Perturbativity, Higgs mass FOPT and Gravitational wave with inert models

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based on *EPJC* 80 (2020) 8, 715 and *PRD* 107 (2023) 5, 055032
with Shulpa Janngid

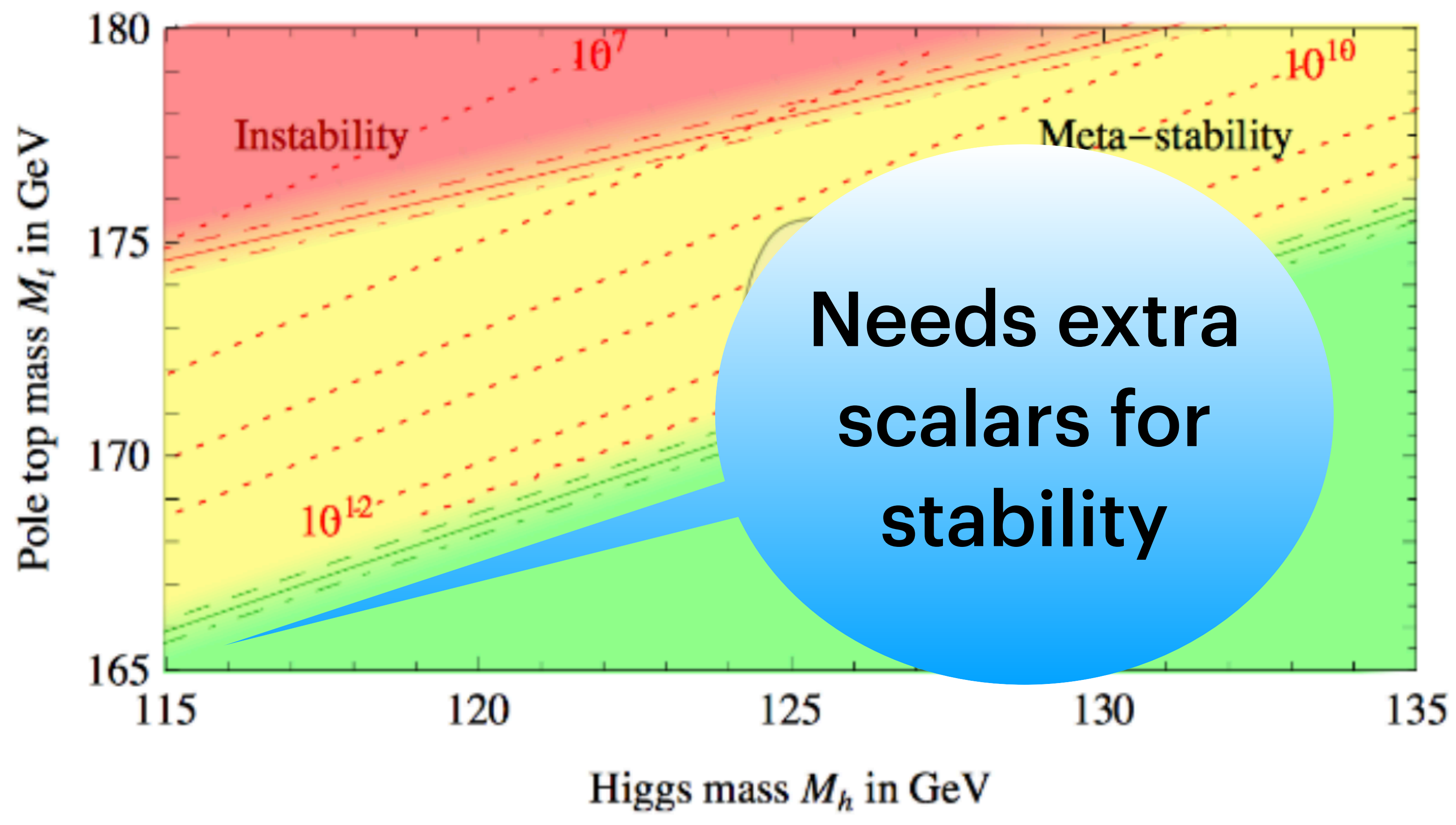
Hearing beyond the standard model with cosmic sources of Gravitational Waves,
ICTS, Bangalore

Plan of my talk

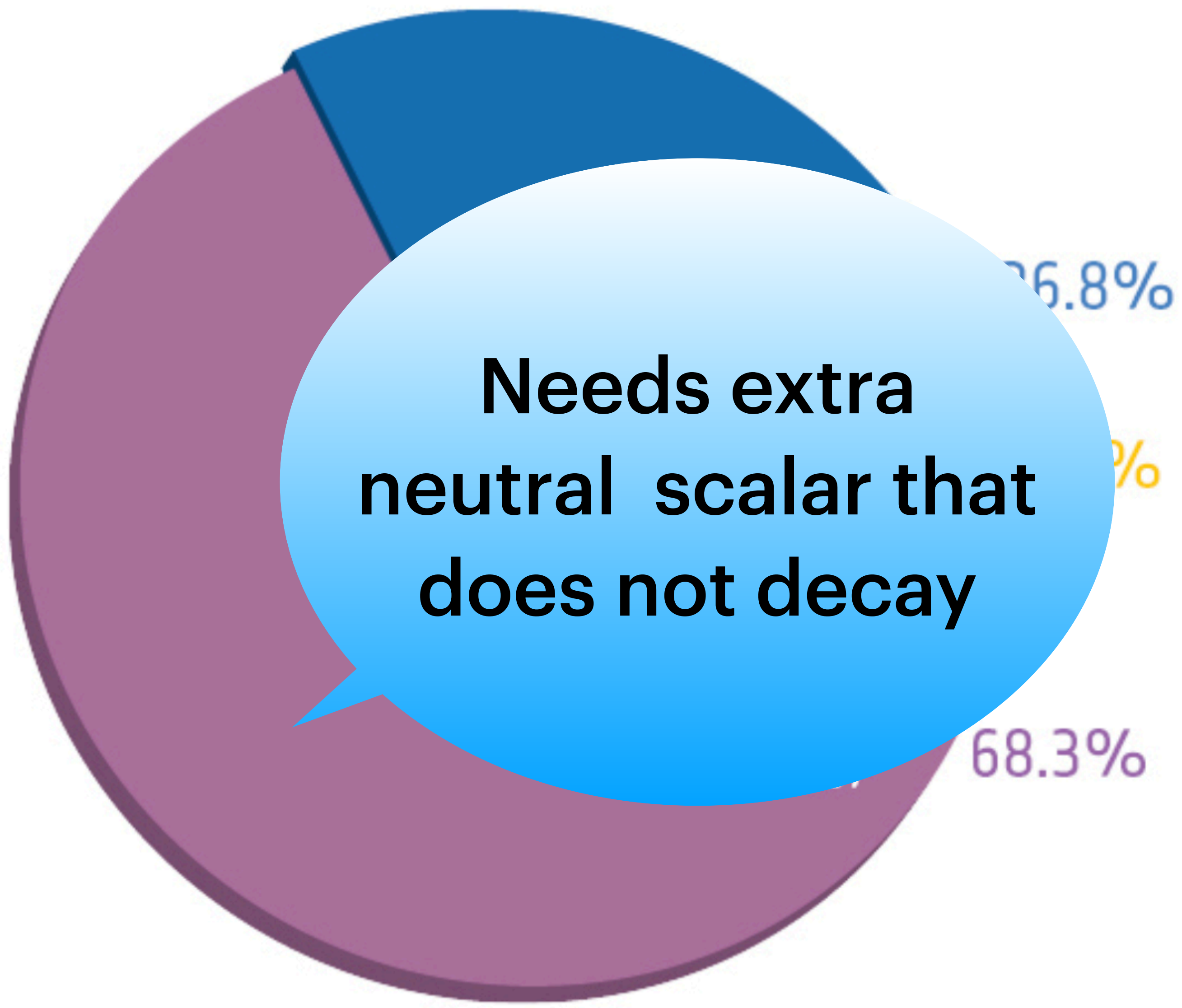
- Vacuum Stability of SM
- Rescue with Inert singlet, doublet and triplet
- Bounds from perturbativity
- First order Phase trannsison in inert singlet and inert triplet
- Upper bounds on bare mass and portal couplings from perturbativity
- Correction from two loop
- Gravitational wave signal
- Bounds from $h \to \gamma\gamma$ measurement

Some basic explanations which may need additional scalar

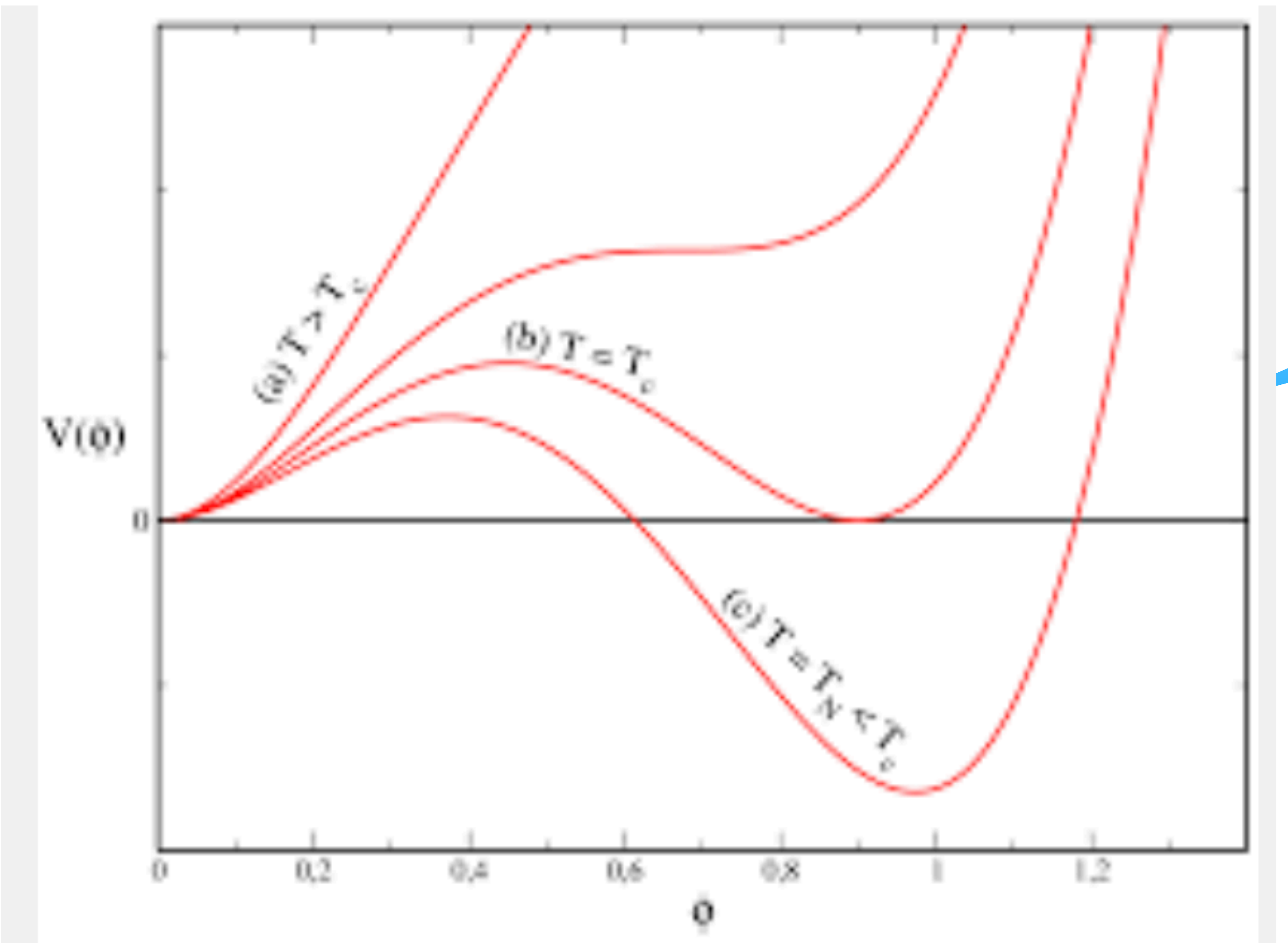
Meta-stable Vacuum in SM



Indirect evidence of Dark Matter



First order Phase Transition



Needs extra scalar

Vacuum Stability

Stability bounds

- Higgs couples to fermions via Yukawa couplings $\mathcal{L}_Y = Y_t \bar{Q} \phi t_R$
- At low field values the top quark contribution is important $\mu \frac{d\lambda}{d\mu} \simeq -\frac{3}{8\pi^2} Y_t^4$
- The solution takes a form, $\lambda(\mu) = \lambda - \frac{3}{8\pi^2} \lambda_t^4 \ln \frac{\mu}{v}$, where at some point we hit $\lambda(\mu) < 0$, leading instability to Higgs potential

$$m_h^2 > \frac{3 m_t^2}{\pi^2 v^2} \ln \frac{\Lambda}{v}$$

- In the Coleman-Weinberg's effective potential approach the RG-improved potential can be written as

$$V_{\rm eff}(h,\mu) \simeq \lambda_{\rm eff}(h,\mu) \frac{h^4}{4}, \quad \text{with } h \gg v\,,$$

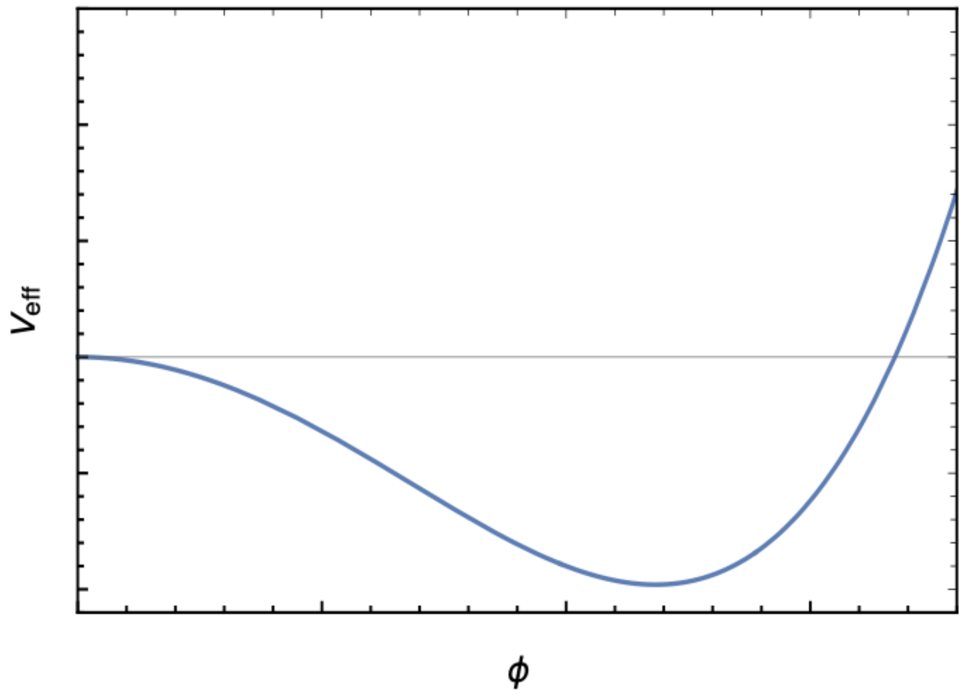
- Where $\lambda_{\rm eff}$ assimilates the loop effects

$$\lambda_{\rm eff}(h,\mu) \simeq \underbrace{\lambda_h(\mu)}_{\text{tree-level}} + \underbrace{\frac{1}{16\pi^2} \sum_{\substack{i=W^\pm,Z,t,\\ h,G^\pm,G^0}} n_i \kappa_i^2 \left[\log \frac{\kappa_i h^2}{\mu^2} - c_i\right]}_{\text{Contribution from SM}} .$$

Stability of the potential

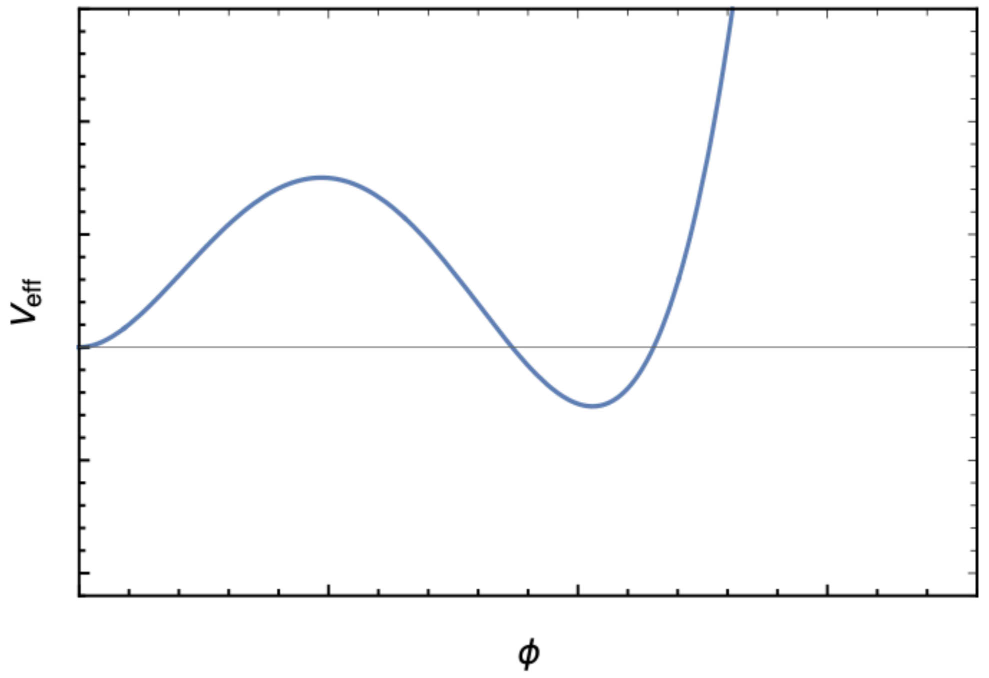
Stability

$$\lambda_{\rm eff} > 0$$



Meta-stability

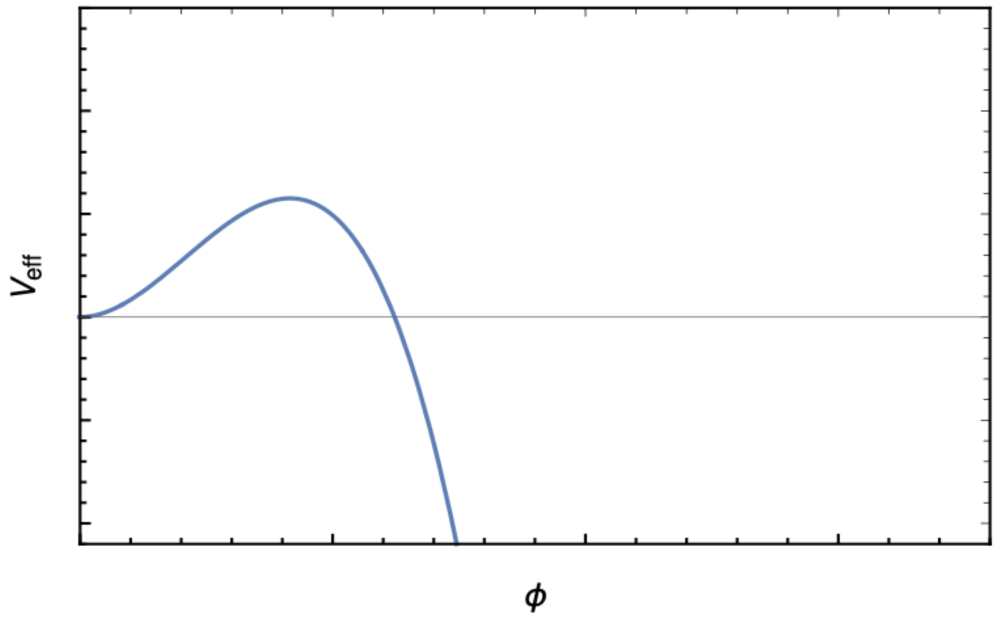
$$0 > \lambda_{\rm eff}(\mu) \gtrsim \frac{-0.065}{1 - 0.01 \log\left(\frac{v}{\mu}\right)}$$

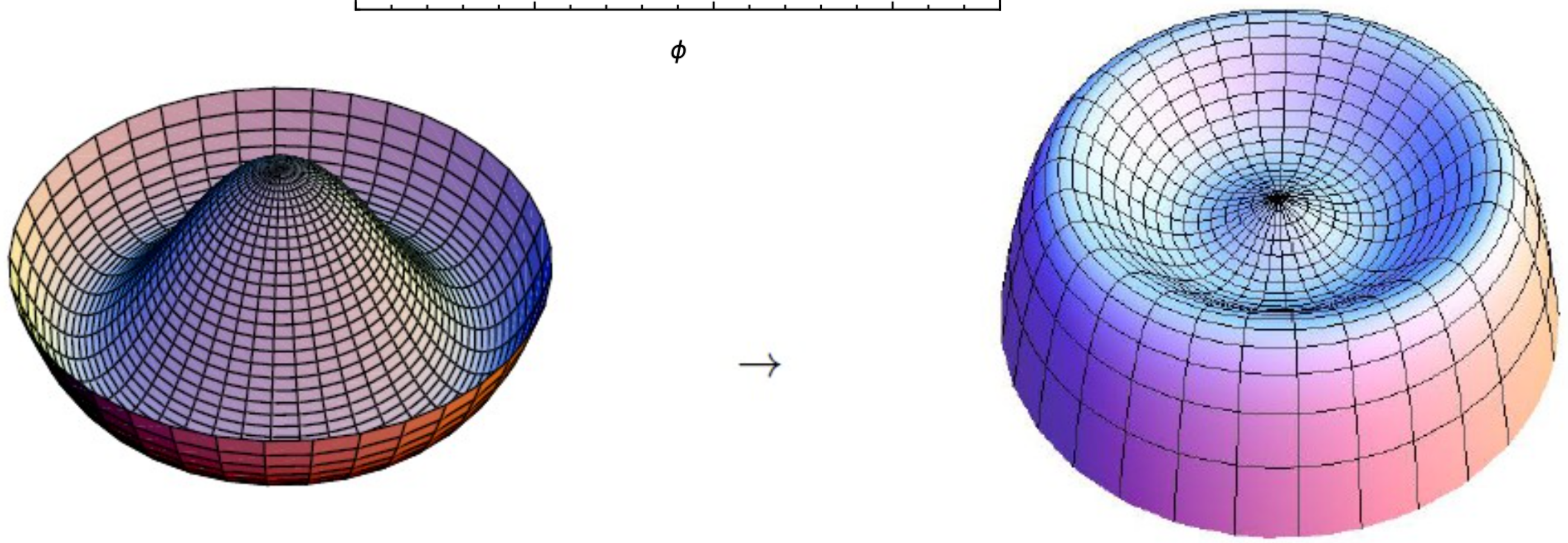


Instability

More negative $\lambda_{\rm eff}$

G. Isidori et. al.: NPB 609 (2001) 387

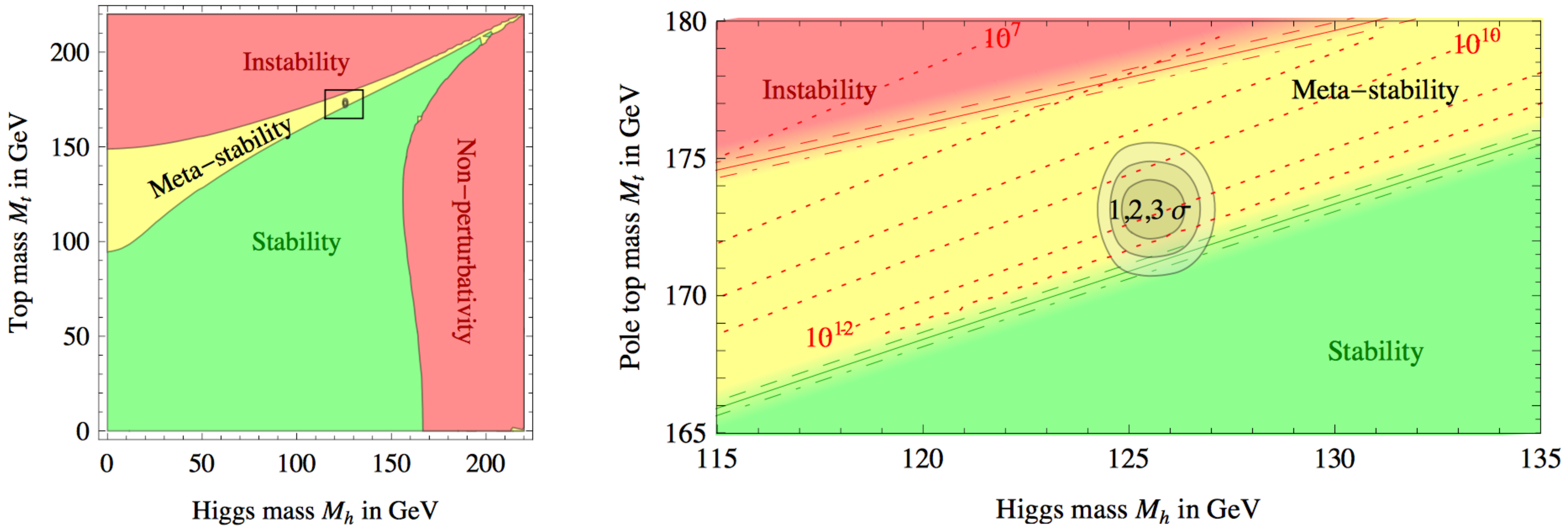




If your mexican hat turns out to be a dog bowl you have a problem...

from A. Strumia

Status of SM



Within the uncertainty of top mass we are in
a metastable vacuum

Degrassi et. al. :JHEP 1208, 098 (2012)

What is the rescue?

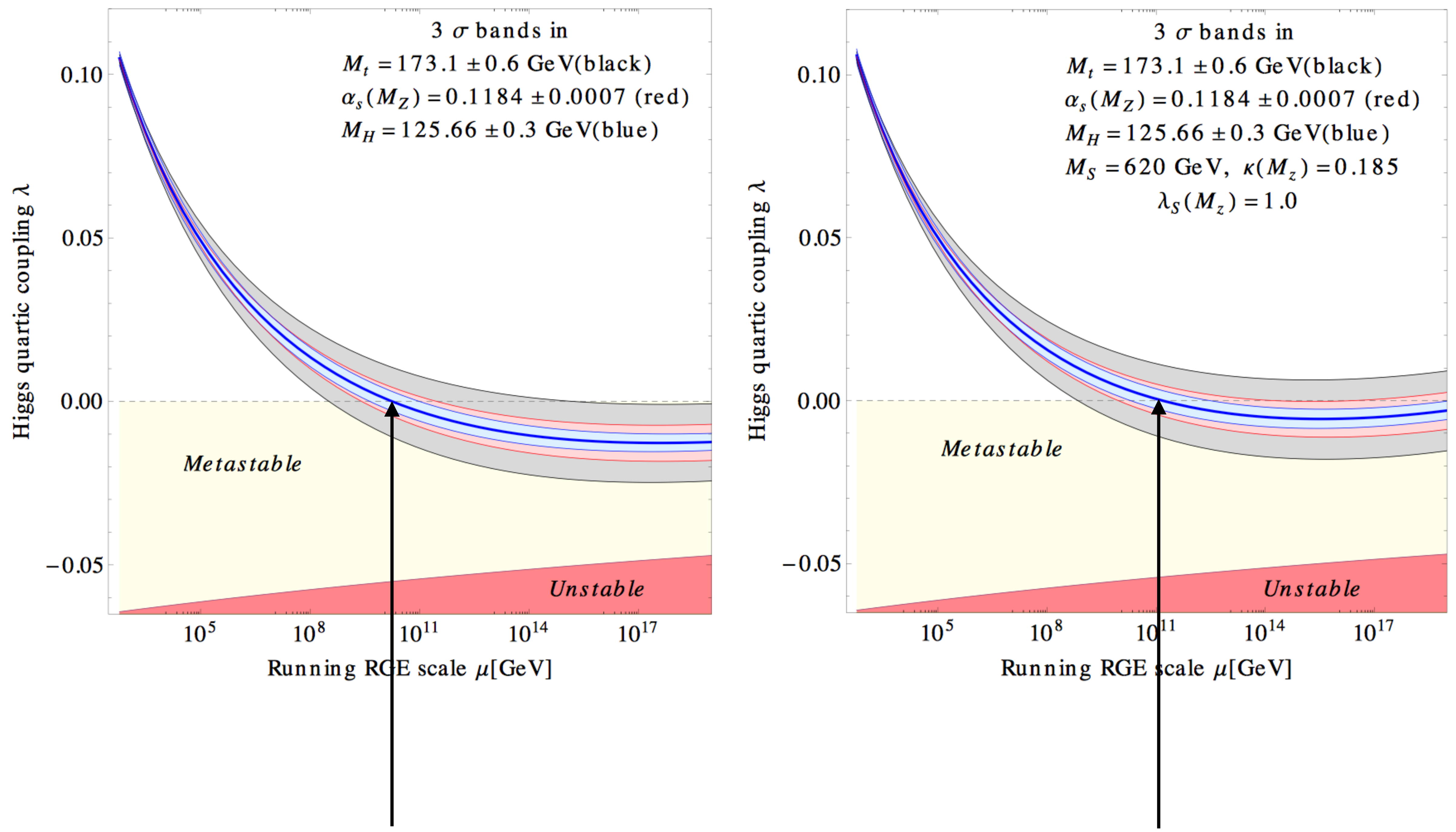
Addition of scalars

- Any scalar extension of SM will enhance the vacuum stability due to positive quantum correction to λ_{eff}
- Singlet extensions are widely studied

Gonderinger et al., Costa et al., Haba et al., Barger et al., Rakshit et al. Baek et al.

SM+ Singlet

$$V(\phi, S) = \mu^2|\phi|^2 + \lambda|\phi|^4 + m_S^2 S^2 + \lambda_{S\phi} S^2|\phi^2| + \lambda_S S^4$$



Khan et al, PRD 90, 113008 (2014)

Cross over region shifted towards higher scale from SM

Addition of scalars: Inert doublet

- We will consider Inert Higgs doublet (Type-I) and Inert Triplet (Y=0) models
- Both the extra SU(2) doublet (Φ_2) and triplet (T) are odd under Z_2 and provide the much needed dark matter candidate
- Inert Higgs doublet:

$$\begin{aligned} \mathrm{V_{scalar}} &= m_{11}^2\Phi_1^\dagger\Phi_1 + m_{22}^2\Phi_2^\dagger\Phi_2 + \lambda_1(\Phi_1^\dagger\Phi_1)^2 + \lambda_2(\Phi_2^\dagger\Phi_2)^2 + \lambda_3(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_2) \\ &+\lambda_4(\Phi_1^\dagger\Phi_2)(\Phi_2^\dagger\Phi_1) + [\lambda_5((\Phi_1^\dagger\Phi_2)^2) + h.c], \end{aligned}$$

$$\Phi_1 = \begin{pmatrix} \phi_1^+ \\ \phi_1^0 \end{pmatrix}, \qquad \Phi_2 = \begin{pmatrix} \phi_2^+ \\ \phi_2^0 \end{pmatrix}$$

← Φ_2 is Z_2 odd, does not get vev

- New Higgs bosons $A, H, H^{\pm}$ are predicted
- The lightest neutral one can be a dark matter candidate

PB, Shilpa Jangid: Eur.Phys.J.C 80 (2020) 8, 715

On various 2HDMs similar studies: Kundu et al., Chakrabarty-Mukhopadhyaya, Khan et al.

Addition of scalars: Inert Triplet

- Inert Triplet model: SM is extended Z_2 odd with a Y=0, SU(2) Triplet T

$$T = \frac{1}{2}\begin{pmatrix} T_0 & \sqrt{2}T^+ \\ \sqrt{2}T^- & -T_0 \end{pmatrix}$$

$$V = m_h^2\Phi^\dagger\Phi + m_T^2 Tr(T^\dagger T) + \lambda_1|\Phi^\dagger\Phi|^2 + \lambda_t(Tr|T^\dagger T|)^2 + \lambda_{ht}\Phi^\dagger\Phi Tr(T^\dagger T)$$

Z_2 odd, does not get vev

- We have $T_0, T^\pm$ extra Higgs bosons which are degenerated at the tree-level
- Breaks by a quantum mass splitting of $\Delta m = (m_{T^\pm} - m_{T^0}) \simeq 166\,\text{MeV}$

Cirelli et al.: NPB753 (2006) 178

- T_0 is dark matter candidate
- $T^\pm \to \pi^\pm T_0$ predicts displaced pion charged track with ~ cm decay length which can be detected at the LHC

PB, Shilpa Jangid: Eur.Phys.J.C 80 (2020) 8, 715

Addition of scalar makes EW vacuum stable

- Unlike fermions, addition of the scalars make the potential more stable
- The RG-improved effective potential gets contributions from IDM/ITM as

$$V_{\rm eff} \;=\; V_0 + V_1^{\rm SM} + V_1^{\rm IDM/ITM}$$

- The effective potential in the SM Higgs direction can be written as

$$V_{\rm eff}(h,\mu) \;\simeq\; \lambda_{\rm eff}(h,\mu)\frac{h^4}{4}, \quad \text{with } h \gg v\,,$$

- The $\lambda_{\rm eff}$ gets positive contributions from extra scalars which counters the negative effect of the top quark

$$\lambda_{\rm eff}\,(h,\mu) \;\simeq\; \underbrace{\lambda_h\,(\mu)}_{\text{tree-level}} + \underbrace{\frac{1}{16\pi^2}\sum_{\substack{i=W^\pm,Z,t,\\ h,G^\pm,G^0}} n_i\kappa_i^2\left[\log\frac{\kappa_i h^2}{\mu^2} - c_i\right]}_{\text{Contribution from SM}} + \underbrace{\frac{1}{16\pi^2}\sum_{i=H,A,H^\pm} n_i\kappa_i^2\left[\log\frac{\kappa_i h^2}{\mu^2} - c_i\right]}_{\text{Contribution from IDM/ITM}};$$

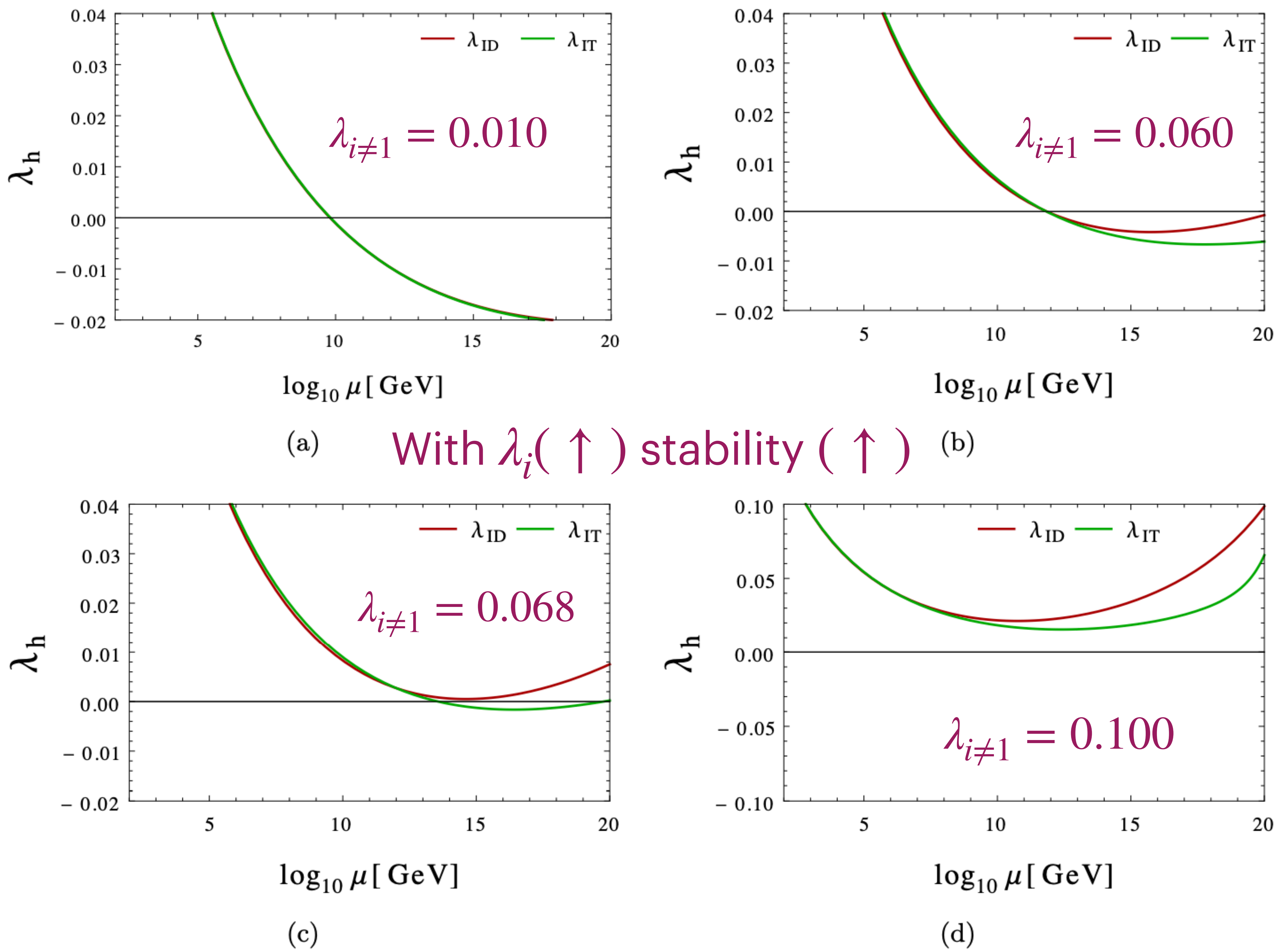
PB, Shilpa Jangid: Eur.Phys.J.C 80 (2020) 8, 715

Addition of scalar makes EW vacuum stable

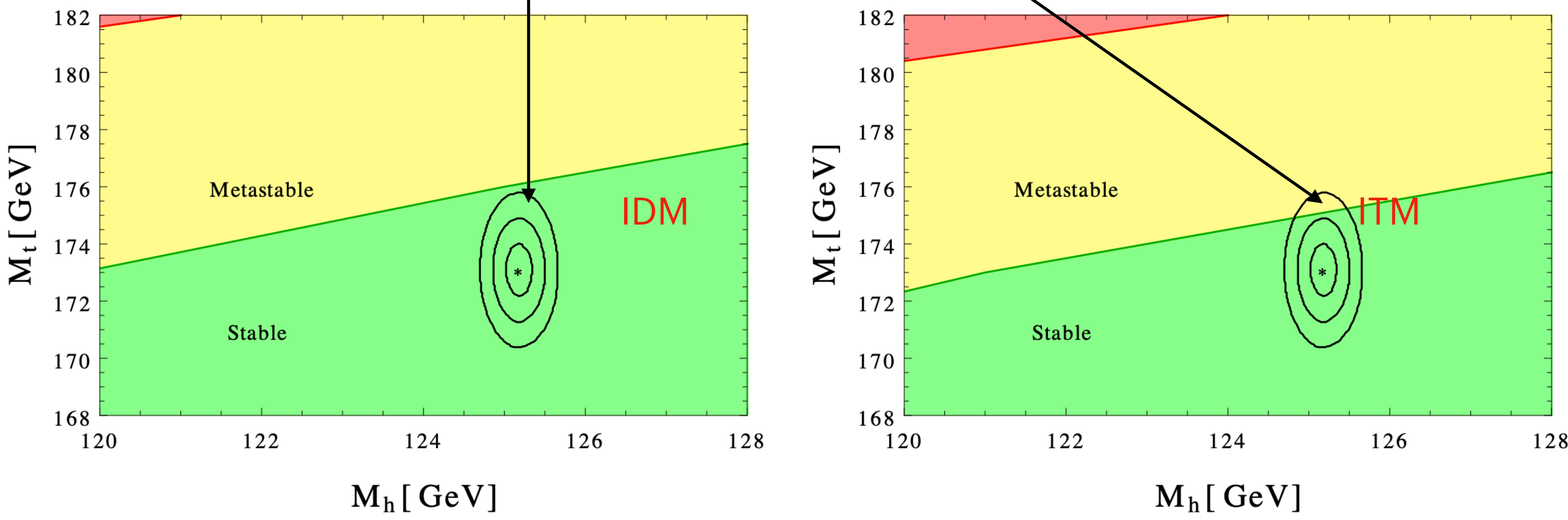
- At one-loop $\lambda_h = \lambda_1$ gets contributions from IDM/ITM and stabilise the vacuum

$$\beta^{\rm IDM}_{\lambda_1} = \frac{1}{16\pi^2}\left[2\lambda_3^2 + 2\lambda_3\lambda_4 + \lambda_4^2 + 4\lambda_5^2\right].$$

$$\beta^{\rm ITM}_{\lambda_1} = \frac{1}{16\pi^2}\left[8\lambda_{ht}^2\right].$$



Mostly in the stable regions



- Higher λ_i are constrained from perturbativity

PB, Shilpa Jangid: EJC 80 (2020) 8, 715

- Models with Type-I, III Seesaw fermions are severely constraints and need extra scalar to stabilise the potential

Type-I →PB, Shilpa Jangid, Bhupal Dev, Arjun Kumar: JHEP 08 (2020) 154
Type-III →PB, Shilpa Jangid, Manimal Mitra: JHEP 02 (2021) 075

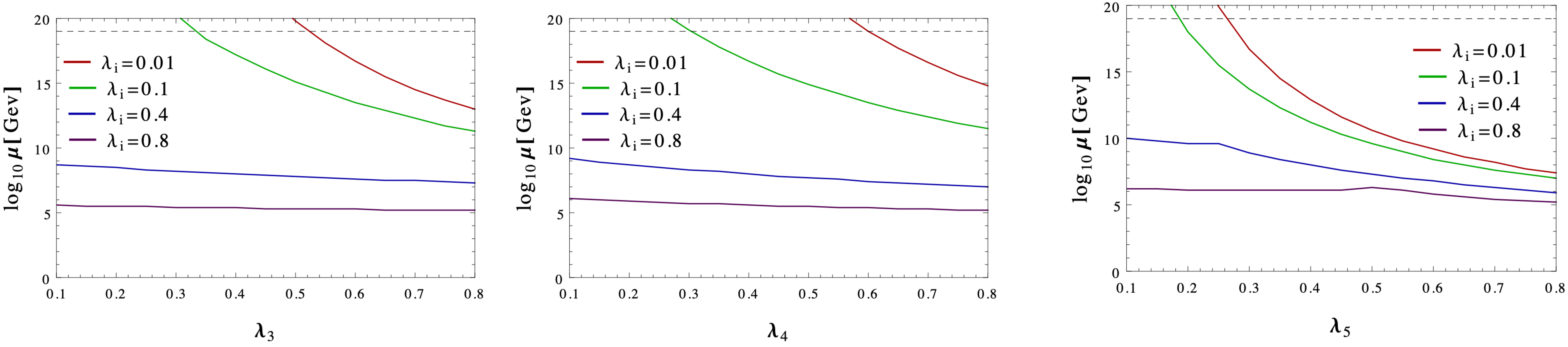
Is there a bound from perturbativity ?

Perturbativity bound on quartic couplings

- The two-loop beta functions gets more positive contributions than one-loop and often hits the perturbativity bounds

$$|\lambda_i| \leq 4\pi, \qquad |g_j| \leq 4\pi,$$

- For inert-doublet scenario we receive perturbativity bounds for all the quadratic couplings



PB, Shilpa Jangid: EJC 80 (2020) 8, 715

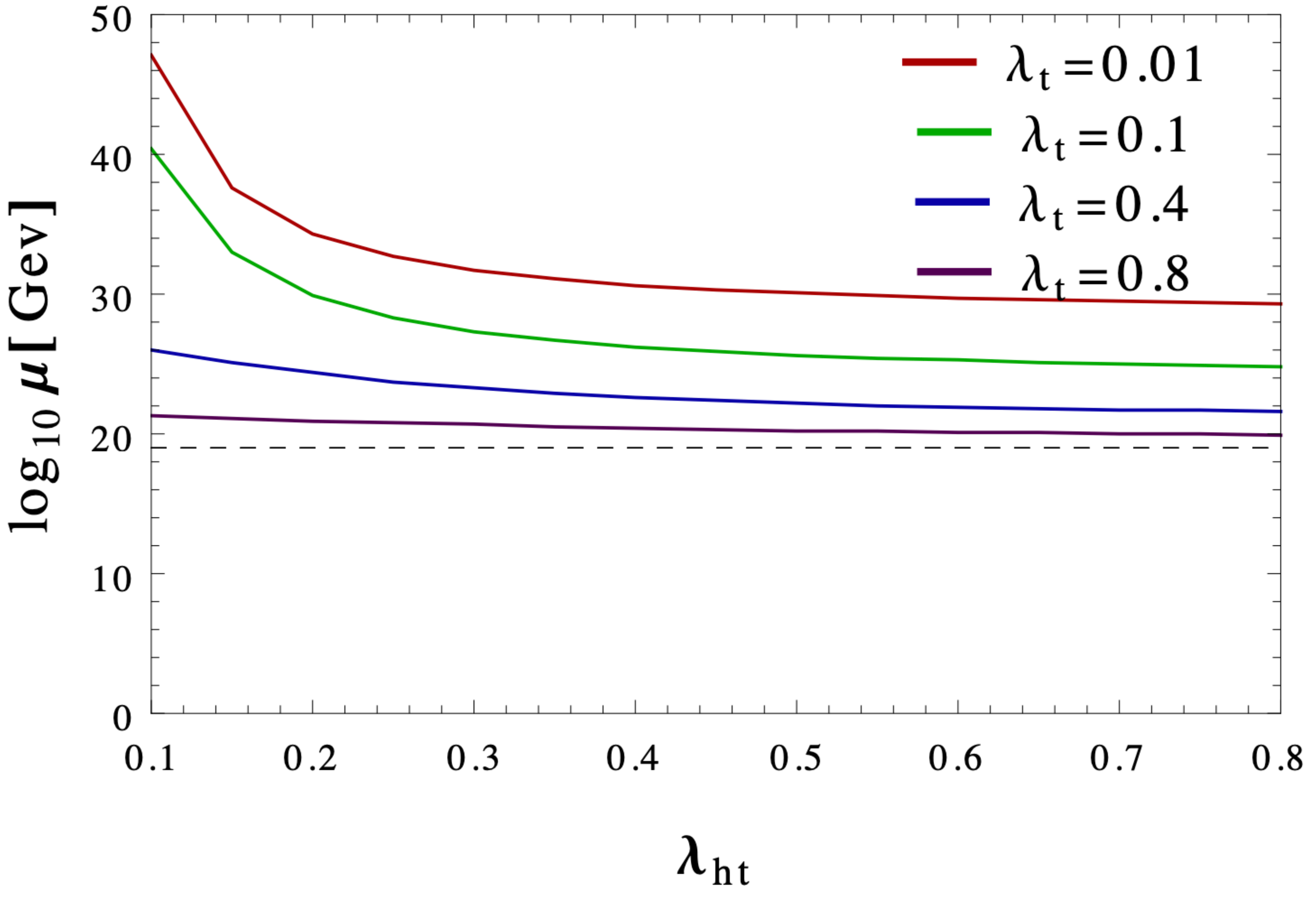
Type-I →PB, Shilpa Jangid, Bhupal Dev, Arjun Kumar: JHEP 08 (2020) 154

Type-III →PB, Shilpa Jangid, Manimal Mitra: JHEP 02 (2021) 075

Perturbativity bound on quartic couplings

- For inert triplet scenario there is only one portal coupling λ_{ht}, which also gets non-perturbative for certain unutial values of the couplings

- Thus larger Higgs quartic coupling g vacuum stability but are constrained there perturbativity



Type-III →PB, Shilpa Jangid, Manimal Mitra: JHEP 02 (2021) 075

Type-I →PB, Shilpa Jangid, Bhupal Dev, Arjun Kumar: JHEP 08 (2020) 15

Has these scalars other motivation ?

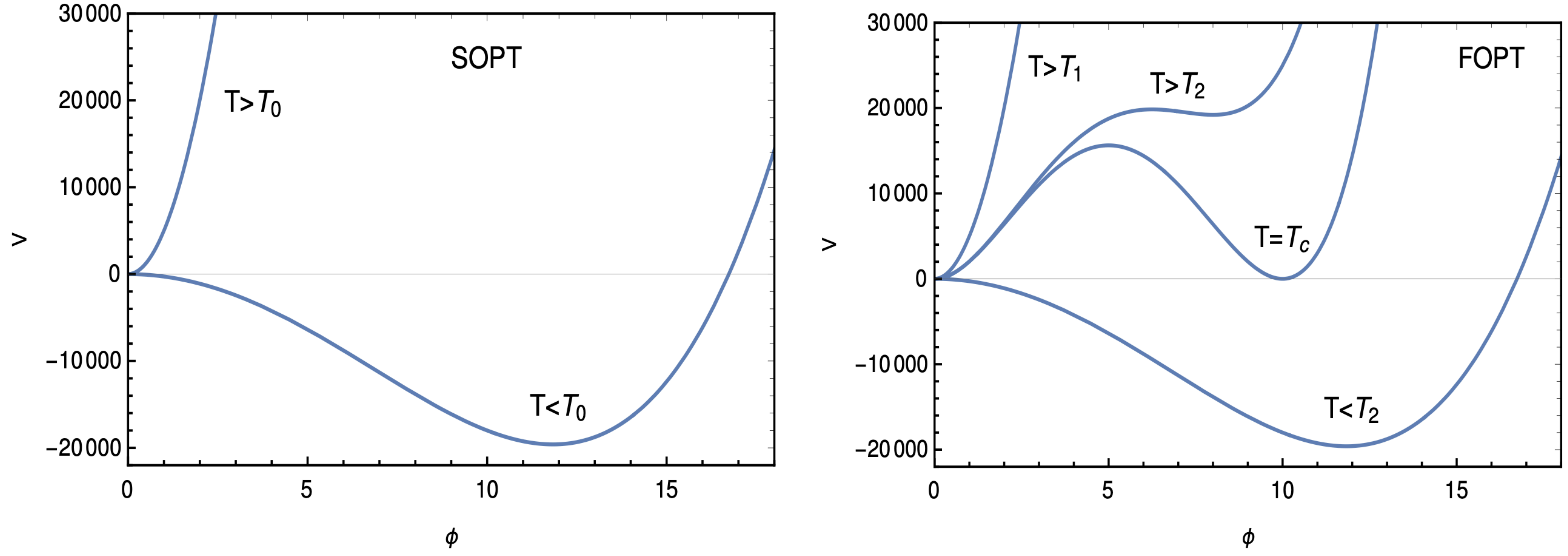
🤔

First order Phase Transition

- Electroweak baryogengesis needs first order phase transition
- If Standard Model electroweak Phase transition is first order then the Higgs mass should have been $\lesssim 50\,\mathrm{GeV}$
- The observed Higgs mass is around 125 GeV
- So there is a need of extra scalar to explain this
- A scalar dark matter is a suitable candidate

Possibility of first order phase transition

- At higher temperature SM potential looks like
  $V(\Phi, T) = D(T^2 - T_0^2)\Phi^2 + \lambda\Phi^4$
- These causes second order phase transition (SOPT)
- However, if there is one negative cubic term then you can get a first order phase transition (FOPT) $V(\Phi, T) = D(T^2 - T_0^2)\Phi^2 + \lambda\Phi^4 - ET\Phi^3$



SM and the possibility of FOPT

- In SM the cubic term is $E \sim \frac{2M_W^3 + M_Z^3}{4\pi v^3} \sim 0.01$
- Which leads to $\lambda \sim 2E \sim 0.02 \implies m_h \sim 49.2\,\mathrm{GeV}$
- However, observed Higgs Boson mass gives
  $m_h = \sqrt{a\lambda}v = 125.5\mathrm{GeV} \implies \lambda \sim 0.13$
- Standard Model phase transition is a smooth cross-over

M.~Gogberashvili, Adv. High Energy Phys. (2018), 4653202

Inert Singlet and triplet extension

- We can add a Z_2-odd SM gauge singlet via

$$V = -\mu^2 H^\dagger H + m_S^2 S^* S + \lambda_1 |H^\dagger H|^2 + \lambda_s |S^* S|^2 + \lambda_{hs} (H^\dagger H)(S^* S),$$

- Here S can serve as dark matter candidate
- Similarly we can extended SM with $Z_2 - \mathrm{odd}, \mathrm{Y} = 0, \mathrm{SU}(2)$ triplet T

$$V = -\mu^2 H^\dagger H + m_T^2 Tr(T^\dagger T) + \lambda_1 |H^\dagger H|^2 + \lambda_t (Tr|T^\dagger T|)^2 + \lambda_{ht} H^\dagger H Tr(T^\dagger T),$$

$$T = \frac{1}{2} \begin{pmatrix} T_0 & \sqrt{2} T^+ \\ \sqrt{2} T^- & -T_0 \end{pmatrix}$$

Thermal potentials

- The one-loop daisy improved finite temperature effective potential can be written as
$$V_{\rm eff}(\phi, T) = V_0(\phi) + V_1(\phi, 0) + \Delta V_1(\phi, T) + \Delta V_{\rm daisy/ring}(\phi, T);$$
- The field dependent masses are dependent on the corresponding Higgs portal couplings

$$M_S^2(\phi) = m_S^2 + \frac{\lambda_{hs}}{2}\phi^2.$$

Singlet

$$M_{T_0}^2(\phi) = m_T^2 + \frac{\lambda_{ht}}{2}\phi^2,$$
$$M_{T\pm}^2(\phi) = m_T^2 + \frac{\lambda_{ht}}{2}\phi^2.$$

Triplet

- Addition of scalars make electroweak vacuum more stable
- Inert extensions provide the dark matter
- Possible FOPT can give rise to EW-baryogenesis and Gravitational wave

Thermal potentials

- The effective potential can be written as

$$V(\phi) = A(T)\phi^2 + B(T)\phi^4 + C(T)(\phi^2 + K^2(T))^{\frac{3}{2}}.$$

Singlet

$$T_1^2 = \frac{2\lambda_{T_1}(\lambda_{hs}\mu_{T_1}^2 + 2\lambda_{T_1} m_S^2)}{\lambda_{hs}\left(\left(\frac{\lambda_{hs}}{6} + \frac{y_t^2}{2}\right)\lambda_{T_1} - \frac{\lambda_{hs}^3}{64\pi^2} - \frac{2\lambda_{T1}^2}{3\lambda_{hs}}(\lambda_{hs} + 2\lambda_s)\right)},$$

$$T_2^2 = \frac{1}{2\alpha}(\Lambda^2(T2) + \sqrt{\Lambda^4(T2) - 16\alpha\mu_{T2}^4}),$$

$$\alpha = \left(\frac{\lambda_{hs}}{6} + \frac{y_t^2}{2}\right)^2 - \frac{1}{24\pi^2}\lambda_{hs}^2\left(\lambda_{hs} + 2\lambda_s\right),$$

$$\Lambda^2(T) = \frac{1}{4\pi^2}\lambda_{hs}^2 m_S^2 + 4\left(\frac{\lambda_{hs}}{6} + \frac{y_t^2}{2}\right)\mu_T^2.$$

Triplet

$$T_1^2 = \frac{6144\pi^2\lambda_{T_1}(\lambda_{ht}\mu_{T_1}^2 + 2\lambda_{T_1} m_T^2)}{\lambda_{ht}\left(3072\pi^2\left(\frac{\lambda_{ht}}{4} + \frac{y_t^2}{2}\right)\lambda_{T_1} - 27\lambda_{ht}^3 - \frac{2048\pi^2\lambda_{T1}^2}{\lambda_{ht}}(2\lambda_{ht} + 4\lambda_t)\right)},$$

$$T_2^2 = \frac{1}{\alpha}(\Lambda^2(T2) + \sqrt{\Lambda^4(T2) - 65536\alpha\mu_{T2}^4}),$$

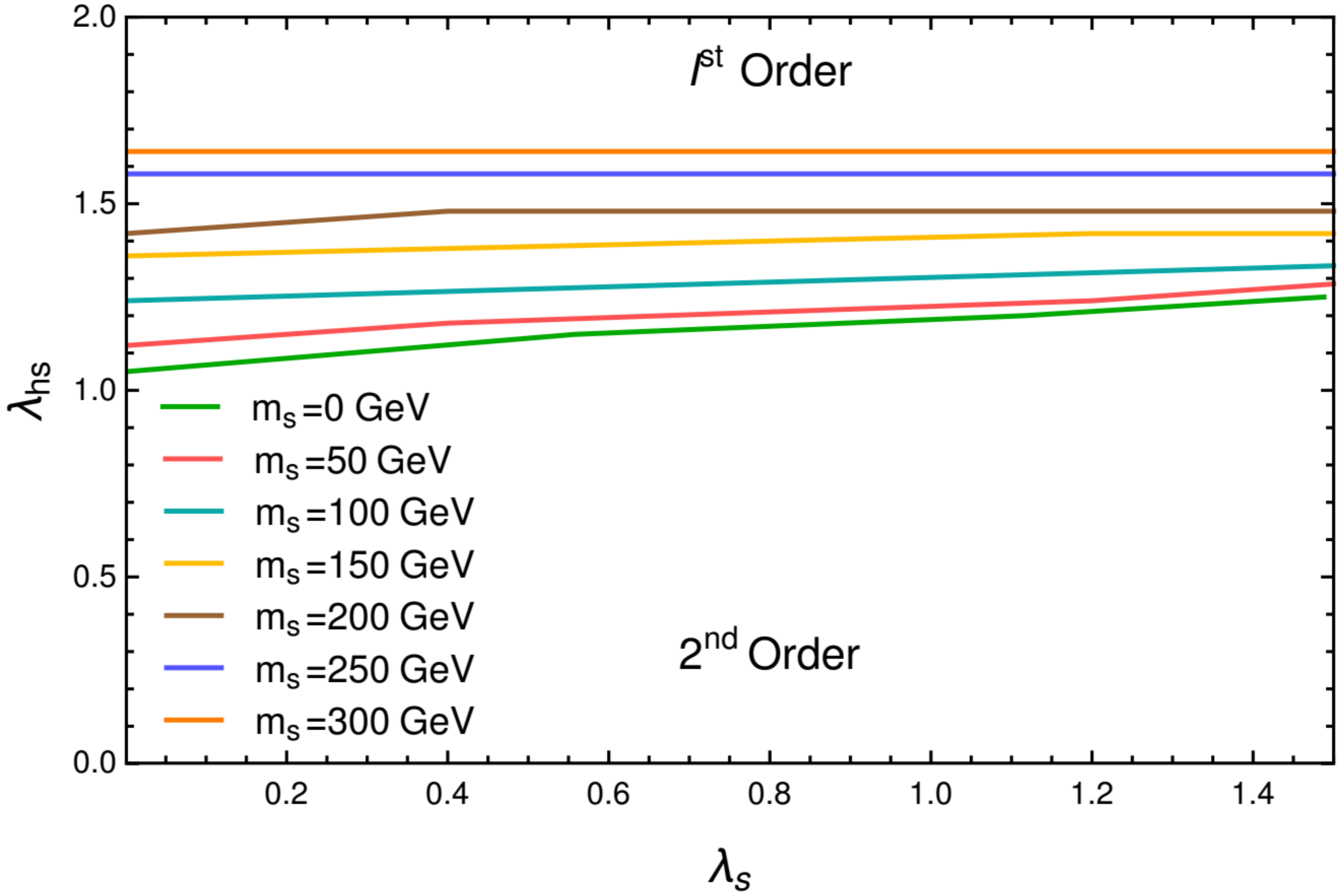
$$\alpha = \left(\frac{\lambda_{ht}}{4} + \frac{y_t^2}{2}\right) - \frac{3}{128\pi^2}\lambda_{ht}^2\left(2\lambda_{ht} + 4\lambda_t\right),$$

$$\Lambda^2(T) = 9\lambda_{ht}^2 m_T^2 + 256\left(\frac{\lambda_{ht}}{4} + \frac{y_t^2}{2}\right)\mu_T^2.$$

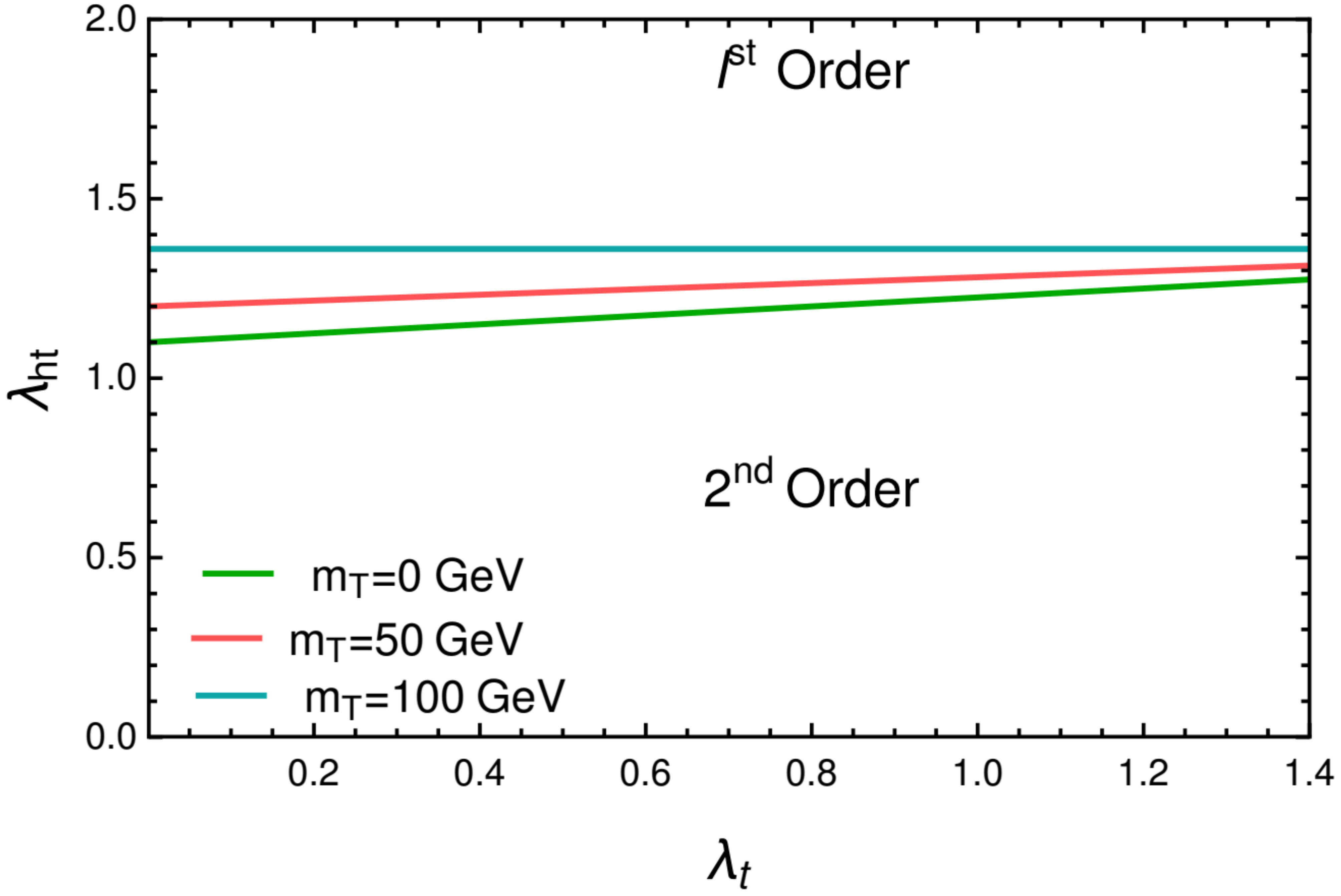
- For first order phase transition we look for regions $T_1 = T_2$

FOPT with ISM and ITM

- $T_1 = T_2$ lines for ISM and ITM are shown by the coloured lines



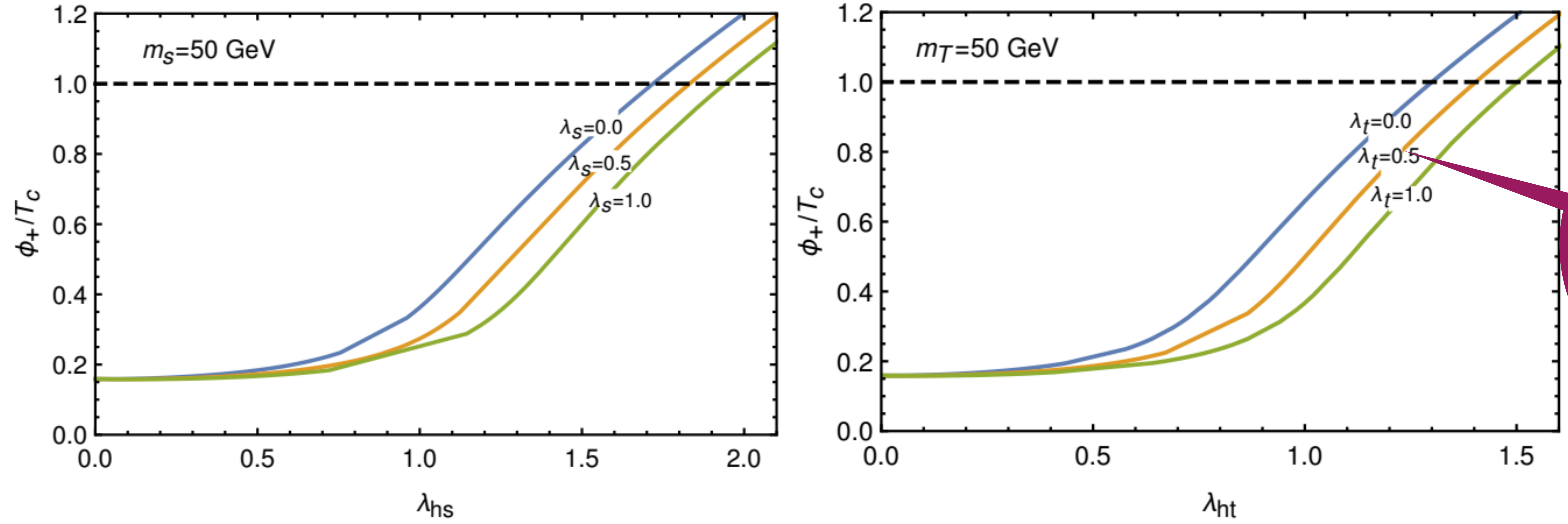
(a) Singlet



(b) Triplet

FOPT with ISM and ITM

- The condition for strongly first order phase traction can be taken to be $\dfrac{\Phi_+(T_c)}{T_c} \geq 1$

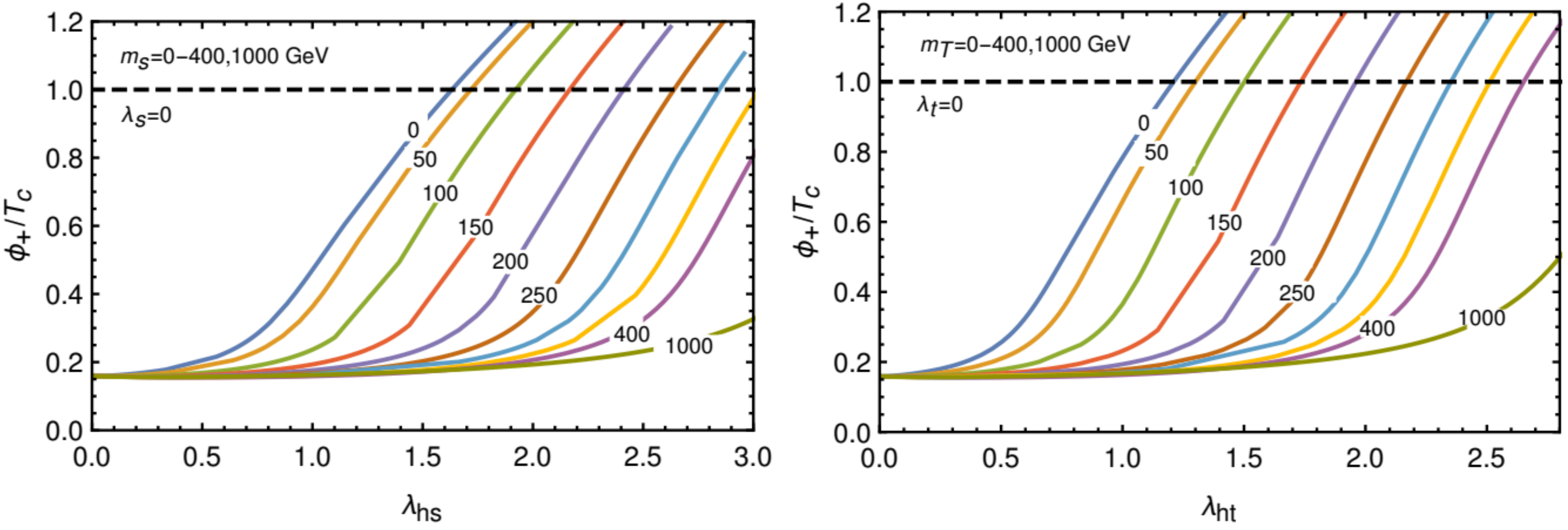


(a) Singlet (b) Triplet

Lower Higgs Portal couplings leads to SOPT

- As singlet has lesser DOF as compared to triplet, needs larger Higgs portal coupling to have SFOPT

FOPT with ISM and ITM



(a) Singlet (b) Triplet

- A variation of soft mass parameter shows higher mass needs larger Higgs portal couplings for FOPT
- $\frac{\phi_+(T_c)}{T_c} > 1$ demands: For the singlet: $m_S \geq 350\,\text{GeV} \implies \lambda_{\text{hs}} \geq 3.0$ and for the triplet: $m_T \geq 400\,\text{GeV} \implies \lambda_{\text{ht}} \geq 2.6$

Constraints from one-loop perturbativity

- The one-loop beta functions take the following form for ISM and ITM

- $\beta_{\lambda_1} = \beta_{\lambda_1}^{\text{SM}} + \Delta\beta_{\lambda_1}^{\text{ISM/ITM}}$, where $\beta_{\lambda_1}^{\text{ISM}} = 4\lambda_{hs}^2, \quad \beta_{\lambda_1}^{\text{ITM}} = 8\lambda_{ht}^2$
- $\beta_{\lambda_t} = \dfrac{1}{16\pi^2}[-24g_2^2\lambda_t + 88\lambda_t^2 + 8\lambda_{ht}^2 + \dfrac{3}{2}g_2^4]$
- $\beta_{\lambda_{ht}} = \dfrac{1}{16\pi^2}[-\dfrac{9}{10}g_1^2\lambda_{ht} - \dfrac{33}{2}g_2^2\lambda_{ht} + 12\lambda\lambda_{ht} + 16\lambda_{ht}^2 + 24\lambda_{ht}\lambda_t + 6y_t^2\lambda_{ht} + \dfrac{3}{4}g_2^4]$
- $\beta_{\lambda_s} = \dfrac{1}{16\pi^2}[20\lambda_s^2 + 8\lambda_{hs}^2]$
- $\beta_{\lambda_{hs}} = \dfrac{1}{16\pi^2}[\lambda_{hs}(12\lambda - \dfrac{9}{2}g_2^2 + 18y_t^2 + 8\lambda_{hs} + 8\lambda_s - \dfrac{9}{10}g_1^2)]$

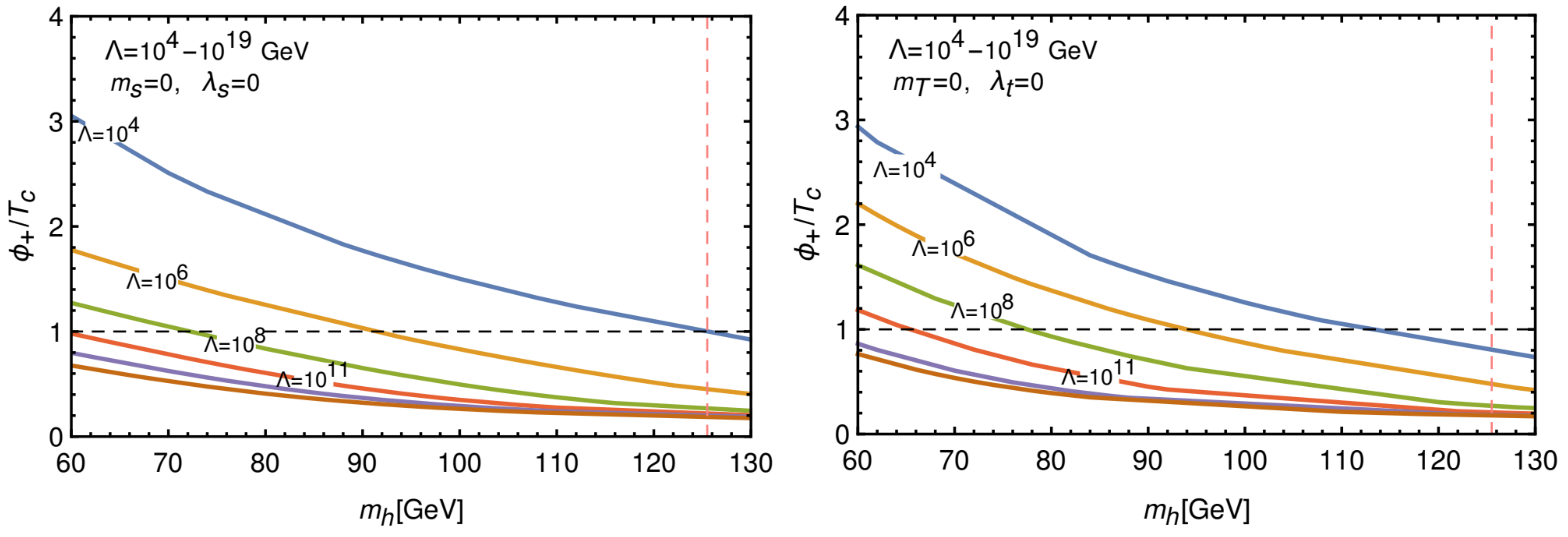
Constraints from one-loop perturbativity

- Depending on the perturbative scale the maximum values of $\lambda_{hs/ht}$ are restricted at the EW scale
- For a lower perturbative scale (Λ), a higher value of the Higgs-portal coupling is allowed
- For more DOF in ITM as compared to ISM, the value are more restricted in the former case
- The change of two-loop perturbativity of course, the results differ due to some negative contributions in the two-loop beta functions

Λ (GeV)	$\lambda_{hs} = \lambda_{hs}^{max}$	$\lambda_{ht} = \lambda_{ht}^{max}$
	m_t (GeV)	m_t (GeV)
	173.2	173.2
10^4	1.6545	1.3710
10^6	0.7290	0.7067
10^8	0.5120	0.4873
10^{11}	0.4780	0.3477
10^{16}	0.3090	0.2490
10^{19}	0.2370	0.2180

One-loop perturbativity limit

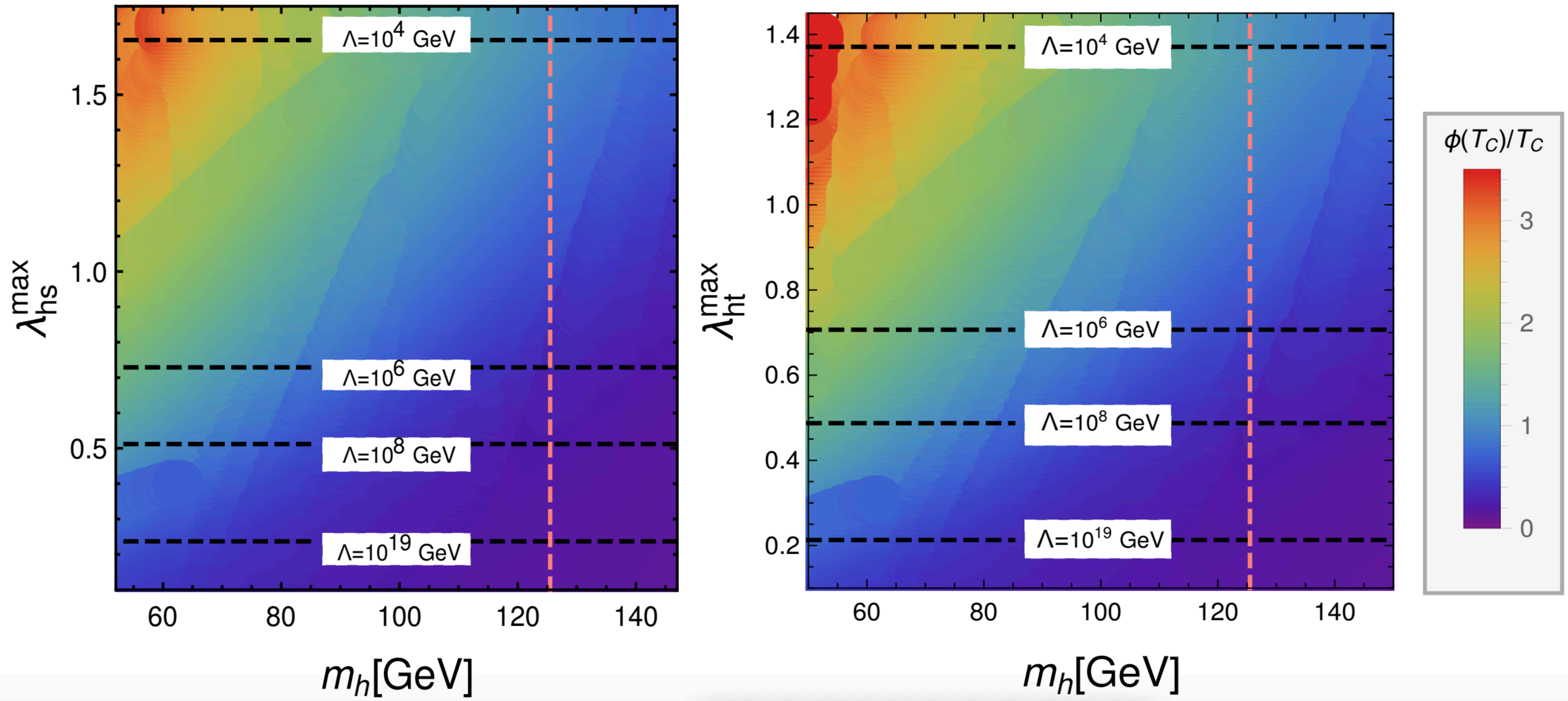
FOPT with ISM and ITM



(a) Singlet (b) Triplet

- For the $\lambda_{hs/ht} = \lambda_{hs/ht}^{max}$ for each scale vs the SM Higgs mass for $m_{s/T} = 0, \lambda_{s/t} = 0$
- Only for singlet with $\Lambda = 10^4\,\mathrm{GeV}$, the SM Higgs mass can reach 125.5 GeV

FOPT with ISM and ITM



- Except for the singlet with $\Lambda = 10^4\,\mathrm{GeV}$ and a SM Higgs mass of 125.5 GeV, FOPT is not possible
- Two-loop results may give a breather

Constraints from two-loop perturbativity

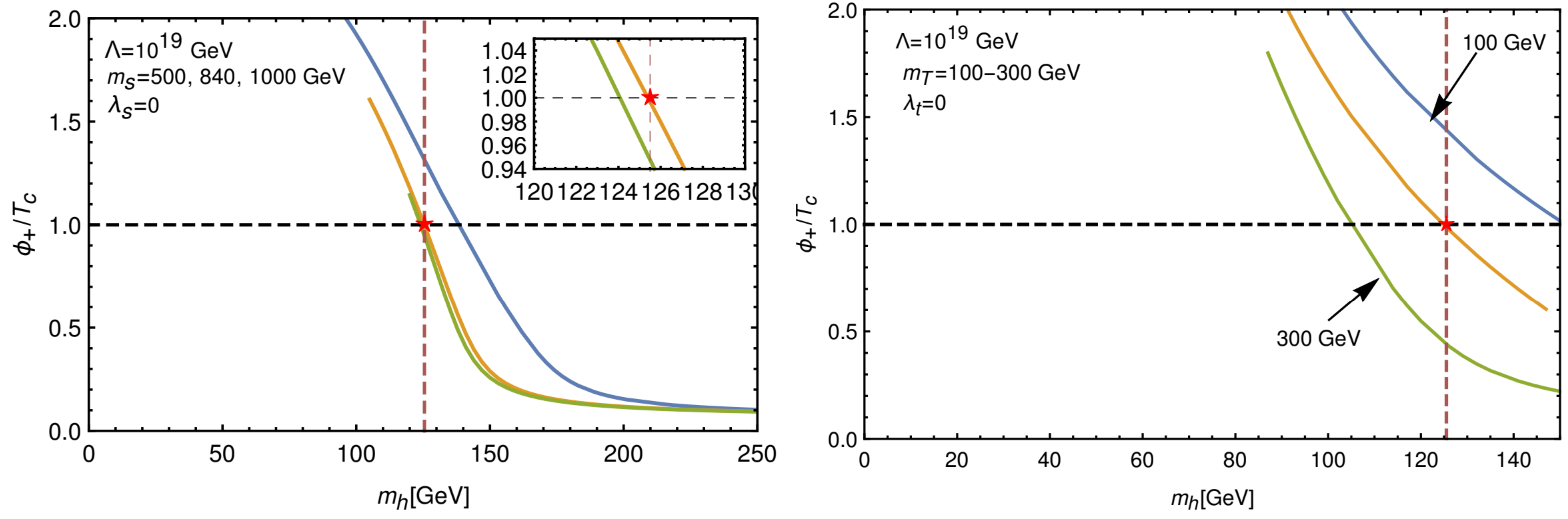
- Unlike one-loop, here λ_1 hits Landau pole before
- However, the growth of λ_1 slows down due to negative contributions proportional to $\lambda_1^3, \lambda_1\lambda_{ht}^2, \lambda_{ht}^3$
- Similar negative contributions can be observed for the singlet case also, which are proportional to $\lambda_1^3, \lambda_1\lambda_{hs}^2, \lambda_{hs}^3$
- However for Planck scale perturbativity, the singlet has almost double allowed value compared to the triplet
- This is due to additional positive contributions of the type $g_2^4\lambda_{ht}, g_2^2\lambda_{ht}$, which are absent in case of the singlet

Two-loop perturbativity limit

Λ (GeV)	λ_{hs}^{max}	λ_{ht}^{max}
10^{19}	4.00	1.95

Higgs mass FOPT with Planck scale perturbativity

- For Planck scale perturbativity of $\lambda_{hs/ht}$ with $\lambda_{s/t} = 0$

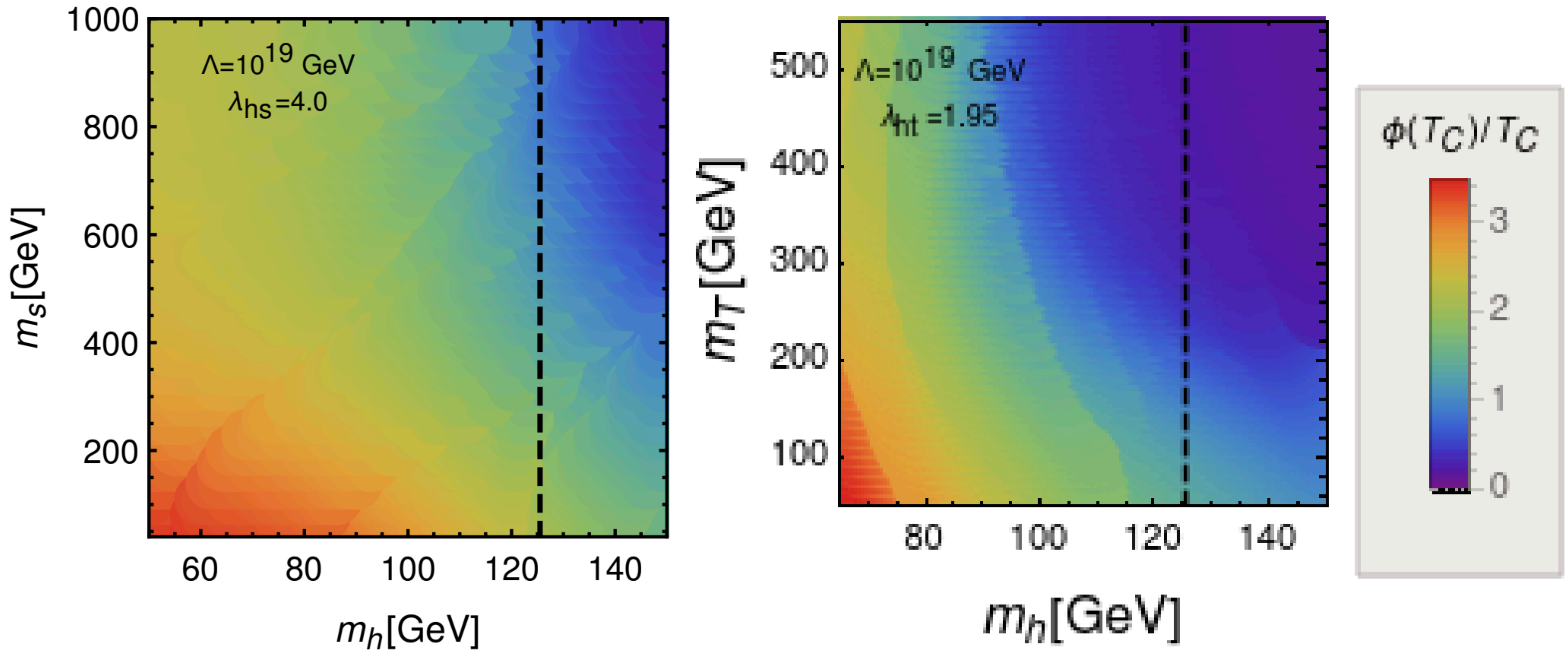


(a) Singlet (b) Triplet

- Singlet and triplet masses are carried for 500, 840, 1000 GeV and 300. 200,100 GeV, respectively

Higgs mass FOPT with Planck scale perturbativity

- For Planck scale perturbativity of $\lambda_{hs/ht} = 4.0(1.95)$



(a) Singlet (b) Triplet

- For FOPT with correct Higgs mass: $m_s \leq 840, m_T \leq 193$ GeV

Two-loop beta function for ITM

$$
\begin{aligned}
\beta_{\lambda=\lambda_1} = & \frac{1}{16\pi^2}\Bigg[\frac{27}{200}g_1^4 + \frac{9}{20}g_1^2g_2^2 + \frac{9}{8}g_2^4 - \frac{9}{5}g_1^2\lambda_1 - 9g_2^2\lambda_1 + 24\lambda_1^2 + 8\lambda_{ht}^2 + 12\lambda_1\mathrm{Tr}\Big(Y_dY_d^\dagger\Big) + 4\lambda_1\mathrm{Tr}\Big(Y_eY_e^\dagger\Big) \\
& + 12\lambda_1\mathrm{Tr}\Big(Y_uY_u^\dagger\Big) - 6\mathrm{Tr}\Big(Y_dY_d^\dagger Y_dY_d^\dagger\Big) - 2\mathrm{Tr}\Big(Y_eY_e^\dagger Y_eY_e^\dagger\Big) - 6\mathrm{Tr}\Big(Y_uY_u^\dagger Y_uY_u^\dagger\Big)\Bigg] \\
& + \frac{1}{(16\pi^2)^2}\Bigg[-\frac{3411}{2000}g_1^6 - \frac{1677}{400}g_1^4g_2^2 - \frac{317}{80}g_1^2g_2^4 + \frac{277}{16}g_2^6 + \frac{1887}{200}g_1^4\lambda_1 + \frac{117}{20}g_1^2g_2^2\lambda_1 - \frac{29}{8}g_2^4\lambda_1 \\
& + \frac{108}{5}g_1^2\lambda_1^2 + 108g_2^2\lambda_1^2 - 312\lambda_1^3 + 10g_2^4\lambda_{ht} + 32g_2^2\lambda_{ht}^2 - 80\lambda_1\lambda_{ht}^2 - 128\lambda_{ht}^3 \\
& + \frac{1}{20}\Big(-5\Big(64\lambda_1\Big(-5g_3^2 + 9\lambda_1\Big) - 90g_2^2\lambda_1 + 9g_2^4\Big) + 9g_1^4 + g_1^2\Big(50\lambda_1 + 54g_2^2\Big)\Big)\mathrm{Tr}\Big(Y_dY_d^\dagger\Big) \\
& - \frac{3}{20}\Big(15g_1^4 - 2g_1^2\Big(11g_2^2 + 25\lambda_1\Big) + 5\Big(-10g_2^2\lambda_1 + 64\lambda_1^2 + g_2^4\Big)\Big)\mathrm{Tr}\Big(Y_eY_e^\dagger\Big) - \frac{171}{100}g_1^4\mathrm{Tr}\Big(Y_uY_u^\dagger\Big) \\
& + \frac{63}{10}g_1^2g_2^2\mathrm{Tr}\Big(Y_uY_u^\dagger\Big) - \frac{9}{4}g_2^4\mathrm{Tr}\Big(Y_uY_u^\dagger\Big) + \frac{17}{2}g_1^2\lambda_1\mathrm{Tr}\Big(Y_uY_u^\dagger\Big) + \frac{45}{2}g_2^2\lambda_1\mathrm{Tr}\Big(Y_uY_u^\dagger\Big) \\
& + 80g_3^2\lambda_1\mathrm{Tr}\Big(Y_uY_u^\dagger\Big) - 144\lambda_1^2\mathrm{Tr}\Big(Y_uY_u^\dagger\Big) + \frac{4}{5}g_1^2\mathrm{Tr}\Big(Y_dY_d^\dagger Y_dY_d^\dagger\Big) - 32g_3^2\mathrm{Tr}\Big(Y_dY_d^\dagger Y_dY_d^\dagger\Big) \\
& - 3\lambda_1\mathrm{Tr}\Big(Y_dY_d^\dagger Y_dY_d^\dagger\Big) - 42\lambda_1\mathrm{Tr}\Big(Y_dY_u^\dagger Y_uY_d^\dagger\Big) - \frac{12}{5}g_1^2\mathrm{Tr}\Big(Y_eY_e^\dagger Y_eY_e^\dagger\Big) - \lambda_1\mathrm{Tr}\Big(Y_eY_e^\dagger Y_eY_e^\dagger\Big) \\
& - \frac{8}{5}g_1^2\mathrm{Tr}\Big(Y_uY_u^\dagger Y_uY_u^\dagger\Big) - 32g_3^2\mathrm{Tr}\Big(Y_uY_u^\dagger Y_uY_u^\dagger\Big) - 3\lambda_1\mathrm{Tr}\Big(Y_uY_u^\dagger Y_uY_u^\dagger\Big) + 30\mathrm{Tr}\Big(Y_dY_d^\dagger Y_dY_d^\dagger Y_dY_d^\dagger\Big) \\
& - 12\mathrm{Tr}\Big(Y_dY_d^\dagger Y_dY_u^\dagger Y_uY_d^\dagger\Big) + 6\mathrm{Tr}\Big(Y_dY_u^\dagger Y_uY_d^\dagger Y_dY_d^\dagger\Big) - 6\mathrm{Tr}\Big(Y_dY_u^\dagger Y_uY_u^\dagger Y_uY_d^\dagger\Big) \\
& + 10\mathrm{Tr}\Big(Y_eY_e^\dagger Y_eY_e^\dagger Y_eY_e^\dagger\Big) + 30\mathrm{Tr}\Big(Y_uY_u^\dagger Y_uY_u^\dagger Y_uY_u^\dagger\Big)\Bigg].
\end{aligned}
$$

$$
\begin{aligned}
\beta_{\lambda_{ht}} = & \frac{1}{16\pi^2}\Bigg[\frac{3}{4}g_2^4 - \frac{9}{10}g_1^2\lambda_{ht} - \frac{33}{2}g_2^2\lambda_{ht} + 12\lambda_1\lambda_{ht} + 16\lambda_{ht}^2 + 24\lambda_{ht}\lambda_t + 6\lambda_{ht}\mathrm{Tr}\Big(Y_dY_d^\dagger\Big) + 2\lambda_{ht}\mathrm{Tr}\Big(Y_eY_e^\dagger\Big) \\
& + 6\lambda_{ht}\mathrm{Tr}\Big(Y_uY_u^\dagger\Big)\Bigg] \\
& + \frac{1}{(16\pi^2)^2}\Bigg[-\frac{9}{16}g_1^2g_2^4 + \frac{329}{48}g_2^6 + \frac{15}{2}g_2^4\lambda_1 + \frac{1671}{400}g_1^4\lambda_{ht} + \frac{9}{8}g_1^2g_2^2\lambda_{ht} - \frac{1087}{48}g_2^4\lambda_{ht} + \frac{72}{5}g_1^2\lambda_1\lambda_{ht} \\
& + 72g_2^2\lambda_1\lambda_{ht} - 60\lambda_1^2\lambda_{ht} + \frac{12}{5}g_1^2\lambda_{ht}^2 + 44g_2^2\lambda_{ht}^2 - 288\lambda_1\lambda_{ht}^2 - 168\lambda_{ht}^3 + 20g_2^4\lambda_t + 144g_2^2\lambda_{ht}\lambda_t \\
& - 576\lambda_{ht}^2\lambda_t - 544\lambda_{ht}\lambda_t^2 - \frac{1}{4}\Big(3g_2^4 - 45g_2^2\lambda_{ht} + \lambda_{ht}\Big(-160g_3^2 + 192\lambda_{ht} + 288\lambda - 5g_1^2\Big)\Big)\mathrm{Tr}\Big(Y_dY_d^\dagger\Big) \\
& - \frac{1}{4}\Big(-15g_2^2\lambda_{ht} + \lambda_{ht}\Big(-15g_1^2 + 64\lambda_{ht} + 96\lambda_1\Big) + g_2^4\Big)\mathrm{Tr}\Big(Y_eY_e^\dagger\Big) - \frac{3}{4}g_2^4\mathrm{Tr}\Big(Y_uY_u^\dagger\Big) \\
& + \frac{17}{4}g_1^2\lambda_{ht}\mathrm{Tr}\Big(Y_uY_u^\dagger\Big) + \frac{45}{4}g_2^2\lambda_{ht}\mathrm{Tr}\Big(Y_uY_u^\dagger\Big) + 40g_3^2\lambda_{ht}\mathrm{Tr}\Big(Y_uY_u^\dagger\Big) - 72\lambda_1\lambda_{ht}\mathrm{Tr}\Big(Y_uY_u^\dagger\Big) \\
& - 48\lambda_{ht}^2\mathrm{Tr}\Big(Y_uY_u^\dagger\Big) - \frac{27}{2}\lambda_{ht}\mathrm{Tr}\Big(Y_dY_d^\dagger Y_dY_d^\dagger\Big) - 21\lambda_{ht}\mathrm{Tr}\Big(Y_dY_u^\dagger Y_uY_d^\dagger\Big) - \frac{9}{2}\lambda_{ht}\mathrm{Tr}\Big(Y_eY_e^\dagger Y_eY_e^\dagger\Big) \\
& - \frac{27}{2}\lambda_{ht}\mathrm{Tr}\Big(Y_uY_u^\dagger Y_uY_u^\dagger\Big)\Bigg].
\end{aligned}
$$

Two-loop beta function for ISM

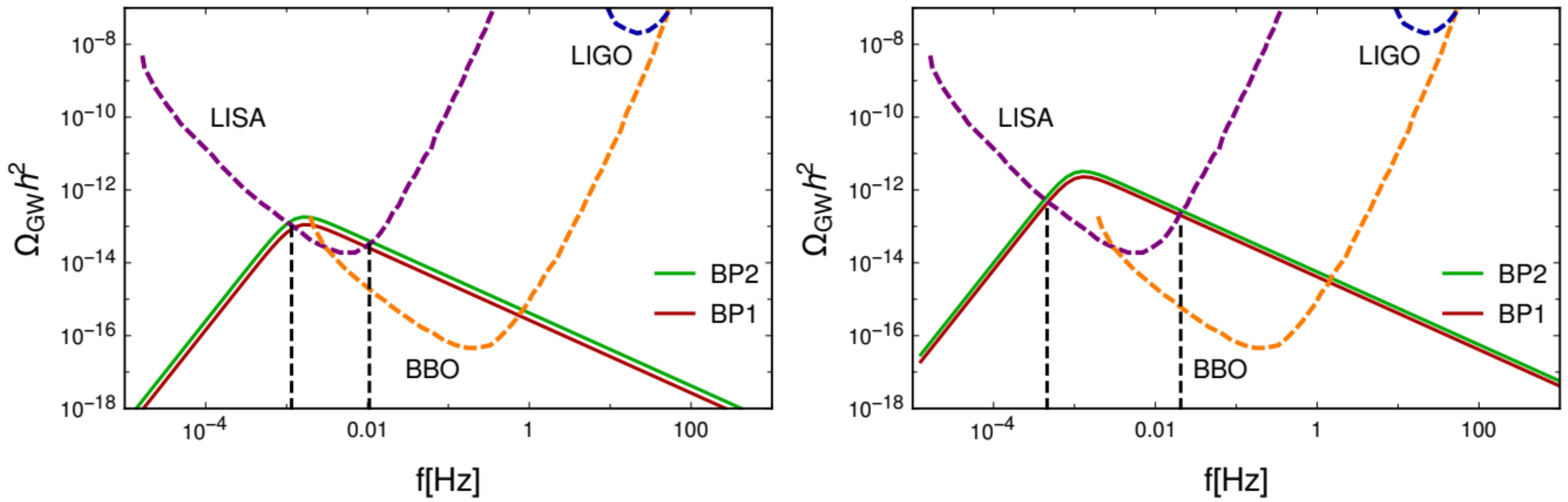
$$
\begin{aligned}
\beta_\lambda^{(1)} = \frac{1}{16\pi^2}\Bigg[& \frac{27}{200}g_1^4 + \frac{9}{20}g_1^2 g_2^2 + \frac{9}{8}g_2^4 - \frac{9}{5}g_1^2\lambda_1 - 9g_2^2\lambda_1 + 24\lambda_1^2 + 4\lambda_{hs}^2 + 12\lambda_1 \mathrm{Tr}\left(Y_d Y_d^\dagger\right) + 4\lambda_1 \mathrm{Tr}\left(Y_e Y_e^\dagger\right) \\
& + 12\lambda_1 \mathrm{Tr}\left(Y_u Y_u^\dagger\right) - 6\mathrm{Tr}\left(Y_d Y_d^\dagger Y_d Y_d^\dagger\right) - 2\mathrm{Tr}\left(Y_e Y_e^\dagger Y_e Y_e^\dagger\right) - 6\mathrm{Tr}\left(Y_u Y_u^\dagger Y_u Y_u^\dagger\right)\Bigg] \\
& + \frac{1}{(16\pi^2)^2}\Bigg[- \frac{3411}{2000}g_1^6 - \frac{1677}{400}g_1^4 g_2^2 - \frac{289}{80}g_1^2 g_2^4 + \frac{305}{16}g_2^6 + \frac{1887}{200}g_1^4\lambda_1 + \frac{117}{20}g_1^2 g_2^2\lambda_1 \\
& - \frac{73}{8}g_2^4\lambda_1 + \frac{108}{5}g_1^2\lambda_1^2 + 108 g_2^2\lambda_1^2 - 312\lambda_1^3 - 40\lambda_1\lambda_{hs}^2 - 32\lambda_{hs}^3 \\
& + \frac{1}{20}\Big(- 5\Big(64\lambda_1\Big(- 5g_3^2 + 9\lambda_1\Big) - 90 g_2^2\lambda_1 + 9g_2^4\Big) + 9g_1^4 + g_1^2\Big(50\lambda_1 + 54 g_2^2\Big)\Big)\mathrm{Tr}\left(Y_d Y_d^\dagger\right) \\
& - \frac{3}{20}\Big(15g_1^4 - 2g_1^2\Big(11g_2^2 + 25\lambda_1\Big) + 5\Big(- 10g_2^2\lambda_1 + 64\lambda_1^2 + g_2^4\Big)\Big)\mathrm{Tr}\left(Y_e Y_e^\dagger\right) - \frac{171}{100}g_1^4\mathrm{Tr}\left(Y_u Y_u^\dagger\right) \\
& + \frac{63}{10}g_1^2 g_2^2\mathrm{Tr}\left(Y_u Y_u^\dagger\right) - \frac{9}{4}g_2^4\mathrm{Tr}\left(Y_u Y_u^\dagger\right) + \frac{17}{2}g_1^2\lambda_1\mathrm{Tr}\left(Y_u Y_u^\dagger\right) + \frac{45}{2}g_2^2\lambda_1\mathrm{Tr}\left(Y_u Y_u^\dagger\right) \\
& + 80 g_3^2\lambda_1\mathrm{Tr}\left(Y_u Y_u^\dagger\right) - 144\lambda_1^2\mathrm{Tr}\left(Y_u Y_u^\dagger\right) + \frac{4}{5}g_1^2\mathrm{Tr}\left(Y_d Y_d^\dagger Y_d Y_d^\dagger\right) - 32 g_3^2\mathrm{Tr}\left(Y_d Y_d^\dagger Y_d Y_d^\dagger\right) \\
& - 3\lambda_1\mathrm{Tr}\left(Y_d Y_d^\dagger Y_d Y_d^\dagger\right) - 42\lambda\mathrm{Tr}\left(Y_d Y_u^\dagger Y_u Y_d^\dagger\right) - \frac{12}{5}g_1^2\mathrm{Tr}\left(Y_e Y_e^\dagger Y_e Y_e^\dagger\right) - \lambda_1\mathrm{Tr}\left(Y_e Y_e^\dagger Y_e Y_e^\dagger\right) \\
& - \frac{8}{5}g_1^2\mathrm{Tr}\left(Y_u Y_u^\dagger Y_u Y_u^\dagger\right) - 32 g_3^2\mathrm{Tr}\left(Y_u Y_u^\dagger Y_u Y_u^\dagger\right) - 3\lambda_1\mathrm{Tr}\left(Y_u Y_u^\dagger Y_u Y_u^\dagger\right) + 30\mathrm{Tr}\left(Y_d Y_d^\dagger Y_d Y_d^\dagger Y_d Y_d^\dagger\right) \\
& - 12\mathrm{Tr}\left(Y_d Y_d^\dagger Y_d Y_u^\dagger Y_u Y_d^\dagger\right) + 6\mathrm{Tr}\left(Y_d Y_u^\dagger Y_u Y_d^\dagger Y_d Y_d^\dagger\right) - 6\mathrm{Tr}\left(Y_d Y_u^\dagger Y_u Y_u^\dagger Y_u Y_d^\dagger\right) \\
& + 10\mathrm{Tr}\left(Y_e Y_e^\dagger Y_e Y_e^\dagger Y_e Y_e^\dagger\right) + 30\mathrm{Tr}\left(Y_u Y_u^\dagger Y_u Y_u^\dagger Y_u Y_u^\dagger\right)\Bigg].
\end{aligned}
$$

$$
\begin{aligned}
\beta_{\lambda_{hs}} = \frac{1}{16\pi^2}\Bigg[& \frac{1}{10}\lambda_{hs}\Big(120\lambda_1 + 20\mathrm{Tr}\left(Y_e Y_e^\dagger\right) - 45 g_2^2 + 60\mathrm{Tr}\left(Y_d Y_d^\dagger\right) + 60\mathrm{Tr}\left(Y_u Y_u^\dagger\right) + 80\lambda_{hs} + 80\lambda_t - 9g_1^2\Big)\Bigg] \\
& + \frac{1}{(16\pi^2)^2}\Bigg[+ \frac{1671}{400}g_1^4\lambda_{hs} + \frac{9}{8}g_1^2 g_2^2\lambda_{hs} - \frac{145}{16}g_2^4\lambda_{hs} + \frac{72}{5}g_1^2\lambda_1\lambda_{hs} + 72 g_2^2\lambda_1\lambda_{hs} - 60\lambda_1^2\lambda_{hs} \\
& + \frac{6}{5}g_1^2\lambda_{hs}^2 + 6g_2^2\lambda_{hs}^2 - 144\lambda_1\lambda_{hs}^2 - 44\lambda_{hs}^3 - 96\lambda_{hs}^2\lambda_t - 40\lambda_{hs}\lambda_t^2 \\
& + \frac{1}{4}\Big(32\Big(- 3\lambda_{hs} + 5g_3^2 - 9\lambda_1\Big) + 45 g_2^2 + 5g_1^2\Big)\lambda_{hs}\mathrm{Tr}\left(Y_d Y_d^\dagger\right) \\
& + \frac{1}{4}\lambda_{hs}\Big(15g_1^2 + 15g_2^2 - 32\Big(3\lambda_1 + \lambda_{hs}\Big)\Big)\mathrm{Tr}\left(Y_e Y_e^\dagger\right) + \frac{17}{4}g_1^2\lambda_{hs}\mathrm{Tr}\left(Y_u Y_u^\dagger\right) + \frac{45}{4}g_2^2\lambda_{hs}\mathrm{Tr}\left(Y_u Y_u^\dagger\right) \\
& + 40 g_3^2\lambda_{hs}\mathrm{Tr}\left(Y_u Y_u^\dagger\right) - 72\lambda_1\lambda_{hs}\mathrm{Tr}\left(Y_u Y_u^\dagger\right) - 24\lambda_{hs}^2\mathrm{Tr}\left(Y_u Y_u^\dagger\right) - \frac{27}{2}\lambda_{hs}\mathrm{Tr}\left(Y_d Y_d^\dagger Y_d Y_d^\dagger\right) \\
& - 21\lambda_{hs}\mathrm{Tr}\left(Y_d Y_u^\dagger Y_u Y_d^\dagger\right) - \frac{9}{2}\lambda_{hs}\mathrm{Tr}\left(Y_e Y_e^\dagger Y_e Y_e^\dagger\right) - \frac{27}{2}\lambda_{hs}\mathrm{Tr}\left(Y_u Y_u^\dagger Y_u Y_u^\dagger\right)\Bigg].
\end{aligned}
$$

Correction of two-loop potential

- The one-loop thermal potential has an accuracy of $\mathcal{O}(g^3)$ but an accuracy of $\mathcal{O}(g^4)$ with two-loop beat function needs a two-loop thermal potential
- One-loop effects the upper bounds on the soft masses
$m_S \lesssim 909, m_T \lesssim 310\,\mathrm{GeV}$
- With two-loop effects the upper bounds on the soft masses
$m_S \lesssim 909, m_T \lesssim 320\,\mathrm{GeV}$

Gravitational waves



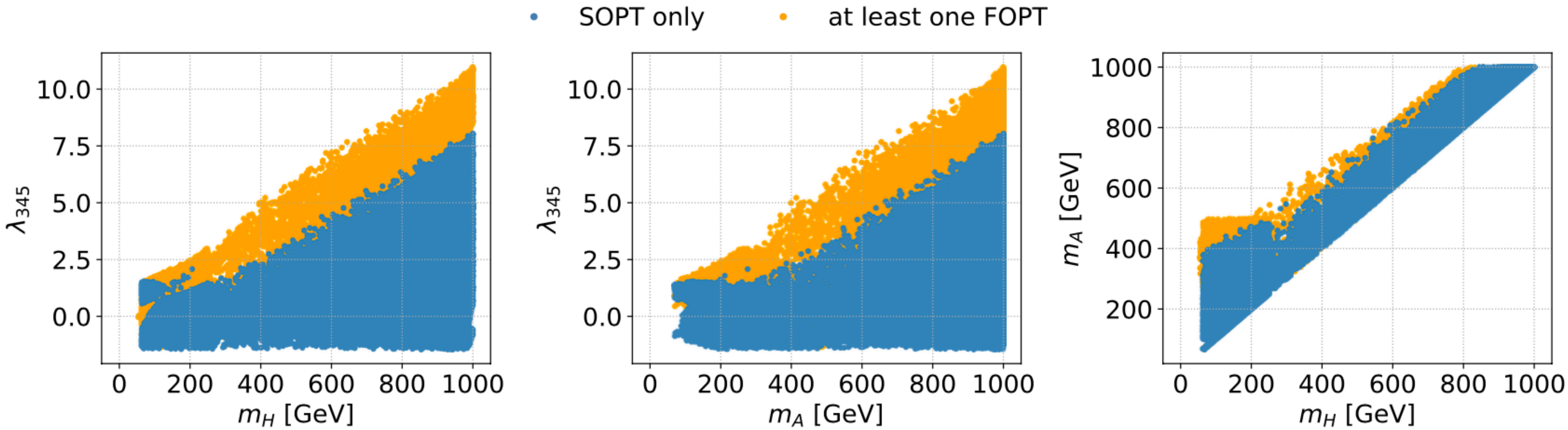
(a) Singlet (b) Triplet

	BP1	BP2
T_n[GeV]	121.03	119.25
α	0.17	0.18
β/H	332.83	327.94
v_n/T_n	1.10	1.16

	BP1	BP2
T_n[GeV]	115.07	113.55
α	0.86	0.89
β/H	284.22	278.87
v_n/T_n	1.16	1.22

- Gravitonal wave frequencies detectable at LISA and BBO are possible

FOPT with inert Doublet

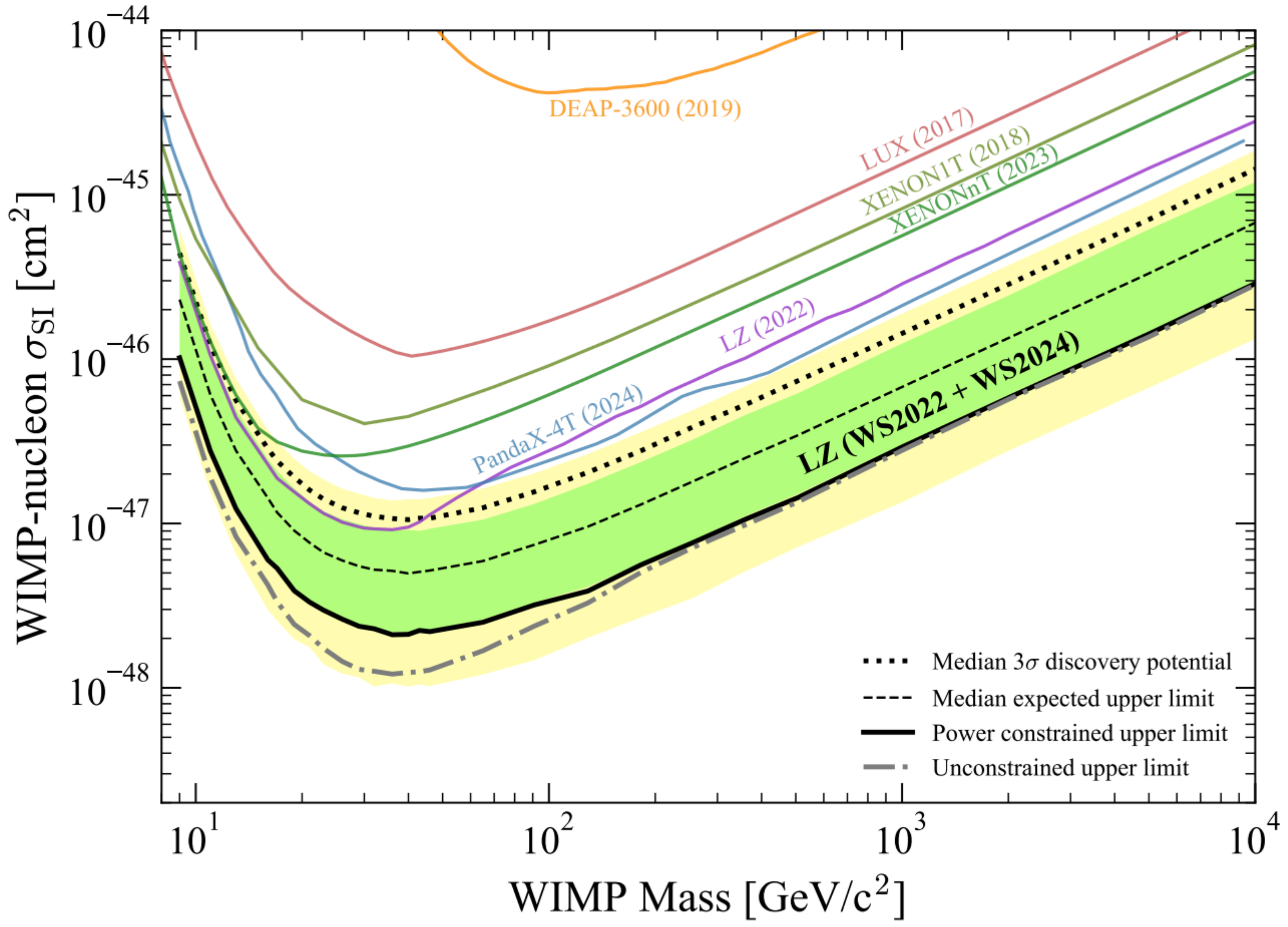


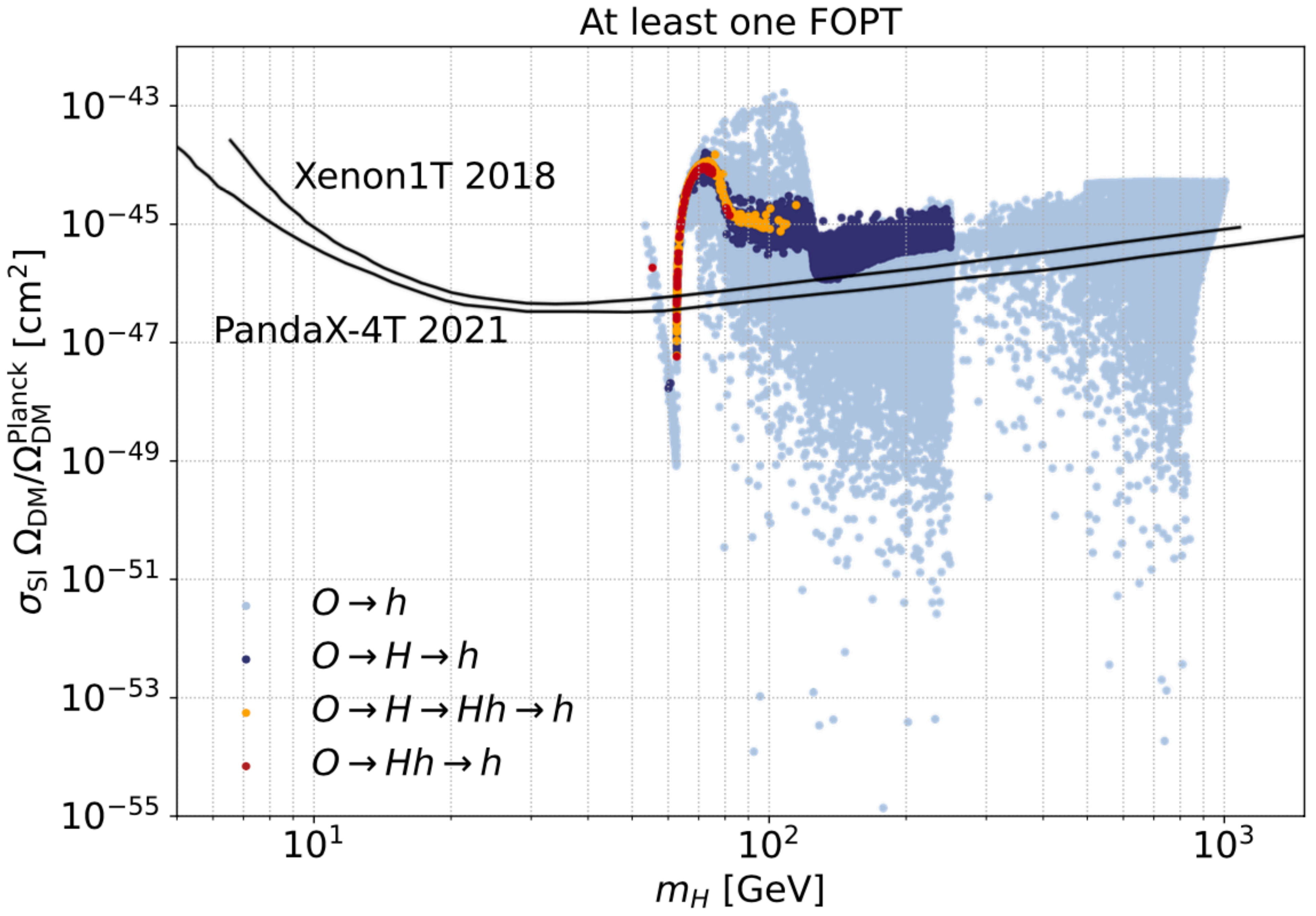
Where $$\lambda_{345} = \lambda_3 + \lambda_4 + \lambda_5,$$

Benincasa, Rose, Kannike and Marzola: *JCAP* 12 (2022) 025

FOPT with inert Doublet

- The points are rescale with DM under abundance I.e. $\frac{\Omega_{DM}}{\Omega^{planck}}$
- However, with current LZWS2022+LZWS2024 bounds, these points may be further ruled out arXiv:2410.17036 [hep-ex]

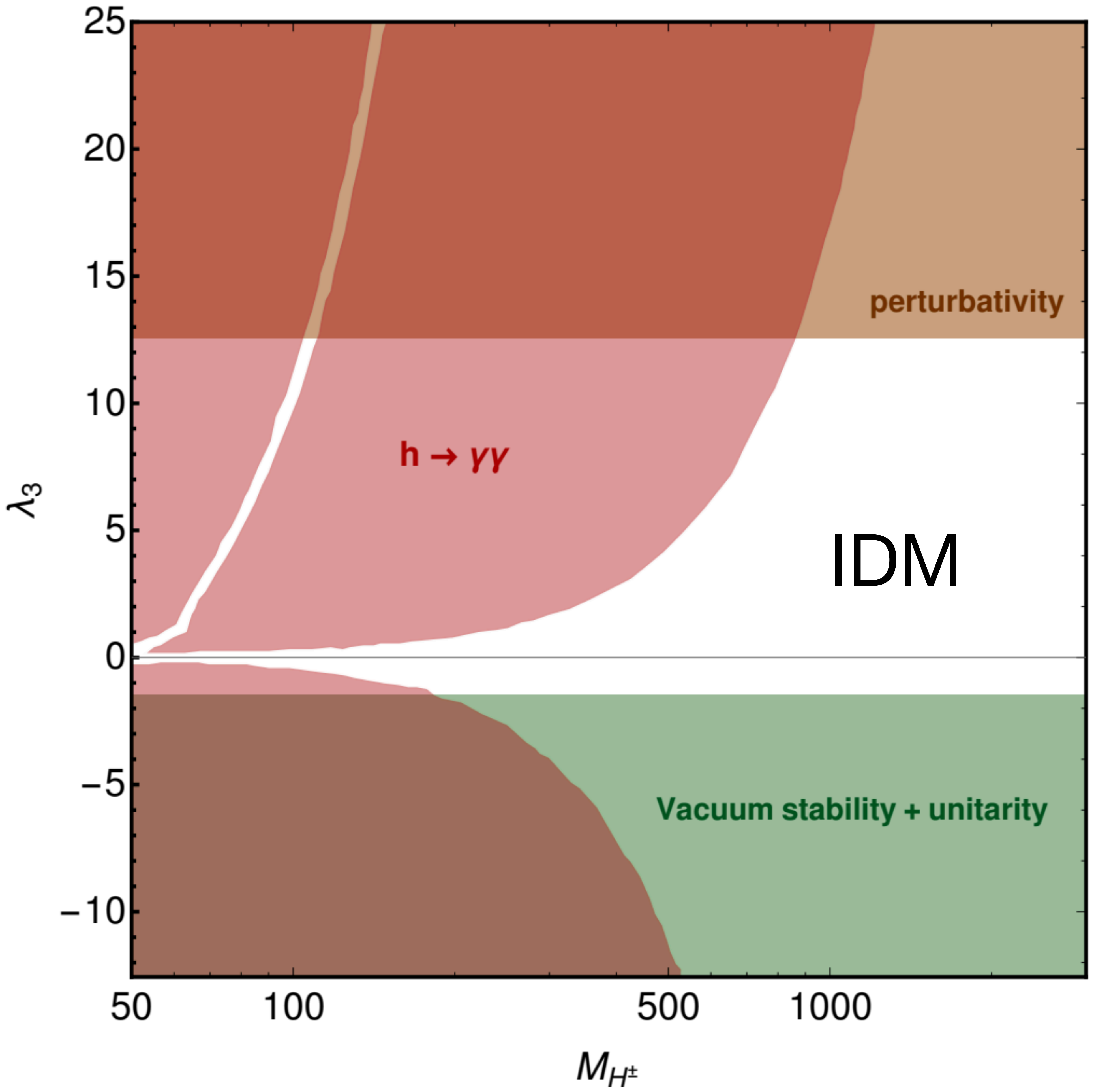




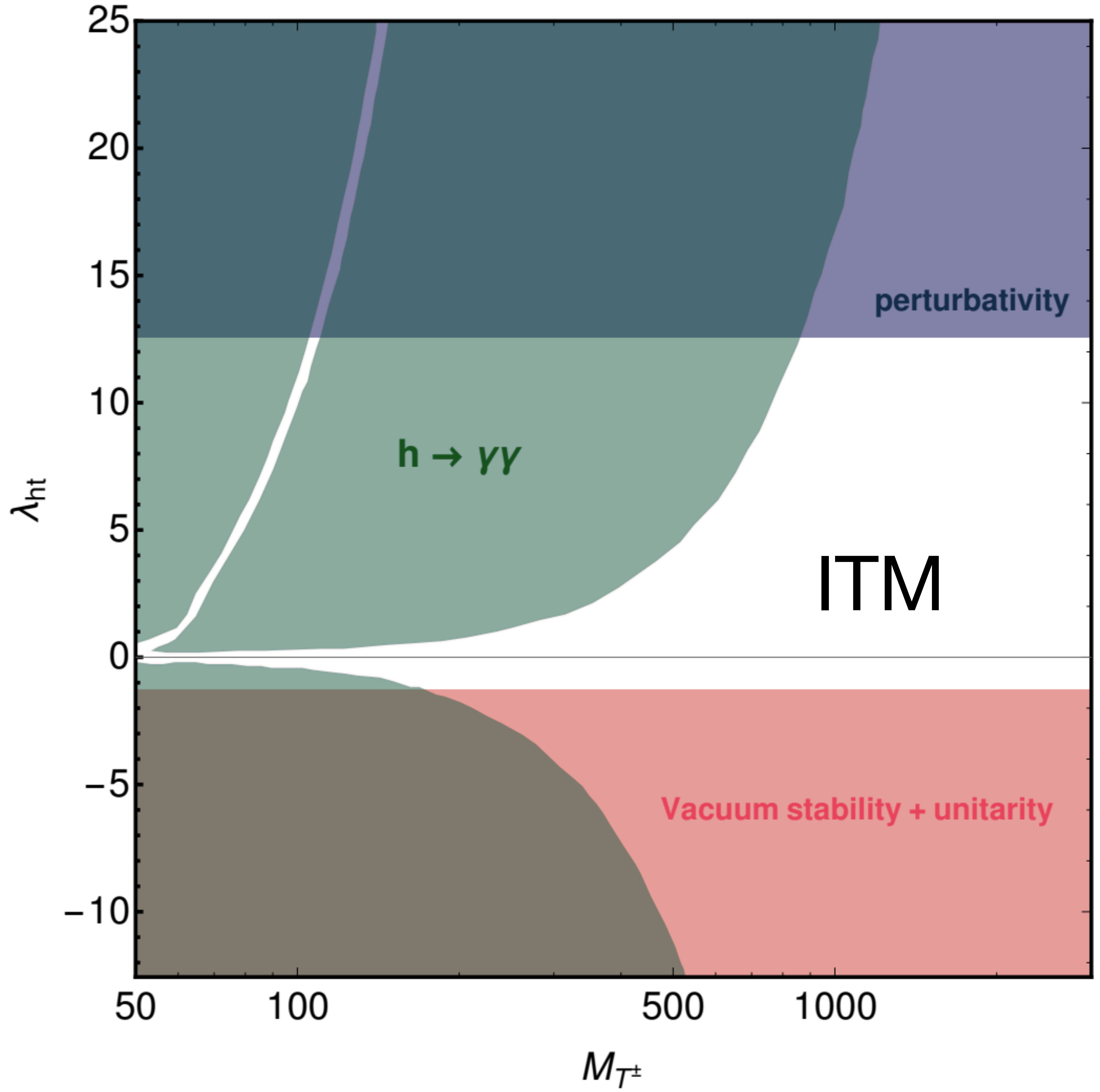
Benincasa, Rose, Kannike and Marzola: *JCAP* 12 (2022) 025

- Models with a charged Higgs like IDM and ITM will further get stringent bounds from $h \to \gamma\gamma$ measurement

$h \to \gamma\gamma$ constraints on IDM and ITM



PB, Mariana Frank, Snehasis Parashar, Chandrima Sen: *JHEP* 03 (2024) 109



PB, Jeonghyeon Song, Snehasis Parashar, Chandrima Sen: *JHEP* 07 (2024) 253

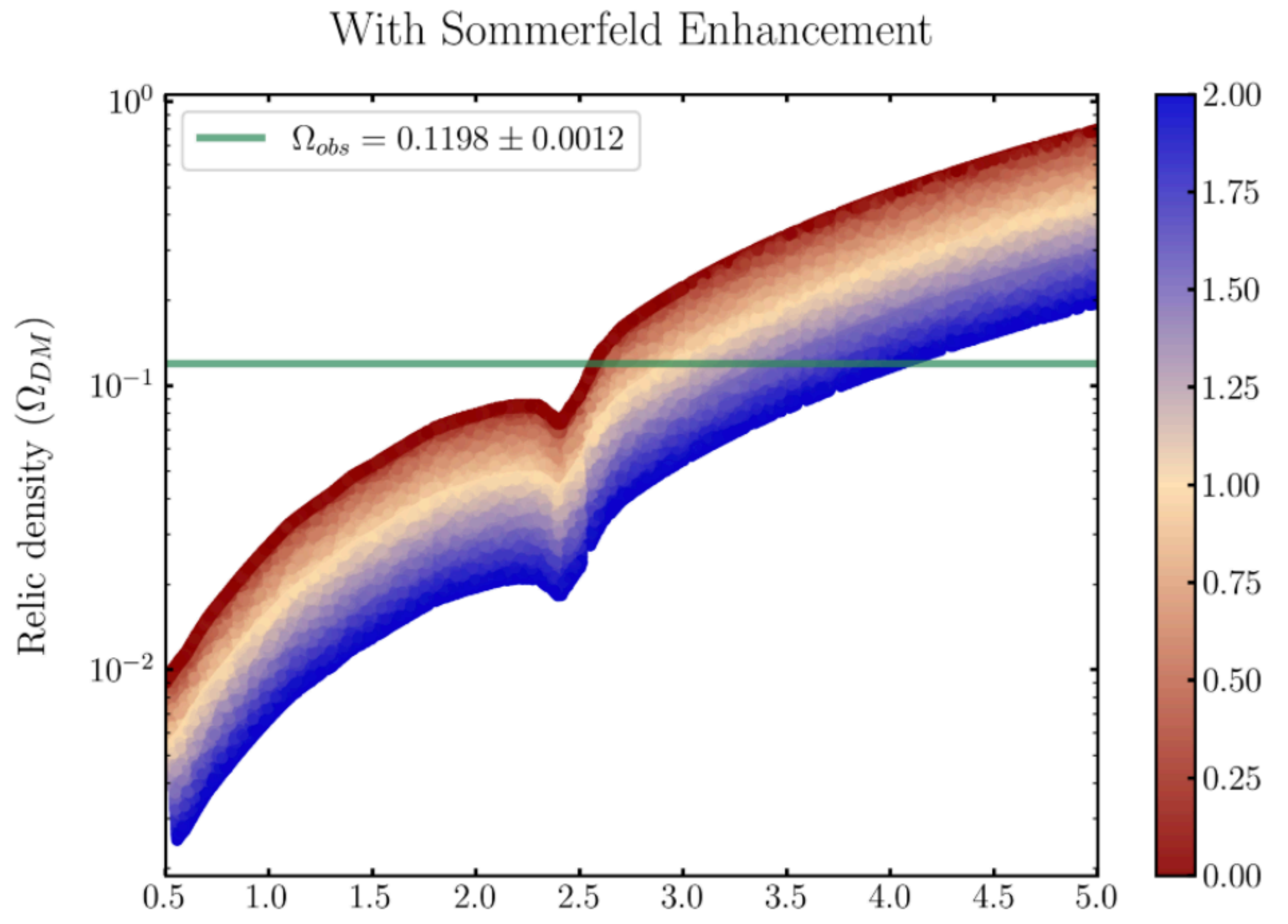
- $\mu_{\gamma\gamma} = 1.04^{+0.10}_{-0.09}$ from ATLAS JHEP 07 (2023) 088

Further constraints

- Sommerfeld enhancement requires more than TeV masses for correct DM relic for ITM

- Similar enhancement will change the parameter shape for IDM .. work in progress

- ISM also suffers heavily from the direct DM bounds



PB, Jeonghyeon Song, Snehasis Parashar, Chandrima Sen: *JHEP* 07 (2024) 253

Cheng-Wei Chiang, Bo-Qiang Lu: JHEP07(2020)082

Cheng-Wei Chiang, Da Huang, Bo-Qiang Lu: JCAP01(2021)035

- There are many other studies on non-inert scalar extensions addressing recent collider constraints

Purushottam, Tathagata, Shubhajit,

Dorival, Kaladharan, Wu, Thomas, ..

Conclusions

- Extra scalar are highly motivated for the stability of the Electroweak vacuum
- It can provide the much needed dark matter
- Models with Seesaw with relatively large Yukawa have potential problem with stability, which can be restored via addition of scalars
- FOPT needs lower mass and larger Higgs portal couplings
- For inert scenarios this corresponds to under abundant DM region
- HESS and FermiLat add further restrictions to low mass values
- Recent LZ bounds constraint the Higgs portal coupling to 5×10^{-3} for $\mathrm{m_{DM}} < 400\,\mathrm{GeV}$
- Models with extra charged scalar is now more constrained from $h \to \gamma\gamma$


THANK
YOU!