
Toward a Quantum Mesoscopic Fluctuation Theory: Progress and Open Questions

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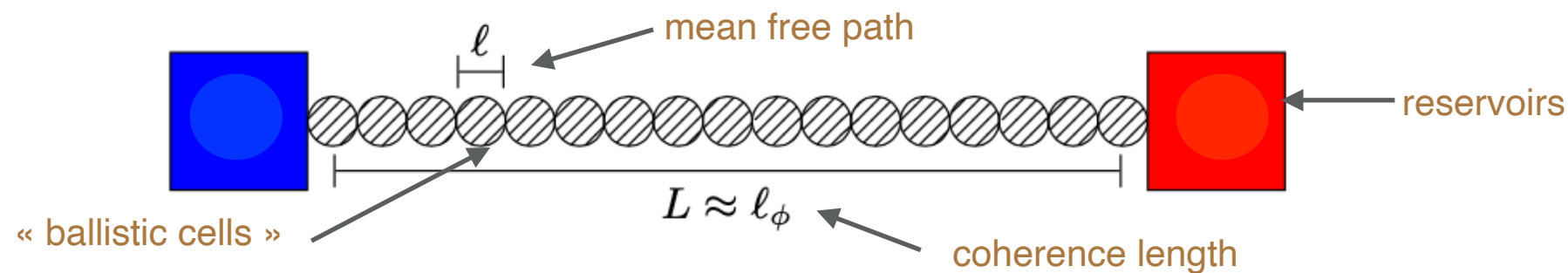


Looking for a « Quantum MFT » ?

► Macroscopic Fluctuation Theory (MFT):

- Fluctuation in out-of-equilibrium « diffusive, classical » systems.
- Large deviation theory (controlled by system size)

► Schematic representation of « mesoscopic » systems coupled to reservoirs



Q-MFT aims => Describing fluctuation of coherent quantum effects at mesoscopic scales and out-of-equilibrium.

—> Is there universality in the fluctuation of quantum coherent effects in out-of-equilibrium diffusive (mesoscopic) many-body systems?

—> **Quantum Mesoscopic Fluctuation Theory (QMFT)...**

Plan:

— Approaching QMFT

- ↪ Quantum lattice processes & Quantum exclusion processes
- ↪ Mean dynamics & asymptotic classicality of transport
- ↪ Beyond the mean / Beyond (fluctuating) hydrodynamics
- ↪ Does a QMFT exist?

— Lesson from QSSEP

- ↪ QSSEP dynamics & fluctuating coherences
- ↪ Scaling limit & free probability

— Including interaction (IQSEP)

- ↪ Interacting QSEP & its phase diagramme
- ↪ MFT & Scaling (field) theory

— Taking the continuum limit

- ↪ QSSEP in the continuum & free processes
- ↪ Free processes beyond hydrodynamics

— A road-map to a QMFT

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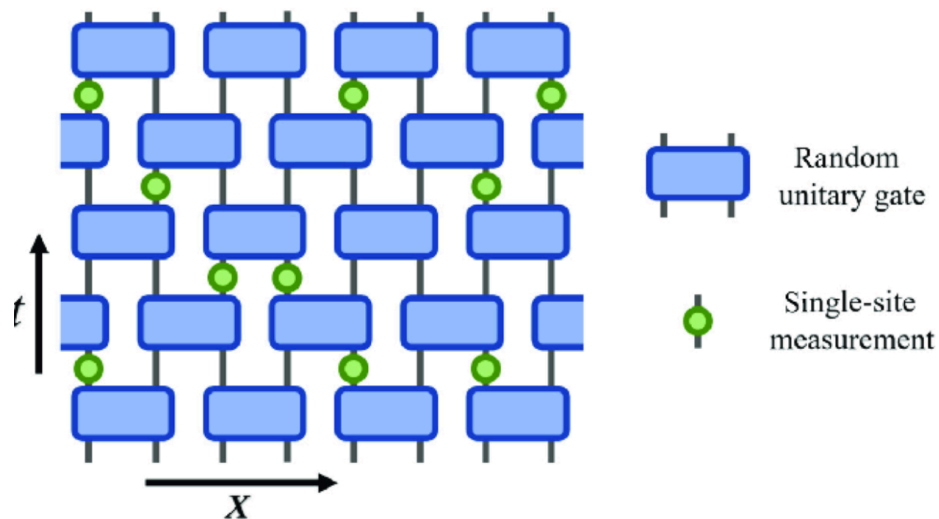
— A road-map to a QMFT

Quantum lattice processes

► QMFT => Looking for simple models of quantum fluctuating lattice diffusion...

- ↪ Extending classical lattice particle processes to the quantum realm
- Stochastic quantum dynamical systems
- Random quantum circuits or Quantum exclusion processes

► Random quantum circuits



[A. Nahum, S. Vijay, J. Haah, 2018]

[V. Khemani, A. Vishwanath, D. Huse, 2018]

[T. Rakovszky, F. Pollmann, C. von Keyserlingk, 2018]

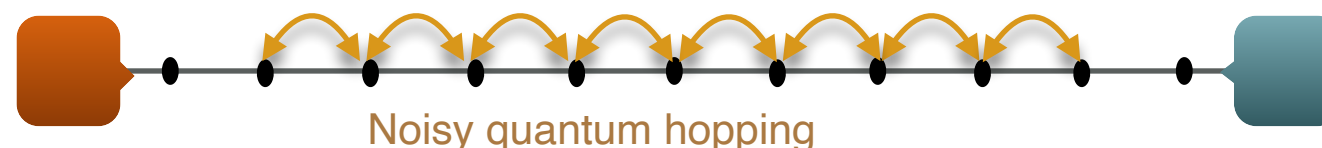
[M. Gullans, D. Huse, 2019]

[...]

► Quantum lattice particle processes:

[cf. Review DB, G. Barraquand, 2026]

Given a classical particle process (with configuration space $\mathcal{C} = \{\vec{n} = \{n_1, \dots, n_N\}\}$) construct a noisy quantum process (with Hilbert space $\mathcal{H} = \text{Vec}\{|\vec{n}\rangle, \vec{n} \in \mathcal{C}\}$) such that the average evolution is the original classical one.



Quantum exclusion processes



► Quantum state stochastic dynamics: $\rho_{t+dt} = e^{-idH_t}\rho_t e^{+idH_t} + \text{bdry}$ [D.B., M. Bauer, T. Jin, 2017]

$$d\rho_t = -i[dH_t, \rho_t] - \underbrace{\frac{1}{2}[dH_t, [dH_t, \rho_t]]}_{\text{drift} \rightsquigarrow \text{mean dynamics}} + \mathcal{L}_{\text{bdry}}(\rho_t)dt$$

Quantum & statistical randomness : $\rho_t(\omega)$ & $\mathbb{E}$

Mean (one-replica) dynamics: $d\bar{\rho}_t = -\frac{1}{2}[dH_t, [dH_t, \bar{\rho}_t]] + \text{bdry} =: \mathcal{L}_{\text{classical}}(\bar{\rho}_t)dt$

Stochastic dynamics beyond the mean: higher moments / higher replica theory

► QSSEP dynamics : quadratic but noisy (interacting version below) [D.B., T. Jin, 2019]

$$dH_t = \sqrt{D} \sum_j (c_{j+1}^\dagger c_j dW_t^j + c_j^\dagger c_{j+1} d\overline{W}_t^j) + \text{boundary (dissipative) processes (injection/extraction)}$$

Mean dynamics = classical SSEP

Lesson 1 : Asymptotic classicality of transport

► Single replica observables are classical (effectively) one-replica
 Mean (one-replica) dynamics: $d\bar{\rho}_t = \mathcal{L}_{\text{classical}}(\bar{\rho}_t)dt$ MFT
 for $\bar{\rho}_t := \mathbb{E}[\rho_t]$ in the continuum

► Classicality of transport fluctuations at large time...

$$\mathbb{P}_{\text{quant};\omega} \left[\begin{array}{c} \text{density or current} \\ \text{profile} \end{array} \right] \underset{\text{a.s.}}{\asymp} e^{-NI_{\text{classical}}[\text{profile}]}$$

 quantum probability distribution of density or current (for each disorder realization)

— Conjecture/Expectation : [D.B., L. Hruza, 2022]
 [E. Mc Culloch, J. De Nardis, S. Gopalakrishnan, R. Vasseur, 2023]

« *Large time fluctuations of the diagonal density properties are a.s. classical...* »

→ checked/proved in QSSEP and variants [J. Costa, P. Ribeiro, A. De Luca, 2025]
 [D.B., T. Jin, S. Scopa, S. Wei, 2025-26]
 in random quantum circuits [E. Mc Culloch, J. De Nardis, S. Gopalakrishnan, R. Vasseur, 2023]

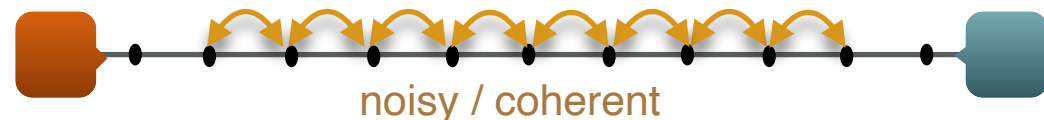
Beyond the mean / Beyond hydrodynamics

► Hydrodynamics (from local conservation law + noise)

$$\partial_t \hat{n}_{x,t} + \partial_x \hat{j}_{x,t} = 0 \implies \partial_t \langle \hat{n}_{x,t} \rangle + \partial_x \langle \hat{j}_{x,t} \rangle = 0 \quad \leftarrow \text{local equilibrium: } \langle \hat{j}_{x,t} \rangle = \mathcal{J}[\langle \hat{n}_{x,t} \rangle] + \text{gradient expansion}$$

=> one-replica, diagonal data.

► Stochastic quantum dynamics:



↔ Off-diagonal coherent effects (interference, entanglement, etc)

— Quantum coherent effects beyond the (classical) mean behavior:



— Stochastic (off-diagonal) « quantum coherences »: higher replica theory

► QMFT => Need for an « off-diagonal hydrodynamics »...

Does a Quantum MFT exists ?

- ▶ Extending to more general quantum exclusion processes
 - ↪ including environment dependent hopping...
- ▶ Exploring QSSEP (and other exclusion processes) in the continuum
 - ↪ as a (non-commutative) stochastic process...
- ▶ Conjectural « universality » in mesoscopic (diffusive) systems ?...
 - ↪ effective description of microscopic models

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Lesson from QSSEP

- QSSEP dynamics : stochastic but quadratic
—> Stochastic process on « coherences »

$$★ \quad G_{ij} = \langle c_j^\dagger c_i \rangle_t = \text{Tr}(\rho_t c_j^\dagger c_i) \quad ★$$

$$\left| \begin{array}{l} G_{t+dt} = e^{-idh_t} G_t e^{+idh_t} \quad + \text{« boundary terms »} \\ dG_t = -i[dh_t, G_t] - \frac{1}{2}[dh_t, [dh_t, G_t]] + \text{bdry terms} \end{array} \right.$$

dh = one-particle hamiltonian

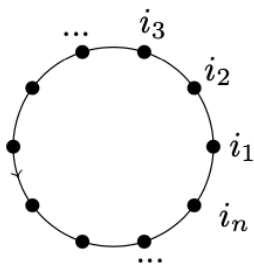
$$dh_t = \begin{pmatrix} 0 & dW^1 & 0 & \dots \\ d\bar{W}^1 & 0 & dW^2 & 0 \\ \vdots & d\bar{W}^2 & 0 & \ddots \end{pmatrix}$$

- Steady mean profile : —> out-of-equilibrium & classical SSEP

$$\left| \begin{array}{l} [n_j] := \mathbb{E}[\langle c_j^\dagger c_j \rangle] = n_a + x(n_b - n_a) \rightarrow \text{current } j \neq 0 \\ \mathbb{E}[G_{ij}] = 0 \quad (i \neq j) : \text{decoherence (in mean)} \end{array} \right.$$

$$\left| \begin{array}{l} \mathbb{E}[|G_{ij}|^2] = N^{-1}(\Delta n)^2 x(1-y) + O(N^{-2}) \\ \mathbb{E}[G_{ij}G_{jk}G_{kj}] = N^{-2}(\Delta n)^2 x(1-2y)(1-z) + O(N^{-3}) \end{array} \right.$$

—> Sub-leading fluctuating coherences (with some patterns)



$$\mathbb{E}[G_{i_1 i_2} G_{i_2 i_p} \cdots G_{i_p i_1}] \sim N^{1-p} g_p(x_1, \dots, x_p)$$

large deviation theory
controlled by system size

infinite coherence length
long range coherent correlations

$$\ell_\phi \approx \infty$$

—> What is the underlying theory in the continuum?

Lesson 2 : Scaling limit & free probability

► Diffusive (continuous) scaling limit

- The limit exists... (with hydrodynamics like equations for coherence fluctuations)
- The matrix of two-point function codes for (some) coherent effects

$$G_{ji} = \text{Tr}(\rho_t c_i^\dagger c_j)$$

random  (spatial) structure 

$$\implies \begin{cases} - \text{Large matrix} \\ - \text{Random matrix} \\ - \text{Structured matrix} \end{cases}$$

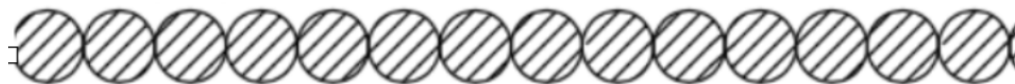
→ need to deal with extended version of RMT for *structured* matrices which is inherently linked to *free probability*

► « Long-range correlations » of coherence fluctuations

- Large deviation principle for coherence fluctuations
- Emergence of *free probability* in noisy mesoscopic systems

— Conjecture/Expectation : [D.B.-L.Hruza, 2022]

« If MFT holds for the local densities then coherence fluctuations fulfill a large deviation principle (with underlying free probability structures)... »



« ballistic cells »

<== Interaction at ultra short space/time scale (mean free path)

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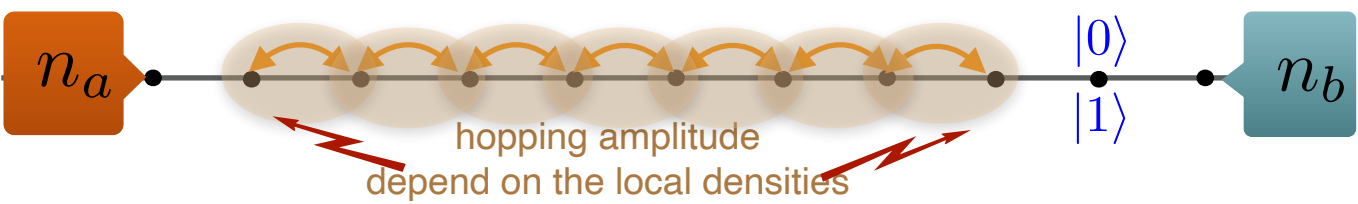
- ↪ QSSEP in the continuum & free processes
- ↪ Free processes beyond hydrodynamics

— A road-map to a QMFT

Interacting QSEP (basics) :

- Stochastic interacting many-body quantum system :

[D.B., F. Huebner, S. Scopa, 2026]



$$\rho_{t+dt} = e^{-idH_t} \rho_t e^{idH_t} + \text{« injection/extraction boundary terms »}$$

$$dH_t = \sum_j \widehat{P_j[\hat{n}]} (c_{j+1}^\dagger c_j dW_t^j + c_j^\dagger c_{j+1} d\bar{W}_t^j)$$

⚡ depending on the local densities

$\ell_{\text{meso}} \approx N^\alpha$

- $\widehat{P_j[\hat{n}]}$ coding for (effective) interactions at ultra short scales (time & length)

$$\left| \begin{array}{ll} \widehat{P_j[\hat{n}]} = \text{constant} & \Rightarrow \text{Quantum SSEP : diffusivity } D_0 \text{ \& } \ell_\phi = \infty \\ \widehat{P_j[\hat{n}]} \neq \text{constant} & \Rightarrow \text{Interacting QSEP : diffusivity } D[n] \text{ \& } \ell_\phi \neq \infty \end{array} \right.$$

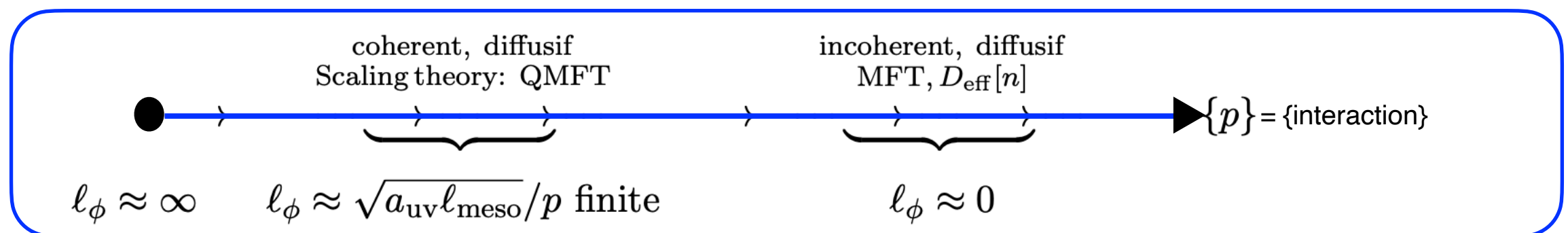
- Effective description for ultra short/fast interaction

Phase diagramme (IQSEP)

[D.B., F. Huebner, S. Scopa, 2026]

- Interaction => Diffusivity $D(n) = \langle P[n]^2 \rangle$ & Coherence length $\ell_\phi \approx \sqrt{a_{uv} \ell_{\text{meso}} / p}$

- Phase diagramme



- Mean/one-replica dynamics projects onto classical particle system => non-linear MFT

- Solvability in the thermodynamic limit (despite non-gaussianity, realisation-wise)

Loop expectation values: $\mathbb{E}[G_{i_1 i_2} \cdots G_{i_n i_1}] \rightarrow$ Infinite BBGKY hierarchy (at finite N)

But it get closed thanks to asymptotic scaling in $1/N$ from MFT + QSSEP

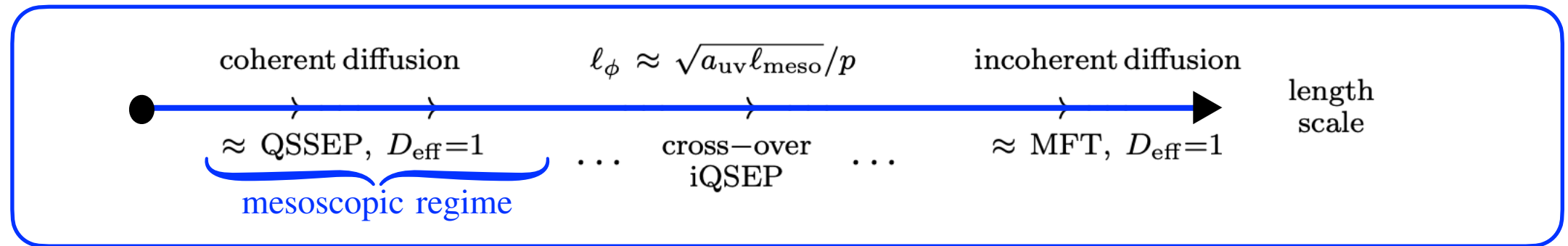
- Effective large scale dynamics of the XXZ spin chain + dephasing noise (one-replica)

$$\widehat{P}_j = \sqrt{D_0} [1 + \lambda(n_{j-1}(1 - n_{j+2}) + n_{j+2}(1 - n_{j-1}))]$$

[A. Christopoulos, J. Da Costa, S. Scopa, Z. Lenarcic, J. De Nardis, D.B., T. Jin, 2026]

IQSEP: Mesoscopic scaling regime

► Scaling theory (as in field theory)



► Mesoscopic scaling regime ($N \rightarrow \infty$, $\lambda \rightarrow 0$, $\ell_\phi \approx \sqrt{a_{\text{uv}} \ell_{\text{meso}}} / \lambda$ fixed)

As in field theory: $\left| \begin{array}{l} \text{Scale the interaction in a way to preserve a finite coherence length} \\ \widehat{P_j[\hat{n}]} = \sqrt{D_0} [1 + \lambda p[\hat{n}_k]_{|k-j| \leq \mathcal{O}(N^\alpha)}] \quad \rightsquigarrow \quad \lambda \approx N^{-1+\alpha/2} \end{array} \right.$

► Hydrodynamic-like equation for the coherence expectations values : QSSEP + mass terms coding for finite coherence length

$$\left[\partial_t - \sum_k \underbrace{(m^2[\bar{n}_{x_k,t}] - \nabla_k^2)}_{\text{local } \ell_\phi^{-2}} \right] g_p(x) = \mathcal{S}_{\text{qssep}}^{\text{contact}}[g_{q < p}](x, t), \quad p \geq 2$$

→ Effective (field) theory: crossover from coherent (UV) to incoherent (IR)

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Taking the continuum limit (QSSEP):

- How to formula the QSSEP dynamics directly in the continuum?

$G = \infty \times \infty$ – matrix & $dh_t = \infty \times \infty$ noisy i.i.d. matrices

- Identifying the underlying structures:

- G is a *structured* random matrix with $\mathbb{E}[G_{ii_1} G_{i_1 i_2} \cdots G_{i_p i}] \sim N^{-p}$

↔ introducing test diagonal matrices $\hat{\Delta}_k$:

$\mathbb{E}[(G \hat{\Delta}_1 G \cdots \hat{\Delta}_p G)_{ii}] \sim O(1) \times \text{diag} = \text{conditioned expectation values}$

↔ conditioning on diagonal matrices = $L_\infty([0,1])$

- dh_t are i.i.d. gaussian matrices with variance $\mathbb{E}[dh_{ij} dh_{ji}] = \delta_{i,j \pm 1} dt$

↔ turn them into conditioned free Brownian motions

with $\mathbb{E}[dh_t \hat{\Delta} dh_t] = dt \hat{\Delta} |_{\text{shifted}} \rightsquigarrow \mathbb{E}[dX_t \Delta dX_t] = \sigma(\Delta) dt$

- | In the continuum, QSSEP is a non-commutative process on an algebra $\mathcal{A}$
generated by a free Brownian motion conditioned over $L_\infty([0,1]) \subset \mathcal{A}$

QSSEP in the continuum :

- QSSEP in the continuum is a non-commutative process driven by a conditioned free Brownian motion

Let X_t be a free Brownian motion, conditioned on $L_\infty([0,1])$, with variance the heat kernel i.e. $\mathbb{E}[dX_t \Delta dX_t] = \sigma_\epsilon(\Delta) dt$, for $\Delta \in L_\infty([0,1])$ and $\sigma_\epsilon := \epsilon^{-1} \exp(\epsilon \partial^2)$

Let $t \rightarrow \phi_t$ be the process defined by

$$d\phi_t = i[dX_t, \phi_t] + \left(\sigma_\epsilon(\mathbb{E}[\phi_t]) - \frac{1}{2} (\sigma_\epsilon(1)\phi_t + \phi_t\sigma_\epsilon(1)) \right) dt \quad (\blacktriangle)$$

Then, $t \rightarrow \phi_t$ (in the limit $\epsilon \rightarrow 0$) QSSEP the scaling limit of QSSEP in the sense that:

$$\lim_{N \rightarrow \infty} \mathbb{E}[(G_s \hat{\Delta}_1 G_s \cdots \hat{\Delta}_p G_s)_{ii}] \Big|_{\substack{i = [xN] \\ s = tN^2}} = \mathbb{E}[\phi_t \Delta_1 \phi_t \cdots \Delta_p \phi_t](x)$$

- Equation $(\blacktriangle)$ is a non-commutative analogue of the noisy heat equation

Simple check on the one-point function: classical diffusion

$$d\mathbb{E}[\phi_t] = L_\epsilon(\mathbb{E}[\phi_t]) dt, \text{ with } L_\epsilon(\Delta) := \underbrace{\sigma_\epsilon(\Delta) - \frac{1}{2}(\sigma_\epsilon(1)\Delta + \Delta\sigma_\epsilon(1))}_{\text{Lindbladian}} = \partial^2 \Delta + O(\epsilon)$$

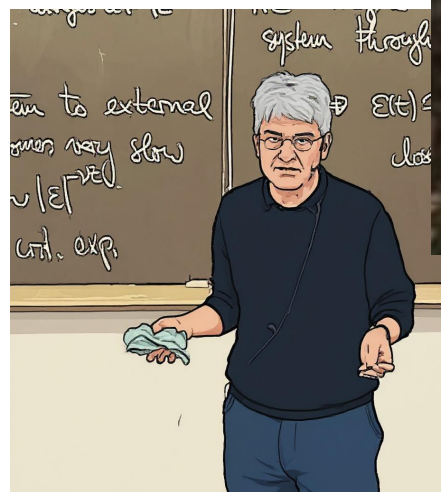
The road-map to a QMFT

- ▶ What is Interacting-QSEP in the continuum?
 - ↪ Toward quantum hydrodynamics for QMFT?...
- ▶ How to define quantum lattice particle processes with general (interacting) free energy?
 - ↪ All Interacting-QSEP have the same free energy (equilibrium steady state)
How to get away from this?...
- ▶ Conjectural « universality »?
 - ↪ Noisy XXZ and more...
- ▶ Alternative field theory description?
 - ↪ Say, sigma model?...
- ▶ Sub-leading (in $1N$)?
 - ↪ Deviation from asymptotic classicality...
- ▶ Add effects of measurements?
- ▶ ?

in progress



in collaboration with



F. Essler



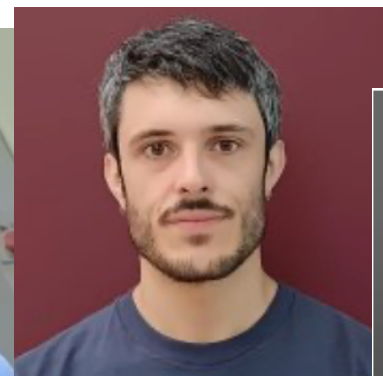
T. Jin



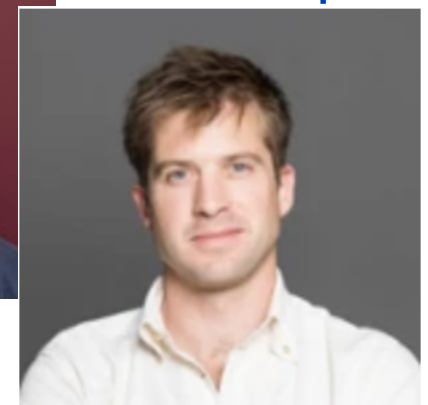
L. Hruza



M. Medenjak



S. Scopa



G. Barraquand



M. Bauer



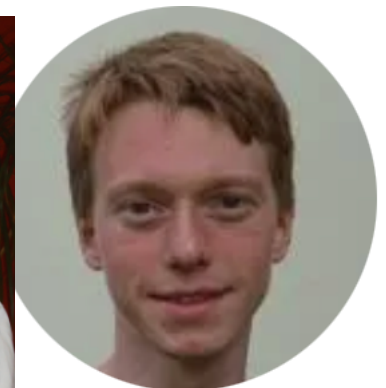
L. Piroli



A. Krajenbrink



Ph. Biane



F. Huebner

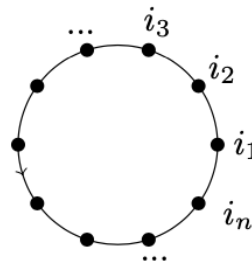
Exact results in QSSEP

Besides those discussed above:

- ▶ Exact loop equations of motion
(from scaling limit of SDE)
- ▶ Invariant measure on loop expectation values
(as free cumulants of auxiliary problem)
- ▶ Large deviation function for density profile and asymptotic classicality
(from structured RMT)
- ▶ Large deviation function for current and asymptotic classicality
(from structured RMT and variational ansatz)
- ▶ Volume law for mean mutual information
(from structured RMT)
- ▶ Large deviation function for entangled men entropy (in the closed case)
(from standard RMT techniques)
- ▶

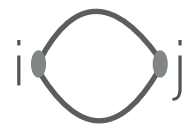
Why free probability?

- Compute the cyclic moments via the cumulant expansion



$$\mathbb{E}[M_{i_1 i_2} \cdots M_{i_n i_1}] = \sum_{\pi \in P_n} C_\pi$$

- N=1, 2, 3 : (as usual...)



$$\mathbb{E}[M_{ij} M_{ji}] = \mathbb{E}[M_{ij} M_{ji}]^c + \mathbb{E}[M_{ij}]^c \mathbb{E}[M_{ji}]^c = N^{-1} [g_2(x, y) + \delta(x - y) g_1(x) g_1(y)]$$

$\delta_{ij} \mathbb{E}[M_{ii}] \quad \delta_{ij} \mathbb{E}[M_{jj}]$
(with $\delta_{ij} \rightsquigarrow N^{-1} \delta(x - y)$)

- N=4 : the role of non-crossing partitions...

$$\mathbb{E}[M_{ij} M_{jk} M_{kl} M_{li}] =$$

$N^{-3} g_4(\cdot)$

$\mathbb{E}[M_{ij} M_{jk}]^c \mathbb{E}[M_{kl} M_{li}]^c$

$\delta_{ik} N^{-2}$

$\mathbb{E}[M_{ij} M_{kl}]^c \mathbb{E}[M_{jk} M_{li}]^c$

$\delta_{ij} \delta_{jk} \delta_{kl} N^{-2}$

sub-leading!

cumulants (partition of edges)

cyclic perm

cyclic perm

- Only the **non-crossing partitions** contribute.

$\implies$ Free probability

classical probability	quantum mechanics	non-commutative probability
$\mathcal{F} = \{\text{functions}\}$	$\mathcal{B}(\mathcal{H}) = \{\text{observables}\}$	$\mathcal{A} = \text{algebra}$
$\mathbb{E}[(\cdot)] = \int d\mu(\cdot)$	$\text{Tr}_{\mathcal{H}}[\rho(\cdot)]$	$\mathbb{E}[(\cdot)] : \mathcal{A} \rightarrow \mathbb{R}$
$\mathbb{E}[(\cdot) X]$	$\text{Tr}_{\mathcal{H}_B}[\rho_{B \otimes D}(\cdot)]$	$\mathbb{E}[(\cdot) \mathcal{D}] : \mathcal{A} \rightarrow \mathcal{D}$
<i>independence</i>	—	<i>freeness</i>
<i>gaussian</i> $C_2(\cdot)$	—	<i>semicircular</i> $\kappa_2(\cdot)$
<i>filtration</i> $\mathcal{F}_t$	—	<i>filtration</i> $\mathcal{A}_t$