NEUTRINOS: INTRODUCTIONS

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Understanding the Universe Through Neutrinos
ICTS
April 22 – May 3, 2024

Tentative Outline for The First Couple of Lectures

1. Brief History of the Neutrino;
2. Neutrino Puzzles – The Discovery of Neutrino Masses;
3. Neutrino Oscillations;

[note: Questions/Suggestions/Complaints are ALWAYS welcome]

Some Neutrino references (WARNING: Biased Sample)

- "Are There Really Neutrinos? – An Evidential History," Allan Franklin, Perseus Books, 2001. Good discussion of neutrino history.
- A. de Gouvêa, "TASI lectures on neutrino physics," hep-ph/0411274;
- R. N. Mohapatra, A. Yu. Smirnov,"Neutrino Mass and New Physics," Ann. Rev. Nucl. Part. Sci. **56**, 569 (2006) [hep-ph/0603118];
- M. C. Gonzalez-Garcia, M. Maltoni, "Phenomenology with Massive Neutrinos," Phys. Rept. **460**, 1 (2008) [arXiv:0704.1800 [hep-ph]];
- C. Giunti and C.W. Kim,"Fundamentals of Neutrino Physics and Astrophysics," Oxford University Press (2007);
- "The Physics of Neutrinos,"V. Barger, D. Marfatia, K. Whisnant, Princeton University Press (2012);
- A. de Gouvêa *et al.*, "Working Group Report: Neutrinos," arXiv:1310:4340;
- A. de Gouvêa, "Neutrino Mass Models," Ann. Rev. Nucl. Part. Sci. **66**, 197 (2016).
- Some lectures at TASI 2020 plus a lot of Snowmass 2021 stuff; look for it in the arXiv.

1 - Brief History of the Neutrino

1. 1896: Henri Becquerel discovers natural radioactivity while studying phosphorescent properties of uranium salts.
   - α rays: easy to absorb, hard to bend, positive charge, mono-energetic;
   - β rays: harder to absorb, easy to bend, negative charge, spectrum?;
   - γ rays: no charge, very hard to absorb.
2. 1897: J.J. Thompson discovers the electron.
3. 1914: Chadwick presents definitive evidence for a continuous β-ray spectrum. Origin unkown. Different options include several different energy loss mechanisms.

It took 15+ years to decide that the "real" β-ray spectrum was really continuous. Reason for continuous spectrum was a total mystery:

- QM: Spectra are discrete;
- Energy-momentum conservation: $N \to N' + e^-$ — electron energy and momentum well-defined.

Nuclear Physics before 1930: nucleus $= n_p p + n_e e^-$.

Example: ^{4}He $= 4p + 2e^-$, works well. However: ^{14}N $= 14p + 7e^-$ is expected to be a fermion. However, it was experimentally known that ^{14}N was a boson!

There was also a problem with the magnetic moment of nuclei: $\mu_N, \mu_p \ll \mu_e$ $(\mu = eh/4mc)$. How can the nuclear magnetic moment be so much smaller than the electron one if the nucleus contains electrons?

SOLUTION: Bound, nuclear electrons are very weird!

This can also be used to solve the continuous β-ray spectrum: energy need not be conserved in nuclear processes! (N. Bohr)

"... This would mean that the idea of energy and its conservation fails in dealing with processes involving the emission and capture of nuclear electrons. This does not sound improbable if we remember all that has been said about peculiar properties of electrons in the nucleus." (G. Gamow, Nuclear Physics Textbook, 1931).

enter the neutrino...

1. 1930: Postulated by Pauli to (a) resolve the problem of continuous β-ray spectra, and (b) reconcile nuclear model with spin-statistics theorem. $\Rightarrow$
2. 1932: Chadwick discovers the neutron.

   neutron $\neq$ Pauli's neutron = neutrino (Fermi);
3. 1934: Fermi theory of Weak Interactions – current-current interaction

   $$\mathcal{H} \sim G_F \left(\bar{p}\Gamma n\right)\left(\bar{e}\Gamma \nu_e\right), \quad \text{where } \Gamma = \{1, \gamma_5, \gamma_\mu, \gamma_\mu\gamma_5, \sigma_{\mu\nu}\}$$

   Way to "see" neutrinos: $\bar{\nu}_e + p \to e^+ + n$. Prediction for the cross-section - too small to ever be observed...
4. 1935: (Yukawa postulates the existence of mesons (pions) as mediators of the nuclear (strong) force: $m_\pi \sim 100$ MeV.)
5. 1936/37: ("Meson" discovered in cosmic rays. Another long, tortuous story. Turns out to be the muon...)
6. 1947: (Marshak, Bethe postulate the 2 meson hypothesis ($\pi \to \mu$). Pion observed in cosmic rays.)

Dear Radioactive Ladies and Gentlemen,

I have come upon a desperate way out regarding the wrong statistics of the ^{14}N and ^{6}Li nuclei, as well as the continuous β-spectrum, in order to save the "alternation law" statistics and the energy law. To wit, the possibility that there could exist in the nucleus electrically neutral particles, which I shall call "neutrons," and satisfy the exclusion principle... The mass of the neutrons should be of the same order of magnitude as the electron mass and in any case not larger than 0.01 times the proton mass. The continuous β-spectrum would then become understandable from the assumption that in β-decay a neutron is emitted along with the electron, in such a way that the sum of the energies of the neutron and the electron is constant...

For the time being I dare not publish anything about this idea and address myself to you, dear radioactive ones, with the question how it would be with experimental proof of such a neutron, if it were to have the penetrating power equal to about ten times larger than a γ-ray.

I admit that my way out may not seem very probable *a priori* since one would probably have seen the neutrons a long time ago if they exist. But only the one who dares wins, and the seriousness of the situation concerning the continuous β-spectrum is illuminated by my honored predecessor, Mr Debye who recently said to me in Brussels: "Oh, it is best not to think about this at all, as with new taxes."

One must therefore discuss seriously every road to salvation. Thus, dear radioactive ones, examine and judge. Unfortunately, I cannot appear personally in Tübingen since a ball... in Zürich... makes my presence here indispensible....

Your most humble servant, W. Pauli

Adapted summary of an English Translation to Pauli's letter dated December 4, 1930, from Ref. 3.

observing the unobservable:

1. 1956: "Discovery" of the neutrino (Reines and Cowan) in the Savannah River Nuclear Reactor site. $\Rightarrow$

   $\bar{\nu}_e + p \to e^+ + n$. Measure positron ($e^+e^- \to \gamma$s) and neutron ($nN \to N^* \to N + \gamma$s) in delayed coincidence in order to get rid of backgrounds.

2. 1958: Neutrino Helicity Measured (Goldhaber et al.). Neutrinos are purely left-handed. Interact only weakly (Parity violated maximally).

   $e^- + ^{152}\text{Eu}(J=0) \to ^{152}\text{Sm}^*(J=1) + \nu \to ^{152}\text{Sm}(J=0) + \nu + \gamma$

3. 1962: The second neutrino: $\nu_\mu \neq \nu_e$ (Lederman, Steinberger, Schwartz at BNL). First neutrino beam.

   $$p + Z \to \pi^+ X \to \mu^+ \nu_\mu \quad \Rightarrow \quad \begin{array}{l} \nu_\mu + Z \to \mu^- + Y \text{ ("always")} \\ \nu_\mu + Z \to e^- + Y \text{ ("never")} \end{array}$$

4. 2001: ν_τ directly observed (DONUT experiment at FNAL). Same strategy: $\nu_\tau + Z \to \tau^- + Y$. ($\tau$-leptons discovered in the 1970's). $\Rightarrow$

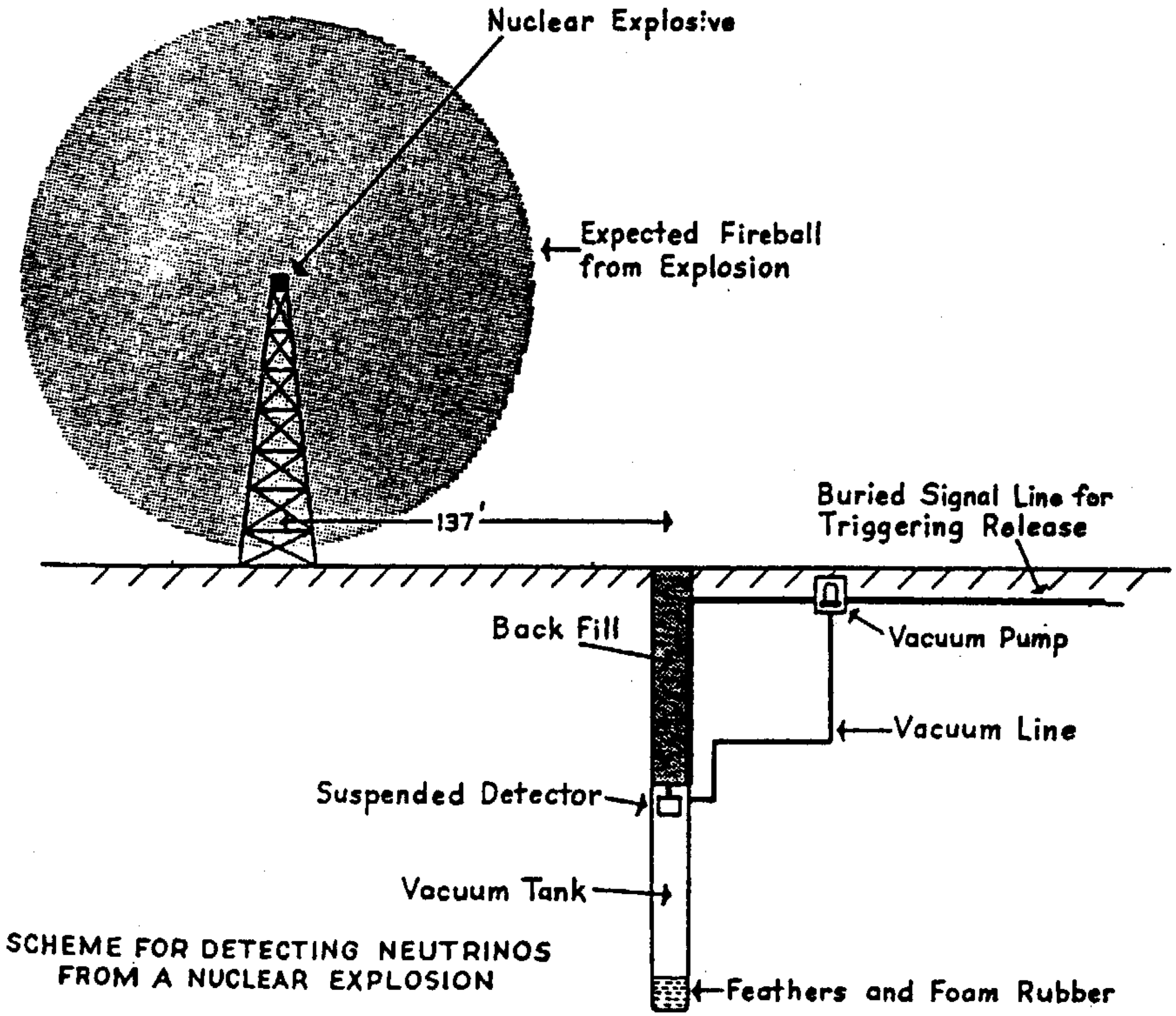


FIGURE 5.1 Scheme for detecting neutrinos from a nuclear e
(Cowan, 1964).

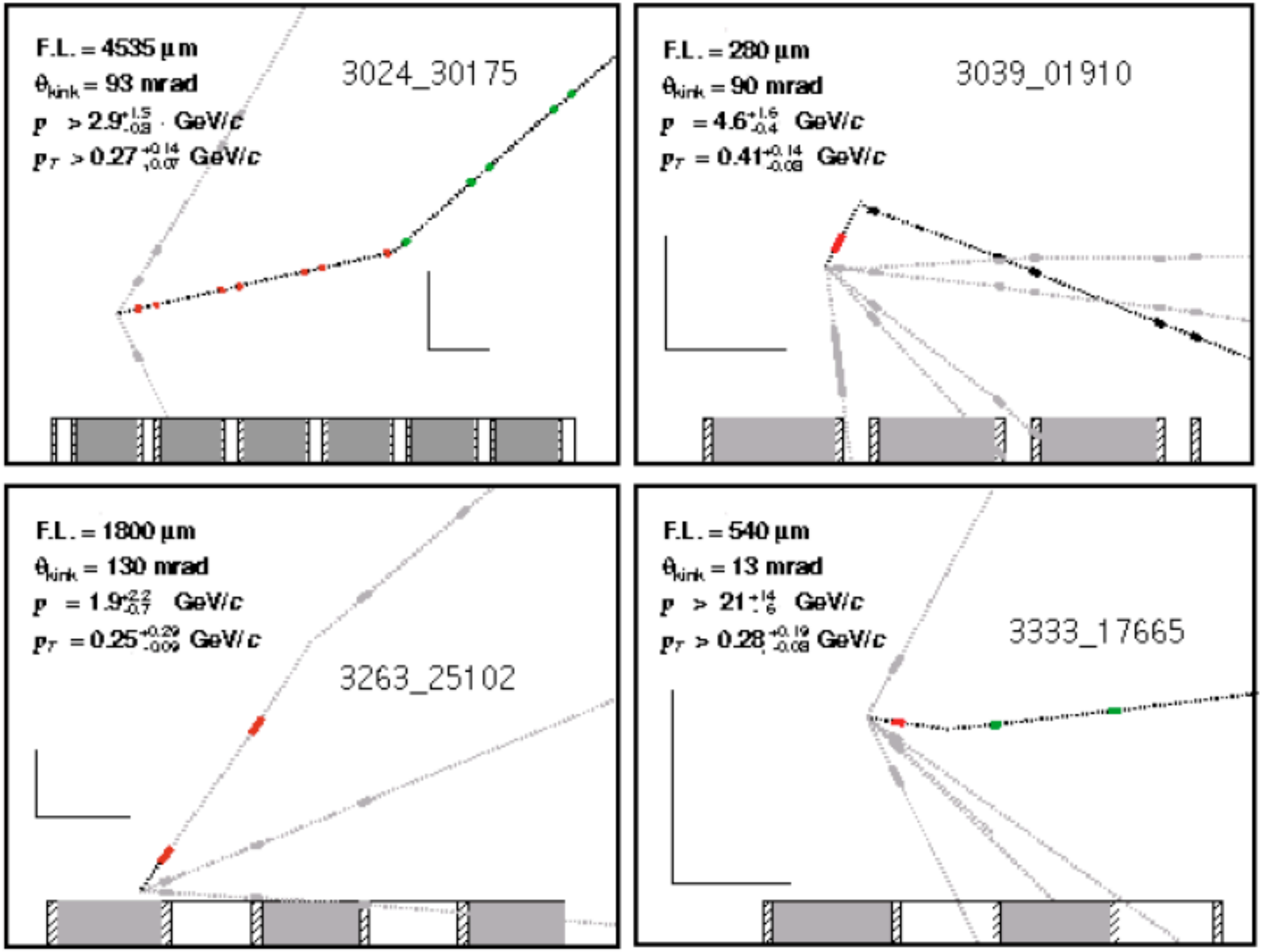


Figure 4-6: The four tau neutrino charged current events. The scale is given by the perpendicular lines (vertical: 0.1 mm, horizontal: 1 mm). The bar on the bottom shows the target material (solid: steel, hatched: emulsion, clear: plastic base).

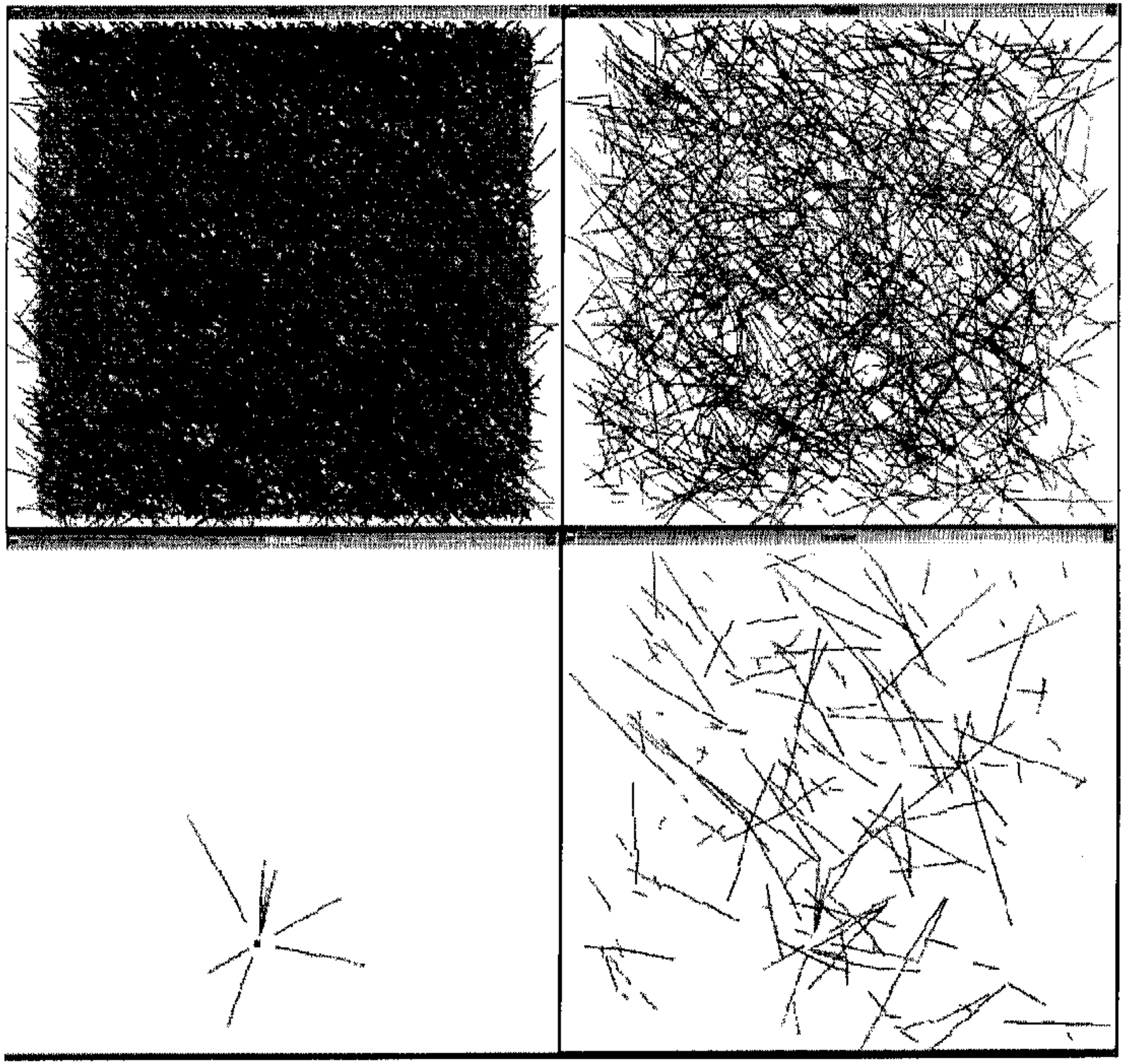
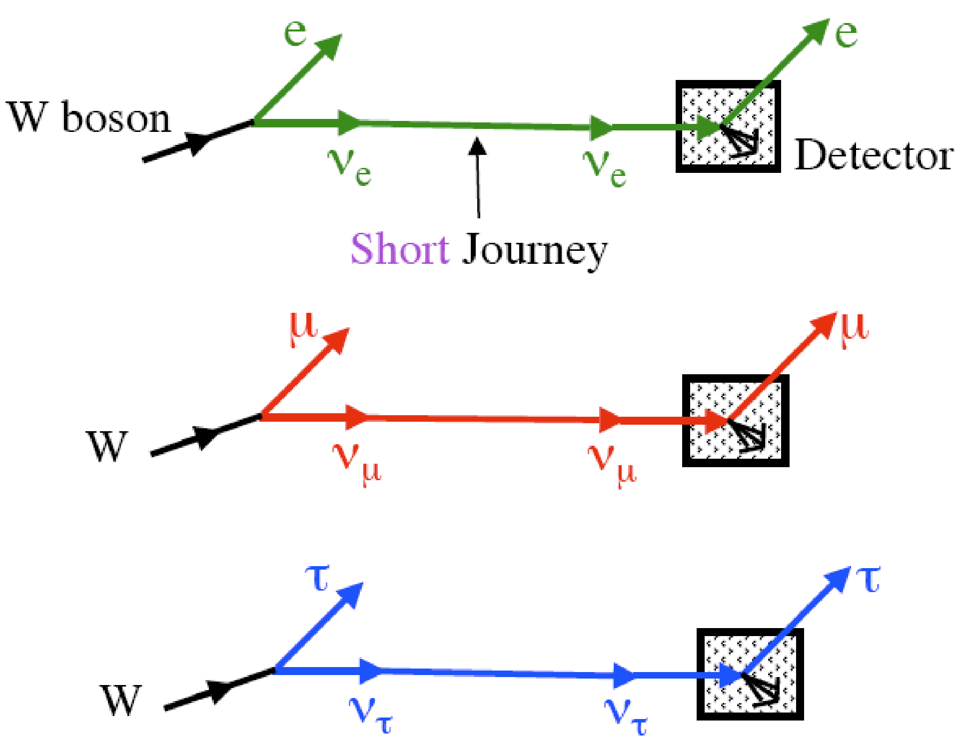


図 5.12: net scan 反応点探索の各段階 (左上から時計回り)。1) 読み込んだ全ての飛跡 ($5 \times 5 mm^2$)、2) 測定領域を突き抜けている飛跡の排除、3) 低運動量の飛跡の排除、4) 一点 ($4\mu m$ 以内) 収束している飛跡

HUNTING FOR

What we Knew of Neutrinos: End of the 20th Century



- come in three flavors (see figure);
- interact only via weak interactions ($W^\pm, Z^0$);
- have ZERO mass – helicity good quantum number;
- ν_L field describes 2 degrees of freedom:
  - left-handed state ν,
  - right-handed state $\bar{\nu}$ (CPT conjugate);
- neutrinos carry lepton number:
  - $L(\nu) = +1$,
  - $L(\bar{\nu}) = -1$.

2– Neutrino Puzzles – 1960's to 2000's

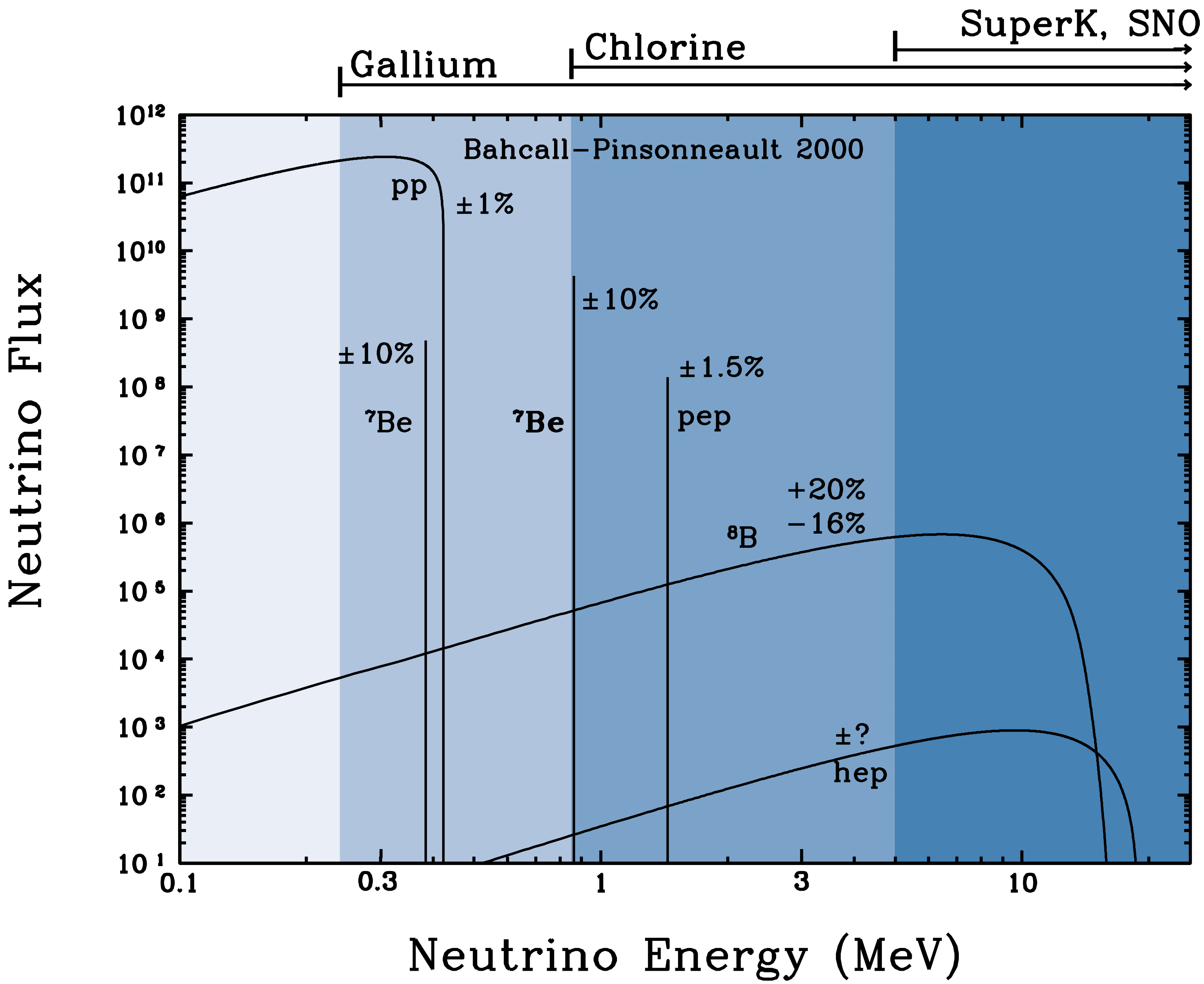
Long baseline neutrino experiments have revealed that neutrinos change flavor after propagating a finite distance, violating the definitions in the previous slide. The rate of change depends on the neutrino energy E_ν and the baseline L.

- $\nu_\mu \to \nu_\tau$ and $\bar{\nu}_\mu \to \bar{\nu}_\tau$ — atmospheric experiments ["indisputable"];
- $\nu_e \to \nu_{\mu,\tau}$ — solar experiments ["indisputable"];
- $\bar{\nu}_e \to \bar{\nu}_{\rm other}$ — reactor neutrinos ["indisputable"];
- $\nu_\mu \to \nu_{\rm other}$ — from accelerator experiments ["indisputable"].

Table 1. Nuclear reactions responsible for producing almost all of the Sun's energy and the different "types" of solar neutrinos (nomenclature): *pp*-neutrinos, *pep*-neutrinos, *hep*-neutrinos, ^{7}Be-neutrinos, and ^{8}B-neutrinos. 'Termination' refers to the fraction of interacting protons that participate in the process.

Reaction	Termination (%)	Neutrino Energy (MeV)	Nomenclature
$p + p \rightarrow {}^2\mathrm{H} + e^+ + \nu_e$	99.96	< 0.423	*pp*-neutrinos
$p + e^- + p \rightarrow {}^2\mathrm{H} + \nu_e$	0.044	1.445	*pep*-neutrinos
${}^2\mathrm{H} + p \rightarrow {}^3\mathrm{He} + \gamma$	100	–	–
${}^3\mathrm{He} + {}^3\mathrm{He} \rightarrow {}^4\mathrm{He} + p + p$	85	–	–
${}^3\mathrm{He} + {}^4\mathrm{He} \rightarrow {}^7\mathrm{Be} + \gamma$	15	–	–
${}^7\mathrm{Be} + e^- \rightarrow {}^7\mathrm{Li} + \nu_e$	15	0.863(90%) 0.386(10%)	^{7}Be-neutrinos
${}^7\mathrm{Li} + p \rightarrow {}^4\mathrm{He} + {}^4\mathrm{He}$		–	–
${}^7\mathrm{Be} + p \rightarrow {}^8\mathrm{B} + \gamma$	0.02	–	–
${}^8\mathrm{B} \rightarrow {}^8\mathrm{Be}^* + e^+ + \nu_e$		< 15	^{8}B-neutrinos
${}^8\mathrm{Be} \rightarrow {}^4\mathrm{He} + {}^4\mathrm{He}$		–	–
${}^3\mathrm{He} + p \rightarrow {}^4\mathrm{He} + e^+ + \nu_e$	0.00003	< 18.8	*hep*-neutrinos

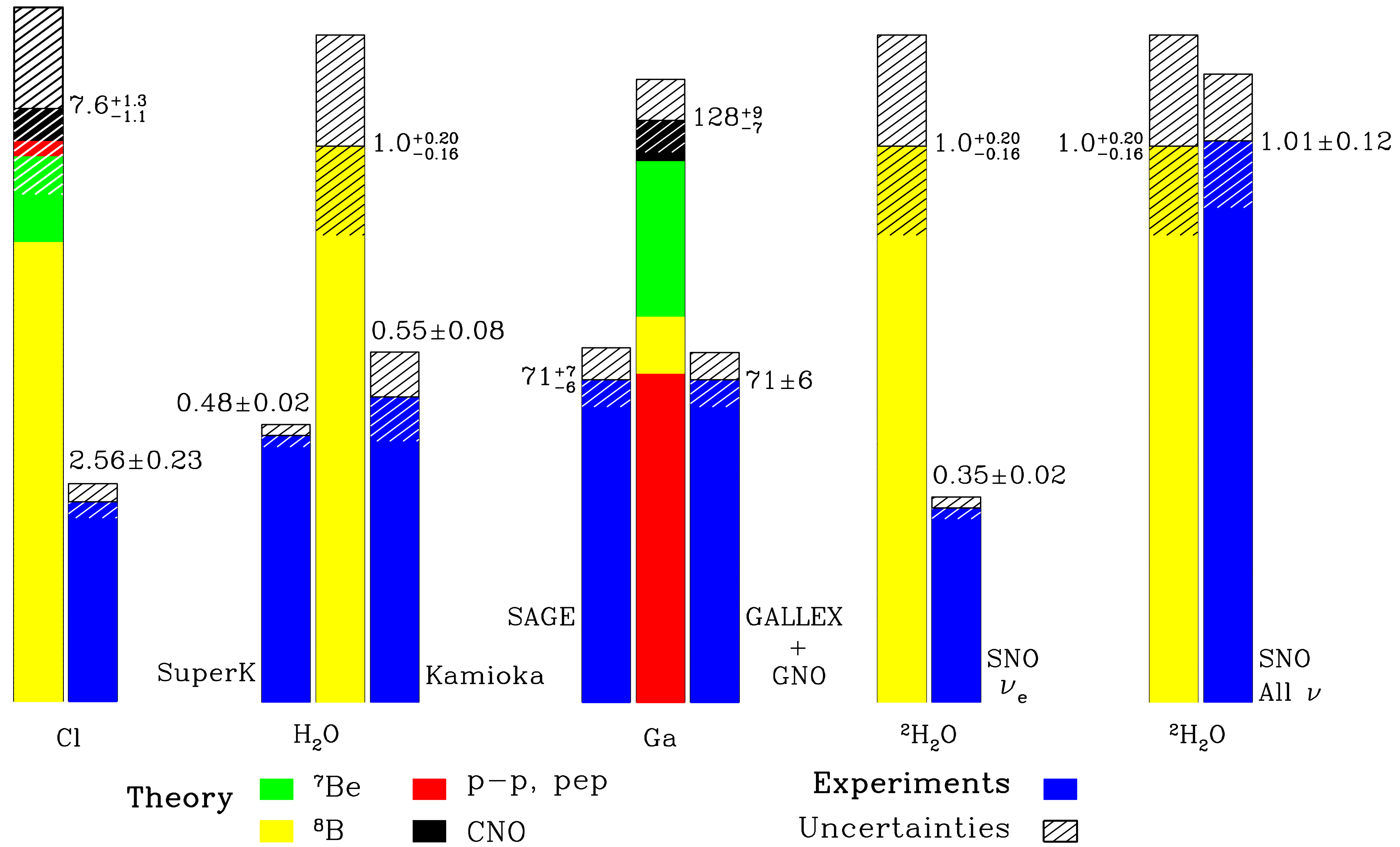
Note: Adapted from Ref. 12. Please refer to Ref. 12 for a more detailed explanation.



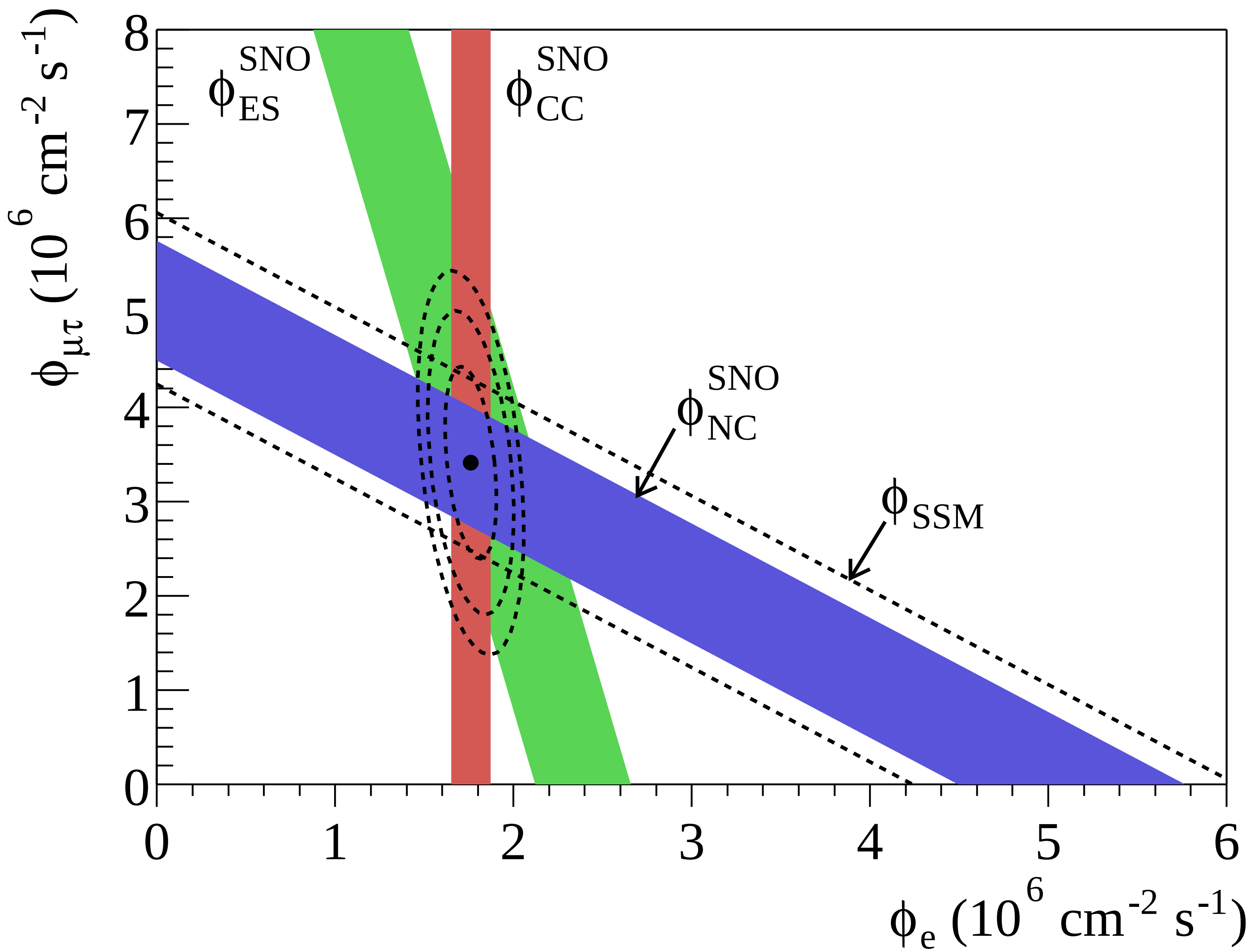
SuperK, SNO
Chlorine
Gallium
Bahcall–Pinsonneault 2000
pp
±1%
±10%
±10%
^{7}Be
^{7}Be
±1.5%
pep
+20%
−16%
^{8}B
±?
hep
Neutrino Flux
Neutrino Energy (MeV)
10^{12}
10^{11}
10^{10}
10^{9}
10^{8}
10^{7}
10^{6}
10^{5}
10^{4}
10^{3}
10^{2}
10^{1}
0.1
0.3
1
3
10

Total Rates: Standard Model vs. Experiment

Bahcall–Pinsonneault 2000



The SNO Experiment: conclusive evidence for flavor change



SNO Measures:

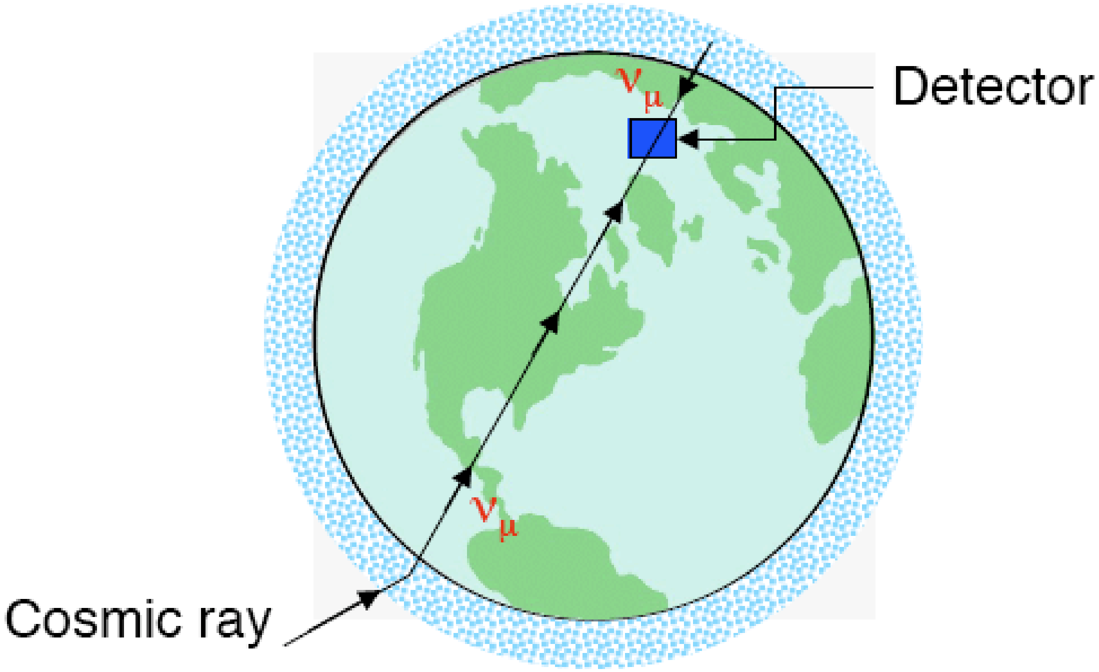
$[CC]\ \nu_e + {}^2H \to p + p + e^-$

$[ES]\ \nu + e^- \to \nu + e^-$

$[NC]\ \nu + {}^2H \to p + n + \nu$

different reactions sensitive to different neutrino flavors.

Atmospheric Neutrinos



Isotropy of the $\gtrsim 2$ GeV cosmic rays + Gauss' Law + No ν_μ disappearance

$$\Rightarrow \quad \frac{\phi_{\nu_\mu}(\mathrm{Up})}{\phi_{\nu_\mu}(\mathrm{Down})} = 1 \ .$$

But Super-Kamiokande finds for $E_\nu > 1.3$ GeV

$$\frac{\phi_{\nu_\mu}(\mathrm{Up})}{\phi_{\nu_\mu}(\mathrm{Down})} = 0.54 \pm 0.04 \ .$$

UP $\neq$ DOWN – neutrinos can tell time! $\rightarrow$ neutrinos have mass.

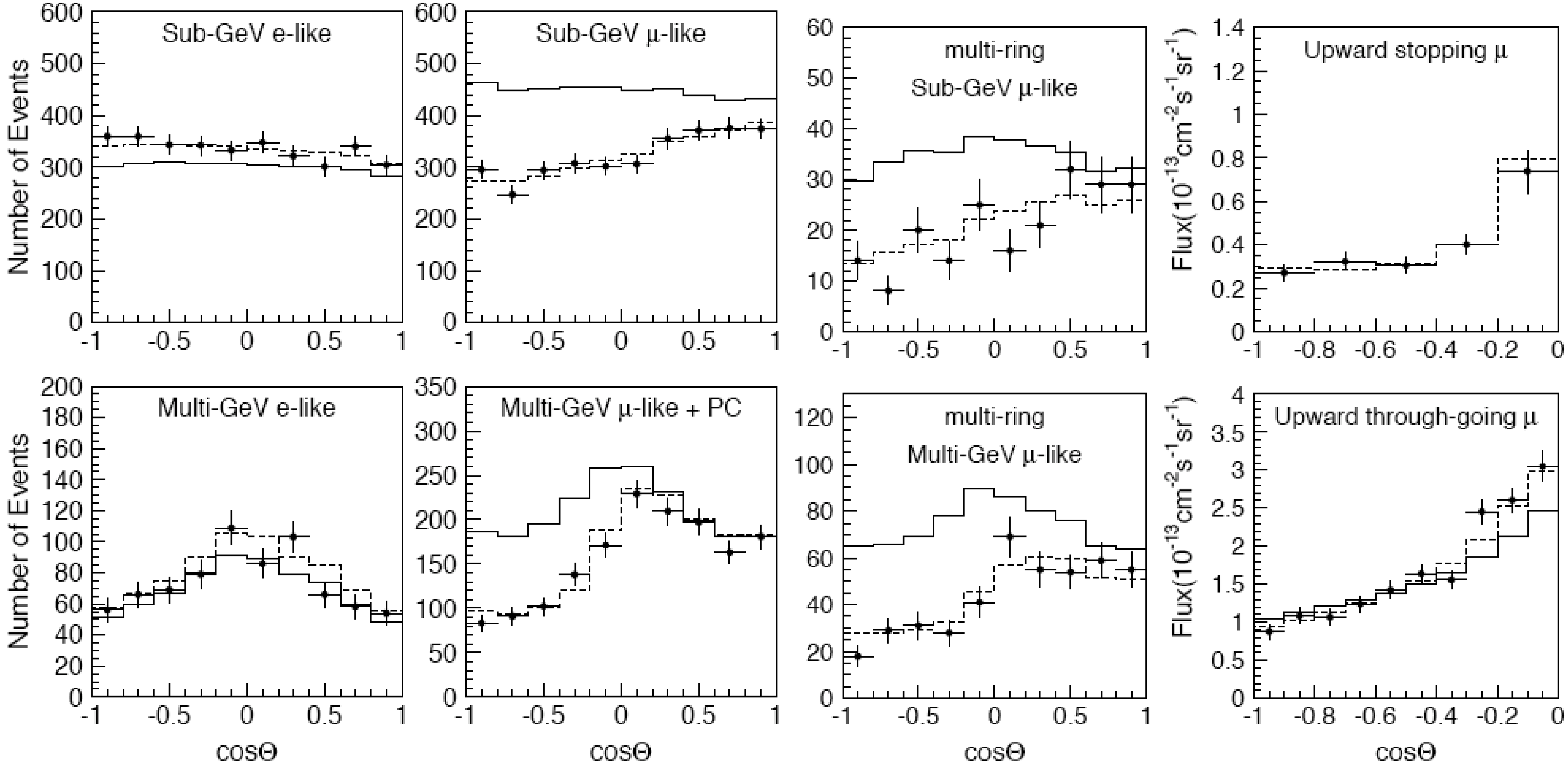


Figure 4. Zenith angle distribution for fully-contained single-ring e-like and μ-like events, multi-ring μ-like events, partially contained events and upward-going muons. The points show the data and the solid lines show the Monte Carlo events without neutrino oscillation. The dashed lines show the best-fit expectations for $\nu_\mu \leftrightarrow \nu_\tau$ oscillations. From M. Ishitsuka [Super-Kamiokande Collaboration], hep-ex/0406076.

3 - Mass-Induced Neutrino Flavor Oscillations

Neutrino Flavor change can arise out of several different mechanisms. The simplest one is to appreciate that, once **neutrinos have mass**, **leptons can mix**. This turns out to be the correct mechanism (certainly the dominant one), and **only** explanation that successfully explains **all** long-baseline data consistently.

Neutrinos with a well defined mass:

$$\nu_1, \nu_2, \nu_3, \ldots \qquad \text{with masses} \;\; m_1, m_2, m_3, \ldots$$

How do these states (neutrino mass eigenstates) relate to the neutrino flavor eigenstates $(\nu_e, \nu_\mu, \nu_\tau)$?

$$\nu_\alpha = U_{\alpha i}\nu_i \qquad \alpha = e, \mu, \tau, \quad i = 1, 2, 3$$

U is a unitary mixing matrix. I'll talk more about it later.

The Propagation of Massive Neutrinos

Neutrino mass eigenstates are eigenstates of the free-particle Hamiltonian:

$$|\nu_i\rangle = e^{-iE_it}|\nu_i\rangle, \qquad E_i^2 - |\vec{p}_i|^2 = m_i^2$$

The neutrino flavor eigenstates are linear combinations of ν_i's, say:

$$\begin{aligned} |\nu_e\rangle &= \cos\theta|\nu_1\rangle + \sin\theta|\nu_2\rangle. \\ |\nu_\mu\rangle &= -\sin\theta|\nu_1\rangle + \cos\theta|\nu_2\rangle. \end{aligned}$$

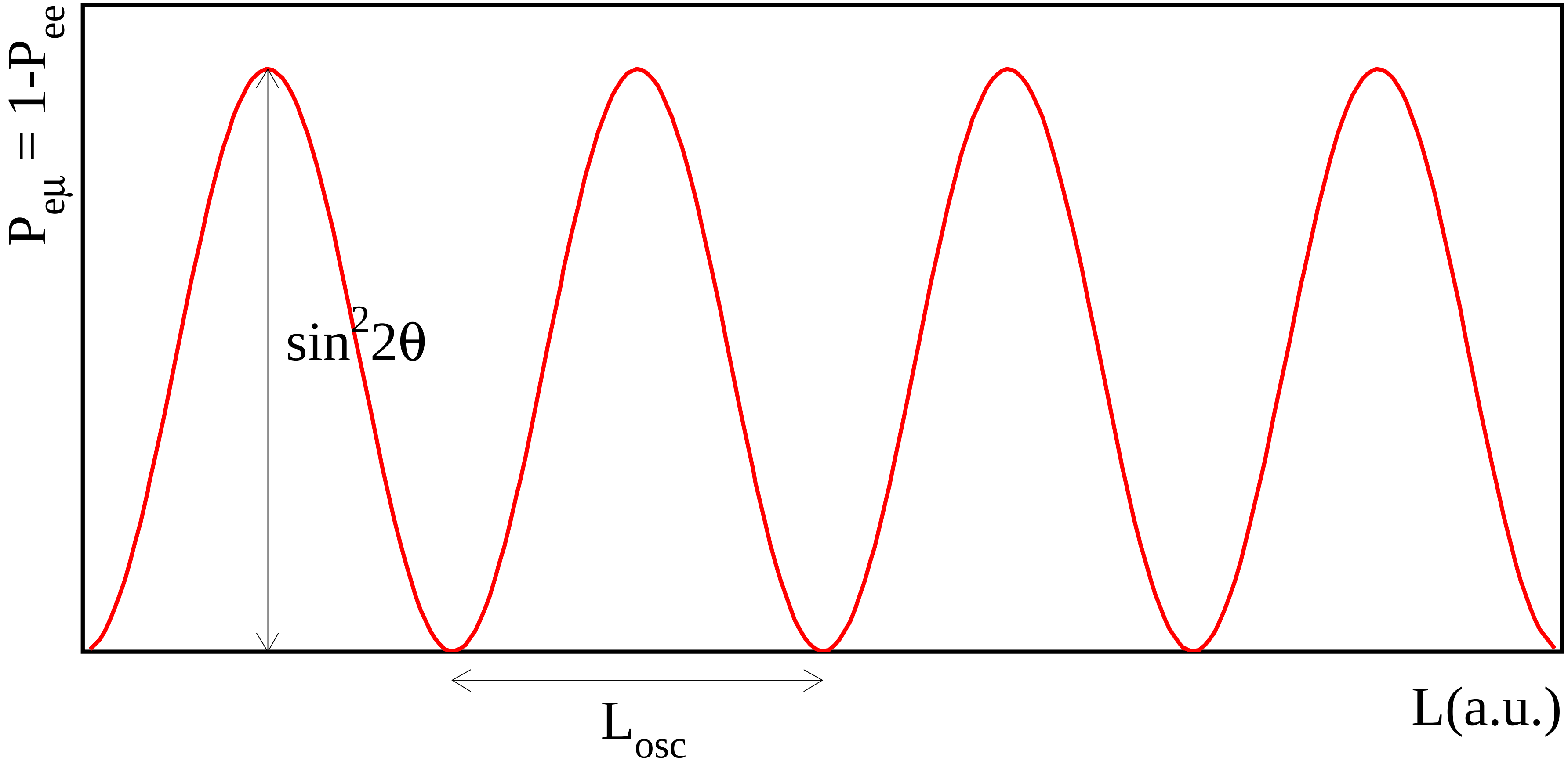
If this is the case, a state produced as a ν_e evolves in vacuum into

$$|\nu(t,\vec{x})\rangle = \cos\theta e^{-ip_1x}|\nu_1\rangle + \sin\theta e^{-ip_2x}|\nu_2\rangle.$$

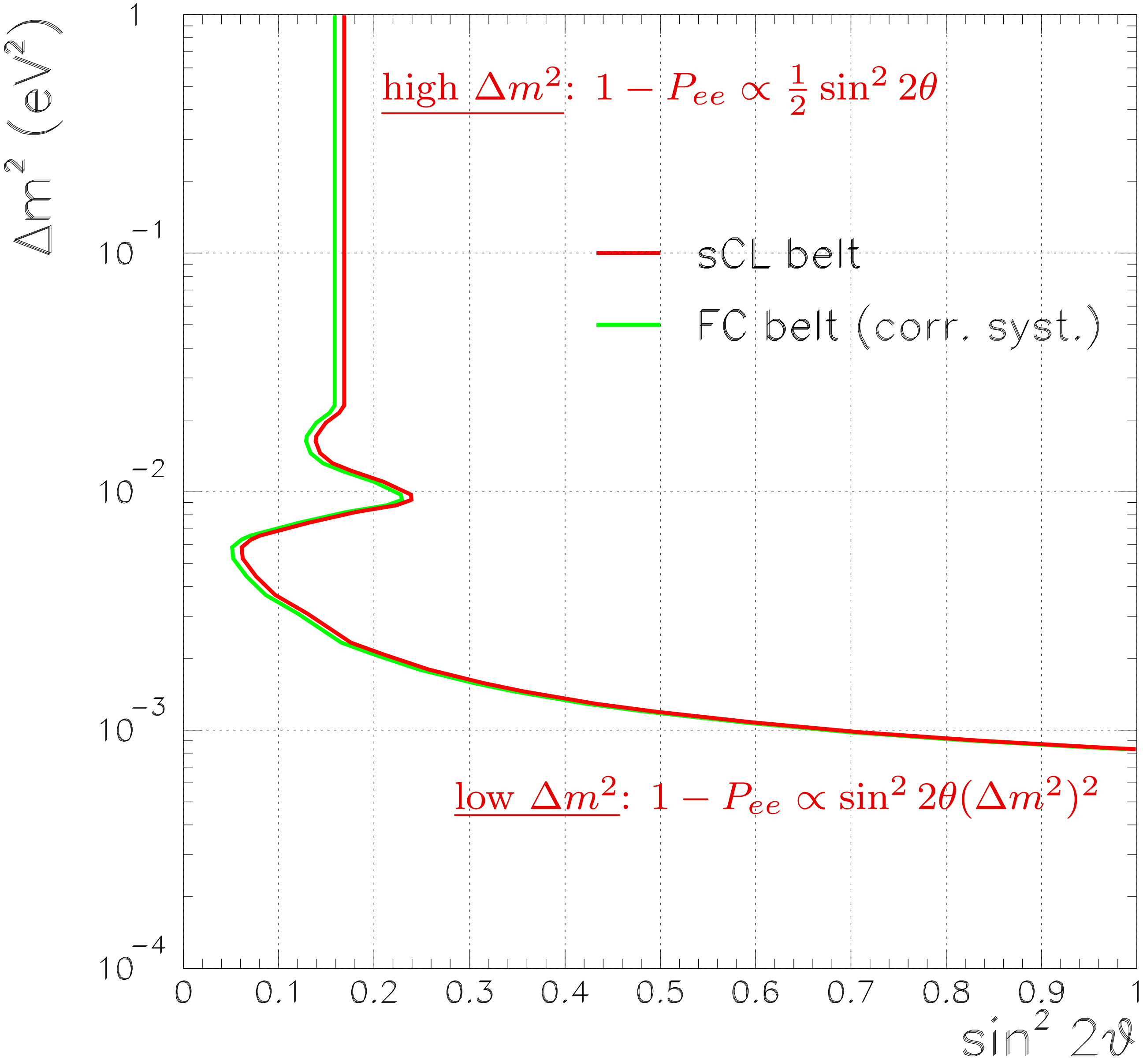
It is trivial to compute $P_{e\mu}(L) \equiv |\langle\nu_\mu|\nu(t,z=L)\rangle|^2$. It is just like a two-level system from basic undergraduate quantum mechanics! In the ultrarelativistic limit (always a good bet), $t \simeq L$, $E_i - p_{z,i} \simeq (m_i^2)/2E_i$, and

$$\boxed{P_{e\mu}(L) = \sin^2 2\theta \sin^2\left(\frac{\Delta m^2 L}{4E_\nu}\right)}$$

oscillation parameters: $\begin{cases} \pi \frac{L}{L_{\rm osc}} \equiv \frac{\Delta m^2 L}{4E} = 1.267 \left(\frac{L}{\rm km}\right) \left(\frac{\Delta m^2}{\rm eV^2}\right) \left(\frac{\rm GeV}{E}\right) \\ \text{amplitude } \sin^2 2\theta \end{cases}$



CHOOZ experiment



$$P_{ee} = 1 - \sin^2 2\theta \sin^2\left(\frac{\Delta m^2 L}{4E}\right)$$

[by-now-old result: $1 - P_{ee} < 0.05$]

There is a long (and oftentimes confused and confusing) history behind this derivation and several others. A comprehensive discussion can be found, for example, in

E.K. Akhmedov, A. Yu. Smirnov, 0905.1903 [hep-ph]

In a nutshell, neutrino oscillations as described above occur whenever

- Neutrino Production and Detection are Coherent $\rightarrow$ cannot "tell" ν_1 from ν_2 from ν_3 but "see" ν_e or ν_μ or ν_τ.
- Decoherence effects due to wave-packet separation are negligible $\rightarrow$ baseline not too long that different "velocity" components of the neutrino wave-packet have time to physically separate.
- The energy released in production and detection is large compared to the neutrino mass $\rightarrow$ so we can assign all of the effect to the neutrino propagation, independent from the production process. Also assures ultra-relativistic approximation good.

$P_{\mu\mu} = 1 - \sin^2 2\theta \sin^2\left(\frac{\Delta m^2 L}{4E}\right)$ Works great for $\sin^2 2\theta \sim 1$ and $\Delta m^2 \sim 10^{-3}$ eV^2

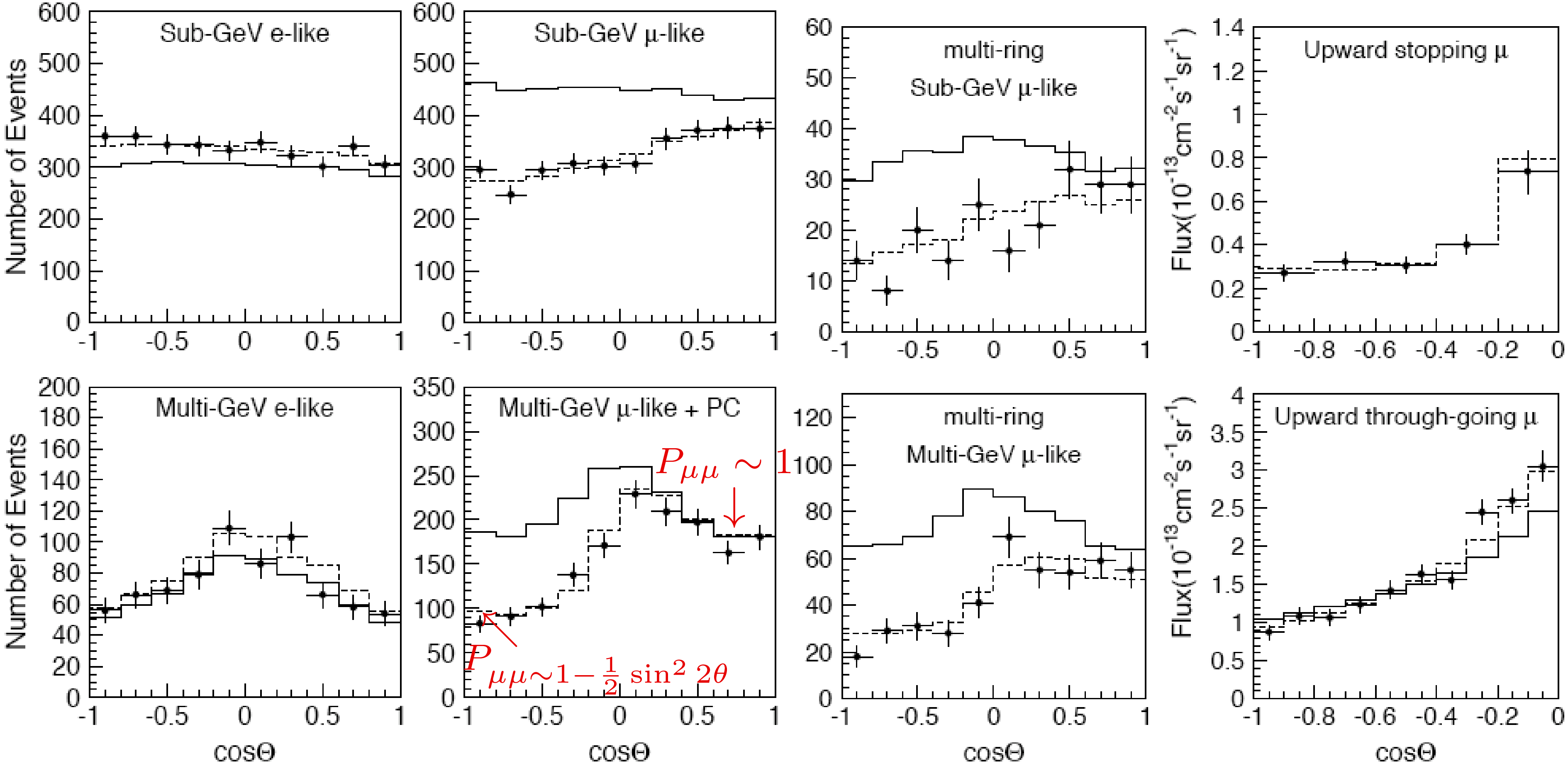
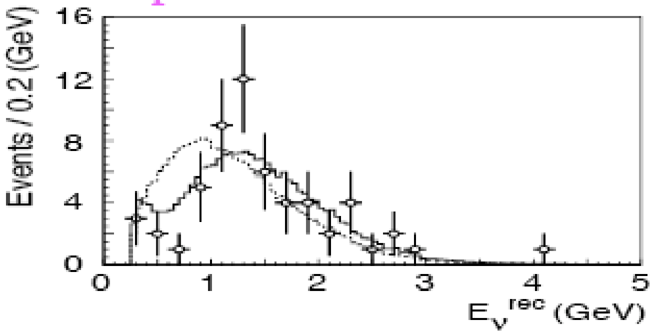


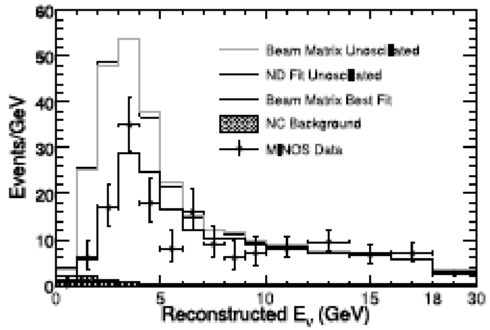
Figure 4. Zenith angle distribution for fully-contained single-ring e-like and μ-like events, multi-ring μ-like events, partially contained events and upward-going muons. The points show the data and the solid lines show the Monte Carlo events without neutrino oscillation. The dashed lines show the best-fit expectations for $\nu_\mu \leftrightarrow \nu_\tau$ oscillations. From M. Ishitsuka [Super-Kamiokande Collaboration], hep-ex/0406076.

K2K	ν_μ at KEK	SK	L=250 km
MINOS	ν_μ at Fermilab	Soundan	L=735 km
Opera/Icarus	ν_μ at CERN	Gran Sasso	L=740 km

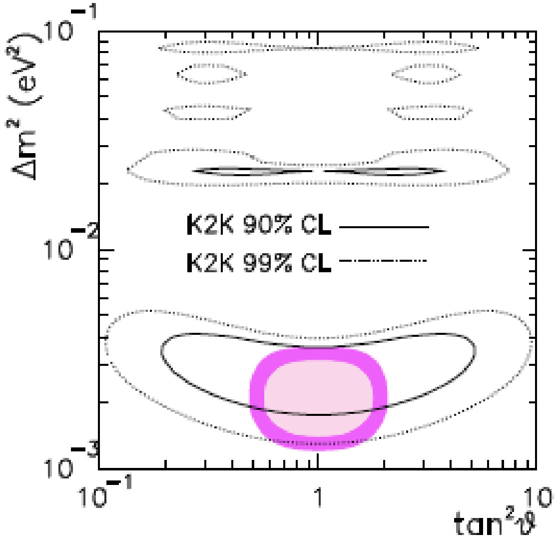
K2K 2004: spectral distortion



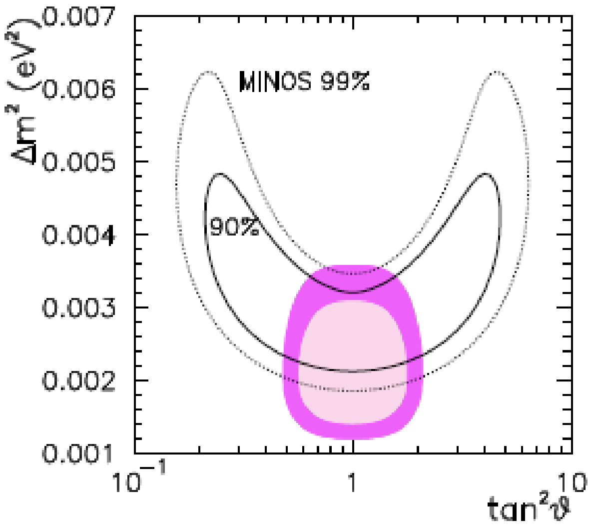
MINOS 2006: spectral distortion



Confirmation of ATM oscillations



Confirmation of ATM oscillations



[Gonzalez-Garcia, PASI 2006]

Matter Effects

The neutrino propagation equation, in the ultra-relativistic approximation, can be re-expressed in the form of a Shrödinger-like equation. In the mass basis:

$$i\frac{\mathrm{d}}{\mathrm{d}L}|\nu_i\rangle = \frac{m_i^2}{2E}|\nu_i\rangle,$$

up to a term proportional to the identity. In the weak/flavor basis

$$i\frac{\mathrm{d}}{\mathrm{d}L}|\nu_\beta\rangle = U_{\beta i}\frac{m_i^2}{2E}U^\dagger_{i\alpha}|\nu_\alpha\rangle.$$

In the 2×2 case,

$$i\frac{\mathrm{d}}{\mathrm{d}L}\left(\begin{array}{c}|\nu_e\rangle \\ |\nu_\mu\rangle\end{array}\right) = \frac{\Delta m^2}{2E}\left(\begin{array}{cc}\sin^2\theta & \cos\theta\sin\theta \\ \cos\theta\sin\theta & \cos^2\theta\end{array}\right)\left(\begin{array}{c}|\nu_e\rangle \\ |\nu_\mu\rangle\end{array}\right),$$

(again, up to additional terms proportional to the 2×2 identity matrix).

Fermi Lagrangian, after a Fiertz rearrangement of the charged-current terms:

$$\mathrm{L} \supset \bar{\nu}_{eL} i\partial_\mu \gamma^\mu \nu_{eL} - 2\sqrt{2}G_F \left(\bar{\nu}_{eL}\gamma^\mu \nu_{eL}\right)\left(\bar{e}_L \gamma_\mu e_L\right) + \ldots$$

Equation of motion for one electron neutrino state in the presence of a non-relativistic electron background, in the rest frame of the electrons:

$$\langle \bar{e}_L \gamma_\mu e_L \rangle = \delta_{\mu 0}\frac{N_e}{2}$$

where $N_e \equiv e^\dagger e$ is the average electron number density (at rest, hence $\delta_{\mu 0}$ term). Factor of $1/2$ from the "left-handed" half.

Dirac equation for a one neutrino state inside a cold electron "gas" is (ignore neutrino mass)

$$(i\partial^\mu \gamma_\mu - \sqrt{2}G_F N_e \gamma_0)|\nu_e\rangle = 0.$$

In the ultrarelativistic limit, (plus $\sqrt{2}G_F N_e \ll E$), dispersion relation is

$$E \simeq |\vec{p}| \pm \sqrt{2}G_F N_e, \qquad + \text{ for } \nu, \quad - \text{ for } \bar{\nu}$$

$$i\frac{\mathrm{d}}{\mathrm{d}L}\left(\begin{array}{c}|\nu_e\rangle\\ |\nu_\mu\rangle\end{array}\right)=\left[\frac{\Delta m^2}{2E}\left(\begin{array}{cc}\sin^2\theta & \cos\theta\sin\theta\\ \cos\theta\sin\theta & \cos^2\theta\end{array}\right)+\left(\begin{array}{cc}A & 0\\ 0 & 0\end{array}\right)\right]\left(\begin{array}{c}|\nu_e\rangle\\ |\nu_\mu\rangle\end{array}\right),$$

$A=\pm\sqrt{2}G_FN_e$ (+ for neutrinos, − for antineutrinos).

Note: Similar effect from neutral current interactions common to all (active) neutrino species → proportional to the identity.

In general, this is hard to solve, as A is a function of L: two-level non-relativistc quantum mechanical system in the presence of time dependent potential.

In some cases, however, the solution is rather simple.

Constant A: good approximation for neutrinos propagating through matter inside the Earth [exception: neutrinos that see Earth's internal structure (the crust, the mantle, the outer core, the inner core)]

$$i\frac{\mathrm{d}}{\mathrm{d}L}\begin{pmatrix} |\nu_e\rangle \\ |\nu_\mu\rangle \end{pmatrix} = \begin{pmatrix} A & \Delta/2\sin 2\theta \\ \Delta/2\sin 2\theta & \Delta\cos 2\theta \end{pmatrix}\begin{pmatrix} |\nu_e\rangle \\ |\nu_\mu\rangle \end{pmatrix}, \quad \Delta \equiv \Delta m^2/2E.$$

$$P_{e\mu} = \sin^2 2\theta_M \sin^2\left(\frac{\Delta_M L}{2}\right),$$

where

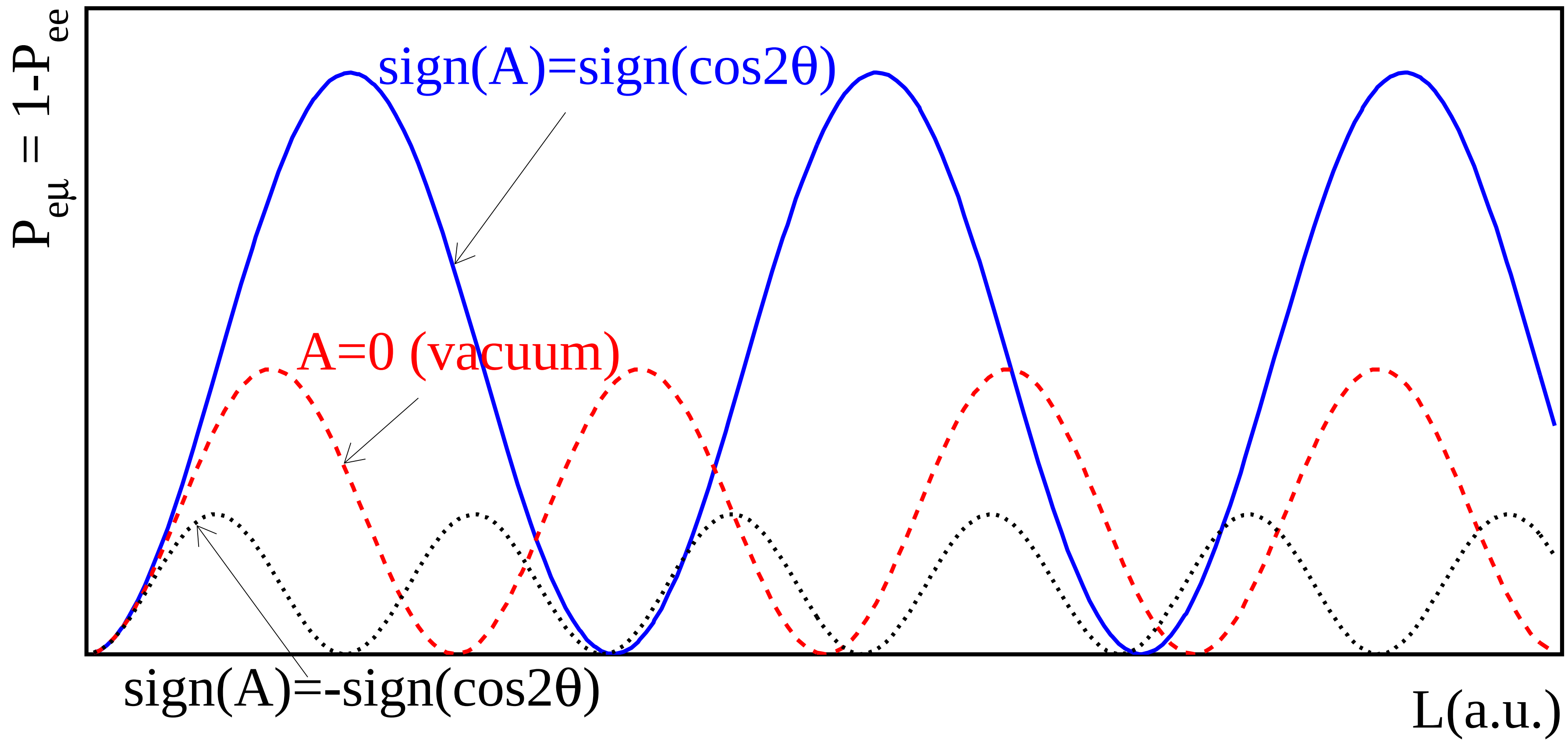
$$\begin{aligned} \Delta_M &= \sqrt{(A-\Delta\cos 2\theta)^2 + \Delta^2\sin^2 2\theta}, \\ \Delta_M \sin 2\theta_M &= \Delta\sin 2\theta, \\ \Delta_M \cos 2\theta_M &= A - \Delta\cos 2\theta. \end{aligned}$$

The presence of matter affects neutrino and antineutrino oscillation differently.

Nothing wrong with this: CPT-theorem relates the propagation of neutrinos in an electron background to the propagation of antineutrinos in a positron background.

Enlarged parameter space in the presence of matter effects.

For example, can tell whether $\cos 2\theta$ is positive or negative.



The MSW Effect

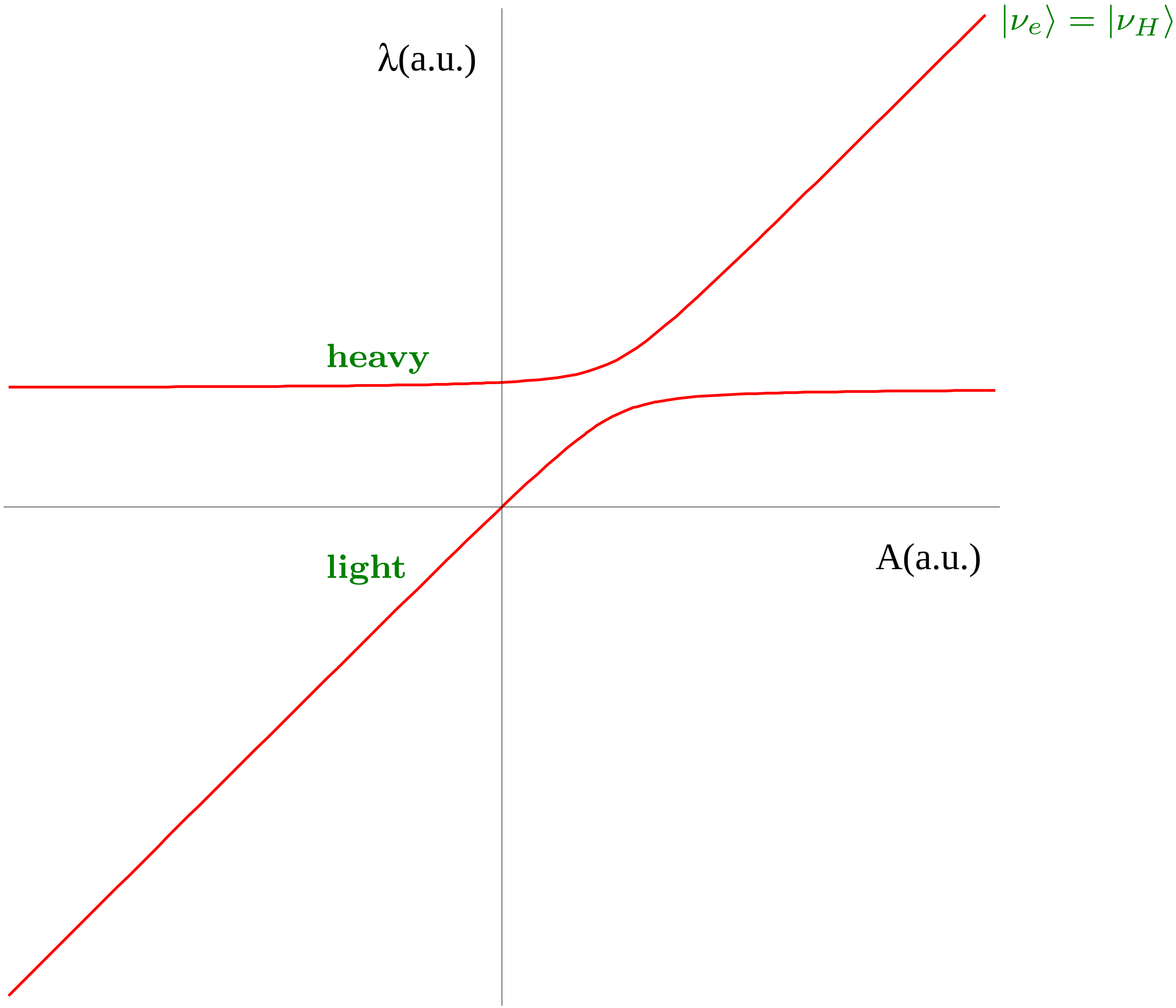
Curiously enough, the oldest neutrino puzzle is the one that is most subtle to explain. This is because solar neutrinos traverse a strongly varying matter density on their way from the center of the Sun to the surface of the Earth.

For the Hamiltonian

$$\left[\Delta\begin{pmatrix}\sin^2\theta & \cos\theta\sin\theta \\ \cos\theta\sin\theta & \cos^2\theta\end{pmatrix} + A\begin{pmatrix}1 & 0 \\ 0 & 0\end{pmatrix}\right],$$

it is easy to compute the eigenvalues as a function of A:

(remember, $\Delta = \Delta m^2/2E$)



$|\nu_e\rangle = |\nu_H\rangle$
λ(a.u.)
heavy
light
A(a.u.)

A decreases "slowly" as a function of L $\Rightarrow$ system evolves adiabatically.

$$|\nu_e\rangle = |\nu_{2M}\rangle \text{ at the core } \to |\nu_2\rangle \text{ in vacuum,}$$
$$P_{ee}^{\rm Earth} = |\langle\nu_e|\nu_2\rangle|^2 = \sin^2\theta.$$

Note that $P_{ee} \simeq \sin^2\theta$ applies in a wide range of energies and baselines, as long as the approximations mentioned above apply —ideal to explain the energy independent suppression of the ^{8}B solar neutrino flux!

Furthermore, large average suppressions of the neutrino flux are allowed if $\sin^2\theta \ll 1$. Compare with $\bar{P}_{ee}^{\rm vac} = 1 - 1/2\sin^2 2\theta > 1/2$.

One can expand on the result above by loosening some of the assumptions. $|\nu_e\rangle$ state is produced in the Sun's core as an *incoherent* mixture of $|\nu_{1M}\rangle$ and $|\nu_{2M}\rangle$. Introduce adiabaticity parameter P_c, which measures the probability that a $|\nu_{iM}\rangle$ matter Hamiltonian state will *not* exit the Sun as a $|\nu_i\rangle$ mass-eigenstate.

$$
\begin{aligned}
|\nu_e\rangle &\rightarrow |\nu_{1M}\rangle, \text{ with probability } \cos^2\theta_M, \\
&\rightarrow |\nu_{2M}\rangle, \text{ with probability } \sin^2\theta_M,
\end{aligned}
$$

where θ_M is the matter angle at the neutrino **production point**.

$$
\begin{aligned}
|\nu_{1M}\rangle &\rightarrow |\nu_1\rangle, \text{ with probability } (1-P_c), \\
&\rightarrow |\nu_2\rangle, \text{ with probability } P_c, \\
|\nu_{2M}\rangle &\rightarrow |\nu_1\rangle \text{ with probability } P_c, \\
&\rightarrow |\nu_2\rangle \text{ with probability } (1-P_c).
\end{aligned}
$$

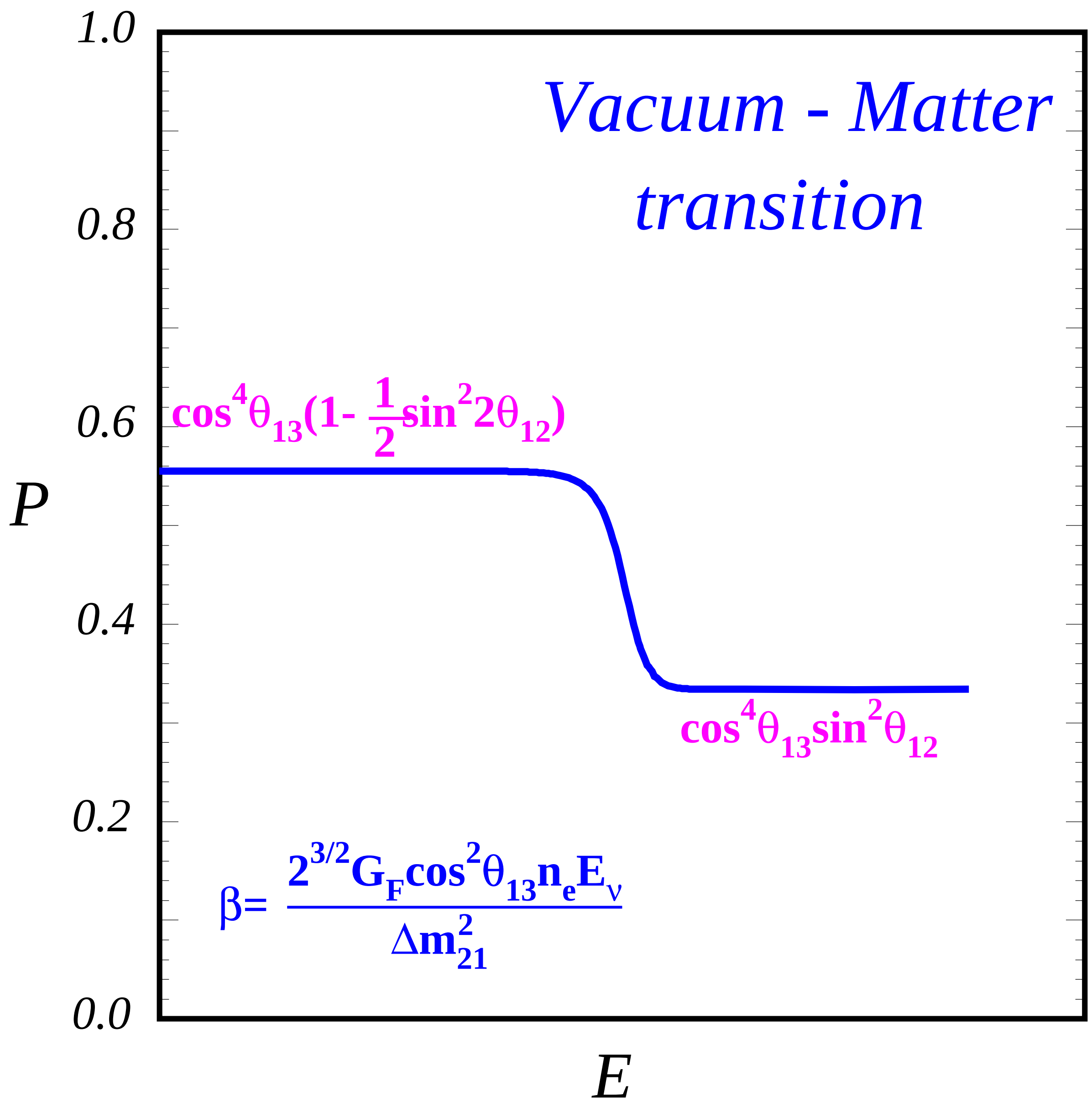
$P_{1e}=\cos^2\theta$ and $P_{2e}=\sin^2\theta$ so

$$
\begin{aligned}
P_{ee}^{\rm Sun} = &\cos^2\theta_M\left[(1-P_c)\cos^2\theta + P_c\sin^2\theta\right] \\
&+\sin^2\theta_M\left[P_c\cos^2\theta+(1-P_c)\sin^2\theta\right].
\end{aligned}
$$

For $N_e = N_{e0}e^{-L/r_0}$, P_c, (crossing probability), is exactly calculable

$$
P_c = \frac{e^{-\gamma\sin^2\theta}-e^{-\gamma}}{1-e^{-\gamma}}, \quad \gamma = 2\pi r_0\Delta. \tag{1}
$$

Adiabatic condition: $\gamma \gg 1$, when $P_c \rightarrow 0$.



We need:

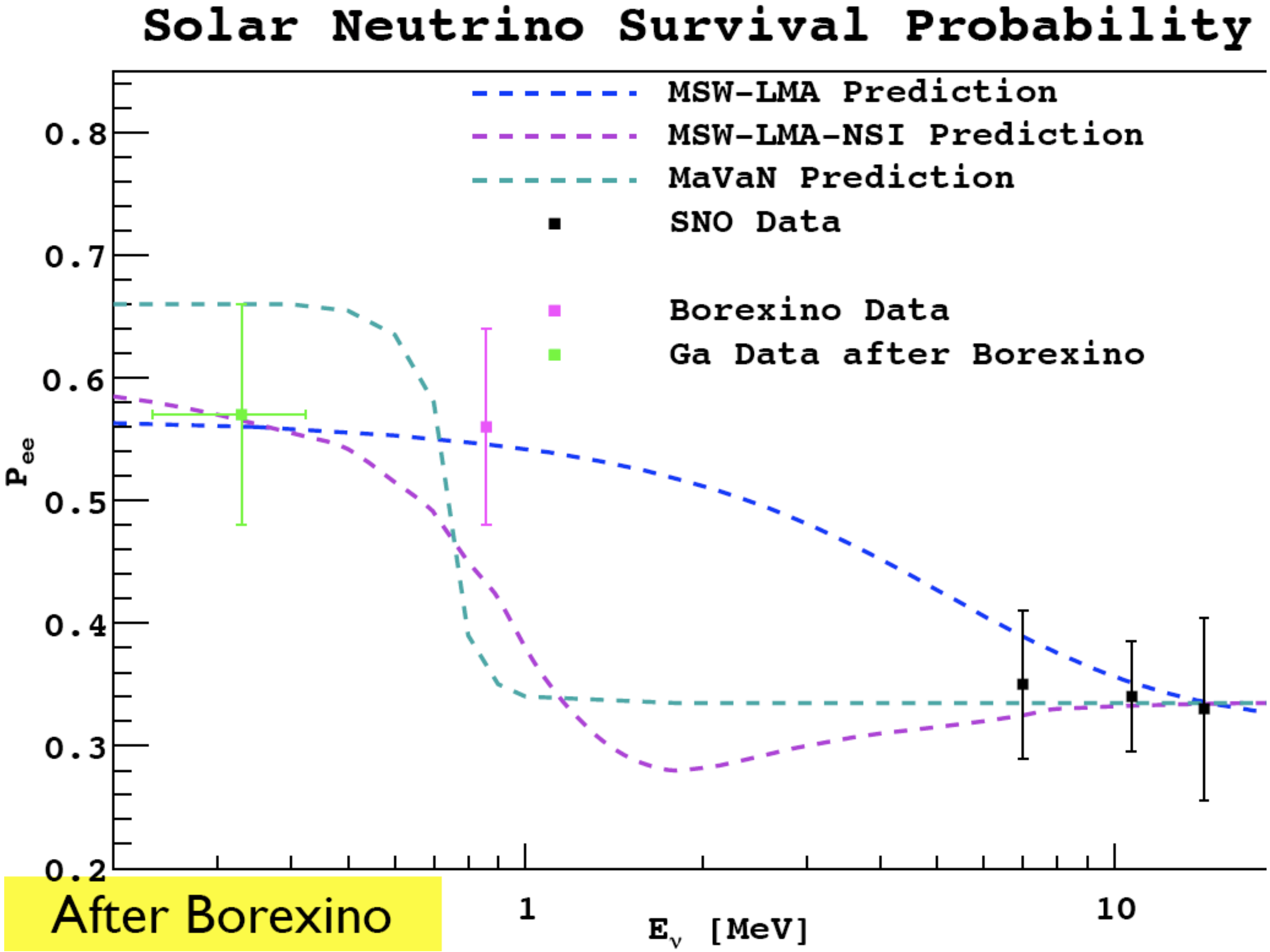
- $P_{ee} \sim 0.3$ (^{8}B neutrinos)
- $P_{ee} \sim 0.6$ (^{7}Be, pp neutrinos)

$\Rightarrow \sin^2\theta \sim 0.3$

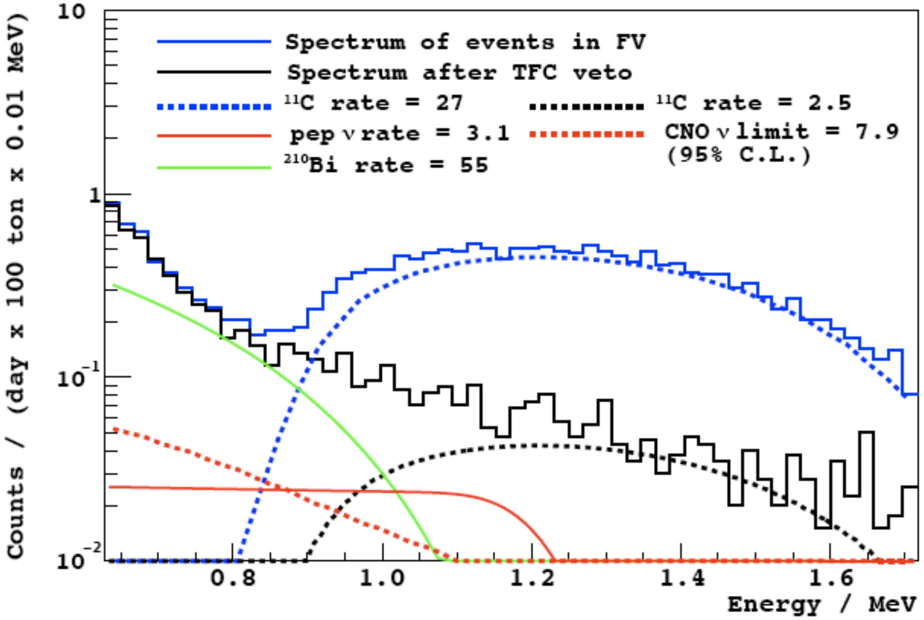
$\Rightarrow \Delta m^2 \sim 10^{-(5 \text{ to } 4)}$ eV^2

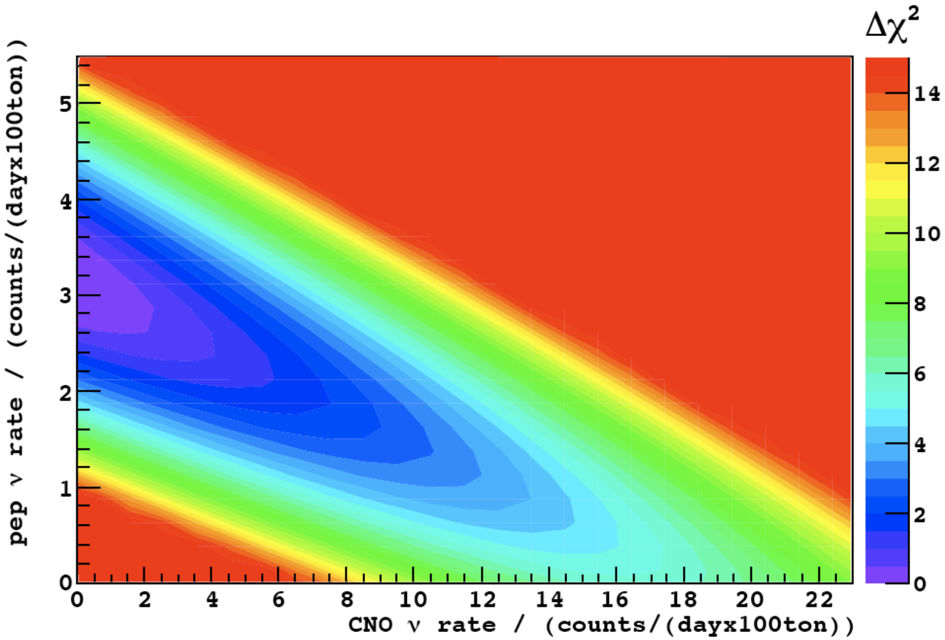
for a long time, there were many other options!

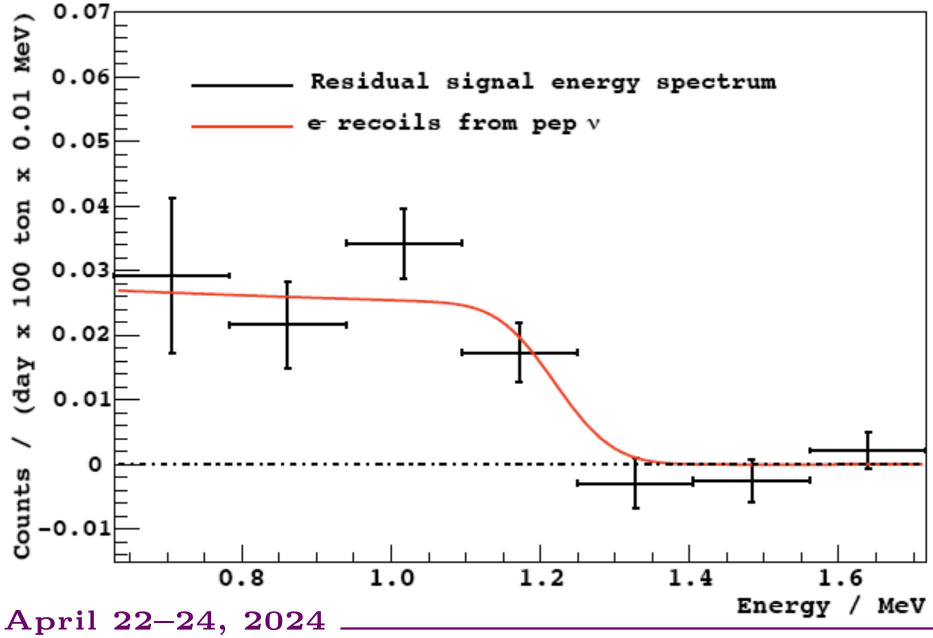
(LMA, LOW, SMA, VAC)

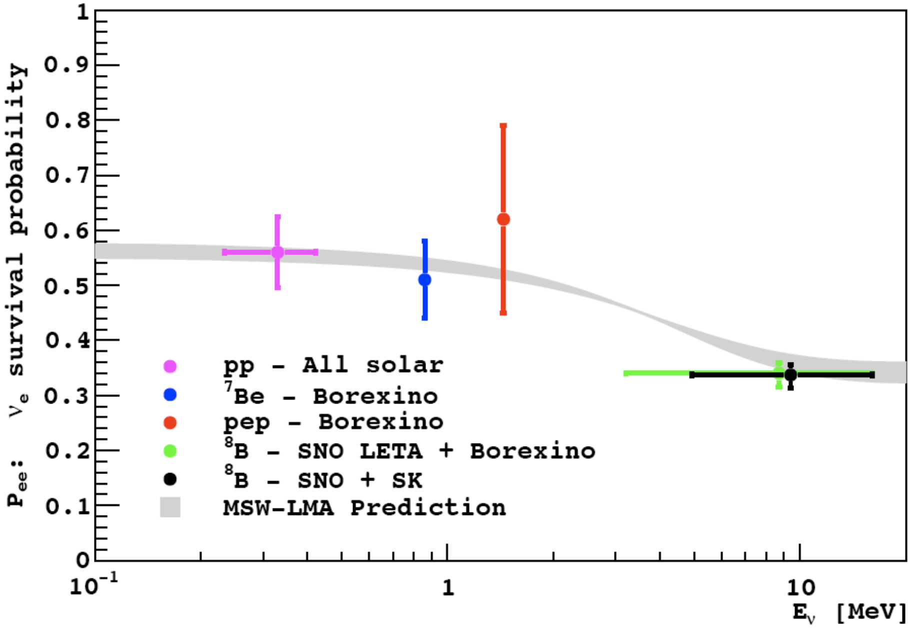

Solar Neutrino Survival Probability
MSW-LMA Prediction
MSW-LMA-NSI Prediction
MaVaN Prediction
SNO Data
Borexino Data
Ga Data after Borexino
0.8
0.7
0.6
0.5
0.4
0.3
0.2
P_{ee}
1
10
E_ν [MeV]
After Borexino

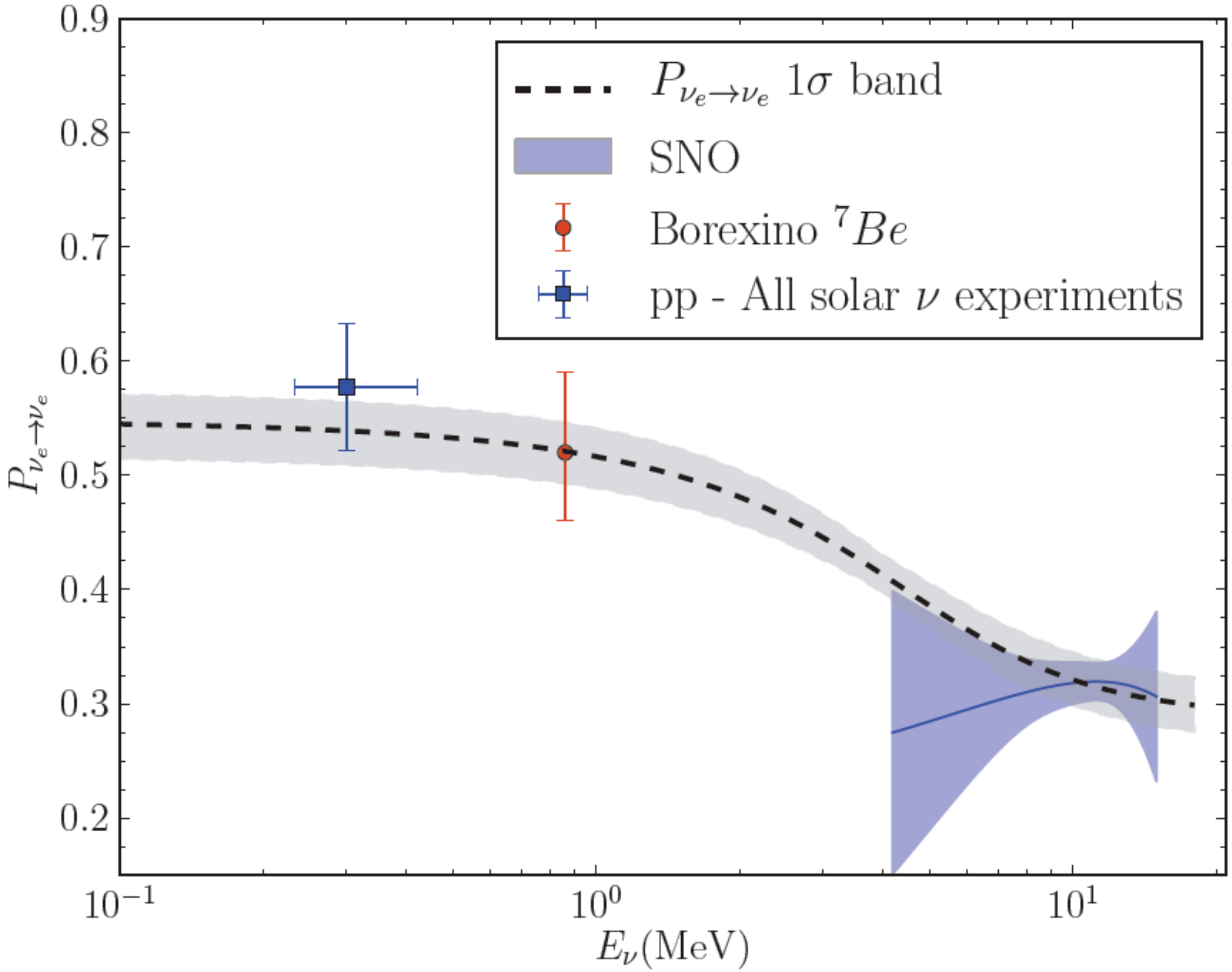
Borexino, 1110.3230

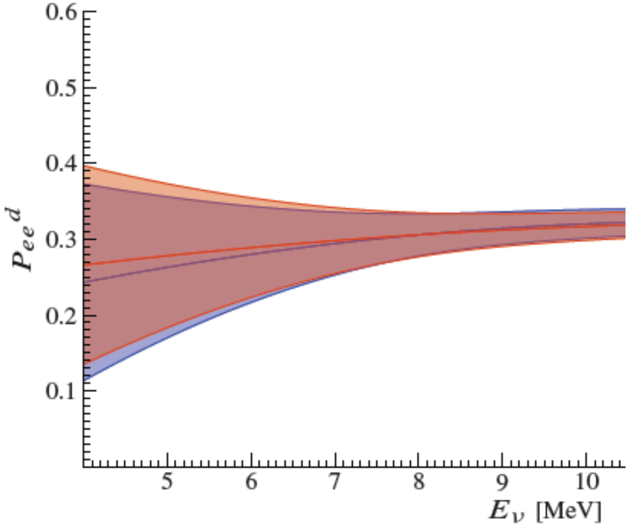


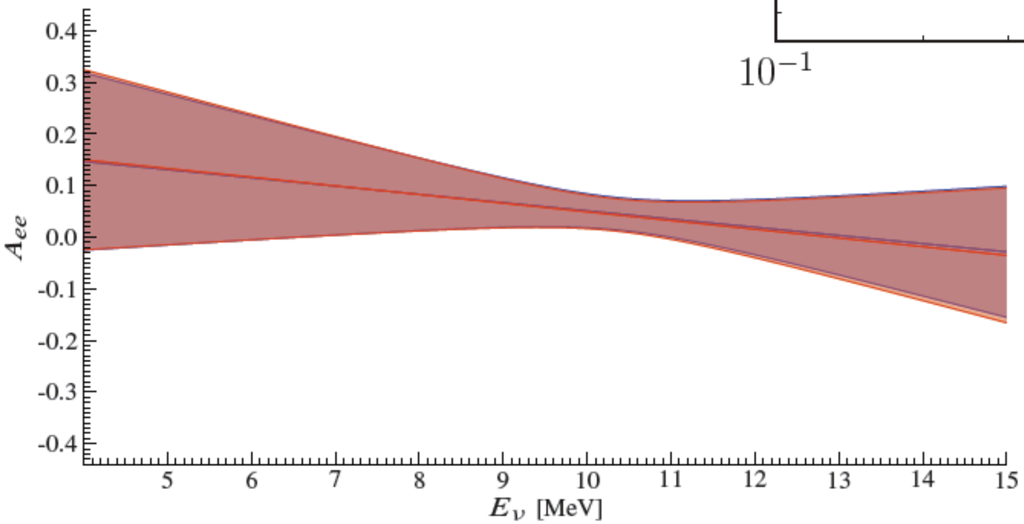










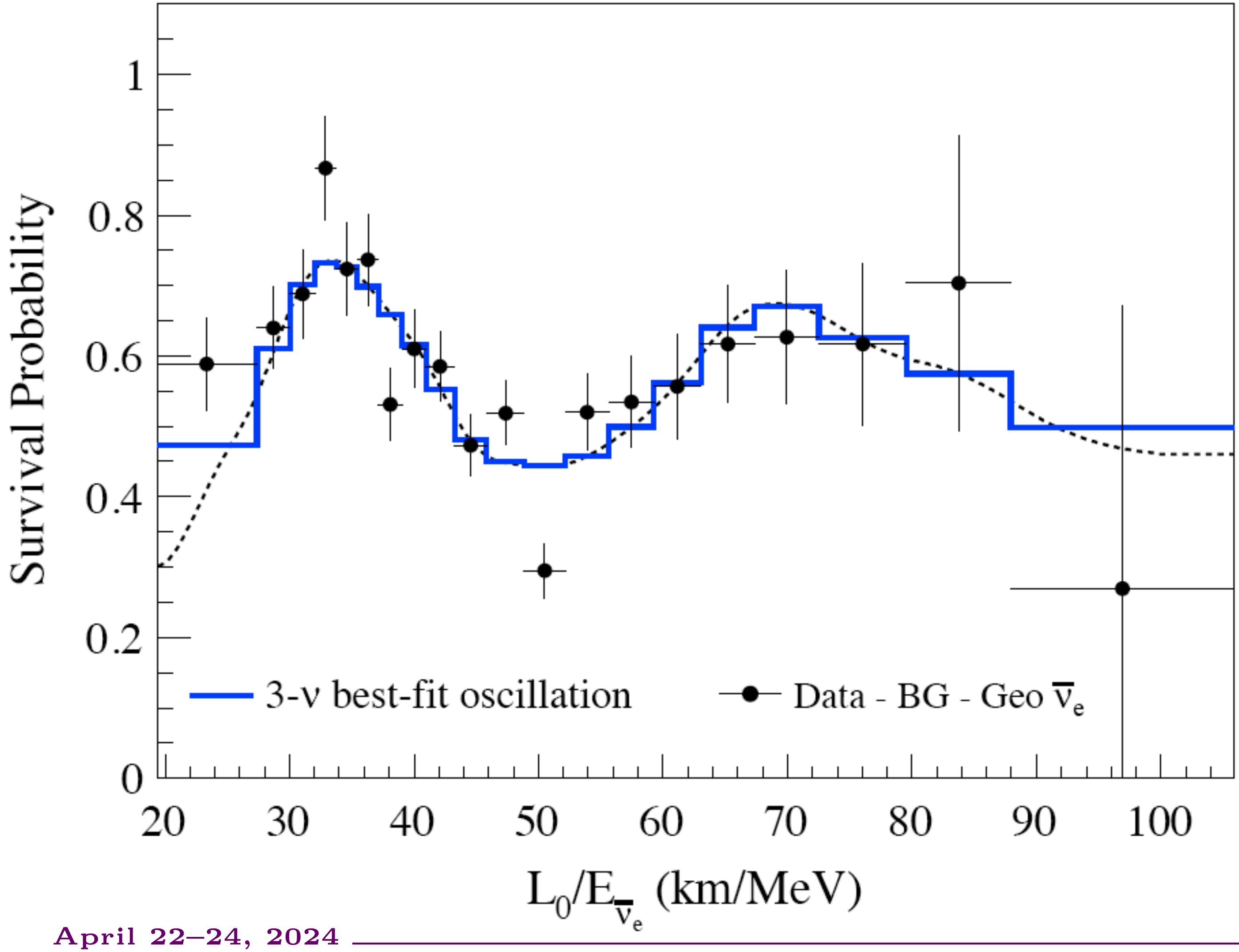


“Final” SNO results, 1109.0763

Solar oscillations confirmed by Reactor experiment: KamLAND

[arXiv:1303.4667]

$$\text{phase} = 1.27\left(\frac{\Delta m^2}{5\times10^{-5}\ \text{eV}^2}\right)\left(\frac{5\ \text{MeV}}{E}\right)\left(\frac{L}{100\ \text{km}}\right)$$

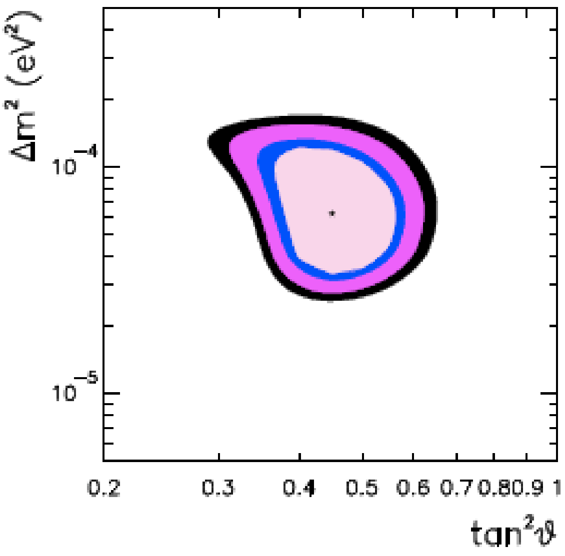


$$P_{ee} = 1 - \sin^2 2\theta \sin^2\left(\frac{\Delta m^2 L}{4E}\right)$$

oscillatory behavior!

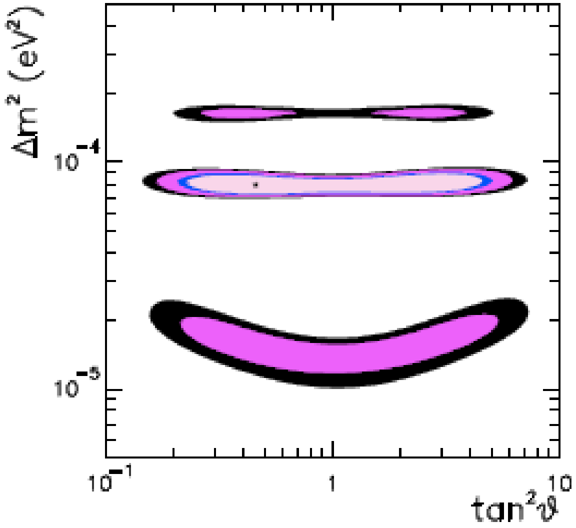
Solar

$\nu_e \rightarrow \nu_{\rm active}$

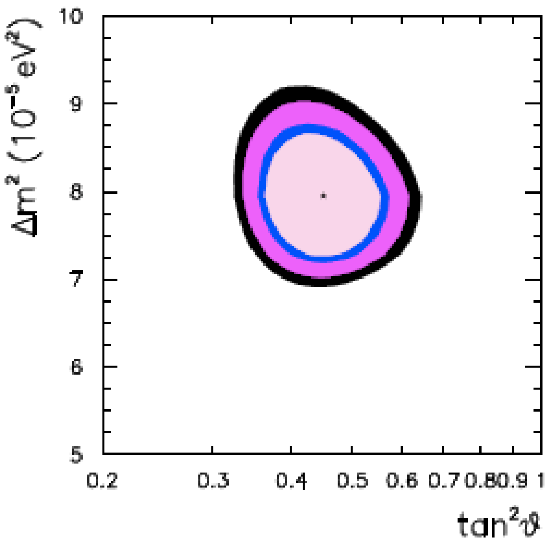


+ KamLAND

$\bar{\nu}_e \not\rightarrow \bar{\nu}_e$



ν_e oscillation parameters compatible with $\overline{\nu}_e$: *Sensible* to assume CPT: $P_{ee} = P_{\bar{e}\bar{e}}$

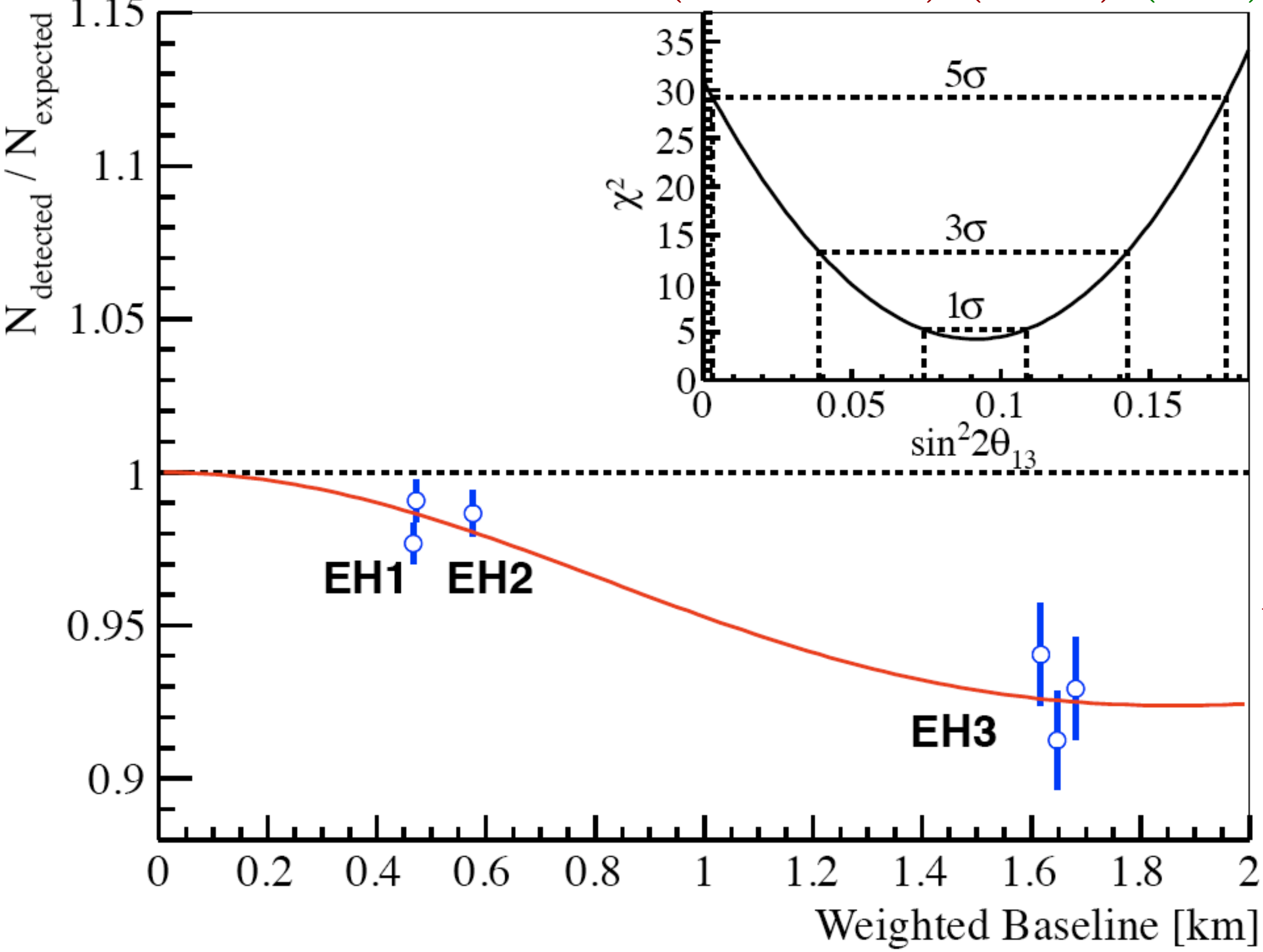


$$\Delta m^2_\odot = (8^{+0.4}_{-0.5}) \times 10^{-5}\ {\rm eV}^2 \quad (1\sigma)$$
$$\tan^2\theta_\odot = 0.45^{+0.05}_{-0.05}$$

[Gonzalez-Garcia, PASI 2006]

Atmospheric Oscillations in the Electron Sector: Daya Bay, RENO, Double Chooz

$$\text{phase}= 0.64\left(\frac{\Delta m^2}{2.5\times10^{-3}\ \text{eV}^2}\right)\left(\frac{5\ \text{MeV}}{E}\right)\left(\frac{L}{1\ \text{km}}\right)$$



$$P_{ee} = 1 - \sin^2 2\theta \sin^2\left(\frac{\Delta m^2 L}{4E}\right)$$

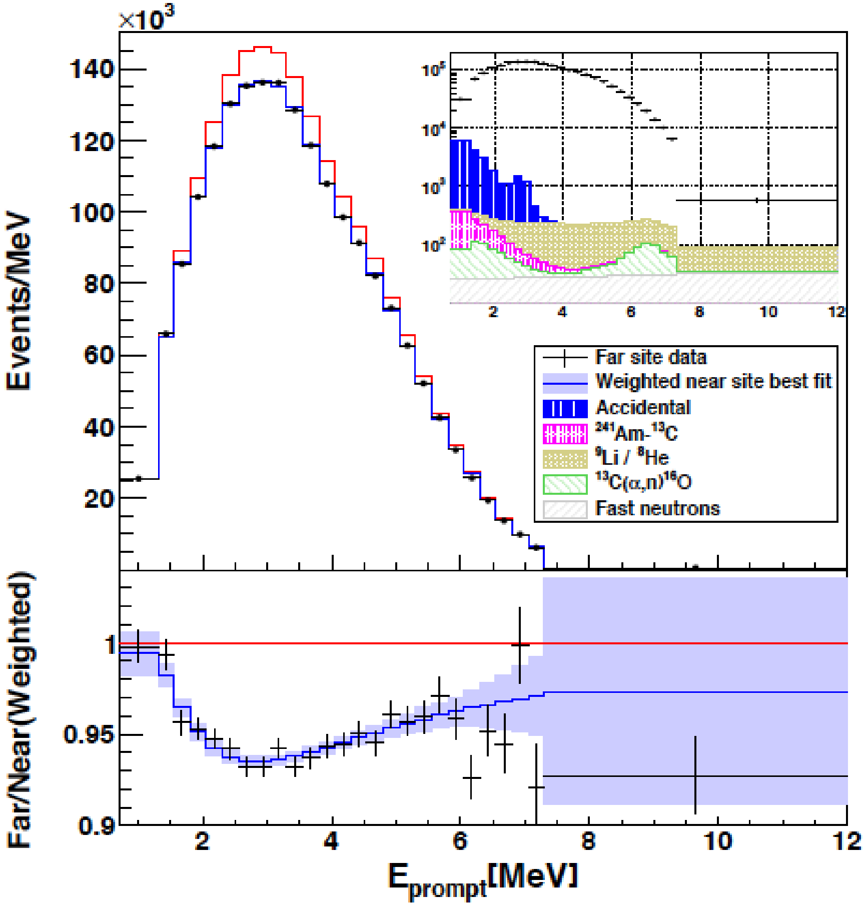


FIG. 3. The background-subtracted spectrum at the far site (black points) and the expectation derived from near-site measurements excluding (red line) or including (blue line) the best-fit oscillation. The bottom panel shows the ratios of data over predictions with no oscillation. The shaded area is the total uncertainty from near-site measurements and the extrapolation model. The error bars represent the statistical uncertainty of the far-site data. The inset shows the background components on a logarithmic scale. Detailed spectra data are provided as Supplemental Material [14].

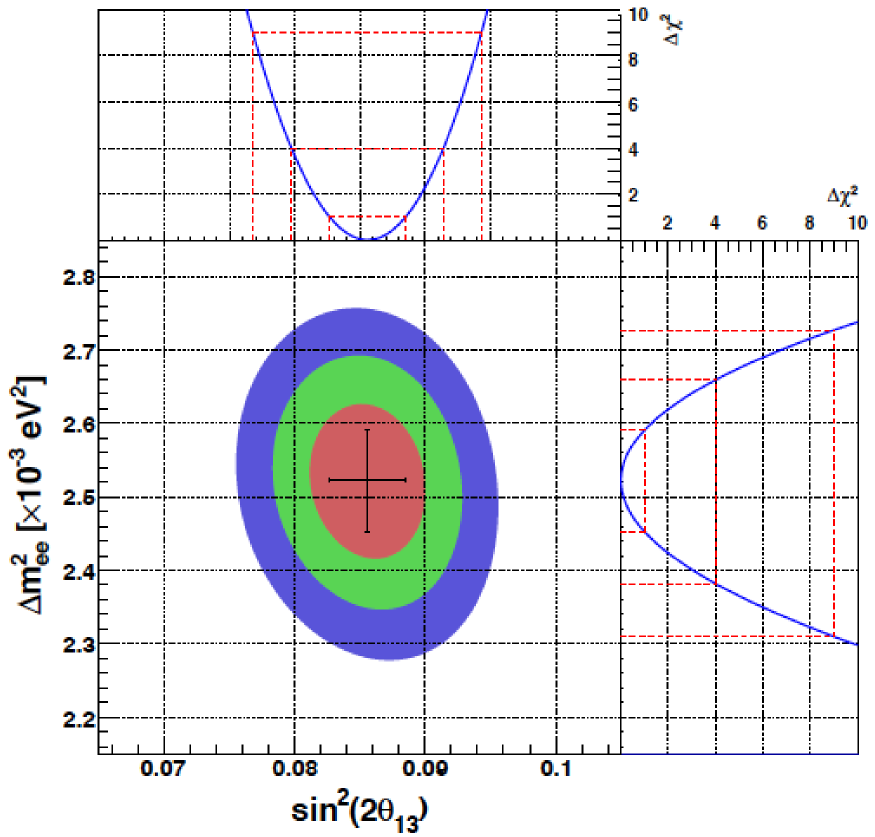


FIG. 4. The 68.3%, 95.5% and 99.7% C.L. allowed regions in the Δm^2_{ee}-$\sin^2 2\theta_{13}$ plane. The one-dimensional $\Delta\chi^2$ for $\sin^2 2\theta_{13}$ and Δm^2_{ee} are shown in the top and right panels, respectively. The best-fit point and one-dimensional uncertainties are given by the black cross.

Summarizing:

Both the solar and atmospheric puzzles can be properly explained in terms of two-flavor neutrino oscillations:

- **solar**: $\nu_e \leftrightarrow \nu_a$ (linear combination of ν_μ and ν_τ): $\Delta m^2 \sim 10^{-4}$ eV^2, $\sin^2\theta \sim 0.3$.
- **atmospheric**: $\nu_\mu \leftrightarrow \nu_\tau$: $\Delta m^2 \sim 10^{-3}$ eV^2, $\sin^2\theta \sim 0.5$ ("maximal mixing").
- **short-baseline reactors:** $\nu_e \leftrightarrow \nu_a$ (linear combination of ν_μ and ν_τ): $\Delta m^2 \sim 10^{-3}$ eV^2, $\sin^2\theta \sim 0.02$.

Putting it all together – 3 flavor mixing:

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{e\tau 2} & U_{\tau 3} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

Definition of neutrino mass eigenstates (who are ν_1, ν_2, ν_3?):

- $m_1^2 < m_2^2$ — $\Delta m^2_{13} < 0$ – Inverted Mass Hierarchy
- $m_2^2 - m_1^2 \ll |m_3^2 - m_{1,2}^2|$ — $\Delta m^2_{13} > 0$ – Normal Mass Hierarchy

$$\tan^2\theta_{12} \equiv \frac{|U_{e2}|^2}{|U_{e1}|^2}; \quad \tan^2\theta_{23} \equiv \frac{|U_{\mu 3}|^2}{|U_{\tau 3}|^2}; \quad U_{e3} \equiv \sin\theta_{13} e^{-i\delta}$$

[For a detailed discussion see AdG, Jenkins, PRD78, 053003 (2008)]

Three Flavor Mixing Hypothesis Fits All* Data Really Well.

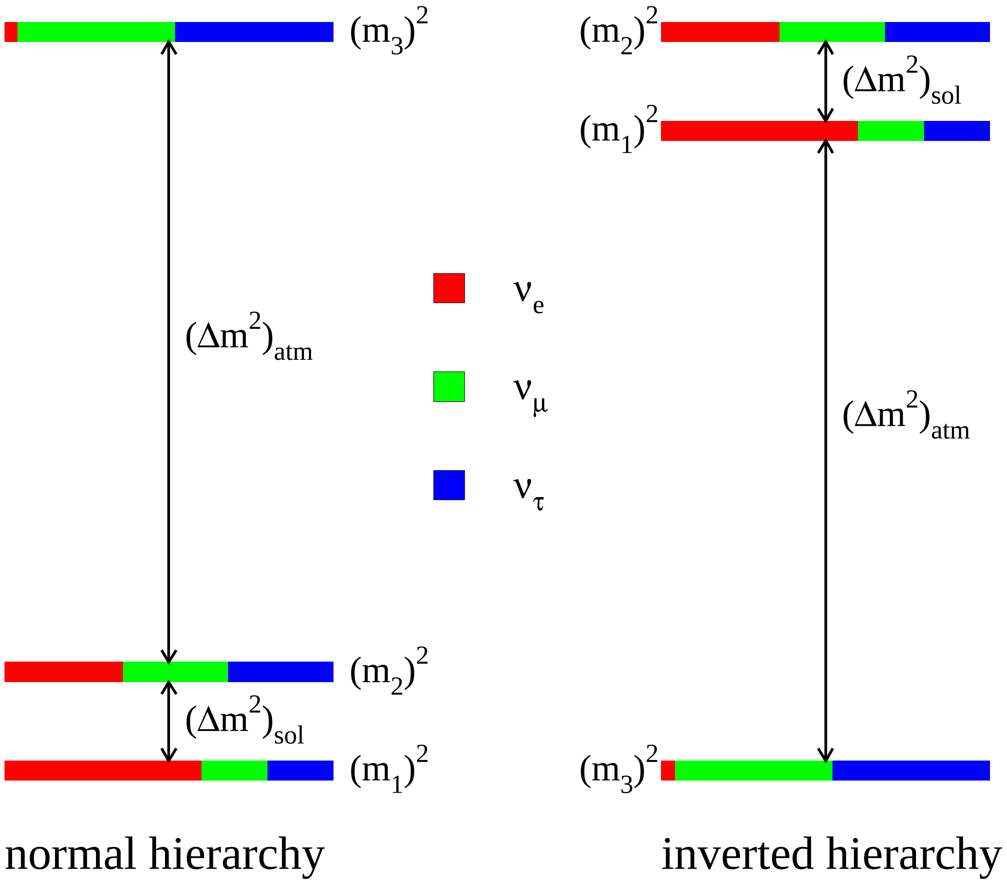
* Modulo short-baseline anomalies.

NuFIT 5.0 (2020)

without SK atmospheric data	Normal Ordering (best fit)		Inverted Ordering ($\Delta\chi^2 = 2.7$)	
	bfp $\pm 1\sigma$	3σ range	bfp $\pm 1\sigma$	3σ range
$\sin^2\theta_{12}$	$0.304^{+0.013}_{-0.012}$	$0.269 \to 0.343$	$0.304^{+0.013}_{-0.012}$	$0.269 \to 0.343$
$\theta_{12}/^\circ$	$33.44^{+0.78}_{-0.75}$	$31.27 \to 35.86$	$33.45^{+0.78}_{-0.75}$	$31.27 \to 35.87$
$\sin^2\theta_{23}$	$0.570^{+0.018}_{-0.024}$	$0.407 \to 0.618$	$0.575^{+0.017}_{-0.021}$	$0.411 \to 0.621$
$\theta_{23}/^\circ$	$49.0^{+1.1}_{-1.4}$	$39.6 \to 51.8$	$49.3^{+1.0}_{-1.2}$	$39.9 \to 52.0$
$\sin^2\theta_{13}$	$0.02221^{+0.00068}_{-0.00062}$	$0.02034 \to 0.02430$	$0.02240^{+0.00062}_{-0.00062}$	$0.02053 \to 0.02436$
$\theta_{13}/^\circ$	$8.57^{+0.13}_{-0.12}$	$8.20 \to 8.97$	$8.61^{+0.12}_{-0.12}$	$8.24 \to 8.98$
$\delta_{CP}/^\circ$	195^{+51}_{-25}	$107 \to 403$	286^{+27}_{-32}	$192 \to 360$
$\frac{\Delta m^2_{21}}{10^{-5}\ \text{eV}^2}$	$7.42^{+0.21}_{-0.20}$	$6.82 \to 8.04$	$7.42^{+0.21}_{-0.20}$	$6.82 \to 8.04$
$\frac{\Delta m^2_{3\ell}}{10^{-3}\ \text{eV}^2}$	$+2.514^{+0.028}_{-0.027}$	$+2.431 \to +2.598$	$-2.497^{+0.028}_{-0.028}$	$-2.583 \to -2.412$

[Esteban *et al*, arXiv:2007.14792, http://www.nu-fit.org]

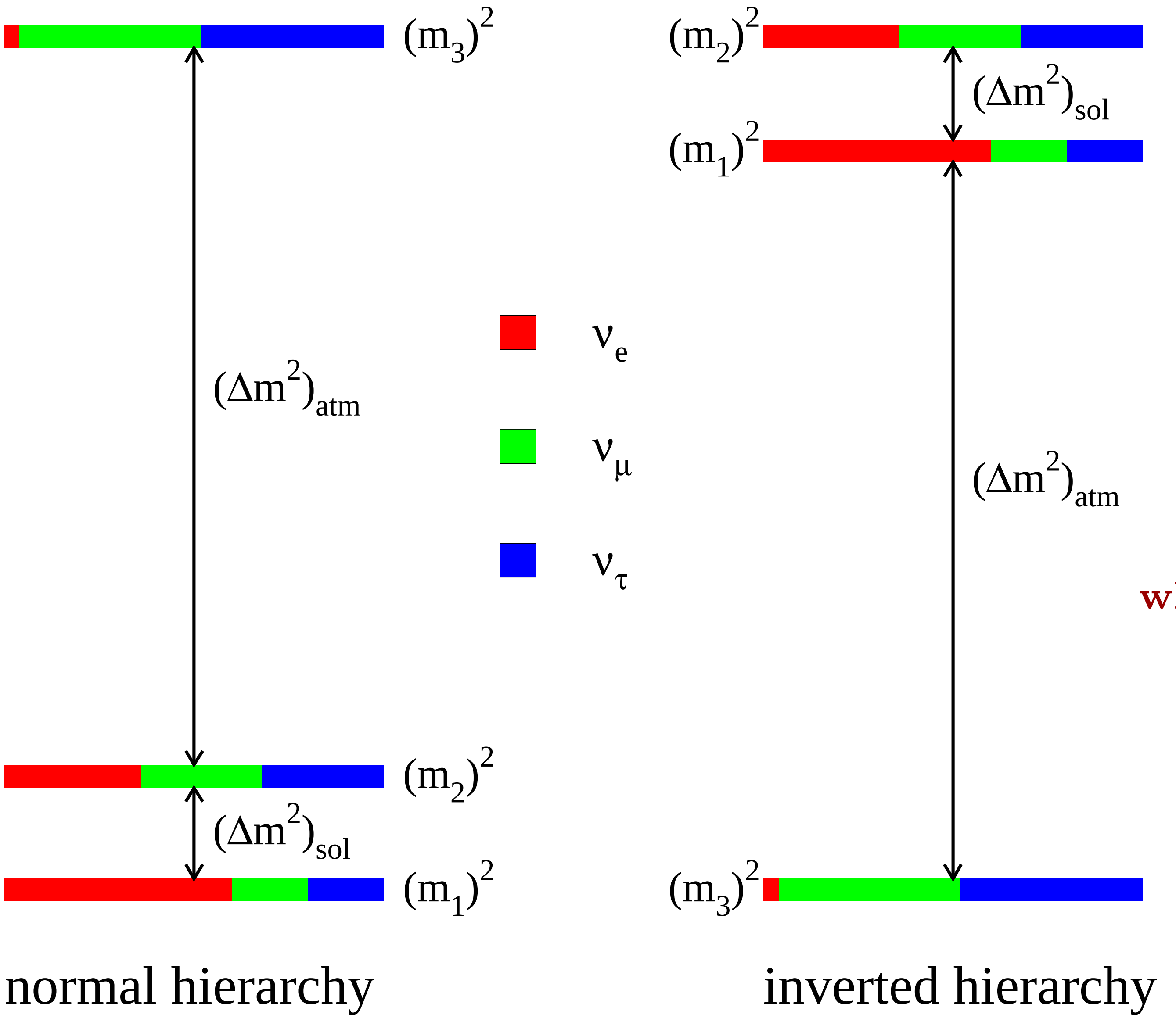
4 - Understanding Neutrino Oscillations: What is Left to Do?



- ~~What is the ν_e component of ν_3?~~ ($\theta_{13} \neq 0$!)
- Is CP-invariance violated in neutrino oscillations? ($\delta \neq 0, \pi$?) [‘yes’ hint]
- Is ν_3 mostly ν_μ or ν_τ? [$\theta_{23} \neq \pi/4$ hint?]
- What is the neutrino mass hierarchy? ($\Delta m^2_{13} > 0$?) [NH hint?]

⇒ All of the above can “only” be addressed with new neutrino oscillation experiments

Ultimate Goal: Not Measure Parameters but Test the Formalism (Over-Constrain Parameter Space)



The Neutrino Mass Hierarchy

which is the right picture?

Why Don't We Know the Neutrino Mass Hierarchy?

Most of the information we have regarding θ_{23} and Δm^2_{13} comes from atmospheric neutrino experiments (SuperK). Roughly speaking, they measure

$$P_{\mu\mu} = 1 - \sin^2 2\theta_{23} \sin^2\left(\frac{\Delta m^2_{13} L}{4E}\right) + \text{ subleading.}$$

It is easy to see from the expression above that the leading term is simply not sensitive to the sign of Δm^2_{13}.

On the other hand, because $|U_{e3}|^2 \sim 0.02$ and $\frac{\Delta m^2_{12}}{\Delta m^2_{13}} \sim 0.03$ are both small, we are yet to observe the subleading effects.

Determining the Mass Hierarchy via Oscillations – the large U_{e3} route

Again, necessary to probe $\nu_\mu \to \nu_e$ oscillations (or vice-versa) governed by Δm^2_{13}. This is the oscillation channel that (almost) all next-generation, accelerator-based experiments are concentrating on, including the ongoing experiments T2K and NOνA.

In vaccum

$$P_{\mu e} = \sin^2\theta_{23}\sin^2 2\theta_{13}\sin^2\left(\frac{\Delta m^2_{13}L}{4E}\right) + \text{ “subleading”},$$

so that, again, this is insensitive to the sign of Δm^2_{13} at leading order. However, in this case, matter effects may come to the rescue.

As I discussed already, neutrino oscillations get modified when these propagate in the presence of matter. Matter effects are sensitive to the neutrino mass ordering (in a way that I will describe shortly) and different for neutrinos and antineutrinos.

If $\Delta_{12} \equiv \frac{\Delta m^2_{12}}{2E}$ terms are ignored, the $\nu_\mu \to \nu_e$ oscillation probability is described, in constant matter density, by

$$P_{\mu e} \simeq P_{e\mu} \simeq \sin^2\theta_{23}\sin^2 2\theta_{13}^{\rm eff}\sin^2\left(\frac{\Delta_{13}^{\rm eff}L}{2}\right),$$

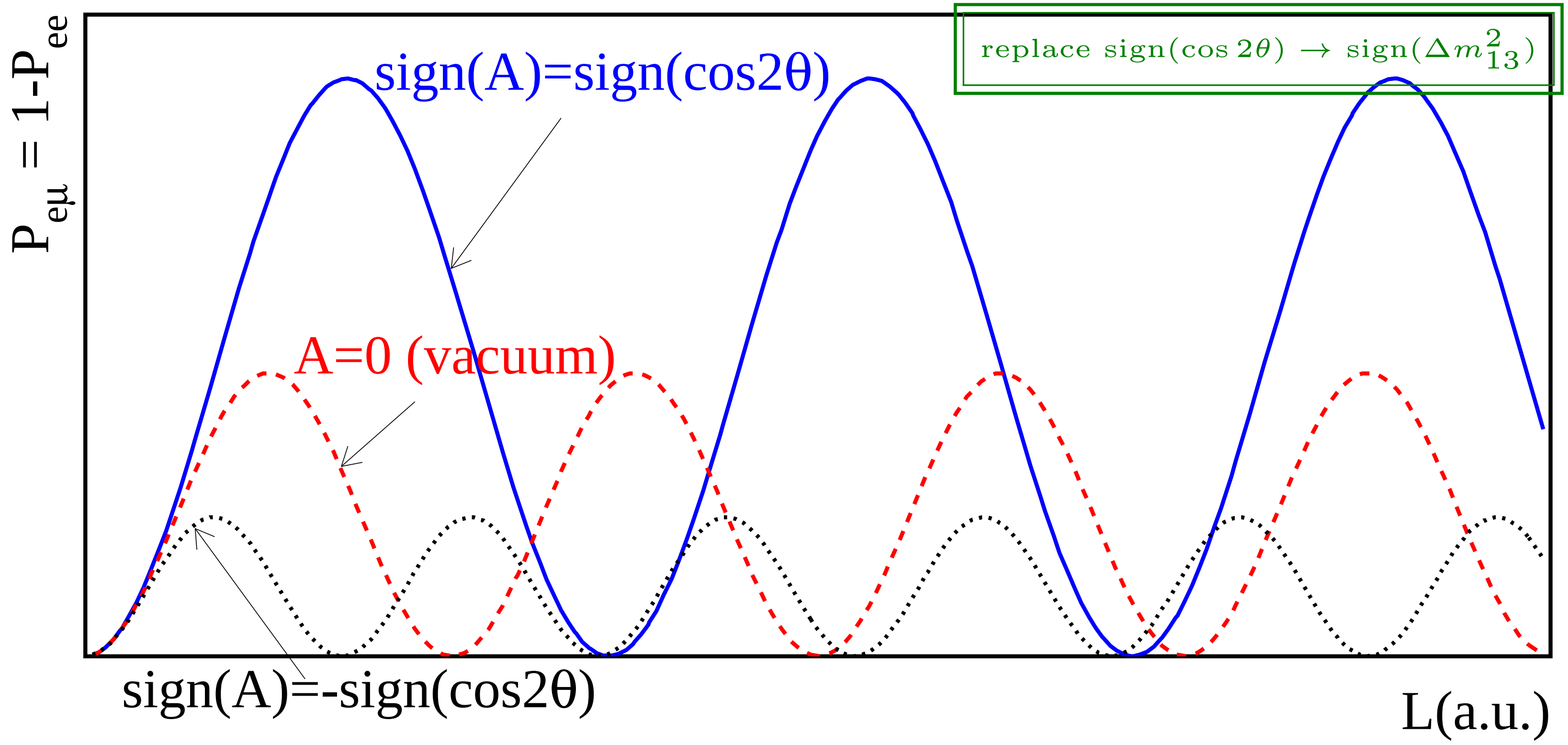
$$\sin^2 2\theta_{13}^{\rm eff} = \frac{\Delta_{13}^2\sin^2 2\theta_{13}}{(\Delta_{13}^{\rm eff})^2},$$

$$\Delta_{13}^{\rm eff} = \sqrt{(\Delta_{13}\cos 2\theta_{13} - A)^2 + \Delta_{13}^2\sin^2 2\theta_{13}},$$

$$\Delta_{13} = \frac{\Delta m^2_{13}}{2E},$$

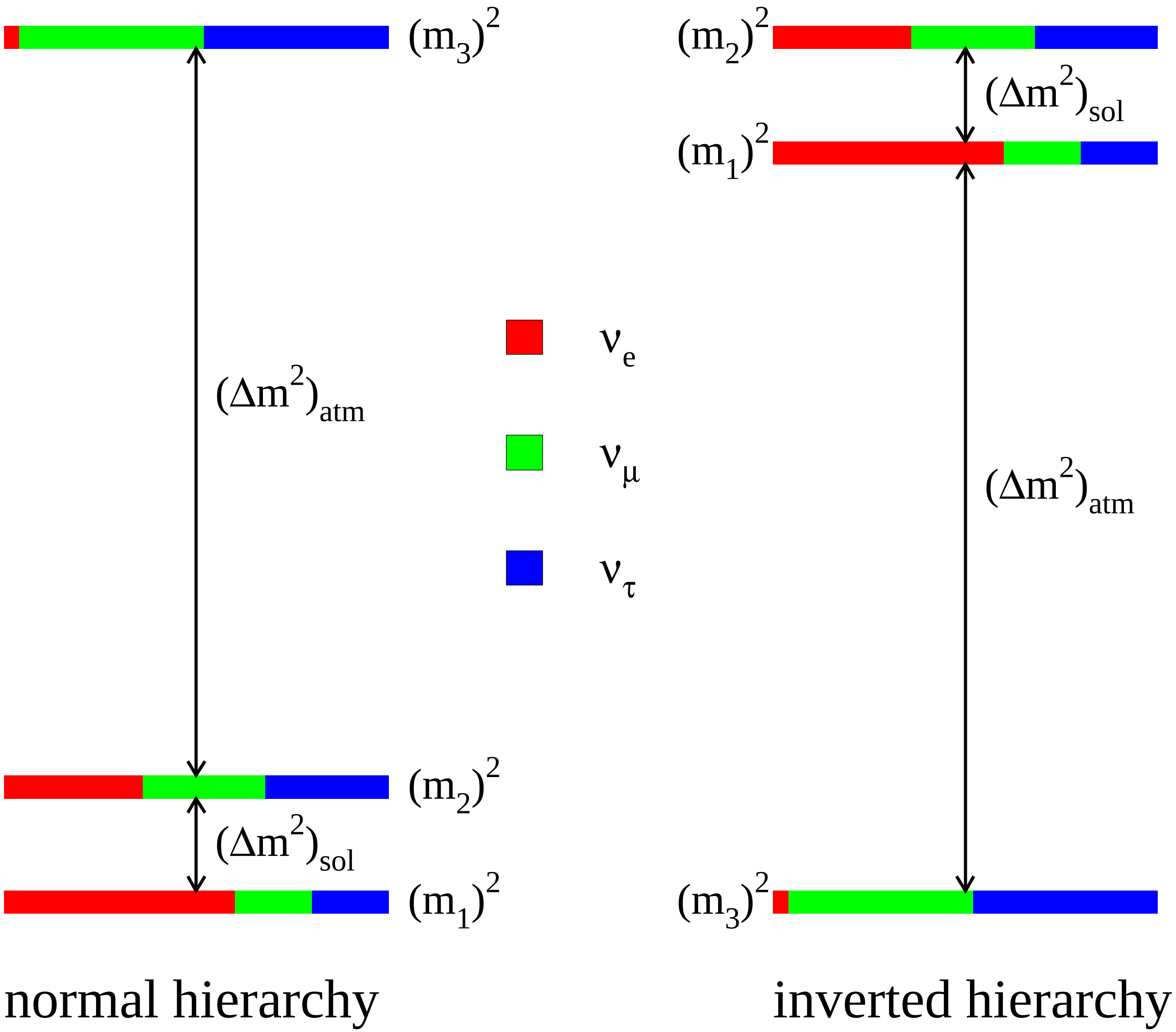
$A \equiv \pm\sqrt{2}G_F N_e$ is the matter potential. It is positive for neutrinos and negative for antineutrinos.

$P_{\mu e}$ depends on the relative sign between Δ_{13} and A. It is different for the two different mass hierarchies, and different for neutrinos and antineutrinos.



Requirements:

- $\sin^2 2\theta_{13}$ large enough – otherwise there is nothing to see!
- $|\Delta_{13}| \sim |A|$ – matter potential must be significant but not overwhelming.
- $\Delta_{13}^{\rm eff} L$ large enough – matter effects are absent near the origin.



The JUNO way:

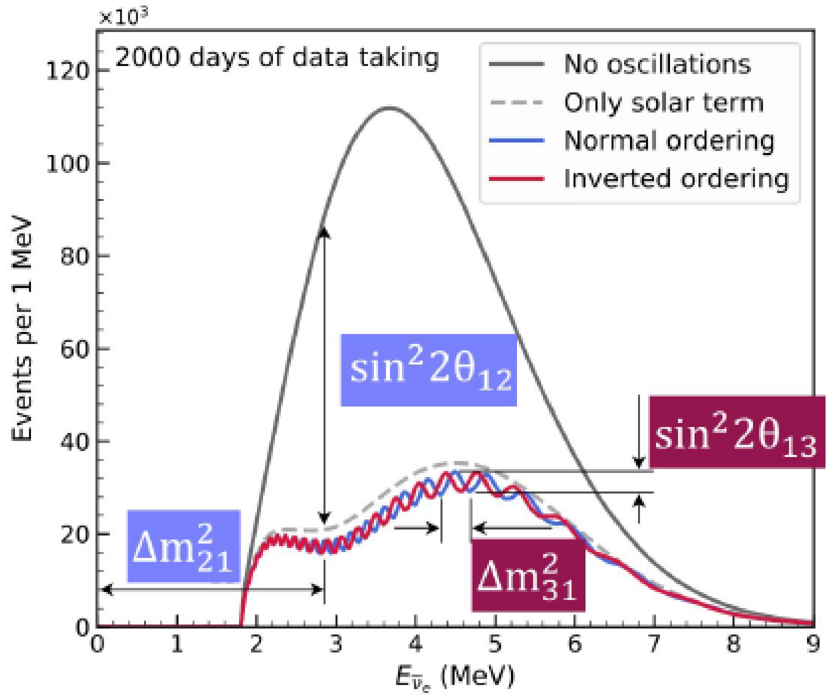
What is largest, $|\Delta m^2_{31}|$ or $|\Delta m^2_{32}|$?

Need to resolve all three oscillation lengths.

Reactor antineutrino oscillation at JUNO

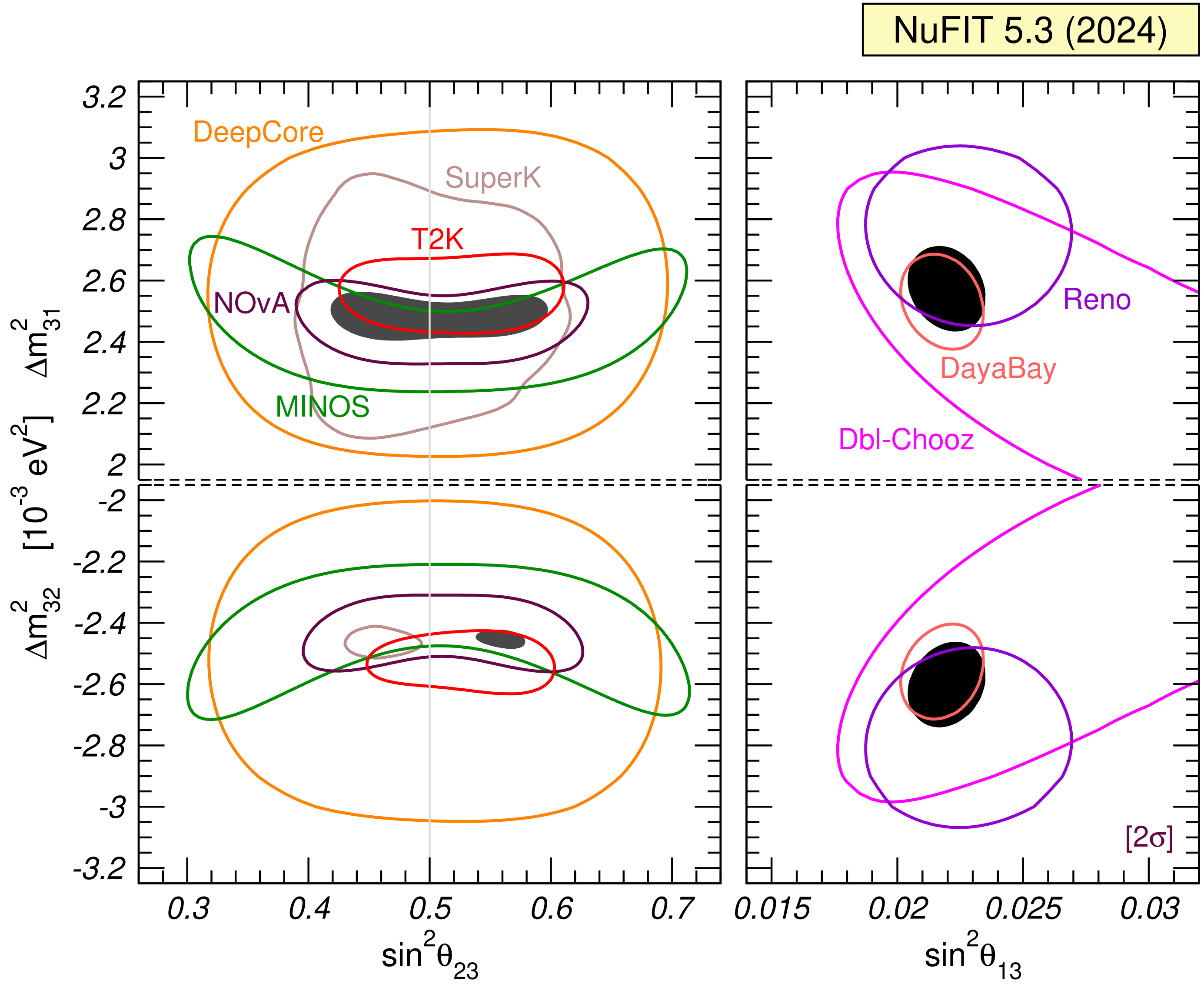
Expect around 22k reactor IBDs & 400 geo IBDs per year, with 1400 $^{9}Li/^{8}He$ β-n decays.

- With six years of data, can determine neutrino mass ordering at 3-4 σ – energy resolution is critical.
- Oscillation parameters $\sin^2\theta_{12}$, Δm^2_{21}, and $|\Delta m^2_{32}|$ can be measured to a precision < 0.6%.
- Antineutrino spectrum will be constrained by JUNO-TAO, a 2.6-ton Gd-LS detector with ~2% resolution, located 30 m from a reactor core.



[from Lebanowski at APS 2022]

$$1 - P_{ee} = C_{21}\sin^2\Delta_{21} + C_{31}\sin^2\Delta_{31} + C_{32}\sin^2\Delta_{32}, \qquad \Delta_{ij} = \Delta m^2_{ij} L/4E$$



NuFIT 5.3 (2024)
DeepCore
SuperK
T2K
NOvA
MINOS
Reno
DayaBay
Dbl-Chooz
[2σ]
3.2
3
2.8
2.6
2.4
2.2
2
-2
-2.2
-2.4
-2.6
-2.8
-3
-3.2
0.3
0.4
0.5
0.6
0.7
0.015
0.02
0.025
0.03
Δm^2_{31} $[10^{-3}\ eV^2]$ Δm^2_{32}
$\sin^2\theta_{23}$
$\sin^2\theta_{13}$

The "Holy Graill" of Neutrino Oscillations – CP Violation

In the old Standard Model, there is only one[a] source of CP-invariance violation:

$\Rightarrow$ The complex phase in V_{CKM}, the quark mixing matrix.

Indeed, as far as we have been able to test, all CP-invariance violating phenomena agree with the CKM paradigm:

- ϵ_K;
- ϵ'_K;
- $\sin 2\beta$;
- etc.

Recent experimental developments, however, provide strong reason to believe that this is not the case: neutrinos have mass, and leptons mix!

[a] modulo the QCD θ-parameter, which will be "willed away" henceforth.

Golden Opportunity to Understand Matter versus Antimatter?

The SM with massive Majorana neutrinos accommodates **five** irreducible CP-invariance violating phases.

- One is the phase in the CKM phase. We have measured it, it is large, and we don't understand its value. At all.
- One is θ_{QCD} term ($\theta G\tilde{G}$). We don't know its value but it is only constrained to be very small. We don't know why (there are some good ideas, however).
- Three are in the neutrino sector. One can be measured via neutrino oscillations. 50% increase on the amount of information.

We don't know much about CP-invariance violation. Is it really fair to presume that CP-invariance is generically violated in the neutrino sector solely based on the fact that it is violated in the quark sector? Why? Cautionary tale: “Mixing angles are small”

CP-invariance Violation in Neutrino Oscillations

The most promising approach to studying CP-violation in the leptonic sector seems to be to compare $P(\nu_\mu \to \nu_e)$ versus $P(\bar{\nu}_\mu \to \bar{\nu}_e)$.

The amplitude for $\nu_\mu \to \nu_e$ transitions can be written as

$$A_{\mu e} = U^*_{e2}U_{\mu 2}\left(e^{i\Delta_{12}} - 1\right) + U^*_{e3}U_{\mu 3}\left(e^{i\Delta_{13}} - 1\right)$$

where $\Delta_{1i} = \frac{\Delta m^2_{1i}L}{2E}$, $i = 2, 3$.

The amplitude for the CP-conjugate process can be written as

$$\bar{A}_{\mu e} = U_{e2}U^*_{\mu 2}\left(e^{i\Delta_{12}} - 1\right) + U_{e3}U^*_{\mu 3}\left(e^{i\Delta_{13}} - 1\right).$$

[remember: according to unitarty, $U_{e1}U^*_{\mu 1} = -U_{e2}U^*_{\mu 2} - U_{e3}U^*_{\mu 3}$]

In general, $|A|^2 \neq |\bar{A}|^2$ (CP-invariance violated) as long as:

- Nontrivial "Weak" Phases: $\arg(U_{ei}^* U_{\mu i}) \to \delta \neq 0, \pi$;
- Nontrivial "Strong" Phases: Δ_{12}, $\Delta_{13} \to L \neq 0$;
- Because of Unitarity, we need all $|U_{\alpha i}| \neq 0 \to$ three generations.

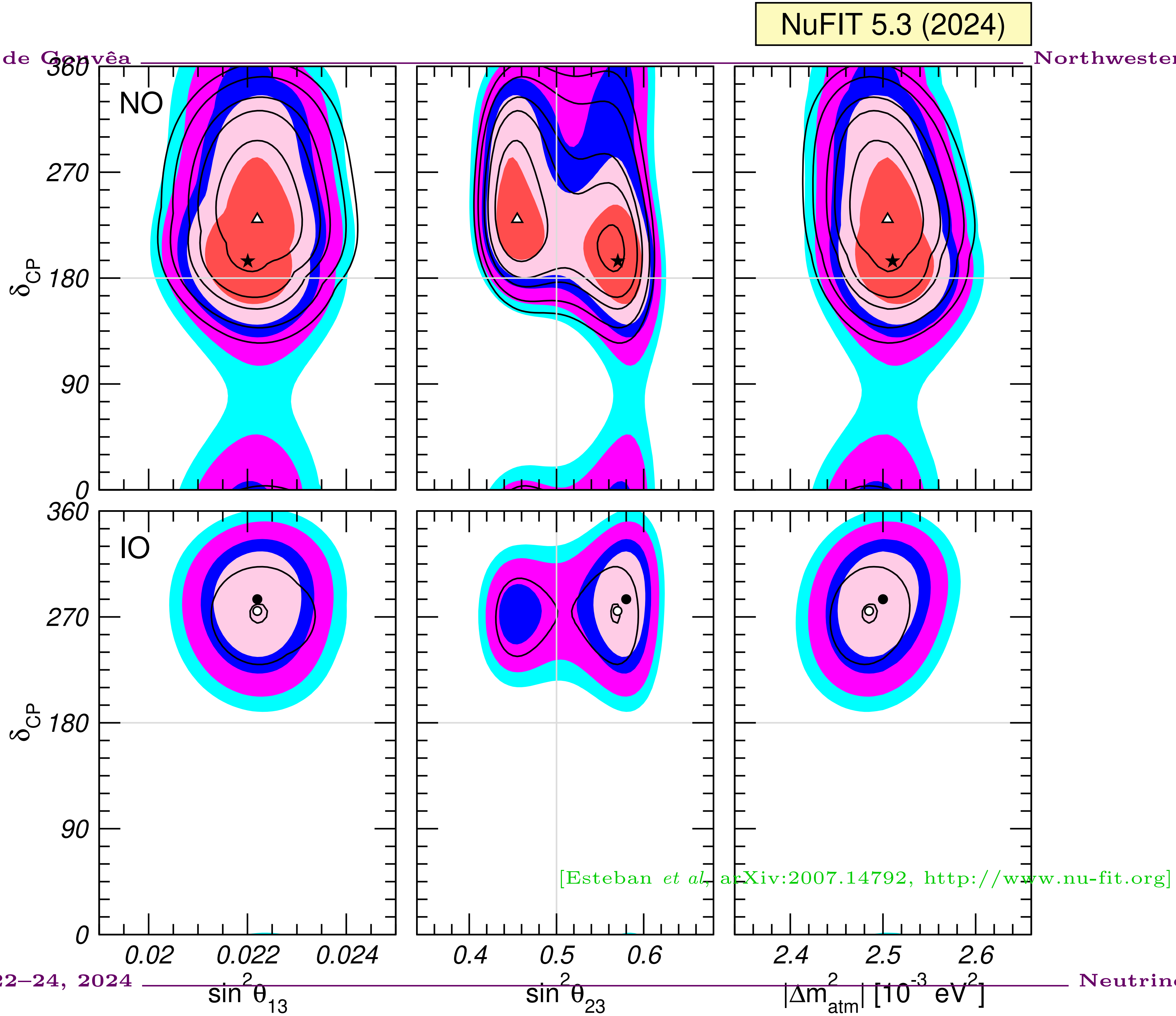
All of these can be satisfied, with a little luck: given that two of the three mixing angles are known to be large, **we need** $|U_{e3}| \neq 0$**. (✓)**

The goal of next-generation neutrino experiments is to determine the magnitude of $|U_{e3}|$. We need to know this in order to understand how to study CP-invariance violation in neutrino oscillations!

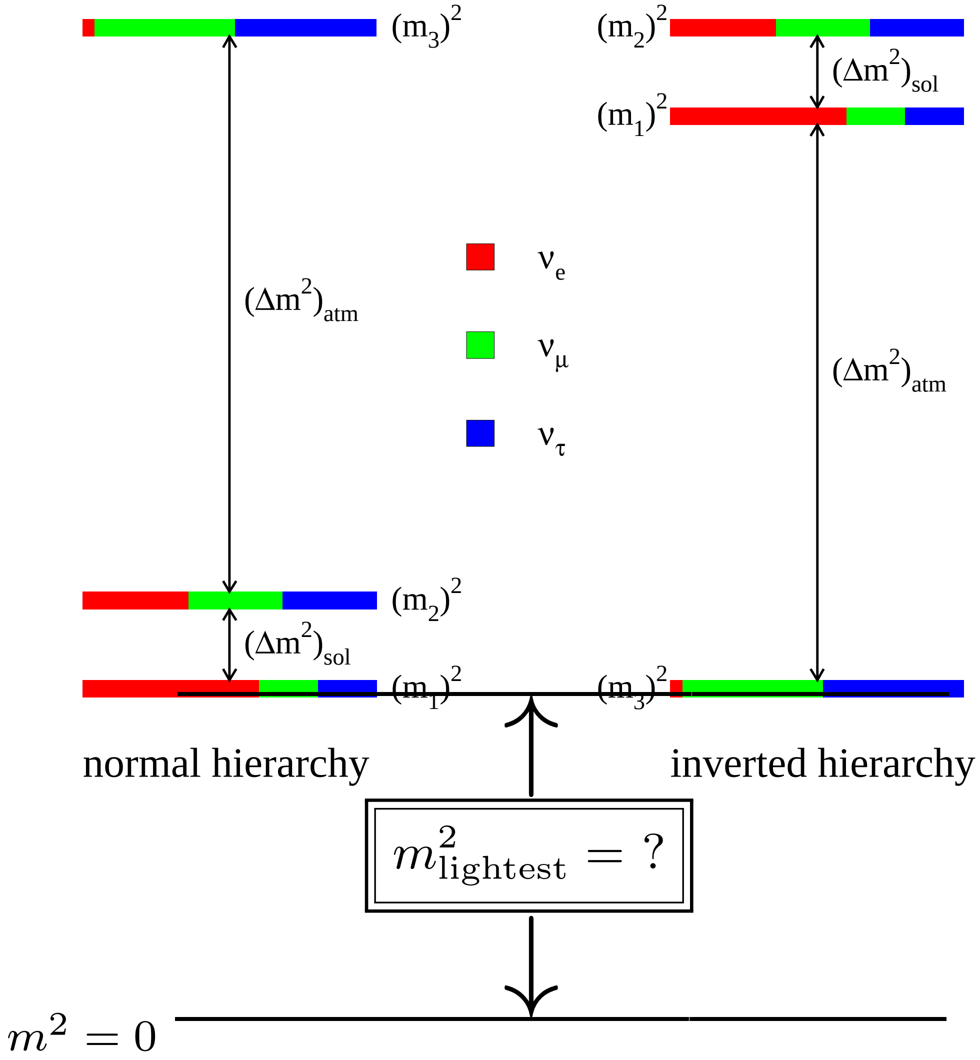
In the real world, life is much more complicated. The lack of knowledge concerning the mass hierarchy, θ_{13}, θ_{23} leads to several degeneracies.

Note that, in order to see CP-invariance violation, we **need** the "subleading" terms!

In order to ultimately measure a new source of CP-invariance violation, we will need to combine different measurements:
– oscillation of muon neutrinos and antineutrinos,
– oscillations at accelerator and reactor experiments,
– experiments with different baselines,
– etc.


NuFIT 5.3 (2024)
NO
IO
δ_{CP}
360
270
180
90
0
0.02
0.022
0.024
0.4
0.5
0.6
2.4
2.5
2.6
$\sin^2\theta_{13}$
$\sin^2\theta_{23}$
$|\Delta m^2_{atm}|$ [10^{-3} eV^2]
[Esteban *et al*, arXiv:2007.14792, http://www.nu-fit.org]

4– What We Know We Don't Know (ii): How Light is the Lightest Neutrino?



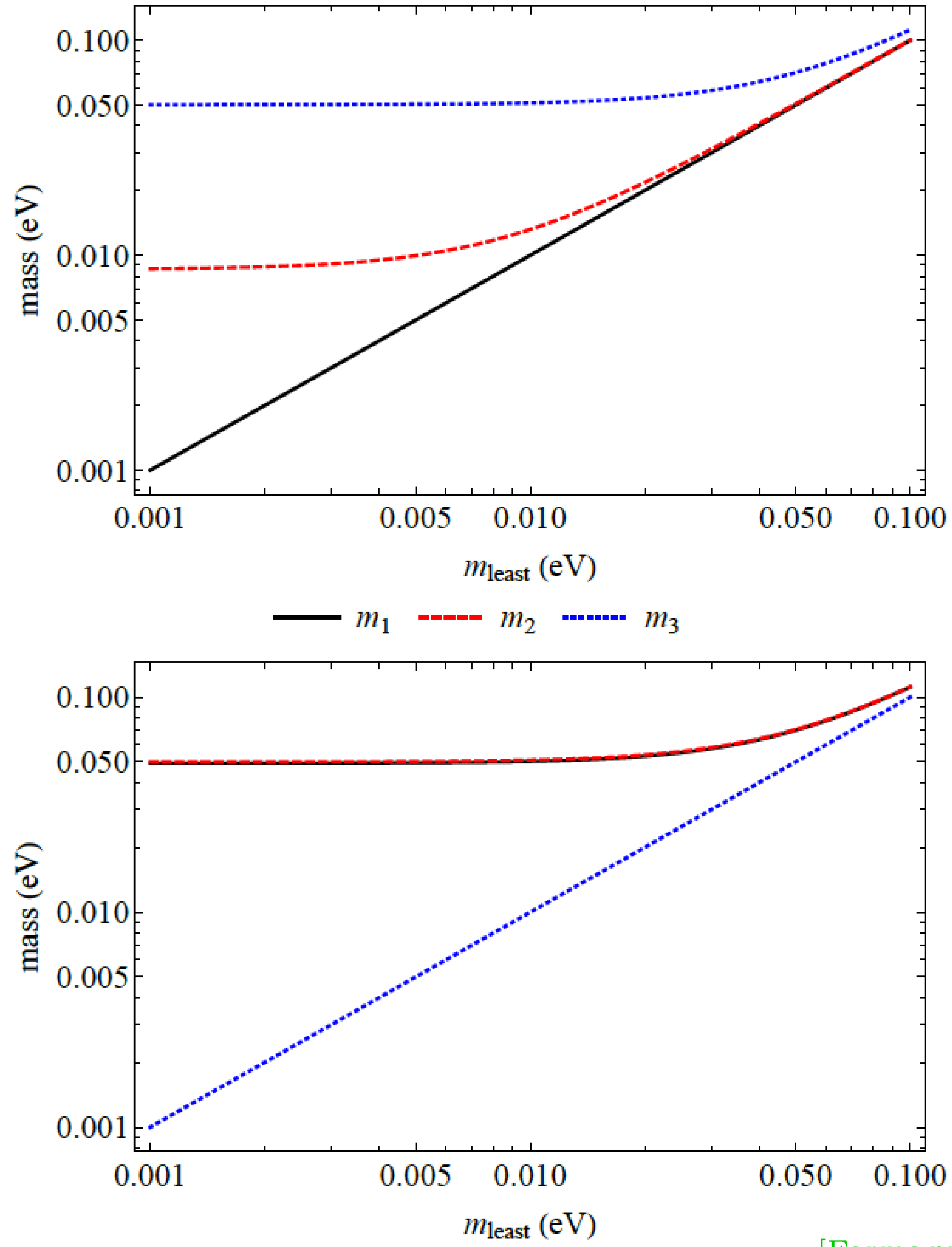
So far, we've only been able to measure neutrino mass-squared differences.

The lightest neutrino mass is only poorly constrained: $m^2_{\rm lightest} < 1~{\rm eV}^2$

qualitatively different scenarios allowed:

- $m^2_{\rm lightest} \equiv 0$;
- $m^2_{\rm lightest} \ll \Delta m^2_{12,13}$;
- $m^2_{\rm lightest} \gg \Delta m^2_{12,13}$.

Need information outside of neutrino oscillations.



[Formaggio, AdG, Robertson, Phys.Rept. 914 (2021)]

FIG. 4: Current best-fit values of the neutrino masses m_1, m_2, m_3 as a function of the lightest neutrino mass, for the normal mass-ordering (top) and the inverted mass ordering (bottom).

The most direct probe of the lightest neutrino mass – precision measurements of β-decay

Observation of the effect of non-zero neutrino masses **kinematically**.

When a neutrino is produced, some of the energy exchanged in the process should be spent by the non-zero neutrino mass.

Typical effects are very, very small – we've never seen them! The most sensitive observable is the electron energy spectrum from tritium decay.

$$^3\mathrm{H} \to {}^3\mathrm{He} + e^- + \bar{\nu}$$

Why tritium? Small Q value, reasonable abundances. Required sensitivity proportional to m^2/Q^2.

In practice, this decay is sensitive to an effective "electron neutrino mass":

$$m^2_{\nu_e} \equiv \sum_i |U_{ei}|^2 m_i^2$$

Experiments measure the **shape** of the end-point of the spectrum, not the value of the end point. This is done by counting events as a function of a low-energy cut-off. note: LOTS of Statistics Needed!

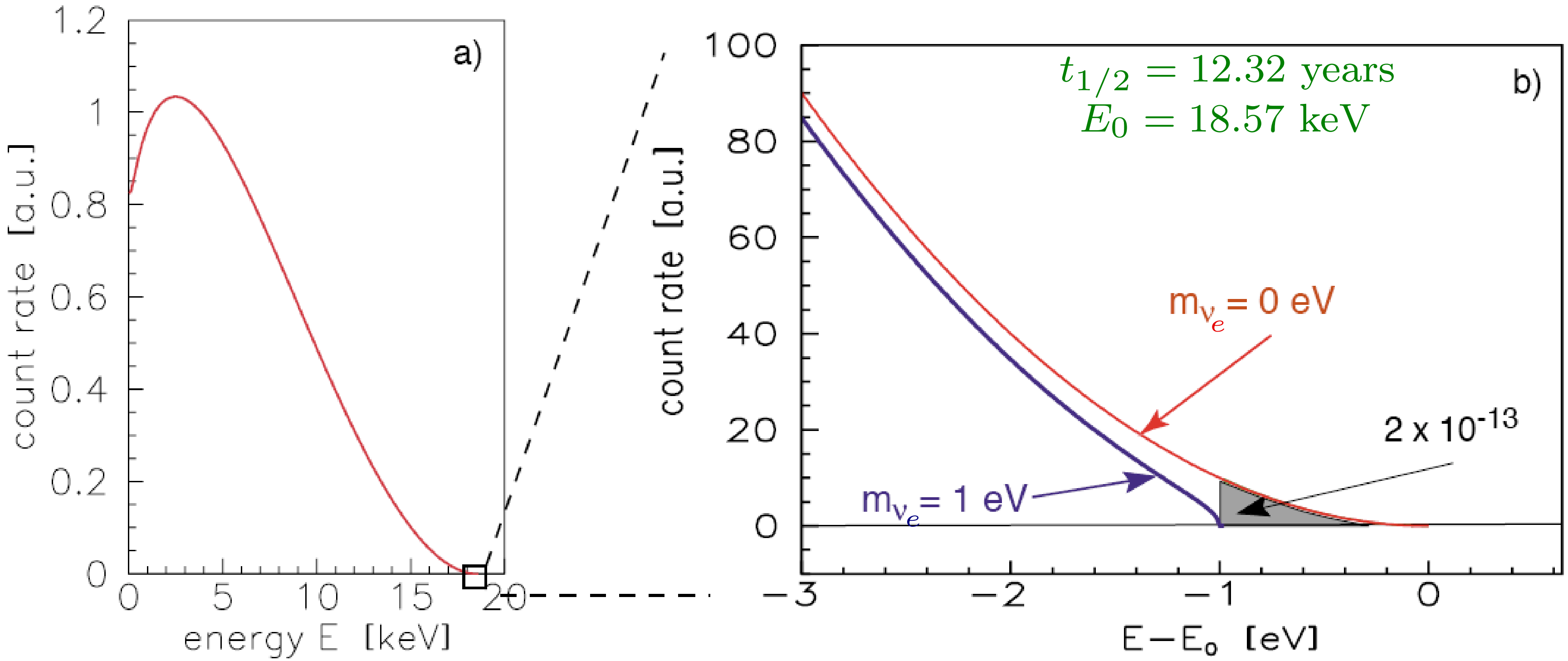


Figure 2: The electron energy spectrum of tritium β decay: (a) complete and (b) narrow region around endpoint E_0. The β spectrum is shown for neutrino masses of 0 and 1 eV.

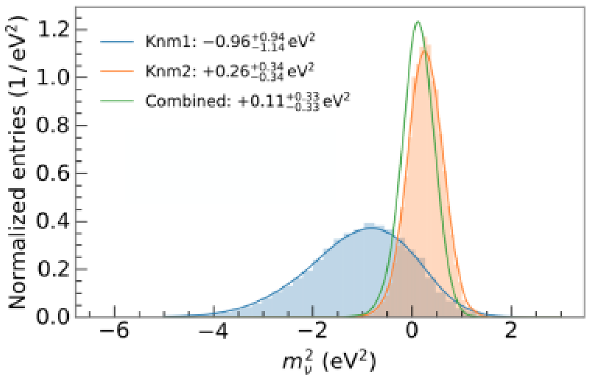
ONGOING: The Karlsruhe Tritium Neutrino (KATRIN) Experiment:

sensitivity $m^2_{\nu_e} > (0.2\text{ eV})^2$

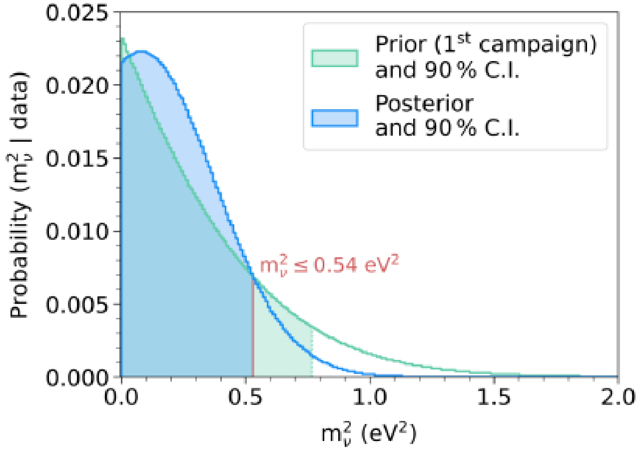
Results Combined KNM1 + KNM2

Nature Physics 18, 160 (2022)

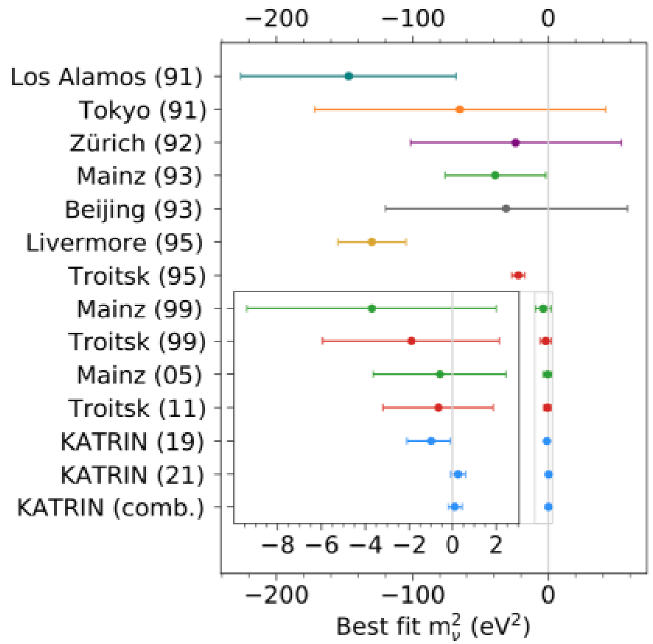
Frequentist likelihoods (multiplication or combined fit)



Bayesian posteriors (KNM1 posterior as KNM2 prior):



Best fit:	$m_\beta^2 = 0.1 \pm 0.3$ eV^2
Limits LT and FC:	$m_\beta < 0.8$ eV (90% CL)
Limits Bayesian:	$m_\beta < 0.73$ eV (90% CI)



2 months KATRIN data better than Mainz, Troitsk

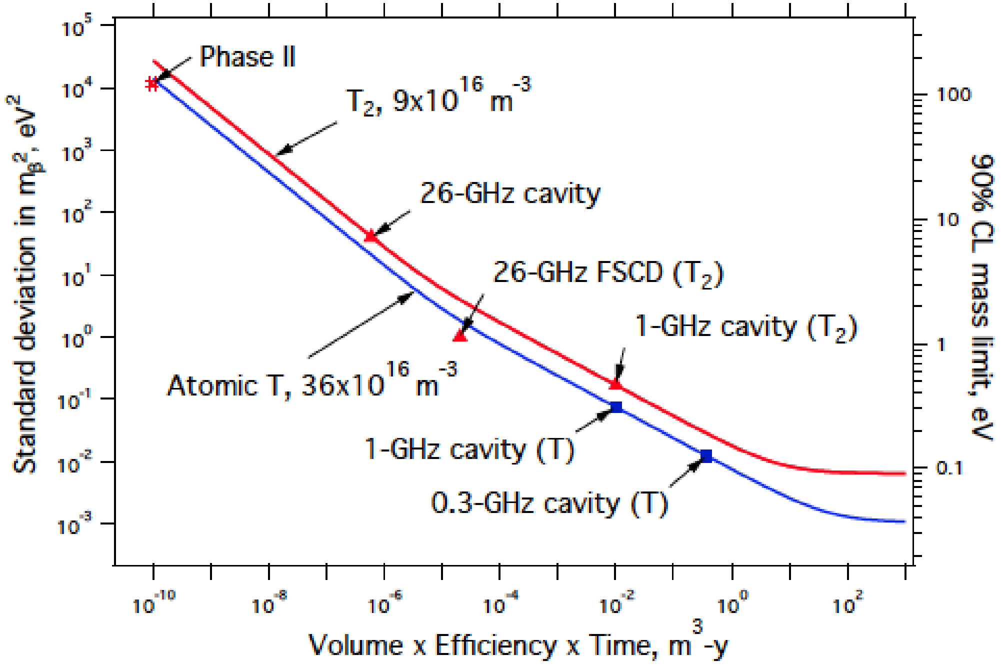
- Statistics x6, systematics x12
- First sub-eV neutrino mass sensitivity in lab
- Multiple independent blind analyses

[Lehnert at APS 2022]

Any experiment with a molecular tritium (T_2) source will have a systematic penalty associated with uncertainty from rotational and vibrational states of the daughter $^3HeT^+$ populated in the decay.

In order to push to the inverted ordering scale, future experiments will need to switch from molecular to atomic sources.

Project 8 aims to evolve to atomic tritium target to overcome this obstacle and push to the inverted ordering scale



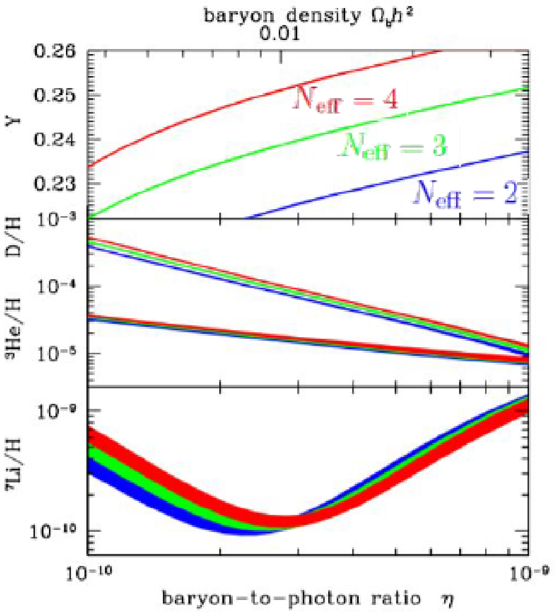
See Snowmass contribution arXiv:2203.07349 for more details

[Formaggio at APS 2022]

Cosmic Neutrinos are Measured through BBN and CMB

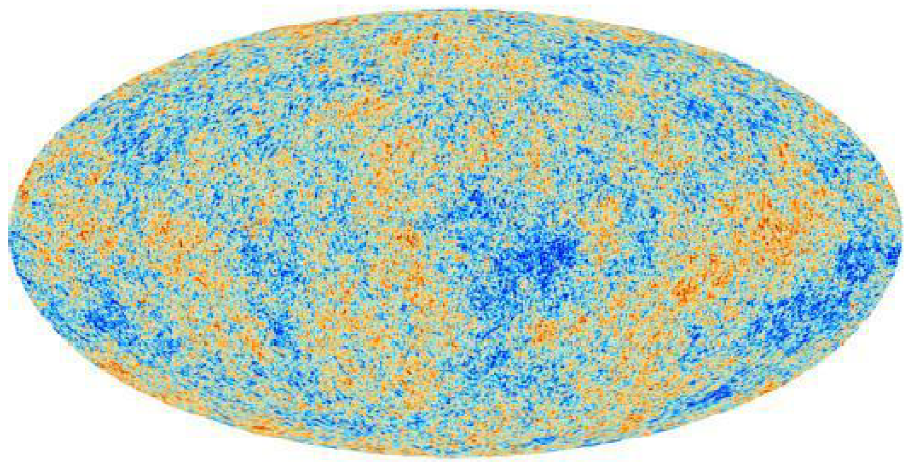
Primordial Abundances

$N_{\text{eff}}^{\text{BBN}} = 2.95 \pm 0.54$ (95% CL)



CMB Measurements

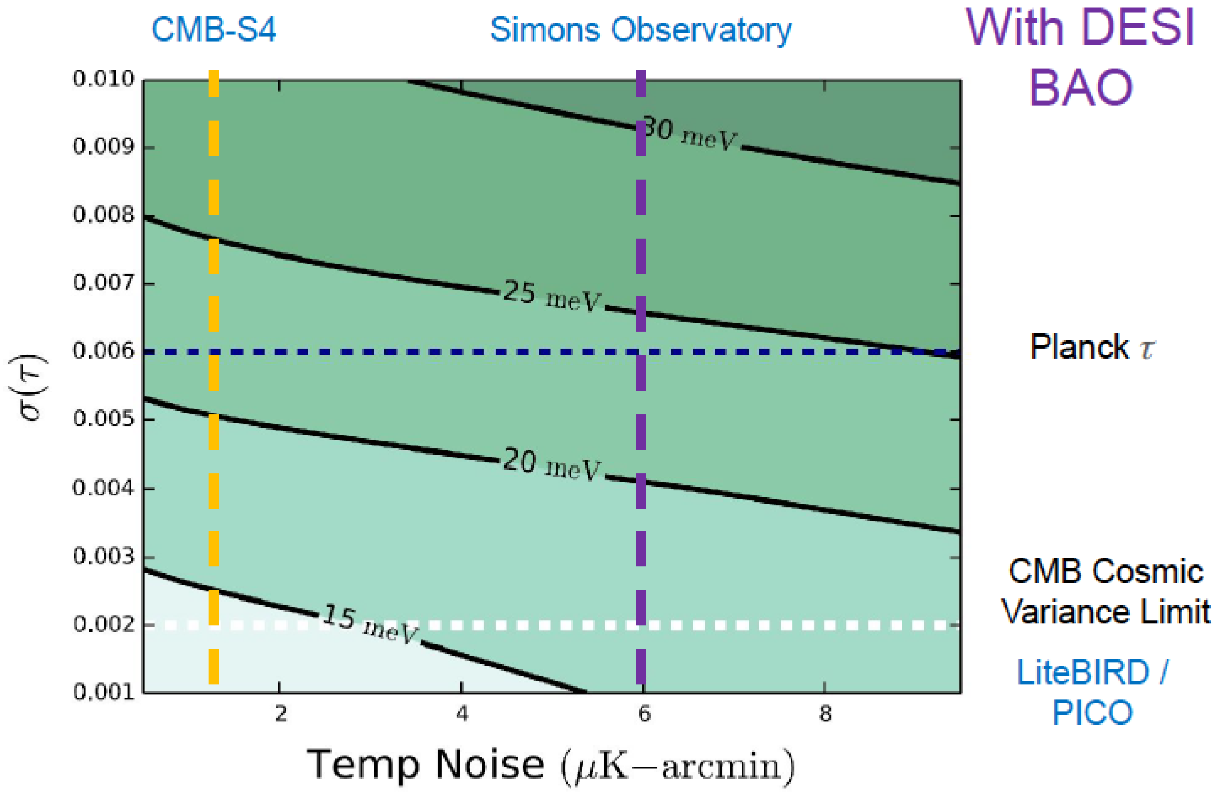
$N_{\text{eff}}^{\text{CMB}} = 2.99 \pm 0.34$ (95% CL)



Cyburt, et al. (2015); Cooke, et al. (2018); Planck (2018)

[Meyers at APS 2022]

CMB + BAO Forecasts for Neutrino Mass Constraints



Current constraint:

$$\sum m_\nu < 120 \text{ meV } (95\% \text{ CL})$$

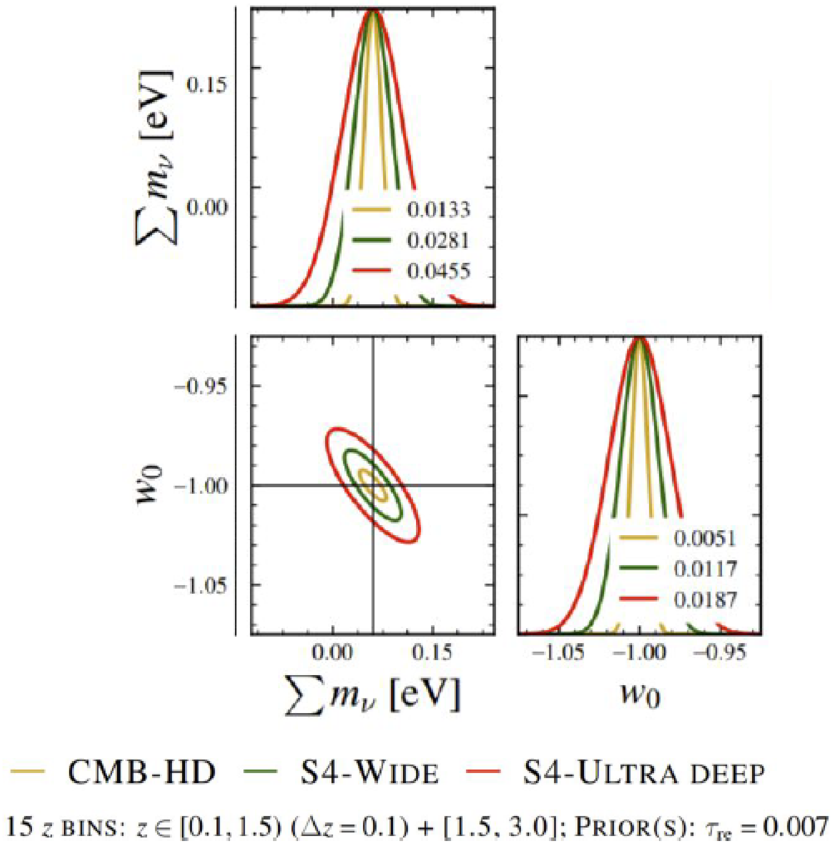
(Planck + BAO)

CMB lensing reconstruction with upcoming surveys, combined with DESI BAO, will enable significant measurement of even minimal $\sum m_\nu$ especially with improved τ measurement

CMB-S4 (2016); Simons Observatory (2018); Planck (2018)

[Meyers at APS 2022]

CMB + Cluster Forecasts for Neutrino Mass Constraints



- The abundance of galaxy clusters is sensitive to the suppression of matter clustering imprinted by massive neutrinos
- Combined with a measurement of the primordial amplitude from the CMB, cluster abundances can measure neutrino mass with similar precision to CMB lensing
- The redshift dependence of the cluster abundance also allows simultaneous constraints on other physics that may impact structure growth, like the dark energy equation of state

Raghunathan, et al (2021)

[Meyers at APS 2022]

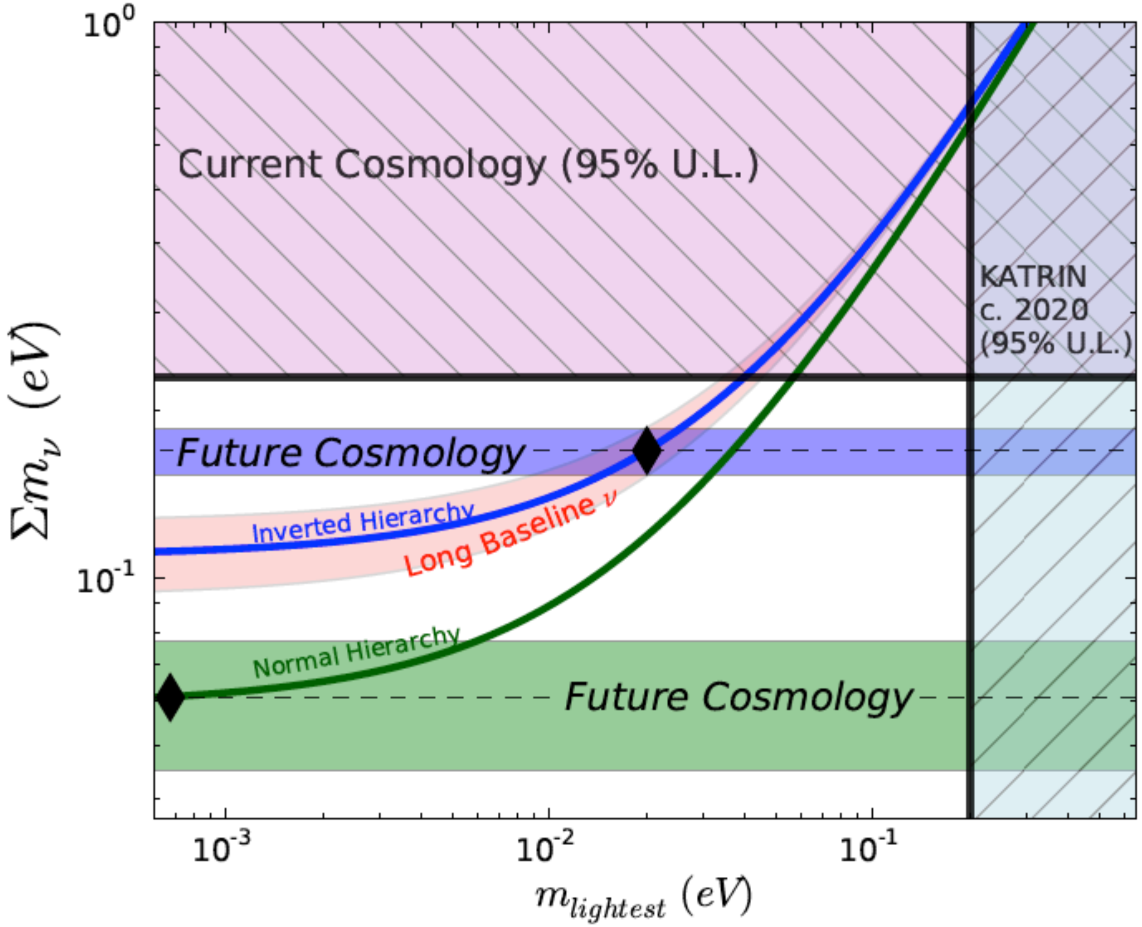


Figure 7. *Current constraints and forecast sensitivity of cosmology to the sum of neutrino masses. In the case of an "inverted hierarchy," with an example case marked as a diamond in the upper curve, future combined cosmological constraints would have a very high-significance detection, with 1-σ error shown as a blue band. In the case of a normal neutrino mass hierarchy with an example case marked as diamond on the lower curve, future cosmology would still detect the lowest* $\sum m_\nu$ *at greater than 3-σ.*

[K. Abazajian *et al.* arXiv:1309.5386]

Caveats for Cosmic Surveys as input for neutrino masses

- Indirect probe of neutrino mass. What we are really measuring are properties of the universe at very large scales as a function of red-shift.
- Degeneracies with other parameters. Lots of quantities are fit for at once. Model dependency. Current bounds can be loosened if there is new particle physics or new ingredients in the early universe.
- Imagine a positive claim from cosmic surveys that neutrino masses are not zero. Would you believe it if you did not know, from oscillations, neutrinos were massive?

Fork on the Road: Are Neutrinos Majorana or Dirac Fermions?

[9 out of 10 theorists agree: "Best" Question in Neutrino Physics Today!]

How Many Degrees of Freedom are There in a Neutrino? (2 versus 4)

A massive charged fermion (s=1/2) is described by 4 degrees of freedom:

$$e^-_L \leftarrow \text{CP} \rightarrow e^+_R$$

$$\updownarrow \text{“Lorentz”}$$

$$e^-_R \leftarrow \text{CP} \rightarrow e^+_L$$

This is referred to as a Dirac fermion. Here, we can talk about

Parity: relates $e^{\pm}_R$ with $e^{\pm}_L$

Charge-Conjugation: relates $e^{\pm}_R$ with $e^{\mp}_R$

(Massless fermions are weird. We can make do with only “half” of them, even if they are charged.)

How Many Degrees of Freedom are There in a Neutrino? (2 versus 4)

For a massive neutral fermion (s=1/2), there are two choices: Dirac ...

$$\nu_L \leftarrow \mathrm{CP} \rightarrow \bar{\nu}_R$$

$$\updownarrow \text{“Lorentz”}$$

$$\nu_R \leftarrow \mathrm{CP} \rightarrow \bar{\nu}_L$$

or Majorana ...

$$\nu_L \leftarrow \mathrm{CP} \rightarrow \nu_R$$

$$\updownarrow \text{“Lorentz”}$$

$$\nu_R \leftarrow \mathrm{CP} \rightarrow \nu_L$$

In the Majorana case, neutrinos are their own antiparticles. This means $\nu_L = \bar{\nu}_L$ and $\nu_R = \bar{\nu}_R$. (Helicity matters!)

Why Don't We Know the Answer to 4 versus 2?

If neutrino masses were indeed zero, this is a nonquestion: there is no distinction between a massless Dirac and Majorana fermion.

Processes that are sensitive to the Majorana versus Dirac nature of the neutrino vanish in the limit $m_\nu \to 0$. Since neutrinos masses are very small, the probability for these to happen is very, very small: $A \propto m_\nu/E$.

Charged-Current Weak Interactions are Purely Left-Handed (Chirality)

What does this mean? For example, In the decay of a muon at rest,

$$\mu^- \to e^- \nu_\mu \bar{\nu}_e,$$

the electrons come out almost 100% polarized:

$$|e^-\rangle \sim |L\rangle + \left(\frac{m_e}{m_\mu}\right)|R\rangle.$$

For the CP-conjugated process, we get the CP-conjugated answer: In the process

$$\mu^+ \to e^+ \bar{\nu}_\mu \nu_e,$$

the positrons come out almost 100% polarized:

$$|e^+\rangle \sim |R\rangle + \left(\frac{m_e}{m_\mu}\right)|L\rangle.$$

Charged-Current Weak Interactions are Purely Left-Handed (Chirality)

When it comes to neutrino production, for example, in pion-decay at rest

$$\pi^+ \to \mu^+ \nu$$

$$|\nu\rangle \sim |L\rangle + \left(\frac{m_\nu}{m_\pi}\right)|R\rangle.$$

For the CP-conjugated process, we get the CP-conjugated answer:

$$\pi^- \to \mu^- + \mathrm{CP}(\nu)$$

the CP-conjugated neutrino state comes out almost 100% polarized:

$$|\mathrm{CP}(\nu)\rangle \sim |R\rangle + \left(\frac{m_\nu}{m_\pi}\right)|L\rangle.$$

(Remember: $m_\nu/m_\pi < 10^{-9}$)

Charged-Current Weak Interactions are Purely Left-Handed (Chirality)

The same goes for neutrino detection. Ignoring neutrino-mass effects

$$\nu_L + X \to e^- + Y$$

and the CP conjugate channel is

$$\mathrm{CP}(\nu_L) + \mathrm{CP}(X) \to e^+ + \mathrm{CP}(Y)$$

So, if we can ignore neutrino masses, left-handed neutrinos are produced together with positively-charged leptons and, when they are detected, they only know how to produce negatively-charged leptons. The opposite goes for the CP-conjugate of the neutrino: these are produced with negatively-charged leptons and, when they are detected, they only know how to produce positively-charged leptons. It does not matter if they are Dirac fermions or Majorana fermions!

Gedanken Experiment, remembering that $m_\nu \neq 0$:

In the scattering process $e^- + X \to \nu_e + X$, the electron neutrino is, in a reference frame where $m \ll E$,

$$|\nu_e\rangle \sim |L\rangle + \left(\frac{m}{E}\right)|R\rangle.$$

If the neutrino is a Majorana fermion, $|R\rangle$ behaves mostly like a "$\bar{\nu}_e$," (and $|L\rangle$ mostly like a "ν_e,") such that the following process could happen:

$$e^- + X \to \nu_e + X, \quad \text{followed by} \quad \nu_e + X \to e^+ + X, \qquad P \simeq \left(\frac{m}{E}\right)^2$$

Lepton number can be violated by 2 units with small probability. Typical numbers: $P \simeq (0.1~\text{eV}/100~\text{MeV})^2 = 10^{-18}$. VERY Challenging!

Global Lepton Number Symmetry

In the massless-neutrino limit, there is a conserved global symmetry we call **Lepton Number**. If we assign the following charges to the leptons

$$L(e^-) = L(\mu^-) = L(\tau^-) = 1 = L(\nu),$$

$$L(e^+) = L(\mu^+) = L(\tau^+) = -1 = L(\mathrm{CP}(\nu)),$$

the total lepton number is always conserved.

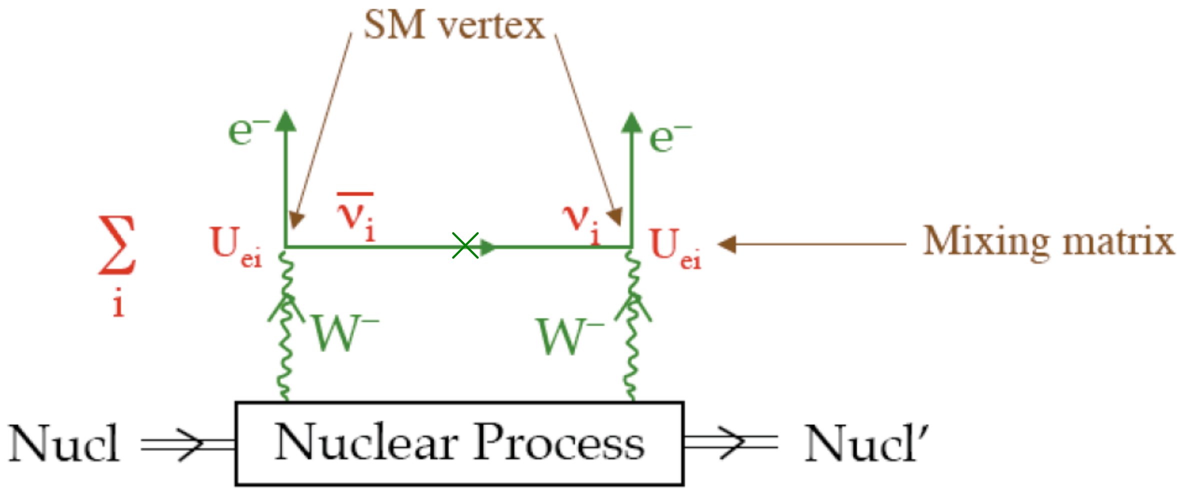
If neutrinos are massive Majorana fermions, we can't assign them ANY quantum number, including lepton number. Hence, lepton number cannot be exactly conserved. If neutrinos are Majorana fermions, lepton number is only approximately conserved. Hence, the "smoking gun" signature of Majorana neutrinos is the observation of LEPTON NUMBER violation.

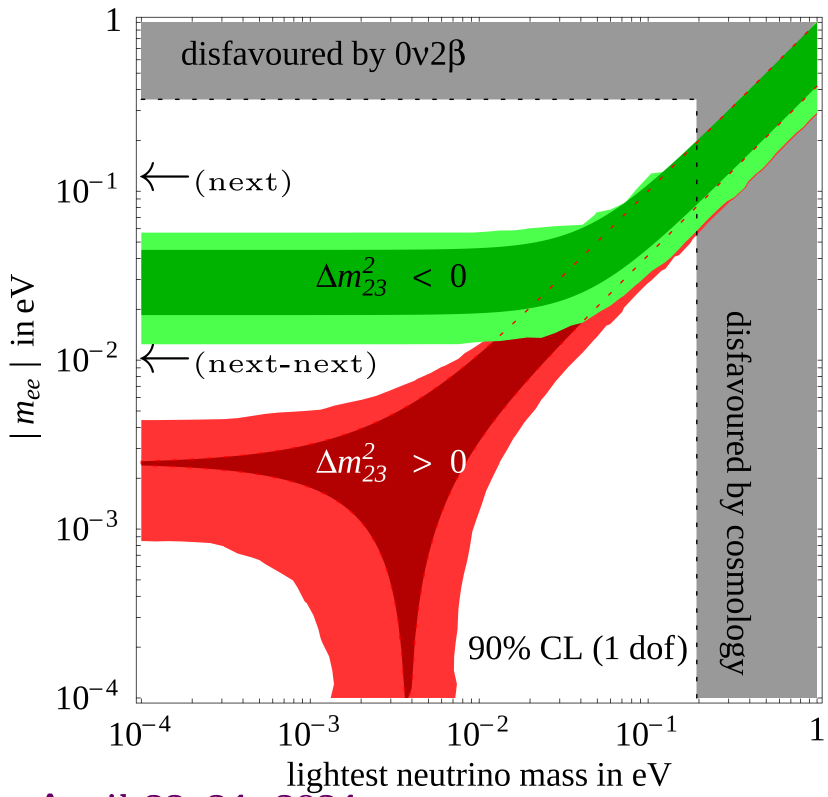
Search for the Violation of Lepton Number (or $B-L$)

Best Bet: search for

Neutrinoless Double-Beta

Decay: $Z \to (Z+2)e^- e^-$





Helicity Suppressed Amplitude $\propto \frac{m_{ee}}{E}$

Observable: $m_{ee} \equiv \sum_i U_{ei}^2 m_i$

⇐ **no longer lamp-post physics!**

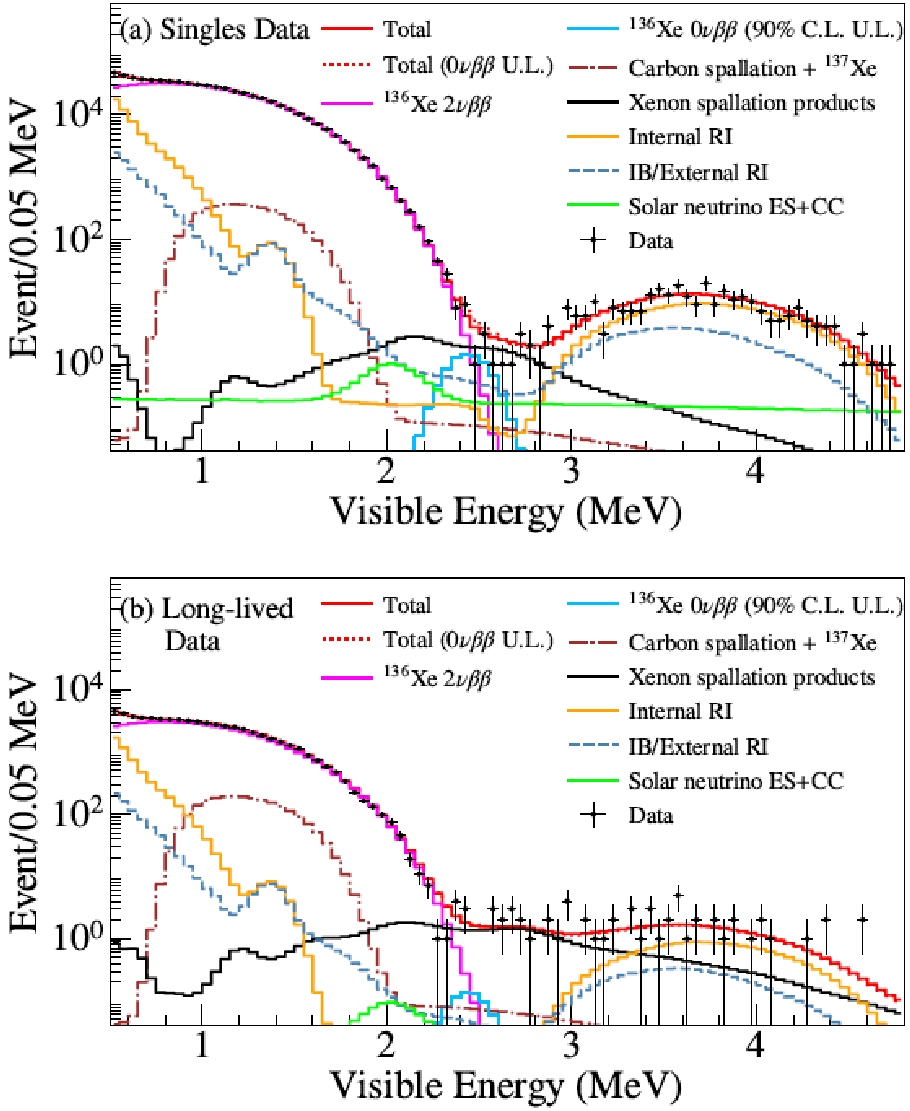


FIG. 2: Energy spectra of selected $\beta\beta$ candidates within a 1.57-m-radius spherical volume drawn together with best-fit backgrounds, the $2\nu\beta\beta$ decay spectrum, and the 90% C.L. upper limit for $0\nu\beta\beta$ decay of (a) singles data (SD), and (b) long-lived data (LD). The LD exposure is about 10% of the SD exposure.

Lots of Experimental Activity!

Moving Towards Ton-Scale Expts.

(LEGEND, CUPID, nEXO, etc)

[KamLAND-Zen Coll. (Abe *et al*), 2203.02139 [hep-ex]]

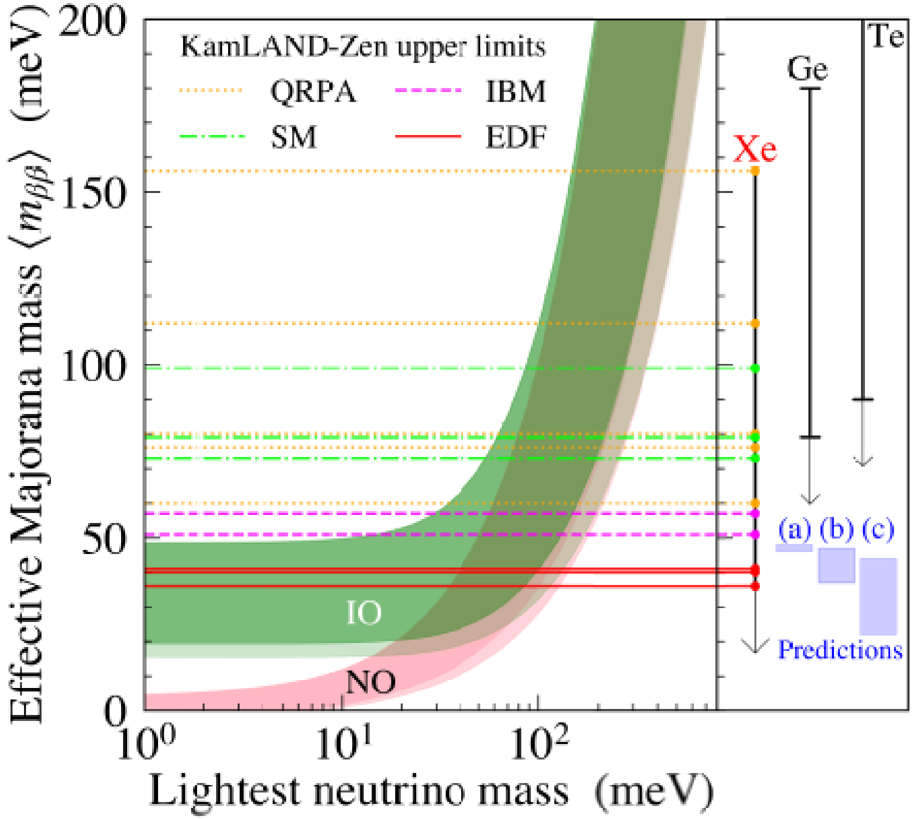


FIG. 4: Effective Majorana neutrino mass $\langle m_{\beta\beta}\rangle$ as a function of the lightest neutrino mass. The dark shaded regions are predictions based on best-fit values of neutrino oscillation parameters for the normal ordering (NO) and the inverted ordering (IO), and the light shaded regions indicate the 3σ ranges calculated from oscillation parameter uncertainties [23, 24]. The regions below the horizontal lines are allowed at 90% C.L. with ^{136}Xe from KamLAND-Zen (this work) considering an improved phase space factor calculation [25, 26] and commonly used nuclear matrix element estimates, EDF [27–29] (solid lines), IBM [30, 31] (dashed lines), SM [32–34] (dot-dashed lines), QRPA [35–39] (dotted lines). The side-panel shows the corresponding limits for ^{136}Xe, ^{76}Ge [40], and ^{130}Te [41], and theoretical model predictions on $\langle m_{\beta\beta}\rangle$, (a) Ref. [2], (b) Ref. [3], and (c) Ref. [4] (shaded boxes), in the IO region.

Lots of Experimental Activity!

Moving Towards Ton-Scale Expts.

(LEGEND, CUPID, nEXO, etc)

[KamLAND-Zen Coll. (Abe *et al*), 2203.02139 [hep-ex]]

Caveats for $0\nu\beta\beta$ as input for neutrino masses

- Indirect probe of neutrino mass;
- Only works if the neutrinos are Majorana fermions;
- Model dependent. While a nonzero rate for $0\nu\beta\beta$ implies neutrinos are massive Majorana fermions, the connection to nonzero neutrino masses can be very indirect. How do we learn that we are measuring what we think we are measuring?
- Real life is hard. Large uncertainties in translating the half-life to the effective neutrino mass (nuclear matrix elements).

How many new CP-violating parameters in the neutrino sector?

If the neutrinos are Majorana fermions, there are more physical observables in the leptonic mixing matrix.

Remember the parameter counting in the quark sector:

9 (3×3 unitary matrix)

$\underline{-5}$ (relative phase rotation among six quark fields)

4 (3 mixing angles and 1 CP-odd phase).

If the neutrinos are Majorana fermions, the parameter counting is quite different: there are no right-handed neutrino fields to “absorb” CP-odd phases:

9 (3 × 3 unitary matrix)

$\underline{-3}$ (three right-handed charged lepton fields)

6 (3 mixing angles and 3 CP-odd phases).

There is CP-invariance violating parameters even in the 2 family case: $4 - 2 = 2$, one mixing angle, one CP-odd phase.

$$\mathcal{L} \supset \bar{e}_L U W^{\mu}\gamma_{\mu}\nu_L - \bar{e}_L(M_e)e_R - \overline{\nu^c_L}(M_\nu)\nu_L + H.c.$$

Write $U = E^{-i\xi/2}U'E^{i\alpha/2}$, where $E^{i\beta/2} \equiv \text{diag}(e^{i\beta_1/2}, e^{i\beta_2/2}, e^{i\beta_3/2})$, $\beta = \alpha, \xi$

$$\mathcal{L} \supset \bar{e}_L U' W^{\mu}\gamma_{\mu}\nu_L - \bar{e}_L E^{i\xi/2}(M_e)e_R - \overline{\nu^c_L}(M_\nu)E^{-i\alpha}\nu_L + H.c.$$

ξ phases can be "absorbed" by e_R,

α phases cannot go away!

on the other hand

Dirac Case:

$$\mathcal{L} \supset \bar{e}_L U W^{\mu}\gamma_{\mu}\nu_L - \bar{e}_L(M_e)e_R - \bar{\nu}_R(M_\nu)\nu_L + H.c.$$

$$\mathcal{L} \supset \bar{e}_L U' W^{\mu}\gamma_{\mu}\nu_L - \bar{e}_L E^{i\xi/2}(M_e)e_R - \bar{\nu}_R(M_\nu)E^{-i\alpha/2}\nu_L + H.c.$$

ξ phases can be "absorbed" by e_R, α phases can be "absorbed" by ν_R,

$$V_{MNS} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{e\tau 2} & U_{\tau 3} \end{pmatrix}' \begin{pmatrix} e^{i\alpha_1/2} & 0 & 0 \\ 0 & e^{i\alpha_2/2} & 0 \\ 0 & 0 & e^{i\alpha_3/2} \end{pmatrix}.$$

It is easy to see that the Majorana phases never show up in neutrino oscillations ($A \propto U_{\alpha i} U^*_{\beta_i}$).

Furthermore, they only manifest themselves in phenomena that vanish in the limit $m_i \to 0$ – after all they are only physical if we "know" that lepton number is broken.

$$A(\alpha_i) \propto m_i/E \quad \to \quad \text{tiny!}$$

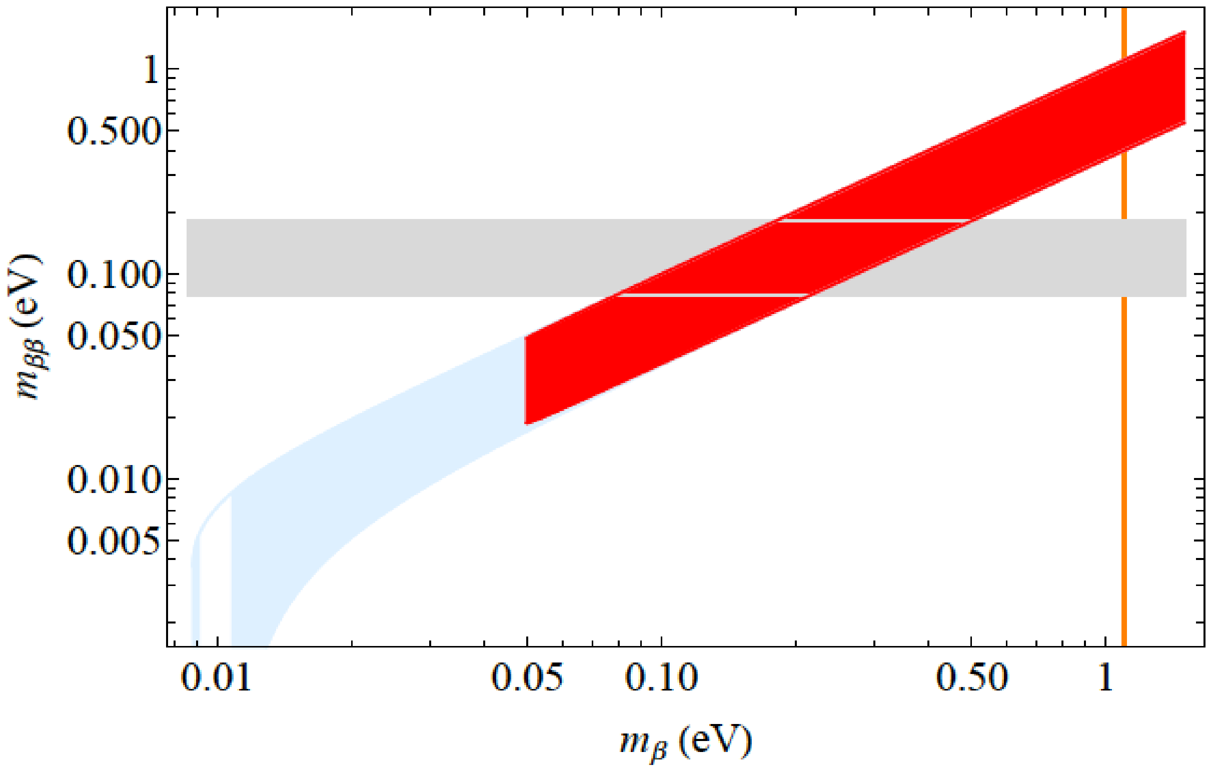


FIG. 5: $m_{\beta\beta}$ as a function of m_β, for both the normal (lighter, blue) and inverted (darker, red) mass orderings. The bands are a consequence of allowing for all possible values of the relative Majorana phases. For everything else, we use the current best-fit values of the oscillation parameters from [29]. The whited-out region inside the light-blue contour is meant to highlight the values of m_β for which $m_{\beta\beta}$ can vanish exactly. We assume the neutrinos are Majorana fermions. If neutrinos are Dirac fermions, $m_{\beta\beta} = 0$. The grey, horizontal band corresponds to the 95% CL upper bound on $m_{\beta\beta}$ from GERDA [37]. The width of the band is a consequence of uncertainties in the nuclear matrix element for the neutrinoless double-beta decay of ^{76}Ge. The vertical line corresponds to the current 90% upper bound on m_β [56].

[Formaggio, AdG, Robertson, Phys.Rept. 914 (2021)]

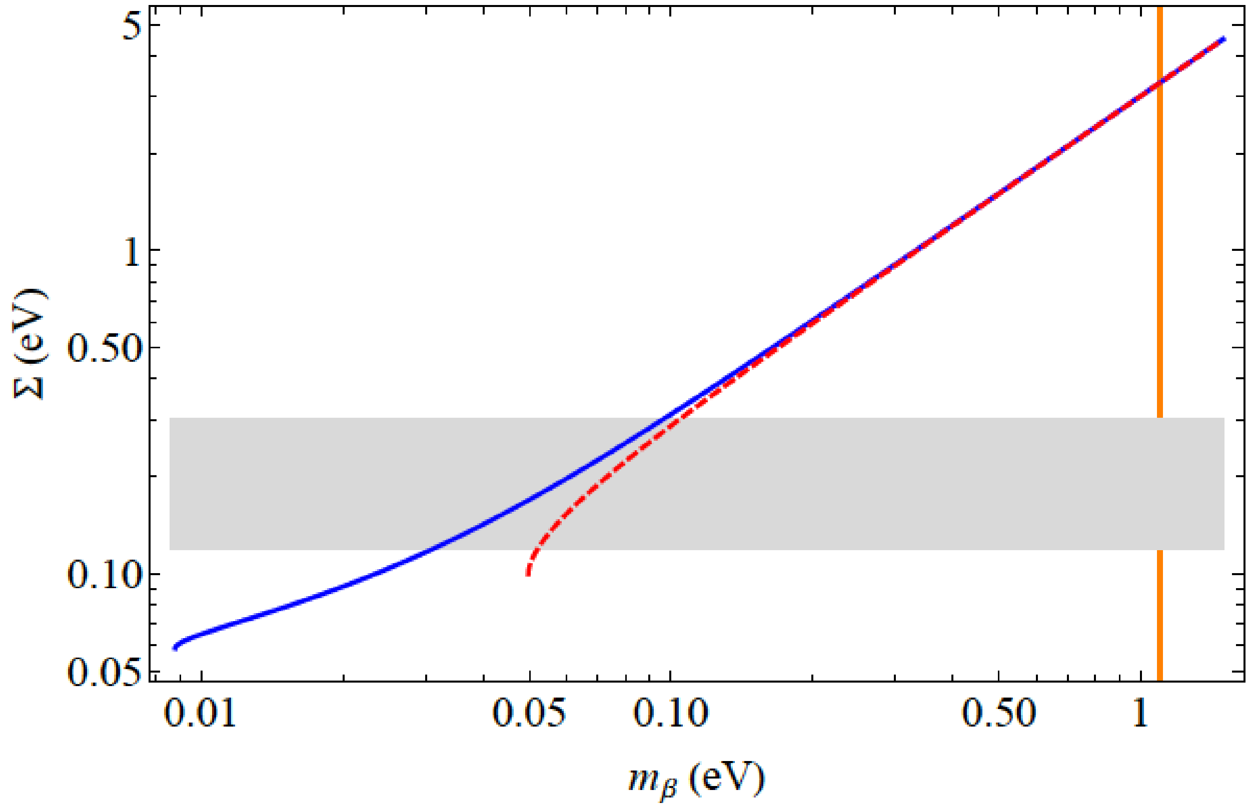
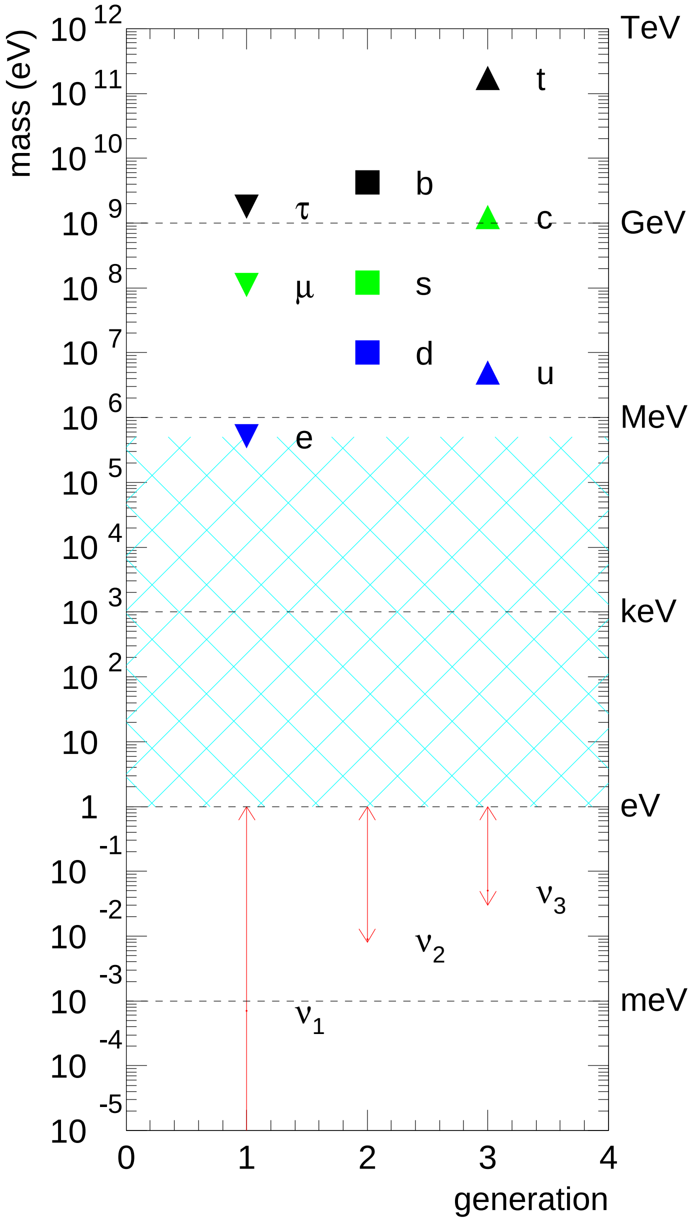


FIG. 6: Σ as a function of m_β, for both the normal (blue,solid) and inverted (red,dashed) mass orderings. We use the current best-fit values of the oscillation parameters from [29]. The, horizontal band corresponds to the range of 95% CL upper bounds on Σ discussed in [60]. Different upper bounds correspond to different ingredients added to the Standard Model of cosmology. The vertical line corresponds to the current 90% upper bound on m_β [56].

[Formaggio, AdG, Robertson, Phys.Rept. 914 (2021)]



NEUTRINOS HAVE MASS

albeit very tiny ones…

SO WHAT?

Nonzero neutrino masses imply the existence of new fundamental fields ⇒ New Particles

We know nothing about these new particles. They can be bosons or fermions, very light or very heavy, they can be charged or neutral, experimentally accessible or hopelessly out of reach...

There is only a handful of questions the standard model for particle physics cannot explain (these are personal. Feel free to complain).

- What is the physics behind electroweak symmetry breaking? (Higgs ✓).
- What is the dark matter? (not in SM).
- Why is there so much ordinary matter in the Universe? (not in SM).
- Why does the Universe appear to be accelerating? Why does it appear that the Universe underwent rapid acceleration in the past? (not in SM).

Neutrino Masses, Higgs Mechanism, and New Mass Scale of Nature

The LHC has revealed that the minimum SM prescription for electroweak symmetry breaking — the one Higgs doublet model — is at least approximately correct. What does that have to do with neutrinos?

The tiny neutrino masses point to three different possibilities.

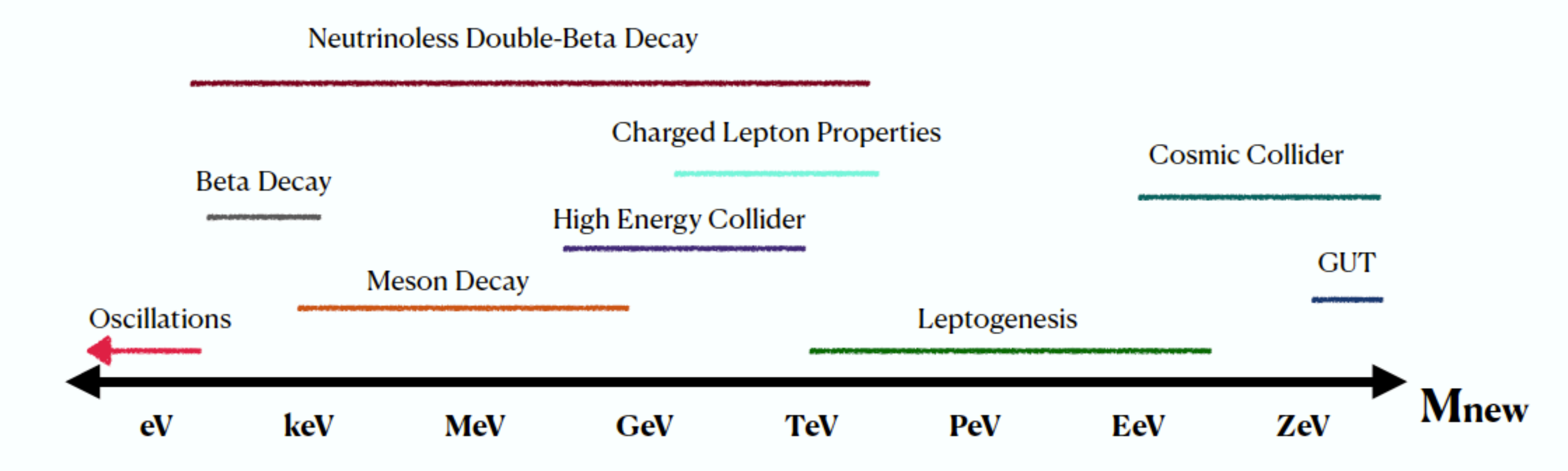
1. Neutrinos talk to the Higgs boson very, very **weakly**. And **lepton-number must be an exact symmetry** of nature (or broken very, very weakly);

2. Neutrinos talk to a **different Higgs** boson – there is a new source of electroweak symmetry breaking!;

3. Neutrino masses are small because there is **another source of mass** out there — a new energy scale indirectly responsible for the tiny neutrino masses, a la the **seesaw mechanism**.

We are going to need a lot of experimental information from all areas of particle physics in order to figure out what is really going on!

What Is the ν Physics Scale? We Have No Idea!



Different Mass Scales Are Probed in Different Ways, Lead to Different Consequences, and Connect to Different Outstanding Issues in Fundamental Physics.

Concluding Remarks

The venerable Standard Model sprung a leak in the end of the last century: neutrinos are not massless! [and we are still trying to patch it. . .]

1. We still **know very little** about the new physics uncovered by neutrino oscillations. In particular, the new physics (broadly defined) can live almost anywhere between sub-eV scales and the GUT scale.

2. **Neutrino masses are very small** – we don't know why, but we think it means something important.

3. **Neutrino mixing is "weird"** – we don't know why, but we think it means something important.

4. What is going on with the **short-baseline anomalies?**

5. There is plenty of **room for surprises**, as neutrinos are very deep probes of all sorts of physical phenomena. Neutrino oscillations are "quantum interference devices," potentially sensitive to whatever else might be out there (keep in mind, neutrino masses might be physics at $\Lambda \simeq 10^{14}$ GeV).