Lectures on quantum aspects of black holes

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KAWS, January 2022

Lecture Plan

- Lecture 1: Generalities on the black hole information problem.
- Lecture 2: Chaos and the near horizon geometry. Traversable wormholes.
- Lecture 3: Longer times.

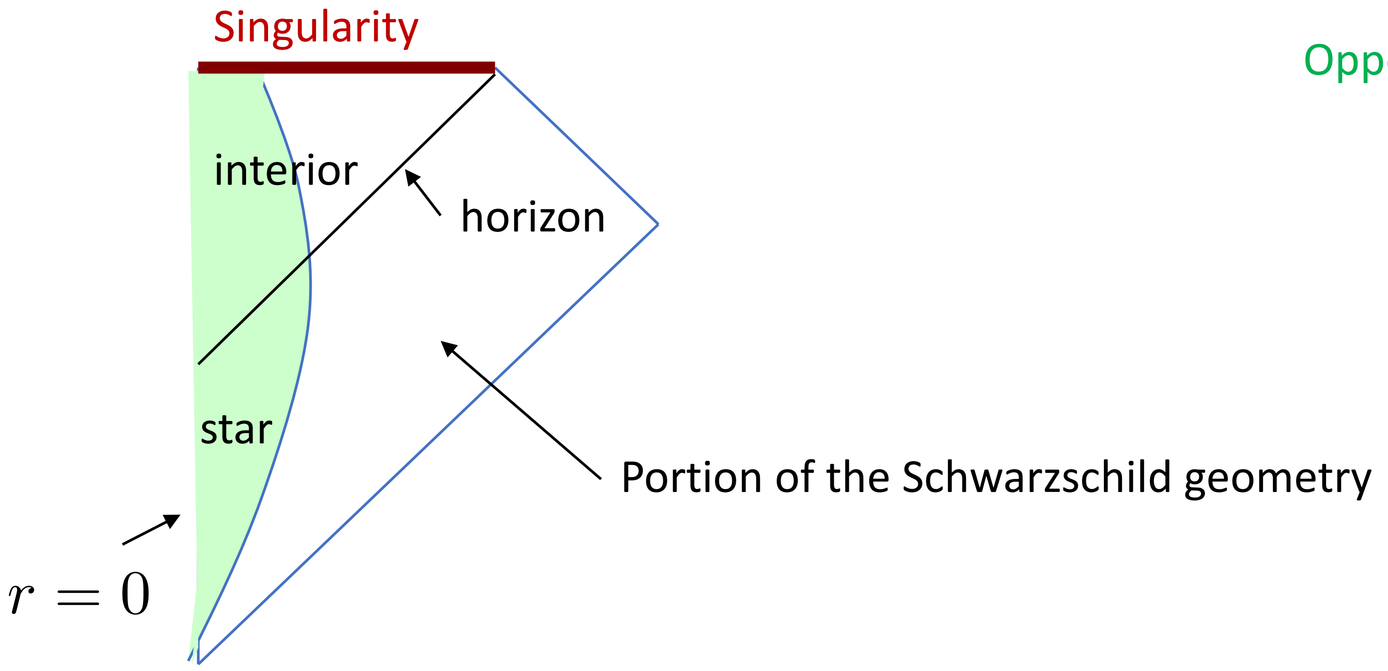
Lecture 1:

The black hole information problem

- Quantum aspects of black holes
- Black hole thermodynamics
- “Central dogma”
- Aspects of the black hole information problem.
- Simple vs complex
- Entropy?
- Entangled black holes.
- Simple 2d gravity theories.

Classical black holes

Geometry of a Black Hole made from collapse

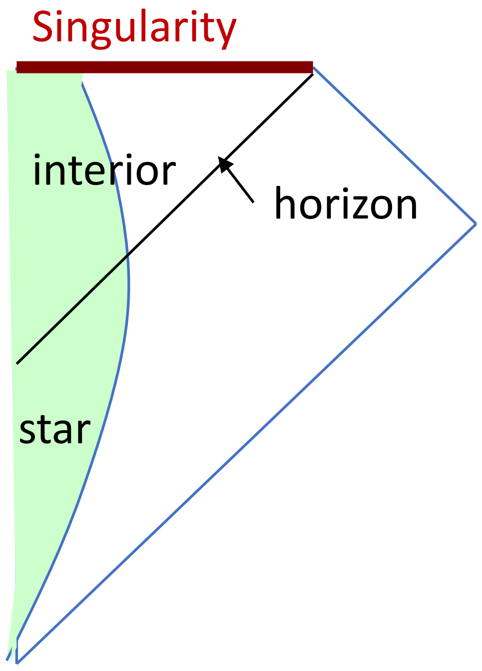


Oppenheimer Snyder 1939

Horizon Area law

Area law: The area of a black hole horizon always increases. → 2nd law of thermodynamics

Hawking



Starts with small area and it grows to larger area

The Schwarzschild solution

Schwarzschild 1917

$$ds^2 = -(1 - \frac{r_s}{r})dt^2 + \frac{dr^2}{(1 - \frac{r_s}{r})} + r^2 d\Omega_2^2$$

$$r_s = 2G_N M/c^2$$

- Classical black holes were confusing!

- It took about 50 years to understand the classical geometry of the Schwarzschild solution.

- Once we include quantum effects, they are even more confusing!

We now move on to quantum mechanical aspects

The theory

Semiclassical gravity

Start from a classical solution, quantize the fields, resolve Einstein's equations with $\langle T_{\mu\nu} \rangle$ in the right hand side, go to higher orders, introducing counterterms to absorb the divergences, etc.

This defines an effective field theory at any order in perturbation theory.
(Number of arbitrary parameters grows with the order in the perturbation theory. In string theory, no arbitrary parameters…).

This is a good framework for approximate computations for distances $L >> l_p$ (in the black hole context: $r_s >> l_p$).

Also used in inflation, to calculate the primordial density fluctuations. Used to explain the universe we see!.

Black holes are hot

- Black holes have a temperature.

Hawking

Surface gravity → redshifted (proper acceleration is infinite,
as is the proper temperature)

$$T = \frac{1}{4\pi r_s} = \frac{\kappa}{2\pi} = \frac{a}{2\pi}$$

- Accelerated observer in Minkowski space → also sees a temperature.

Unruh

More details in Yiming Chen's tutorial

Entropy

- Use first law:

$$dS = \frac{dE}{T} = \frac{dM}{T} \ , \qquad r_s = G_N M/c^2$$

$$\boxed{S = \frac{\text{Area}}{4G_N} = \frac{\text{Area}}{4l_p^2} = \frac{4\pi r_s^2}{4l_p^2}}$$

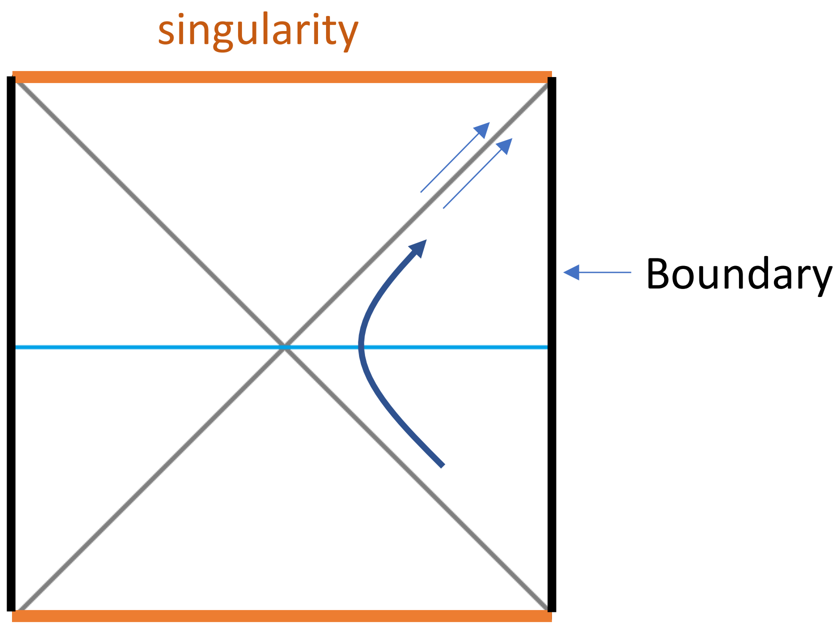
Exercises:

1) Compute the mass and size of a Schwarschild black hole which would look white due to Hawking radiation.

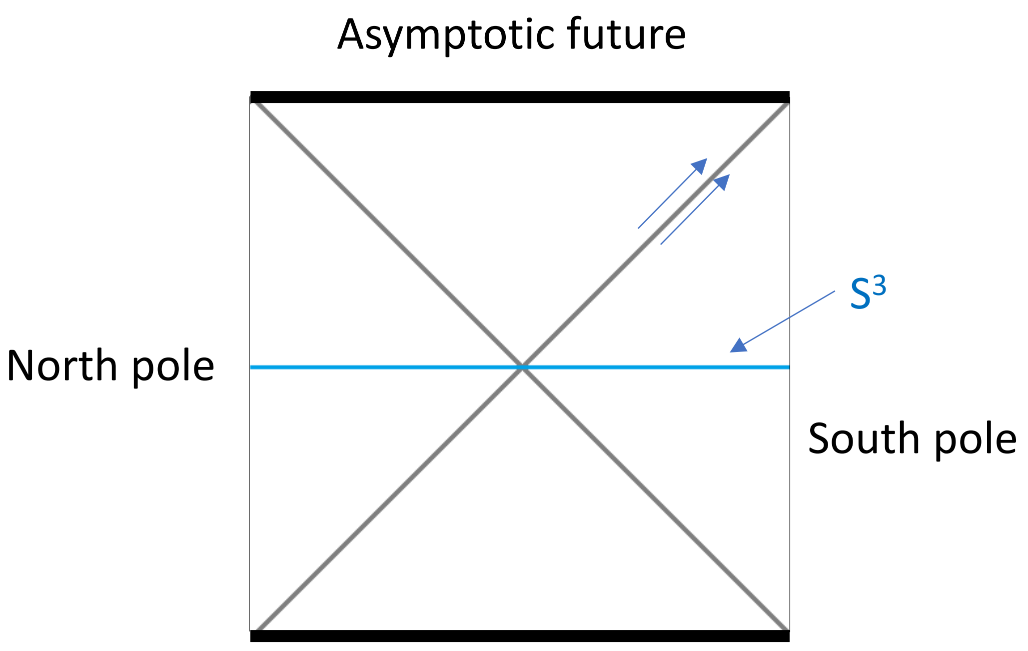
2) Estimate the lifetime of a black hole using the black body radiation formula to estimate the energy emitted by the black hole.

3) Estimate the entropy of a solar mass black hole, a 10^9 solar mass black hole, the entropy of all stars in the universe, and the entropy of the CMB in the observable universe.

Black holes vs de Sitter



AdS black hole



De Sitter space

Boundary time translation → boost in the near horizon region

time translation for the pole observer → boost in the near horizon region

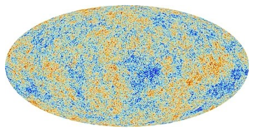
$$T = \frac{\kappa}{2\pi} = \frac{H}{2\pi} = \frac{1}{2\pi R_{dS}}$$

In both cases we have UV modes that become IR modes and thermal radiation. The distances between light rays grows towards the future. Space near the horizon is ``expanding''.

Evolving backwards → becomes ``trans-Planckian''.

De Sitter temperature and us

- According to inflationary theory, the universe was close to de-Sitter at early times.
- It expanded like de Sitter for an amount of time set by a ``clock''= rolling scalar field.
- Temperature → fluctuations in this clock. Different in different regions.
- Caused some regions to expand more than others. → density fluctuations. Small, 10^{-5}. Seen in CMB.

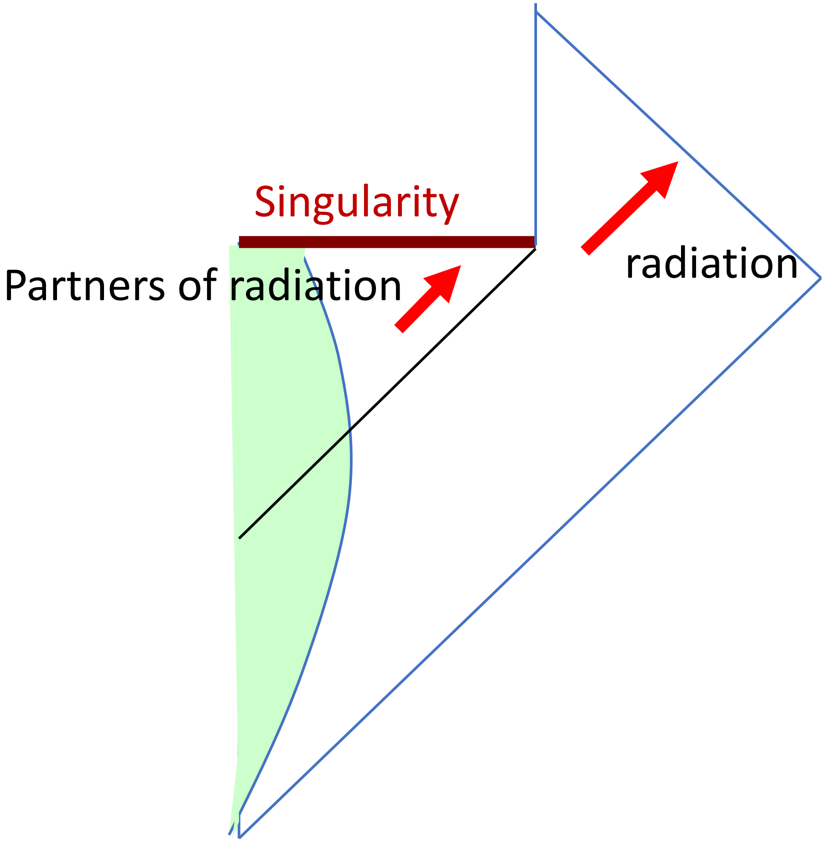


- Fluctuations are necessary to seed the formation of structure, galaxies, stars, planets,..., us.

Even though Hawking radiation has not been seen for black holes, a similar effect is central to our theory of structure formation in cosmology

Conceptual questions raised by Hawking radiation

Geometry of an evaporating black hole made from collapse



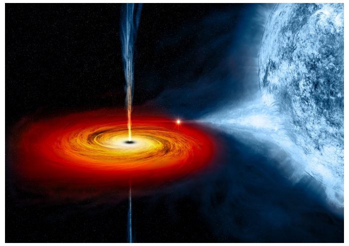
The radiation is entangled with
partners of radiation.

Since we do not measure the interior
we get a large entropy for the radiation.

A pure state seems to go
a mixed state.

Area is decreasing. The 2nd law ?...

Generalized entropy



$$S = \frac{\text{Area}_H}{4G_N} + S_{\text{matter}} = \frac{\text{Area}_H}{4G_N} + S_{\text{QFT}}$$

Bekenstein 70's

Includes the entropy of quantum fields

Bombelli, Koul, Lee,Sorkin 1986

Obeys the 2nd Law

Wall 2010

More entropy in Hawking radiation than in the initial area of the black hole. Process is irreversible.

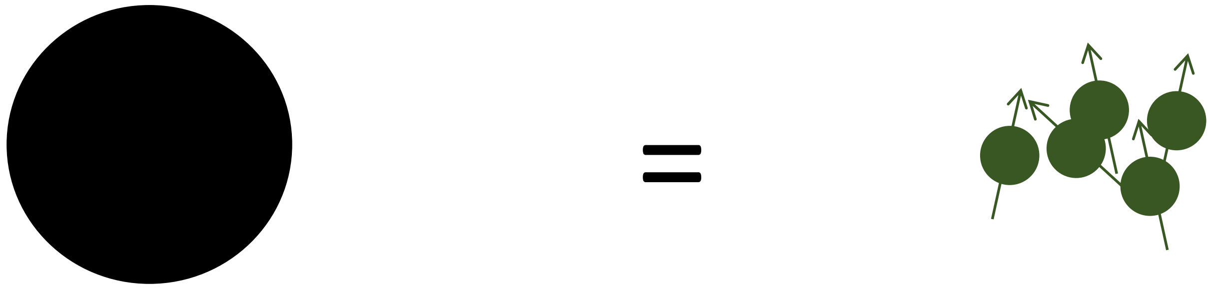
These results have inspired a

Central Hypothesis

Black holes as quantum systems

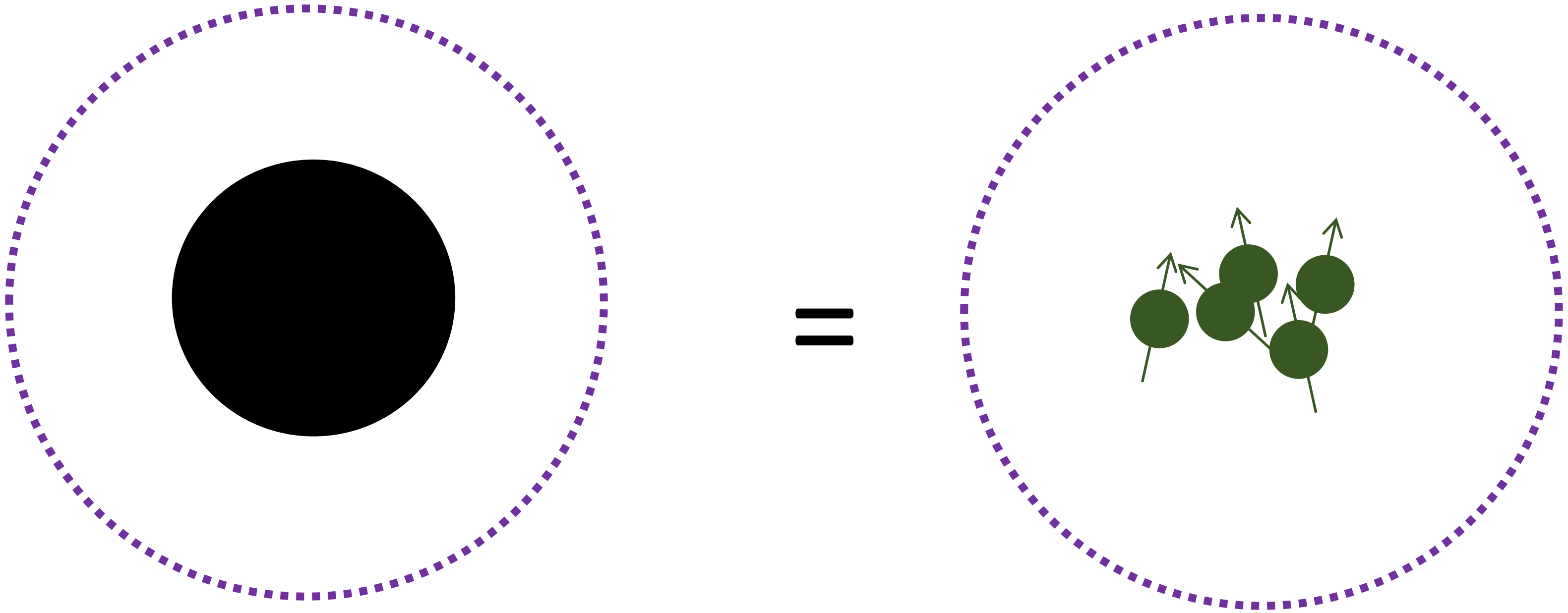
Central ``dogma'', or central hypothesis

- A black hole seen from the outside can be described as a quantum system with S degrees of freedom (qubits). S = Area/4 $(l_p = 1)$
- It evolves according to unitary evolution, seen from outside.



…in other words

- If one includes $A/4G_N$ "mysterious" qubits, then the black hole can be described as an ordinary quantum system.



- No statement about the interior. So far, just a statement about the exterior.

- Confusing words: “black hole” sometimes means the object viewed from outside. Sometimes means the full spacetime geometry including the interior region. We will use the word “black hole” to refer mostly to the black hole as seen from the outside.

The information problems

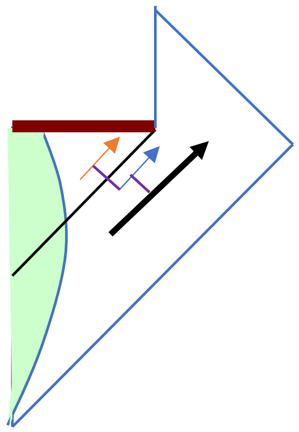
Outside issues

- The outside region starts in a pure state and evolves to a mixed state. We could only restore purity by including the interior modes, but those are not visible from the outside!. Hawking
- Page curve. (see T. Hartman's lectures) Page
- Black hole S-matrix?. If you form a black hole in different ways, how do we compute the amplitudes for different states of the Hawking radiation to come out.
- Long time correlators: If you excite a black hole, they should retain some memory → two point functions should not decay back to zero. JM
- AMPS paradox: If we have an old black hole, its late time Hawking radiation should be entangled with the early Hawking radiation. But late time Hawking radiation is also entangled with late time ingoing Hawking partners → not possible due to the monogamy of entanglement.

Almheiri, Marolf Polchinski, Sully

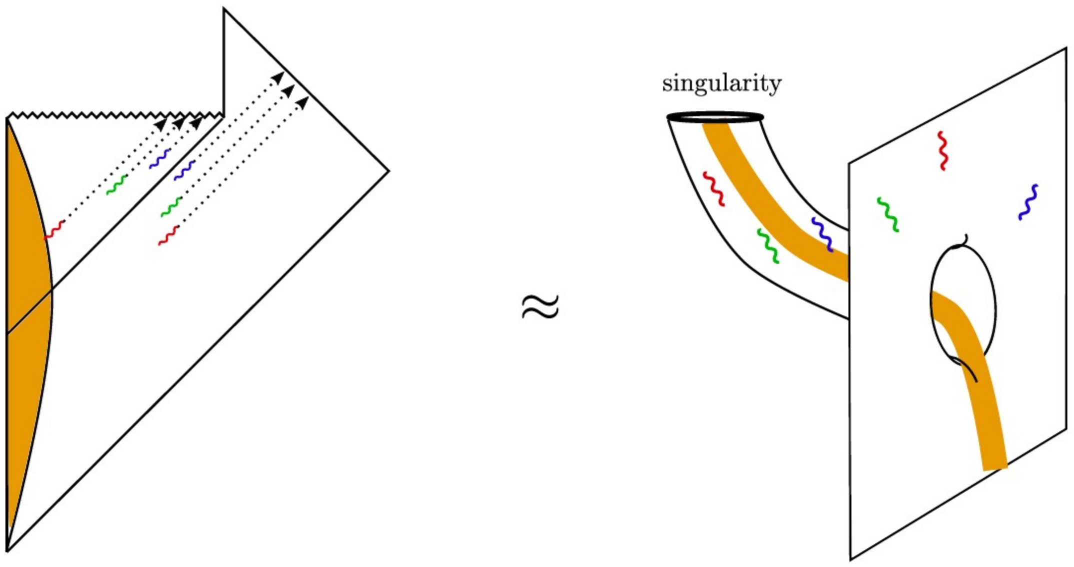
AMPS paradox

- Old black holes that has evaporated past the Page time (when the entropy of Hawking radiation is decreasing.
- The black hole is close to maximally entangled with radiation.
- The radiation that is now emitted should be entangled with the early radiation, so that the total entropy of radiation decreases.
- This radiation should also be entangled with the interior mode.
- Entanglement is ``monogamous" → contradiction…



Why would we insist on the “central dogma” given the arguments against it?

The skeptic's view:



One universe splits of a ``teenage universe''   (big baby universe)

The state is pure if you include both universes, but not if you look only at the original universe.

Evidence in favor

- Black hole thermodynamics. Why should the entropy increase? If it is going to a new universe ?

- Black hole microstate counting for SUSY black holes in string theory.

Strominger, Vafa, … Sen…

- AdS/CFT (or Matrix theory): Conjectured equivalences between gravity and simpler systems. These can be viewed as more precise versions of the ``central dogma'', where we explicitly give the Hamiltonian (CFT on the boundary). But we need to retreat to the boundary of the spacetime.
    - Precision tests of AdS/CFT involving integrability → small fluctuations around the vacuum. Do not probe details of black holes.
    - Tests that use supersymmetry (no black holes).
    - Entropy: Cardy formula, or numerical simulations for the D0 brane case (or BFSS model)

- If it is true, can we understand why it is true directly by following the rules of gravity in the bulk?

- Can we recover the interior geometry from the full quantum theory that describes the outside? (``complementarity'' ? . Modern proposal: entanglement wedge reconstruction hypothesis).

- Can we ``resolve'' the singularity? What does it mean?.

A research proposal

- Understand quantum aspects of black holes.
- Understand the singularity.
- Extract lessons for cosmology.
- Make an experimentally verifiable prediction

Black holes and thermalization

- Simple observables: Expectation values of a finite number of fields (as $G_N \to 0$)

- This simple outside observables thermalize. At late times they approach the values for a black hole in thermal equilibrium.
- This is what is expected for a very entropic system → black hole degrees of freedom interact significantly with each other.
- Beautiful connection between black holes and hydrodynamics, ``fluid/gravity correspondence", etc...

Damour, Son, Policastro, Starinets, Eling, Oz, Bhattacharyya, Minwalla, Rangamani, Haehl, Loganayagam, … many others

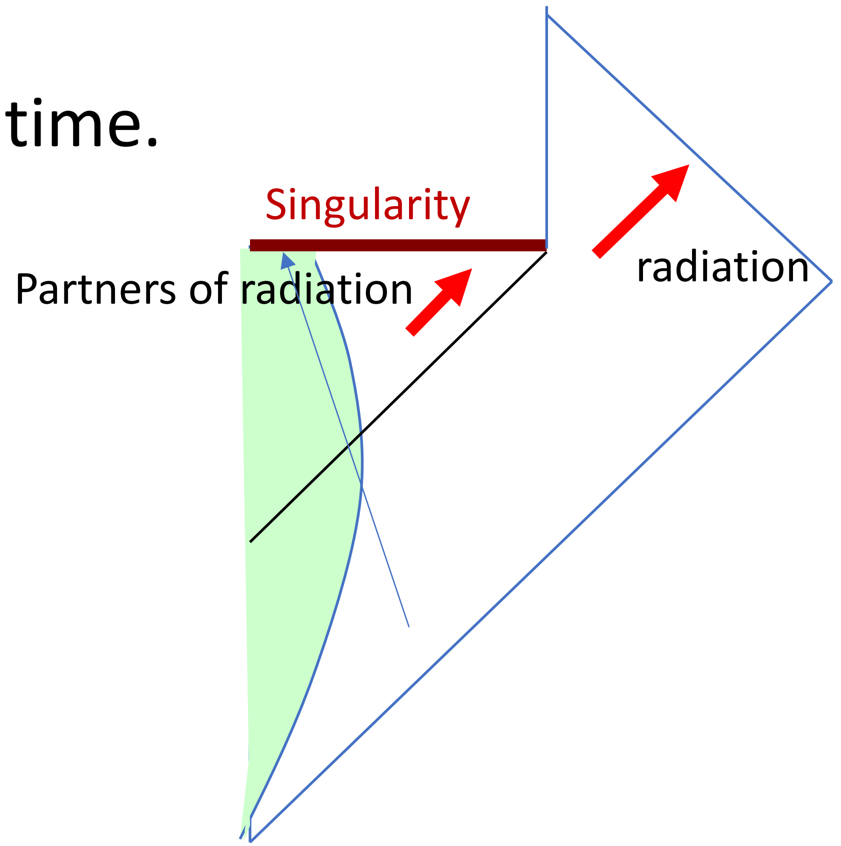
Black hole as a ``mesoscopic'' object

- The information problem involves "mesoscopic" properties. The entropy is large but finite, and we want to reconcile thermodynamics with the unitary quantum behavior.

- Extracting information from Hawking radiation, or doing many of the checks of unitarity, is very hard in practice. It is hard even for relatively simple manybody systems of tens of qubits. This type of experiments require exquisite control.

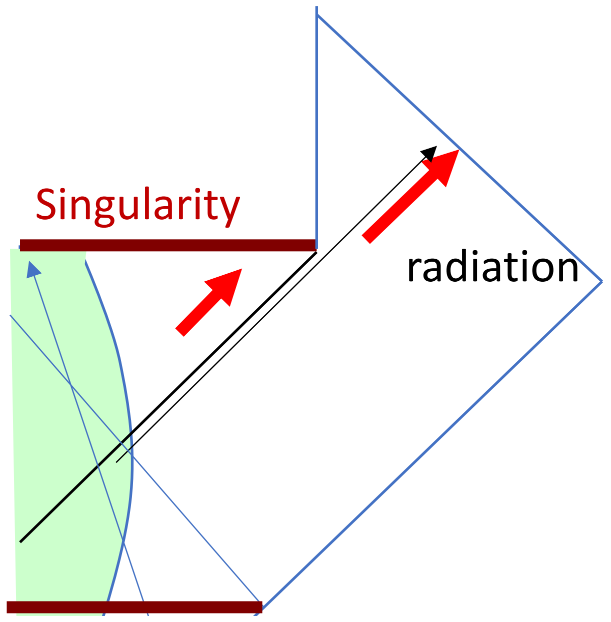
How to extract the information from Hawking radiation?

- Let the black hole evaporate completely.
- Collect all Hawking radiation.
- Evolve (or simulate the evolution) backwards in time.
- The information comes out of the black hole.



Why it is tricky?

- Let the black hole evaporate completely.
- Collect almost all Hawking radiation. Miss one mode.
- Evolve (or simulate the evolution) backwards in time.
- The missed mode leads to a shockwave at the horizon
- The information does not comes out of the black hole.
- It remains behind a horizon.



It is still possible in principle to recover the information, but it is harder….

Can we test that the radiation is pure?

- Make two identical black holes and use the ”swap” test to check whether they are the same pure state.

Harlow-Hayden

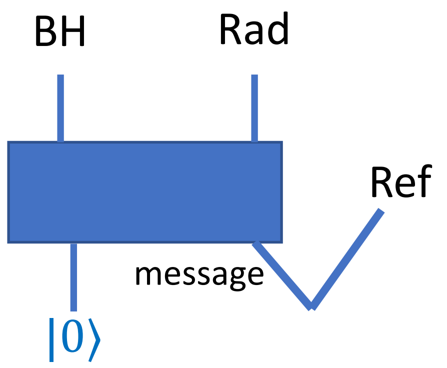
- Practically this is fairly difficult, but it is ``easy’’ by complexity standards. The number of operations is polynomial in the entropy, rather than exponential in the entropy.
- Note that experimentalists that have a ``quantum memory’’ can do interesting things!.

Exercise : Read about the swap test in Wikipedia, or some other reference…

There are things that are more complicated

- Let the black hole evaporate so that the entropy of the remaining black hole is less than half of the thermodynamic entropy of the initial black hole.
- If you sent an unknown message into the initial black hole, we expect to recover it from the Hawking radiation.
- Let us explain why…

The message is in the Hawking radiation

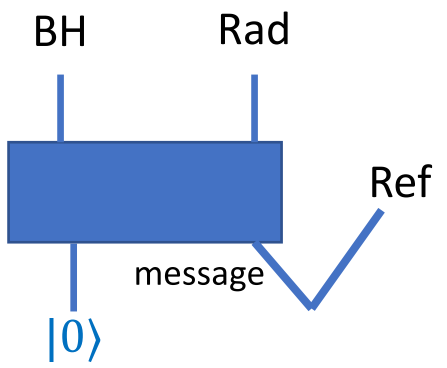


Main tool:

Consider a factorized Hilbert space: $H = H_A \times H_B$, $\dim(H_A) \ll \dim(H_B)$

Somewhat generic pure state in H → ρ_A is nearly maximally mixed. Page

The message is in the Hawking radiation



Pure state in $H_A \times H_B$, $\dim(H_A) \ll \dim(H_B)$
→ ρ_A is nearly maximally mixed.

B = Rad, A = BH + Ref , $\dim(H_{rad}) \gg \mathrm{Area}_{BH} + S_{Ref}$

We want to argue that: $I(\mathrm{Ref}, \mathrm{BH}) \sim 0$, $I(\mathrm{Rad}, \mathrm{Ref}) \sim 2S(\mathrm{Ref})$ → Message is in the radiation

$$I(\mathrm{BH}, \mathrm{Ref}) = S(\mathrm{BH}) + S(\mathrm{Ref}) - S(\mathrm{BH} \cup \mathrm{Ref}) \sim 0$$

used $S(\mathrm{BH} \cup \mathrm{Ref}) \sim S(\mathrm{BH}) + S(\mathrm{Ref})$

$$I(\mathrm{Rad}, \mathrm{Ref}) = S(\mathrm{Rad}) + S(\mathrm{Ref}) - S(\mathrm{Rad} \cup \mathrm{Ref}) \sim 2S(\mathrm{Ref})$$

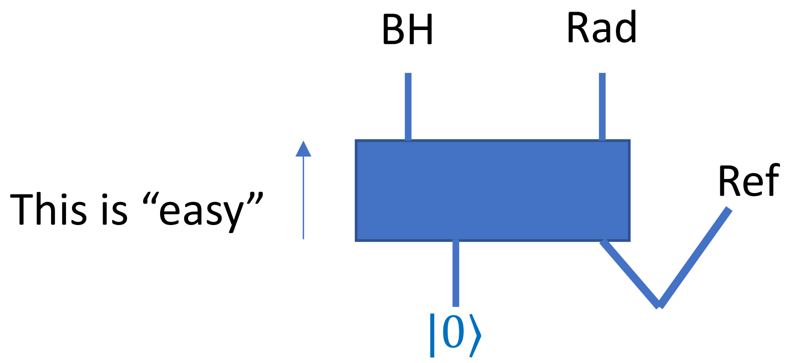
$S(\mathrm{Rad}) \sim S(\mathrm{BH}) + S(\mathrm{Ref})$, $S(\mathrm{Rad} \cup \mathrm{Ref}) = S(\mathrm{BH})$

But is is exponentially difficult to extract it!.

Harlow, Hayden; Kim, Tang, Preskill

Much more difficult than evolving backwards in time!

Complexity is likely relevant for black hole discussions!



There is a "Python's lunch conjecture" stating more precisely which operations in gravity are supposed to be very complex. It is related to the existence of non-minimal quantum extremal surfaces in the calculations of entropies.

Brown, Gharibyan, Penington, Susskind

A very important tool for recent developments

A formula for the ``fine grained entropy”

Two notions of entropy

- Fine grained entropy. (Also called Von Neuman entropy, or quantum entropy, or ``entanglement'' entropy)

$$S = -\mathrm{Tr}[\rho \log \rho]$$

Remains constant under unitary time evolution.

- Coarse grained entropy = thermodynamic entropy. Arises from ``sloppiness"

$$S = \max_{\hat{\rho}} \left(-\mathrm{Tr}[\hat{\rho} \log \hat{\rho}] \right) \ , \qquad \mathrm{Tr}[A\hat{\rho}] = \mathrm{Tr}[A\rho]$$

Obeys 2nd law.

Subset of observables, ``simple observables", eg. A = E,Q,..

Fine grained gravitational entropy

$$S = \min_X \left\{ \text{ext}_X \left[\frac{\text{Area(X)}}{4G_N} + S_{\text{semi-cl}}(\Sigma) \right] \right\}$$

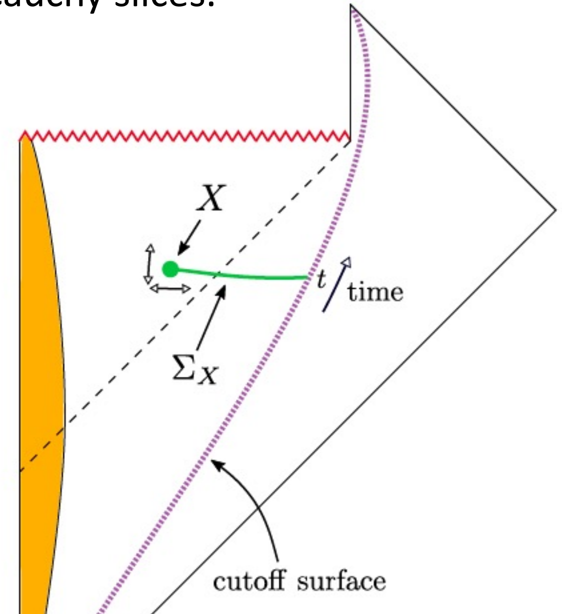
Ryu-Takayanagi 2006
Hubeny, Rangamani, Takayanagi 2007
Faulkner, Lewkowycz, JM 2013
Engelhardt, Wall 2014

Follows from AdS/CFT rules:
Lewkowycz, JM , Faulkner,Dong,...

The final surface is called minimal quantum extremal surface.

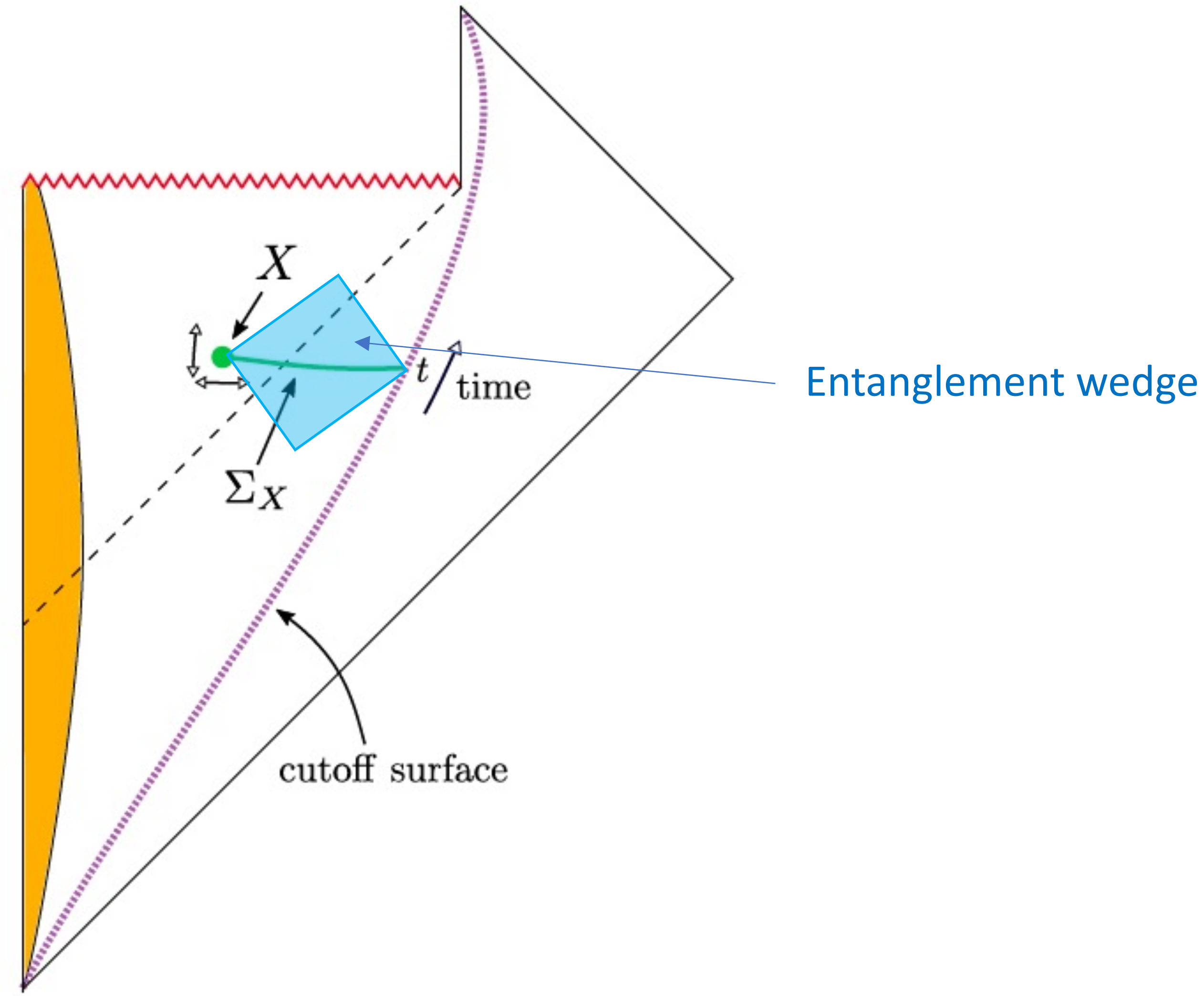
Also maxi-min: minimize along a spatial slice (Cauchy slice) and then maximize among all possible Cauchy slices.

Wall; Akers, Engelhardt, Penington, Usatyuk.



We are allowed to take the surface to the inside. It depends on the geometry of the interior

Entanglement wedge



Entanglement wedge reconstruction hypothesis

- The quantum system describes everything that is included in its entanglement wedge.

- We can recover the state of a (probe) qubit inside the entanglement wedge.

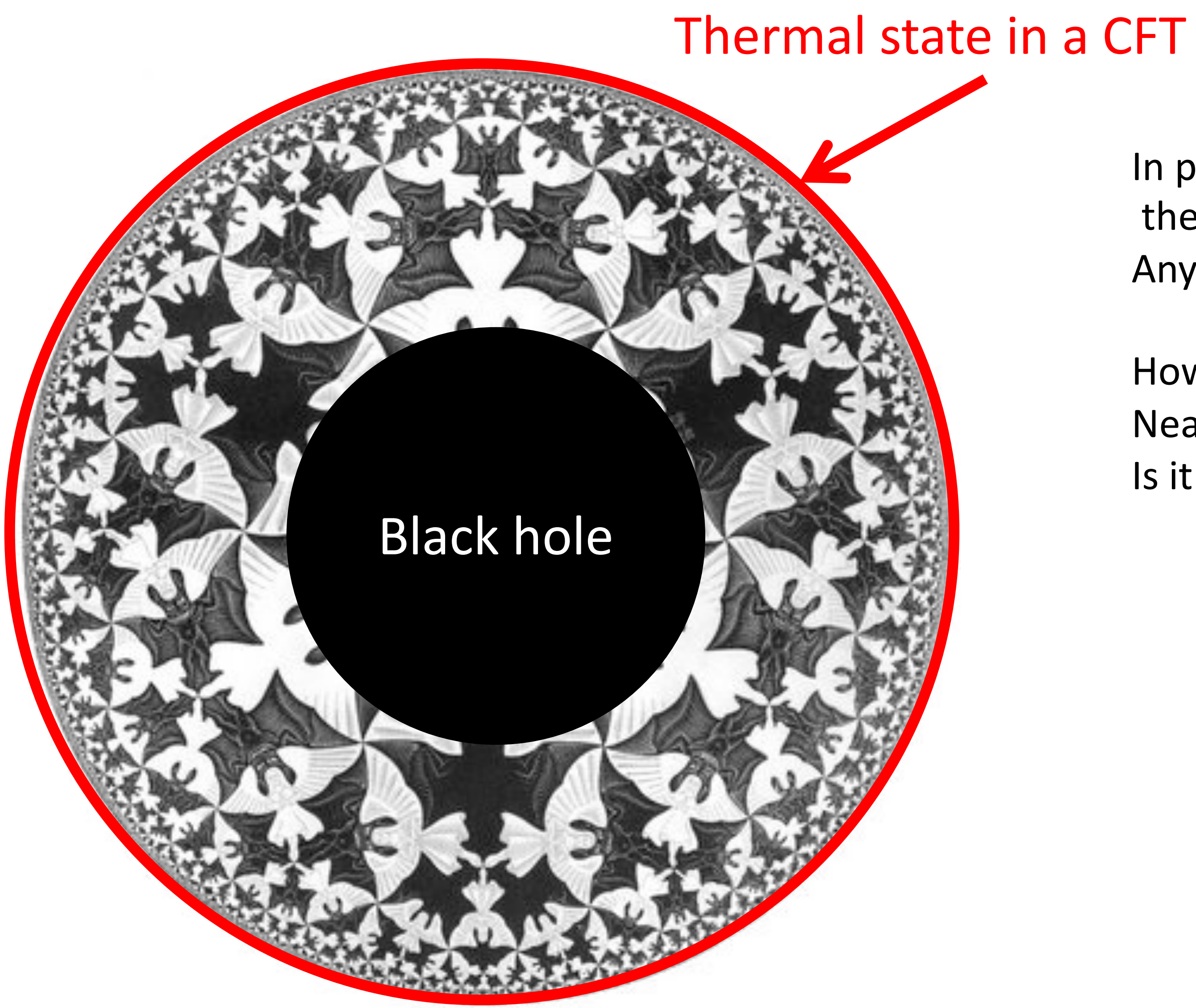
- Recovery is state dependent (subspace dependent) and similar to quantum error correction.

Czech, Karczmarek, Nogueira, Van Raamsdonk, Wall, Headrick, Hubeny, Lawrence, Rangamani, Almheiri, Dong, Harlow, Jafferis, Lewkowycz, J.M., Suh, Wall, Faulkner….

Tom Hartman will discuss this in more detail in his lectures…

It is a key difference with previous ideas on black holes (eg. ``complementarity'').
In particular, the quantum system of the ``central dogma''
might not describe the full interior of the black hole.

AdS/CFT (or BFSS matrix theory)



In principle allows us to do any computation, if we use the boundary theory.
Any computation that involve observables far away.

How do we describe an observer in the interior?
Near the singularity?
Is it even possible ? → It depends…

Simple remarks on the entropy

- Part of the problem is that we do not know how to describe the black hole microstates from the gravity point of view.
- I will describe now a simple idea and its limitations…

Entropy of quantum fields around the horizon

- The vacuum state in QFT is highly entangled at short distances.
- When we look at the fields outside the horizon → find them in a thermal state.
- We can compute their entropy. We find that it is UV divergent.

Entropy of quantum fields around the horizon

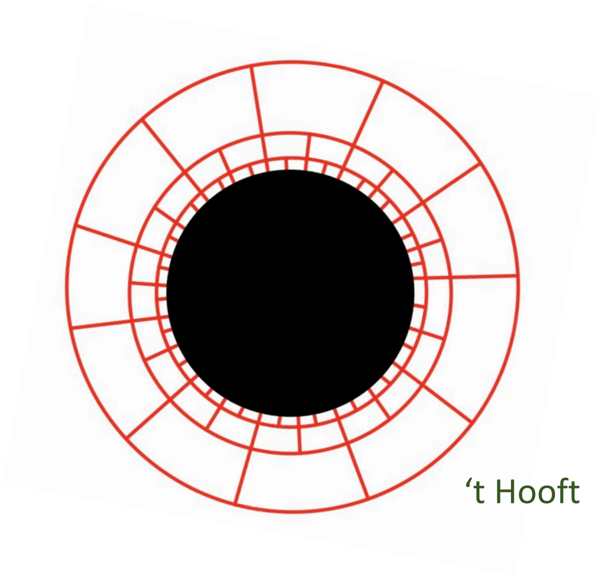
$$ds^2 = -f\,dt^2 + \frac{dr^2}{f} + r^2 d\Omega_2^2 \ , \qquad f = (1 - \frac{r_s}{r}) \sim x^2$$

$$= f\left[-dt^2 + \frac{dr^2}{f^2} + \frac{r^2}{f} d\Omega_2\right]$$

$$= f\left[-dt^2 + r_s^2 \frac{dx^2 + d\Omega^2}{x^2} + \cdots\right] \ , \qquad f \sim \frac{r - r_s}{r_s} = x^2 \ll 1$$

[...]= optical metric = time x Hyperbolic space. Divergence from x → 0, or r→ r_s.

$$S \propto \text{Area} \int \frac{dx}{x^3} \sim \frac{1}{x_c^2} = \frac{\text{Area}}{\epsilon^2} = \int dV s(T_{\text{proper}})$$



't Hooft

Entropy of quantum fields around the horizon

$$S = \frac{\text{Area}}{\epsilon^2} + \cdots$$

Looks a bit like

$$S = \frac{\text{Area}}{4 l_p^2} + \cdots$$

But semiclassical gravity is valid when we choose $\epsilon \gg l_p$!

We can absorb this divergence as a counterterm: a renormalization of G_N

Susskind - Uglum

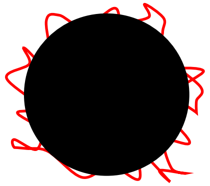
The conservative statement is that this is a correction to the area formula.

It cannot be pushed to $\epsilon \sim l_p$ within the validity of semiclassical gravity.

Could it work if we did a computation in a UV finite theory such as string theory?

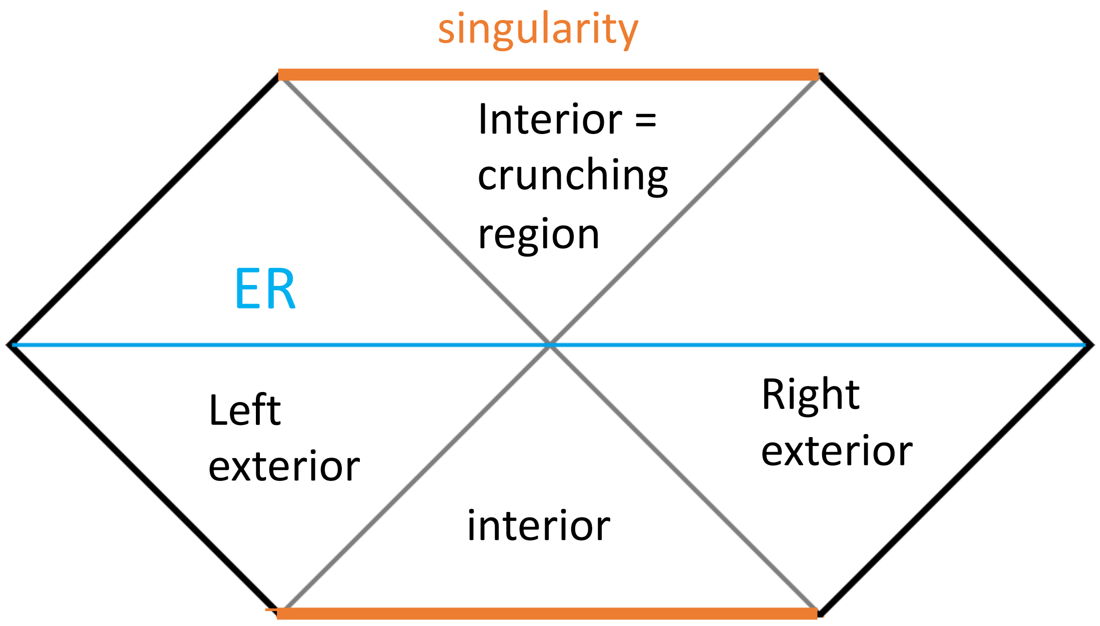
→ Entropy would come form a gas of string with ends on the horizon. No precise computation....

Susskind - Uglum



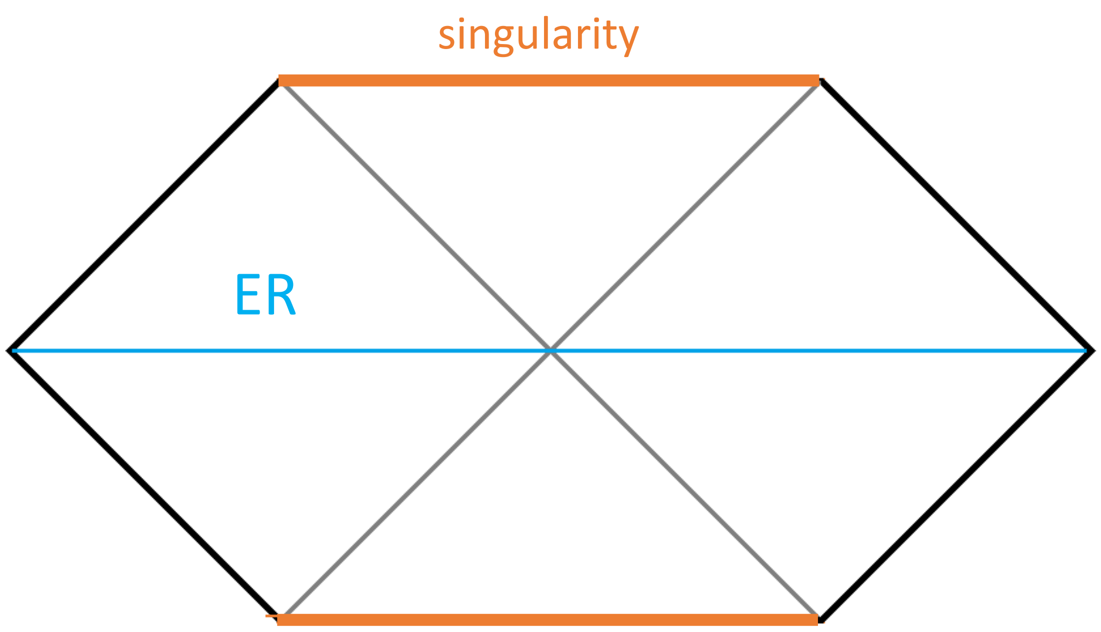
Let’s go back to the Schwarzschild solution

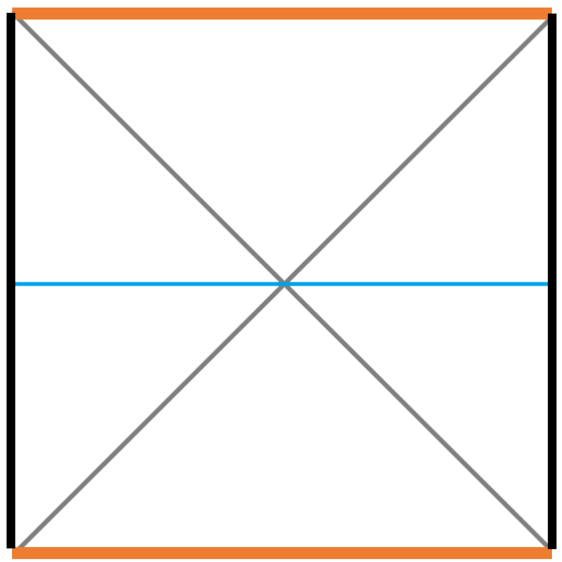
Full Schwarzschild solution



Eddington, Lemaitre, Einstein, Rosen, Finkelstein, Kruskal

Full Schwarzschild solution



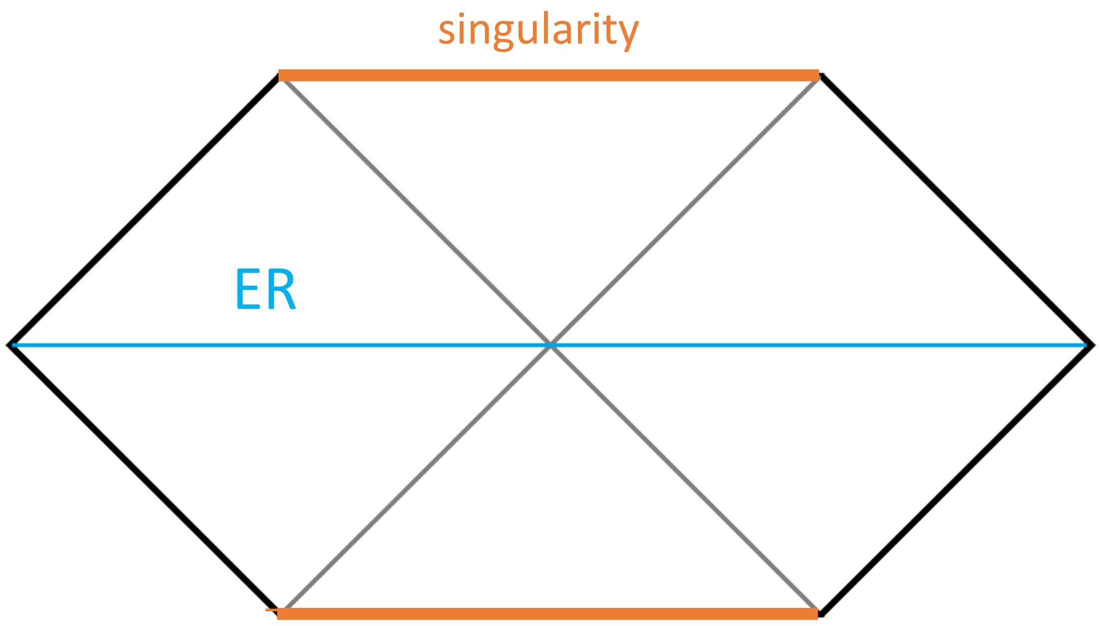


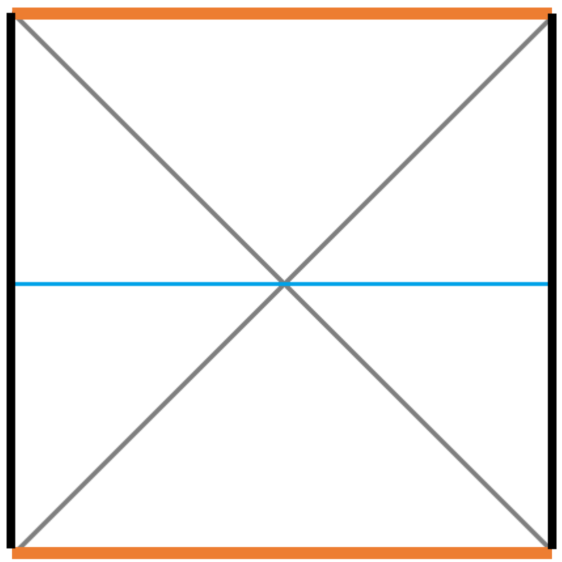
Vacuum solution. No matter.
Two exteriors, sharing the interior. Non-traversable = cannot send a signal between the two asymptotic regions.

We do not know how to easily make it from initial data that does not have the black holes.

- What should we make of these?
- One option is to ignore them.
- Can we interpret them in the context of the ``central dogma’’.

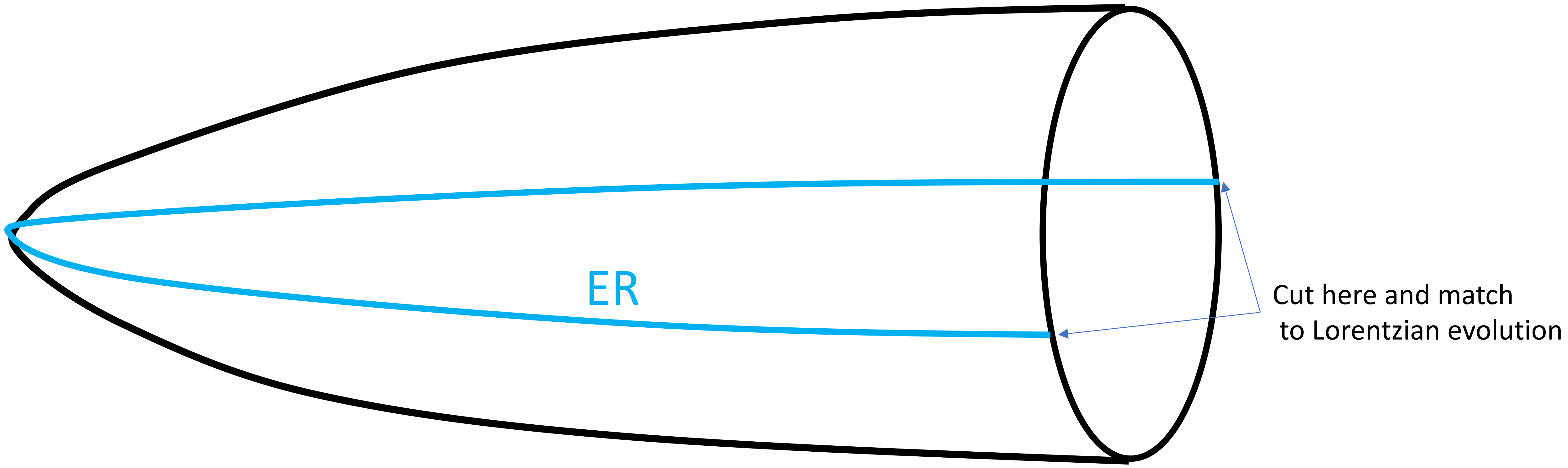
Full Schwarzschild solution = entangled states





$$|TFD\rangle = \sum_n e^{-\beta E_n/2} |\bar{E}_n\rangle |E_n\rangle$$

A particular entangled
State, EPR


ER
Cut here and match
to Lorentzian evolution

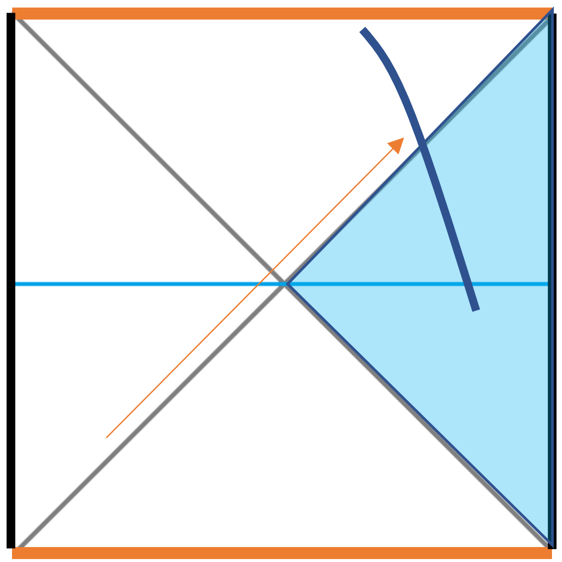
ER = EPR

- The full Schwarzschild solution = entangled state.
- Bulk connectivity is related to entanglement.
- If we vary the entangled sate → vary the bulk state → typically it will become longer.

- Is this true for any system ? Or only for particular large N systems in particular states?
- Is there a geometry for a "typical" entangled state?

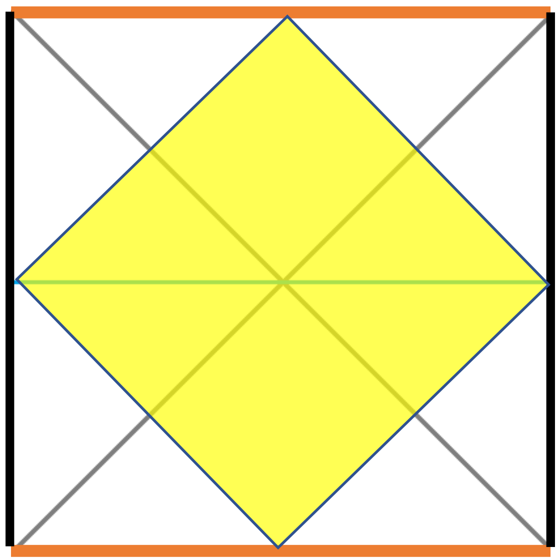
Some interesting lessons

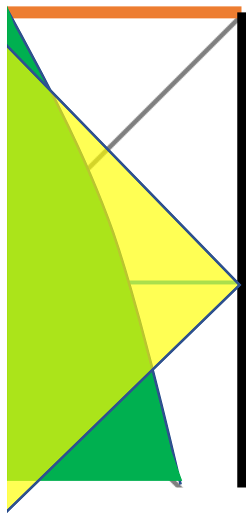
- Entanglement can lead to geometric connections.
- Entanglement wedge of one side.
- Only the exterior.
- The interior of the black hole might not ``belong’’ to the quantum system describing the exterior.
- We need the other side to describe the interior. (It is not a matter of precision.)
- Do not know what will happen to you when you cross the horizon.



A side comment

States vs spacetimes





TFD = "Wheeler-de-Witt patch" = spacelike separated points.

The full geometry also represents the evolution of the TFD state with two decoupled Hamiltonians.

A black hole is not a state, it is a state together with a particular evolution

Simple models

Simple black holes

- One might be tempted to think of quantum gravity as a theory of interacting gravitons.
- However, black holes exist in theories of gravity that have no propagating gravitons.
- We can have black holes in two dimensional theories of gravity. We can think of these as the spherically symmetric reduction of higher dimensional theories. (The three dimensional case is also very simple).
- Two dimensional theories are perturbatively renormalizable theories of gravity. But they have issues when we sum over topologies.

A particularly simple 2d theory of gravity

$$S = \frac{\phi_0}{4\pi} \int \sqrt{g} R + \frac{1}{4\pi} \int \sqrt{g} \phi (R + 2)$$

JT gravity (Jackiw-Teitelboim)

$$+ S_{\rm mat}(g, \chi)$$

Purely topological term

This is an approximation to 4d gravity when we consider the near horizon geometry of a 4d (or higher d) near extremal charged black hole. But we can also consider it as a theory on its own right.

A particularly simple 2d theory of gravity

$$S = \frac{\phi_0}{4\pi}\int \sqrt{g}R + \frac{1}{4\pi}\int \sqrt{g}\phi(R+2) \; + S_{\mathrm{mat}}(g,\chi)$$

JT gravity (Jackiw-Teitelboim)

Purely topological term

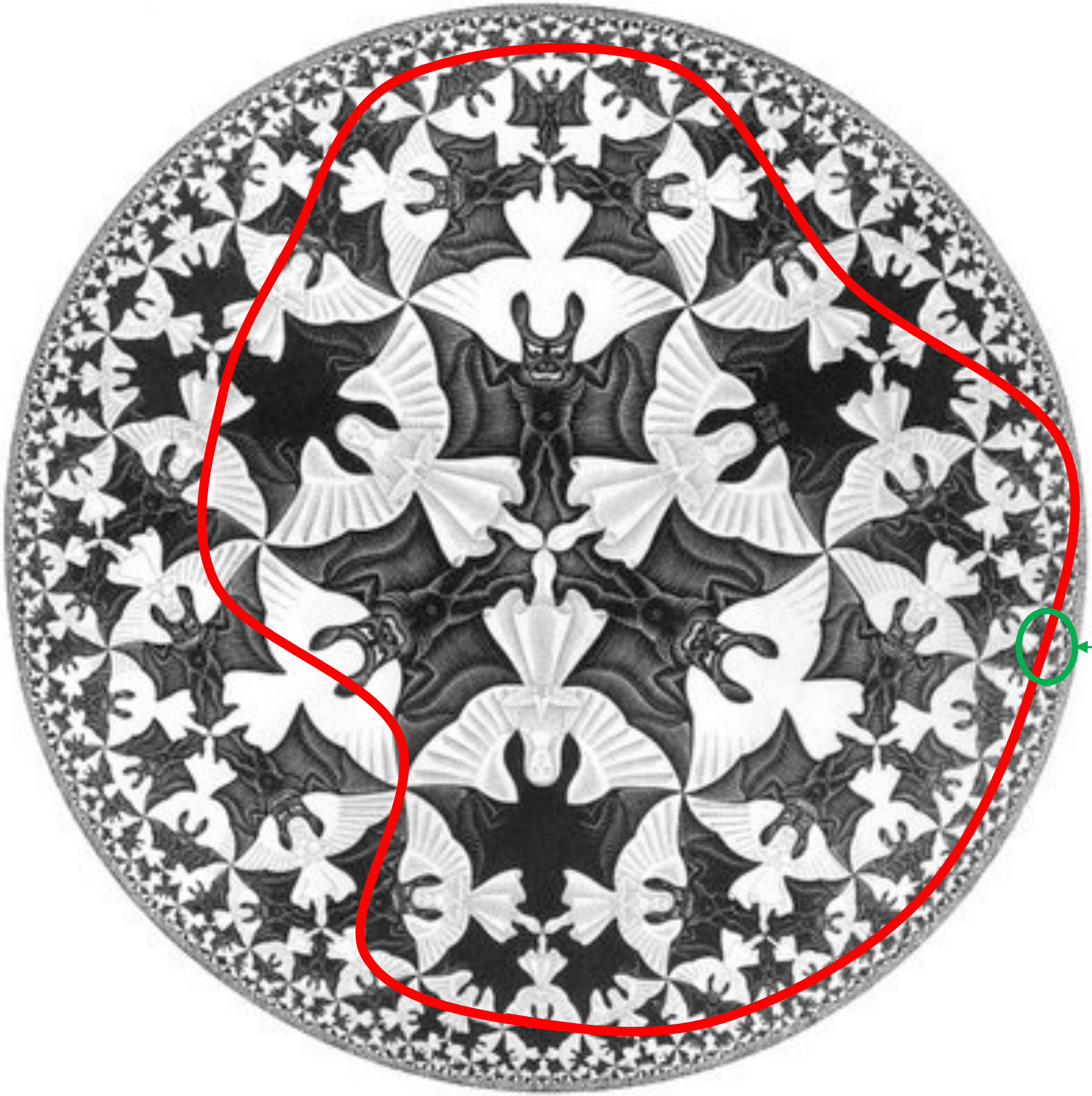
Equations of motion for ϕ → metric is locally H_2
Equations of motion for the metric → Fix ϕ up to a few integration constants.

No propagating bulk gravitons. But there is a dynamical boundary mode.

Nearly AdS$_2$ gravity

Euclidean black hole

Region inside the red line



Infinite number
of other
configurations with the same
boundary length.

Locally the same

Schwarzian action from Nearly AdS_2 gravity

No bulk propagating modes, only a boundary mode

$$S = \int d^2x\sqrt{g}\phi(R+2) - 2\frac{\phi_r}{\epsilon^2}\int duK \to$$

$$S = \frac{\phi_r\beta}{\epsilon^2} - \phi_r\int_0^\beta duSch(t(u),u)$$

Parametrization of the boundary curve

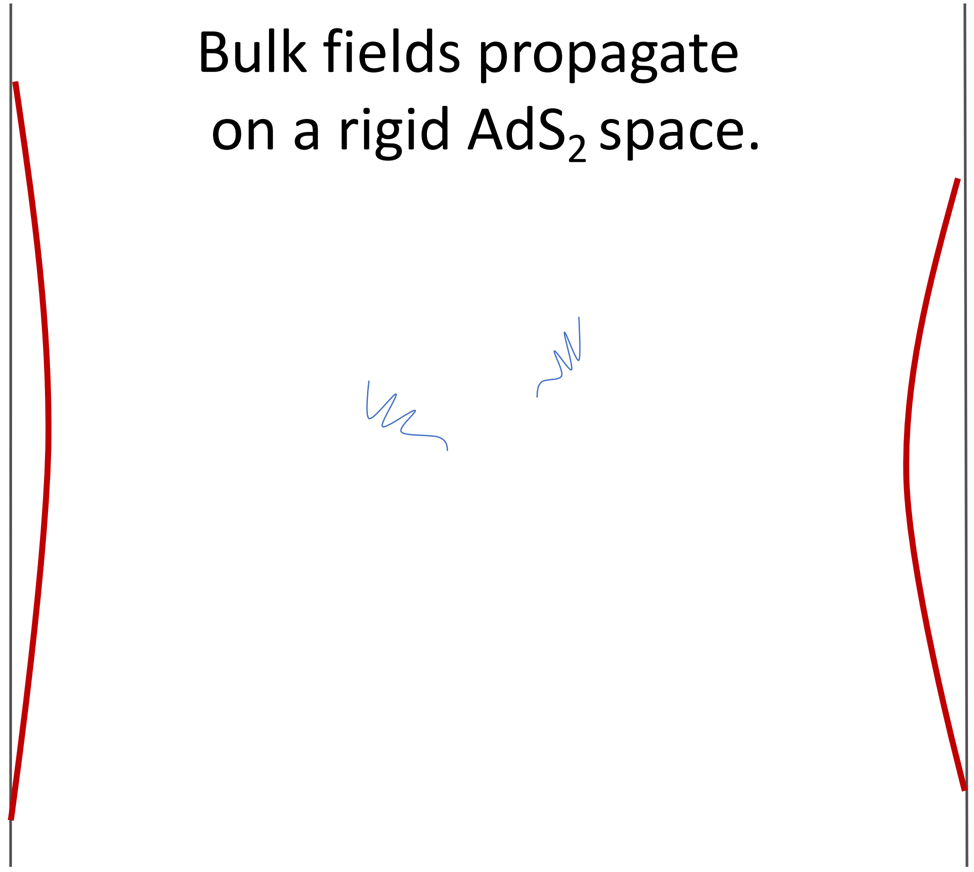
$$ds^2 = \frac{-dt^2+dz^2}{z^2}$$

u = (rescaled) proper length.

Boundary time

Dynamics

Bulk fields propagate on a rigid AdS_2 space.

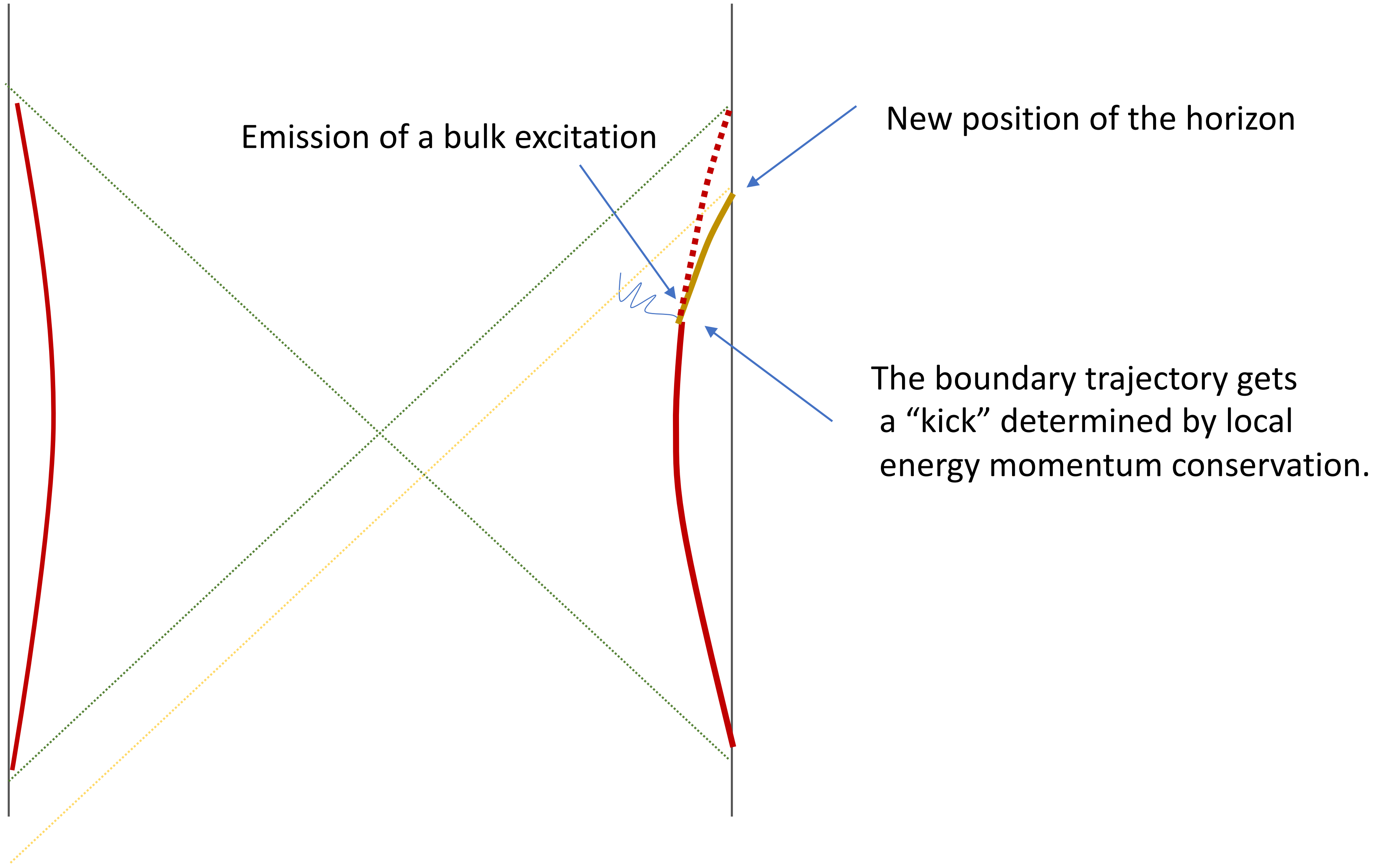


Boundaries also move in a rigid AdS_2 space, following local dynamical laws.

Schwarzian action describes this motion.

The left particle + right particle + bulk fields should form an SL(2) invariant combination (what remains of the reparametrization constraints of GR).

Dynamics

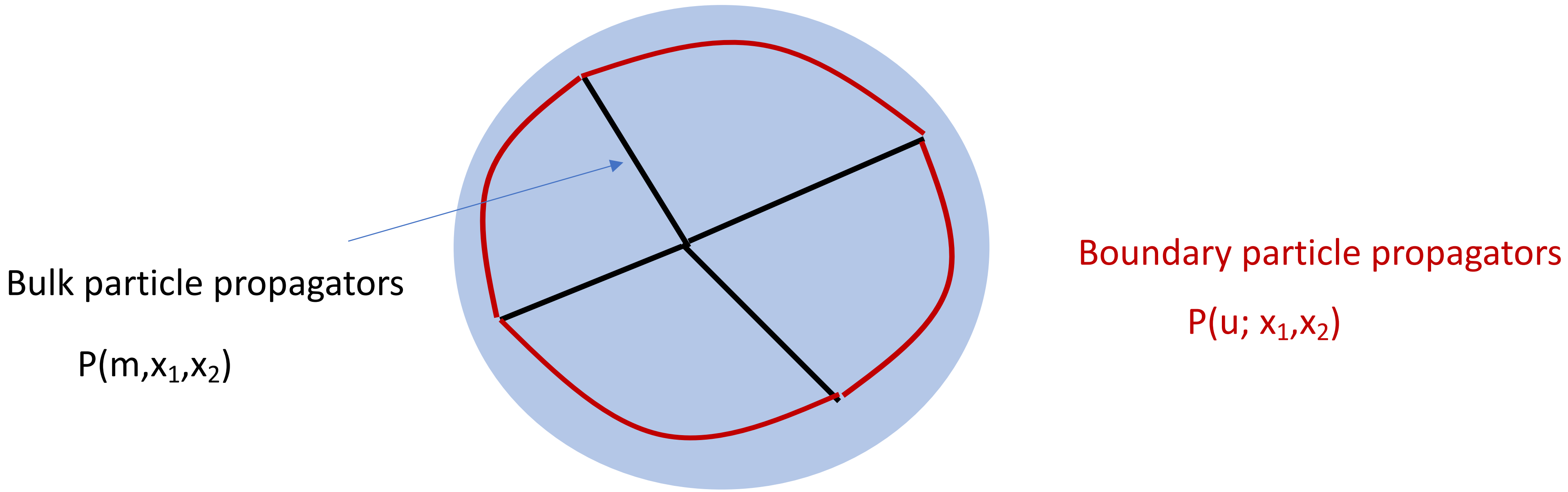


Emission of a bulk excitation
New position of the horizon
The boundary trajectory gets
a “kick” determined by local
energy momentum conservation.

Perturbative quantum gravity

- The motion of the boundary particle can be exactly quantized.

- This enables us to compute all perturbative quantum gravity corrections to various correlation functions.



Z. Yang
Kitaev Suh

Two point function

Bagrets, Altland, Kamenev;
Z. Yang
Kitaev Suh

$$\langle O(0)O(u)\rangle = \int dsds'\rho(s)\rho(s')\exp\left[-s^2u - s'^2(\beta-u)\right]|\Gamma(\Delta+is+is')\Gamma(\Delta+is-is')|^2$$

$$\rho(s) = s\sinh 2\pi s$$

$$\langle O(0)O(u)\rangle \sim Z(\beta)\frac{1}{u^{2\Delta}} \quad \text{for} \quad u \ll 1$$

$$\langle O(0)O(u)\rangle \sim \frac{1}{u^{3/2}(\beta-u)^{3/2}}\,, \qquad \text{for} \quad u,\ \beta \ll 1$$

We can also send
u → i t to go to Lorentzian time

Compare to

$$\langle O(0)O(u)\rangle \propto \int dEdE'\rho(E)\rho(E')e^{-uE-(\beta-u)E'}|\langle E|O|E'\rangle|^2$$

- This includes al perturbative gravity corrections.
- There is still the possibility of topology change, this involves a new parameter e^{-S_0} which was not appearing before.
- For the pure JT gravity theory it is possible to evaluate the terms in the genus expansion → same results as a certain random matrix model.

Saad, Shenker, Stanford

End of lecture 1