Planar and nonlinear Hall transport in quasi-2D systems

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QTT Research Group: interests

Berry curvature induced transport and optical phenomena

[Linear and non-linear Hall effects, quantum anomalies, new transport/optical effects…]

New collective modes in bulk and on surface

[Chiral plasmons, new collective modes in bulk, surface plasmons]

First principle based exploration of novel quantum materials

[Topological materials, 2D materials, topology + FE, Magnetism…, transport, excitons]

Novel electronic devices

[Negative Capacitance FET, 2D transistors..]

Geometrical properties of electron wave-packets

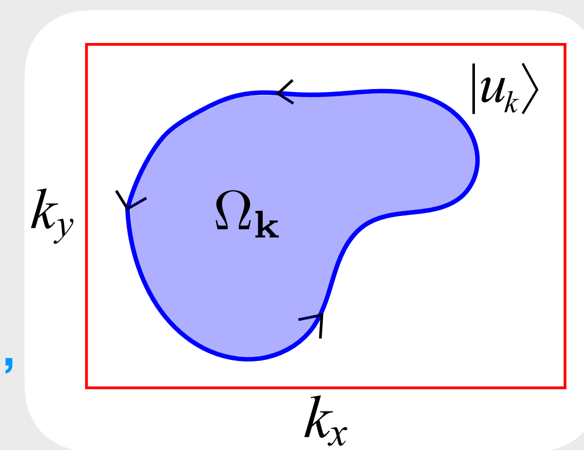
- One of the most interesting ideas in Quantum Mechanics
- Became prominent around 1984, 57 years after the discovery of QM

Wave-function moving around the Brillouin zone generates an irreducible phase - the Berry phase

$$\Phi_{\text{Berry}} = \oint \mathbf{\Omega_k} \cdot \hat{n}\ dk_x dk_y$$

'Magnetic field' in momentum space

Connected to 'degenerate band crossing' or 'band inversion' points in the BZ



$$\Phi_{\text{Magnetic Flux}} = \oint \mathbf{B} \cdot \hat{n}\ dxdy$$

Magnetic field in real space

Connected to 'magnetic monopoles' and circulating currents in real space

Gives rise to very interesting geometrical phenomena in quantum materials

Curvature in Brillouin zone

Topological properties

Novel surface/edge states

Anomalous velocity

$(\mathbf{E} \times \mathbf{\Omega_k})$

Modified Eq. of motion

Novel transport phenomena

(AHE, NAHE….)

Orbital magnetization

(Finite size wavepacket)

Valley polarization

OMM induced Hall effect

Quantum geometry of Bloch electrons -> Transport

Translation and self-rotation of a finite width electron wave packet with Bloch electron dynamics

↓

Geometrical properties: **Berry connection, Berry curvature, orbital magnetic moment, quantum metric, metric connection**

↓

These couple to external perturbations (E, B, T-gradient) in interesting ways. Generate new phenomena in transport and optical experiments.

Boltzmann Transport theory

Eq. of motion, 1 band picture

Quantum Kinetic theory

Density matrix, multi-band picture

Berry Curvature => Novel transport phenomena

Anomalous velocity
[Anomalous Hall, valley Hall effect]

Chiral magnetic velocity
[chiral magnetic effect,
Chiral anomaly in WSM]

$$\dot{\mathbf{r}} = D_{\mathbf{k}} \left(\tilde{\mathbf{v}}_{\mathbf{k}} + \frac{e}{\hbar}(\mathbf{E} \times \mathbf{\Omega}_{\mathbf{k}}) + \frac{e}{\hbar}(\tilde{\mathbf{v}}_{\mathbf{k}} \cdot \mathbf{\Omega}_{\mathbf{k}})\mathbf{B} \right)$$

Lorentz force

$$\hbar\dot{\mathbf{k}} = D_{\mathbf{k}} \left(-e\mathbf{E} - e(\tilde{\mathbf{v}}_{\mathbf{k}} \times \mathbf{B}) - \frac{e^2}{\hbar}(\mathbf{E} \cdot \mathbf{B})\mathbf{\Omega}_{\mathbf{k}} \right)$$

$$D_{\mathbf{k}} \equiv [1 + \tfrac{e}{\hbar}(\mathbf{B} \cdot \mathbf{\Omega}_{\mathbf{k}})]^{-1}$$

BC modifies the semiclassical Eq. of motion
of wave packets in crystals

=> induces novel transport effects

Berry force
[Magneto-electric coupling,
Negative MR, Planar Hall effect,
'chiral anomaly' like physics]

Playground: the problem of transport conductivity

Modelling electrical conductivity in transport measurements

	Non-interacting physics	Many body physics
No Disorder	**Band geometry** Linear: Anomalous Hall effect Non-linear: i) Berry curvature Dipole ii) Quantum metric dipole (BCP..) Third order: ???? AHE in systems with P and T	**BG + electron correlations** Linear: Superfluid stiffness Nonlinear: ????
Disorder	**Extrinsic: disorder + BG + asymmetric scattering** Linear: weak localization, extrinsic AHE Non-Linear: extrinsic AHE, ????	**Disorder + BG + Correlation effects** Linear: Kondo effect, Many body localization, ???? Nonlinear: ????

Hall effect without magnetic field: Beyond the linear regime

In case we do not have an anomalous Hall

or symmetric (Drude) Hall response

in the linear response regime

What is the dominant Hall response in the non-linear regime

Are there novel longitudinal responses beyond nonlinear Drude?

Linear

Third-order

$$j_a = \sum_b \sigma^{(1)}_{ab} E_b + \sum_{bc} \sigma^{(2)}_{abc} E_b E_c + \sum_{bcd} \sigma^{(3)}_{abcd} E_b E_c E_d + \ldots$$

Second-order

Problem of second order nonlinear DC conductivity?

Full problem => Band geometry + Disorder (Symm + Asymm) + strong correlation effects

Partial solution => Band geometry + symmetric scattering effects

PHYSICAL REVIEW B **108**, L201405 (2023)

Letter

Intrinsic nonlinear conductivities induced by the quantum metric

Kamal Das[1,2,*,†] Shibalik Lahiri,[1,3,*,‡] Rhonald Burgos Atencia,[4,5,§] Dimitrie Culcer,[4,5,∥] and Amit Agarwal[1,¶]

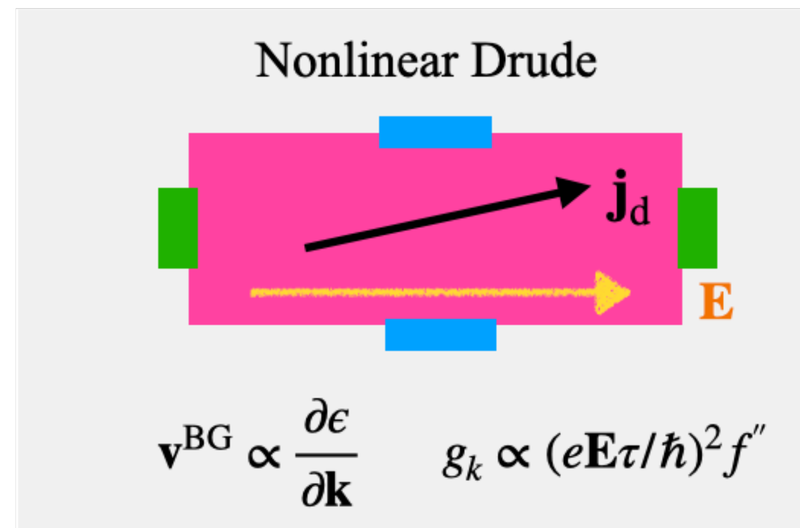
Nonlinear Zero B Hall effect: Second order Responses

$$\mathbf{j} = \sum_{\mathbf{k}} \text{velocity } [\mathbf{v}] \times \text{non eq. distribution function } [g_{\mathbf{k}}]$$

Correction in the distribution function =>

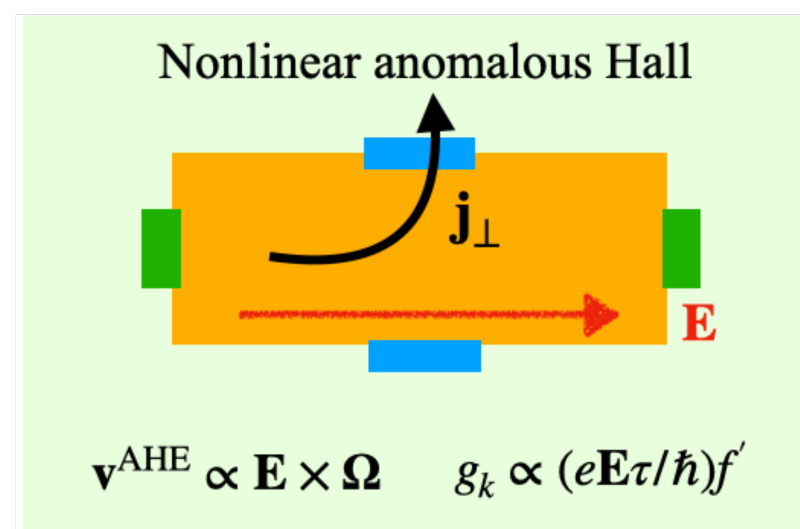
Non-linear Drude response

(Second order correction in the distribution function)



Berry Curvature Dipole

(First order correction in the distribution function
+ Anomalous velocity)



Nonlinear Zero B Hall effect: Second order Responses

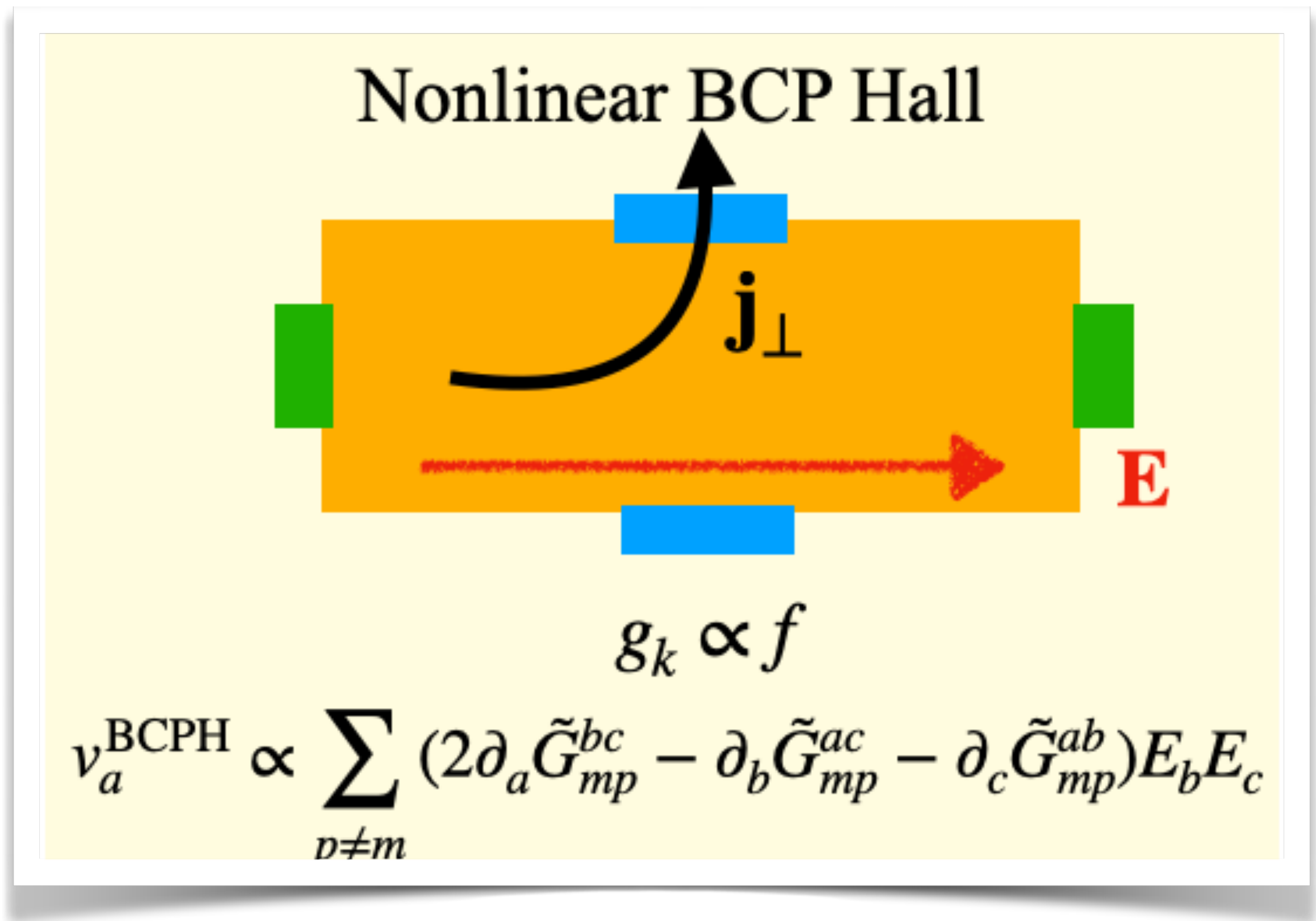
E-induced correction in the Berry curvature and anomalous velocity

+

Intrinsic or unperturbed distribution function

$$\boldsymbol{v}^{\rm AHE} = e(\boldsymbol{E}\times\boldsymbol{\Omega})/\hbar$$

$$\boldsymbol{v}^{\rm BCPH} = e(\boldsymbol{E}\times\boldsymbol{\Omega}^{E})/\hbar$$



Berry connection polarizability (quantum metric) induced Hall response

Quantum geometry of Bloch electrons - I

Distance between nearby states => quantum geometric tensor

$$ds^2 = \langle \psi(\mathbf{k}+d\mathbf{k}) - \psi(\mathbf{k}) | \psi(\mathbf{k}+d\mathbf{k}) - \psi(\mathbf{k}) \rangle = \left\langle \frac{\partial \psi(\mathbf{k})}{\partial k_a} \middle| \frac{\partial \psi(\mathbf{k})}{\partial_{k_b}} \right\rangle dk_a dk_b$$

a/b => coordinate axis

$$\mathcal{Q}_{ab} \equiv \left\langle \frac{\partial \psi(\mathbf{k})}{\partial k_a} \middle| \frac{\partial \psi(\mathbf{k})}{\partial_{k_b}} \right\rangle \equiv \mathcal{G}_{ab} - \frac{i}{2}\Omega_{ab}$$

Geometric tensor ($\mathcal{Q}_{ab}$)

Quantum metric ($\mathcal{G}_{ab}$)

Berry curvature (Ω_{ab})

All topological indexes => the BZ integrals of functions of Berry curvature

Topological materials => Novel surface states with interesting properties

BC/QM => New transport and optical phenomena (AHE, valley HE, NL Hall)

Quantum geometry of Bloch electrons - II

From single band picture <=> multi band picture

Band resolved band geometric quantities

Berry connection

$$\mathcal{R}_{mp}(\boldsymbol{k}) = i\langle u^m_{\boldsymbol{k}}|\partial_{\boldsymbol{k}} u^p_{\boldsymbol{k}}\rangle$$

Geometric tensor

$$\mathcal{Q}^{cb}_{mp} = \mathcal{R}^c_{mp}\mathcal{R}^b_{pm}$$

Quantum Metric

$$\mathcal{G}^{cb}_{mp} = \{\mathcal{R}^c_{pm}, \mathcal{R}^b_{mp}\}/2$$

Berry curvature

$$\Omega^{cb}_{mp} = i[\mathcal{R}^c_{pm}, \mathcal{R}^b_{mp}]$$

$$\mathcal{Q}^{cb}_{m} = \sum_{p\neq m} \mathcal{Q}^{cb}_{mp}$$

Nonlinear Zero B Hall effect: Second order Responses

A natural question

Is that the complete story?

Or, are there missing terms?

How do we capture all possible nonlinear terms systematically,
on the same footing
even in a simple relaxation time approximation?

Quantum Kinetic theory: Non-linear transport/optics

E field - matter interaction: $\mathcal{H}_{\rm int}(\boldsymbol{k},t) = \mathcal{H}_0(\boldsymbol{k}) + U + e\boldsymbol{r}\cdot\boldsymbol{E}(t)$

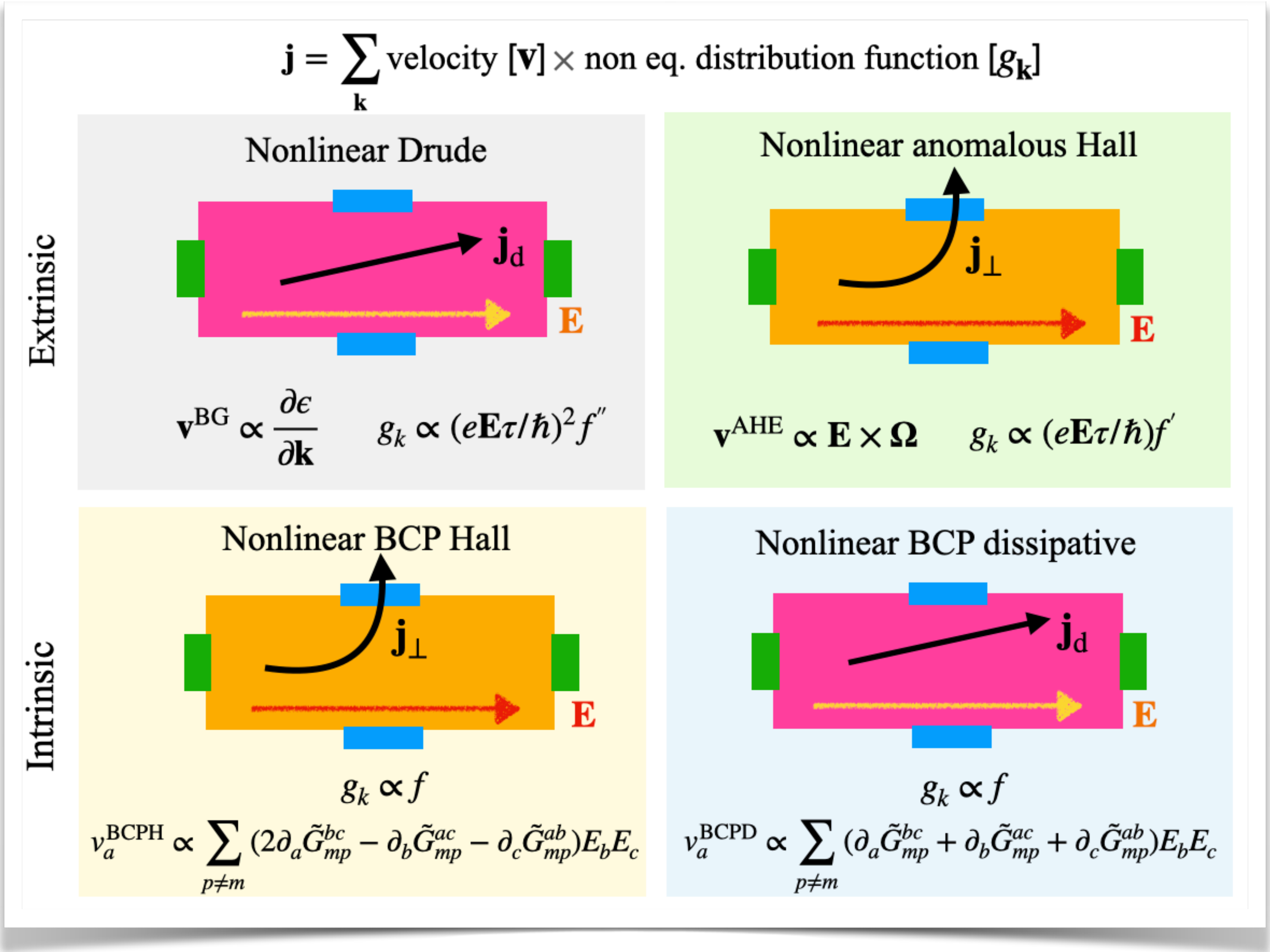
Velocity operator: $\mathbf{v} = \dot{\mathbf{r}} = \frac{1}{i\hbar}[\mathbf{r}, \mathcal{H}_{\rm int}]$

Current expectation value: $\mathbf{J} = \mathrm{Trace}[\mathbf{v}\rho]$

Evolution of density matrix: $\frac{d\rho(\boldsymbol{k},t)}{dt} + \frac{i}{\hbar}[\mathcal{H}_{\rm int}(\boldsymbol{k},t), \rho(\boldsymbol{k},t)] = 0$

- Work in the basis states of H_0
- 'r' operator => Berry connection
- EOM of density matrix => Covariant derivative

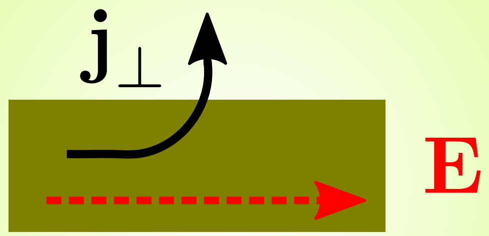
Nonlinear Zero B Hall effect: Second order Responses



+ Asymmetric scattering contributions (side jump and skew scattering)

Berry curvature dipole induced Hall conductivity

Nonlinear anomalous Hall



$\mathbf{v}^{\mathrm{AHE}} = \mathbf{E} \times \boldsymbol{\Omega}$ $\quad$ $g_{\mathbf{k}} \propto (eE\tau/\hbar) f'$

- Hall effect in time-reversal symmetric systems
- Induced by Berry curvature dipole
- Inversion symmetry breaking + asymmetry of band structure is essential

PRL **115,** 216806 (2015)

Quantum Nonlinear Hall Effect Induced by Berry Curvature Dipole in Time-Reversal Invariant Materials

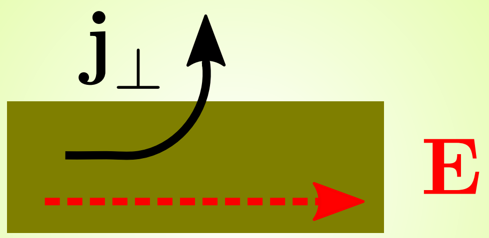
Inti Sodemann and Liang Fu

Berry curvature dipole

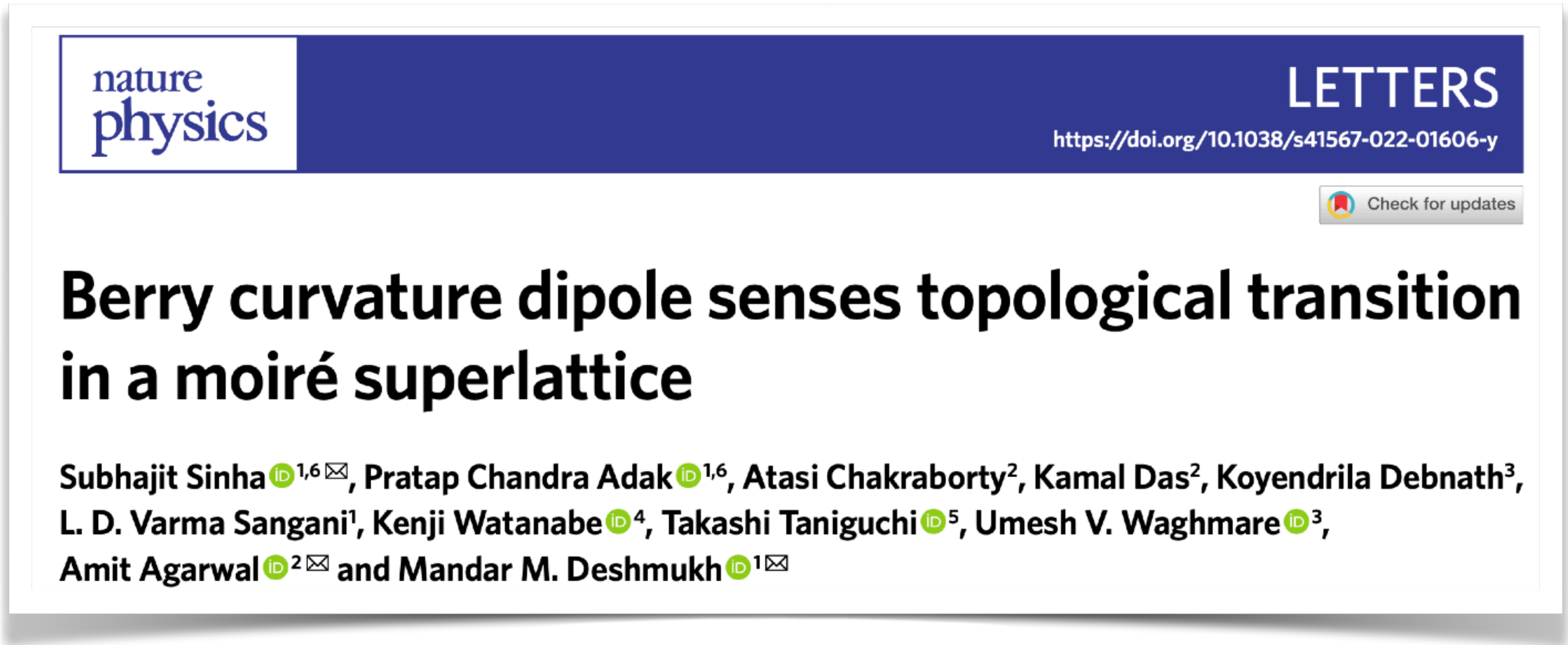
$$\chi_{\mathrm{abc}} = -\varepsilon_{adc} \frac{e^3 \tau}{2(1 + i\omega\tau)} \int_k f_0 (\partial_b \Omega_d).$$

Berry curvature dipole induced Hall conductivity

Nonlinear anomalous Hall



$\mathbf{v}^{\mathrm{AHE}} = \mathbf{E} \times \mathbf{\Omega}$ $\qquad g_{\mathbf{k}} \propto (eE\tau/\hbar) f'$


nature physics
LETTERS
https://doi.org/10.1038/s41567-022-01606-y
Check for updates

Berry curvature dipole senses topological transition in a moiré superlattice

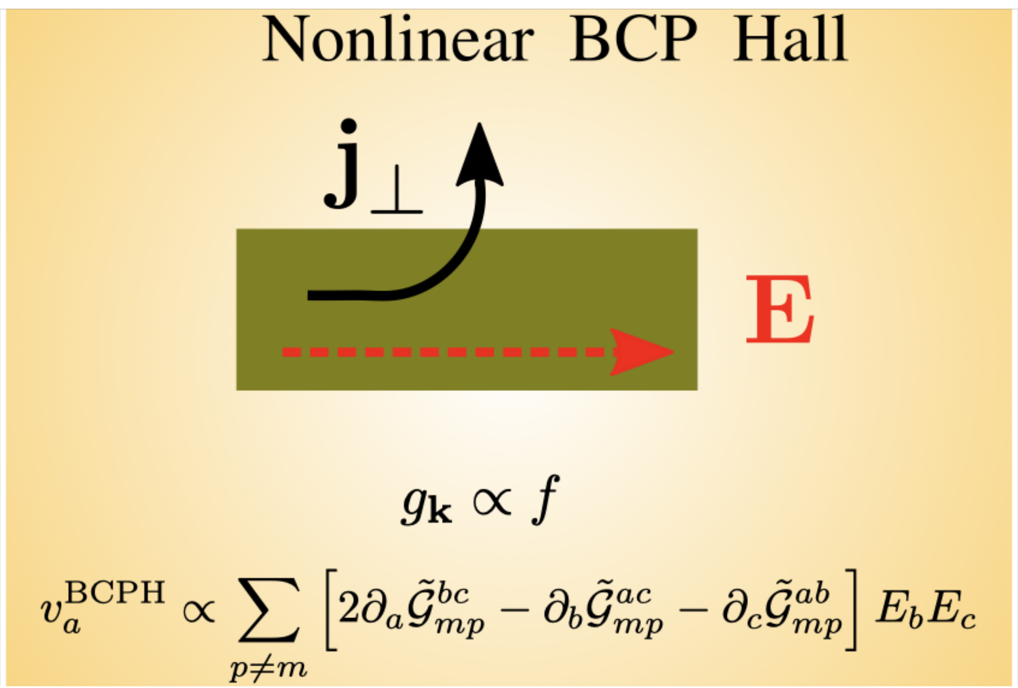
Subhajit Sinha[1,6], Pratap Chandra Adak[1,6], Atasi Chakraborty[2], Kamal Das[2], Koyendrila Debnath[3], L. D. Varma Sangani[1], Kenji Watanabe[4], Takashi Taniguchi[5], Umesh V. Waghmare[3], Amit Agarwal[2] and Mandar M. Deshmukh[1]

Topological phase transitions are hard to detect (valley Chern type)

=> We demonstrate that the BCD changes sign across the transition

=> NL Hall measurement can detect topological phase transitions

Intrinsic: Berry curvature polarizability Hall current



PRL **112,** 166601 (2014) PHYSICAL REVIEW LETTERS week ending 25 APRIL 2014

Field Induced Positional Shift of Bloch Electrons and Its Dynamical Implications

Yang Gao,[1] Shengyuan A. Yang,[2] and Qian Niu[1,3]

– E filed induced correction to Berry connection: $\mathcal{A}_a^E(\boldsymbol{k}) = G_{ab}(\boldsymbol{k}) E_b$

– This induces corrections in An. velocity => Second order intrinsic Hall response

This corrections is significant in systems where the Berry curvature vanishes
For example in (PT) symmetric systems (bipartite antiferromagnets)
in P and T symmetric systems (graphene) => NL valley response

PHYSICAL REVIEW LETTERS **127,** 277202 (2021)

Intrinsic Second-Order Anomalous Hall Effect and Its Application in Compensated Antiferromagnets

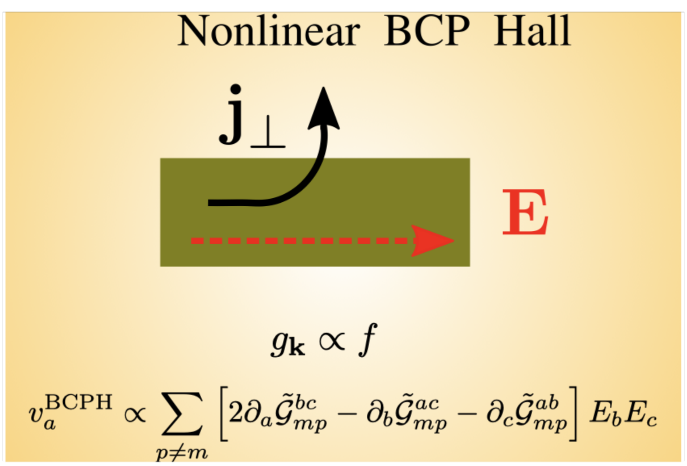
Huiying Liu,[1] Jianzhou Zhao,[1,2,*] Yue-Xin Huang,[1] Weikang Wu,[1,3] Xian-Lei Sheng,[4] Cong Xiao,[5,6,7,†] and Shengyuan A. Yang[1]

PHYSICAL REVIEW LETTERS **127,** 277201 (2021)

Intrinsic Nonlinear Hall Effect in Antiferromagnetic Tetragonal CuMnAs

Chong Wang,[1] Yang Gao,[2,3,*] and Di Xiao[1,†]

Intrinsic: Berry curvature polarizability Hall current



Recent observation

Article

Quantum-metric-induced nonlinear transport in a topological antiferromagnet

https://doi.org/10.1038/s41586-023-06363-3

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Accepted: 22 June 2023

Naizhou Wang[1], Daniel Kaplan[2], Zhaowei Zhang[1], Tobias Holder[2], Ning Cao[3], Aifeng Wang[3], Xiaoyuan Zhou[3], Feifei Zhou[1], Zhengzhi Jiang[1], Chusheng Zhang[1], Shihao Ru[1], Hongbing Cai[1,4], Kenji Watanabe[5], Takashi Taniguchi[6], Binghai Yan[2]✉ & Weibo Gao[1,4,7]✉

RESEARCH

RESEARCH ARTICLE

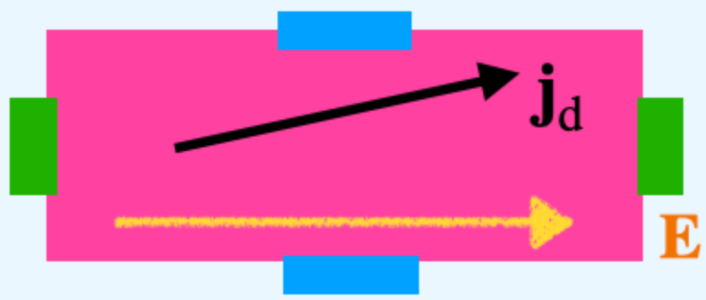
TOPOLOGICAL MATTER

Quantum metric nonlinear Hall effect in a topological antiferromagnetic heterostructure

Anyuan Gao[1], Yu-Fei Liu[1,2], Jian-Xiang Qiu[1], Barun Ghosh[3], Thaís V. Trevisan[4,5], Yugo Onishi[6], Chaowei Hu[7], Tiema Qian[7], Hung-Ju Tien[8], Shao-Wen Chen[2], Mengqi Huang[9], Damien Bérubé[1], Houchen Li[1], Christian Tzschaschel[1], Thao Dinh[1,2], Zhe Sun[1,10], Sheng-Chin Ho[1], Shang-Wei Lien[8], Bahadur Singh[11], Kenji Watanabe[12], Takashi Taniguchi[12], David C. Bell[13,14], Hsin Lin[15], Tay-Rong Chang[8,16,17], Chunhui Rita Du[9], Arun Bansil[3], Liang Fu[6], Ni Ni[7], Peter P. Orth[4,5], Qiong Ma[10,18], Su-Yang Xu[1]*

Intrinsic: Quantum metric dipole induced current

Nonlinear BCP dissipative



$$g_k \propto f$$

$$v_a^{\mathrm{BCPD}} \propto \sum_{p \neq m} (\partial_a \tilde{G}^{bc}_{mp} + \partial_b \tilde{G}^{ac}_{mp} + \partial_c \tilde{G}^{ab}_{mp}) E_b E_c$$

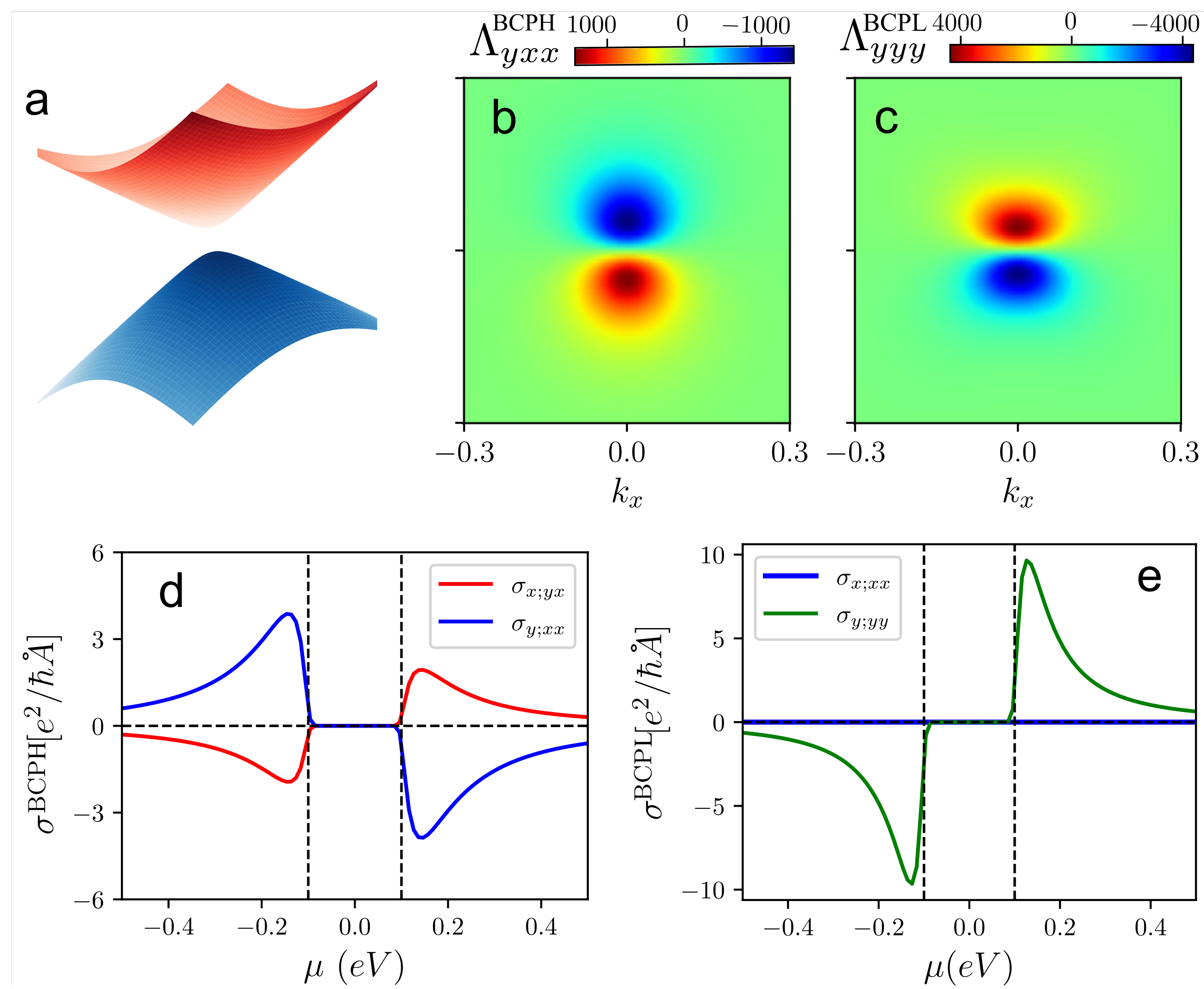
PHYSICAL REVIEW B **108**, L201405 (2023)

Letter

Intrinsic nonlinear conductivities induced by the quantum metric

Kamal Das,[1,2,*,†] Shibalik Lahiri,[1,3,*,‡] Rhonald Burgos Atencia,[4,5,§] Dimitrie Culcer,[4,5,∥] and Amit Agarwal[1,¶]

$$\mathcal{H} = v_F(k_x\sigma_y - k_y\sigma_x) + v_t k_y \sigma_0 + \Delta\sigma_z$$

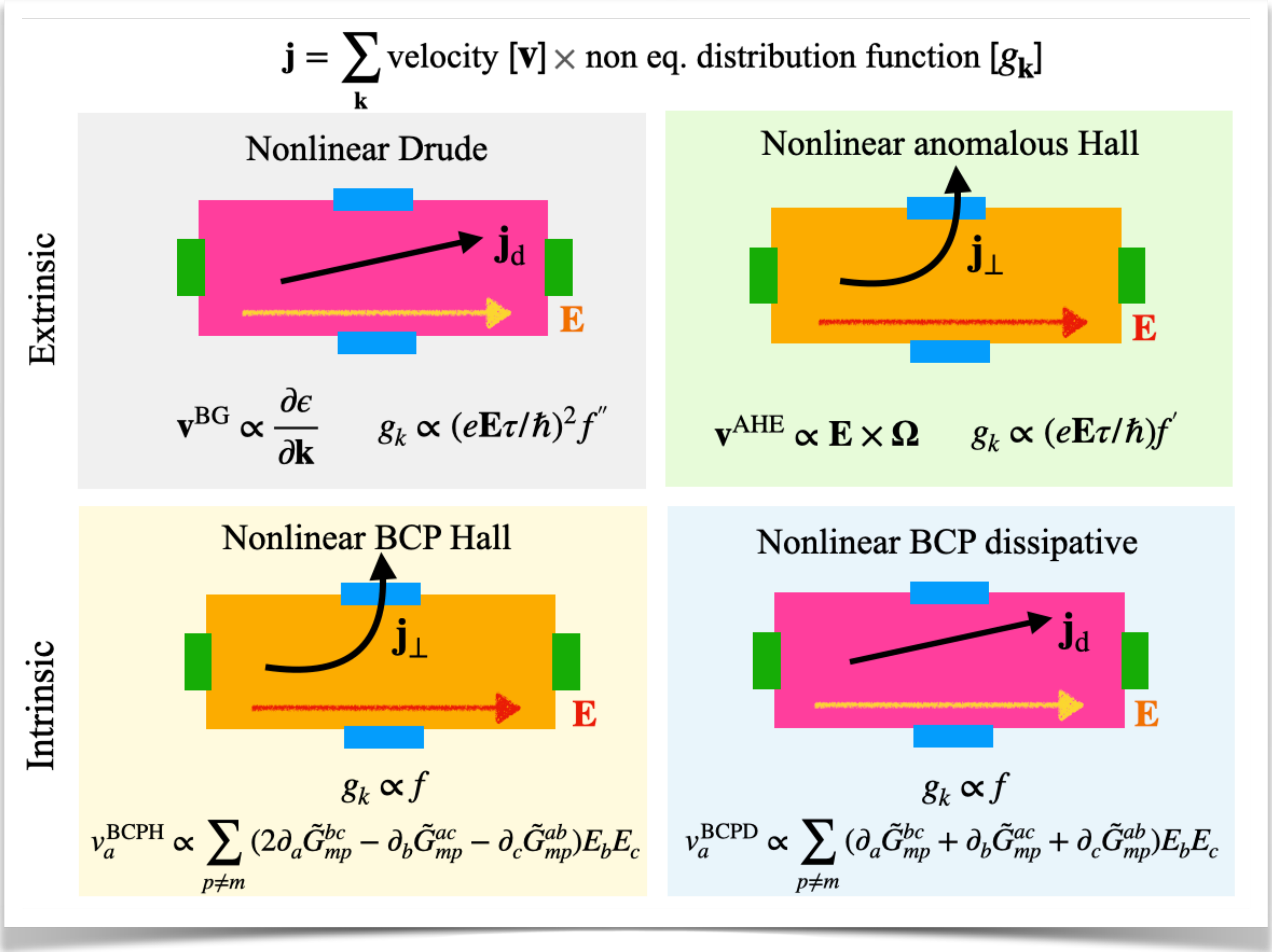


New NL intrinsic response: longitudinal as well as transverse

Interband coherence effects in longitudinal transport

Not very well understood from the semiclassical calculation framework

Nonlinear Zero B Hall effect: Second order Responses

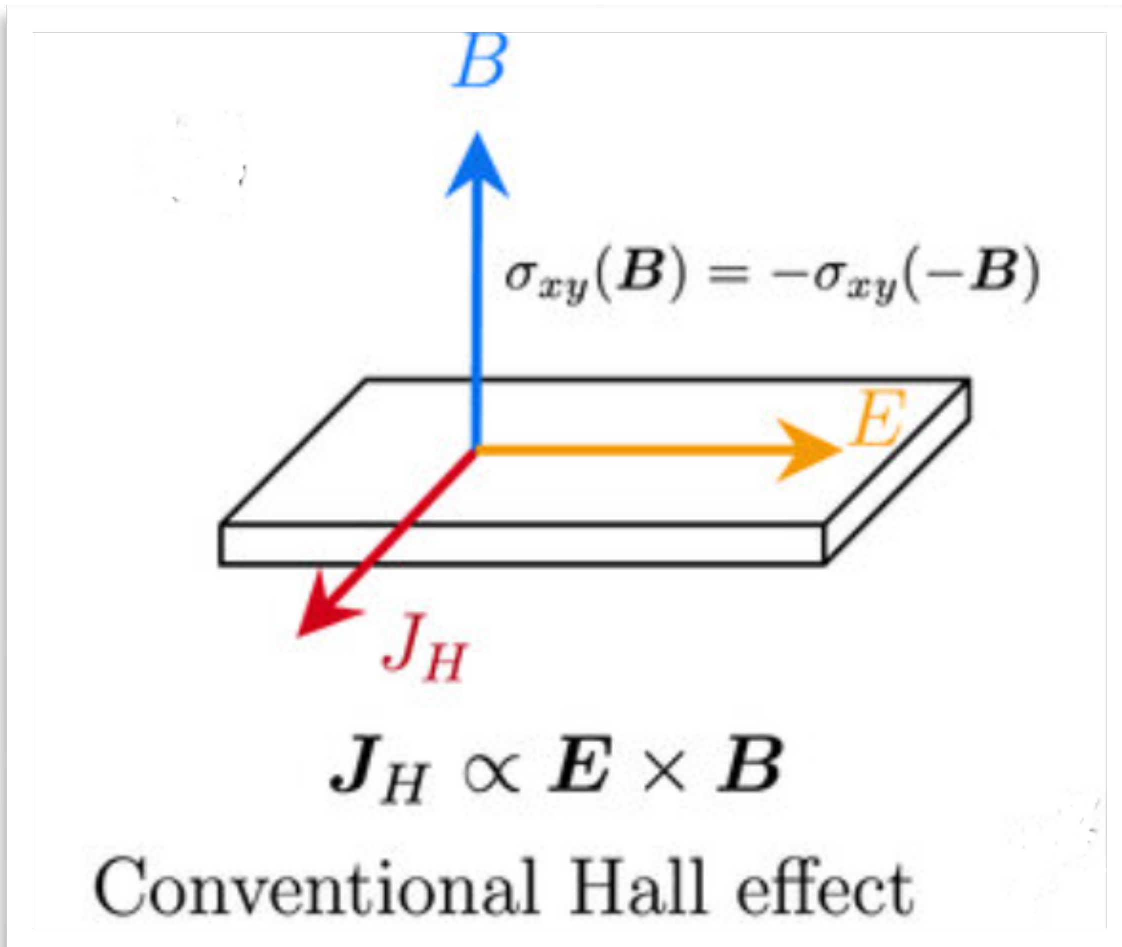


+ Asymmetric scattering contributions (side jump and skew scattering)

Planar Hall Effect in Quasi-Two-Dimensional Materials

Koushik Ghorai,* Sunit Das,* Harsh Varshney, and Amit Agarwal†
Department of Physics, Indian Institute of Technology Kanpur, Kanpur-208016, India

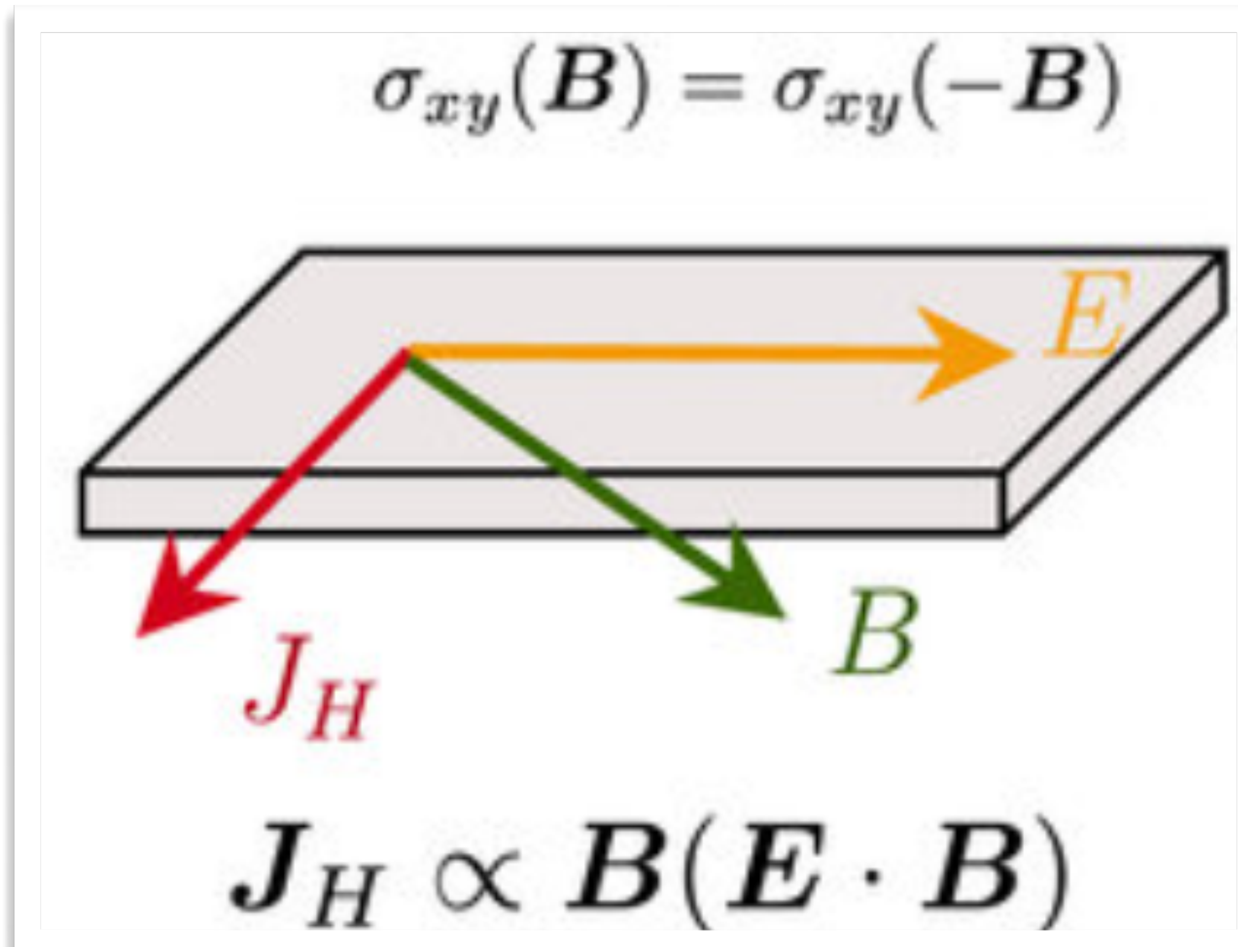
Lorentz Hall effect



Transverse Hall voltage generated by orthogonal electric and magnetic field

Lorentz force

Planar Hall effect



In-plane transverse voltage in presence of in-plane electric and magnetic field

Chiral magnetic velocity
+ Berry force

Chiral magnetic velocity and Berry force

Chiral magnetic velocity
[chiral magnetic effect,
Chiral anomaly in WSM]

Anomalous velocity
[Anomalous Hall, valley Hall effect]

$$\dot{\mathbf{r}} = D_{\mathbf{k}} \left(\tilde{\mathbf{v}}_{\mathbf{k}} + \frac{e}{\hbar}(\mathbf{E} \times \mathbf{\Omega}_{\mathbf{k}}) + \frac{e}{\hbar}(\tilde{\mathbf{v}}_{\mathbf{k}} \cdot \mathbf{\Omega}_{\mathbf{k}})\mathbf{B} \right)$$

Lorentz force

$$\hbar\dot{\mathbf{k}} = D_{\mathbf{k}} \left(-e\mathbf{E} - e(\tilde{\mathbf{v}}_{\mathbf{k}} \times \mathbf{B}) - \frac{e^2}{\hbar}(\mathbf{E} \cdot \mathbf{B})\mathbf{\Omega}_{\mathbf{k}} \right)$$

$$D_{\mathbf{k}} \equiv [1 + \tfrac{e}{\hbar}(\mathbf{B} \cdot \mathbf{\Omega}_{\mathbf{k}})]^{-1}$$

BC modifies the semiclassical Eq. of motion
of wave packets in crystals

=> induces novel transport effects

Berry force
[Magneto-electric coupling,
Negative MR, Planar Hall effect,
'chiral anomaly' like physics]

Planar Hall effect in 3D systems

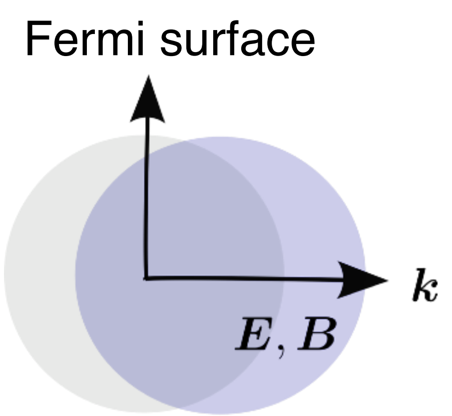
Dynamics of electrons

$$\dot{\mathbf{r}}_\lambda = D_\lambda \left[\mathbf{v}_\lambda + \frac{e}{\hbar}\mathbf{E}\times\mathbf{\Omega}_\lambda + \frac{e}{\hbar}(\mathbf{v}_\lambda\cdot\mathbf{\Omega}_\lambda)\mathbf{B}\right]$$

$$\hbar\dot{\mathbf{k}}_\lambda = D_\lambda \left[-e\mathbf{E} - e\mathbf{v}_\lambda\times\mathbf{B} - \frac{e^2}{\hbar}(\mathbf{E}\cdot\mathbf{B})\mathbf{\Omega}_\lambda\right]$$

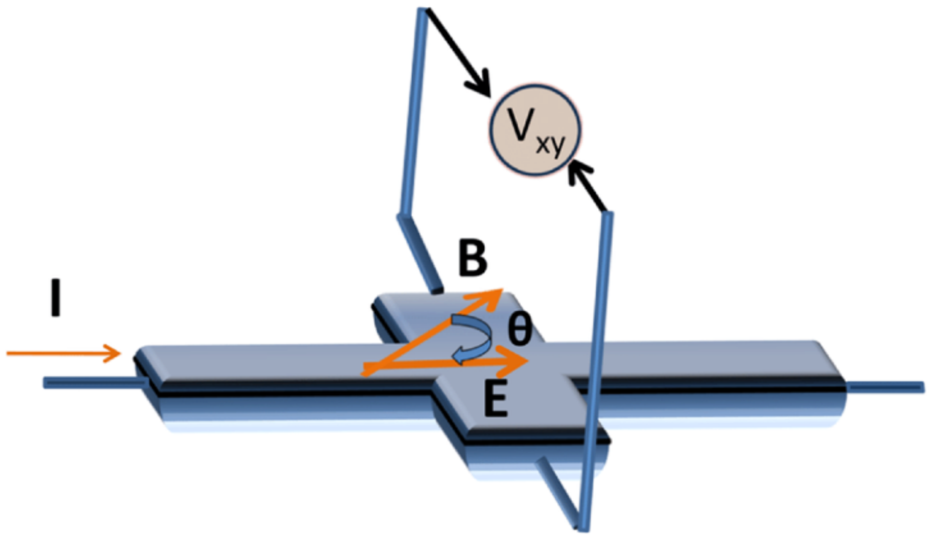
Non-equilibrium distribution function

$$g_\lambda = f_\lambda - e\tau D_\lambda\left(\mathbf{v}_\lambda\cdot\mathbf{E} + \tfrac{e}{\hbar}\left(\mathbf{v}_\lambda\cdot\mathbf{\Omega}_\lambda\right)\mathbf{E}\cdot\mathbf{B}\right)\left(-\frac{\partial f_\lambda}{\partial\epsilon_\lambda}\right)$$

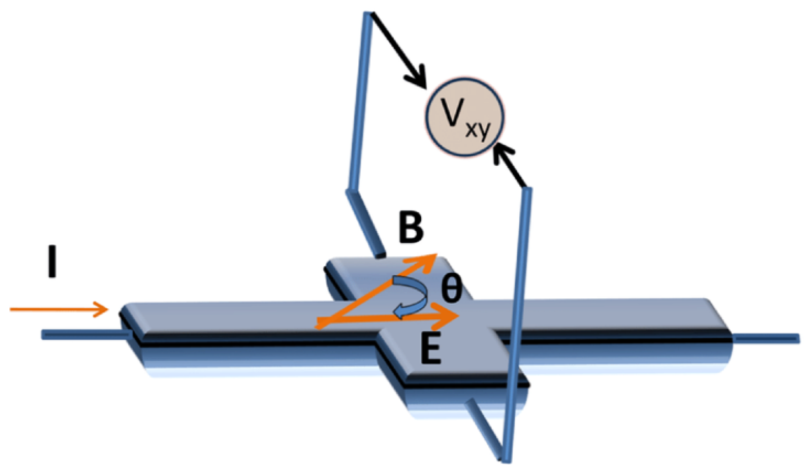


Electronic current

$$\mathbf{j}^e = -e\int[d\mathbf{k}]D^{-1}\,\dot{\mathbf{r}}\;g_{\mathbf{k}}$$



Theory of Planar Hall effect in 3D systems



- The charge current definition:

Chiral magnetic velocity

'Berry force'

$$\mathbf{j}_e = -e\int [d\mathbf{k}]\dot{\mathbf{r}}_\lambda g_\lambda$$

$$= -e\int [d\mathbf{k}]D_\lambda\left[\mathbf{v}_\lambda + \frac{e}{\hbar}\mathbf{E}\times\mathbf{\Omega}_\lambda + \frac{e}{\hbar}(\mathbf{v}_\lambda\cdot\mathbf{\Omega}_\lambda)\mathbf{B}\right]\left[-e\tau D_\lambda\left(\mathbf{v}_\lambda\cdot\mathbf{E} + \frac{e}{\hbar}\left(\mathbf{v}_\lambda\cdot\mathbf{\Omega}_\lambda\right)\mathbf{E}\cdot\mathbf{B}\right)\right]\left(-\frac{\partial f^\chi_\lambda}{\partial\epsilon_\lambda}\right)$$

- The expression for planar Hall effect:

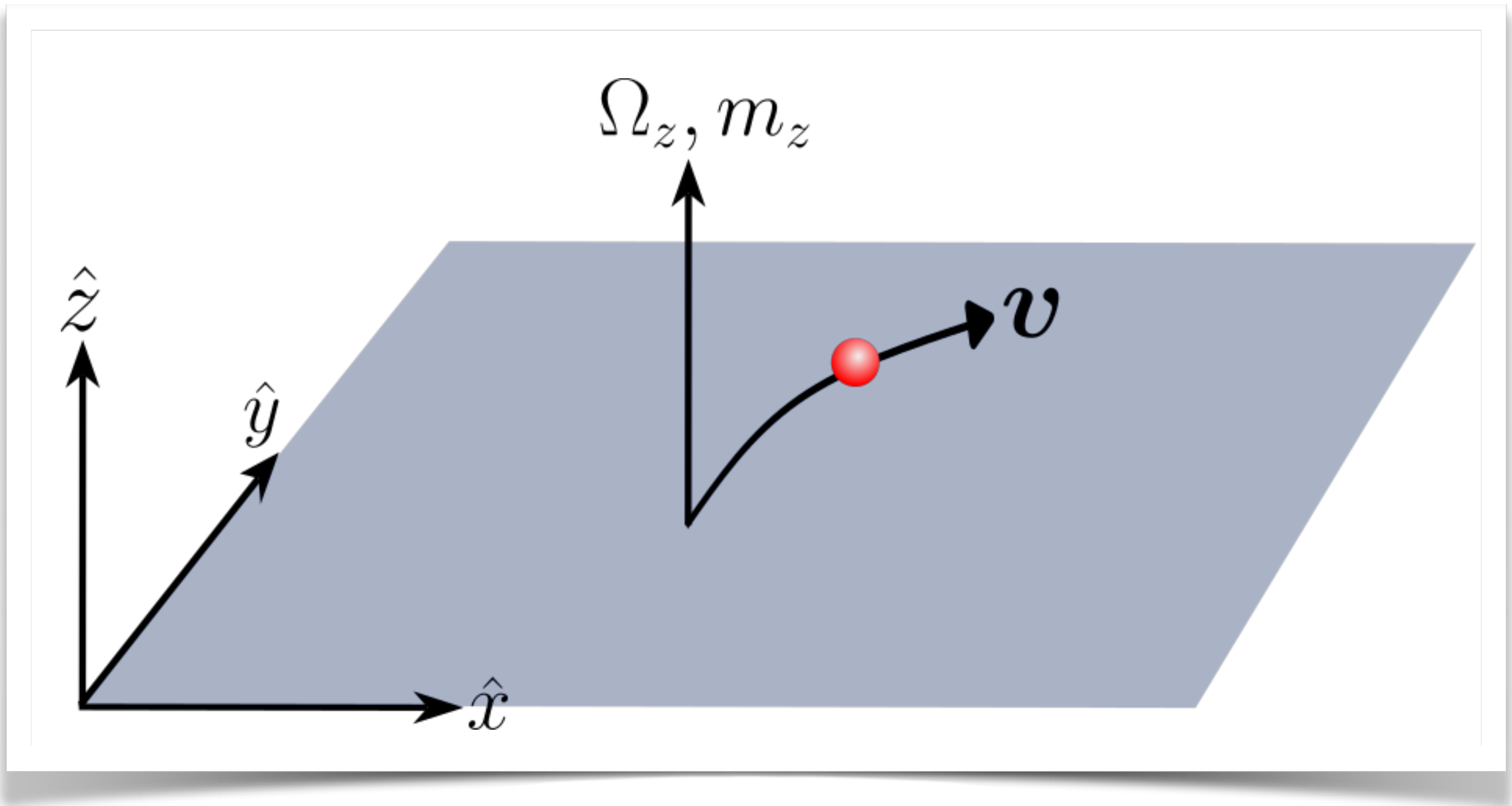
$$\mathbf{j}_e = -\frac{e^2\tau}{\hbar}\int [d\mathbf{k}](\mathbf{v}_\lambda\cdot\mathbf{\Omega}_\lambda)(\mathbf{v}_\lambda\cdot\mathbf{E})\mathbf{B}\left(-\frac{\partial f^0_\lambda}{\partial\epsilon_\lambda}\right) - \frac{e^3\tau}{\hbar}\int [d\mathbf{k}](\mathbf{v}_\lambda\cdot\mathbf{\Omega}_\lambda)^2(\mathbf{E}\cdot\mathbf{B})\mathbf{B}\left(-\frac{\partial f^0_\lambda}{\partial\epsilon_\lambda}\right)$$

Linear in B

Quadratic in B

- For co-planar configuration of **E** and **B**, both of these terms gives the planar Hall response.

What about 2D systems?



$\boldsymbol{v}_{n\boldsymbol{k}} \perp \boldsymbol{\Omega}_{n\boldsymbol{k}}$ ⇨ Chiral magnetic velocity = 0 in 2D systems

No Berry curvature induced Planar Hall effect

Possibility of finite planar BC in 2D systems

2D systems with two or more atomic layers

+

Interlayer tunnelling of electrons

=> Finite inplane Berry Curvature and Orbital magnetic moment

$$\boldsymbol{\Omega}_{n\boldsymbol{k}}^{\text{planar}} = 2\hbar \sum_{n' \neq n} \frac{\text{Re}(\boldsymbol{v}_{nn'} \times \boldsymbol{\mathcal{Z}}_{n'n})}{(\varepsilon_{n\boldsymbol{k}} - \varepsilon_{n'\boldsymbol{k}})}, \qquad \boldsymbol{m}_{n\boldsymbol{k}}^{\text{planar}} = e \sum_{n' \neq n} \text{Re}(\boldsymbol{v}_{nn'} \times \boldsymbol{\mathcal{Z}}_{n'n}).$$

$$\boldsymbol{\mathcal{Z}}_{n'n} = \hat{\boldsymbol{z}} \langle u_{n'\boldsymbol{k}} | \mathcal{Z} | u_{n\boldsymbol{k}} \rangle$$

Origin of finite planar BC in 2D systems

Conventional Berry Curvature:

$$\boldsymbol{\Omega}_{n\boldsymbol{k}} = i\hbar^2 \sum_{n'\neq n} \frac{\boldsymbol{v}_{nn'} \times \boldsymbol{v}_{n'n}}{(\varepsilon_{n\boldsymbol{k}} - \varepsilon_{n'\boldsymbol{k}})^2}$$

Redefine out-of-plane velocity matrix element:

$$v^z_{n'n} = (1/\hbar)\, \langle u_{n'\boldsymbol{k}} |\, \partial_{k_z} H_{\boldsymbol{k}} \,| u_{n\boldsymbol{k}} \rangle \equiv (1/\hbar)\, \langle u_{n'\boldsymbol{k}} |\, i[H_{\boldsymbol{k}}, Z] \,| u_{n\boldsymbol{k}} \rangle = (i/\hbar)(\varepsilon_{n'\boldsymbol{k}} - \varepsilon_{n\boldsymbol{k}}) Z_{n'n}$$

Interlayer tunnelling can give rise to out of plane velocity matrix

Planar Berry curvature
in quasi-2D systems

$$\boldsymbol{\Omega}^{\text{planar}}_{n\boldsymbol{k}} = 2\hbar \,\text{Re} \sum_{n'\neq n} \frac{\boldsymbol{v}_{nn'} \times \boldsymbol{Z}_{n'n}}{(\varepsilon_{n\boldsymbol{k}} - \varepsilon_{n'\boldsymbol{k}})}$$

Similarly, planar orbital magnetic moment:

$$\boldsymbol{m}^{\text{planar}}_{n\boldsymbol{k}} = e \,\text{Re} \sum_{n'\neq n} \boldsymbol{v}_{nn'} \times \boldsymbol{Z}_{n'n}$$

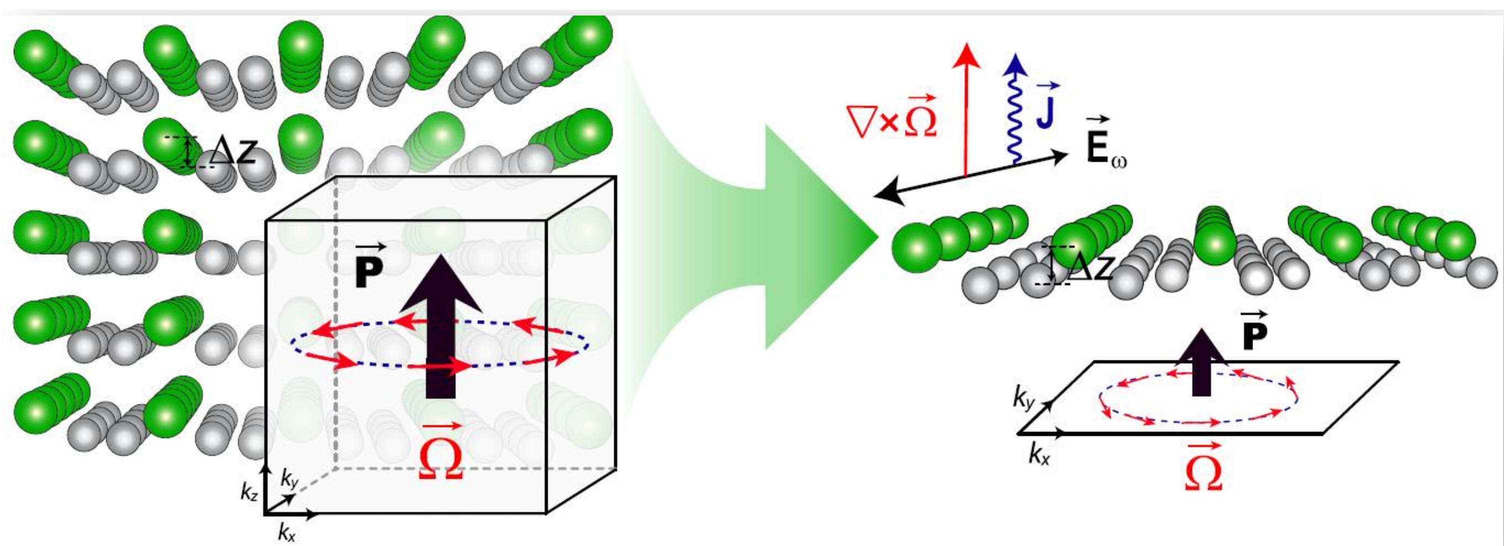
Origin of finite planar BC in 2D systems

PHYSICAL REVIEW B **104**, L081114 (2021)

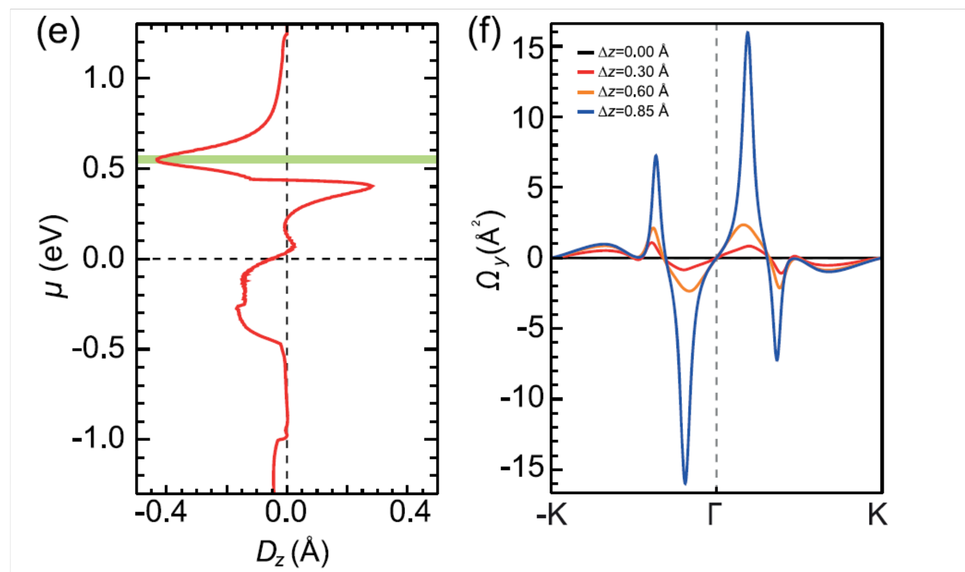
Letter

Vertical transverse transport induced by hidden in-plane Berry curvature in two dimensions

Kyoung-Whan Kim,[1,*] Hogyun Jeong,[2,*] Jeongwoo Kim,[3,†] and Hosub Jin[2,‡]



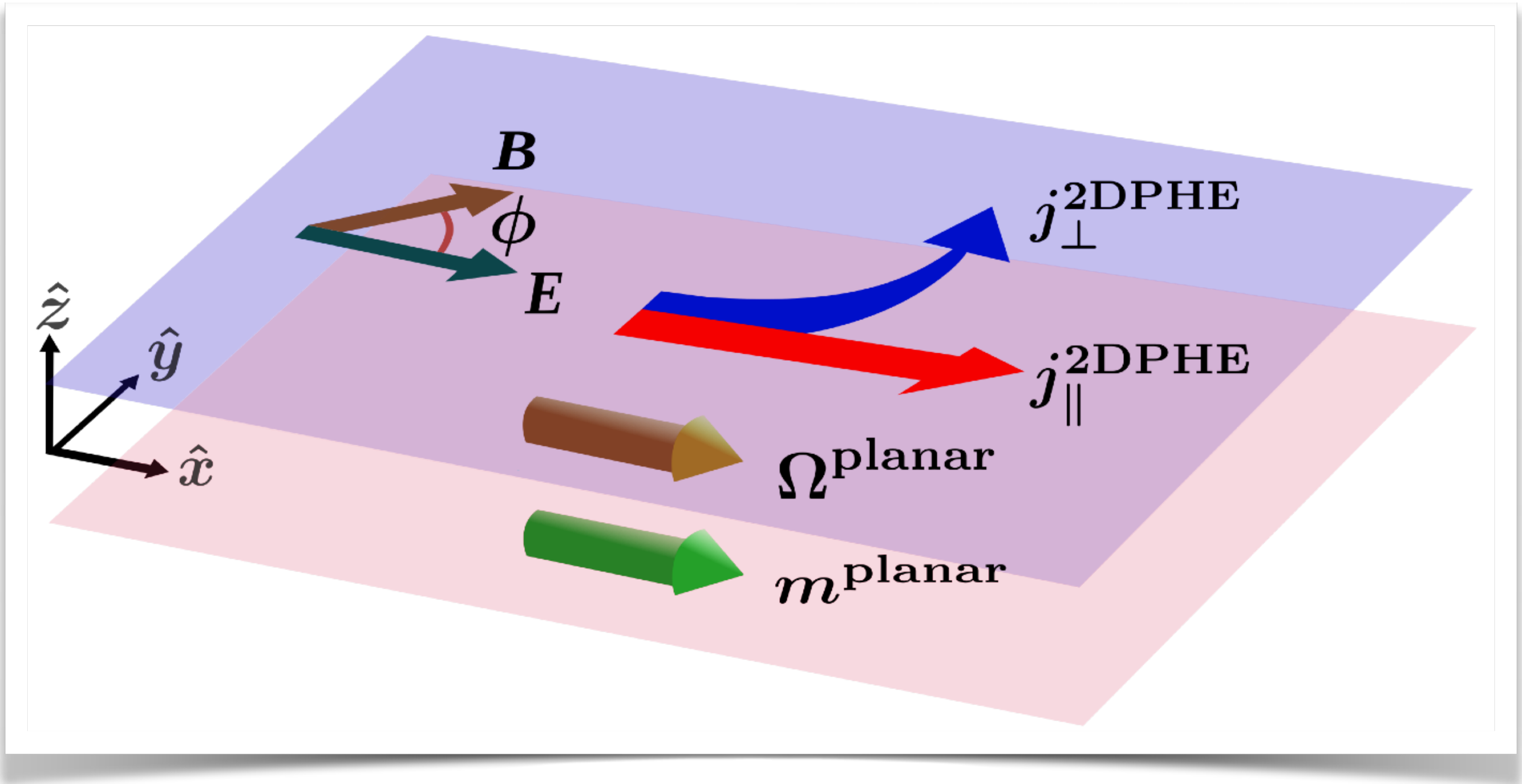
During dimensional reduction, the helical texture of in-plane BC is preserved for polar BiAg2 monolayer (Phys. Rev. B **104**, L081114)



Out-of-plane BCD and planar BC in BiAg2 monolayer

Phys. Rev. B **104**, L081114

Planar Hall effect in quasi-2D systems



Planar Current: $j_a = \tau \chi_{ab;c} E_b B_c + \tau \chi_{ab;cd} E_b B_c B_d$

Planar Hall effect in **quasi**-2D systems

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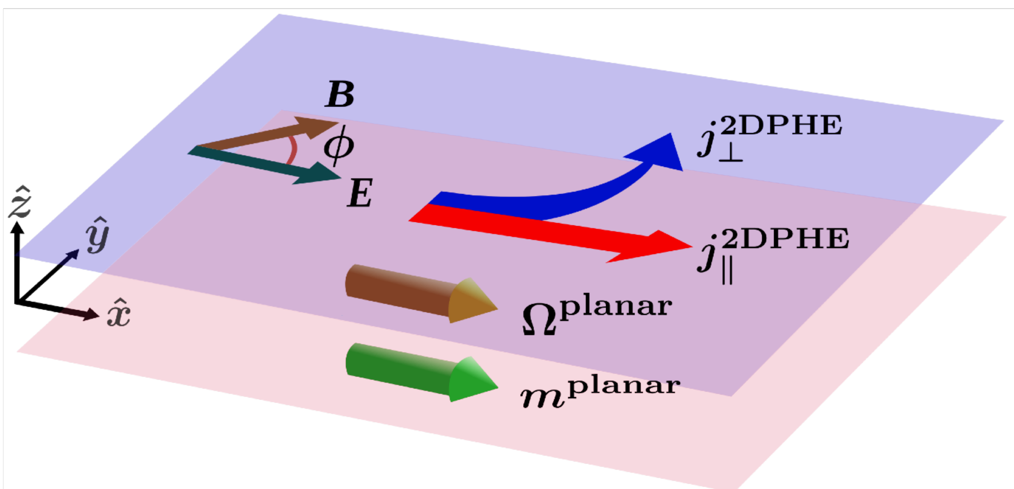


$$\sigma_{\|} = \tau B(\chi_{xx;x}\cos\phi + \chi_{xx;y}\sin\phi) + \tau B^2(\chi_{xx;xx}\cos^2\phi + \chi_{xx;yy}\sin^2\phi + \chi_{xx;xy}\sin\phi\cos\phi)\ , \quad (9)$$

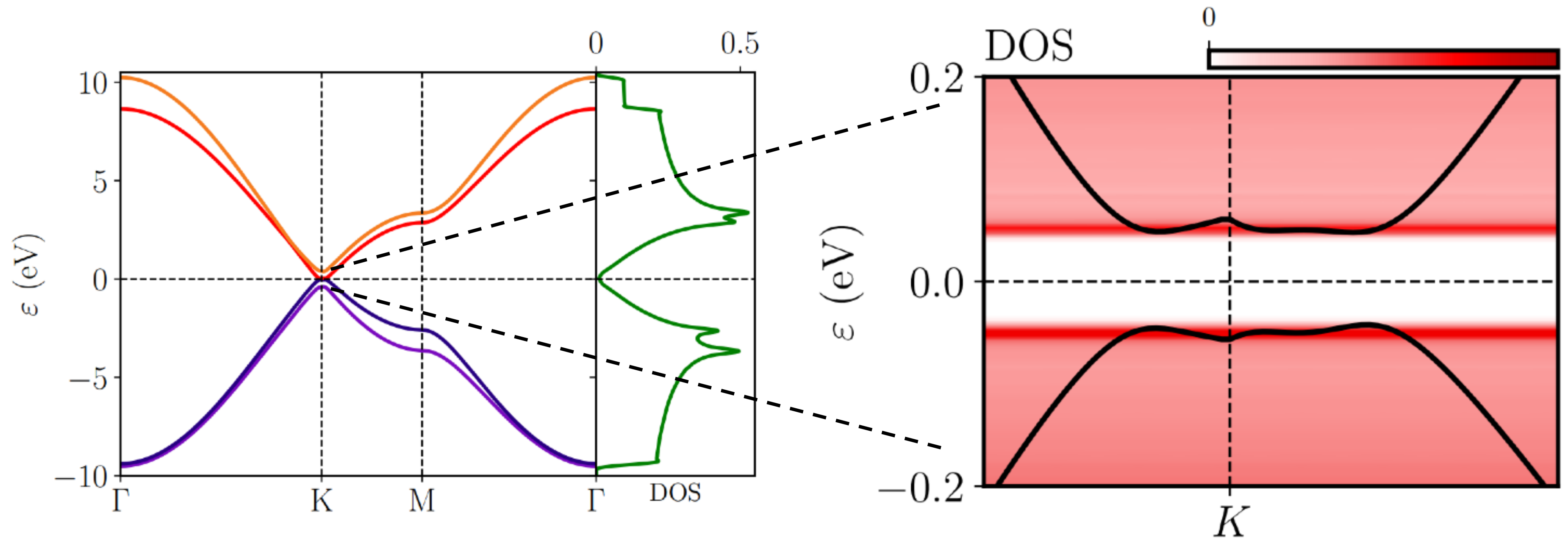
$$\sigma_{\perp} = \tau B(\chi_{yx;x}\cos\phi + \chi_{yx;y}\sin\phi) + \tau B^2(\chi_{yx;xx}\cos^2\phi + \chi_{yx;yy}\sin^2\phi + \chi_{yx;xy}\sin\phi\cos\phi)\ . \quad (10)$$

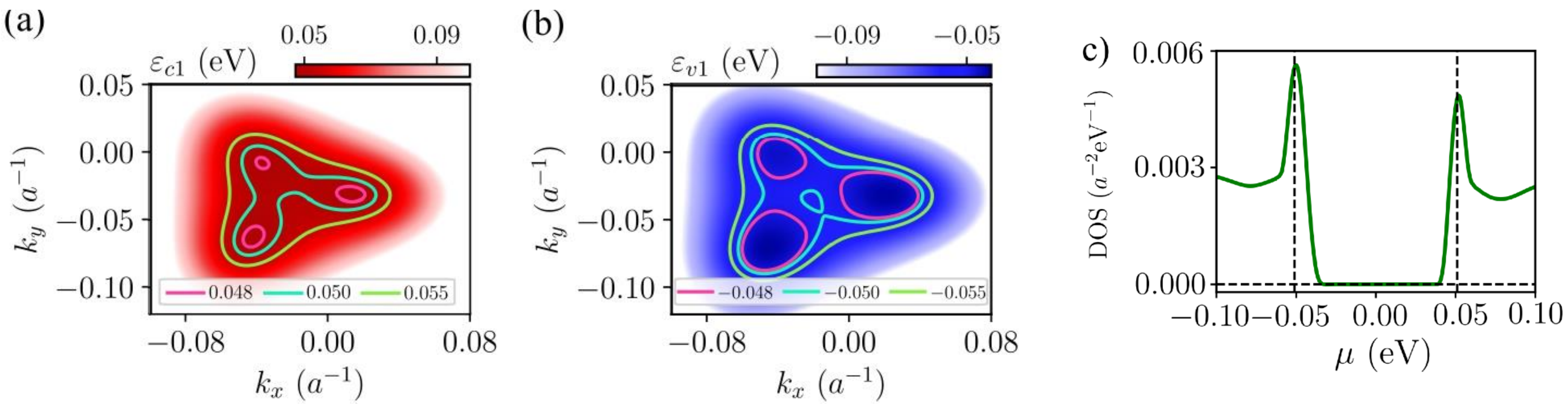
B-linear planar Hall conductivity vanishes in $\mathcal{T}$-symmetric systems.

Finiteness of $\chi_{xx;xy}$, $\chi_{yx;xx}$, $\chi_{yx;yy}$ requires broken $\mathcal{M}_x$,$\mathcal{M}_y$ symmetries.

Longitudinal	Transverse	$\mathcal{P}$	$\mathcal{T}$	$\mathcal{M}_x$	$\mathcal{M}_y$	$\mathcal{C}_{3z}$
$\chi_{xx;x}$	$\chi_{yx;y}$	✓	✗	✓	✗	✓
$\chi_{xx;y}$	$\chi_{yx;x}$	✓	✗	✗	✓	✓
$\chi_{xx;xx}$, $\chi_{xx;yy}$	$\chi_{yx;xy}$	✓	✓	✓	✓	✓
$\chi_{xx;xy}$	$\chi_{yx;xx}$, $\chi_{yx;yy}$	✓	✓	✗	✗	✓

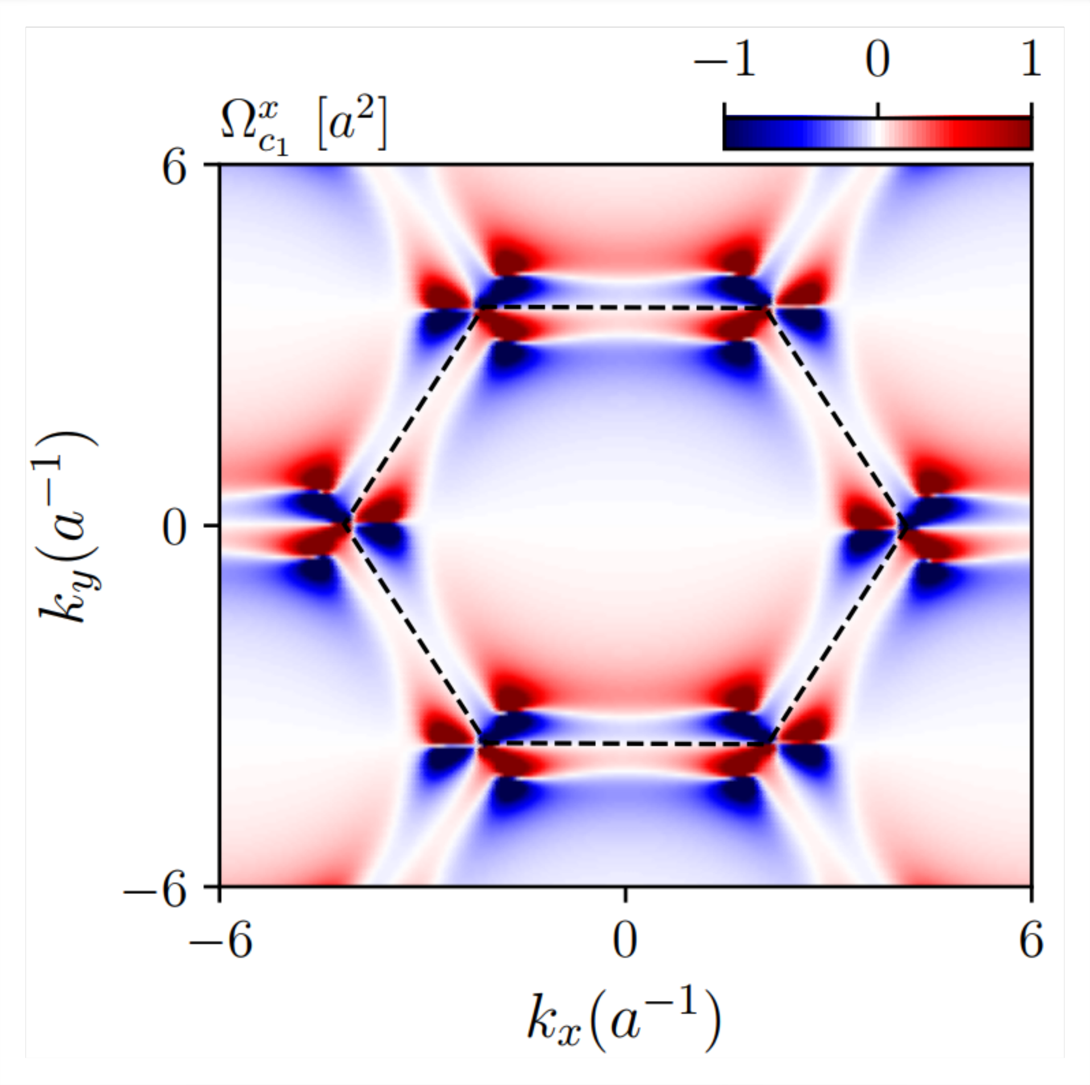
2D-Planar Hall effect in bilayer graphene

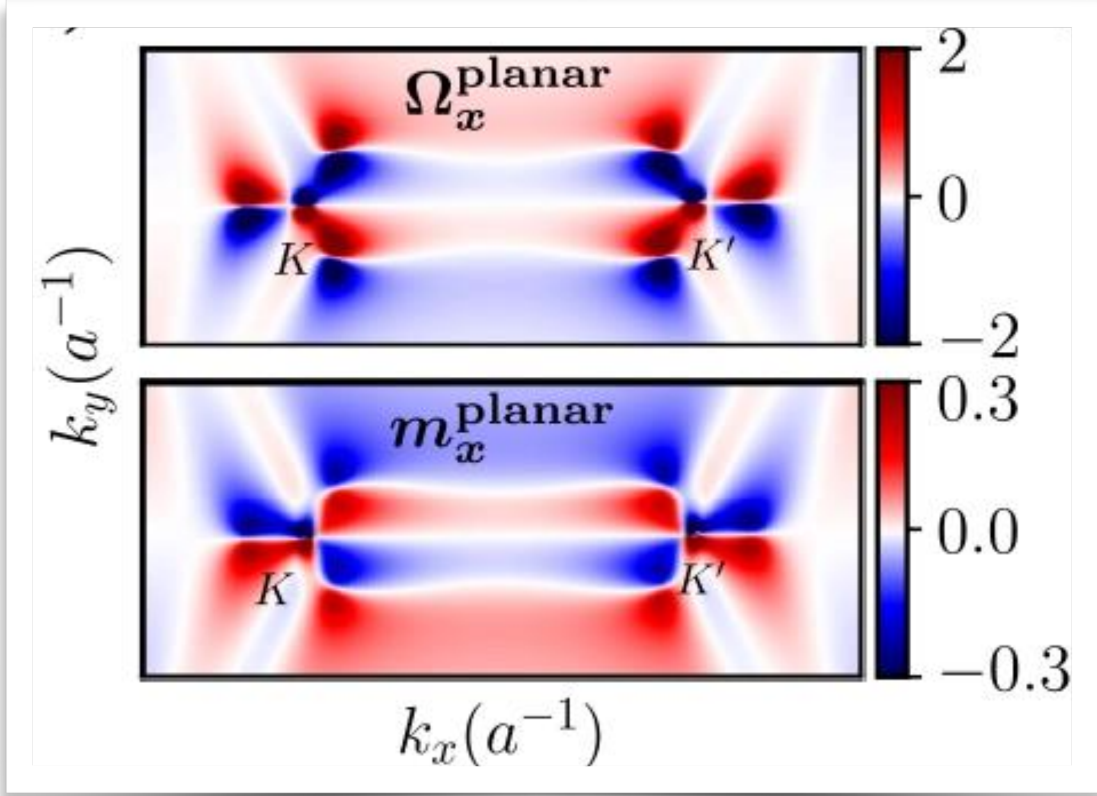




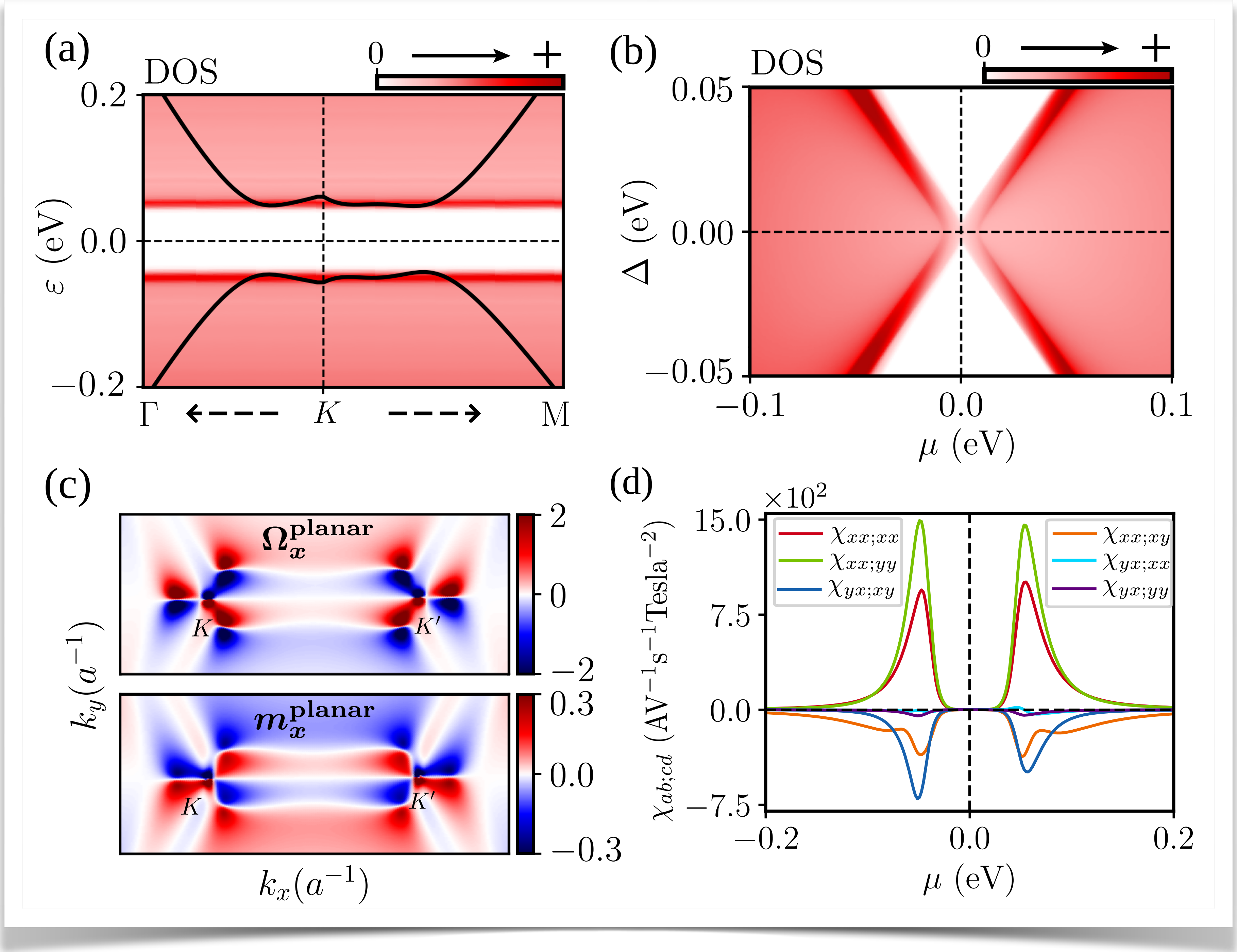
2D-Planar Hall effect in bilayer graphene

Planar BC and OMM in Bilayer graphene

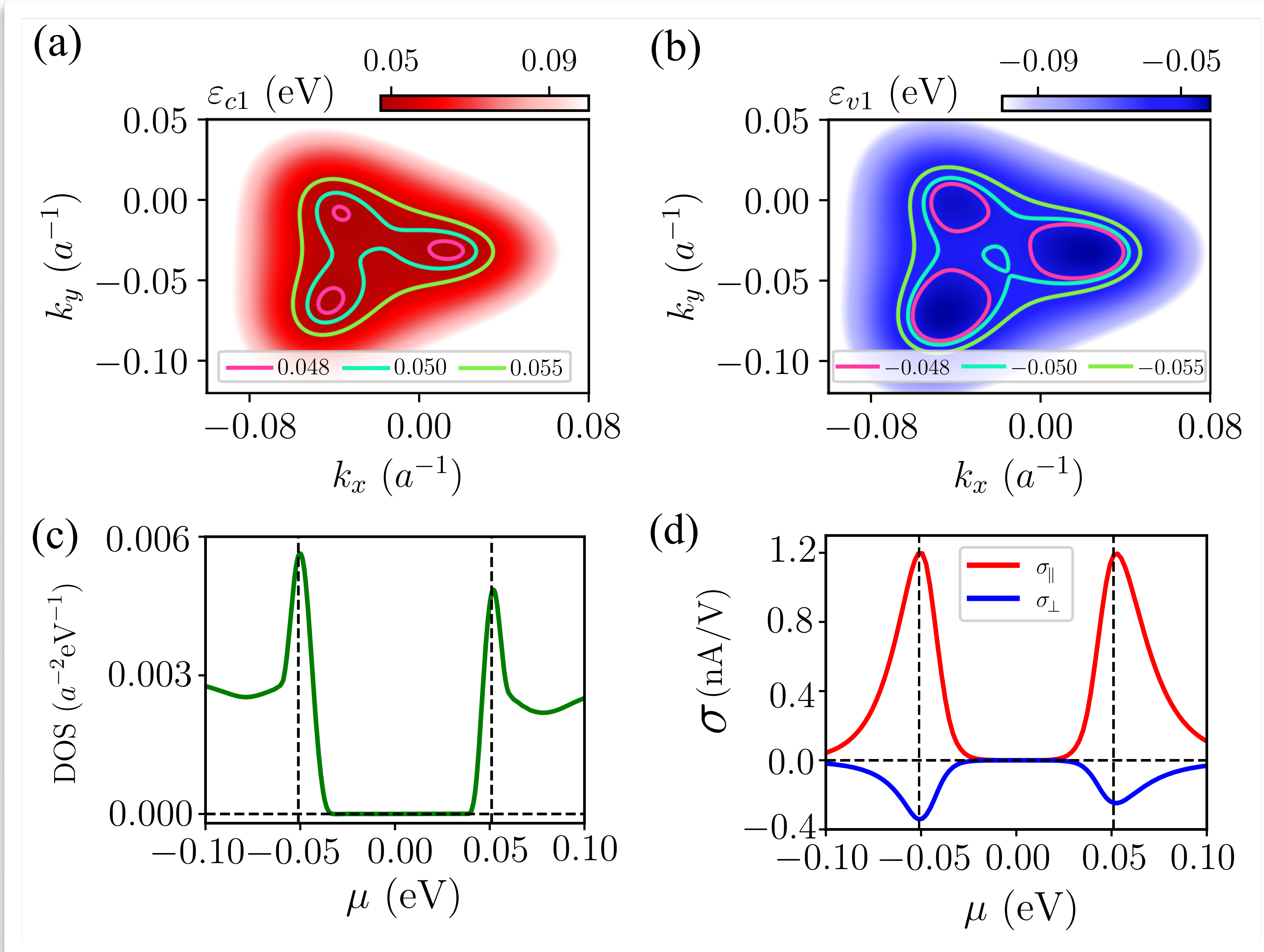




2D-Planar Hall effect in bilayer graphene



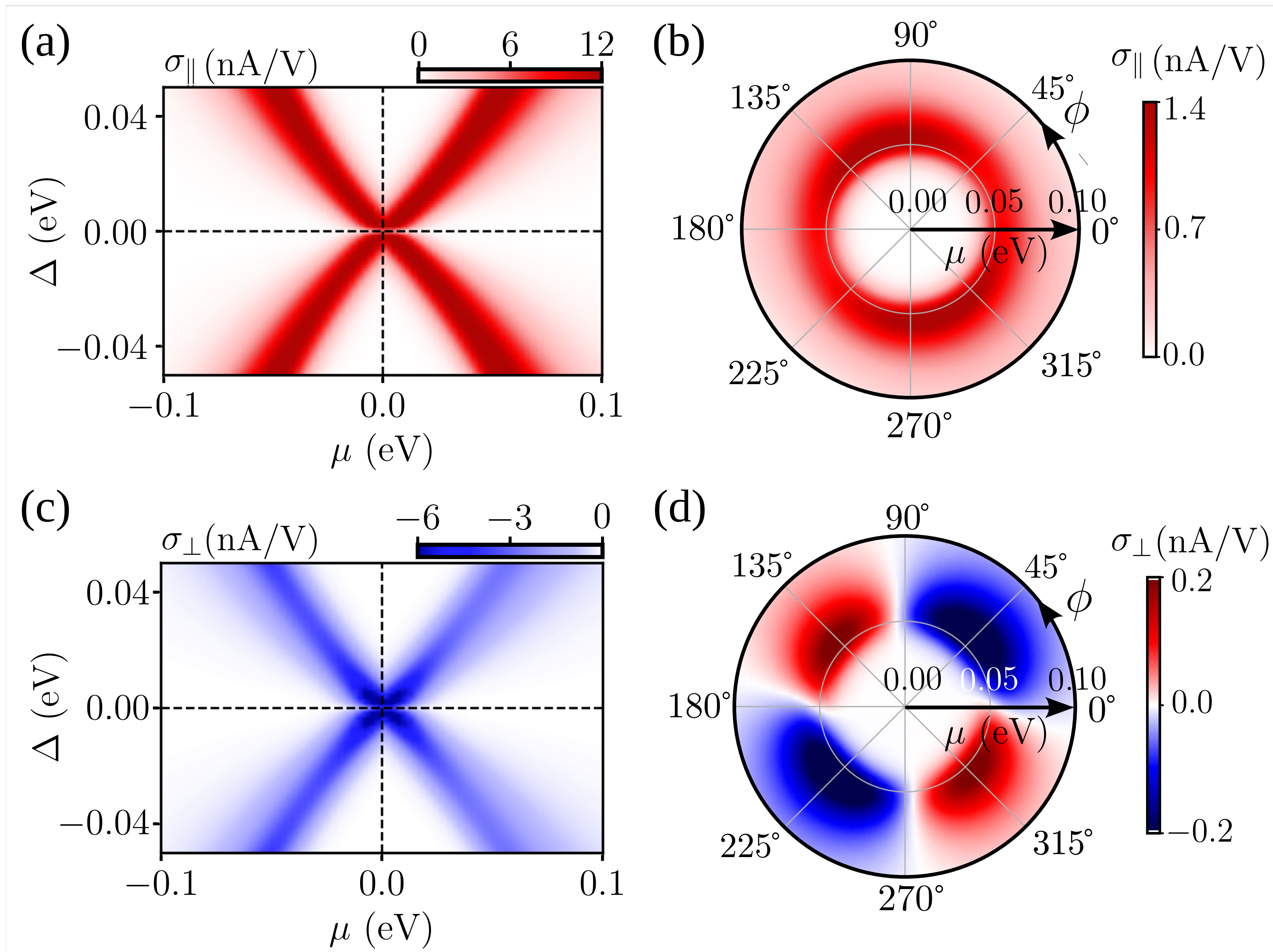
2D-Planar Hall effect in bilayer graphene



2D-Planar Hall effect in bilayer graphene

$$\sigma_{\parallel} = \tau B(\chi_{xx;x}\cos\phi + \chi_{xx;y}\sin\phi) + \tau B^2(\chi_{xx;xx}\cos^2\phi + \chi_{xx;yy}\sin^2\phi + \chi_{xx;xy}\sin\phi\cos\phi)\ , \tag{9}$$

$$\sigma_{\perp} = \tau B(\chi_{yx;x}\cos\phi + \chi_{yx;y}\sin\phi) + \tau B^2(\chi_{yx;xx}\cos^2\phi + \chi_{yx;yy}\sin^2\phi + \chi_{yx;xy}\sin\phi\cos\phi)\ . \tag{10}$$



Variation of 2DPHE with doping, displacement field, and angle

2D Planar Hall effect: Possibilities

Planar Nernst, and planar Seebeck effect in 2D systems..

What about planar spin Hall effect in 2D systems?

What about planar spin-thermoelectric effect in 2D systems?

What about similar physics in bosonic systems?

What about in-plane quantum metric and its implications?