Majorana Zero Modes in d+id' Superconductors

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[Mercado, Sahoo, Franz, Phys. Rev. Lett. 128, 137002 (2022)]

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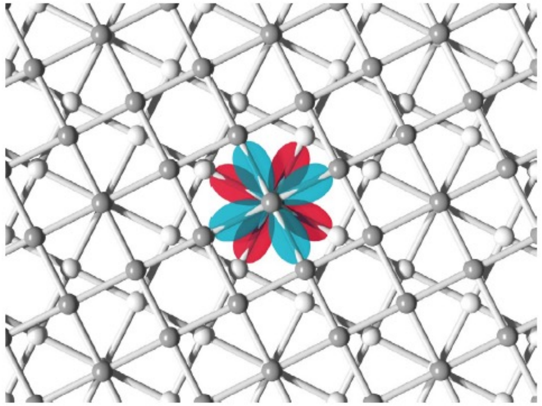
Motivation

[O. Can, T. Tummuru, R. P. Day, I. Elfimov,
A. Damascelli, and M. Franz, Nat. Phys. 17, 519 (2021)]

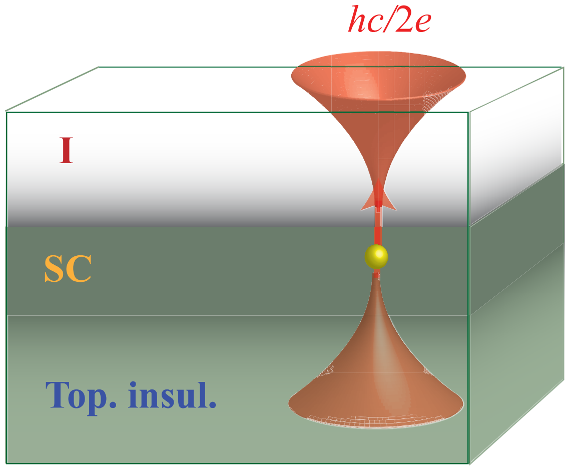


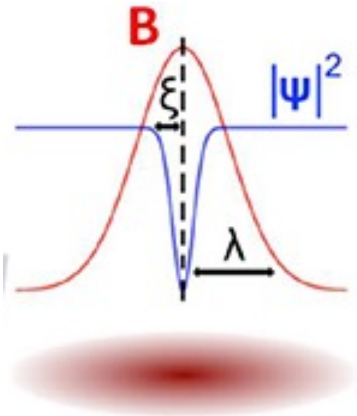
- Twisting d-wave superconductors (SCs) results in the time-reversal symmetry broken d+id' SC order close to 45^0.

- Is this SC topological? Yes, Class C (Chern number 2)
  [M Sato, Y Ando, Rep. Prog. Phys. 80, 076501 (2017)]

- Can there be Majorana zero modes in this SC? No

[L. Fu, C. Kane, PRL 102, 216403 (2009)]

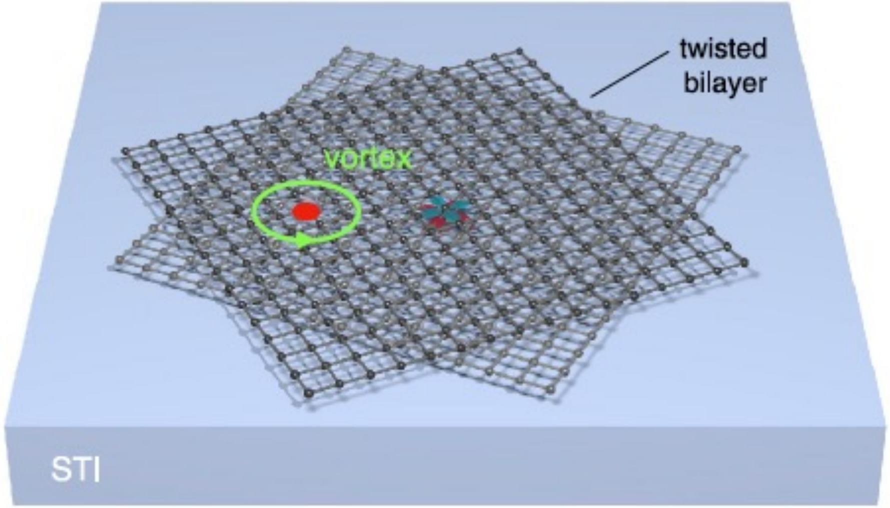


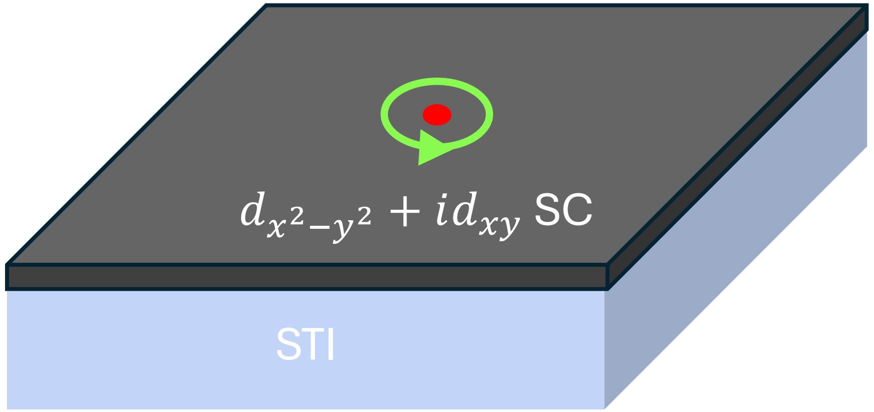


The lower coherence length (ξ) (a function of SC gap) of the BCS superconductors makes it difficult to resolve Caroli-de Gennes-Matricon (CdGM) states from the Majorana zero mode.

Other examples of high-Tc SC showing Majorana zero modes $FeSe_xTe_{1-x}$, $NbSe_2$ / Bi_2Te_3.

The Fu-Kane set-up





- Discuss zero mode wave function in the continuum model for the surface
- Discuss zero mode at the vortex core in the lattice model
- Summary and outlook

Introduction

Fu-Kane Model

$$\int d^2\mathbf{r}\ \Psi^\dagger(\mathbf{r})\ H\ \Psi(\mathbf{r}) \qquad \Psi = \begin{pmatrix} \psi_\uparrow \\ \psi_\downarrow \\ \psi_\downarrow^\dagger \\ -\psi_\uparrow^\dagger \end{pmatrix} \qquad H = \begin{pmatrix} -iv\boldsymbol{\sigma}.\boldsymbol{\nabla} & \Delta_0 e^{i\phi} \\ \Delta_0 e^{-i\phi} & iv\boldsymbol{\sigma}.\boldsymbol{\nabla} \end{pmatrix}$$

A 4x4 BdG Hamiltonian for Dirac surface states with proximity induced S-wave SC order

Symmetry: Particle-hole symmetry, $\sigma_y \tau_y K$ Time-reversal symmetry, $i\sigma_y K$

Here the zero mode is a non-degenerate zero energy solution, which is also an eigen state of particle-hole symmetry.

Simple decoupled differential equation for each spin

$$\Psi_0^+(r,\theta) = \begin{pmatrix} 0 \\ i \\ 1 \\ 0 \end{pmatrix} e^{-\int_0^r dr' \Delta_0(r')/v}$$

Wavefunction mostly localized localized at $r = 0$.

[L. Fu, C. Kane, PRL 102, 216403 (2009)]

Fu-Kane model for d+id' Superconductor

4x4 BdG Hamiltonian for strong Topological insulator surface proximitized with SC order

$$H_{\mathbf{k}} = \begin{pmatrix} h_{\mathbf{k}} - \mu & \Delta_{\mathbf{k}} \\ \Delta_{\mathbf{k}}^{\dagger} & -h_{\mathbf{k}}^{\mathcal{T}} + \mu \end{pmatrix}$$

$$\Delta_{\mathbf{k}} = \Delta \left(k_x^2 - k_y^2\right) + i\Delta'(2k_x k_y)$$

$$E_{\mathbf{k}}^2 = (v|\mathbf{k}| \pm \mu)^2 + |\Delta_{\mathbf{k}}|^2$$

Note that only finite chemical potential leads to a fully gapped superconductor,

BdG Hamiltonian in real space

$$H = \begin{pmatrix} -iv\boldsymbol{\sigma}.\boldsymbol{\nabla} - \mu & \hat{\Delta} \\ \hat{\Delta}^{\dagger} & iv\boldsymbol{\sigma}.\boldsymbol{\nabla} + \mu \end{pmatrix}$$

Particle-hole symmetric

No time-reversal symmetry

Zero mode solution at the vortex core

The $d_{x^2-y^2} + i\, d_{xy}$ gap operator for a single vortex placed at the origin

$$\hat{\Delta} = -a_0^2 \left(\{\partial_+, \{\partial_+, \Delta(\mathbf{r})\}\} + \frac{i}{2}[\partial_+^2 \Theta(\mathbf{r})] \right) \qquad \partial_\pm = e^{\pm i\varphi}\left(-i\partial_r \pm \frac{\partial_\varphi}{r}\right)$$

where vortex is centered at the origin $\quad \Delta(\mathbf{r}) = \Delta_0(r)\, e^{in\varphi}$

[O Vafek, A Melikyan, M. Franz, Z. Tešanović, Phys. Rev. B 63, 134509 (2001)]

Due to particle-hole symmetry the zero-mode solution has a single unknown spinor, $u = \binom{u_\uparrow}{u_\downarrow}$, and $v = i\sigma^y u^*$.

We take following ansatz

$$(-iv\boldsymbol{\sigma}.\nabla - \mu)\, u + i\sigma^y \hat{\Delta} u^* = 0 \qquad u(r,\phi) = e^{il\varphi}\begin{pmatrix} e^{-i\frac{\pi}{4}}\chi_\uparrow(r) \\ e^{i\left(\frac{\pi}{4}+\varphi\right)}\chi_\downarrow(r) \end{pmatrix}$$

[M Cheng, R. M. Lutchyn, V. Galitski, S Das Sarma, PRB 82, 094504, (2010)]

Zero mode solution at the vortex core (Contd.)

Equations to solve for n=1 vortex

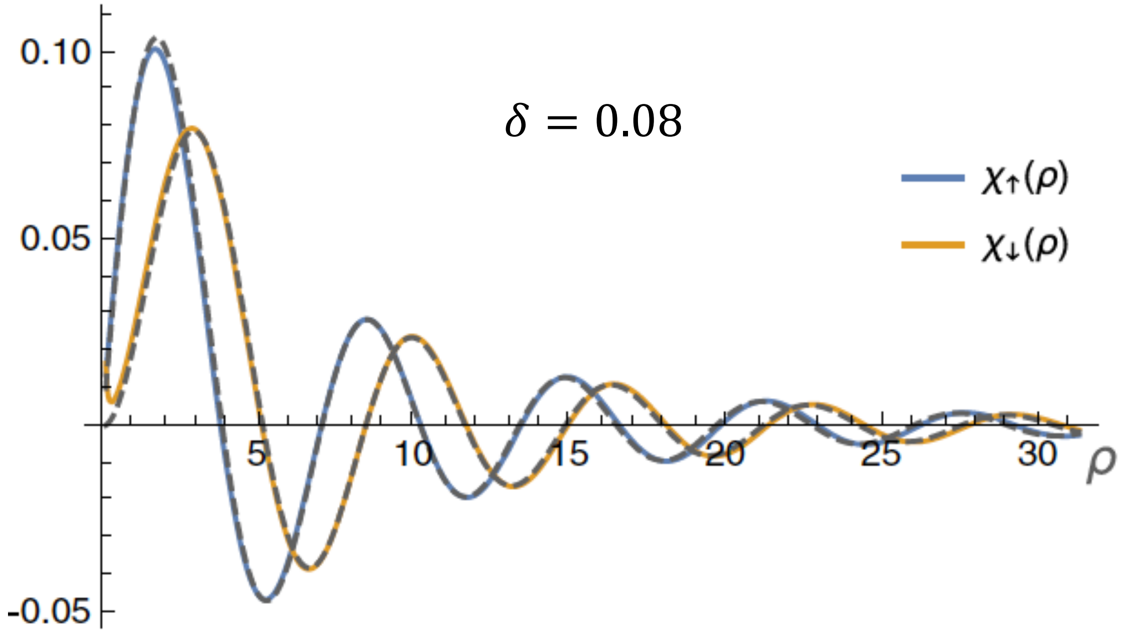
$$\chi_\uparrow = \left[\left(\partial_\rho + \frac{2}{\rho}\right) - \delta\left(\partial_\rho^2 + \frac{2}{\rho}\partial_\rho - \frac{1}{4\rho^2}\right)\right]\chi_\downarrow$$

$$\chi_\downarrow = -\left[\left(\partial_\rho - \frac{1}{\rho}\right) - \delta\left(\partial_\rho^2 - \frac{1}{4\rho^2}\right)\right]\chi_\uparrow$$

$$\rho = \left(\frac{\mu}{v}\right) r \qquad \delta = \Delta_0 \mu a_0^2 / v^2$$

These equations can be analytically solved in the asymptotic limit ($\rho \gg 1$)

$$\chi_\infty(\rho) = A e^{-\rho\delta} \begin{pmatrix} J_1(\rho) \\ J_2(\rho) \end{pmatrix}$$



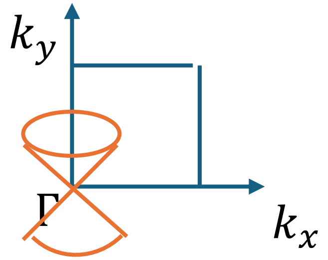
Non-zero chemical potential results in oscillating zero mode solution, but oscillations decay exponentially.

[Mercado, Sahoo, Franz, Phys. Rev. Lett. 128, 137002 (2022)]

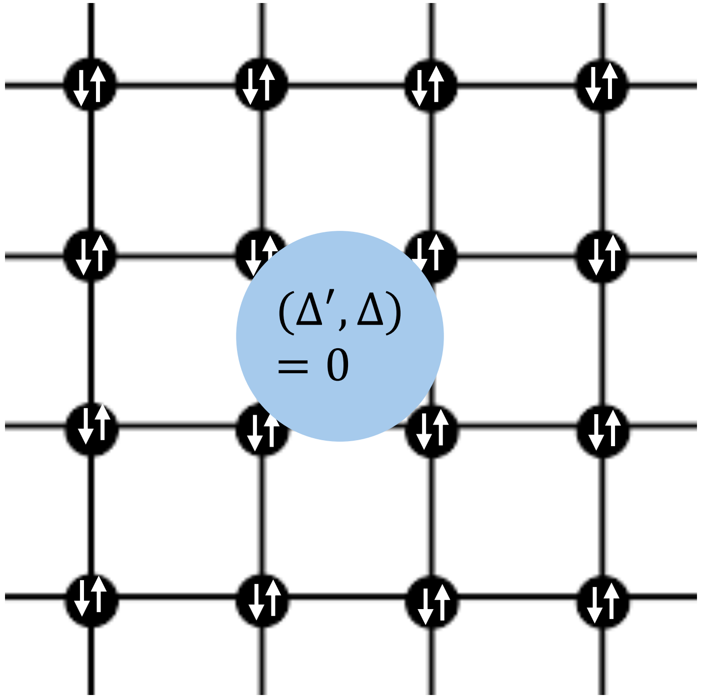
Lattice model simulation results

$$H_{\mathbf{k}}^{lat} = \begin{pmatrix} h_{\mathbf{k}} - \mu & \Delta_{\mathbf{k}} \\ \Delta_{\mathbf{k}}^{\dagger} & -h_{\mathbf{k}}^{\mathcal{T}} + \mu \end{pmatrix}$$

$$h_{\mathbf{k}}^{lat} = \lambda\left(\sigma^y \sin(k_x) - \sigma^x \sin(k_y)\right) + M\sigma^z\left(2 - \cos(k_x) - \cos(k_y)\right)$$



$$\Delta_{\mathbf{k}}^{lat} = \Delta\left(\cos(k_x) - \cos(k_y)\right) + 2i\Delta' \sin(k_x)\sin(k_y)$$



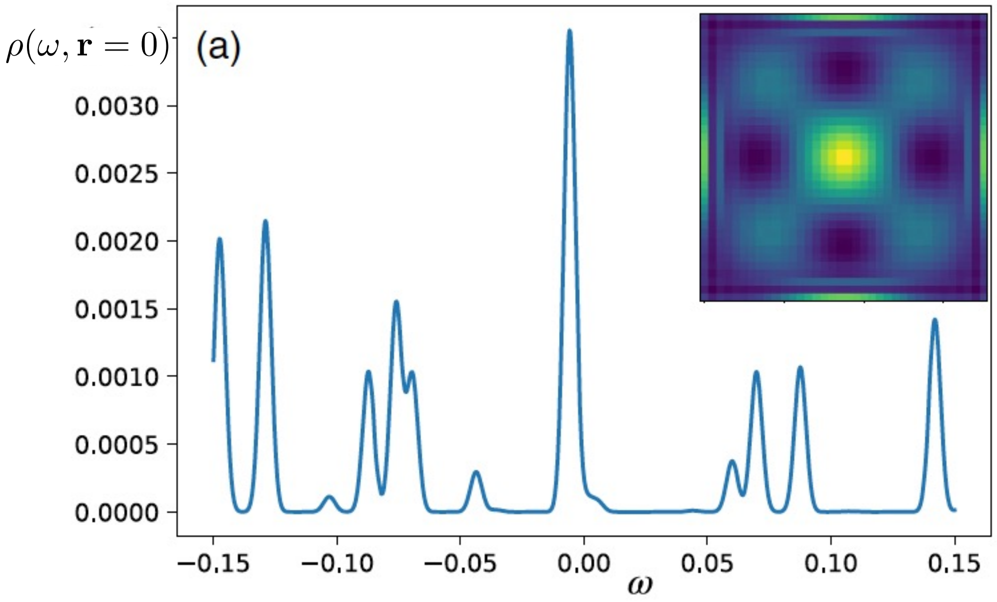
Fourier transform to get

- Spin flip nearest neighbor hopping
- Onsite Zeeman spin energy splitting
- Nearest neighbor magnetic field term
- Nearest neighbor and second nearest neighbor singlet pairing

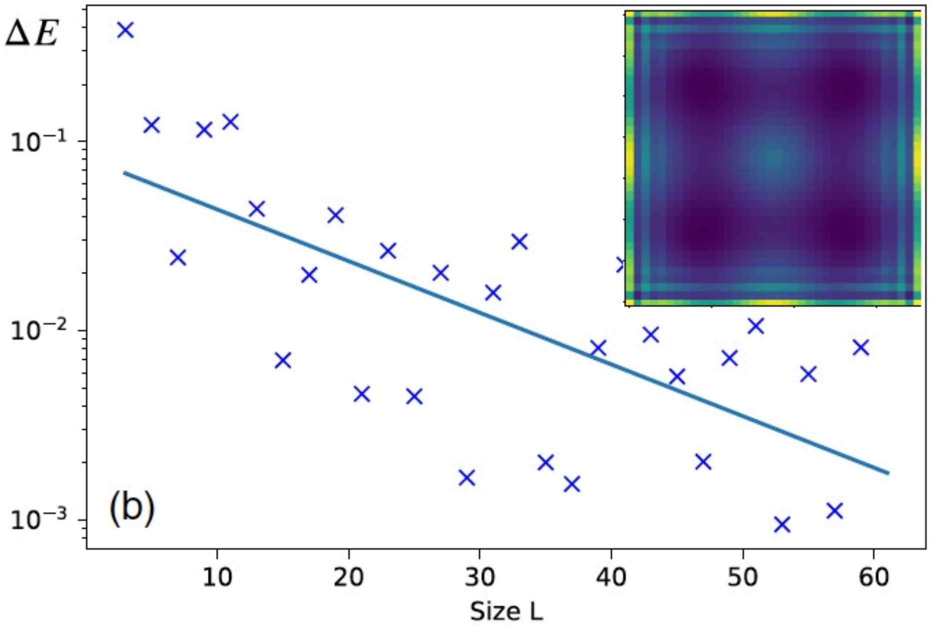
[Mercado, Sahoo, Franz, Phys. Rev. Lett. 128, 137002 (2022)]

Lattice model simulation results (Contd.)

LDOS, $\rho(\omega, \mathbf{r}) = \sum_n \left[|u_n(\mathbf{r})|^2 \delta(\omega - \epsilon_n) + |v_n(\mathbf{r})|^2 \delta(\omega + \epsilon_n) \right]$



LDOS plotted at the vortex core for L=35, M=0.5, μ=-0.3, Δ= 0.15, Δ′= 0.10.



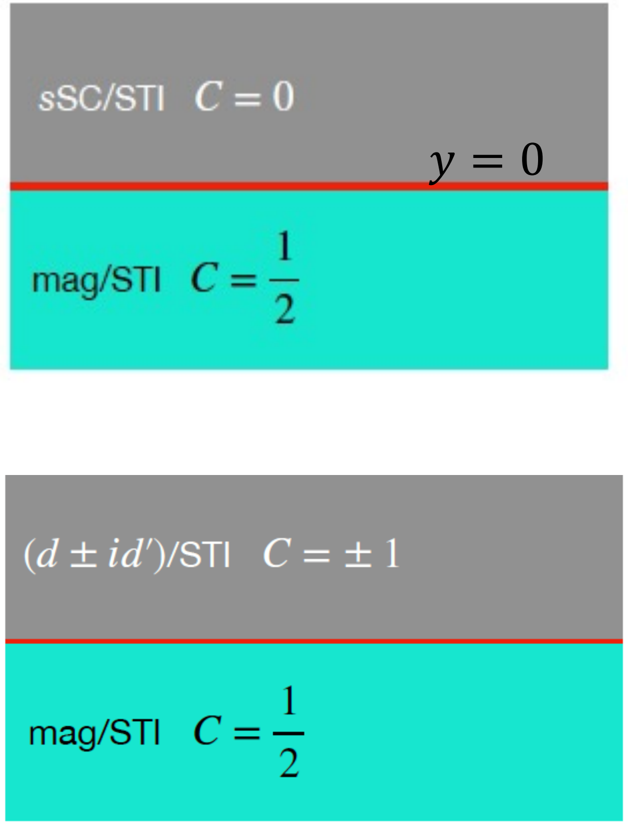
Energy split in log scale as a function of L

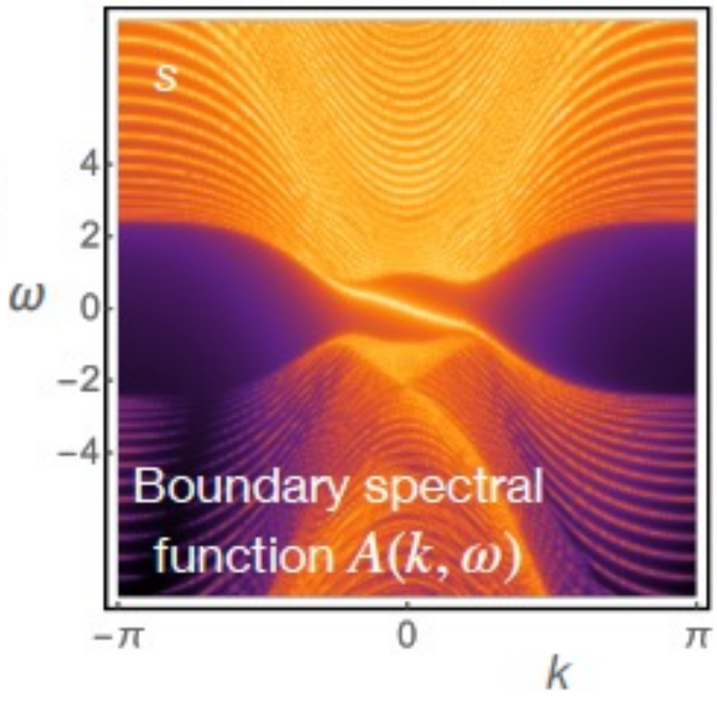
[Mercado, Sahoo, Franz, Phys. Rev. Lett. 128, 137002 (2022)]

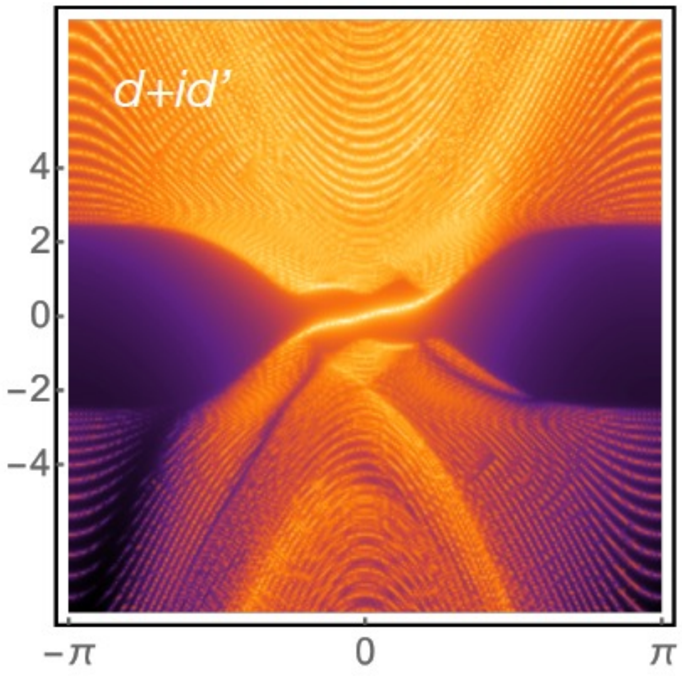
Interface Majorana modes for Heterostructures

$$A(k,\omega) = \ln[-\mathrm{im}([(\omega + i\delta)\mathrm{I}_{4Mx4M} - H(k)_{4Mx4M}]^{-1}(y=0))]$$

$\Delta C = -\frac{1}{2}$ (M=70) $\Delta C = +\frac{1}{2}$







Summary

- Appearance of Majorana zero modes at the core of Abrikosov vortex was discussed both using real-space continuum model, and lattice simulation.
- There is some advantage of probing Majorana zero modes in a Fu-Kane setup with high-Tc superconductors i.e the Majorana Zero modes can possibly be distinguished from the CdGM states.

$$\delta E \approx \frac{\Delta_{TI}}{N_{CdGM}} = \frac{1}{\rho(\mu)\pi R_v^2} = \frac{2\Delta_{topo}^2}{\rho(\mu)\hbar^2 v_F^2}$$

Outlook

- In the toy model we assumed that the $d_{x^2-y^2} + id_{xy}$ superconducting order and gap can be induced on the TI, but it will depend on the nature of interaction at the interface, with more complication coming from the twisted Bi2212 structure.

Bi2212 / Bi_2Se3 (or Bi_2Te_3)

- Nature Communications volume 3, Article number: 1056 (2012)
- Nature Physics volume 9, pages621–625 (2013)
- Phys. Rev. B **90**, 085128 (2014)

Thank you