Metastable behaviour of the spin-S Ising and Blume-Capel ferromagnets

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Collaborators

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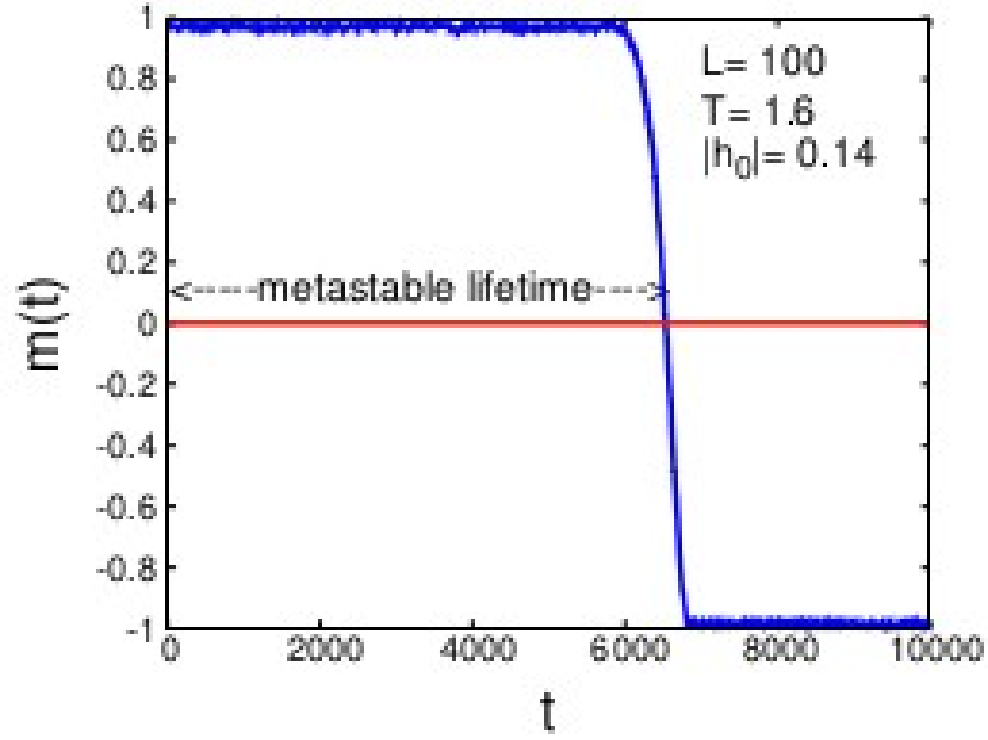
M. Naskar, M. Acharyya, E. Vatansever, N. G. Fytas, Phys Rev E, 104 (2021) 014107

Metastability

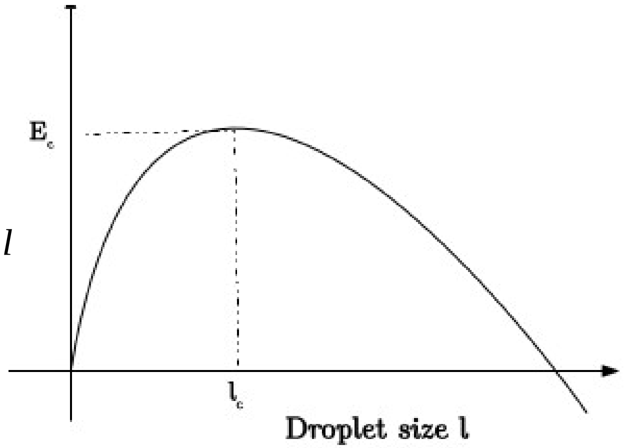


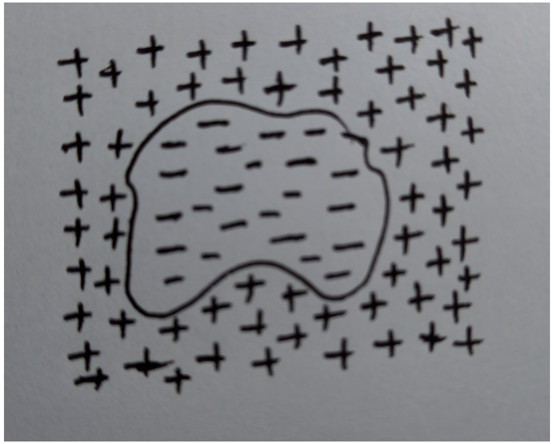
Classical theory of nucleation

$$E_l = -2hl + \sigma l^{\frac{(d-1)}{d}}$$

Energy of formation of droplet (l)

E_l





R. Becker and W. Doring, Ann. Phys. (Leipzig) 24 (1935) 719

Growth of supercritical droplets

$$E_c = \frac{K_d \sigma^d}{h^{d-1}}$$

$$n_c \sim \exp\left(-\frac{E_c}{k_B T}\right) \sim \exp\left(-\frac{K_d \sigma^d}{k_B T h^{d-1}}\right)$$

$$I \sim n_c \; ; \quad \tau_n \sim \frac{1}{I}$$

$$\tau_n \sim \exp\left(\frac{K_d \sigma^d}{k_B T h^{d-1}}\right)$$

Growth of Single droplet

$$\tau_c \sim \exp\left(\frac{K_d \sigma^d}{k_B T (d+1) h^{d-1}}\right)$$

Coalescence of multiple droplets

Model system

$$H=-\frac{J}{s^2}\sum_{ij} S_i^z S_j^z+\frac{D}{s^2}\sum_i \left(S_i^z\right)^2-\frac{h}{s}\sum_i S_i^z$$

$$S^z=\left(-S,-S+1,-S+2,\ldots\ldots\ldots +S\right)$$

$D=0\ (\ Spin-S\ \ Ising\ \ model)$ $\qquad$ $D\neq 0\ (\ Spin-S\ \ Blume-Capel\ \ model)$

Monte Carlo simulation

Metropolis single spin flip algorithm

$$P\left(S^z_{old} \rightarrow S^z_{new}\right) = Min\left[1, \exp\left(-\frac{\delta E}{k_B T}\right)\right]$$

Metastable lifetime (MC results)

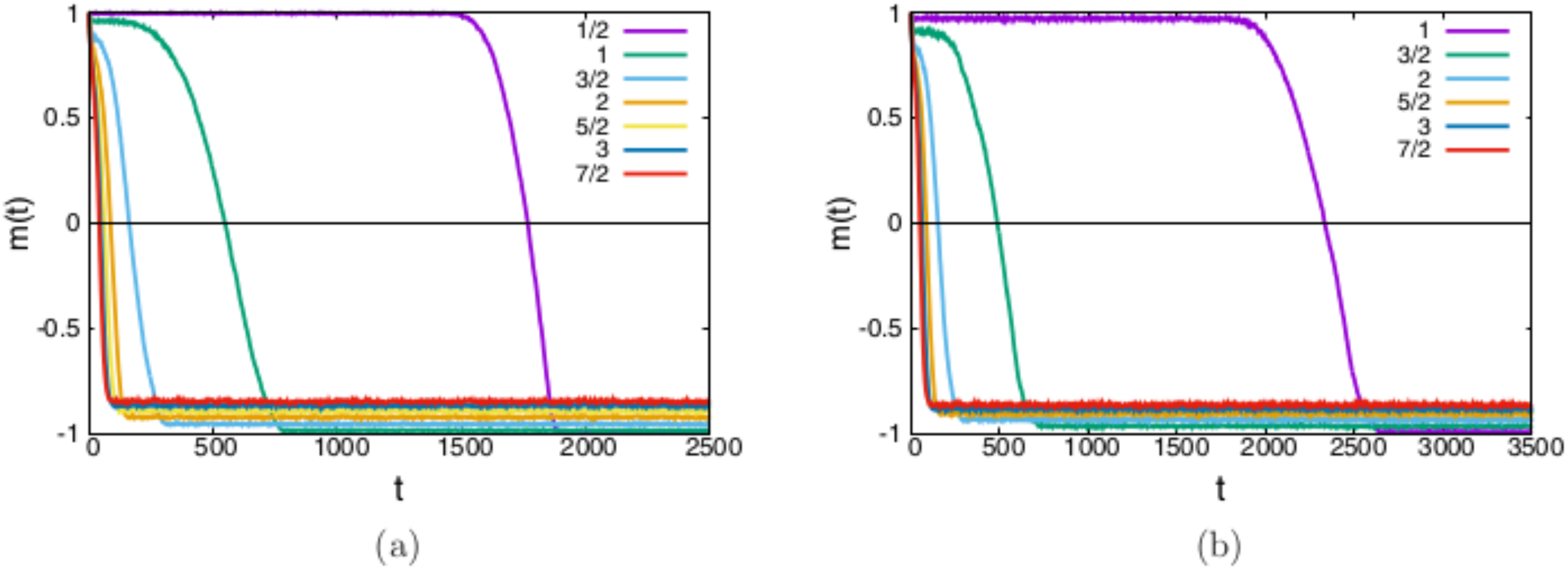


FIG. 3. (a) Variation of the magnetization with time (in MCSS) for Ising models at $T = 1.0$ and $h = -0.4$. Reversal times for the spin-$\{1/2, 1, 3/2, 2, 5/2, 3, 7/2\}$ systems are $\tau = 1767, 549, 163, 87, 63, 48$, and 42 MCSS, respectively. (b) Analogous results for Blume-Capel models at $T = 0.8$ and $h = -0.4$. Reversal times for the spin-$\{1, 3/2, 2, 5/2, 3, 7/2\}$ systems are $\tau = 2332, 487, 153, 88, 63$, and 51 MCSS, respectively. In both panels, typical benchmark curves obtained from a single run are shown for illustrative reasons.

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Metastable lifetime (Theory)

The multidroplet/coalescence regime

$$\log(\tau_c) \sim \frac{1}{(d+1)h^{d-1}}$$

The single droplet/nucleation regime

$$\log(\tau_n) \sim \frac{1}{h^{d-1}}$$

d dimension

h applied magnetic field

J. D. Gunton and M. Droz, Introduction to the theory of metastable and stable states (Springer-Verlag, 1983)

Metastable lifetime as a function of applied field

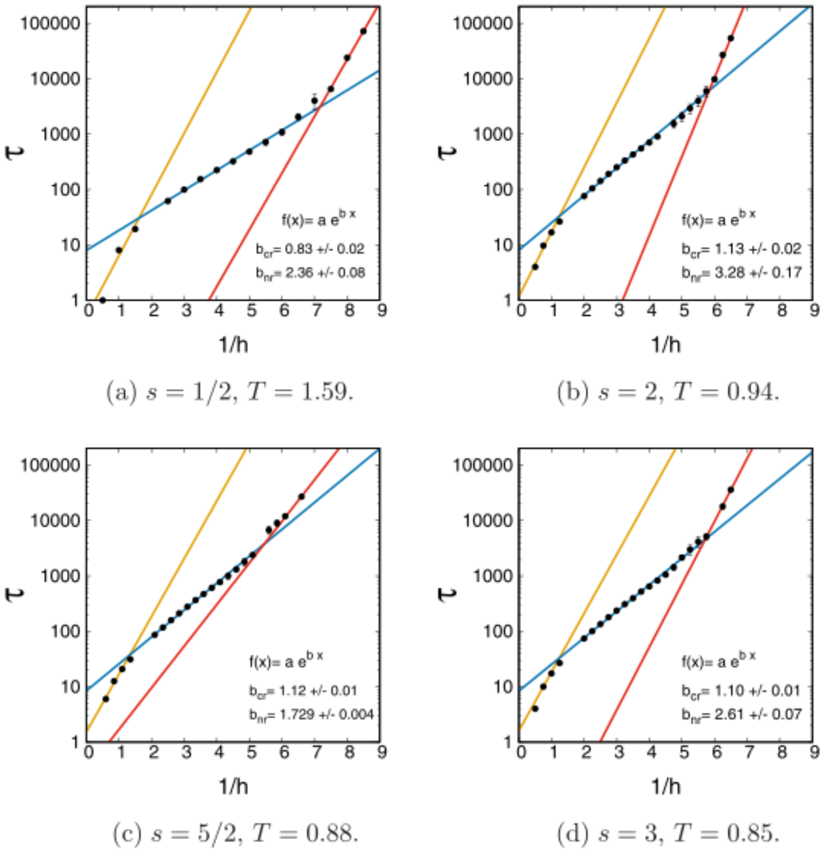
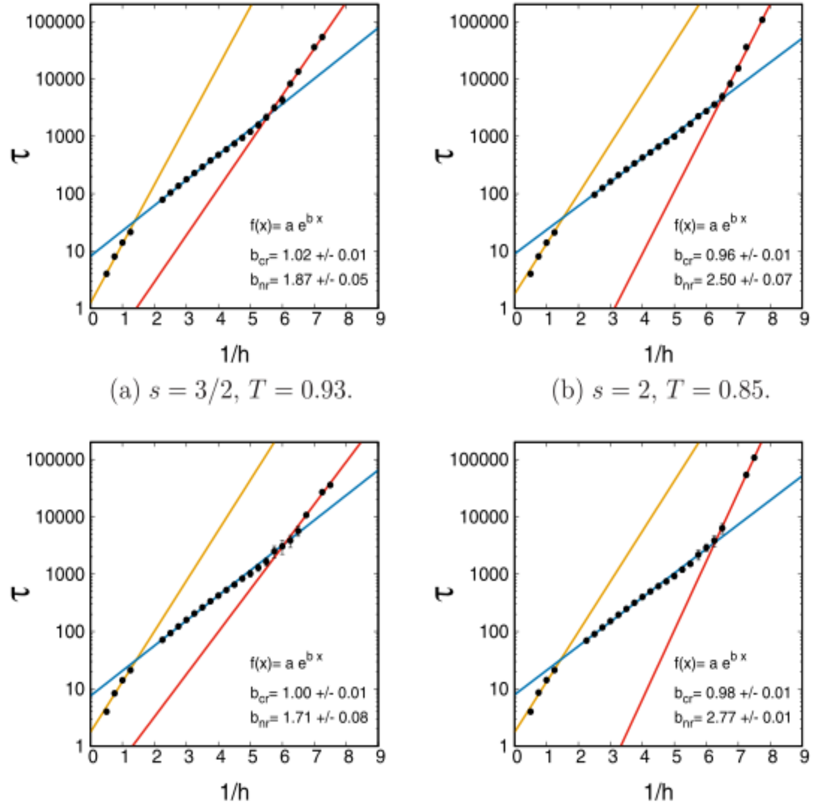


FIG. 4. Mean reversal time as a function of the inverse magnetic field at $T = 0.7T_L^*$ for four spin-s Ising models, as indicated in the panels. Results averaged over 1000 samples. In cases in which the error bars are not visible, this is due to being smaller than the symbol size used. Note the appearance of three different regimes: (i) strong-field regime (yellow line), (ii) coalescence regime (blue line), and (iii) nucleation regime (red line), which are identified with different slopes. Note the logarithmic scale in the vertical axis. See also Table II.

Metastable lifetime as a function of applied field



(a) $s = 3/2,\ T = 0.93.$ (b) $s = 2,\ T = 0.85.$

(c) $s = 5/2,\ T = 0.79.$ (d) $s = 3,\ T = 0.76.$

FIG. 5. Same as in Fig. 4 for four spin-s Blume-Capel models. Again, in some of the data points error bars are smaller than the symbol size. See also Table III.

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How does the metastable volume fraction decay ?

Avrami's law

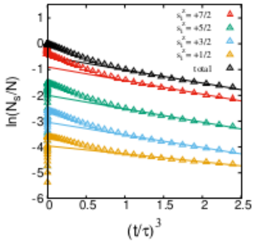
$$\phi \sim \exp\left(-c\left(t/\tau\right)^{d+1}\right)$$

Metastable volume fraction

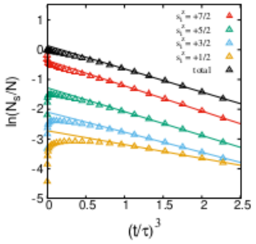
$$\phi = \frac{N_+}{N}$$

M. Avrami, J. Chem. Phys. 7 (1939) 1103

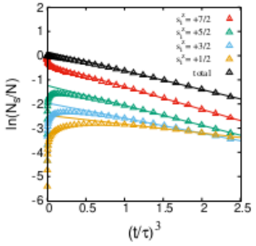
Decay of metastable volume fraction (Ising)



(a) $h = -0.2,\ \tau = 1915 \pm 291$ MCSS.



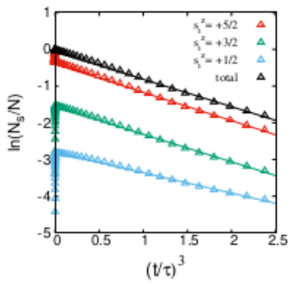
(b) $h = -0.5,\ \tau = 74 \pm 1$ MCSS.



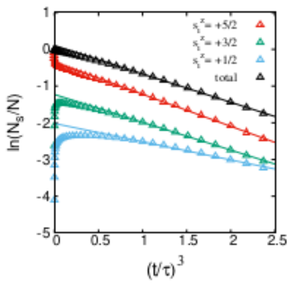
(c) $h = -0.8,\ \tau = 27 \pm 1$ MCSS.

FIG. 6: Variation of $\ln(N_s/N)$ of the spin components $s_i^z = \{7/2, 5/2, 3/2, 1/2\}$ and also of the total versus $(t/\tau)^3$ for the $s = 7/2$ Ising system in the presence of three different strengths of the applied field. N_s is the number of particular spin state s_i^z, N is the total number of spins, and τ the reversal time. For the case of black open triangles, N_s is the total number of the four spin components. The temperature is set to $T = 0.7T_L^*$ and the fits shown by the solid lines correspond to times $t > \tau$. Results averaged over 1000 samples are shown with the size of error bars being smaller than that of the symbol size used.

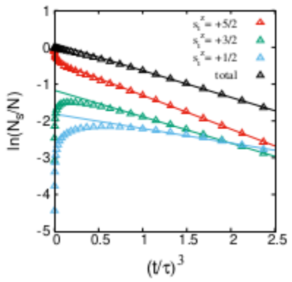
Decay of metastable volume fraction (Blume-Capel)



(a) $h = -0.2,\ \tau = 1019 \pm 139$ MCSS.



(b) $h = -0.5,\ \tau = 54 \pm 1$ MCSS.



(c) $h = -0.8,\ \tau = 23 \pm 1$ MCSS.

FIG. 7: Similar to Fig. 6 for the spin components $s_i^z = \{5/2, 3/2, 1/2\}$ and also for the total of them for the $s = 5/2$ Blume-Capel model. Again, as in Fig. 6 the size of error bars is smaller than the symbol size.

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Concluding remarks

1. The MC results of metastable lifetime of Spin-S Ising and Blume-Capel ferromagnet are in good agreement with the classical theory of nucleation.

2. The MC results of the decay of metastable volume fraction are in good agreement with the Avrami's law.

Thank you