Constraining the Inflationary Magnetic Field and Reheating via GWs

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Program: HEARING BEYOND THE STANDARD MODEL WITH COSMIC SOURCES OF GRAVITATIONAL WAVES (ICTS-TIFR)

Content

Introduction and Motivation

Existence of Magnetic Fields

- Our Universe is fully Magnetized.

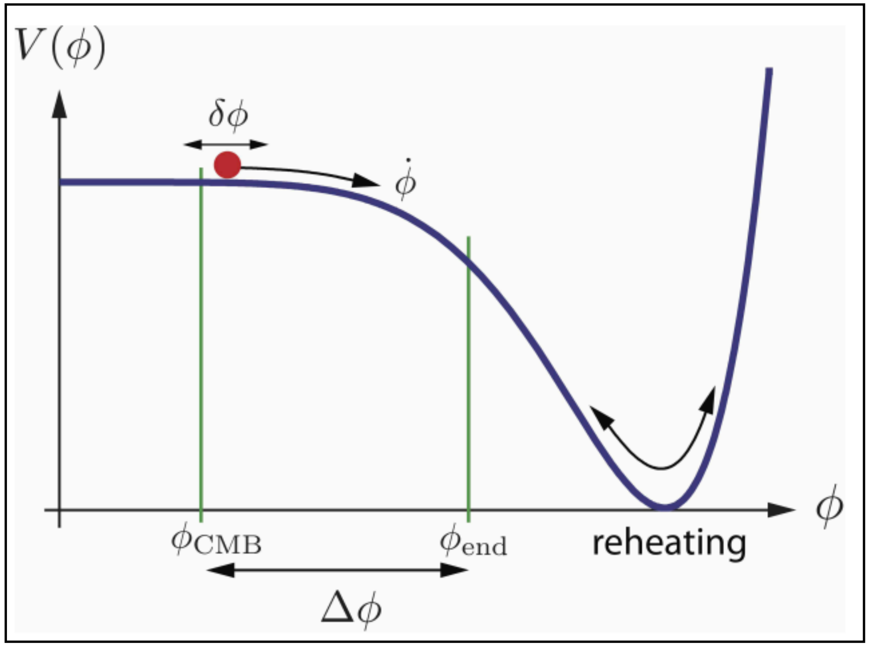
- The existence of the Magnetic field is not strange, but explaining the origin of the large-scale magnetic field (1 kpc $[10^{-14} - 10^{-6}\ G]$ to larger than 1 Mpc $[10^{-16} - 10^{-9}\ G]$) without introducing extra dynamics is challenging.

The magnetic field in the Whirlpool Galaxy (M51). (Credit: NASA)

Introduction and Motivation

Why Reheating is Important?

- The process converts the energy of the inflaton field into a thermal bath of particles, paving the way for primordial nucleosynthesis and subsequent structure formation.
- Reheating provides a natural setting for producing dark matter candidates and exploring scenarios beyond the Standard Model (BSM), such as baryogenesis or axion production.
- The dynamics of reheating can amplify or seed primordial magnetic fields, influencing the evolution of large-scale cosmic magnetism it can also influence the GW spectrum (ref to Prof. L. Sriram Kumar Talk).



Defining Some Useful Parameters

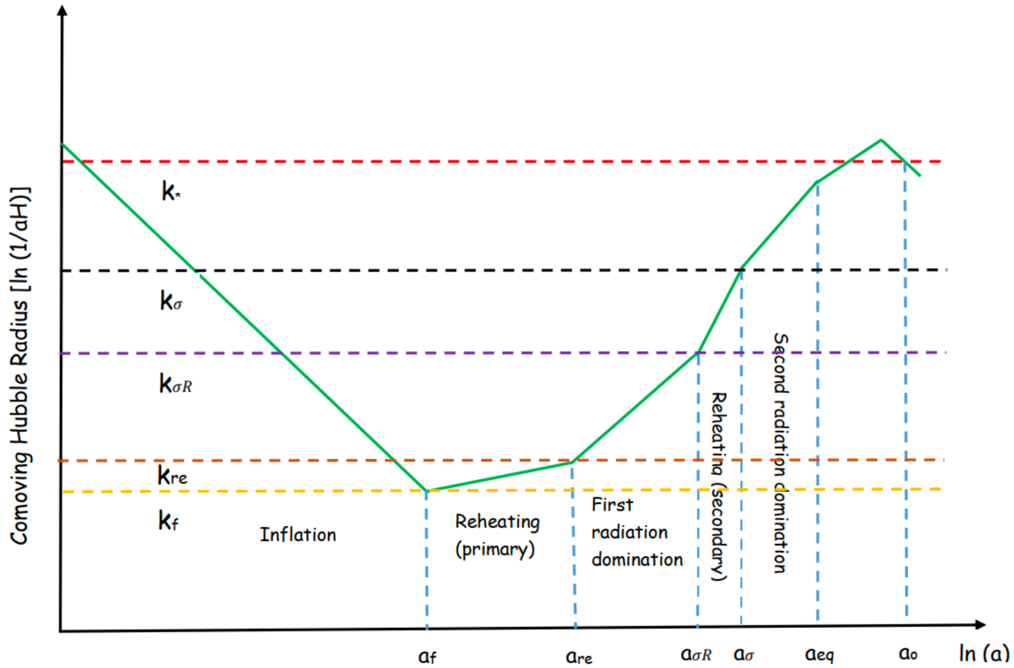
- The largest mode that left the horizon at the end of Inflation

$$(k_{\rm end}/a_0) = \left(\frac{43}{11g_{\rm re}}\right)^{1/3} \left(\frac{\pi^2 g_{\rm re}}{90}\right)^{\alpha} \frac{H_{\rm end}^{1-2\alpha} T_{\rm re}^{4\alpha-1} T_0}{M_{\rm pl}^{2\alpha}}$$

- The lowest possible mode re-entering the horizon at the end of reheating

$$(k_{\rm re}/a_0) \simeq 3.9 \times 10^6 \left(\frac{T_{\rm re}}{10^{-2}\ {\rm GeV}}\right) {\rm Mpc}^{-1}$$

- In de Sitter Inflationary background, Hubble is nearly constant and can be expressed as $H_{\rm end} = \pi M_{\rm pl}\sqrt{rA_s/2}$



$A_s = 2.1 \times 10^{-9}$ $\alpha(w_{re}) = 1/3(1+w_{re})$

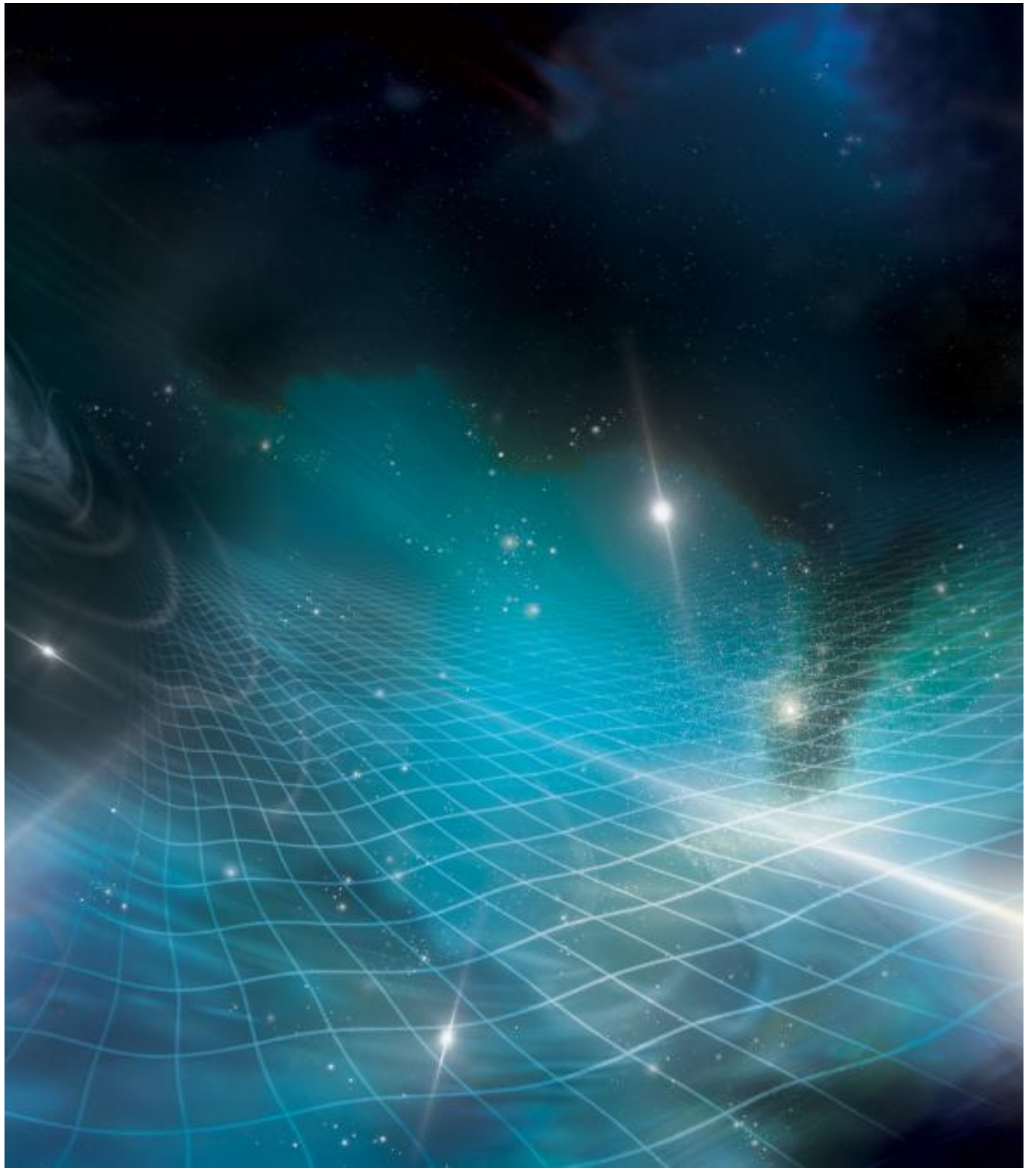
$M_{\rm pl} =$ reduced plank mass

$r =$ tensor-to-scalar ration

$$(r_{0.05} \leq 0.036 \Rightarrow H_{\rm end} \simeq 10^{-5} M_{\rm pl})$$

M. R. Haque, D. Maity, T. Paul, and L. Sriramkumar, Phys. Rev. D 104, 063513 (2021);
A.Chakraborty, S.M, D.Maity, arXiv:2408.07767

Gravitational Waves



Generation and Evolution of GWs

EoM for the tensor perturbation in Fourier mode

$$h_{\mathbf{k}}^{\lambda\prime\prime} + 2\frac{a'}{a}h_{\mathbf{k}}^{\lambda\prime} + k^2 h_{\mathbf{k}}^{\lambda} = \mathcal{S}_{\mathbf{k}}^{\lambda},$$

Polarization Index

C. Caprini and L. Sorbo, JCAP 10, 056
R. Sharma, K. Subramanian, and T. R. Seshadri, Phys. Rev. D 101, 103526 (2020);
S. Maiti, D. Maity, and L. Sriramkumar, arXiv:2401.01864 [gr-qc]

For Electromagnetic field, the source term is given by

$$\mathcal{S}_{\mathbf{k}}^{\lambda}(\eta) = -\frac{2}{M_{\rm pl}^2} e_{\lambda}^{ij}(\mathbf{k}) \int \frac{\mathrm{d}^3\mathbf{q}}{(2\pi)^{3/2}} [E_i(\mathbf{q},\eta)E_j(\mathbf{k}-\mathbf{q},\eta) + B_i(\mathbf{q},\eta)B_j(\mathbf{k}-\mathbf{q},\eta)].$$

Polarization tensor

We defined the power spectrum of tensor fluctuations as

$$\mathcal{P}^{\lambda}(k,\eta) = \frac{k^3}{2\pi^2}\langle h_{\mathbf{k}}^{\lambda} h_{\mathbf{k}'}^{\lambda *}\rangle \delta(\mathbf{k}+\mathbf{k}') = \mathcal{P}_{\rm PRI}^{\lambda} + \mathcal{P}_{\rm SEC}^{\lambda}$$

Inflationary Production from Quantum fluctuations

$$\propto \frac{H_{\rm end}^2}{M_{\rm pl}^2}$$

Ref:Prof. L.SriramKumar

Secondary Production due to EM field

Computing the Dimensionless GWs energy density

The tensor power spectrum at a specific time η_f

S. Maiti, D. Maity, and L. Sriramkumar, arXiv:2401.01864 [gr-qc]

$$\mathcal{P}^{\lambda}_{\rm SEC}(k,\eta_f) = \frac{2}{M_{\rm P}^4}\left[\int_{\eta_i}^{\eta_f} d\eta_1 a^2(\eta_1)\mathcal{G}_k(\eta_f,\eta_1)\right]^2 \int_0^\infty \frac{dq}{q}\int_{-1}^{1} d\mu \frac{f(\mu,\beta,\lambda)}{[1+(q/k)^2-2\mu(q/k)]^{3/2}}$$
$$\times\, \mathcal{P}^{\lambda}_{\rm B}(q,\eta_1)\mathcal{P}^{\lambda}_{\rm B}(|\mathbf{k}-\mathbf{q}|,\eta_1) + \mathcal{P}^{\lambda}_{\rm E}(q,\eta_1)\mathcal{P}^{\lambda}_{\rm E}(|\mathbf{k}-\mathbf{q}|,\eta_1),$$

Where $f(\mu,\beta,\lambda)$ defined as

$$\boxed{\begin{aligned} f(\mu,\beta,\lambda) &= (1+\lambda\mu)^2(1+\lambda\beta)^2/16 \qquad &\textbf{(Helical)}\\ &= 2(1+\mu^2)(1+\beta^2) \qquad &\textbf{(Non-helical)}\end{aligned}}$$

$$\mu = \hat{\boldsymbol{k}}\cdot\hat{\boldsymbol{q}} \text{ and } \beta = (\widehat{\boldsymbol{k}-\boldsymbol{q}})\cdot\hat{\boldsymbol{k}}$$

Gw spectral energy density is defined as

$$\boxed{\Omega_{\rm gw}(k,\eta) = \frac{\rho_{\rm gw}(k,\eta)}{\rho_c(\eta)} = \frac{1}{12}\frac{k^2\mathcal{P}_{\rm T}(k,\eta)}{a^2(\eta)H^2(\eta)}}$$

Primary + Secondary

Origin of seed Magnetic field

1. Primordial Quantum Fluctuations
   - Inflationary Dynamics : $(f(\phi, R)F_{\mu\nu}F^{\mu\nu}; f(\phi, R)F_{\mu\nu}\tilde{F}^{\mu\nu}; \ldots)$
2. Quantum Effect in the Early Universe
   - Non-liner effect in Quantum Electrodynamics
   - Existence Axion-like(ALP) particles : $(\chi F_{\mu\nu}\tilde{F}^{\mu\nu})$
3. Phase Transition and Topological Defects
   - Cosmic Strings
   - Domain Walls Collision
4. Gravitational and Curvature Effects
   - Curvature Perturbations (also source the PBHs)
   - Inflationary Gravitational Waves

Origin of seed Magnetic field

Helical Magnetic field

1. Primordial Quantum Fluctuations
   - Inflationary Dynamics : $(f(\phi, R)F_{\mu\nu}F^{\mu\nu}; f(\phi, R)F_{\mu\nu}\tilde{F}^{\mu\nu}; \ldots)$

Non-helical Magnetic field

2. Quantum Effect in the Early Universe
   - Non-liner effect in Quantum Electrodynamics
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For Particular types of coupling

$$f(\phi, \mathscr{R}) \propto a^n \quad a_i < a < a_e$$

Ratra, Bharat 1992

$$k\eta << 1$$

$$\mathcal{P}_{\rm B}^{\lambda}(k\eta) \propto H_{\rm end}^4 \tilde{\mathcal{B}}(\lambda, n)(k\eta)^{n_{\rm B}}$$

GW spectral energy density at present day

S. Maiti, D. Maity, and L. Sriramkumar,
arXiv:2401.01864 [gr-qc]

GW spectrum for Primary prductions

$\mathcal{O}(1)$

$$\Omega_{\mathrm{GW}}^{\mathrm{PRI}}(k)h^2 \simeq \frac{\Omega_{\mathrm{r}}h^2 g_0^{s\,1/3}}{6 g_{\mathrm{re}}^{1/3}} \frac{H_{\mathrm{I}}^2}{M_{\mathrm{Pl}}^2} \times \begin{cases} 1 & k < k_{\mathrm{re}}, \\ \mathcal{D}_1 \left(\frac{k}{k_{\mathrm{re}}}\right)^{-n_w} & k > k_{\mathrm{re}}, \end{cases}$$

Secondary production due to Magnetic fields

For $w_{\mathrm{re}} < 1/3$

$$\Omega_{\mathrm{GW}}^{\mathrm{SEC}}(k)h^2 \simeq \frac{\Omega_{\mathrm{r}}h^2 g_0^{s\,1/3}}{6 g_{\mathrm{re}}^{1/3}} \left(\frac{H_{\mathrm{I}}\tilde{\mathcal{B}}}{M_{\mathrm{Pl}}}\right)^4 \left(\frac{k_{\mathrm{re}}}{k_{\mathrm{e}}}\right)^{2n_{\mathrm{B}}} \mathcal{F}_{n_{\mathrm{B}}}(k) \times \begin{cases} (k/k_{\mathrm{re}})^{2n_{\mathrm{B}}} & k_* < k < k_{\mathrm{re}}, \\ (k/k_{\mathrm{re}})^{2n_{\mathrm{B}}-n_w} & k_{\mathrm{re}} < k < k_{\mathrm{e}}, \end{cases} \quad ($$

For $w_{\mathrm{re}} \geq 1/3$

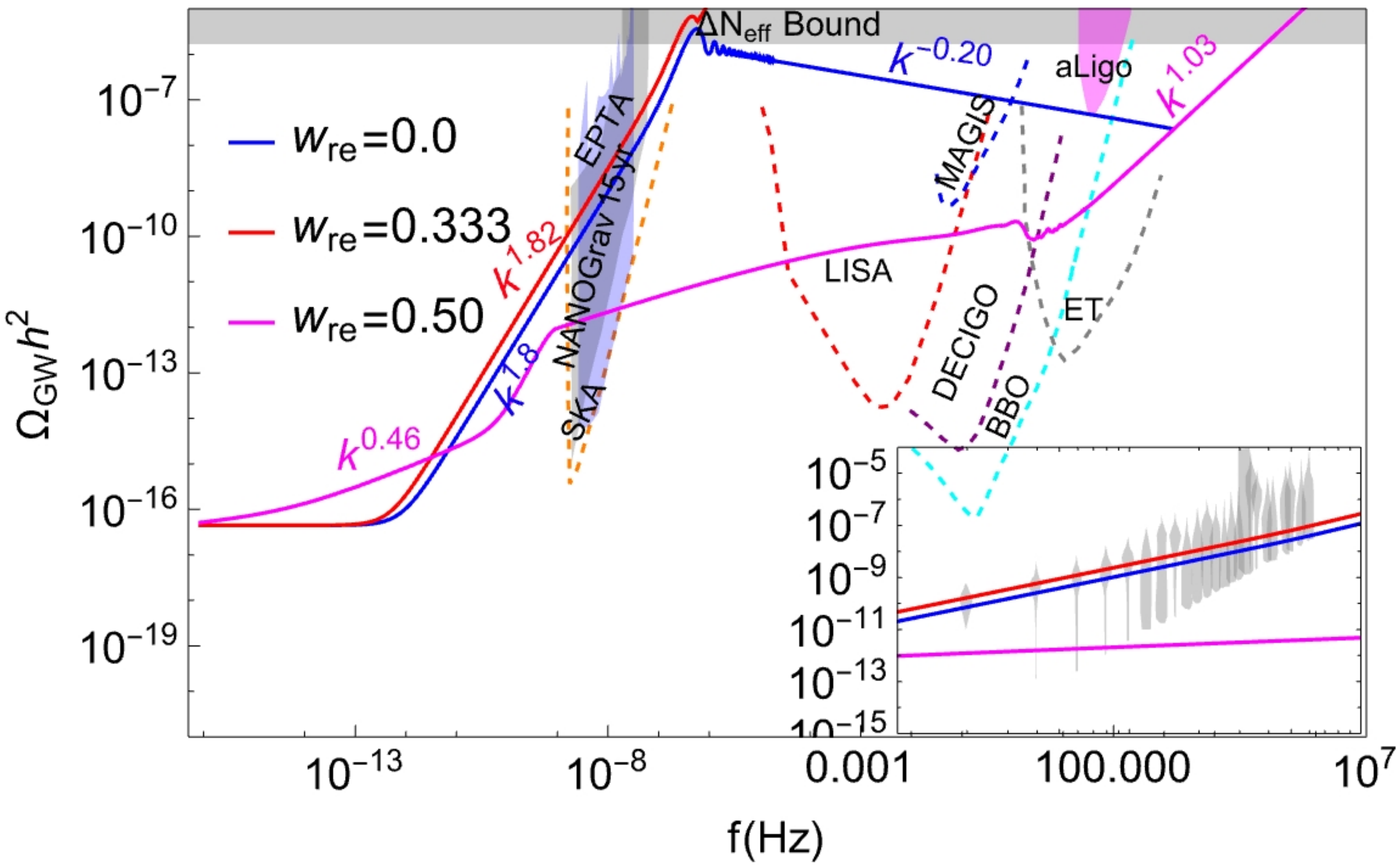
$$\Omega_{\mathrm{GW}}^{\mathrm{SEC}}(k)h^2 \simeq \frac{\Omega_{\mathrm{r}}h^2 g_0^{1/3}}{6 g_{\mathrm{re}}^{1/3}} \left(\frac{H_{\mathrm{I}}\tilde{\mathcal{B}}}{M_{\mathrm{Pl}}}\right)^4 \left(\frac{k_{\mathrm{re}}}{k_{\mathrm{e}}}\right)^{2(n_{\mathrm{B}}+n_w)} \mathcal{F}_{n_{\mathrm{B}}} \times \begin{cases} \mathcal{A}_1 (k/k_{\mathrm{re}})^{2n_{\mathrm{B}}} & k_* < k < k_{\mathrm{SB1}}, \\ \mathcal{I}_2(k)(k_{\mathrm{re}}/k_\nu)^2 (k/k_{\mathrm{re}})^{2n_{\mathrm{B}}+2} & k_{\mathrm{SB1}} < k < k_\nu, \\ \mathcal{I}_2(k)(k/k_{\mathrm{re}})^{2n_{\mathrm{B}}} & k_\nu < k < k_{\mathrm{re}}, \\ \mathcal{A}_2 (k/k_{\mathrm{re}})^{2n_{\mathrm{B}}-|n_w|} & k > k_{\mathrm{re}}, \end{cases} \quad ($$

Where $\mathcal{F}_{n_{\mathrm{B}}}$. Is defined as

$$\mathcal{F}_{n_{\mathrm{B}}}(k) \simeq \frac{32}{3n_{\mathrm{B}}}\left[1 - \left(\frac{k_*}{k}\right)^{n_{\mathrm{B}}}\right] + \frac{2\alpha}{3}\left[\left(\frac{k_{\mathrm{e}}}{k}\right)^{2n_{\mathrm{B}}-3} - 1\right]$$

$\alpha = 56/5(3n_{\mathrm{B}} - 3)$ $\quad n_w = 2(1 - 3w_{\mathrm{re}})/(1 + 3w_{\mathrm{re}})$

GW spectrum for Present day



Benchmark Point

w_{re}	$\log_{10}(\tilde{\mathcal{B}})$	$\log_{10}(T_{\rm re}/\mathrm{GeV})$	$n_{\rm B}$
0	2.91	−0.51	0.90
1/3	2.97	−0.55	0.91
1/2	1.76	4.1	0.23

Bound on $\Delta N_{\rm eff}$

If we treate GW energy density as an excess radiation component then we can connect it with extra relativistic degree of freedom quantity as $\Delta N_{\rm eff} = N_{\rm eff} - N_{\rm SM}$

$$\Omega_{\rm gw} h^2 = \frac{7}{8}\left(\frac{4}{11}\right)^{4/3} \Omega_\gamma h^2 \Delta N_{\rm gw} \simeq 1.6 \times 10^{-6}\left(\frac{\Delta N_{\rm gw}}{0.284}\right)$$

$$\Omega_\gamma h^2 = 2.47 \times 10^{-5}$$

(Present-day photon energy density)

Where

$$\Omega_{\rm gw} h^2 = \int_{k_{\rm min}}^{k_{\rm end}} \frac{dk}{k} \Omega_{\rm gw}(k) h_0^2$$

Total produced GW energy density

Comparison with the PTA data:

We implement our model In PTARcade

A. Mitridate, D. Wright, R. von Eckardstein, T. Schr¨oder,J. Nay, K. Olum, K. Schmitz, and T. Trickle- arXiv:2306.16377 (2023)

As we have seen from the previous figure, $w_{\rm re} < 1/3$ it is most favored by the bound and the tensor-to-scalar ratio $r_{0.05} < 0.036$.

Parameters	$\log_{10}(T_{\rm re}/{\rm GeV})$	$w_{\rm re}$	$\log_{10}(\tilde{\mathcal{B}})$	Bayesian
Prior	$\mathcal{U}[-2.0, 1]$	$\mathcal{U}[0, 0.333]$	$\mathcal{U}[0, 5]$	$\mathcal{B}_{\rm X,Y}$
$n_{\rm B} = 1.0$	$-0.55^{+0.26}_{-0.08}$	$0.11^{+0.13}_{-0.09}$	$0.02^{+0.03}_{-0.03}$	35.62 ± 8.53
$n_{\rm B} = 0.75$	$-0.47^{+0.27}_{-0.12}$	$0.13^{+0.12}_{-0.10}$	$0.94^{+0.03}_{-0.03}$	20.37 ± 3.21
$n_{\rm B} = 0.5$	$-0.45^{+0.21}_{-0.13}$	$0.15^{+0.12}_{-0.10}$	$1.81^{+0.04}_{-0.03}$	3.7 ± 0.55

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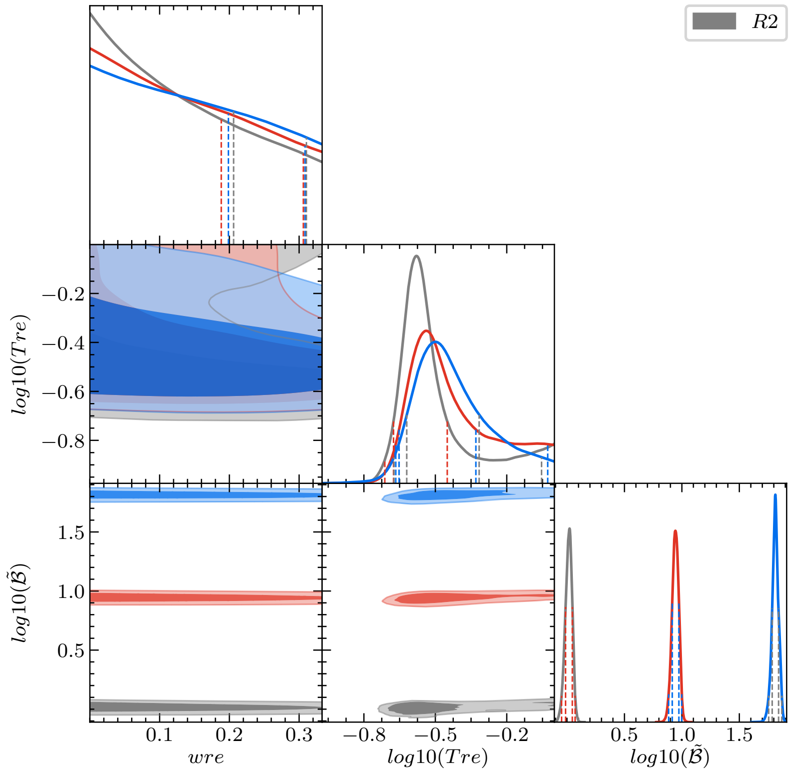

Takehome message

- Gravitational wave detection can constrain inflationary magnetic fields, their initial profiles, and reheating scenarios.
- The post-inflationary reheating era is pivotal for magnetic field evolution and the resulting gravitational wave spectra.
- A strong blue-tilted magnetic field from inflation can generate significant gravitational waves, potentially detectable by future sensitivity curves, and may also explain PTA signals with suitable reheating scenarios.

Thank you for your attention.