



Application of Particle Filter on Data Assimilation

Prashant Kumar

Scientist-SF Atmospheric Sciences Division Space Applications Centre, ISRO <u>prashant22@sac.isro.gov.in</u>

ICTS Workshop on Data Assimilation in Weather and Climate Models, 16 May 2024

NWP models use most data from met-satellites through Assimilation Today, more than 95% observations for weather prediction are provided by satellites

NWP models use most data from met-satellites through Assimilation Today, more than 95% observations for weather prediction are provided by satellites



ICTS Workshop on Data Assimilation

NWP models use most data from met-satellites through Assimilation Today, more than 95% observations for weather prediction are provided by satellites



NWP models use most data from met-satellites through Assimilation Today, more than 95% observations for weather prediction are provided by satellites







THIS IS ONLY 5-7% OF WHAT SATELLITES OBSERVE !!













Aim of Data Assimilation

Too few and irregular (in space and time) observations, too many model grids.

Goal: To produce a regular, physically consistent best state of the atmosphere or system



(Daley, 1991)

✓ Objective Analysis

Cost of Computation

- ✓ Objective Analysis
- ✓ Optimum Interpolation (OI)

Cost of Computation

- ✓ Objective Analysis
- ✓ Optimum Interpolation (OI)
- ✓ Variation Method: 3 D-Var

4 D-Var

Cost of Computation

- ✓ Objective Analysis
- ✓ Optimum Interpolation (OI)
- ✓ Variation Method: 3 D-Var

4 D-Var

✓ Kalman Filter (e.g. EnKF, LETKF)

Hybrid Methods (e.g. 3D/4DEnVar)

- ✓ Objective Analysis
- ✓ Optimum Interpolation (OI)
- ✓ Variation Method: 3 D-Var

4 D-Var

- ✓ Kalman Filter (e.g. EnKF, LETKF)
- ✓ Particle Filter

Hybrid Methods (e.g. 3D/4DEnVar)



- ✓ Objective Analysis
- ✓ Optimum Interpolation (OI)
- ✓ Variation Method: 3 D-Var

4 D-Var

- ✓ Kalman Filter (e.g. EnKF, LETKF)
- ✓ Particle Filter Hy

 \checkmark

Hybrid Methods (e.g. 3D/4DEnVar)

Modern Era: Beyond Linear & Gaussian Constraints.

ICTS Workshop on Data Assimilation

Sources of errors

Errors in the First Guess

Errors in the model (Numerical Approx., Physics, etc.)

Errors in Observations

Sources of errors

Errors in the First Guess

Errors in the model (Numerical Approx., Physics, etc.)

Errors in Observations

- Limitations of Data Assimilation Techniques
- Errors can be random and/or systematic errors
- Intrinsic predictability limitations

Example 1: Add datatype with unrealistic low *R*.
OSE/OSSE shows negative impact.
Adjoint sensitivity shows this datatype contributes most to forecast error reduction.

(CGMS, 2018)

Example 1: Add datatype with unrealistic low *R*.
OSE/OSSE shows negative impact.
Adjoint sensitivity shows this datatype contributes most to forecast error reduction.

Adjoint sensitivity measures impact in this setup.

CGMS, 2018)

Example 1: Add datatype with unrealistic low R.
OSE/OSSE shows negative impact.
Adjoint sensitivity shows this datatype contributes most to forecast error reduction.

Adjoint sensitivity measures impact in this setup.

Example 2: Add two identical datatypes, first one, then other.

OSE show first has large impact, second small. Adjoint sensitivity shows they have the same impact.

CGMS, 2018)

Example 1: Add datatype with unrealistic low R.
OSE/OSSE shows negative impact.
Adjoint sensitivity shows this datatype contributes most to forecast error reduction.

Adjoint sensitivity measures impact in this setup.

Example 2: Add two identical datatypes, first one, then other.

OSE show first has large impact, second small. Adjoint sensitivity shows they have the same impact. OSEs are sensitive to order of changes.

CGMS, 2018)

Example 1: Add datatype with unrealistic low R.
OSE/OSSE shows negative impact.
Adjoint sensitivity shows this datatype contributes
most to forecast error reduction.

Adjoint sensitivity measures impact in this setup.

Example 2: Add two identical datatypes, first one, then other.

OSE show first has large impact, second small. Adjoint sensitivity shows they have the same impact. OSEs are sensitive to order of changes.

(CGMS, 2018)

Both are correct! But both are open to misinterpretation.

DA method versus Resolution and Prediction Time



Prediction time horizon

Conditional Probability Distribution:

Conditional Probability Distribution:

Background:
$$P(x/x_b) = \frac{1}{(2\Pi)^{n/2}|B|} e^{-\frac{1}{2}\left[\left(x-x_b\right)^T B^{-1}\left(x-x_b\right)\right]}$$

-

Conditional Probability Distribution:

Background:
$$P(x/x_b) = \frac{1}{(2\Pi)^{n/2}|B|} e^{-\frac{1}{2}\left[\left(x-x_b\right)^T B^{-1} \left(x-x_b\right)\right]}$$

Observations: $\mathbf{P}(\mathbf{y}/\mathbf{x}) = \frac{1}{(2\Pi)^{m/2} |R|} e^{\frac{-1}{2} \left[\left(y - Hx_{r} \right)^{T} R^{-1} \left(y - Hx_{r} \right) \right]}$

where m < n

ICTS Workshop on Data Assimilation

Since the above two distribution are mutually exclusive (independent), their joint Probability Distribution is the product of the two Gaussian distribution

Since the above two distribution are mutually exclusive (independent), their joint Probability Distribution is the product of the two Gaussian distribution Joint Probability Distribution $L(x) \propto P(x/x_{b}) * P(y/x)$

Since the above two distribution are mutually exclusive (independent), their joint Probability Distribution is the product of the two Gaussian distribution Joint Probability Distribution $L(x) \propto P(x/x_{\rm b}) * P(y/x)$ Original problem is multiplication!

Since the above two distribution are mutually exclusive (independent), their joint Probability Distribution is the product of the two Gaussian distribution

Joint Probability Distribution

 $L(x) \propto P(x/x_b) * P(y/x)$

Original problem is multiplication !

Bayes Theorem tells that data assimilation is a multiplication problem: given the prior and the likelihood, the solution is the point-wise multiplication of the two.

Cost Function is defined as the M.L. estimate of the product of two exponential as,

$$J(x) = \frac{1}{2}(x - x_b)^T B^{-1}(x - x_b) + \frac{1}{2}(y - Hx)^T R^{-1}(y - Hx)$$

Minimizing the cost function:

$$\partial J / \partial x = 0$$

$$B^{-1}(x - x_b) - H^T R^{-1}(y - Hx) = 0$$

ICTS Workshop on Data Assimilation

Cost Function is defined as the M.L. estimate of the product of two exponential as,

$$J(x) = \frac{1}{2}(x - x_b)^T B^{-1}(x - x_b) + \frac{1}{2}(y - Hx)^T R^{-1}(y - Hx)$$

Minimizing the cost function:

$$\partial J / \partial x = 0$$

$$B^{-1}(x - x_b) - H^T R^{-1}(y - Hx) = 0$$

But, constraints simplify it as Linear optimization problem

Drawbacks of OI, 3D-Var, 4D-Var

- ✓ Background error Covariances (B) update is omitted
- Ideally the B should be flow-dependent (e.g. Error of the day)

Drawbacks of OI, 3D-Var, 4D-Var

- Background error Covariances (B) update is omitted
- Ideally the B should be flow-dependent (e.g. Error of the day)

Kalman Filter (Computationally Expensive)

 Use Model equations to propagate B forward in time.

$$B \longrightarrow B(t)$$

 \checkmark Analysis step as in OI

ICTS Workshop on Data Assimilation

 $\vec{u} \quad \vec{v} \quad \vec{\theta} \quad \vec{p} \quad \vec{q}$ $\vec{u} \quad \mathbf{B}_{uu} \quad \mathbf{B}_{uv} \quad \mathbf{B}_{u\theta} \quad \mathbf{B}_{up} \quad \mathbf{B}_{uq}$ $\vec{v} \quad \mathbf{B}_{uv} \quad \mathbf{B}_{vv} \quad \mathbf{B}_{v\theta} \quad \mathbf{B}_{vp} \quad \mathbf{B}_{vq}$ $\vec{\theta} \quad \mathbf{B}_{u\theta} \quad \mathbf{B}_{v\theta} \quad \mathbf{B}_{\theta\theta} \quad \mathbf{B}_{\theta p} \quad \mathbf{B}_{\theta q}$ $\vec{p} \quad \mathbf{B}_{up} \quad \mathbf{B}_{vp} \quad \mathbf{B}_{\theta p} \quad \mathbf{B}_{pq}$ $\vec{q} \quad \mathbf{B}_{uq} \quad \mathbf{B}_{vq} \quad \mathbf{B}_{\theta q} \quad \mathbf{B}_{pq} \quad \mathbf{B}_{qq}$

Ensemble Kalman Filter (Possible)


Issues in Ensemble Kalman Filter

The success of the EnKF depends on size of the ensembles to give an accurate estimate of the sample mean and covariance.

Issues in Ensemble Kalman Filter

The success of the EnKF depends on size of the ensembles to give an accurate estimate of the sample mean and covariance.

But for large scale problems, ensemble under sampling cause major problems:

Issues in Ensemble Kalman Filter

The success of the EnKF depends on size of the ensembles to give an accurate estimate of the sample mean and covariance.

But for large scale problems, ensemble under sampling cause major problems:

- ✓ underestimated ensemble variance,
- ✓ filter divergence,
- errors in estimated correlations, in particular
 Spurious long-range correlations

ICTS Workshop on Data Assimilation

Use more ensemble members (see Miyoshi et al.
 2014)
 How many ensembles are sufficient ?

 Use more ensemble members (see Miyoshi et al. 2014)
 How many ensembles are sufficient ?
 Who Knows?

1. Use more ensemble members (see Miyoshi et al. 2014)

How many ensembles are sufficient? Who Knows?

2. Localization

- Addresses problem of spurious correlations
- Increases the effective ensemble size

1. Use more ensemble members (see Miyoshi et al. 2014)

How many ensembles are sufficient? Who Knows?

2. Localization

- Addresses problem of spurious correlations
- Increases the effective ensemble size
- 3. Inflation
- Addresses problem of filter divergence

1. Use more ensemble members (see Miyoshi et al. 2014)

How many ensembles are sufficient ? Who Knows?

2. Localization

- Addresses problem of spurious correlations
- Increases the effective ensemble size
- 3. Inflation
- Addresses problem of filter divergence
- 4. Combine ensemble with variational approaches

1. Use more ensemble members (see Miyoshi et al. 2014)

How many ensembles are sufficient ? Who Knows?

2. Localization

- Addresses problem of spurious correlations
- Increases the effective ensemble size

3. Inflation

- Addresses problem of filter divergence
- 4. Combine ensemble with variational approaches
 These are known as hybrid methods (Operational Techniques)

✓ All EnKF methods (~ 30-100) using Gaussian assumption.

✓ All EnKF methods (~ 30-100) using Gaussian assumption.

Is this a valid assumption when the dynamical model is non-linear?

- ✓ All EnKF methods (~ 30-100) using Gaussian assumption.
- Is this a valid assumption when the dynamical model is non-linear?
- ✓ The optimality of the Kalman filters is no longer preserved when applied to non-linear problems.
- \checkmark This is a common issue for all ensemble filters whose analysis step is based on the equations of the Kalman filter.

- ✓ All EnKF methods (~ 30-100) using Gaussian assumption.
- Is this a valid assumption when the dynamical model is non-linear?
- ✓ The optimality of the Kalman filters is no longer preserved when applied to non-linear problems.
- \checkmark This is a common issue for all ensemble filters whose analysis step is based on the equations of the Kalman filter.

\checkmark This is a very hot topic!

And what if the distributions are non-Gaussian?

Fully non-Gaussian data-assimilation methods exist, but none has proven to work for large-scale systems.

And what if the distributions are non-Gaussian?

- Fully non-Gaussian data-assimilation methods exist, but none has proven to work for large-scale systems.
- A few do have potential among which the Particle Filter.

And what if the distributions are non-Gaussian?

- Fully non-Gaussian data-assimilation methods exist, but none has proven to work for large-scale systems.
- A few do have potential among which the Particle Filter.
- It has been shown that the 'curse of dimensionality' has been cured....

How does Gaussianity comes into DA

Is non–Gaussianity relevant in DA

ICTS Workshop on Data Assimilation

How does Gaussianity comes into DA

Reasons for non-Gaussianity: Nonlinear models

If X_n is Gaussian, then $X_{n+1} = \mathcal{M}(X_n) + R_{n+1}$ is generally not Gaussian as soon as \mathcal{M} is nonlinear (whether R is Gaussian or not). Example

 $X_{n+1} = X_n^2 + R_{n+1},$ X_n and R_{n+1} standard Gaussian.



How does Gaussianity comes into DA

Reasons for non–Gaussianity: Nonlinear observations

If X_n is Gaussian and $Y_n = \mathcal{H}(X_n) + S_n$, then $X_n | Y_n$ is generally not Gaussian as soon as \mathcal{H} is nonlinear (whether S is Gaussian or not). Example

 $Y_n = X_n^2 + S_n,$ X_n and S_n standard normal.

Data assimilation: general formulation



ICTS Workshop on Data Assimilation









with
$$w_i = rac{p_d\left(d|\psi_i
ight)}{\sum_i p_d\left(d|\psi_i
ight)}$$
 the

the weights.

No Linear Assumption, Fully Non-linear, No need of TL & AD Model, FG is not change (Balence)

What are these weights?

• The weight w_i is the normalised value of the pdf of the observations given model state \mathcal{X}_i .

What are these weights?

- The weight w_i is the normalised value of the pdf of the observations given model state \mathcal{X}_i .
- For Gaussian distributed variables it is given by:

$$w_i \propto p(y|x_i)$$

$$\propto \exp\left[-\frac{1}{2}\left(y - H(x_i)\right)R^{-1}\left(y - H(x_i)\right)\right]$$

No explicit need for state covariances

• 3DVar and 4DVar need a good error covariance of the prior state estimate: complicated

No explicit need for state covariances

- 3DVar and 4DVar need a good error covariance of the prior state estimate: complicated
- The performance of Ensemble Kalman filters relies on the quality of the sample covariance, forcing artificial inflation and localisation.

No explicit need for state covariances

- 3DVar and 4DVar need a good error covariance of the prior state estimate: complicated
- The performance of Ensemble Kalman filters relies on the quality of the sample covariance, forcing artificial inflation and localisation.
- Particle filter doesn't have this problem, but...

Standard Particle filter



The standard particle filter is degenerate for moderate ensemble size in moderate-dimensional systems.

ICTS Workshop on Data Assimilation

Particle Filter degeneracy: resampling

• With each new set of observations the old weights are multiplied with the new weights.

Particle Filter degeneracy: resampling

- With each new set of observations the old weights are multiplied with the new weights.
- Very soon only one particle has all the weight...

Particle Filter degeneracy: resampling

- With each new set of observations the old weights are multiplied with the new weights.
- Very soon only one particle has all the weight...
- Solution:

Resampling : duplicate high-weight particles, proposal density, etc.

However: degeneracy

• For large-scale problems with lots of observations SIR method is still degenerate:

However: degeneracy

- For large-scale problems with lots of observations SIR method is still degenerate:
- Only a few particles get high weights; the other weights are negligibly small.
However: degeneracy

- For large-scale problems with lots of observations SIR method is still degenerate:
- Only a few particles get high weights; the other weights are negligibly small.
- However, we can enforce almost equal weight for all particles.

However: degeneracy

- For large-scale problems with lots of observations SIR method is still degenerate:
- Only a few particles get high weights; the other weights are negligibly small.
- However, we can enforce almost equal weight for all particles.

Equal-Weight Particle Filter

Frequent Filter Restart &

Local Particle Filter



ICTS Workshop on Data Assimilation

Particle Filter Assimilation of INSAT-3D TIR Channel All-Sky TB

JGR Atmospheres

Research Article 🔂 Free Access

Assimilating INSAT-3D Thermal Infrared Window Imager Observation With the Particle Filter: A Case Study for Vardah Cyclone

Prashant Kumar 🔀, Munn V. Shukla

First published: 28 January 2019 | https://doi.org/10.1029/2018JD028827 | Citations: 17

Generally not use for assimilation in NWP model

Generally not use for assimilation in NWP model

A hybrid data-assimilation method is designed for very severe cyclonic storm "Vardah,"

Generally not use for assimilation in NWP model

A hybrid data-assimilation method is designed for very severe cyclonic storm "Vardah,"

3D-Var method is used to assimilate control observations (Conv., Satellites, etc.)

Generally not use for assimilation in NWP model

A hybrid data-assimilation method is designed for very severe cyclonic storm "Vardah,"

3D-Var method is used to assimilate control observations (Conv., Satellites, etc.)

Particle filter method is used to assimilate all-sky Thermal IR observations from INSAT-3D satellite.

Various particles (or ensembles) are prepared using diverse WRF model physics.

Various particles (or ensembles) are prepared using diverse WRF model physics.

To implement particle filter, INSAT-3D thermal IR window channel 1 (TIR-1; center wavelength 11 μ m) measured brightness temperature (BT) and cloud mask product are used to assign appropriate weights to different particles to reduce model uncertainties.

Various particles (or ensembles) are prepared using diverse WRF model physics.

To implement particle filter, INSAT-3D thermal IR window channel 1 (TIR-1; center wavelength 11 μ m) measured brightness temperature (BT) and cloud mask product are used to assign appropriate weights to different particles to reduce model uncertainties.

This step is followed by **resampling step** in which new particles are generated from high weight particles using stochastic kineticenergy backscatter scheme (SKEBS) method in which dynamical variables are perturbed into the model physics. Schematic of INSAT-3D measured thermal infrared-1 BT and cloud-mask product assimilation using particle filter.



• Particles having higher weights are resampled at the observation time.

- Particles having higher weights are resampled at the observation time.
- In this step, new particles are generated from large weight particles using stochastic kinetic-energy backscatter scheme (SKEBS) to avoid rapid filter degeneracy.

- Particles having higher weights are resampled at the observation time.
- In this step, new particles are generated from large weight particles using stochastic kinetic-energy backscatter scheme (SKEBS) to avoid rapid filter degeneracy.
- The advantage of SKEBS scheme is that it perturbs the dynamic state directly, and perturb dynamical variables feed into the physical parameterizations (model physics).

- Particles having higher weights are resampled at the observation time.
- In this step, new particles are generated from large weight particles using stochastic kinetic-energy backscatter scheme (SKEBS) to avoid rapid filter degeneracy.
- The advantage of SKEBS scheme is that it perturbs the dynamic state directly, and perturb dynamical variables feed into the physical parameterizations (model physics).
- In this way, the total numbers of particles are again same (92 here) at observation time step.

- Particles having higher weights are resampled at the observation time.
- In this step, new particles are generated from large weight particles using stochastic kinetic-energy backscatter scheme (SKEBS) to avoid rapid filter degeneracy.
- The advantage of SKEBS scheme is that it perturbs the dynamic state directly, and perturb dynamical variables feed into the physical parameterizations (model physics).
- In this way, the total numbers of particles are again same (92 here) at observation time step.
- The idea is to focus the particles toward high probability regions of the target pdf, so that the number of particles required for a good approximation of target pdf remains manageable with fewer dimensions as compared to actual model space.



Simulated TC Landfall Error is better than IMD (14.7N, 80.0E) & SCORPIO (15.2N, 80.0E) predicted operational track forecasts from 00 UTC 10Dec2016.



Simulated TC Landfall Error is better than IMD (14.7N, 80.0E) & SCORPIO (15.2N, 80.0E) predicted operational track forecasts from 00 UTC 10Dec2016.

Track of the storm center from WCNT (blue line), WPF (red line) experiments along with IMD observed best track (black line)





Six-hourly track errors in the simulated cyclone track (in kilometers).



Vertical structure of RMSD in (a) humidity, (c) temperature, and (e) wind speed in WCNT experiments when compared with ECMWF analyses and vertical distribution of improvement parameter in (b) humidity, (d) temperature, and (f) wind speed in WPF experiments over WCNT experiments.

Earth and Space Science

Research Article 🔂 Open Access

Assimilation of the Rain Gauge Measurements Using Particle Filter

Prashant Kumar 🔀

First published: 23 September 2020 | https://doi.org/10.1029/2020EA001212 | Citations: 3

- Rainfall forecast from the NWP model is one of the
 most crucial and least accurate parameter compared to
 other parameters, e.g. temperature and humidity.
- ✓ Improving initial condition in precipitating regions is important for advancing the skill of the NWP models.

- Rainfall forecast from the NWP model is one of the
 most crucial and least accurate parameter compared to
 other parameters, e.g. temperature and humidity.
- ✓ Improving initial condition in precipitating regions is important for advancing the skill of the NWP models.
- Assimilation of cloud and precipitation data in the NWP model is very preliminary, and limited to use of clear-sky data only (e.g. IR radiance assimilation).

- Rainfall forecast from the NWP model is one of the
 most crucial and least accurate parameter compared to
 other parameters, e.g. temperature and humidity.
- ✓ Improving initial condition in precipitating regions is important for advancing the skill of the NWP models.
- Assimilation of cloud and precipitation data in the NWP model is very preliminary, and limited to use of clear-sky data only (e.g. IR radiance assimilation).
- Rainfall Assimilation is a direct way to use precipitation observations in the NWP model.

Previous Work on Rainfall Assimilation

- 1) Assimilation of JAXA GSMaP & TRMM 3B42 Rainfall using 4D-Var: Need of appropriate QC (JGR, 2014)
- 2) Assimilation of INSAT-3D HE Rainfall: Impact of real-time rainfall and sensitivity study for Heavy rainfall event, Need of appropriate first guess (QJRMS, 2016)
- 3) Objective: PF Assimilation of IMD observed Rainfall

Assimilation using 4D-Var Method



4D-Var Cost Function $J(x_{0}) = \frac{1}{2} (x_{0} - x_{0}^{b})^{T} [P_{0}^{b}]^{-1} (x_{0} - x_{0}^{b}) + \frac{1}{2} \sum_{t=1}^{n} (y_{t} - H_{t} x_{t})^{T} [R_{t}]^{-1} (y_{t} - H_{t} x_{t})$ 3D-Var \rightarrow 4D-Var: $H \rightarrow HM; H^{T} \rightarrow M^{T}H^{T}$ The solution of 4D-Var is

$$\boldsymbol{x}^{a} = \boldsymbol{x}^{b} + \boldsymbol{B}\boldsymbol{M}^{T}\boldsymbol{H}^{T}\left[\boldsymbol{H}\left(\boldsymbol{M}\boldsymbol{B}\boldsymbol{M}^{T}\right)\boldsymbol{H}^{T} + \boldsymbol{R}\right]^{-1}\left[\boldsymbol{y} - \boldsymbol{H}\boldsymbol{M}\boldsymbol{x}^{b}\right]$$

IMD Rainfall Assimilation using Particle Filter



Mean and median are plotted using dark line and dark dash lines respectively. ICTS Workshop on Data Assimilation

Particle Filter Assimilation (Cont ...)

Individual particles are shown by light blue and red lines. Mean is plotted by dark line.



The Deep Data Assimilation for Geophysical Model

Design of Analog and Deep Forecasting Operators Classical Method (RK4) Truth Analog Analog Forecasting Methods (Lguensat et al. 2017) LSTM Locally_Constant Analogs Successors Locally_Incremental Locally Linear Catalog Deep Forecasting Method (LSTM) t + dt Time Output Layer

<u>Highlights</u>

- A DeepDA method has been proposed in which Deep Learning based sampling of the model dynamics is combined with ensemblebased assimilation techniques.
- The DeepDA method is a non-linear extension of the Analog Data Assimilation (AnDA) method (Lguensat, MWR, 2017).
- Results suggested that the proposed DeepDA is highly computationally efficient with sufficient skill against AnDA method.

A Study towards ML developed Low-Cost NWP Ensembles



Standard deviation with different enseble members for EnKF and PF filters with small and large catalog.



Which method is right for you?

	Var	KF	EnKF	EnVar	Hybrid	PF
Non-Gaussian	X	Х	Х	Х	X	\checkmark
Large system	\checkmark	X	\checkmark	\checkmark	\checkmark	\checkmark
Need info on analysis error	X	\checkmark	\checkmark	X	X	\checkmark
TLM/ adjoint needed	\checkmark	\checkmark	Х	(√X)	(√X)	X
Model expensive to run	\checkmark	\checkmark	\checkmark	\checkmark	\checkmark	X
(no more than 50-100 runs)						
Easily parallelizable	Х	Х	\checkmark	Х	X	\checkmark

NCEO, UK (7-10 May 2024)

Which problem you need to Attack?





MathsPhysicsForecastingDecision(Science)Support

