c$



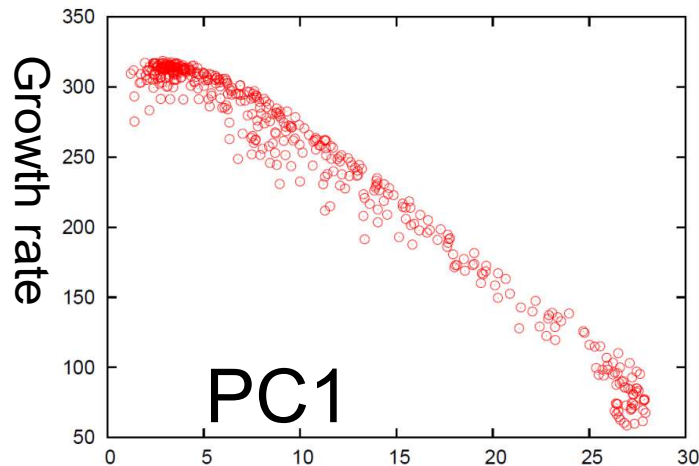
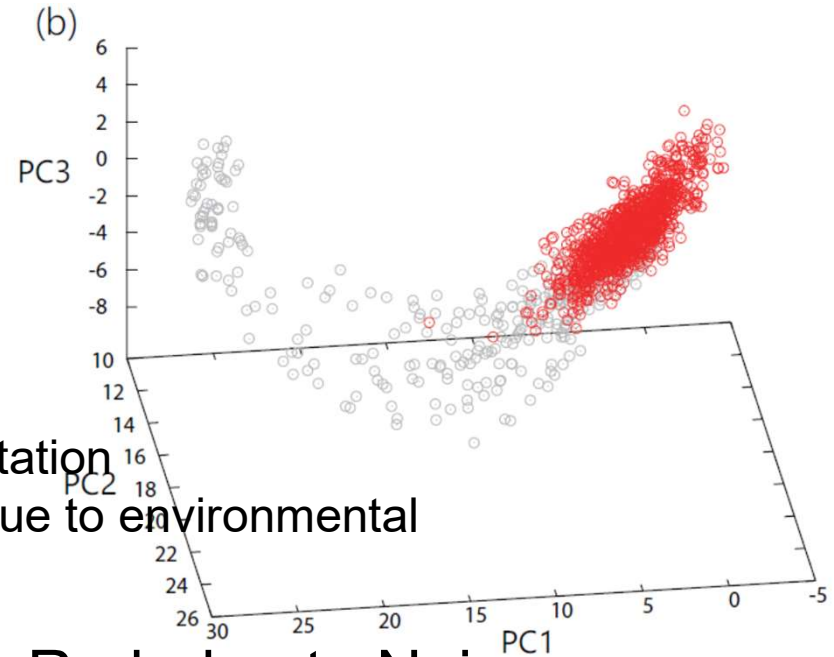
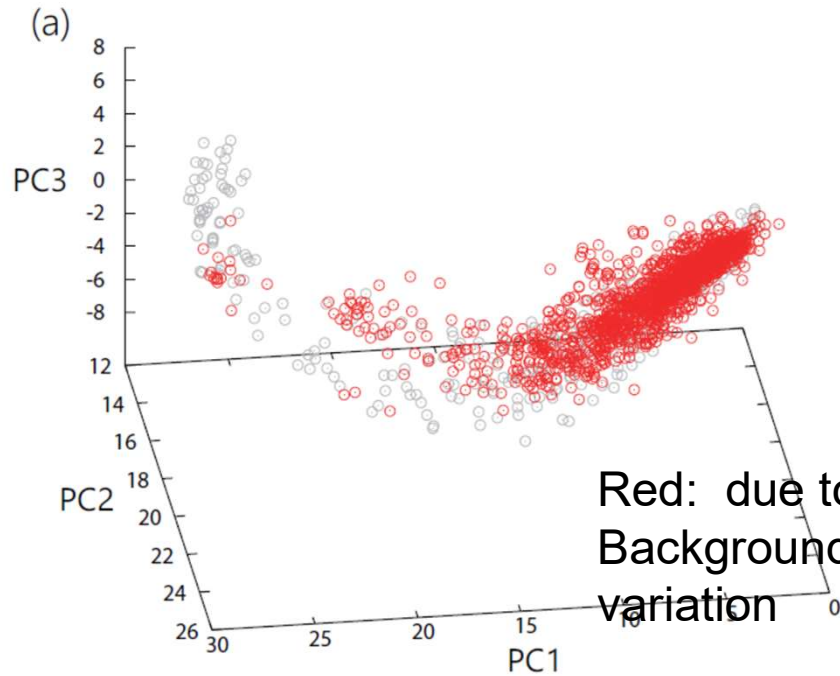
Vip-Vg relationship across traits (phenotypes)

$Vg(i)$: Variation of i-th expression due to mutation

$Vip(i)$: Variation due to noise in dynamics



Recall...



**?Phenotypic change occurs
along a common slow-manifold**

Vg-Vip proportionality is explained by the slow manifold Hypothesis

Evolution occurs along this slow manifold \mathbf{w}^0

$$V_{ip}(i) = (\mathbf{w}_i^0)^2 \langle \delta X^2 \rangle_{noise}$$

$$V_g(i) = (\mathbf{w}_i^0)^2 \langle \delta X^2 \rangle_{mutation}$$

→ $V_g(i)/V_{ip}(i) =$ independent of i

(here we do not need the growth-rate constraint, only slow-manifold constraint is needed)

Vg-Vip relationship ← Changes both by (environmental) noise and (genetic) mutations are constrained along the direction

Need further studies to establish the present theory

(i) Further Confirmation by Experiments

(ii) Confirmation by Models (Universality)/Condition?

Catalytic Reaction Cell Model

Gene regulation network Model (Inoue, KK arXiv)

Spin-glass Models (Sakata et al., PRL 2020)

evolve spin Hamiltonian $J_{ij}S_iS_j$ to achieve certain configuration
dimensional reduction at replica symmetric phase

Protein Model/Data? (Tang et al., PLoSCB, PRR2020)

critical state to satisfy robustness and plasticity

(iii) Theory for dimensional reduction? –1 or few dim?

outliers in eigenvalues – **separation of slow modes**,

Renormalization Group???

Projection to Collective Modes?

Why Slow Dominant Mode Evolved????

Time-scale difference

→ Separation of control/controlled is possible which allows for evolvability

(If many degrees of the similar time scale interfere, not easy to directional change) *too many cocks spoil the broth*

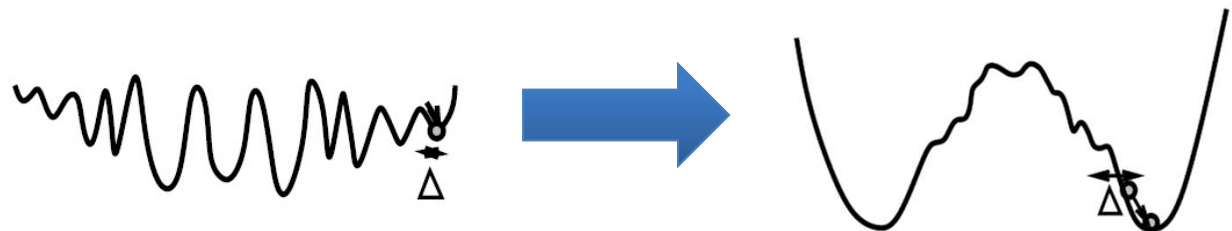
Result of evolution but fosters evolvability

Cf Kohsokabe, KK, JExpZoolologyB 2016

Expanded Linearity in terms Potential picture:

robustness evolved (get out of error catastrophe)

Cf, KK PLoS One 2007



Related Issues

(1) Why Genotype/ Phenotype are separated

i.e., origin of central dogma as symmetry breaking
(information/function)

(Takeuchi, Hoegeg, KK, Nat. Comm 2018;
Takeuchi, KK Proc Roy Soc B 2019)

(2) Beyond Steady-Growth state, cf stationary state

State with $\mu=0$? Extension to no-exponential growth?

Transition from exponential growth to non-growth?

(cf, Himeoka, KK, Phys Rev X 2017)

No longer low-dim?

*cf. Happy families are all alike; every unhappy family is unhappy
in its own way. (Anna Karenina)*

(3) Evolution-Development Congruence?

correspondence in process; (Kohsokabe, KK, JEZ B 2016)

(4) Cell-cell interaction → Coexistence of Diverse

species by leaked chemicals (Yamagishi et al. bioRxiv)

origin of reproduction

Replication of catalytic molecules catalyzed by others

Evolution of catalytic activity k (through replication error):

While working as catalyst, the molecule is not replicated

→ Selfishness preferable $k \downarrow$ --- molecular level

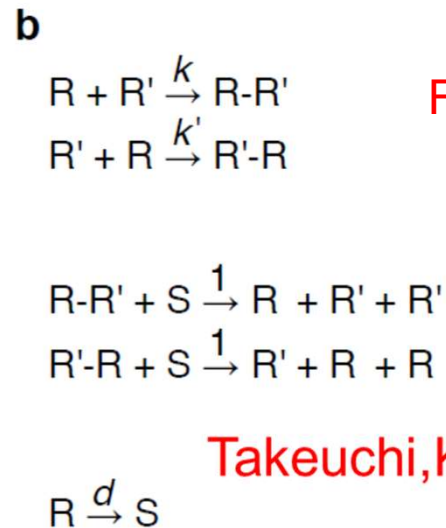
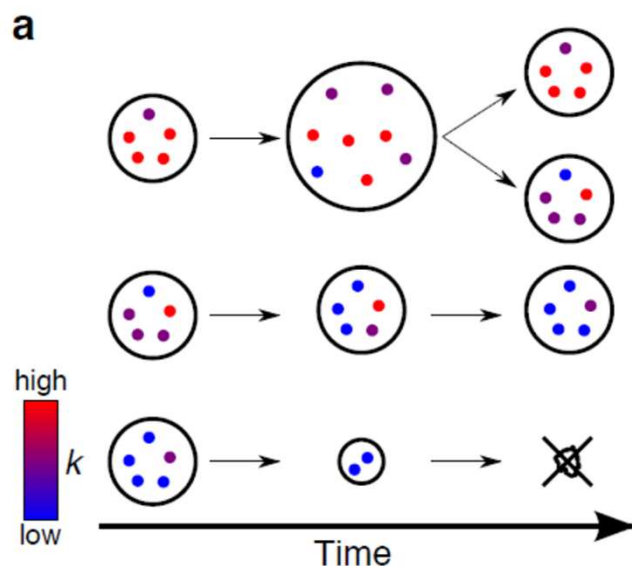
→Extinct → remedy Put molecules into a protocell

Protocell divides when the total # of molecules reaches N

those with smaller $\langle k \rangle$ are selected out

Cell level $\langle k \rangle \uparrow$

Conflicting Multilevel Evolution



Formation of complex

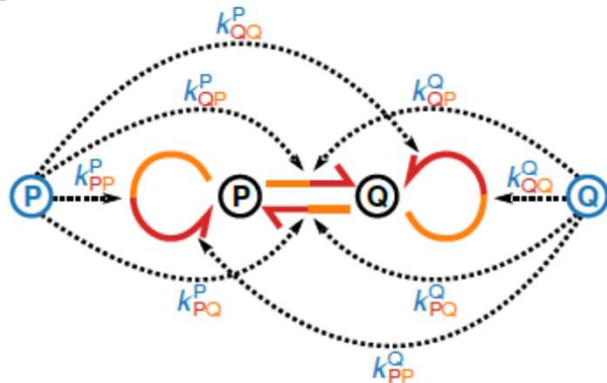
each molecule needs catalysts for replication

Takeuchi, KK, Hogeweg ProcRoy.Soc.2016

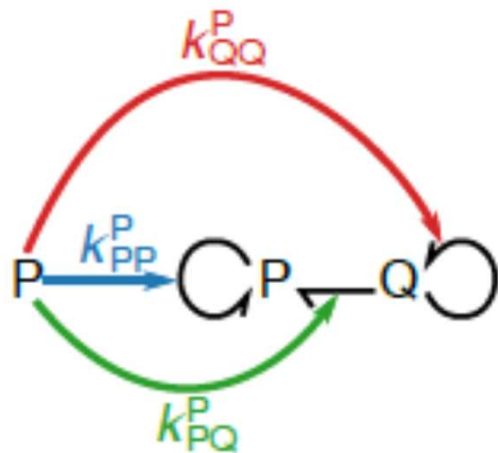
Origin of 'Central Dogma' -- allow for cells with larger N

Two-species (P,Q) → Symmetry breaking to
Functional (catalytic) vs information (template) molecules

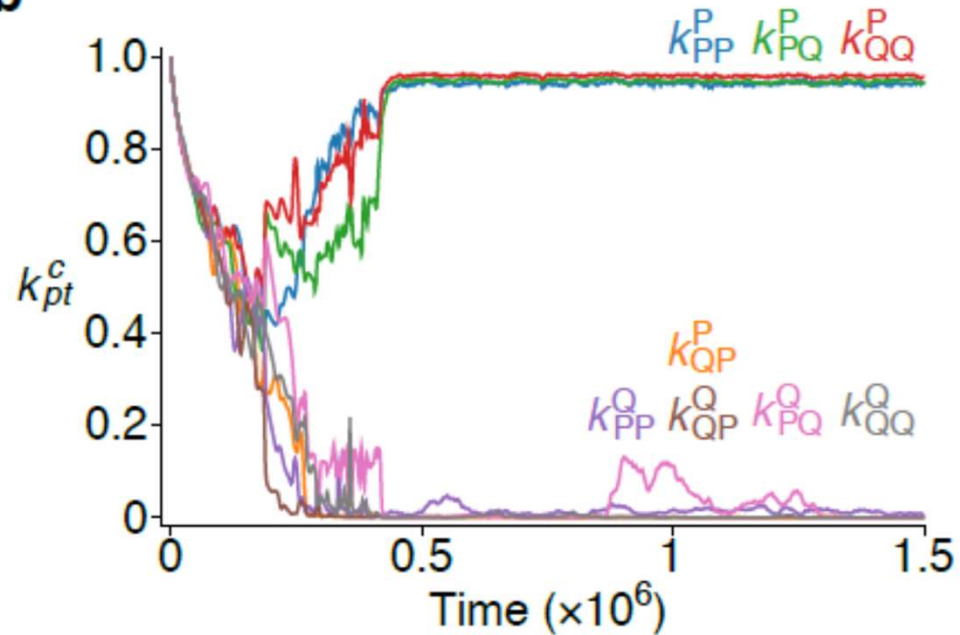
a



meaning	tail	arrow	head
reaction	template	→	product
catalysis	catalyst	→	reaction



b

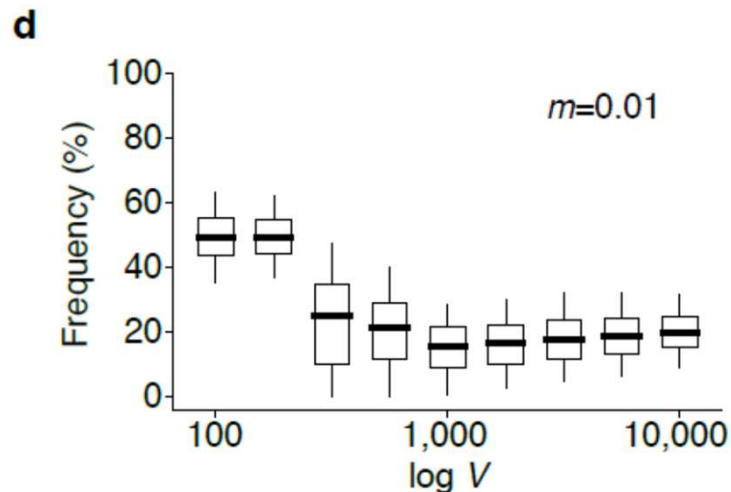
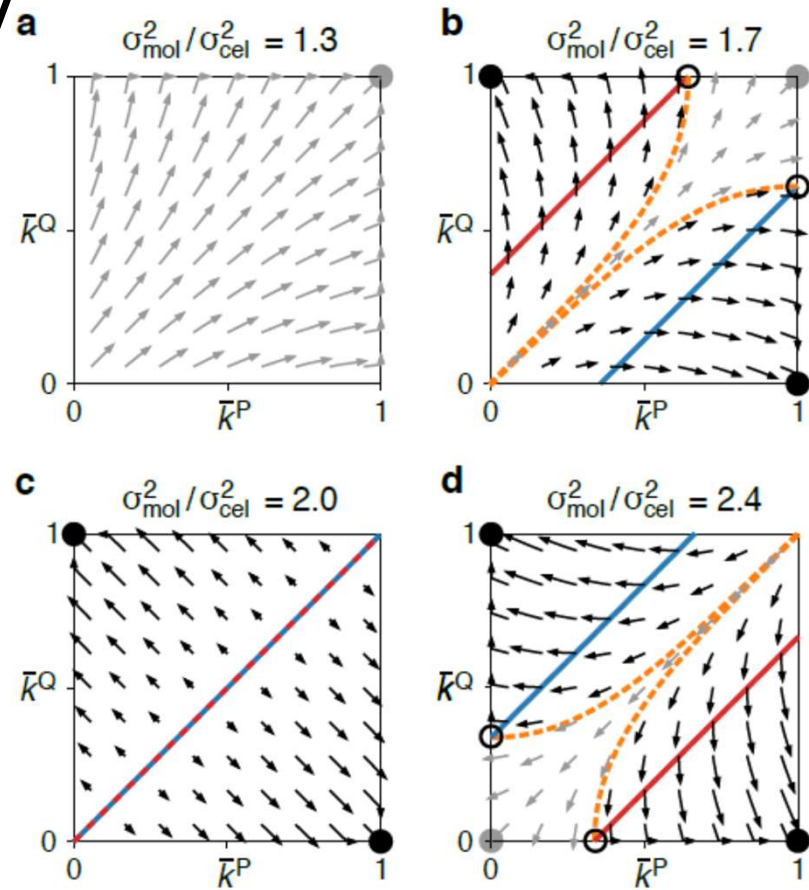


Takeuchi, KK,
BioRxiv 2019

Theoretical Explanation by Price equation (with multilevel)

$$\Delta \bar{k}^P \approx \bar{\omega}^P (\beta_P^P \sigma_{\text{cel}}^2 - \gamma_P^P \sigma_{\text{mol}}^2) + \bar{\omega}^Q \beta_P^Q \sigma_{\text{cel}}^2$$

$$\Delta \bar{k}^Q \approx \bar{\omega}^P \beta_Q^P \sigma_{\text{cel}}^2 + \bar{\omega}^Q (\beta_Q^Q \sigma_{\text{cel}}^2 - \gamma_Q^Q \sigma_{\text{mol}}^2)$$



Intuitively: loss of template information
 → Not as a unit for selection
 → No longer evolution to lose catalysis

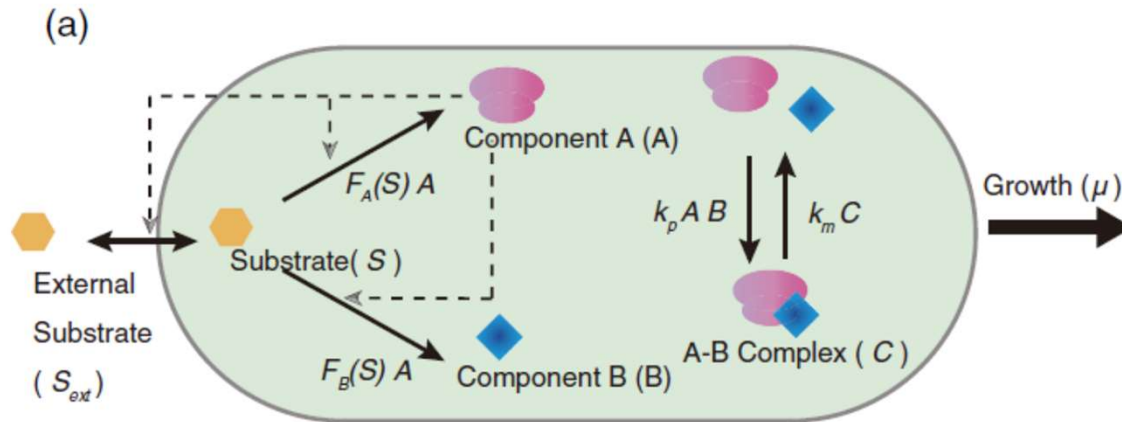
Takeuchi, KK,
 BioRxiv 2019

Transition to Stationary Phase (“Sleeping”) State

Minimal(?) Model

Himeoka, KK 2017 Phys Rev X

Autocatalytic growth by Active (eg Ribosomal) protein + Waste Molecules by replication error etc



Waste+Active
Form a
Complex C

$$\frac{dS}{dt} = -F_A(S)A - F_B(S)A + A(S_{\text{ext}} - S) - \mu S,$$

$$\frac{dA}{dt} = F_A(S)A - G(A, B, C) - d_A A - \mu A,$$

$$\frac{dB}{dt} = F_B(S)A - G(A, B, C) - d_B B - \mu B,$$

$$\frac{dC}{dt} = G(A, B, C) - d_C C - \mu C,$$

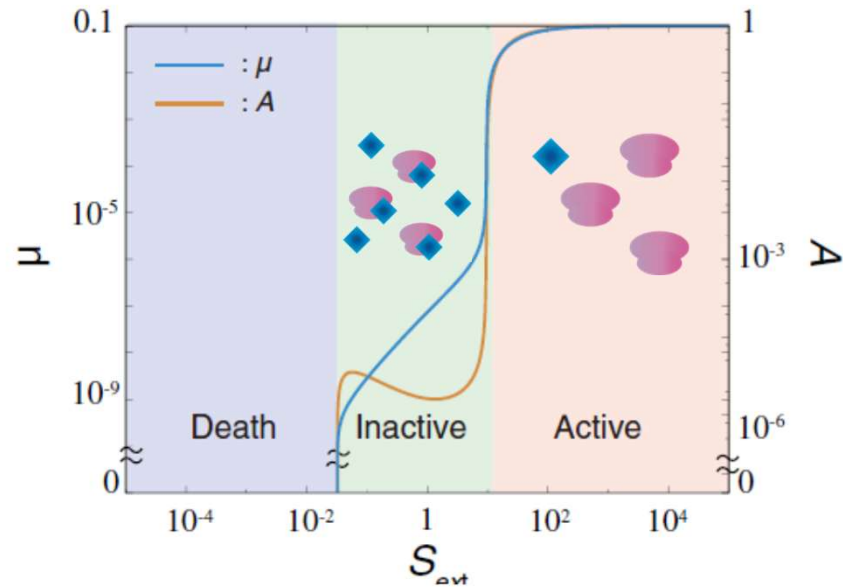
$$G(A, B, C) = k_p AB - k_m C.$$

Low Nutrient
→ Slow
Accumulation
of Waste

$$\left[\frac{d}{dS} \right] \left\{ \frac{F_A(S)}{F_B(S)} \right\} > 0.$$

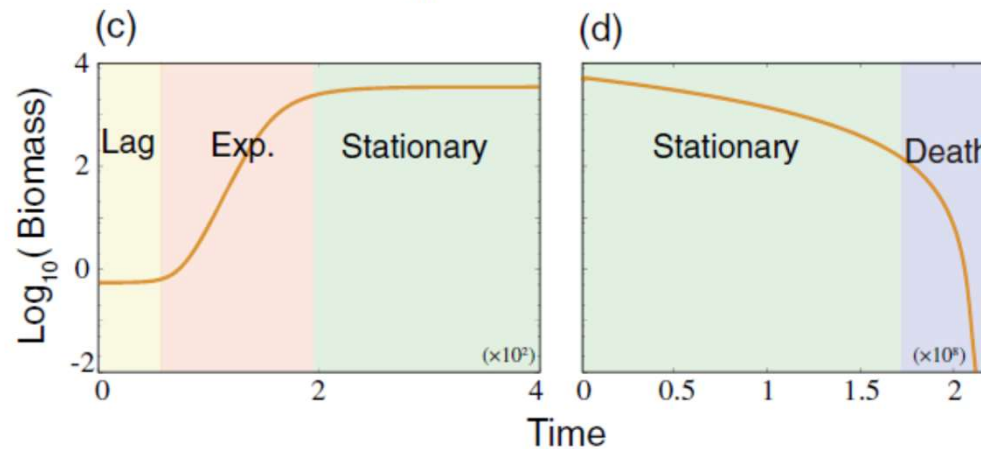
we assume that this ratio increases

Activator for Growth + Inhibition by Waste → Transition to Sleeping state with $\mu \sim 0$ upon nutrient depletion



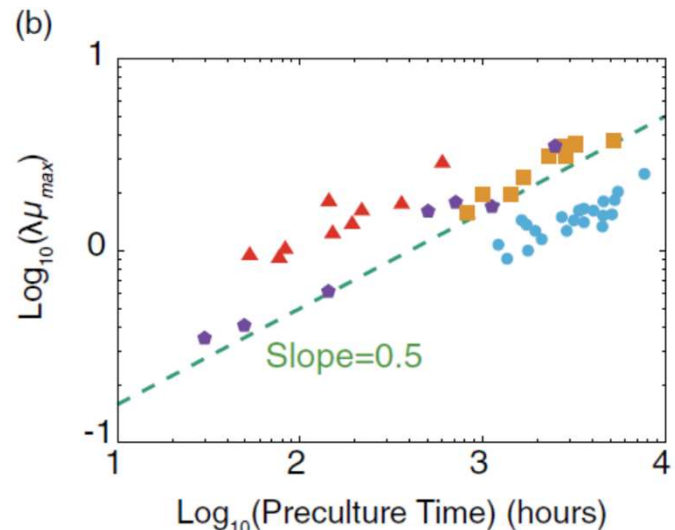
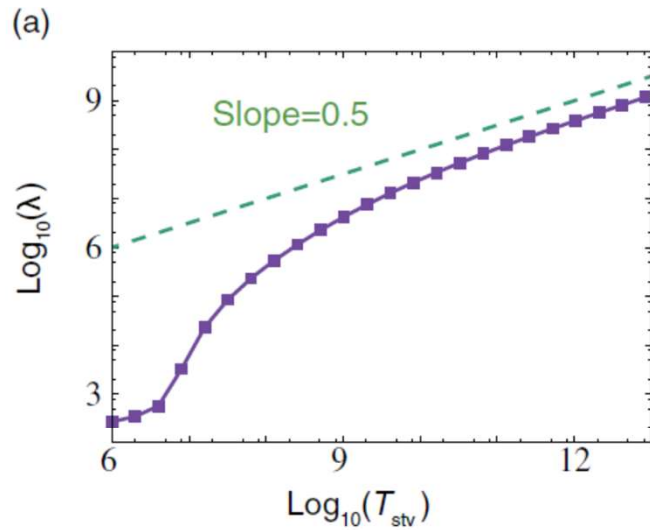
Most Active Proteins are trapped in Complex

Active Proteins are protected

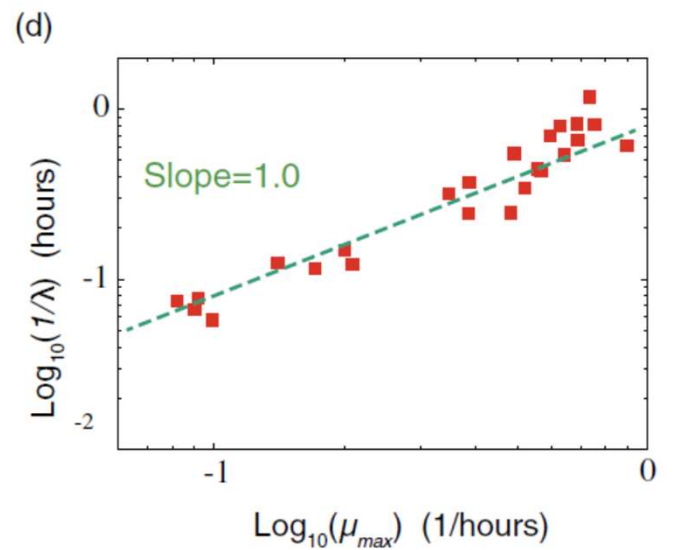
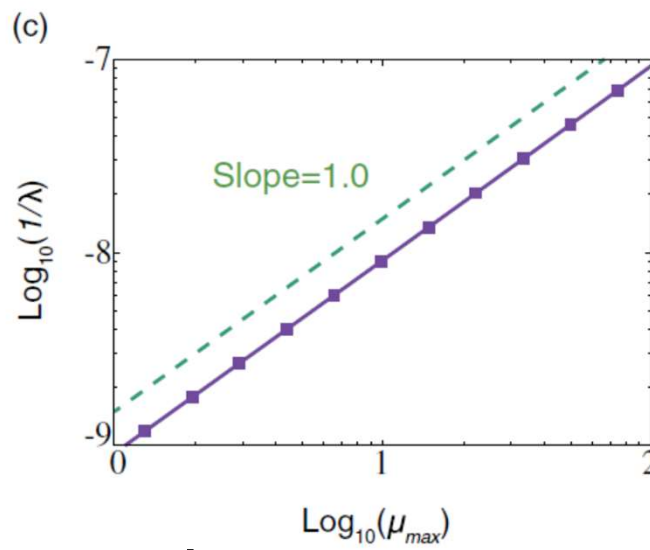


Transition from exponentially growing state to suppressed growth state (growth rate reduced to 5-6 digits)

→ Waste Inhibits the growth (and degradation) by forming a Complex



$$\lambda \propto \sqrt{T_{\text{stv}}}$$



$$\lambda \propto 1/\mu_{\text{max}}$$

Starvation

$$A + B \leftrightarrow C \quad \dot{B}(t) \propto F_B(S)/B \quad dB^2/dt \sim \text{const.} \quad B(t) \propto \sqrt{t}$$

$$k_p AB \sim k_m C$$

Lag time (recovery)

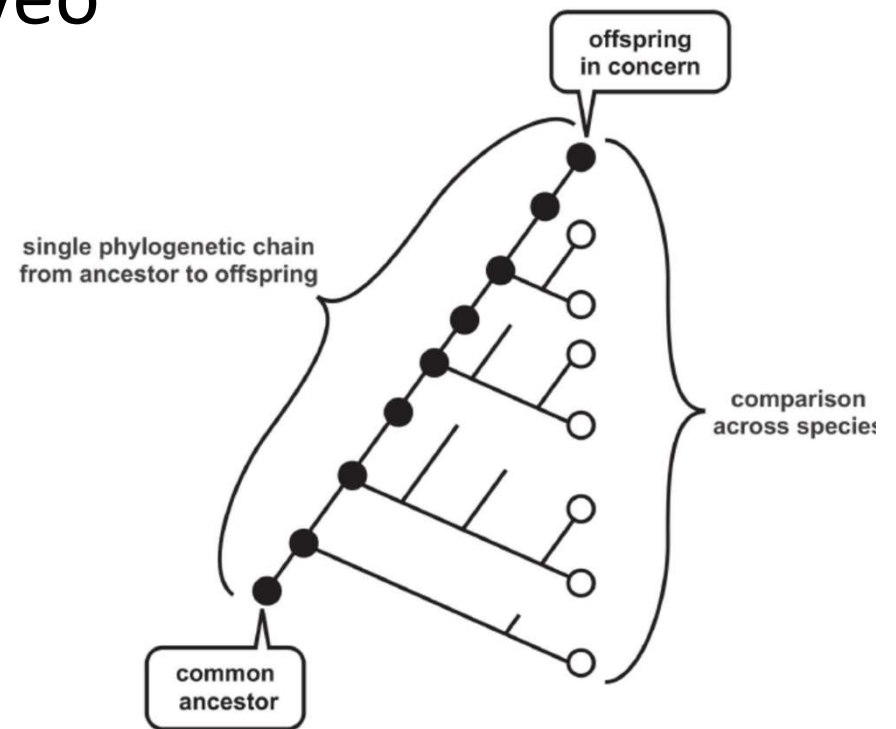
$$\mu B = F_A AB \propto B/B = \text{const}$$

$$\mu = F_A A \text{ and } A \propto 1/B, \quad \dot{B}(t) \propto -\text{const} \quad B(t) \sim B(0) - \text{const} \times t$$

(3) Evolution-Development Congruence?

- Discussed by Haeckel as ontogeny recapitulates phylogeny but too inaccurate, and dismissed
- ?But maybe some relationship between the two
- Merit in numerical evo-deveo

Consistency between
Processes of
different-scale (not states)



Numerical Evolution of development

Cells in 1-dim line

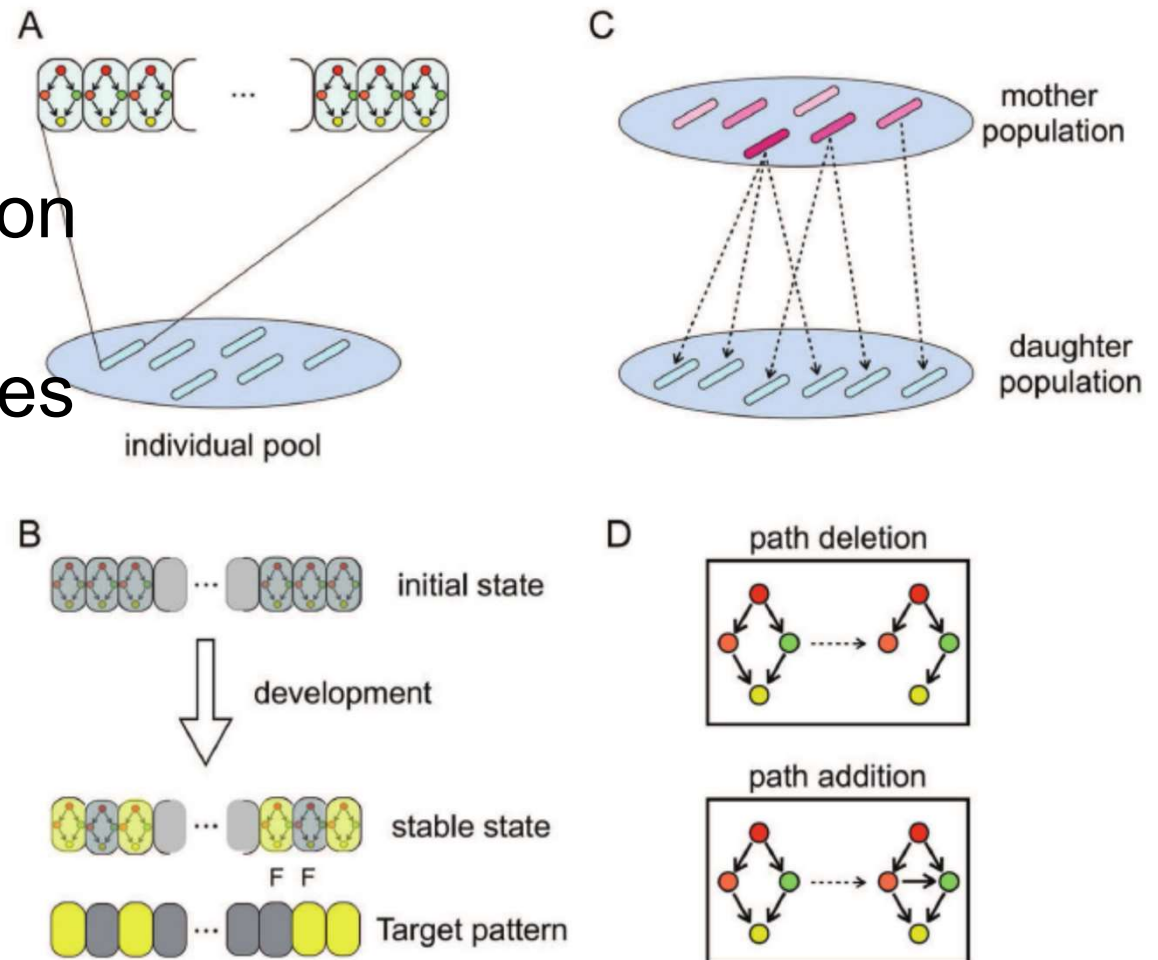
Each cell has protein expression dynamics by GRN

External morphogen gradient for input genes

diffusion of proteins

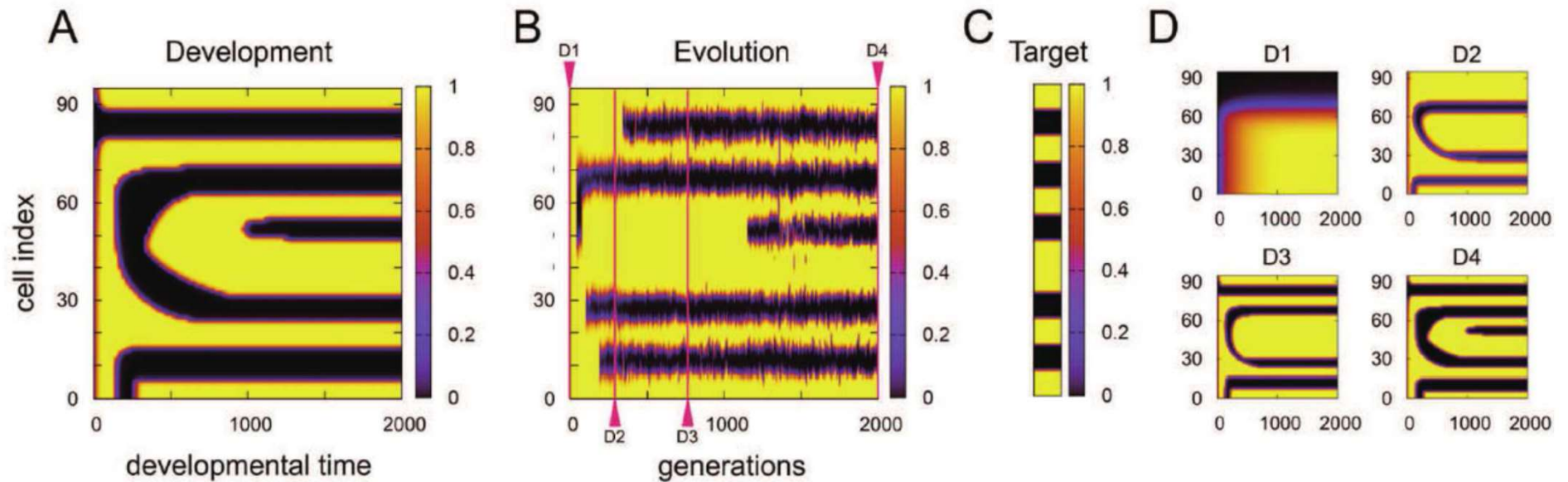
Evolve GRN by mutation

Fitness: Given target pattern for output genes



Kohsokabe & KK
J Exp Zoology B
(2016)

Evo-devo congruence
topology (+ ordering) of stripe pattern
formation agrees,

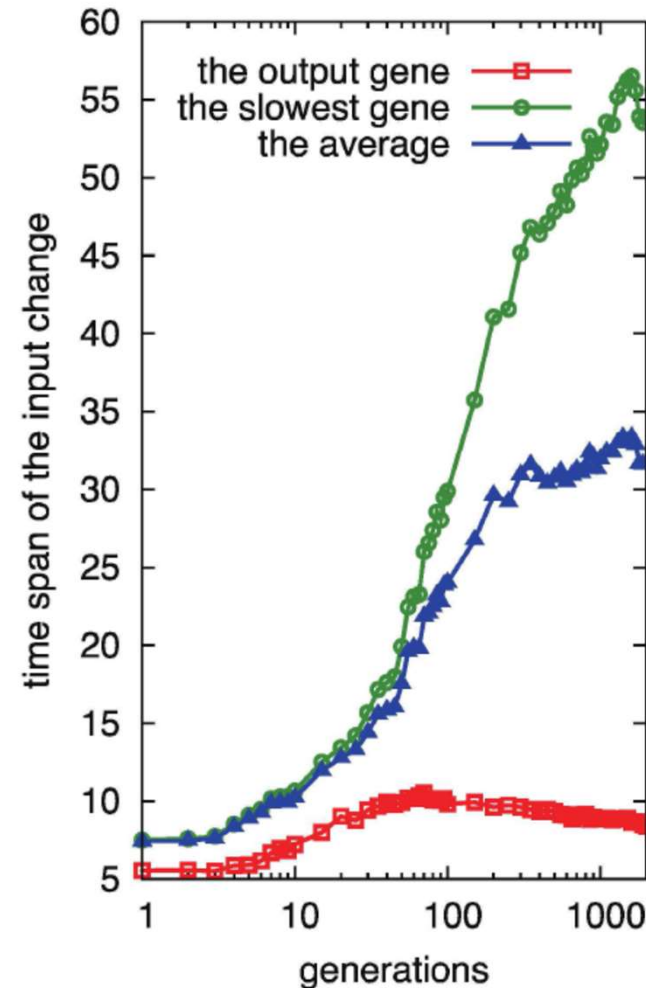


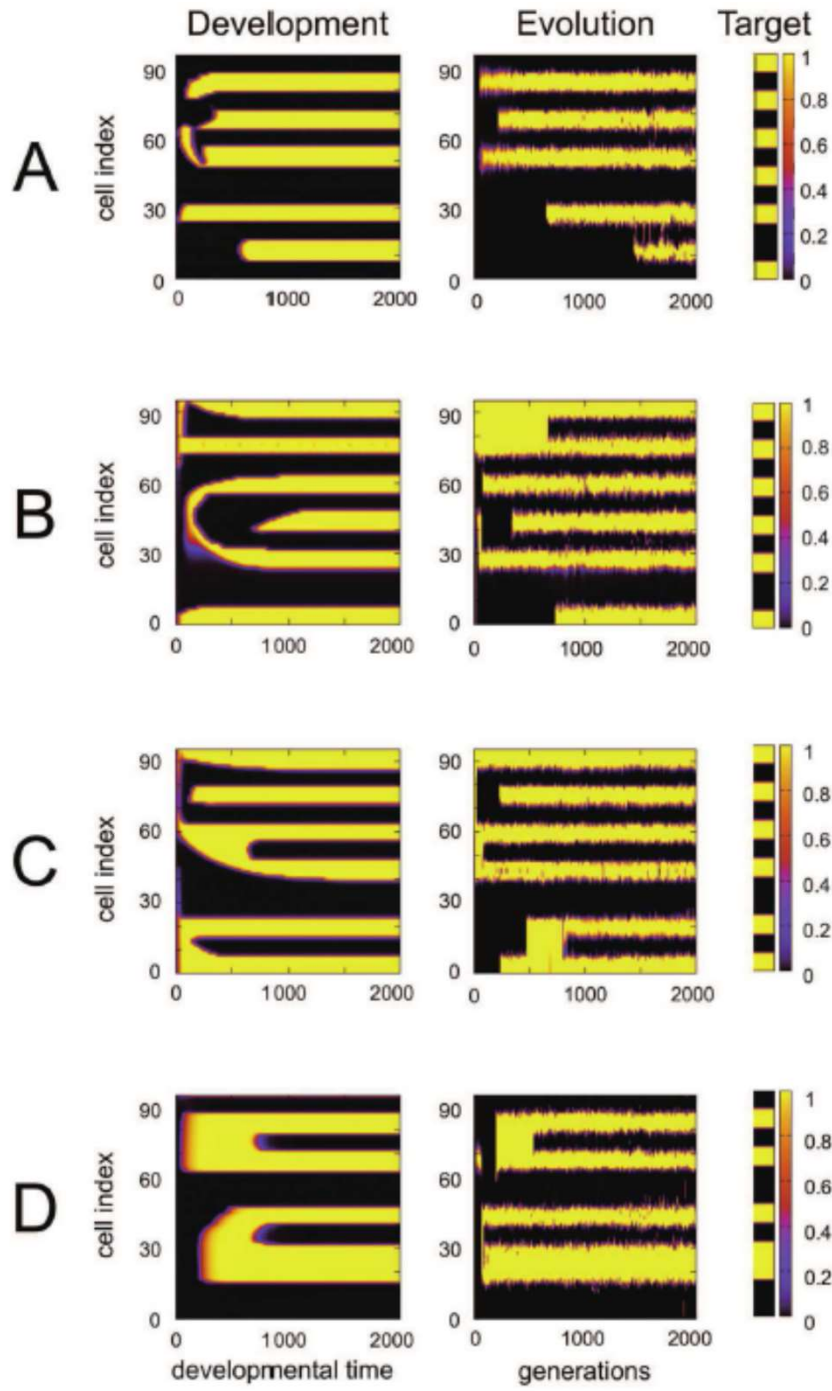
Why congruence?

both evo and devo consist of quasistationary regime + epoch for rapid stripe formation

Evolution -- punctuated equilibrium (need time for relevant mutation)

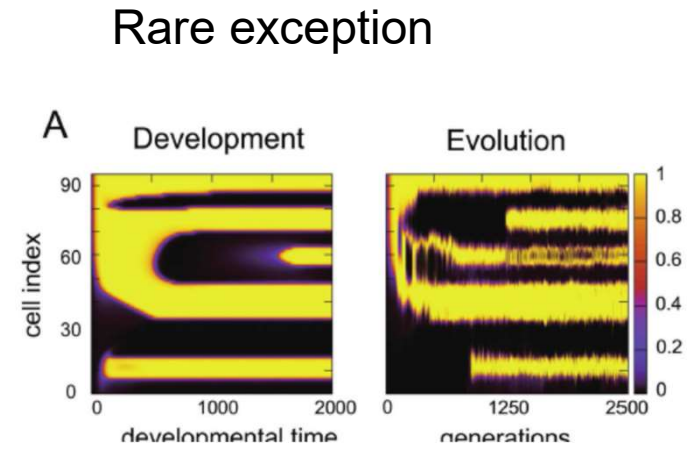
Development – emergence of genes whose expression change slowly and control the output expression → works as a “ bifurcation parameter”





Comparison between evo and devo

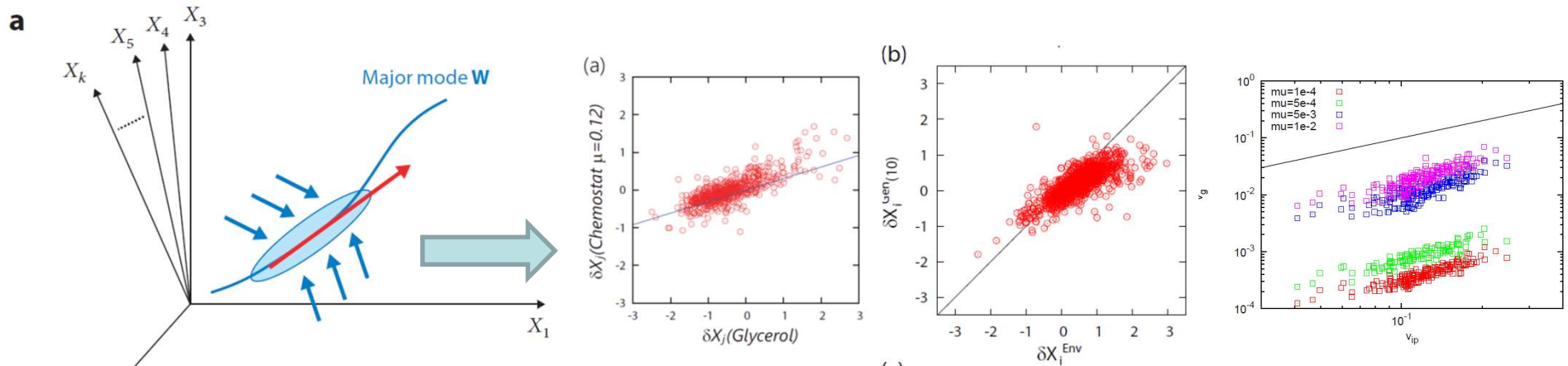
For most (95%) examples, good evo-devo congruence



Summary

Low-dimensional structure formed from high-dimensional phenotypic space ← robustness

(Furusawa, KK, Phys Rev E, 2018; KK, Furusawa, Ann Rev Biophys 2018; Sato, KK, PRR 2020; Sakata, KK, PRL 2020)



collaborator

**Chikara Furusawa ;
Takuya Sato**

Universal law for adaptation

(KK Furusawa Yomo PRX2015)

Evolutionary LeChatelier Principle

(Furusawa KK Interface 2015)

Vg-Vip Law (→ direction in phenotypic evolution)

