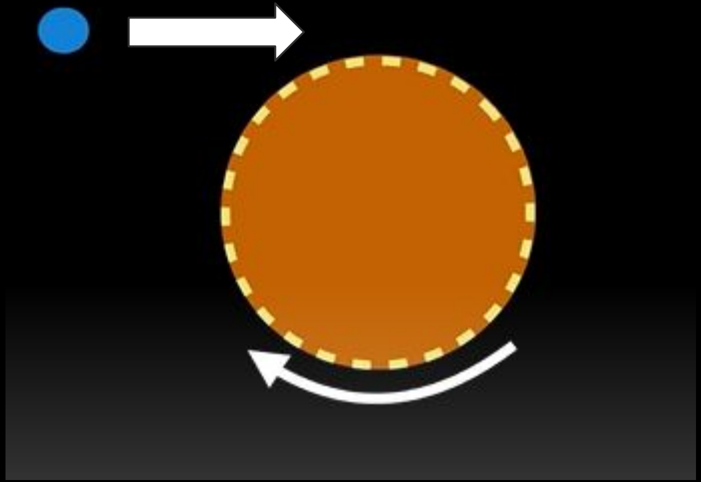
Seeing the invisible with black hole superradiance sourced gravitational waves

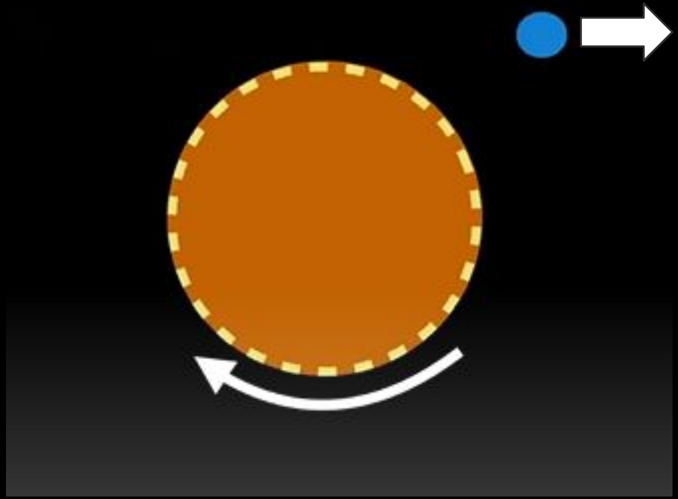
Indra Kumar Banerjee
IISER Berhampur

06/01/2025

Hearing Beyond the Standard Model with Cosmic Sources of Gravitational Waves
International Center for Theoretical Sciences

Superradiance

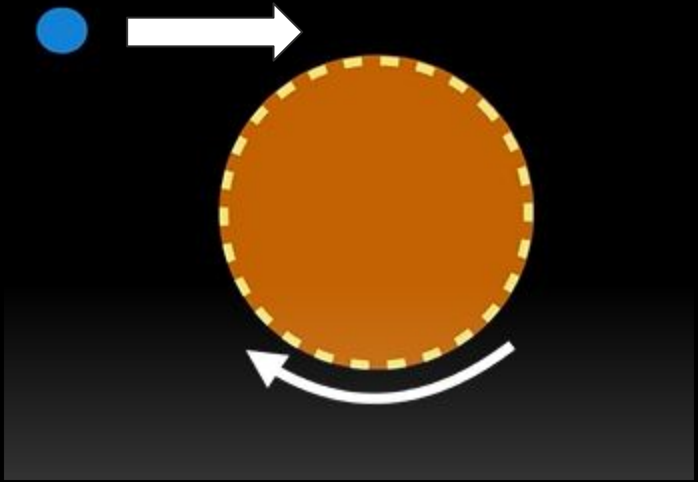


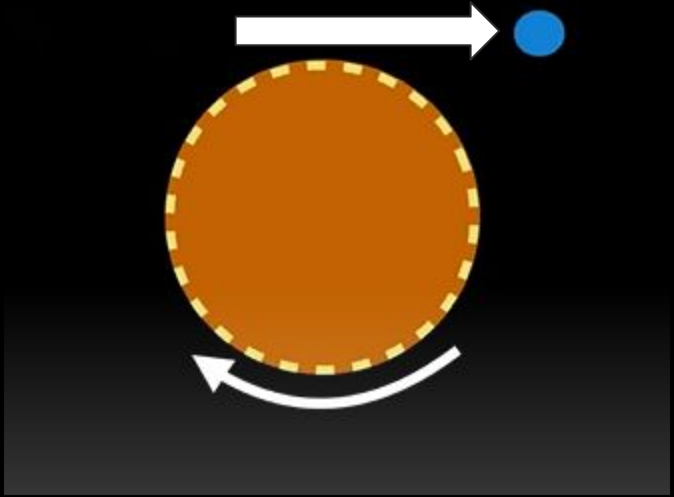


Rotating cylinder with dissipation or absorption.

Cartoon taken from talk titled “Black holes superradiance of self-interacting scalar field” by Oliver Simon

Superradiance



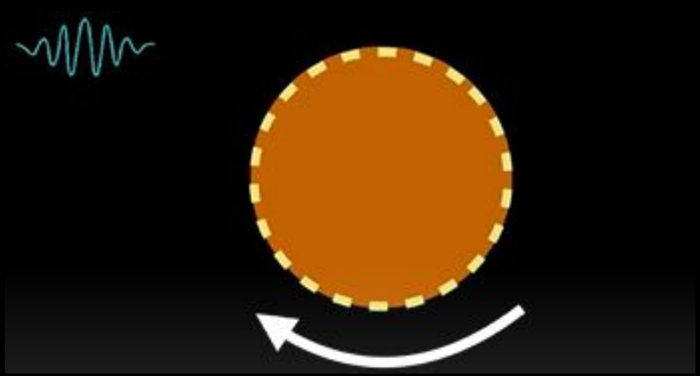


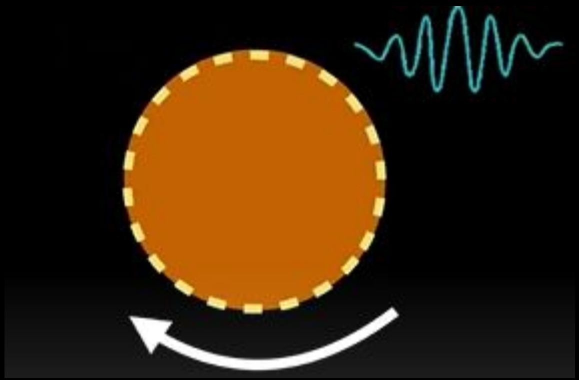
$$\Omega_c > v_\phi$$

Absorption causes the enhancement of velocity

Cartoon taken from talk titled "Black holes superradiance of self-interacting scalar field" by Oliver Simon

Wave Superradiance

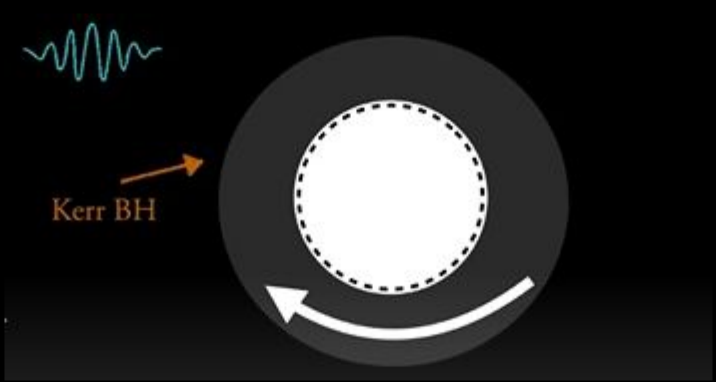


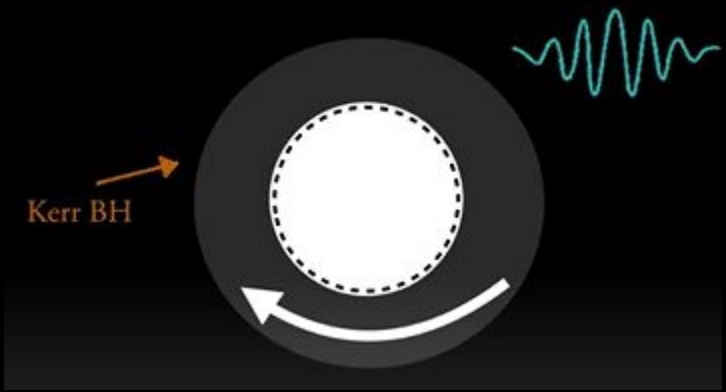


$$\Omega_c > \omega/m$$

Cartoon taken from talk titled “Black holes superradiance of self-interacting scalar field” by Oliver Simon

Black hole Superradiance



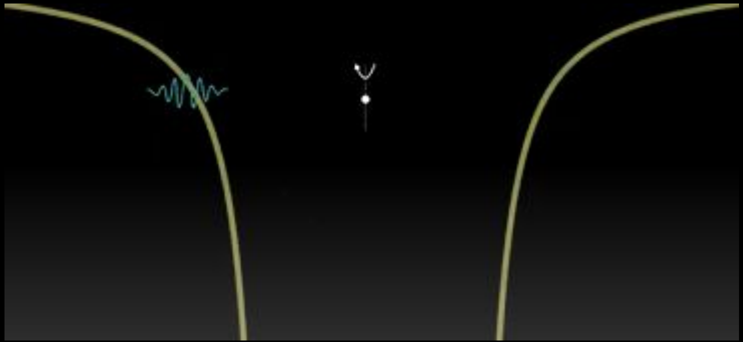


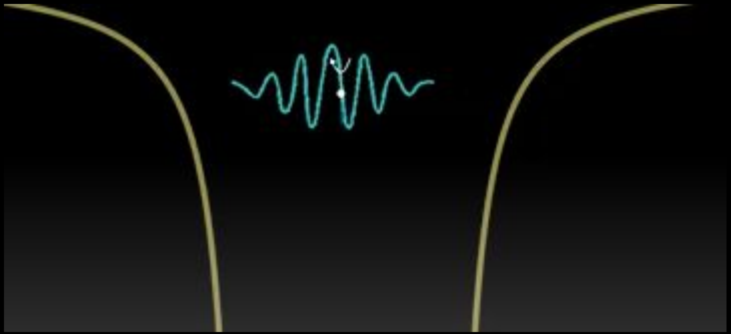
$$\Omega_{\mathrm{BH}} > \omega/m$$

Cartoon taken from talk titled "Black holes superradiance of self-interacting scalar field" by Oliver Simon

BH Superradiance: Massive Particles

$$V(r) = -\frac{GM_{\mathrm{BH}}\mu}{r}$$





$$V(r) = -\frac{GM_{\mathrm{BH}}\mu}{r} = -\frac{\alpha}{r}$$

Gravitational fine-structure Constant

This makes the massive bound states very similar to the Hydrogen atom problem with the states characterized with (*n, l, m*)

Cartoon taken from talk titled "Black holes superradiance of self-interacting scalar field" by Oliver Simon

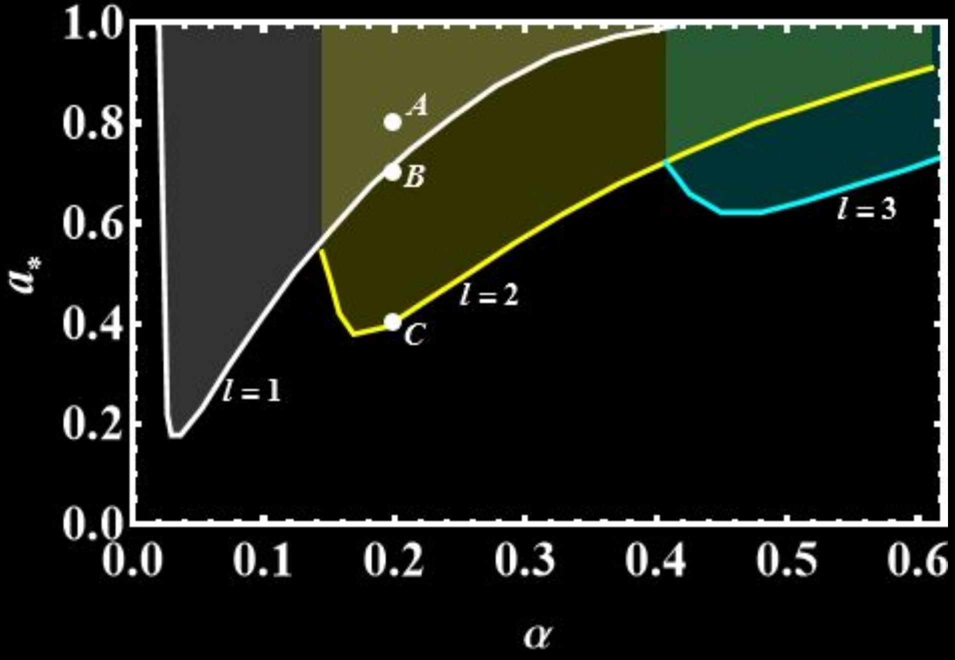
Gravitational Atom

- **Cloud size :** $$r_{\text{cloud}} \approx \frac{n^2}{\alpha^2} r_g$$
- **One particle energy :** $$E \approx \mu \left(1 - \frac{\alpha^2}{2n^2}\right) + i\Gamma^{\text{SR}}_{nlm}$$

Superradiance rate

1. Most efficient at $l = m$
2. $\boldsymbol{\Gamma}^{\text{SR}}_{\mathbf{211}} \gg \boldsymbol{\Gamma}^{\text{SR}}_{\mathbf{322}} \gg \ldots$

Gravitational Atom: Evolution



- $\alpha \approx 0.05 \left(\frac{M_{\rm BH}}{M_\odot}\right)\left(\frac{\mu}{7\times10^{-19}\ \text{GeV}}\right)$
- $t_{AB} \ll t_{BC}$
- $\alpha = 0.2\ \text{ and }\ \mu = 6\times10^{-22}\ \text{GeV}$

  $t_{AB} \sim 1\ \text{yr}\ \text{ and }\ t_{BC} \sim 10^6\ \text{yr}$

Gravitational Wave Generation

Annihilation of the scalars

Transition of energy levels

Gravitational Wave Generation

Annihilation of the scalars

- $\text{GW frequency} \sim 2\mu$

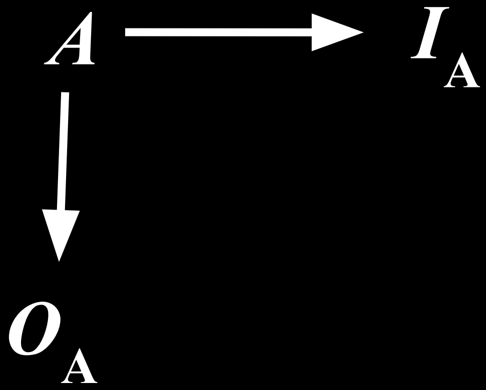
Transition of energy levels JCAP 11 (2024) 045

- $(3,1,1) \rightarrow (2,1,1)$
- $\Delta\omega = \frac{5}{72}\alpha^2\mu$

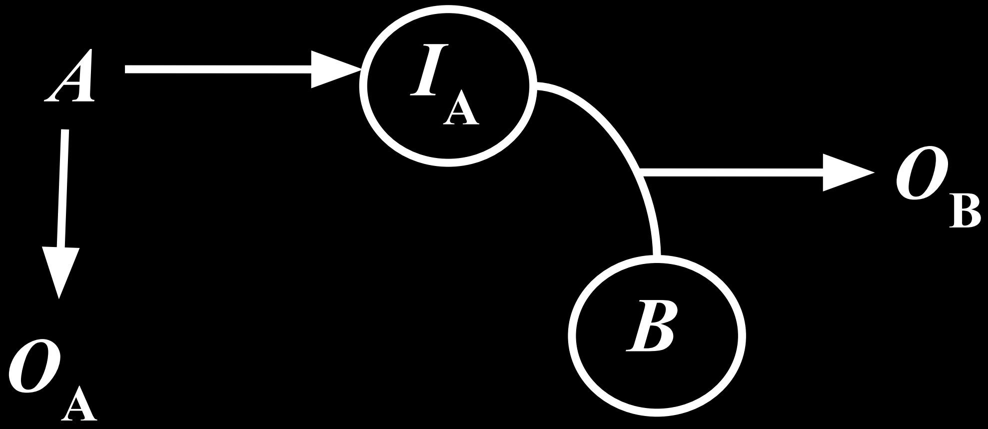
$$\tau_{\rm GW} = 0.48\left(\frac{\mu}{10^{-21}\ \mathrm{GeV}}\right)^{-9}\left(\frac{M_{\rm BH}}{20M_\odot}\right)^{-8}\ \mathrm{yr}$$

$$E_{\rm GW} = \frac{\alpha M_{\rm BH}}{3\mu}\left(1 - e^{-t/\tau_{\rm GW}}\right)\Delta\omega$$

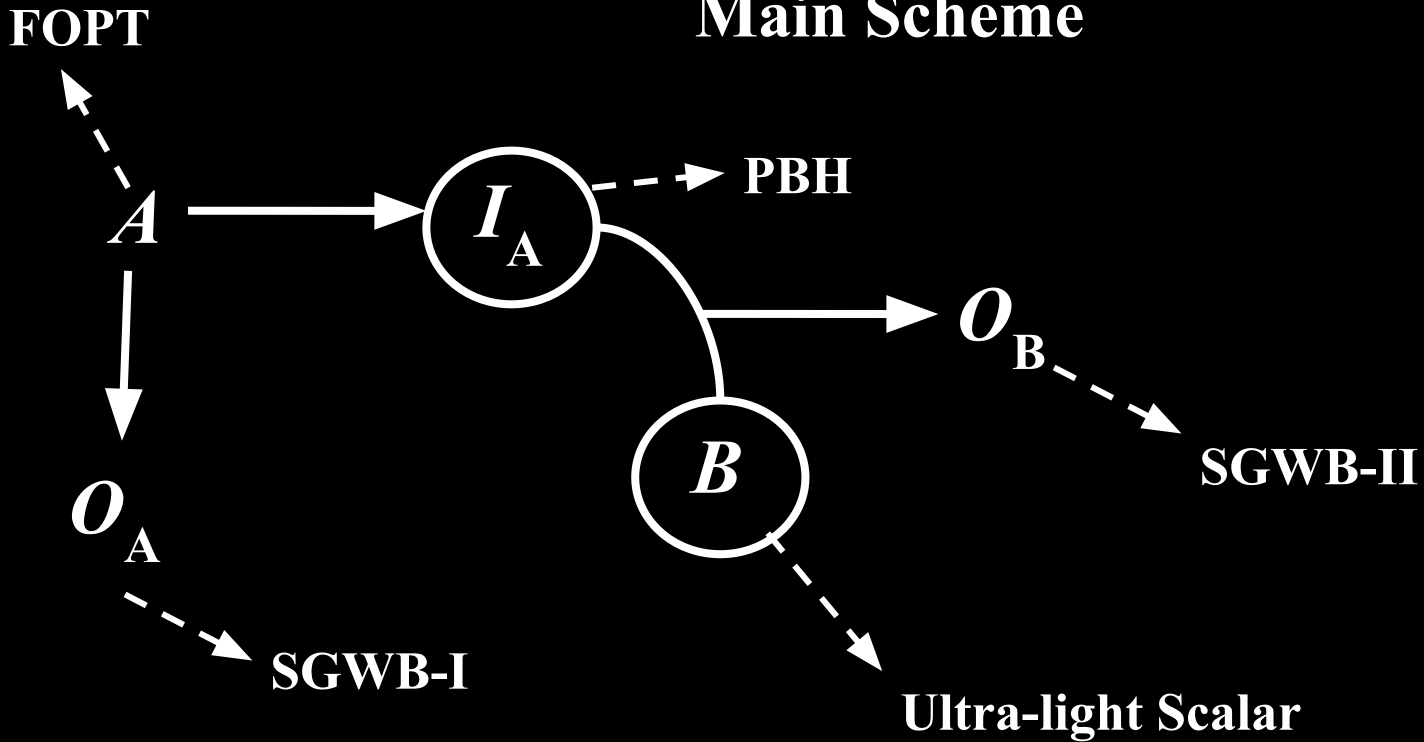
Main Scheme



Main Scheme



Main Scheme



JCAP 11 (2024) 045, IKB, Ujjal Kumar Dey

First Order Phase Transition

Talks by Ryusuke Jinno, Francesco Resigno, Andrew Long, Thomas Konstandin

Important Variables: $\alpha,\ \beta/H,\ T_{\mathrm{reh}},\ v_w, \ldots$

GW Generation: Main contribution from bubble wall collision

Kosowsky, Turner '93
Lewicki et. al '22

$$\Omega_{\mathrm{GW}}(f) = \frac{1}{\rho_c}\frac{d\rho_{\mathrm{GW}}}{d\ln f}$$

$$= 1.67\times 10^{-5}\left(\frac{H}{\beta}\right)^2\left(\frac{\kappa\alpha}{1+\alpha}\right)^2\frac{0.11v_w^2}{0.42+v_w^3}\left(\frac{100}{g_*}\right)^{1/3}\frac{3.8(f/f_p)^{2.8}}{1+2.8(f/f_p)^{3.8}}$$

$$f_p = \frac{0.62}{1.8-0.1v_w+v_w^2}\left(\frac{\beta}{H}\right)\frac{T}{100\ \mathrm{GeV}}\left(\frac{g_*}{100}\right)^{1/6}\times 1.65\times 10^{-5}\ \mathrm{Hz}$$

First Order Phase Transition

Origin of Primordial Black Holes: Collapse of overdense regions[1], collapse from the curvature perturbations created due to the fluctuation of bubble nucleation[2], etc.

$$\mathcal{P}_{coll} = \exp\left[-0.5646(\beta/H)^{1.466}(1+\delta_c^{0.6639\beta/H})\right]$$

$$f_{\rm PBH} = \left(\frac{\mathcal{P}_{coll}}{2.2\times10^{-8}}\right)\left(\frac{T_{\rm reh}}{140\ {\rm MeV}}\right)$$

$$M_{\rm PBH} = \left(\frac{20}{g_*(T_{\rm reh})}\right)^{1/2}\left(\frac{T_{\rm reh}}{140\ {\rm MeV}}\right)^{-2}$$

2305.04942

- [1] 2106.05637, 2212.14037, 2305.04924, 2305,04942, …
- [2] 2402.04158

SGWB due to BH Superradiance

- The initial spin of PBHs created from FOPT in radiation dominated universe is very small → we consider merger remnants with $\chi \sim 0.7$

JHEP (2024) 006, IKB, Ujjal Kumar Dey

Astro. J. Lett. 704 (2009) L40

$$\Omega_{\rm GW}^{\rm sup} = \frac{f}{\rho_c} \int dz \frac{dt}{dz} \int \tau p(\chi) d\chi \frac{dE_s}{df_s}$$

$$\tau = \int \frac{dm_1}{m_1} \frac{dm_2}{m_2} \frac{d\tau}{d\ln m_1 d\ln m_2},$$

$$\frac{d\tau}{d\ln m_1 d\ln m_2} = 14.8 {\rm yr}^{-1} {\rm Gpc}^{-3} f(m_1) f(m_2) \frac{(m_1+m_2)^{10/7}}{(m_1 m_2)^{5/7}} \left(\frac{\delta_{\rm loc}}{10^8}\right) \left(\frac{v_0}{10 {\rm km/s}}\right)^{-11/7},$$

$$\frac{dt}{dz} = \frac{1}{H_0 \sqrt{\Omega_{\rm M}(1+z)^3 + \Omega_\Lambda}(1+z)},$$

$$\frac{dE_s}{df_s} = E_{\rm GW} \delta\left(f(1+z) - f_s\right).$$

JCAP 11 (2024) 045, Phys. Dark. Univ. 36 (2022) 101009

SGWB due to BH Superradiance

- The initial spin of PBHs created from FOPT in radiation dominated universe is very small → we consider merger remnants with $\chi \sim 0.7$

JHEP (2024) 006, IKB, Ujjal Kumar Dey

Astro. J. Lett. 704 (2009) L40

$$\Omega_{\rm GW}^{\rm sup} = \frac{f}{\rho_c}\int dz \frac{dt}{dz}\int \tau p(\chi) d\chi \frac{dE_s}{df_s}$$

$\frac{dE_s}{df_s}$ → $f_s = \frac{\mu c^2}{\pi \hbar}$

$p(\chi)$ → $p(\chi) = \delta(\chi - 0.7)$

$$\tau = \int \frac{dm_1}{m_1}\frac{dm_2}{m_2}\frac{d\tau}{d{\rm ln}m_1 d{\rm ln}m_2},$$

$$\frac{d\tau}{d{\rm ln}m_1 d{\rm ln}m_2} = 14.8{\rm yr}^{-1}{\rm Gpc}^{-3} f(m_1) f(m_2) \frac{(m_1+m_2)^{10/7}}{(m_1 m_2)^{5/7}}\left(\frac{\delta_{\rm loc}}{10^8}\right)\left(\frac{v_0}{10{\rm km/s}}\right)^{-11/7},$$

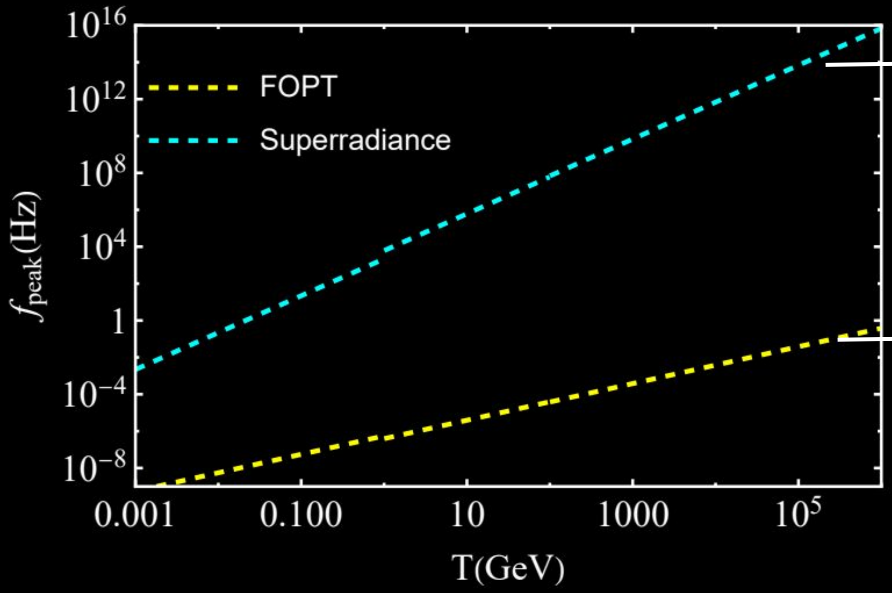
$$\frac{dt}{dz} = \frac{1}{H_0\sqrt{\Omega_{\rm M}(1+z)^3+\Omega_\Lambda}(1+z)},$$

$$\frac{dE_s}{df_s} = E_{\rm GW}\delta\left(f(1+z) - f_s\right).$$

JCAP 11 (2024) 045, Phys. Dark. Univ. 36 (2022) 101009

SGWB: The two components

- **The peak frequencies:** $f_p^{SR} = 6.843 \times 10^3 \left(\frac{\alpha}{0.5}\right)\left(\frac{T_{reh}}{GeV}\right)^2\left(\frac{g_*}{100}\right)^{1/2}$ Hz

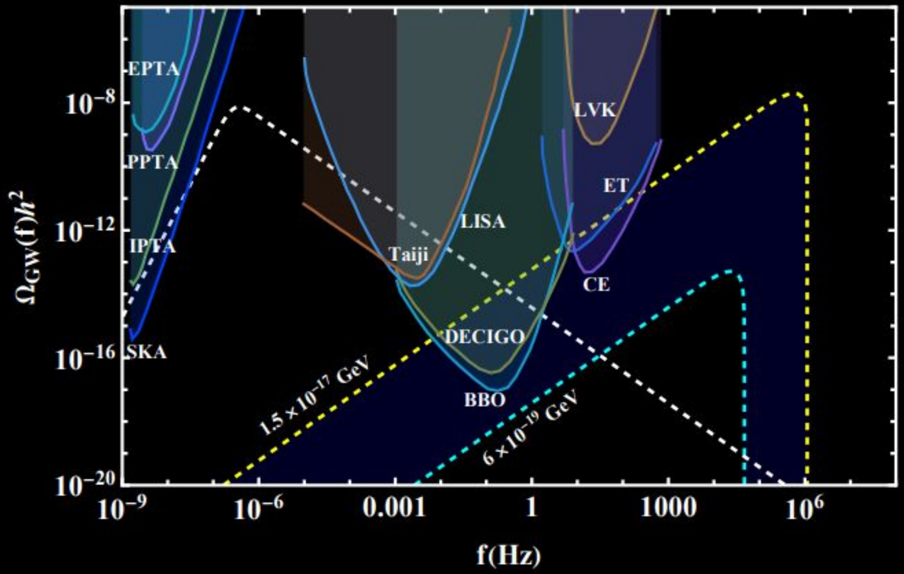


$\alpha = 0.25$

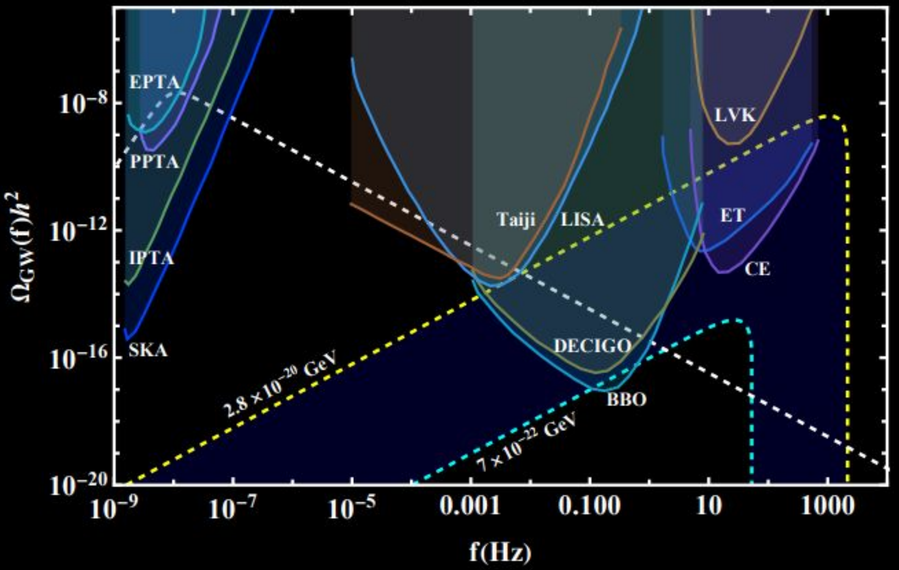
$\beta/H = 9$

SGWB: The two components

- **The ‘cumulative’ spectra:**



$T_{\text{reh}} = 1\ \text{GeV}$



$T_{\text{reh}} = 0.039\ \text{GeV}$

Summary and Future Prospects

- BH superradiance can act as a very efficient source of gravitational waves.
- Confirm or rule out the mechanism of PBH creation from FOPT or the existence of ultra-light scalars of certain mass.
- One can also consider transient monochromatic GW signals from these PBHs (memory!). Talks by Subhendra Mohanty, Ashoke Sen, Arpan Hait
- Non-standard cosmological scenarios can have implications on PBH spin which may change the spectra.

THANK YOU