MEASURING SPEED OF GRAVITY AND DISPERSION USING GW370817 IN A SINGLE COSMIC EXPLORER

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Hearing beyond the standard model with cosmic sources of Gravitational Waves
ICTS, Bandalore
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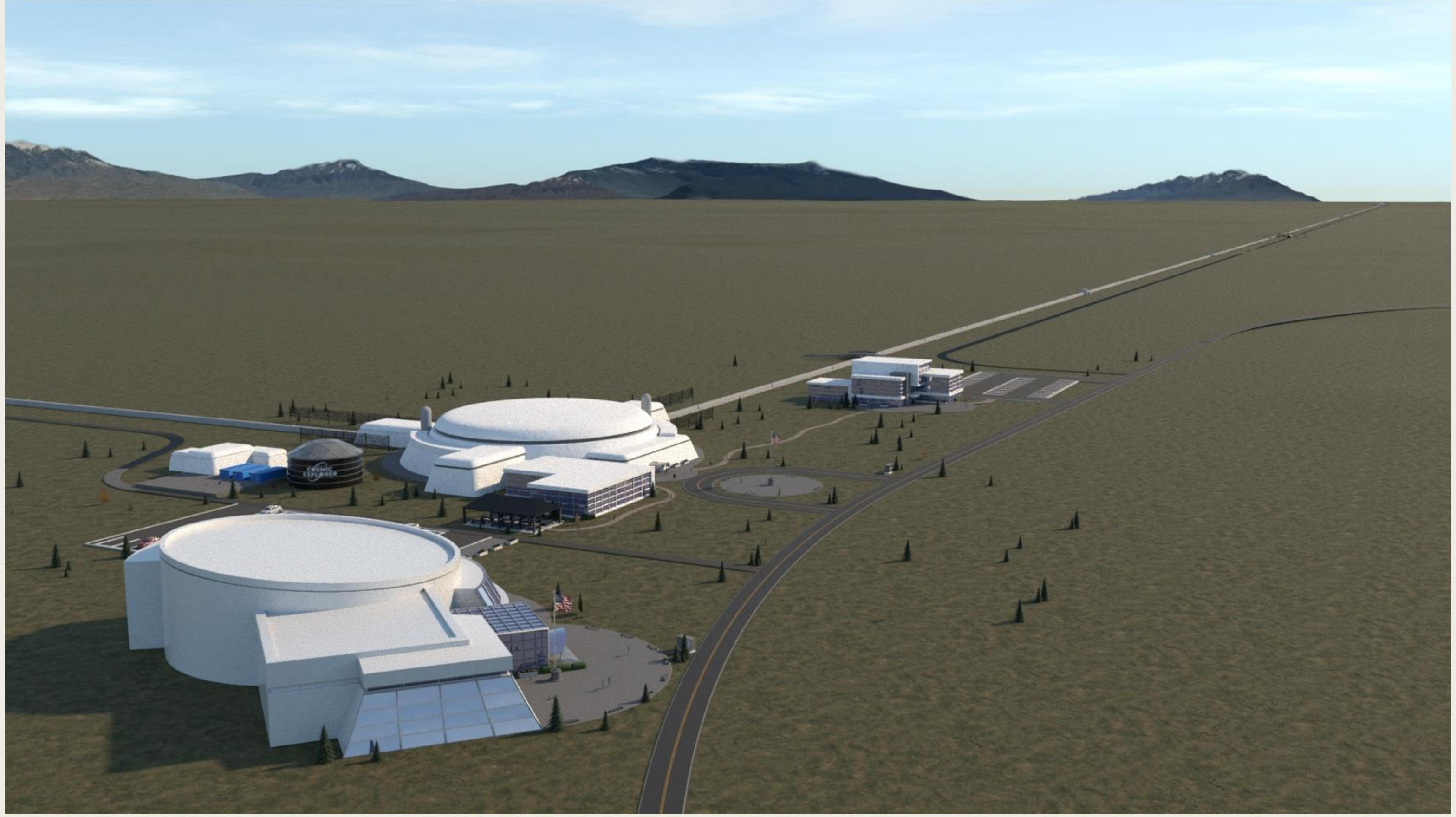
Outline

- Introduction to Cosmic Explorer
- Detection of GWs from Compact Binary Coalescences
- Parameter Estimation
- Measuring speed of gravity by GWs alone
- Detector-Size effects
- GW370817
- Higher-Order Modes
- The injected waveform
- Results: Speed of Gravity
- Frequency Dependent Dispersion Relation
- Results: Dispersion
- Conclusion

Cosmic Explorer

- Next-generation GW observatory
- Two L-shaped detectors with arm length 20 km and 40 km
- We use only one 40 km CE for the study



(Credit: Angela Nguyen, Virginia Kitchen, Eddie Anaya, California State University Fullerton)

Cosmic Explorer

- Next-generation GW observatory
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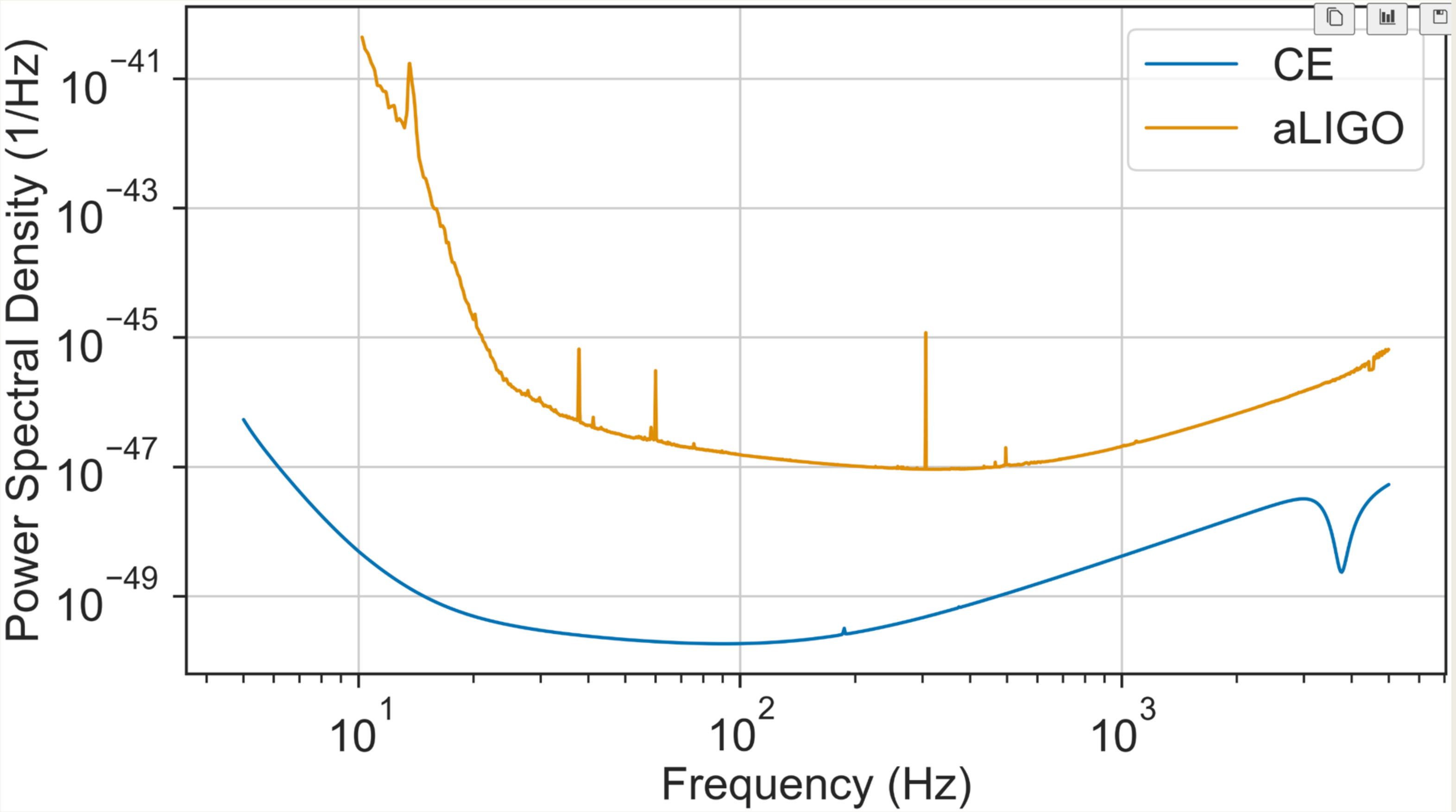
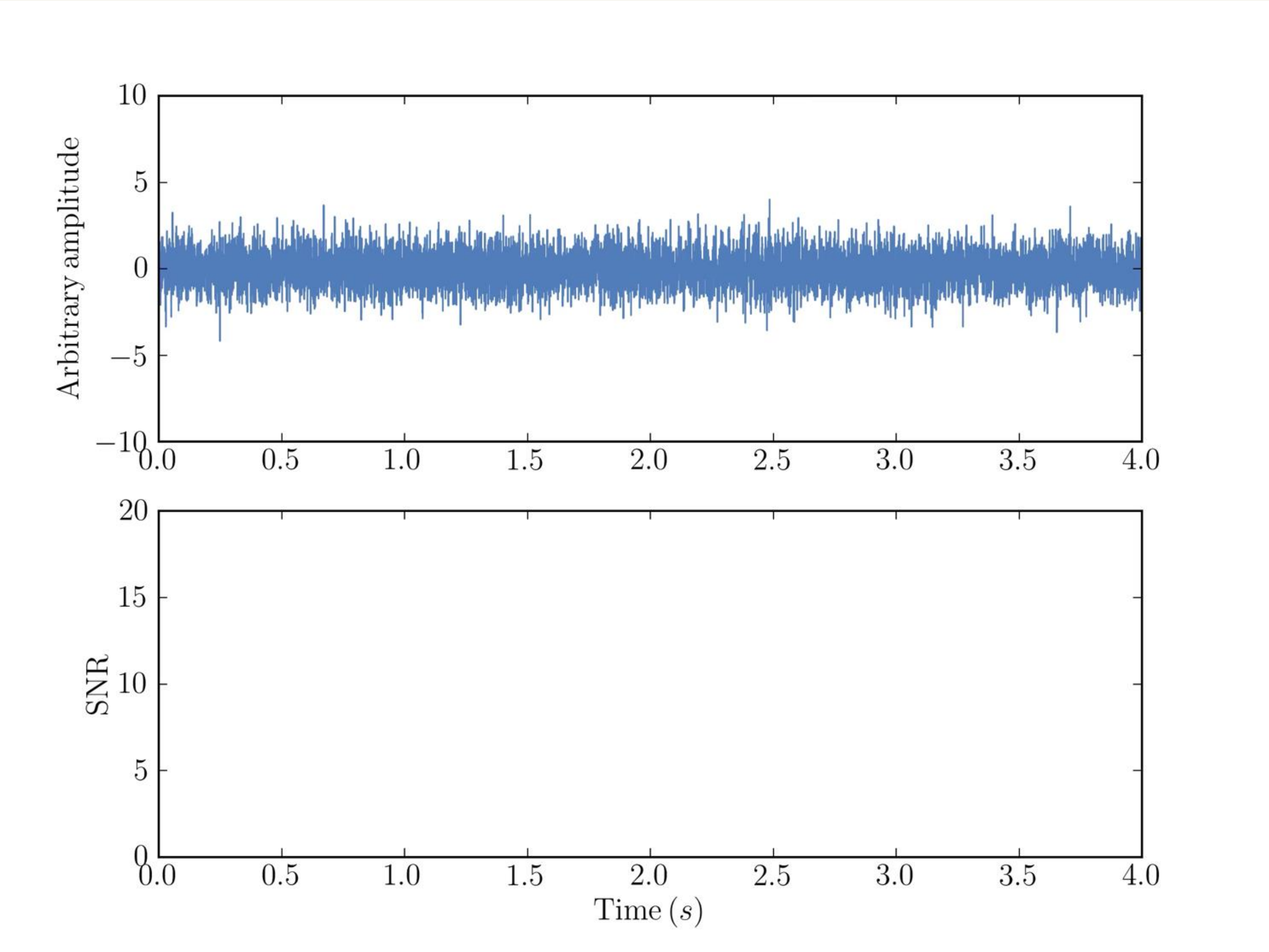


What would these detectors detect?



$$\mathrm{SNR} = \sqrt{4 \int_{f_{\min}}^{f_{\max}} \frac{|\tilde{d}(f)\tilde{h}^*(f)|^2}{S_n(f)}\, df}$$

Credit: Ryan Magee

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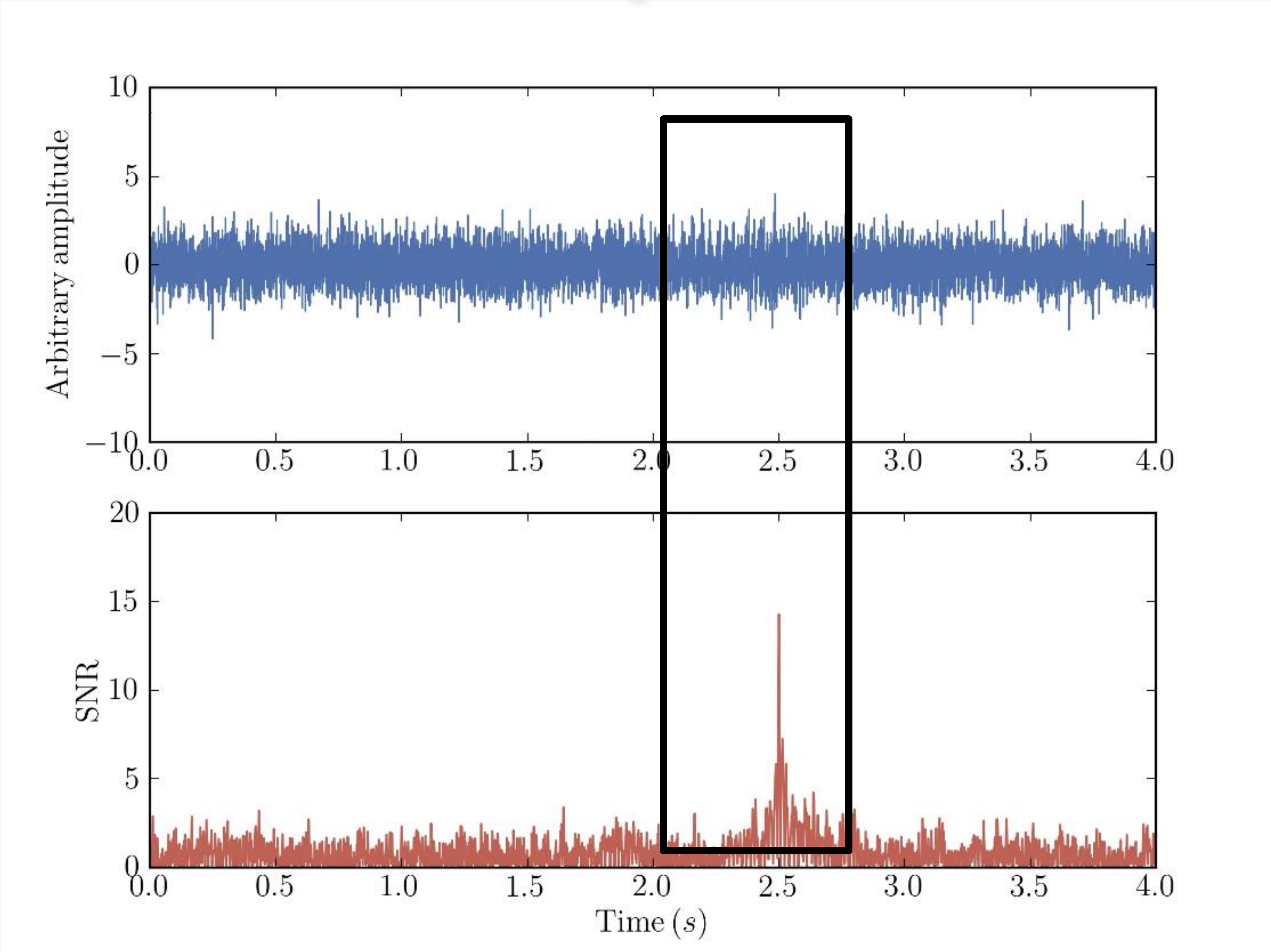


Once the part of the data with the signal is identified, we can proceed to parameter estimation.

Parameter Estimation

Parameters like mass, spin, sky-position

- The observed signal has a GW and noise

$$d(t) = h(t) + n(t)$$

- The noise distribution is assumed stationary and gaussian and so we can define a likelihood function

$$\mathcal{L}(\vec{\theta}; d(t)) = \exp\left(-\frac{1}{2}\langle d(t) - h(t,\vec{\theta}) | d(t) - h(t,\vec{\theta})\rangle\right)$$

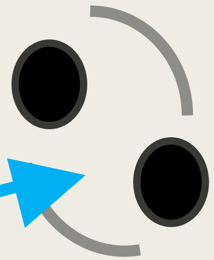
- We need the distribution of parameters given the data, or the posterior probability density which can be obtained by Bayes theorem

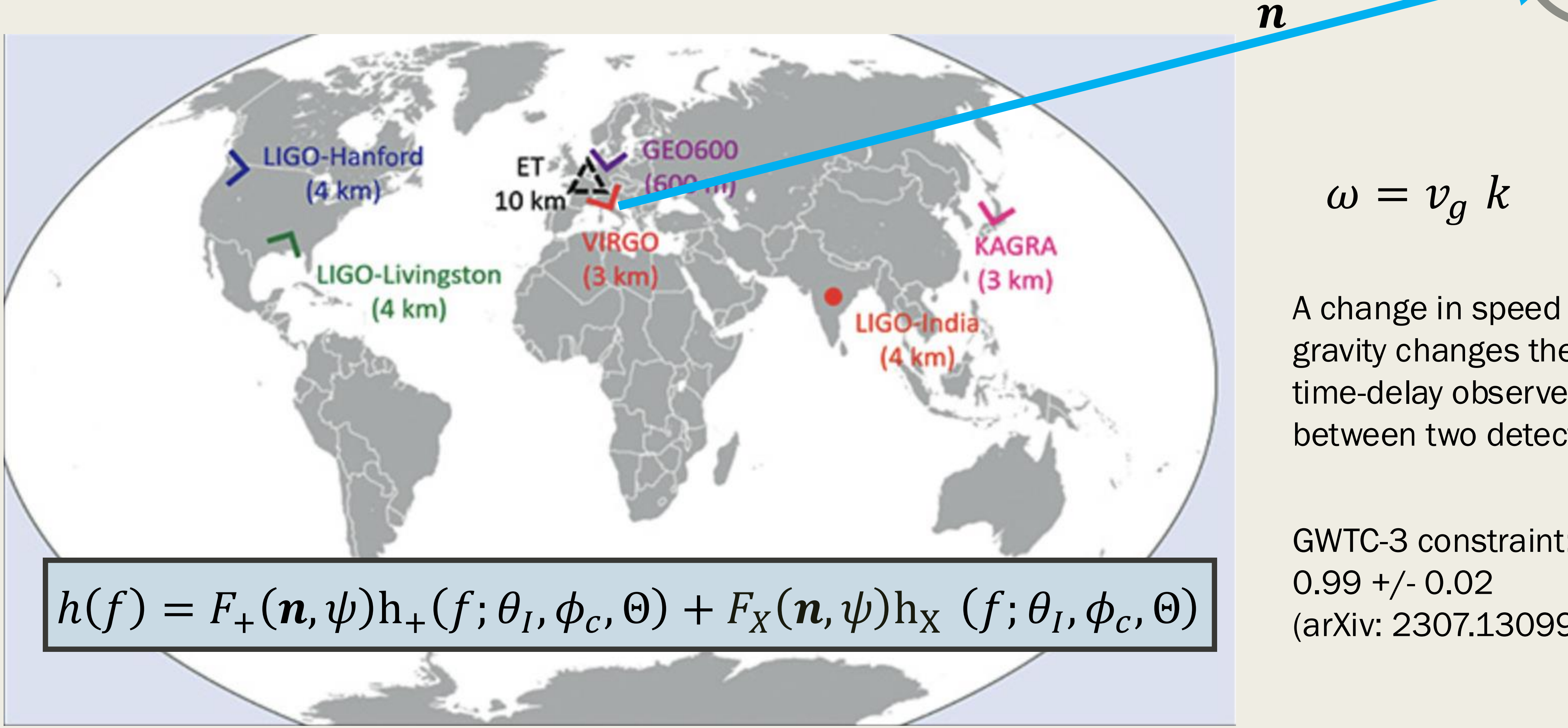
$$p(\vec{\theta}|d) = \frac{\mathcal{L}(\vec{\theta}; d)\pi(\vec{\theta})}{\mathcal{Z}}$$

$$\langle a|b\rangle = 4\Re\int_{f_{\rm low}}^{f_{\rm high}} \frac{\tilde{a}(f)\tilde{b}^*(f)}{S_n(f)} df$$

We use a nested sampler to sample the posterior space and approximate the likelihood using Relative Binning.

How do we measure speed of gravity?





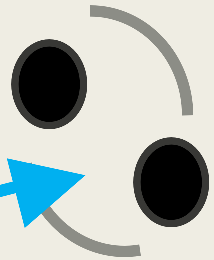
$$h(f) = F_{+}(\boldsymbol{n},\psi)\mathrm{h}_{+}(f;\theta_I,\phi_c,\Theta) + F_{X}(\boldsymbol{n},\psi)\mathrm{h}_{X}\ (f;\theta_I,\phi_c,\Theta)$$

$$\omega = v_g\ k$$

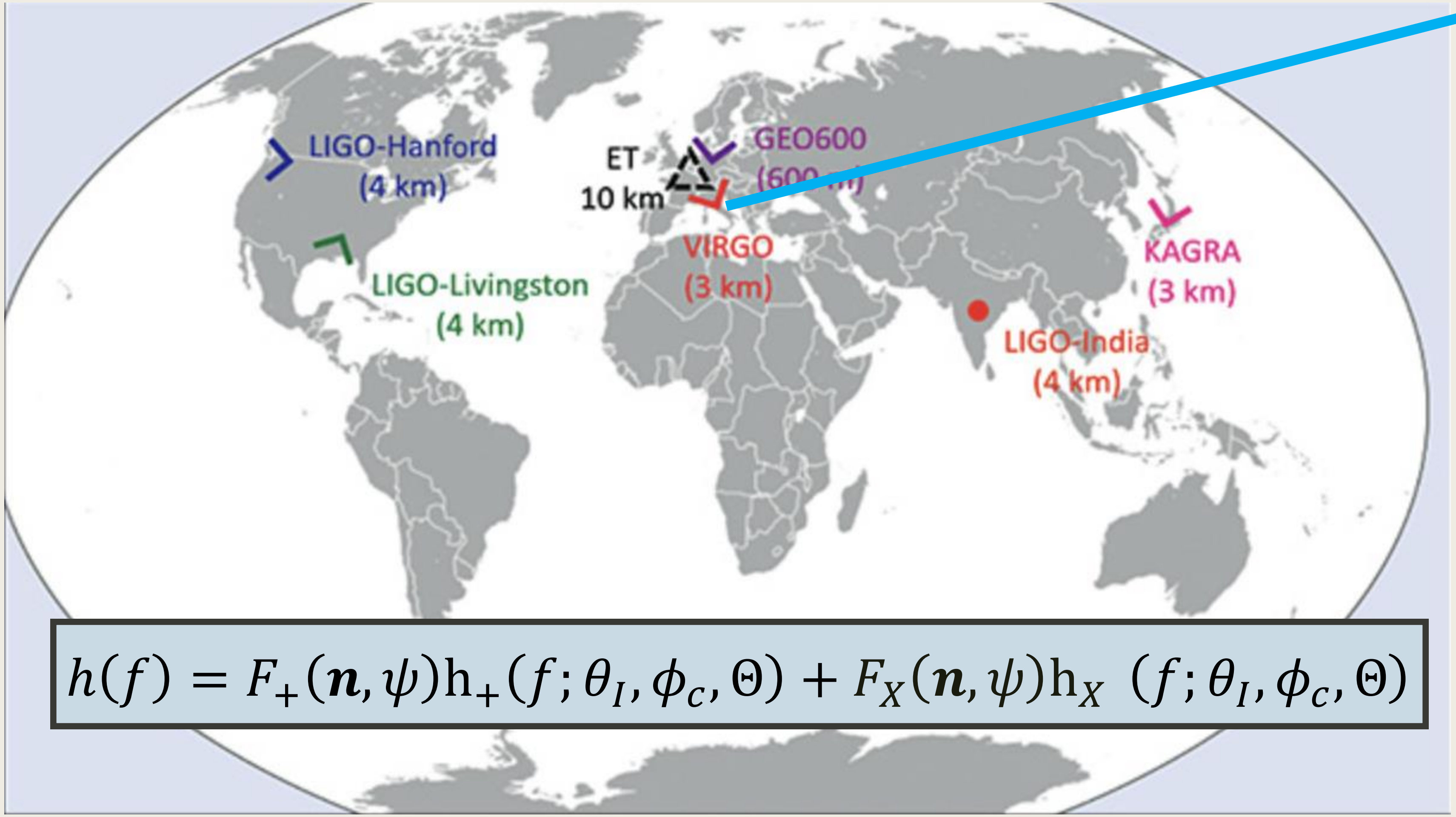
A change in speed of gravity changes the time-delay observed between two detectors.

GWTC-3 constraint:
0.99 +/- 0.02
(arXiv: 2307.13099)

How do we measure speed of gravity?



$\boldsymbol{n}$



$$h(f) = F_+(\boldsymbol{n},\psi)\mathrm{h}_+(f;\theta_I,\phi_c,\Theta) + F_X(\boldsymbol{n},\psi)\mathrm{h}_X\ (f;\theta_I,\phi_c,\Theta)$$

$$\omega = v_g\ k$$

If the signal lasts long $\boldsymbol{n}$ changes with time even if we have only detector.

*With multiple detectors or long signals we can measure the **group velocity** of GWs.*

Measuring speed of gravity using 1 CE: Detector Size

- LIGO:

$$f \sim 10\ - 1000\, Hz \quad L_{arm} \sim 4\, km \quad \frac{f L_{arm}}{c} \sim 0.0003 - 0.01$$

- CE:

$$f \sim 5\ - 2000\, Hz \quad L_{arm} \sim 40\, km \quad \frac{f L_{arm}}{c} \sim 0.01 - 0.2$$

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As the wavelength of a gravitational wave becomes comparable to the size of detector arms, the photon in the detector moves in a variable gravitational field.

Measuring speed of gravity using 1 CE: Detector Size

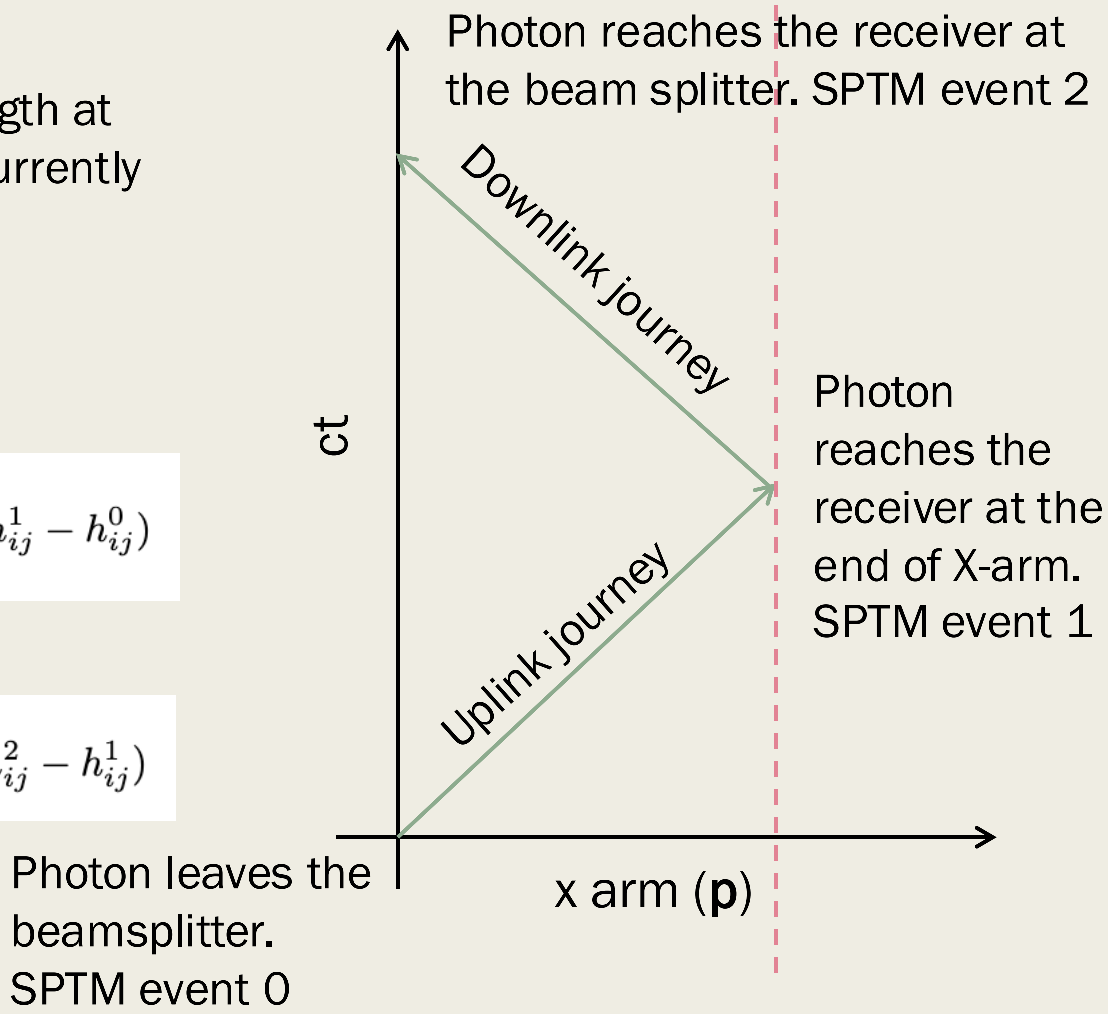
Length of the detector is comparable to the GW wavelength at high frequencies. So, the static approximation that we currently use is not valid.
Let's consider a photon in the x-arm of a detector.

Redshift of a photon for the uplink journey

$$\frac{\nu_0 - \nu_1}{\nu_1} = \frac{1}{2}\frac{\hat{\mathrm{p}}^i\hat{\mathrm{p}}^j}{1-\hat{\mathbf{p}}\cdot\hat{\mathbf{n}}}(h^1_{ij} - h^0_{ij})$$

Redshift of a photon for the downlink journey

$$\frac{\nu_1 - \nu_2}{\nu_2} = \frac{1}{2}\frac{\hat{\mathrm{p}}^i\hat{\mathrm{p}}^j}{1+\hat{\mathbf{p}}\cdot\hat{\mathbf{n}}}(h^2_{ij} - h^1_{ij})$$

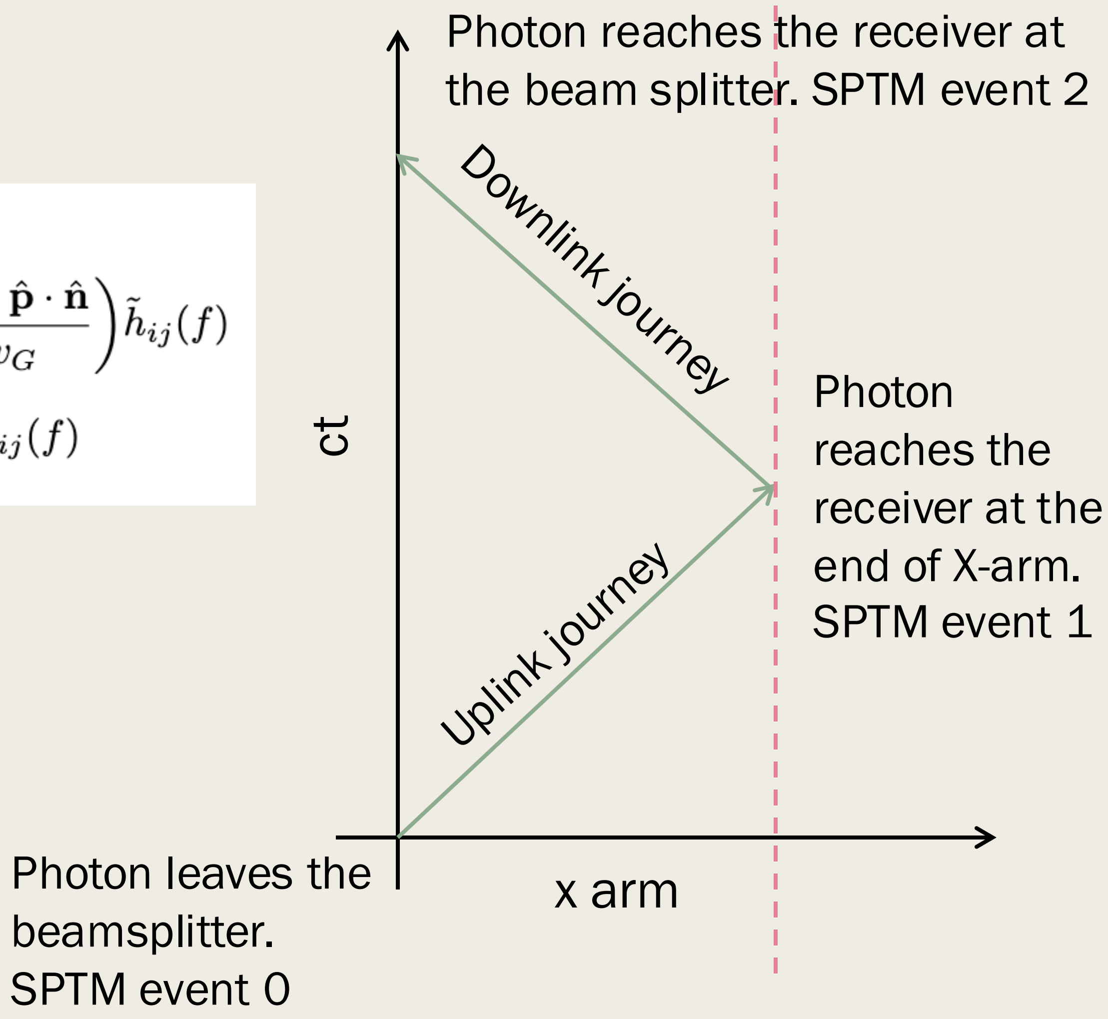


Measuring speed of gravity using 1 CE: Detector Size

$$
\begin{aligned}
h^0_{ij} &= h_{ij}(t) \\
h^1_{ij} &= h_{ij}\left(t + \frac{L}{v_G}(1 - \hat{\mathbf{p}}\cdot\hat{\mathbf{n}})\right) \\
h^2_{ij} &= h_{ij}\left(t + \frac{2L}{v_G}\right)
\end{aligned}
$$

F. T.

$$
\begin{aligned}
\tilde{h}^0_{ij} &= \tilde{h}_{ij}(f) \\
\tilde{h}^1_{ij} &= \exp\left(2\pi i f L \frac{1 - \hat{\mathbf{p}}\cdot\hat{\mathbf{n}}}{v_G}\right)\tilde{h}_{ij}(f) \\
\tilde{h}^2_{ij} &= \exp\left(\frac{4\pi i f L}{v_G}\right)\tilde{h}_{ij}(f)
\end{aligned}
$$



Measuring speed of gravity using 1 CE: Detector Size

Photon reaches the receiver at the beam splitter. SPTM event 2

$$
\begin{aligned}
\frac{\nu_0-\nu_2}{\nu_0} &= \frac{\nu_0-\nu_1}{\nu_0}+\frac{\nu_1-\nu_2}{\nu_0}\\
&\simeq \frac{\nu_0-\nu_1}{\nu_1}+\frac{\nu_1-\nu_2}{\nu_2}\\
&= \frac{1}{2}\hat{\mathrm{p}}^i\hat{\mathrm{p}}^j\tilde{h}_{ij}(f)\left[\frac{\exp\left(\frac{4\pi i f L}{v_G}\right)-\exp\left(2\pi i f L\frac{1-\hat{\mathbf{p}}\cdot\hat{\mathbf{n}}}{v_G}\right)}{1+\hat{\mathbf{p}}\cdot\hat{\mathbf{n}}}-\frac{\exp\left(2\pi i f L\frac{1-\hat{\mathbf{p}}\cdot\hat{\mathbf{n}}}{v_G}\right)-1}{1-\hat{\mathbf{p}}\cdot\hat{\mathbf{n}}}\right]\\
&= \frac{2\pi i f L}{v_G}D(\hat{\mathbf{p}}\cdot\hat{\mathbf{n}},2\pi i f L/v_G)\hat{\mathrm{p}}^i\hat{\mathrm{p}}^j\tilde{h}_{ij}(f)
\end{aligned}
$$

Photon leaves the beamsplitter. SPTM event 0

x arm

$$D(\boldsymbol{p}.\boldsymbol{n}, fL/c) = ½\, e^{2\pi i f L/c}\left\{e^{\frac{i\pi f L(1-\boldsymbol{p}.\boldsymbol{n})}{c}}\operatorname{sinc}\left[\frac{\pi f L(1+\boldsymbol{p}.\boldsymbol{n})}{c}\right] + e^{-\frac{i\pi f L(1+\boldsymbol{p}.\boldsymbol{n})}{c}}\operatorname{sinc}\left[\frac{\pi f L(1-\boldsymbol{p}.\boldsymbol{n})}{c}\right]\right\}$$

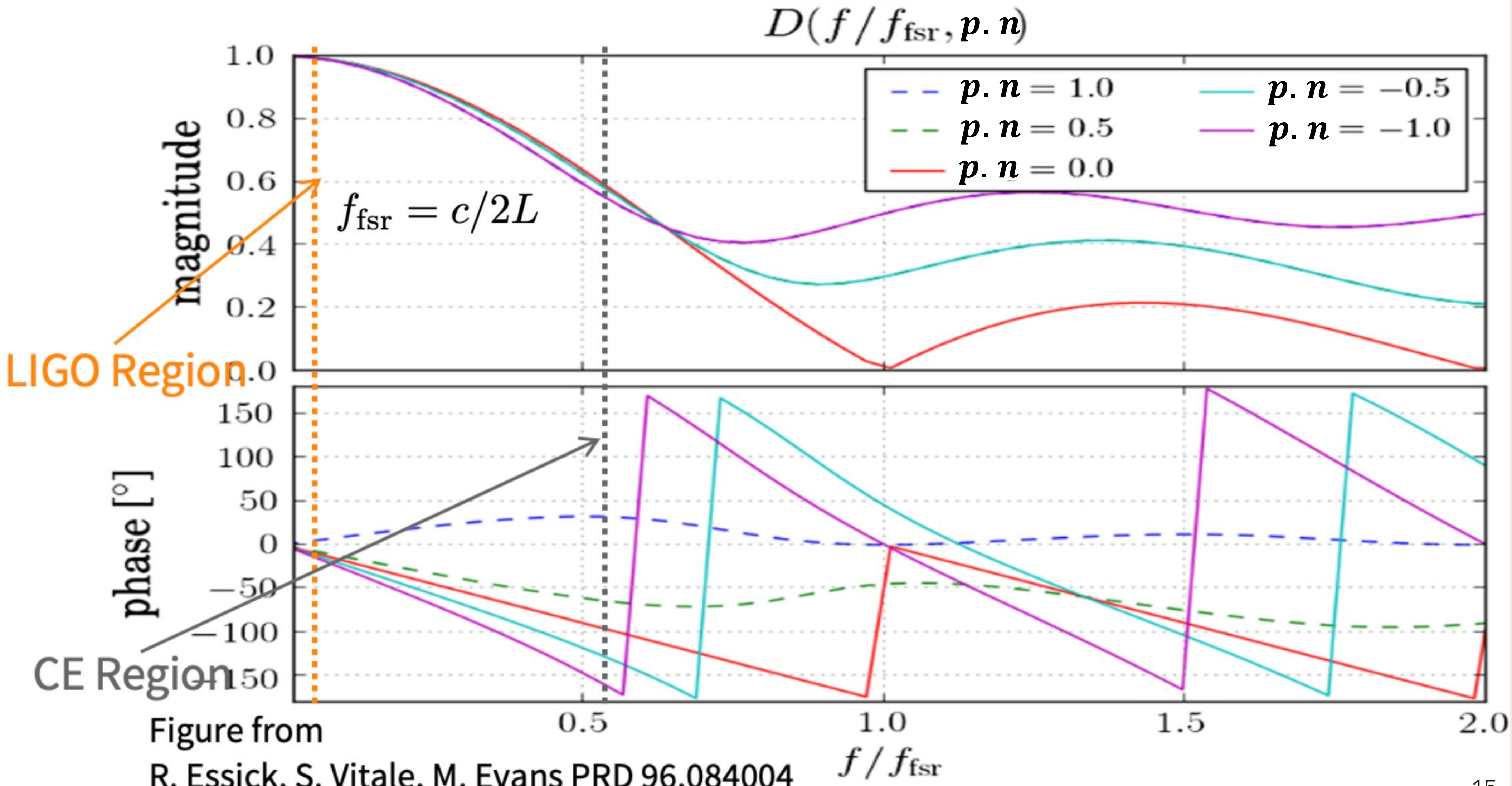


Figure from
R. Essick, S. Vitale, M. Evans PRD 96,084004

GW370817

- Simulated GW170817 like event in a single 40 km Cosmic Explorer.
- The inclination angle however is set to be edge on to amplify higher modes of radiation and decrease the SNR.
- The SNR of the injected signal is 1000.

Parameter	$\Theta_{GW370817}$
Chirp Mass ($\mathcal{M}_z$)	1.20994 $M_\odot$
Mass Ratio (q)	0.918
χ_1^z	0
χ_2^z	0
Right Asc. (RA)	3.44616
Declination (Dec)	-0.408084
Incl. Angle ($\theta_{\rm jn}$)	$\pi/2$
Pol. Angle (ψ)	2.212
Phase (ϕ)	5.180
Time at CE (t_{CE})	1187008882.45 s
Lum. Distance (d_L)	46.395 Mpc

What are higher-order modes?

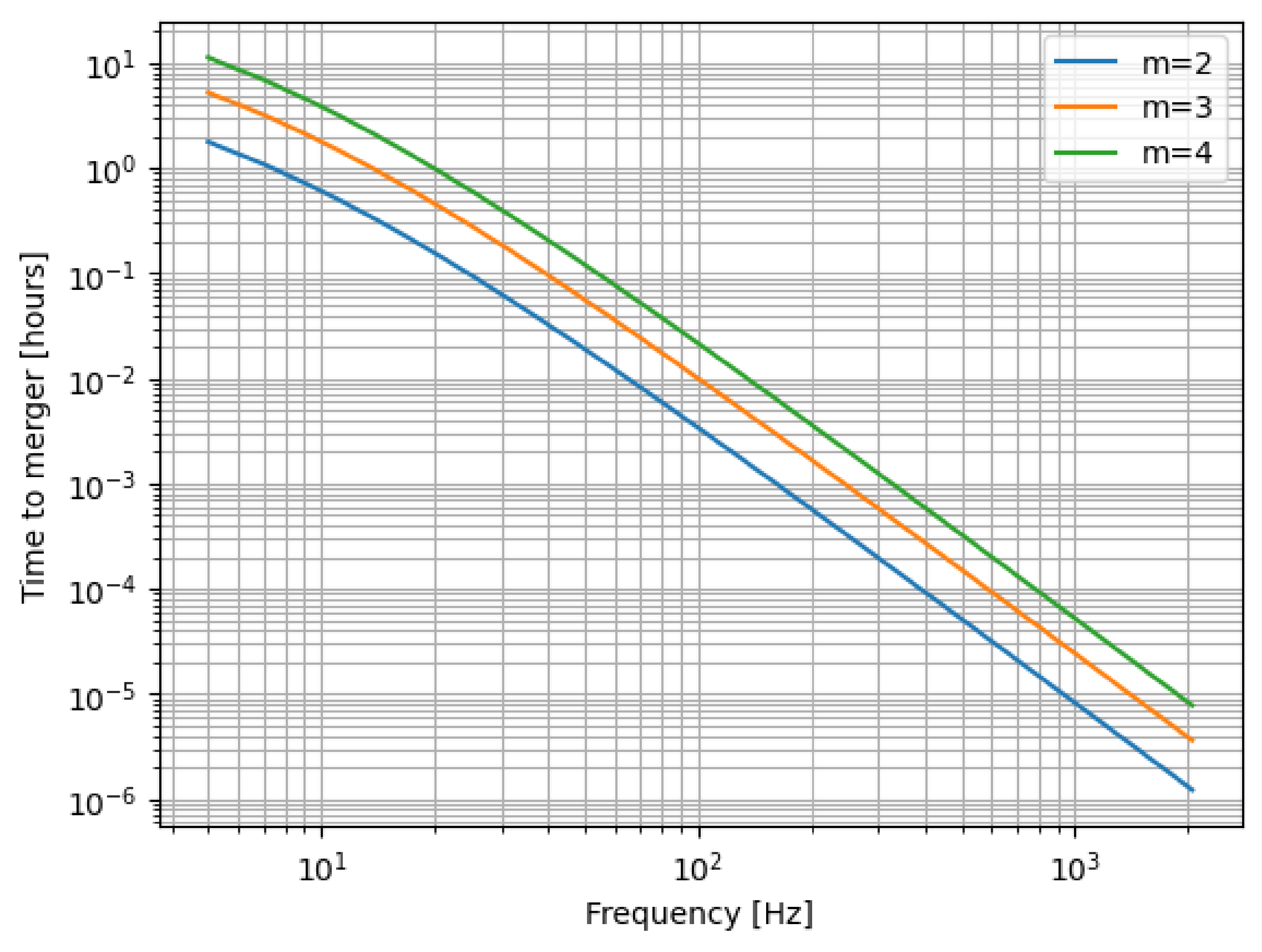
GW polarizations are often written in terms of modes in spherical harmonic basis with the dominant mode being l=m=2.

$$h_+ - i h_\times = \sum_{\ell=2}^{\infty} \sum_{m=-\ell}^{\ell} {}_{-2}Y_{\ell m}(\iota, \phi)\, h_{\ell m} \,,$$

For face-on cases there are no higher modes.

What are higher-order modes?

In PN theory the time to merger for a l,m mode from frequency f is equal to the time to merger for a (2,2) mode from frequency 2f/m.



The waveform

$$h(t) = F_{+}(\boldsymbol{n}(t),\psi)\sum_{l,m}\mathrm{h}_{l,m+}(t;\theta_I,\phi_c,\Theta) + F_{\times}(\boldsymbol{n}(t),\psi)\sum_{l,m}\mathrm{h}_{l,m\times}(t;\theta_I,\phi_c,\Theta)$$

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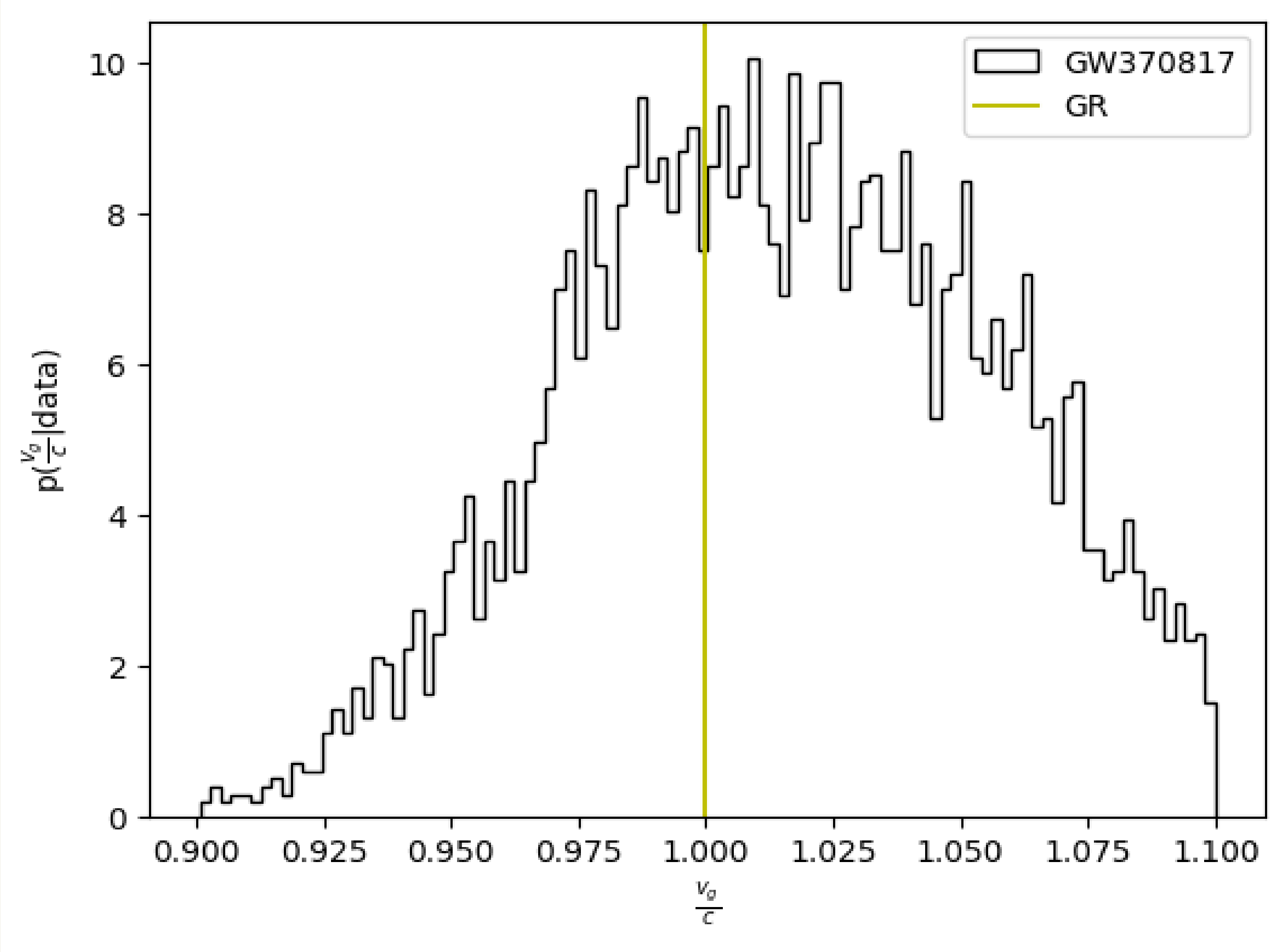
$$h(f) = \sum_{m} F_{+}(\boldsymbol{n}(t^m(f)),\psi)\sum_{l}\mathrm{h}_{l,m+}(f;\theta_I,\phi_c,\Theta) + \sum_{m} F_{\times}(\boldsymbol{n}(t^m(f)),\psi)\sum_{l}\mathrm{h}_{l,m\times}(f;\theta_I,\phi_c,\Theta)$$

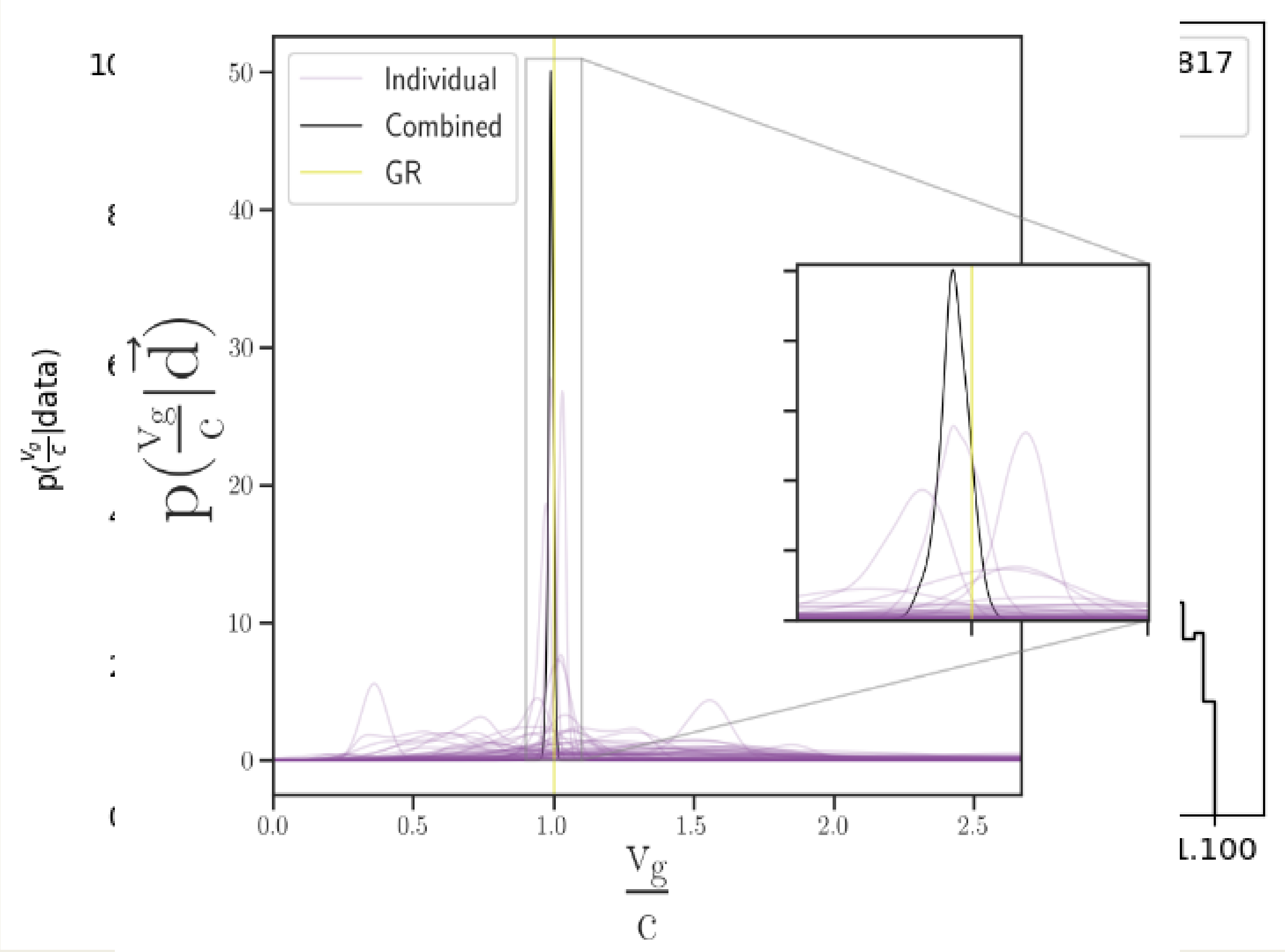
We inject the waveform for GW370817 and see how well we can recover the waveform. The injected GWs propagate at the speed of light.

Results: v_g



$$v_g = 1.01 \pm 0.04\, c$$

Results: v_g



Comparable to inference using 41 real GW events from GWTC-1 and GWTC-2

$v_g = 0.99 \pm 0.02\ c$

(arXiv: 2307.13099)

Frequency-dependent dispersion relation

Most alternative theories predict a frequency dependent dispersion relation.

Use a phenomenological model

$E^2 = p^2c^2 + A_\alpha p^\alpha c^\alpha$; $\alpha = 0$, 0.5, 1, 1.5, 2.5, 3, 3.5, 4

- Massive Graviton $\alpha = 0$.
- Multi-fractal Spacetime $\alpha = 2.5$.
- Doubly special Relativity $\alpha = 3$
- Extra-dimension, Horava-Lifshitz, standard model extension $\alpha = 4$
- Also performed runs with α = 0.5, 1, 1.5, 3.5

Assumptions

- No change in the local wave-zone.
- Effects are linear in A_α

Effective phase correction

$$\delta\Phi_\alpha(f) = \mathrm{sign}(A_\alpha) \begin{cases} \dfrac{\pi D_\mathrm{L}}{\alpha - 1} \lambda_{A,\mathrm{eff}}^{\alpha-2} \left(\dfrac{f}{c}\right)^{\alpha-1}, & \alpha \neq 1 \\ \dfrac{\pi D_\mathrm{L}}{\lambda_{A,\mathrm{eff}}} \ln\left(\dfrac{\pi G \mathcal{M}^\mathrm{det} f}{c^3}\right), & \alpha = 1 \end{cases} .$$

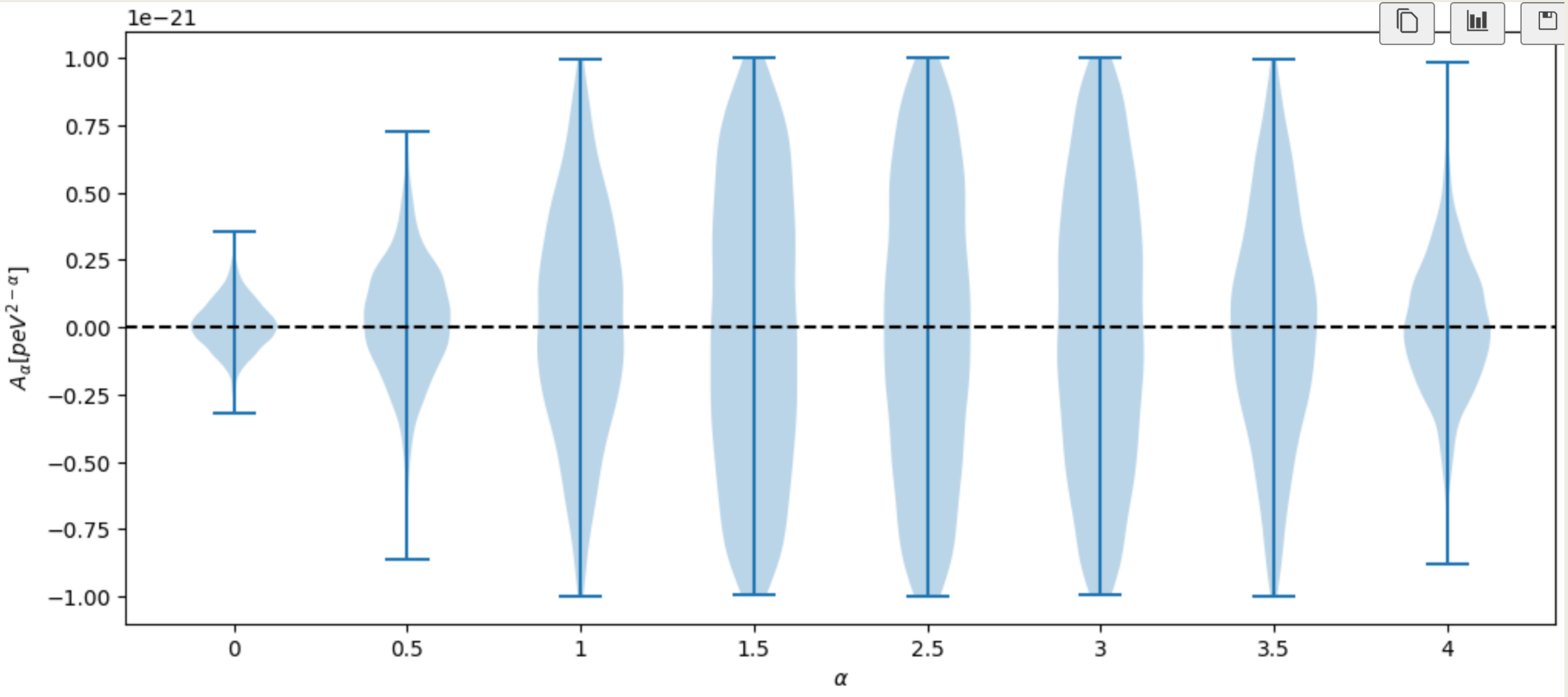
$$\lambda_A := hc|A_\alpha|^{1/(\alpha-2)}$$

$$\lambda_{A,\mathrm{eff}} := \left[\frac{(1+z)^{1-\alpha} D_\mathrm{L}}{D_\alpha}\right]^{1/(\alpha-2)} \lambda_A$$

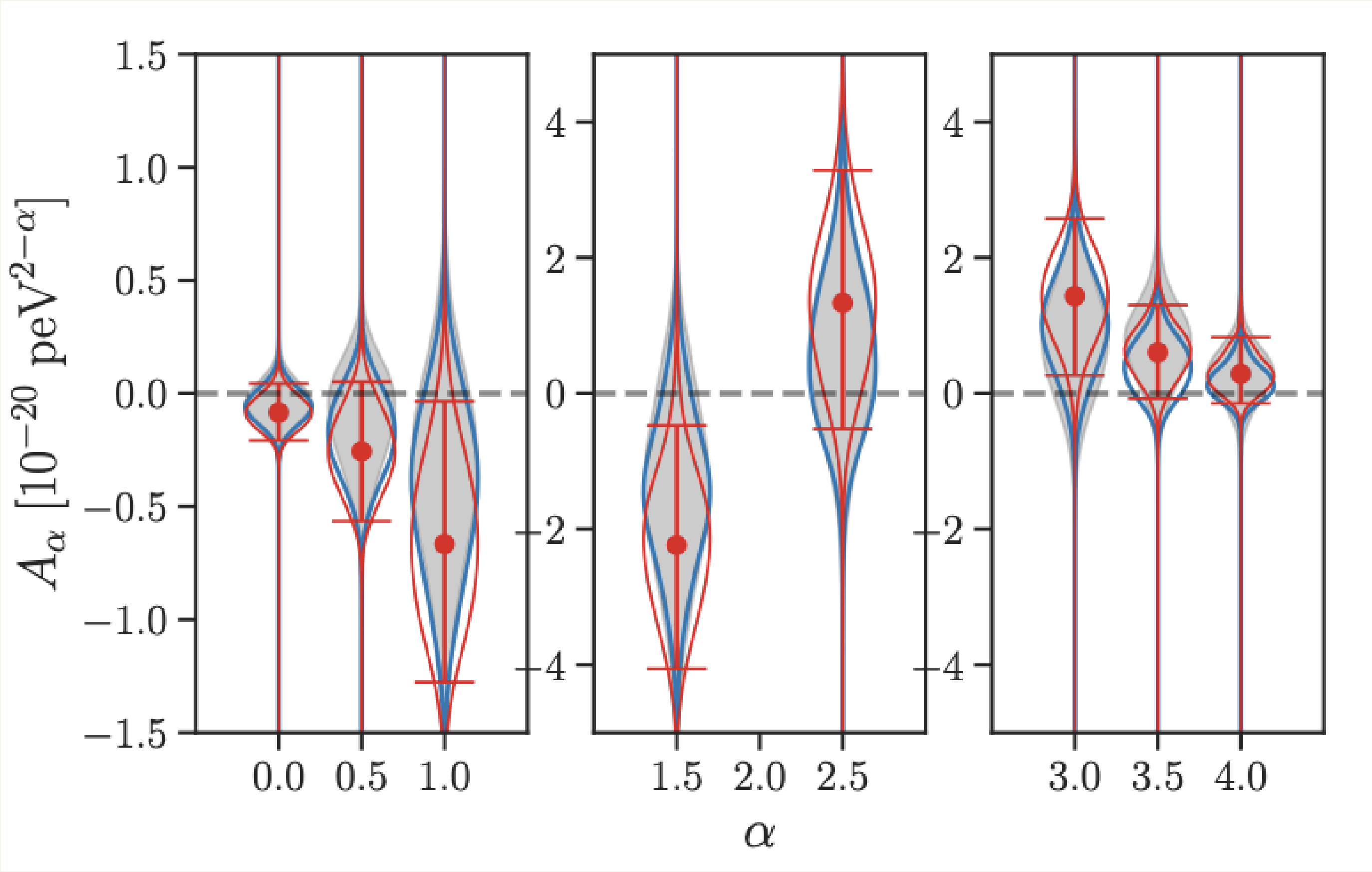
$$D_\alpha = \frac{(1+z)^{1-\alpha}}{H_0} \int_0^z \frac{(1+\bar{z})^{\alpha-2}}{\sqrt{\Omega_\mathrm{m}(1+\bar{z})^3 + \Omega_\Lambda}} d\bar{z} ,$$

Terms in phase are constrained much better!!

Results: Dispersion measurement in GW370817



Comparison: Dispersion measurement in GWTC3



- Our results are better by an order of magnitude when compared to current constraints.
- Note that we use only 1 event compared to 100 in GWTC3!!

arXiv: 2112.06861

Conclusion

- We demonstrate that it is possible to obtain constraints (comparable to GWTC3 results) for speed of gravity and dispersion using one loud event in Cosmic Explorer.
- The framework developed for parameter estimation is very general and can be used as long as one can generate h(f) efficiently. This framework will be made public very soon.
- Beyond GR effects can be mimicked by errors in waveform modelling and astrophysical approximations. Better understanding of these effects are required.

Thank You

Questions?

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