Planar and nonlinear Hall transport in quasi-2D systems

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Berry curvature induced transport and optical phenomena

[Linear and non-linear Hall effects, quantum anomalies, new transport/optical effects…]

New collective modes in bulk and on surface

[Chiral plasmons, new collective modes in bulk, surface plasmons]

First principle based exploration of novel quantum materials [Topological materials, 2D materials, topology + FE, Magnetism…, transport, excitons]

Novel electronic devices

[Negative Capacitance FET, 2D transistors..]

Geometrical properties of electron wave-packets

- One of the most interesting ideas in Quantum Mechanics
- Became prominent around 1984, 57 years after the discovery of QM

Wave-function moving around the Brillouin zone generates an irreducible phase - the Berry phase

$$
\Phi_{\text{Berry}} = \oint \Omega_{\mathbf{k}} \cdot \hat{n} \, dk_x dk_y
$$

'Magnetic field' in momentum space

Connected to 'degenerate band crossing' or 'band inversion' points in the BZ

$$
\Phi_{\text{Magnetic Flux}} = \oint \mathbf{B} \cdot \hat{n} \, dxdy
$$

Magnetic field in real space

Connected to 'magnetic monopoles' and circulating currents in real space

Gives rise to very interesting geometrical phenomena in quantum materials

Curvature in Brillouin zone

Topological properties Novel surface/edge states **Anomalous velocity** $(E \times \Omega_{\mathbf{k}})$

Modified Eq. of motion Novel transport phenomena (AHE, NAHE….)

Orbital magnetization (Finite size wavepacket) Valley polarization OMM induced Hall effect

Quantum geometry of Bloch electrons -> Transport

Translation and self-rotation of a finite width electron wave packet with Bloch electron dynamics

Geometrical properties: **Berry connection, Berry curvature, orbital magnetic moment, quantum metric, metric connection**

These couple to external perturbations (E, B, T-gradient) in interesting ways. Generate new phenomena in transport and optical experiments.

Boltzmann Transport theory | Quantum Kinetic theory

Eq. of motion, 1 band picture Density matrix, multi-band picture

Berry Curvature => Novel transport phenomena ics of the center of the carrier wave-packet (location at \sim

| Annual management | |
|---|--------------------------|
| Annualous velocity | |
| [Anomalous Hall, valley Hall effect] | [chiral magnetic effect] |
| $\dot{\mathbf{r}} = D_{\mathbf{k}} \left(\tilde{\mathbf{v}}_{\mathbf{k}} + \frac{e}{\hbar} (\mathbf{E} \times \mathbf{\Omega}_{\mathbf{k}}) + \frac{e}{\hbar} (\tilde{\mathbf{v}}_{\mathbf{k}} \cdot \mathbf{\Omega}_{\mathbf{k}}) \mathbf{B} \right)$ | |
| $\dot{\mathbf{r}} = D_{\mathbf{k}} \left(-e\mathbf{E} - e(\tilde{\mathbf{v}}_{\mathbf{k}} \times \mathbf{B}) - \frac{e^2}{\hbar} (\mathbf{E} \cdot \mathbf{B}) \mathbf{\Omega}_{\mathbf{k}} \right)$ | |
| $D_{\mathbf{k}} \equiv [1 + \frac{e}{\hbar} (\mathbf{B} \cdot \mathbf{\Omega}_{\mathbf{k}})]^{-1}$ | |

BC modifies the semiclassical Eq. of motion **Berry rorce**
Expressed by expression and the international method of the canonisms ~v˜^k = rk✏˜k, where ˜✏^k = ✏^k m^k *·* B is the electronic dispersion modified by the intrinsic orbital magnetic moof wave packets in crystals [Magneto-electric coupling, BC modifies the semiclassical Eq. of motion of wave packets in crystals

=> induces novel transport effects

megative MR, Planar Hall effect, **Berry force** 'chiral anomaly' like physics]

Chiral magnetic velocity

Playground: the problem of transport conductivity

Modelling electrical conductivity in transport measurements

Non-interacting physics Many body physics

Disorder

Disorder

Hall effect without magnetic field: Beyond the linear regime

In case we do not have an anomalous Hall or symmetric (Drude) Hall response in the linear response regime

What is the dominant Hall response in the non-linear regime Are there novel longitudinal responses beyond nonlinear Drude?

Problem of second order nonlinear DC conductivity?

Full problem => Band geometry + Disorder (Symm + Asymm) + strong correlation effects

Partial solution => Band geometry + symmetric scattering effects

Letter

Kamal Das ,

Intrinsic nonlinear conductivities induced by the quantum metric

PHYSICAL REVIEW B **108**, L201405 (2023)

Kamal Das [•],^{1,2,*,†} Shibalik Lahiri,^{1,3,*,‡} Rhonald Burgos Atencia,^{4,5,§} Dimitrie Culcer,^{4,5,∥} and Amit Agarwal [•]^{1,¶}

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1,2,*,† Shibalik Lahiri,1,3,*,‡ Rhonald Burgos Atencia,4,5,§ Dimitrie Culcer,4,5,[∥] and Amit Agarwal 1,¶

Nonlinear Zero B Hall effect: Second order Responses

Correction in the distribution function =>

Non-linear Drude response

(Second order correction in the distribution function)

Berry Curvature Dipole

(First order correction in the distribution function + Anomalous velocity)

QM induced velocity as the origin of BCPD current:—

stant, we have also the absolute the absolute temperature and *interest in the complete interest in the complete control in the complete in* the complete order of the complete order of the complete order of the complete or

E-induced correction in the Berry curvature and anomalous velocity + Intrinsic or unperturbed distribution function order distribution function *f*² = *e*²⌧ ²@*b*@*cfEbE^c* where density matrix as a sum of four parts \mathcal{C} four parts \mathcal{C} four parts \mathcal{C} and **interpretent in the original and the o**rturbed distribution function **E** ruibed distribution function. nduced correction in the Berry curvature and anomalous velocity ing non-equilibrium distribution function function \pm Intrinsic or unperturbed distribution funct band gradient velocity (*v*BG

Berry connection polarizability (quantum metric) induced Hall response

Quantum geometry of Bloch electrons - I A*mn* ⌘ *Rmn* = *i*h *^m*(k)*|*r^k *ⁿ*(k)i

Distance between nearby states => quantum geometric tensor θ

$$
ds^{2} = \langle \psi(\mathbf{k} + d\mathbf{k}) - \psi(\mathbf{k}) | \psi(\mathbf{k} + d\mathbf{k}) - \psi(\mathbf{k}) \rangle = \left\langle \frac{\partial \psi(\mathbf{k})}{\partial k_{a}} \middle| \frac{\partial \psi(\mathbf{k})}{\partial_{k_{b}}} \right\rangle \, dk_{a} dk_{b}
$$

 $a/b \Rightarrow$ coordinate axis

All topological indexes => the BZ integrals of functions of Berry curvature m^k = h (k)*|*r ⇥ v*|* (k)i

Topological materials => Novel surface states with interesting properties \mathcal{A} . Semiclassical transport with Berry curvature and orbital magnetization \mathcal{A}

BC/QM => New transport and optical phenomena (AHE, valley HE, NL Hall)

Quantum geometry of Bloch electrons - II the Nth order term in the NTH order term in the density matrix, provide term in the density matrix, ρ ðkarendi er þeir er þeir að framstæði.
Dæmi , is series of the electric field strength, ^jðtÞ ¼ ^P^N ^j ns − 11 \Box Quantum geometry of Bloch electrons . chemical potential potential potential network **Quantum geometry of Bloch electrons - II**

From single band picture <=> multi band picture Explicitly calculating the SH current for a d dimensional related to the quantum geometric properties of the wave From single band picture <=> multi band pictu for the state $\frac{1}{\sqrt{2\pi}}$

related to the quantum geometric properties of the wave

Band resolved band geometric quantities nd res \overline{a} τ ¼ l han $\mathop{\mathsf {band}}$ geometric qua Band resolved band geometric quantities $2S$ contributions to the peaks stem from Fermi surface terms

Herry connection

 $\mathcal{R}_{mp}(\bm{k})=i\langle u^{m}_{\bm{k}}|\partial_{\bm{k}}u^{p}_{\bm{k}}\rangle$

Up to linear order in the external field strength, the external field strength, the external field strength, t
Contract strength, the external field strength, the external field strength, the external field strength, the Geometric tensor Quantum Metric Berry curvature under the exchange of the last two indices. This is achieved via the relation, ^σabc ^¼ ^σacb ^¼ [−]1=ð2πÞ^dðe³=ℏ²^Þ

111. TV
1

 \mathbf{r}

corresponding dc counterparts [21], we have denoted the a dobrie a de poem and the Sh conductives are defined to be seen and the Sh conductivity of the Sh conducti Ω uantum Matric Ω orny curvature

 $\mathcal{L}_{nn}^b = i[\mathcal{R}_{nm}^c, \mathcal{R}_{mn}^b]$ μ $\lbrack \quad \rho m, \quad m\rho \rbrack$ $\{b_{mp}\}/2$ $\Omega_{mp}^{cb} = i[\mathcal{R}_{pm}^c, \mathcal{R}_{mp}^b]$ $_{mp}^{cb}=i[\mathcal{R}_{pm}^{c},\mathcal{R}_{m}^{b}]$ $\genfrac{}{}{0pt}{}{b}{mp}$

$$
\mathcal{Q}^{cb}_m = \sum_{p \neq m} \mathcal{Q}^{cb}_{mp}
$$

$$
\mathcal{Q}_{mp}^{cb}=\mathcal{R}_{mp}^c\mathcal{R}_{pm}^b \qquad \quad \mathcal{G}_{m_l}^{cb}
$$

and symplectic connection $\mathcal{L}^{\mathcal{L}}$ terms as Ra $\mathcal{L}^{\mathcal{L}}$

and symplectic connection (\overline{C}) terms as Ra \overline{C} terms as Ra \overline{C}

Geometric tensor
\n
$$
Q_{mp}^{cb} = \mathcal{R}_{mp}^{c} \mathcal{R}_{pm}^{b}
$$
 $Q_{mp}^{cb} = \{\mathcal{R}_{pm}^{c}, \mathcal{R}_{mp}^{b}\}/2$ $Q_{mp}^{cb} = i[\mathcal{R}_{pm}^{c}, \mathcal{R}_{mp}^{b}]$

Nonlinear Zero B Hall effect: Second order Responses

A natural question

Is that the complete story?

Or, are there missing terms?

How do we capture all possible nonlinear terms systematically, on the same footing even in a simple relaxation time approximation?

Quantum Kinetic theory: Non-linear transport/optics \mathbf{A} . Seminal transport with Berry curvature and orbital magnetization \mathbf{A}

F field – matter interaction: $\mathcal{H}_{\text{int}}(\mathbf{k},t) = \mathcal{H}_0(\mathbf{k}) + U + e \mathbf{r} \cdot \mathbf{E}(t)$ ction: $\mathcal{H}_{\text{int}}(\boldsymbol{k},t) = \mathcal{H}_0(\boldsymbol{k}) + U + e \boldsymbol{r} \cdot \boldsymbol{E}(t)$

$$
\text{Velocity operator:} \qquad \mathbf{v} = \dot{\mathbf{r}} = \frac{1}{i\hbar} [\mathbf{r}, \mathcal{H}_{int}]
$$

Current expectation value:

$$
\mathbf{J} = \text{Trace}[\mathbf{v}\rho]
$$

dt ⁺

is given by? ?

Evolution of density matrix:
$$
\frac{d\rho(\mathbf{k},t)}{dt} + \frac{i}{\hbar}[\mathcal{H}_{\text{int}}(\mathbf{k},t),\rho(\mathbf{k},t)] = 0
$$

Im [h*n|*@*^kaH|n*⁰

|@*^kbH|n*i]

|@*^kbH|n*i]

ih*n*⁰

|@*^kbH|n*i]

|@*^kbH|n*i]

(✏*ⁿ* ✏*ⁿ*0)² *,* (5)

*H*int(*k, t*) = *H*0(*k*) + *U* + *er · E*(*t*) (1)

- \overline{a} asis states of H₀ **.**
Work in the basis states of H₀ ⁼ ² Im [h*n|*@*^kaH|n*⁰
- r operator => Berry connection
Ford for the orbital magnetic moment tensor or similar magnetic moment tensor moment tensor moment tensor and ten (✏*ⁿ* ✏*ⁿ*0)² *,* (5) (✏*ⁿ* ✏*ⁿ*0)² *,* (5) • 'r' operator => Berry connection
- ~ ovariant aerivative **production in the set of the set of** where *n* is the band index with *i*ndex with *H*_{*n*}i connection

Similar moment is the original moment of the orbital moment of the is given by? ? **1.** EOM of density matrix => Covariant derivative • EOM of density matrix => Covariant derivative

Nonlinear Zero B Hall effect: Second order Responses

+ Asymmetric scattering contributions (side jump and skew scattering)

Berry curvature dipole induced Hall conductivity

follows:

Hall effect in time-reversal symmetric systems

O Induced by Berry curvature dipole

Inversion symmetry breaking + asymmetry of band structure is essential

20 NOVEMBER 2015

in Time-Reversal Invariant Materials

PRL 115, 216806 (2015) The presence of the factor only guarantee that only guarantee that $\frac{1}{2}$ and $\frac{1}{2}$ and $\frac{1}{2}$ and $\frac{1}{2}$ and $\frac{1}{2}$ and $\frac{1}{2}$ and $\frac{1}{2}$ and

PRL 115, 216806 (2015) PHYSICAL REVIEW LETTERS week ending

Quantum Nonlinear Hall Effect Induced by Berry Curvature Dipole in Time-Reversal Invariant Materials \Box $\frac{1}{2}$ in this reversal invariant in the low temperature is response in the so that th

Inti Sodemann and Liang Fu Department of Physics, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139, USA a Fermi Liquid property and Liquid property and Liquid property as \mathbb{R}^n .

It is well as nonversion that a nonversion that a nonversion time-reversion time- U_{eff} van van die diponstrate that Hall-like cond-order response to the second-order response to the Berry curvature dipole **Berry curvature** dipole

$$
\chi_{abc} = -\varepsilon_{adc} \frac{e^3 \tau}{2(1 + i\omega \tau)} \int_k f_0(\partial_b \Omega_d).
$$

Berry curvature dipole induced Hall conductivity

Topological phase transitions are hard to detect (valley Chern type)

=> We demonstrate that the BCD changes sign across the transition

=> NL Hall measurement can detect topological phase transitions

Intrinsic: Berry curvature polarizability Hall current nolarizability Hall curront **intrinsic contribution to the second-order AHE is most contribution to the second-order AHE is most contributio**
The second-order AHE is most contribution to the second-order and contribution to the second-order and contr

DOI: 10.1103.1112.112.112.112.1666.112.1666.112.1666.112.16660 => Second order intrinsic Hall response [9–14]. Moreover, its resulting field correction to the Berry $-$ This induces corrections in An. velocity $B_{\rm{C}}$ tensor $\Gamma_{\rm{C}}$ tensor $\Gamma_{\rm{C}}$ and with index n, BCP can be a band with index

as an anomalous velocity and a modification of the phasesystems wnere the Be $t = t$ modynamic and transport properties $\frac{1}{2}$. ric systams (hinartita) orbital magnetic moment, the Berry curvature provides the essential ingredient for a full theory of the electron response \mathbf{f} rems (arapne $\mathcal{F}_{\mathcal{F}}$ response functions such as electric polarizations such as $\mathcal{F}_{\mathcal{F}}$ or magnetic field. The electric nonlinear anomalous Hall cy curvature vanishes electric polarizability and require the system of the system to have both the t_1 spatial inversion symmetry breaking. The symmetry breaking. The symmetry breaking. The symmetry breaking. The symmetry t_1 murer romagnets) hall effect o symmetry restrictions, and it competes with the ordinary Anm ^a ^ðkÞAmn ^b ðkÞ For example in (PT) symmetric systems (bipartite antiferromagnets) allou recnance alley response semiclassical theory also provides straightforward methods stems where the Berry curvature orpar me annie
ne) => NL valley m≠n response This corrections is significant in systems where the Berry curvature vanishes in P and T symmetric systems (graphene) => NL valley response This corrections is sionificant in systems where the Rerry cury transport response in the DC limit. The DC limits of the DC limits of the Bordes entire terms of the two contributions of conductivity arises from the second-order correction to the second-order correction to the second-order correct
The second-order correction to the second-order correction to the second-order correction to the second-order

Intrinsic: Berry curvature polarizability Hall current $\frac{1}{2}$

Article

Quantum-metric-induced nonlinear transport in a topological antiferromagnet

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Naizhou Wang¹, Daniel Kaplan², Zhaowei Zhang¹, Tobias Holder², Ning Cao³, Aifeng Wang³, Xiaoyuan Zhou³, Feifei Zhou¹, Zhengzhi Jiang¹, Chusheng Zhang¹, Shihao Ru¹, Hongbing Cai^{1,4}, Kenji Watanabe⁵, Takashi Taniguchi⁶, Binghai Yan^{2⊠} & Weibo Gao^{1,4,7⊠}

Nonlinear BCP Hall ${\bf E}$ $g_{\mathbf{k}} \propto f$ $v^{\rm BCPH}_a \propto \, \sum\limits_\text{ } \left[2 \partial_a \tilde{\mathcal{G}}^{bc}_{mp} - \partial_b \tilde{\mathcal{G}}^{ac}_{mp} - \partial_c \tilde{\mathcal{G}}^{ab}_{mp} \right] E_b E_c$

Recent observation

order correction to the distribution function function function function and the anoma-

RESEARCH

RESEARCH ARTICLE

TOPOLOGICAL MATTER

Quantum metric nonlinear Hall effect in a topological antiferromagnetic heterostructure \mathbf{r} and \mathbf{r} and \mathbf{r} and \mathbf{r}

Anyuan Gao¹, Yu-Fei Liu^{1,2}, Jian-Xiang Qiu¹, Barun Ghosh³, Thaís V. Trevisan^{4,5}, Yugo Onishi⁶, Chaowei Hu⁷,
Tiema Qian⁷, Hung-Ju Tien⁸, Shao-Wen Chen², Menggi Huang⁹, Damien Bérubé¹, Houchen Li¹, Christian Tzschaschel¹, Thao Dinh^{1,2}, Zhe Sun^{1,10}, Sheng-Chin Ho¹, Shang-Wei Lien⁸, Bahadur Singh¹¹, Kenji Watanabe¹², Takashi Taniguchi¹², David C. Bell^{13,14}, Hsin Lin¹⁵, Tay-Rong Chang^{8,16,17}, Chunhui Rita Du⁹, Arun Bansil³, Liang Fu⁶, Ni Ni⁷, Peter P. Orth^{4,5}, Oiong Ma^{10,18}, Su-Yang Xu^{1*}

Intrinsic: Quantum metric dipole induced current $\mathbf{B} = \mathbf{B} \mathbf{B} + \mathbf{B$ metric dipole induced current metric o

Letter

Kamal Das ,

New NL intrinsic response: longitudinal as well as transverse

> Interband coherence effects in longitudinal transport *^a* + *j*int*,*od *^a* + *j*int*,*oo *^a* . The corresponding

Not very well understood from the semiclassical calculation framework

Tilted massive Dirac system:— We choose tilted PHYSICAL REVIEW B **108**, L201405 (2023)

Dirac system as it o⊥erent insights into different into different into different into different into different
Different into different into different into different into different into different into different into diffe \mathbf{R} increase noninear conductivities made by the quantum metric **Intrinsic nonlinear conductivities induced by the quantum metric**

Dirac system, and *PT* preserving antiferromagnets.

Kamal Das \bullet , ^{1,2,*,†} Shibalik Lahiri, ^{1,3,*,‡} Rhonald Burgos Atencia, ^{4,5,§} Dimitrie Culcer, ^{4,5,∥} and Amit Agarwal \bullet ^{1,¶}

¹*Department of Physics, Indian Institute of Technology Kanpur, Kanpur 208016, India* ²*Department of Condensed Matter Physics, Weizmann Institute of Science, Rehovot 7610001, Israel*

$$
\mathcal{H} = v_F(k_x \sigma_y - k_y \sigma_x) + v_t k_y \sigma_0 + \Delta \sigma_z
$$

1,2,*,† Shibalik Lahiri,1,3,*,‡ Rhonald Burgos Atencia,4,5,§ Dimitrie Culcer,4,5,[∥] and Amit Agarwal 1,¶

Nonlinear Zero B Hall effect: Second order Responses

+ Asymmetric scattering contributions (side jump and skew scattering)

Planar Hall Effect in Quasi-Two-Dimensional Materials

Koushik Ghorai,[∗] Sunit Das,[∗] Harsh Varshney, and Amit Agarwal† *Department of Physics, Indian Institute of Technology Kanpur, Kanpur-208016, India*

The planar Hall e μ ect in 3D systems is an e μ ective probe for the intervals of their Berry curvature, topology, to

Lorentz Hall effect Planar Hall effect

Transverse Hall voltage generated by orthogonal electric and magnetic field

Lorentz force

In-plane transverse voltage in presence of in-plane electric and magnetic field

> **Chiral magnetic velocity + Berry force**

Chiral magnetic velocity and Berry force ics of the center of the center of the center of the center of the carrier wave-packet (location at the center of the center

| Annual management | |
|---|--------------------------|
| Annualous velocity | |
| [Anomalous Hall, valley Hall effect] | [chiral magnetic effect] |
| $\dot{\mathbf{r}} = D_{\mathbf{k}} \left(\tilde{\mathbf{v}}_{\mathbf{k}} + \frac{e}{\hbar} (\mathbf{E} \times \mathbf{\Omega}_{\mathbf{k}}) + \frac{e}{\hbar} (\tilde{\mathbf{v}}_{\mathbf{k}} \cdot \mathbf{\Omega}_{\mathbf{k}}) \mathbf{B} \right)$ | |
| $\dot{\mathbf{r}} = D_{\mathbf{k}} \left(-e\mathbf{E} - e(\tilde{\mathbf{v}}_{\mathbf{k}} \times \mathbf{B}) - \frac{e^2}{\hbar} (\mathbf{E} \cdot \mathbf{B}) \mathbf{\Omega}_{\mathbf{k}} \right)$ | |
| $D_{\mathbf{k}} \equiv [1 + \frac{e}{\hbar} (\mathbf{B} \cdot \mathbf{\Omega}_{\mathbf{k}})]^{-1}$ | |

BC modifies the semiclassical Eq. of motion **Berry rorce**
Expressed by expression and the international method of the canonisms ~v˜^k = rk✏˜k, where ˜✏^k = ✏^k m^k *·* B is the electronic dispersion modified by the intrinsic orbital magnetic moof wave packets in crystals [Magneto-electric coupling, BC modifies the semiclassical Eq. of motion of wave packets in crystals

=> induces novel transport effects

megative MR, Planar Hall effect, **Berry force** 'chiral anomaly' like physics]

Chiral magnetic velocity

Electronic current $\mathbf{j}^e = -e$ Z $\left[\frac{d\mathbf{k}}{D}^{-1} \mathbf{\dot{r}} \mathbf{g_k}\right]$

Theory of Planar Hall effect in 3D systems

 ${\bf v}_{\lambda}\cdot {\bf \Omega}_{\lambda}\neq 0.$

No Berry curvature induced Planar Hall effect

Possibility of finite planar BC in 2D systems

2D systems with two or more atomic layers + Interlayer tunnelling of electrons Planar Berry curvature and OMM

=> Finite inplane Berry Curvature and Orbital magnetic moment

$$
\Omega_{nk}^{\text{planar}} = 2\hbar \sum_{n' \neq n} \frac{\text{Re}(v_{nn'} \times \mathcal{Z}_{n'n})}{(\varepsilon_{nk} - \varepsilon_{n'k})}, \qquad m_{nk}^{\text{planar}} = e \sum_{n' \neq n} \text{Re}(v_{nn'} \times \mathcal{Z}_{n'n}).
$$
\n
$$
\mathcal{Z}_{nk}^{\text{planar}} = e \sum_{n' \neq n} \text{Re}(v_{nn'} \times \mathcal{Z}_{n'n}).
$$
\n
$$
\mathcal{Z}_{n'n} = \hat{z} \langle u_{n'k} | \mathcal{Z} | u_{nk} \rangle
$$
\n
$$
j^{\text{planar}} = -e^2 \tau \int_{nk} \mathcal{D}_k \tilde{v}_k (\tilde{v}_k \frac{\hat{z}_k}{\hat{z}_k}) \underbrace{\partial_{\tilde{z}} \tilde{f}_0 \stackrel{\mathbf{B}}{=} \mathcal{C}_{\tilde{h}}^3}_{\tilde{h}_k \text{ Qplanar} \tilde{h}^{\text{QPhuar}}}
$$
\n
$$
- \frac{e^3 \tau}{\hbar} (\mathbf{E} \cdot \mathbf{B}) \int_{nk} \mathcal{D}_k \tilde{v}_k (\tilde{v}_k \cdot \Omega_k) \frac{\partial_{\tilde{z}} \tilde{f}_0}{\partial_{\tilde{z}} \tilde{f}_0} - \frac{e^4 \tau}{\hbar^2} (\mathbf{E} \cdot \mathbf{B}) \mathbf{B} \int_{nk} \mathcal{D}_k (\tilde{v}_k \cdot \Omega_k) \frac{\partial_{\tilde{z}} \tilde{f}_0}{\partial_{\tilde{z}} \tilde{f}_0}.
$$

 $\iota\boldsymbol{k}$

 $^3\tau$

Origin of finite planar BC in 2D systems

Conventional Berry Curvature:

$$
\Omega_{n\mathbf{k}} = i\hbar^2 \sum_{n' \neq n} \frac{v_{nn'} \times v_{n'n}}{(\varepsilon_{n\mathbf{k}} - \varepsilon_{n'\mathbf{k}})^2}
$$

Redefine out-of-plane velocity matrix element:

 $v_{n'n}^z = (1/\hbar) \langle u_{n'k} | \partial_{k_z} H_{k} | u_{n k} \rangle \equiv (1/\hbar) \langle u_{n'k} | i[H_{k}, Z] | u_{n k} \rangle = (i/\hbar) (\varepsilon_{n'k} - \varepsilon_{n k}) Z_{n'n}$

Interlayer tunnelling can give rise to out of plane velocity matrix

Planar Berry curvature in quasi-2D systems

$$
\Omega_{n\mathbf{k}}^{\text{planar}} = 2\hbar \text{ Re} \sum_{n' \neq n} \frac{v_{nn'} \times Z_{n'n}}{(\varepsilon_{n\mathbf{k}} - \varepsilon_{n'\mathbf{k}})}
$$

Similarly, planar orbital magnetic moment:

$$
\boxed{\boldsymbol{m}_{n\boldsymbol{k}}^{\textrm{planar}}=e\;\textrm{Re}\sum_{n'\neq n}\boldsymbol{v}_{nn'}\times\boldsymbol{Z}_{n'n}}
$$

Origin of finite planar BC in 2D systems

During dimensional reduction, the helical texture of in-plane BC is T_A and T_A transport of $\frac{1}{100}$ preserved for polar BiAg2 monolayer (Phys. Rev. B 104, L081114)

Planar Hall effect in **quasi**-2D systems

Planar Current: $j_a = \tau \chi_{ab;c} E_b B_c + \tau \chi_{ab;cd} E_b B_c B_d$

Planar Hall effect in quasi-2D systems obtain the longitudinal and transverse 2DPHE currents: tween the *E* and *B* is important for exploring its oriin quasi-cu sysiems.

j2DPHERE 1999 PREMIE 1999 PREMIE 1999
Premier 1999
Premier 1999 Premier 1999 Premie

Planar Current: $j_a = \tau \chi_{ab;c} E_b B_c + \tau \chi_{ab;cd} E_b B_c B_d$

$$
\sigma_{\parallel} = \tau B(\chi_{xx;x} \cos \phi + \chi_{xx;y} \sin \phi) + \tau B^2(\chi_{xx;xx} \cos^2 \phi + \chi_{xx;yy} \sin^2 \phi + \chi_{xx;xy} \sin \phi \cos \phi), \qquad (9)
$$

work with the field configuration described in Fig. 1 to

$$
\sigma_{\perp} = \tau B(\chi_{yx;x} \cos \phi + \chi_{yx;y} \sin \phi) + \tau B^2(\chi_{yx;xx} \cos^2 \phi + \chi_{yx;yy} \sin^2 \phi + \chi_{yx;xy} \sin \phi \cos \phi).
$$
 (10)

(and Table S1 in SM [33]) provide a complete character-

These equations and the symmetry restrictions in Table I

? ⁼ ⌧*B*(*yx*;*^x* cos ⁺ *yx*;*^y* sin) + ⌧*B*²(*yx*;*xx* cos²

tems with an in-plane mirror or an in-plane two-fold ro-fold ro-fold ro-fold ro-fold ro-fold ro-fold ro-fold rotty vanisnes in *f*-symmetric systems. ity vanishes in τ -symmetric systems. $t_{\rm c}$ with an in-plane mirror or an in-plane two-fold ro-fold ro-f

is entirely captured by *xx*;*xx*, and *xx*;*yy* (*yx*;*xy*) with yy requires broken \mathcal{M}_x , \mathcal{M}_y symmetries. $\sum_{i=1}^{n}$ is equal to $\sum_{i=1}^{n}$ (*x*;*x*;*y*, *x*;*y* (*y*) with $\sum_{i=1}^{n}$

2D-Planar Hall effect in bilayer graphene

2D-Planar Hall effect in bilayer graphene graphene, only the *B2* planar response

Planar BC and OMM in Bilayer graphene

2D-Planar Hall effect in bilayer graphene

2D-Planar Hall effect in bilayer graphene

2D-Planar Hall effect in bilayer graphene work with the field configuration described in Fig. 1 to obtain the longitudinal and the longitudinal and the longitudinal and transverse 2DPH currents in the longitud $\mathcal{H}_{\mathcal{A}}$ is a vertical displace-displace-displace-displace-displace-displace-displace-displace-displace-displace-displacement in bilaver araphen **Post in priceyor graphent** $\mathbf{C} = \mathbf{C} \cdot \mathbf{C} \cdot \mathbf{C}$ ^k ⁼ ⌧*B*(*xx*;*^x* cos ⁺ *xx*;*^y* sin) + ⌧*B*²(*xx*;*xx* cos²

$$
\sigma_{\perp} = \tau B(\chi_{yx;x} \cos \phi + \chi_{yx;y} \sin \phi) + \tau B^2(\chi_{yx;xx} \cos^2 \phi + \chi_{yx;yy} \sin^2 \phi + \chi_{yx;xy} \sin \phi \cos \phi).
$$
 (10)

its armchair direction, which we represent by *M^x* (see

Blg, only the party of the Blg, only the Blg, only the Blg, only the 2DPHE in Blg, only the 2DPHE in Blg, only the 2DPHE in Blg, or an and the Blg, or an anti-

inducing a planar-BC and planar-OMM in systems with

Variation of 2DPHE with doping, displacement field, and angle

2D Planar Hall effect: Possibilities

Planar Nernst, and planar Seebeck effect in 2D systems..

What about planar spin Hall effect in 2D systems?

What about planar spin-thermoelectric effect in 2D systems?

What about similar physics in bosonic systems?

What about in-plane quantum metric and its implications?