Standard Model: Example

Fields	$SU(3)_c$	$SU(2)_L$	$U(1)_Y$	$SU(2)_L \times SU(2)_R$
φ	1	2	1/2	(1, 1)
Q_L^p	3	2	1/6	(2, 1)
u_R^p	$\bar{3}$	1	2/3	(1, 2)
d_R^p	$\bar{3}$	1	-1/3	(1, 2)
e_R^p	1	1	-1	(1, 2)
l_L^p	1	2	-1/2	(2, 1)
$B_{\mu\nu}$	1	1	0	(3, 3)
$W_{\mu\nu}^I$	1	3	0	(3, 3)
$G_{\mu\nu}^a$	8	1	0	(3, 3)
D_μ	1	1	0	(2, 2)

Argument of Plethystics

$$B_l \to \frac{1}{r} Bl^r \chi^{(4)}_{(1,0)}(D^r, \alpha^r, \beta^r)$$

$$Br \to \frac{1}{r} Br^r \chi^{(4)}_{(0,1)}(D^r, \alpha^r, \beta^r)$$

$$Wl \to \frac{1}{r} Wl^r \chi^{(4)}_{(1,0)}(D^r, \alpha^r, \beta^r) \chi_{SU(2)_3}(z_1^r)$$

$$Wr \to Wl\left[\chi^{(4)}_{(1,0)} \to \chi^{(4)}_{(0,1)}\right]$$

$$Gl \to \frac{1}{r} Gl^r \chi^{(4)}_{(1,0)}(D^r, \alpha^r, \beta^r) \chi_{SU(3)_8}(z_1^r, z_2^r)$$

$$Gr \to Gl\left[\chi^{(4)}_{(1,0)} \to \chi^{(4)}_{(0,1)}\right]$$

$$\varphi \to \frac{1}{r} \varphi^r \chi^{(4)}_{(0,0)}(D^r, \alpha^r, \beta^r) \chi_{SU(2)_2}(z_1^r)\, z^{1/2}$$

$$\varphi^\dagger \to \frac{1}{r} \varphi^{\dagger\, r} \chi^{4}_{(0,0)}(D^r, \alpha^r, \beta^r) \chi_{SU(2)_{\bar{2}}}(z_1^r)\, z^{-1/2}$$

$$e \to \frac{(-1)^{r+1}}{r} e^r \chi^{(4)}_{(0,1/2)}\, z^{-r}$$

$$u \to \frac{(-1)^{r+1}}{r} u^r \chi^{(4)}_{(0,1/2)}\, z^{2r/3} \chi_{SU(3)_{\bar{3}}}(z_2^r, z_3^r)$$

$$d \to \frac{(-1)^{r+1}}{r} d^r \chi^{(4)}_{(0,1/2)}\, z^{-r/3} \chi_{SU(3)_{\bar{3}}}(z_2^r, z_3^r)$$

$$L \to \frac{(-1)^{r+1}}{r} L^r \chi^{(4)}_{(1/2,0)}\, z^{-r/2} \chi_{SU(2)_2}(z_1^r)$$

$$Q \to (-1)^{r+1}/r\; Q^r \chi^{(4)}_{(1/2,0)}\, z^{r/6} \chi_{SU(3)}(z_2^r, z_3^r) \chi_{SU(2)}(z_1^r)$$

$$PE[\varphi, \psi, X, D, \alpha, \beta, z_1, z_2, z_3, z]$$

$$= \exp\left[\sum_{\varphi}\sum_{r=1}^{\infty} \varphi^r \frac{\chi_\varphi(D^r, \alpha^r, \beta^r, z_1^r, z_2^r, z_3^r, z^r)}{r}\right]$$

$$\times \exp\left[\sum_{\psi}\sum_{r=1}^{\infty} (-1)^{r+1} \frac{\psi^r \chi_\psi(\cdots)}{r}\right]$$

$$\times \exp\left[\sum_{X}\sum_{r=1}^{\infty} X^r \frac{\chi_X(\cdots)}{r}\right]$$

where, $\varphi \in \{\varphi, \varphi^\dagger\}$

$\psi \in \{e, u, d, L, Q, e^\dagger, u^\dagger, d^\dagger, L^\dagger, Q^\dagger\}$

$X \in \{B_l, B_r, W_l, W_r, G_l, G_r\}$

Polynomial $\rightarrow$ Covariant Operators

Symmetry: SM Gauge $\otimes$ $\underbrace{SU(2)_l \otimes SU(2)_r}_{\text{Lorentz}}$

$$\gamma^\mu = \begin{pmatrix} 0 & \sigma^\mu_{\alpha\dot\beta} \\ \bar\sigma^{\mu\dot\alpha\beta} & 0 \end{pmatrix}, \quad \gamma^5 = \begin{pmatrix} -\mathbb{1} & 0 \\ 0 & \mathbb{1} \end{pmatrix}$$

$$\sigma^\mu = (\mathbb{1}, \sigma^i), \quad \bar\sigma^\mu = (\mathbb{1}, -\sigma^i)$$

$$\psi = \begin{pmatrix} \chi_\alpha \\ \xi^{\dagger\dot\alpha} \end{pmatrix}, \quad \bar\psi = \psi^\dagger\gamma^0 = \begin{pmatrix} \xi^\alpha & \chi^\dagger_{\dot\alpha} \end{pmatrix}$$

$$\psi_L = \begin{pmatrix} \chi_\alpha \\ 0 \end{pmatrix}, \quad \bar\psi_L = \begin{pmatrix} 0 & \chi^\dagger_{\dot\alpha} \end{pmatrix}$$

$$\psi_R = \begin{pmatrix} 0 \\ \xi^{\dagger\dot\alpha} \end{pmatrix}, \quad \bar\psi_R = \begin{pmatrix} \xi^\alpha & 0 \end{pmatrix}$$

$$X_{L,\mu\nu} = \frac{1}{2}\left(X_{\mu\nu} - i \tilde{X}_{\mu\nu} \right) \qquad \Big| \; \tilde{X}_{\mu\nu} = \frac{1}{2} \epsilon_{\mu\nu\rho\sigma} X^{\rho\sigma}$$

$$(X_L)_{\alpha\beta} = \sigma^{\mu}_{\alpha\dot{\beta}} \, \bar{\sigma}^{\nu\dot{\beta}\kappa} \, \epsilon_{\kappa\beta} \, X_{L,\mu\nu}$$

$$X_{R,\mu\nu} = \frac{1}{2}\left(X_{\mu\nu} + i \tilde{X}_{\mu\nu} \right)$$

$$(X_R)^{\dot{\alpha}\dot{\beta}} = \bar{\sigma}^{\mu\dot{\alpha}\kappa} \, \sigma^{\nu}_{\kappa\dot{\kappa}} \, \epsilon^{\dot{\kappa}\dot{\beta}} \, X_{R,\mu\nu}$$

$$\phi \equiv (0,0), \quad \psi_L \equiv (\tfrac{1}{2},0), \quad \psi_R \equiv (0,\tfrac{1}{2}),$$

$$D \equiv (\tfrac{1}{2},\tfrac{1}{2}), \quad X_L \equiv (1,0), \quad X_R \equiv (0,1).$$

$$\mathcal{O} \equiv \phi^{p} \, \psi_L^{q_1} \, \psi_R^{q_2} \, D^{r} \, X_L^{s_1} \, X_R^{s_2}$$

$$\Rightarrow (0,0) \equiv (0,0)^{p} (\tfrac{1}{2},0)^{q_1} (0,\tfrac{1}{2})^{q_2} (\tfrac{1}{2},\tfrac{1}{2})^{r} (1,0)^{s_1} (0,1)^{s_2}$$

$$\Rightarrow [M]^d = [M]^p \, [M]^{3q_1/2} \, [M]^{3q_2/2} \, [M]^{\gamma} \, [M]^{2\delta_1} \, [M]^{2\delta_2}$$

$$\Rightarrow d = p + \frac{3}{2}(q_1 + q_2) + \gamma + 2(\delta_1 + \delta_2)$$

Explicit examples in Covariant form

* **Total derivative**

$$D^2 \to D_\mu D^\mu, \quad \phi D^2 \to D_\mu (D^\mu \phi)$$

$$D^4 \to (D_\mu D^\mu)^2$$

$$D^2 X_L + D^2 X_R \to D_\mu D_\nu X^{\mu\nu} + D_\mu D_\nu \tilde{X}^{\mu\nu}$$

$$\phi D^4 \to D_\mu D_\nu D^\nu (D^\mu \phi)$$

$$\phi^2 D^4 \to D_\mu D^\mu (D^\nu \phi)(D_\nu \phi)$$

* bi-linear in ψ

$$\psi^2_{L,R} \longrightarrow \psi^T_{L,R} C \psi_{L,R} ; \bar{\psi}_{R,L} \psi_{L,R} ;$$

$$\psi^T_{L,R} C \sigma^{\mu\nu} \psi_{L,R} , \bar{\psi}_{R,L} \sigma^{\mu\nu} \psi_{L,R}$$

$$\psi_L \psi_R \to \bar{\psi}_L \gamma^\mu \psi_L , \bar{\psi}_R \gamma^\mu \psi_R$$

$$\bar{\psi}_L \gamma^\mu \psi_L D_\mu \equiv \psi_L \psi_R D$$

$$\bar{\psi}_L \gamma^\mu \psi_L \phi D_\mu \phi \equiv \psi_L \psi_R \phi^2 D$$

$$\bar{\psi}_L \gamma^\mu \psi_L \bar{\psi}_R \gamma_\mu \psi_R \equiv \psi_L^2 \psi_R^2$$

$$\bar{\psi}_R \sigma^{\mu\nu} \psi_L X_{\mu\nu} \equiv \psi_L^2 X_L$$

$$\bar{\psi}_R \psi_L \phi \equiv \psi_L^2 \phi ,$$

$$\bar{\psi}_R \psi_L \bar{\psi}_R \psi_L \equiv \psi_L^4$$

* **Field Strength**

$$X_L^2 + X_R^2 \to X_{\mu\nu} X^{\mu\nu} + \tilde{X}^{\mu\nu} X^{\mu\nu}$$

$$X_L^3 + X_R^3 \to X^{\mu}{}_{\nu} X^{\nu}{}_{\kappa} X^{\kappa}{}_{\mu} + \tilde{X}^{\mu}{}_{\nu} X^{\nu}{}_{\kappa} X^{\kappa}{}_{\mu}$$

$$X_L^4 + X_L^2 X_R^2 + X_R^4 \to (X_{\mu\nu} X^{\mu\nu})(X_{\kappa\lambda} X^{\kappa\lambda})$$
$$+ (\tilde{X}_{\mu\nu} X^{\mu\nu})(X_{\kappa\lambda} X^{\kappa\lambda}) + (\tilde{X}_{\mu\nu} X^{\mu\nu})(\tilde{X}_{\kappa\lambda} X^{\kappa\lambda})$$

$$D^2 X_{L,R}^2 \equiv (D_\mu X^{\mu\nu})^2,$$

$$X_{L,R} \phi^2 D^2 \equiv (D_\mu X^{\mu\nu})(\phi D_\nu \phi)$$

$$D^2 X_L X_R \equiv (D_\nu D_\mu X^{\mu\kappa} \tilde{X}^{\nu}{}_{\kappa}).$$

Explicit redundancies

* **IBP**:

$$i D_\mu (\bar{\psi}_{L,R} \gamma^\mu \psi_{L,R}\, \varphi^\dagger \varphi)$$

$$= (\bar{\psi}_{L,R} \gamma^\mu i \overleftrightarrow{D}_\mu \psi_{L,R}) \varphi^\dagger \varphi + \bar{\psi}_{L,R} \gamma^\mu \psi_{L,R} (\varphi^\dagger i \overleftrightarrow{D}_\mu \varphi)$$

where, $i \overleftrightarrow{D}_\mu \equiv i D_\mu - i \overleftarrow{D}_\mu$

* **EOM**:

① **$\psi_L \psi_R \phi^2 D$**:

$$(\bar{\psi}_{L,R} \gamma^\mu i \overleftrightarrow{D}_\mu \psi_{L,R})\, \varphi^\dagger \varphi$$

$$(\bar{\psi}_{L,R} \gamma^\mu \psi_{L,R}) (\varphi^\dagger i \overleftrightarrow{D} \varphi)$$

$$\propto \bar{\psi}_{L,R} \psi_{R,L}\, \phi (\varphi^\dagger \varphi) \equiv \psi^2_{L,R} \varphi^3$$

② $\psi_{L,R}^2 \phi D^2$:

$$D^2\varphi = c_1\varphi + c_2\,\varphi(\varphi^\dagger\varphi) + c_3\,\bar\psi_L\psi_R + h.c.$$

$$(\bar\psi_L\psi_R)\,D^2\varphi$$
$$= c_1(\bar\psi_L\psi_R\varphi) + c_2(\bar\psi_L\psi_R\varphi)(\varphi^\dagger\varphi)$$
$$+ c_3(\bar\psi_L\psi_R)(\bar\psi_R\psi_L)$$

③ $D^2 X_{L,R}^2,\ D^2 X_L X_R$:

$$D_\mu\tilde{X}^{\mu\nu} = 0,$$

$$D_\mu X^{\mu\nu} = \bar\psi_{L,R}\gamma^\nu\psi_{L,R} + \varphi^\dagger i\overleftrightarrow{D}^\nu\varphi$$

$$(D_\mu X^{\mu\nu})^2 = a_1(\bar\psi_{L,R}\gamma_\nu\psi_{L,R})(D_\mu X^{\mu\nu})$$
$$+ a_2(\varphi^\dagger i\overleftrightarrow{D}_\nu\varphi)(D_\mu X^{\mu\nu})$$

$$= b_1 \left(\bar{\psi}_{L,R}\gamma^\nu \psi_{L,R}\right)^2 + b_2 \left(\varphi^\dagger i \overleftrightarrow{D}^\nu \varphi\right)^2$$

$$+ b_3 \left(\varphi^\dagger i \overleftrightarrow{D}_\nu \varphi\right)\left(\bar{\psi}_{L,R}\gamma^\nu \psi_{L,R}\right)$$

$$(D_\mu X^{\mu\nu})^2, \quad (D_\mu X^{\mu\nu})(D_\rho \tilde{X}^{\rho\nu})$$

$$\xrightarrow{\text{IBP}} [D_\mu, D_\nu]\, X^{[\mu\kappa} \, X_\kappa{}^{\nu]},$$

$$[D_\mu, D_\nu]\, X^{[\mu\kappa} \, \tilde{X}_\kappa{}^{\nu]}$$

$$\equiv X_{\mu\nu} X^{\mu\kappa} X_\kappa{}^\nu, \quad X_{\mu\nu} X^{\mu\kappa} \tilde{X}_\kappa{}^\nu$$

④ $\varphi^2 X_{L,R} D^2$:

$$\left(\varphi^\dagger i \overleftrightarrow{D}_\nu \varphi\right) D_\mu X^{\mu\nu} = a' \left(\bar{\psi}_{L,R}\gamma_\nu \psi_{L,R}\right)\left(\varphi^\dagger i \overleftrightarrow{D}^\nu \varphi\right)$$

$$+ b' \left(\varphi^\dagger i \overleftrightarrow{D}_\nu \varphi\right)\left(\varphi^\dagger i \overleftrightarrow{D} \varphi\right)$$

⑤ $\psi_L \psi_R X_{L,R} D$:

(i) $X^{\mu\nu}\left(\bar{\psi}_{L,R}\gamma_\mu D_\nu \psi_{L,R}\right)$

$$= X^{\mu\nu}\left(\bar{\psi}_{L,R}\sigma_{\mu\nu}\psi_{R,L}\right)\varphi \equiv \psi^2 \varphi X$$

(ii) $\left(D_\mu X^{\mu\nu}\right)\left(\bar{\psi}_{L,R}\gamma_\nu \psi_{L,R}\right)$

$$= c_1'(\bar{\psi}\gamma^\nu\psi)(\bar{\psi}\gamma_\nu\psi)$$

$$+ c_2'(\varphi^\dagger i\overleftrightarrow{D}^\nu\varphi)(\bar{\psi}\gamma_\nu\psi)$$

$$\equiv \psi_{L,R}^4 \,/\, \psi_L^2\psi_R^2 + \psi_L\psi_R\varphi^2 D$$

Symmetry generators and their identities

$$(\sigma^{\mu\nu})_\alpha{}^\beta = (\sigma^\mu)_{\alpha\dot\beta}(\bar\sigma^\nu)^{\dot\beta\beta}$$

$$(\bar\sigma_{\mu\nu})^{\dot\beta}{}_{\dot\alpha} = (\bar\sigma_\mu)^{\beta\dot\beta}\,(\sigma_\nu)_{\beta\dot\alpha}$$

$$(\sigma^\mu)_{\alpha\dot\alpha}(\sigma_\mu)_{\beta\dot\beta} = 2\,\epsilon_{\alpha\beta}\,\epsilon_{\dot\alpha\dot\beta}$$

$$(\sigma^\mu)_{\alpha\dot\alpha}\,(\bar\sigma_\mu)^{\dot\beta\beta} = 2\,\delta_\alpha^\beta\,\delta_{\dot\alpha}^{\dot\beta}$$

$$(\bar\sigma^\mu)^{\dot\alpha\alpha}\,(\bar\sigma_\mu)^{\dot\beta\beta} = 2\,\epsilon^{\alpha\beta}\,\epsilon^{\dot\alpha\dot\beta}$$

$$[\sigma^\mu\bar\sigma^\nu + \sigma^\nu\bar\sigma^\mu]_\alpha{}^\beta = 2\,g^{\mu\nu}\,\delta_\alpha^\beta$$

$$\tau^I_{ij}\,\tau^I_{kl} = 2\,\delta_{il}\,\delta_{jk} - \delta_{ij}\,\delta_{kl} \Rightarrow SU(2)$$

$$T^A_{ij}\,T^A_{kl} = \frac{1}{2}\,\delta_{il}\,\delta_{jk} - \frac{1}{6}\,\delta_{ij}\,\delta_{kl} \Rightarrow SU(3)$$

ψ^4:

$$(\bar{d}\gamma^\mu T^A d)(\bar{Q} T^A \gamma_\mu Q)$$
$$= (\bar{d} d)(\bar{Q} Q) - \frac{1}{3}(\bar{d} Q)(\bar{Q} d)$$

$$(\bar{d}\gamma^\mu d)(\bar{Q}\gamma_\mu Q) = 2(\bar{d} Q)(\bar{Q} d)$$

$$(\bar{L}\sigma_{\mu\nu} e)(\bar{Q}\sigma^{\mu\nu} u)$$
$$= 4(\bar{L} e)(\bar{Q} u) - 8(\bar{L} u)(\bar{Q} e)$$

ϕ^6: $(H^\dagger \tau^I H)(H^\dagger \tau^I H) = (H^\dagger H)^2$

SMEFT op. bases

(1) WARSAW: ϕ^6, ψ^4, $\phi^4 D^2$, $\psi^2\phi^3$, $\phi^2 X^2$, X^3, $\psi^2\phi^2 D$.

(2) SILH: ϕ^6, $\phi^4 D^2$, $\psi^2\phi^3$, $\phi^2 X^2$, X^3, $\phi^2 X D^2$, $X^2 D^2$

(3) Additional Class: $\psi^2\phi D^2$, $\psi^2 X D$.