Infrared regularization of the Lorentzian IKKT matrix model and the emergence of expanding universe

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Workshop on "Nonperturbative and Numerical Approaches to Quantum Gravity, String Theory and Holography"
International Centre for Theoretical Science, Bangalore, India
August 22 - September 2, 2022

Ref.) J.N. and Asato Tsuchiya, JHEP 1906 (2019) 077 [arXiv:1904.05919 [hep-th]]
J.N., proceedings of CORFU21, e-Print: 2205.04726 [hep-th]
Anagnostopoulos-Azuma-Hatakeyama-Hirasawa-J.N.-Papadoudis-Tsuchiya, work in progress

IKKT matrix model

Ishibashi, Kawai, Kitazawa, Tsuchiya, 1996

a conjectured nonperturbative formulation of superstring theory

$$S_{\mathrm{b}} = -\frac{1}{4g^2}\mathrm{tr}([A_\mu, A_\nu][A^\mu, A^\nu])$$

$$S_{\mathrm{f}} = -\frac{1}{2g^2}\mathrm{tr}(\Psi_\alpha(\mathcal{C}\Gamma^\mu)_{\alpha\beta}[A_\mu, \Psi_\beta])$$

SO(9,1) symmetry

$N \times N$ Hermitian matrices

$A_\mu \quad (\mu = 0, \cdots, 9)$ Lorentz vector

$\Psi_\alpha \quad (\alpha = 1, \cdots, 16)$ Majorana-Weyl spinor

Lorentzian metric $\eta = \mathrm{diag}(-1, 1, \cdots, 1)$
is used to raise and lower indices.

Wick rotation $(A_0 = -iA_{10}\ , \quad \Gamma^0 = i\Gamma_{10})$

⇒ Euclidean matrix model SO(10) symmetry

Anagnostopoulos, et al. JHEP 06 (2020) 069 , arXiv: 2002.07410 [hep-th]

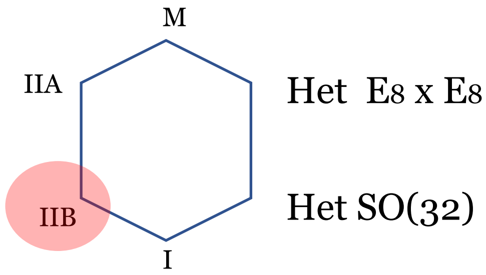
Crucial properties of the IKKT matrix model

as a nonperturbative formulation of superstring theory

- The connection to perturbative formulations can be seen manifestly by considering type IIB superstring theory in 10d.

  worldsheet action, light-cone string field Hamiltonian, etc.

- It is expected to be a nonperturbative formulation of <u>the unique theory</u> underlying the web of string dualities.



- The model has $10D\,\mathcal{N}=2$ SUSY, which cannot be realized in quantum field theories without gravity.
  The low energy effective theory should inevitably include quantum gravity !

  In the SUSY algebra, translation is realized as $A_\mu \mapsto A_\mu + \alpha_\mu \mathbf{1}$,

  which suggests that the space-time is represented as the eigenvalue distribution of A_μ .

Geometry emerges from matrix degrees of freedom dynamically in this approach .

Plan of the talk

0. Introduction
1. Brief review of the Euclidean IKKT model
2. Regularizing the Lorentzian IKKT model
3. How to investigate the model
4. Results of the complex Langevin simulations
5. Summary and discussions

1. Brief review of the Euclidean IKKT model

the Euclidean IKKT model

"Wick rotation" : $A_0 = -iA_{10}$

$$Z_{\mathrm{E}} = \int dA\, d\Psi\, e^{-(S_{\mathrm{b}}+S_{\mathrm{f}})} = \int dA\, e^{-S_{\mathrm{b}}}\, \mathrm{Pf}\mathcal{M}(A)$$

$S_{\mathrm{b}} \propto \mathrm{tr}\,(F_{\mu\nu})^2$ positive semi-definite!

$F_{\mu\nu} = -i\,[A_\mu, A_\nu]$: Hermitian

Euclidean model is well defined without any cutoff.

Krauth-Nicolai-Staudacher ('98), Austing-Wheater ('01)

$\mathrm{Pf}\mathcal{M}(A)$: complex valued

➡ Fluctuation of the phase becomes milder for lower dimensional configs.

A possible mechanism for SSB of SO(10) J.N.-Venizzi ('00)

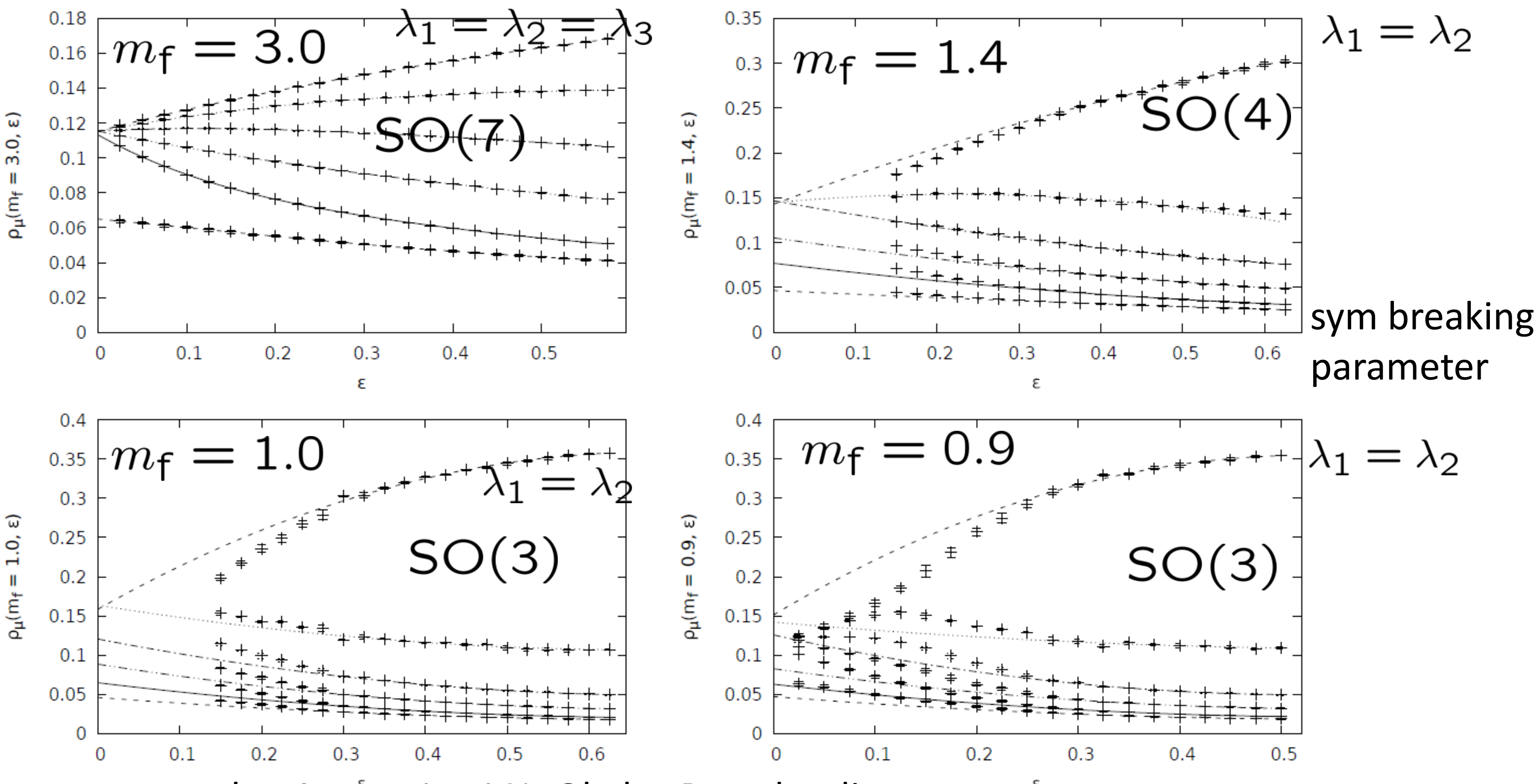
Difficult for Monte Carlo simulation due to the sign problem.

➡ complex Langevin method Parisi ('83), Klauder ('83)

Results for the Euclidean IKKT model $\mathrm{SO}(10) \overset{\text{SSB}}{\rightarrow} \mathrm{SO}(3)$

SSB of SO(10) observed by decreasing the deformation parameter m_{f}.

ten eigenvalues of $T_{\mu\nu} = \frac{1}{N}\mathrm{tr}\,(A_\mu A_\nu)$



Anagnostopoulos-Azuma-Ito-J.N.-Okubo-Papadoudis,
JHEP 06 (2020) 069 , arXiv: 2002.07410 [hep-th]

(Arpith Kumar's talk on 8/27)

2. Regularizing the Lorentzian IKKT model

Partition function of the Lorentzian IKKT model

partition function

$$Z_{\mathsf{L}} = \int dA\, d\Psi\, e^{i(S_{\mathsf{b}}+S_{\mathsf{f}})} = \int dA\, e^{iS_{\mathsf{b}}}\, \mathsf{Pf}\,\mathcal{M}(A)$$

This seems to be natural from the connection to the worldsheet theory.

c.f.)
$$S = \int d^2\xi \sqrt{g} \left(\frac{1}{4}\{X^\mu, X^\nu\}^2 + \frac{1}{2}\bar{\Psi}\gamma^\mu\{X^\mu, \Psi\} \right)$$

$$\xi_0 \equiv -i\xi_2$$

(The worldsheet coordinates should also be Wick-rotated.)

Regularizing the Lorentzian model

- Unlike the Euclidean model,
  the Lorentzian model is NOT well defined as it is.

$$Z_{\mathsf{L}} = \int dA\, d\Psi\, e^{i(S_{\mathsf{b}}+S_{\mathsf{f}})} = \int dA\, e^{iS_{\mathsf{b}}} \underbrace{\mathsf{Pf}\,\mathcal{M}(A)}$$

pure phase factor

polynomial in A

real valued unlike Euclidean

- Wick rotation

$$S_{\mathsf{b}} \mapsto \tilde{S}_{\mathsf{b}} = N e^{i\frac{\pi}{2}u} \left\{ \frac{1}{2} e^{-i\pi u} \mathsf{tr}\,[\tilde{A}_0, \tilde{A}_i]^2 - \frac{1}{4} \mathsf{tr}\,[\tilde{A}_i, \tilde{A}_j]^2 \right\}$$

on the worldsheet

in the target space

This corresponds to deforming the integration contour in the Lorentzian model.

$$\begin{cases} A_0 = \mathsf{e}^{i\frac{\pi}{8}u - i\frac{\pi}{2}u} \tilde{A}_0 = \mathsf{e}^{-i\frac{3\pi}{8}u} \tilde{A}_0 \\ A_i = \mathsf{e}^{i\frac{\pi}{8}u} \tilde{A}_i = \mathsf{e}^{i\frac{\pi}{8}u} \tilde{A}_i \end{cases}$$

$u = 0$: Lorentzian

$u = 1$: Euclidean

Path deformed theory is well-defined for $0 < u \leq 1$

(Yuhma Asano '19, private communication)

$$e^{iS_{\mathrm{b}}(A)} = e^{-S(\tilde{A})} \qquad \begin{cases} A_0 = \mathrm{e}^{-i\frac{3}{8}\pi u}\tilde{A}_0 \\ A_i = \mathrm{e}^{i\frac{1}{8}\pi u}\tilde{A}_i \end{cases} \qquad \tilde{F}_{\mu\nu} = -i[\tilde{A}_\mu, \tilde{A}_\nu]$$

$$S(\tilde{A}) \sim 2 e^{i\frac{\pi}{2}(1-u)} \mathrm{tr}\,(\tilde{F}_{0i})^2 + e^{-i\frac{\pi}{2}(1-u)} \mathrm{tr}\,(\tilde{F}_{ij})^2$$

positive real part for $0 < u \leq 1$

$$\boxed{\mathrm{Re}\, S(\tilde{A}) \geq 0}$$

$S(\tilde{A})$: real positive at $u = 1$ (Euclidean).

According to Cauchy's theorem,
$\langle \mathcal{O}(\mathrm{e}^{-i\frac{3}{8}\pi u}\tilde{A}_0, \mathrm{e}^{i\frac{1}{8}\pi u}\tilde{A}_i)\rangle_u$ is independent of u.

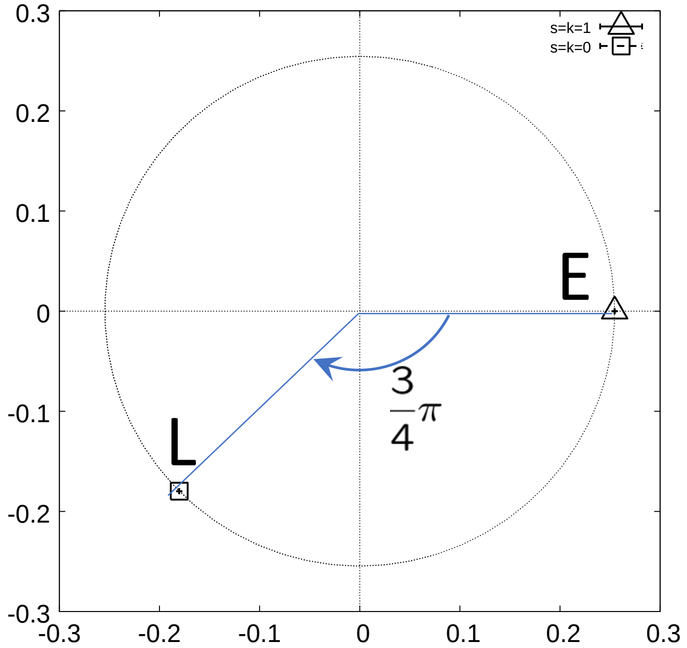
⇨ If we define the Lorentzian model by taking the $u \to +0$ limit,

$$\langle \mathcal{O}(A_0, A_i)\rangle_{\mathrm{L}} = \langle \mathcal{O}(\mathrm{e}^{-i\frac{3\pi}{8}}\tilde{A}_0, \mathrm{e}^{i\frac{\pi}{8}}\tilde{A}_i)\rangle_{\mathrm{E}}$$

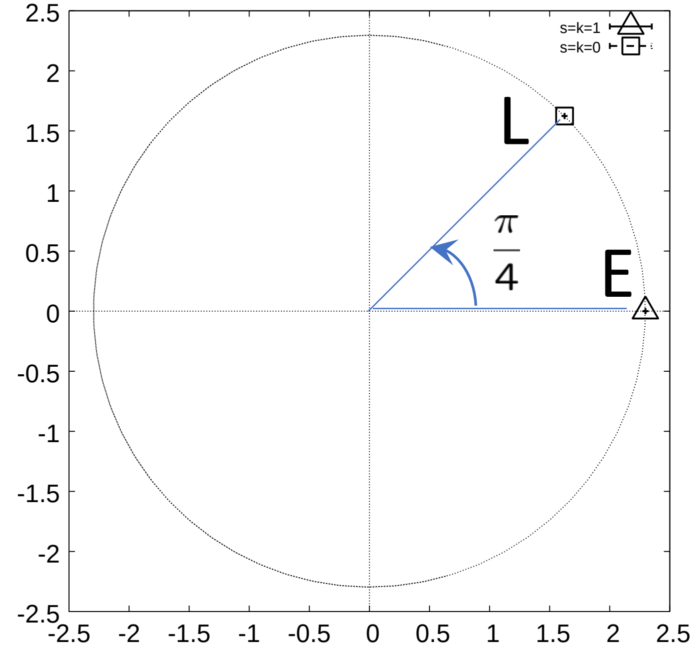
Confirmation of the equivalence by CL simulation

10D bosonic model

$$\left\langle \frac{1}{N}\mathrm{tr}(A_0)^2 \right\rangle_{\mathrm{L}} = e^{-\frac{3\pi}{4}i} \left\langle \frac{1}{N}\mathrm{tr}(\tilde{A}_0)^2 \right\rangle_{\mathrm{E}}$$



$$\left\langle \frac{1}{N}\mathrm{tr}(A_i)^2 \right\rangle_{\mathrm{L}} = e^{\frac{\pi}{4}i} \left\langle \frac{1}{N}\mathrm{tr}(\tilde{A}_i)^2 \right\rangle_{\mathrm{E}}$$



$$\begin{cases} A_0 = \mathrm{e}^{-i\frac{3}{8}\pi u}\tilde{A}_0 \\ A_i = \mathrm{e}^{i\frac{1}{8}\pi u}\tilde{A}_i \end{cases}$$

$u = 0 \mapsto u = 1$

Lorentzian Euclidean

The emergent space-time is complex and has Euclidean signature!

➡ Can we regularize the Lorentzian IKKT model in a different manner?

Introducing a Lorentz invariant mass term

$$Z = \int dA \, e^{i(S_{\mathrm{b}}+S_{\mathrm{m}})} \mathrm{Pf}\mathcal{M}(A)$$

Anagnostopoulos-Azuma-Hatakeyama-Hirasawa-Ito-J.N.-Papadoudis-Tsuchiya, work in progress

$$\boxed{S_{\mathrm{m}} \; = \; \frac{1}{2} N\gamma \left\{ \mathrm{tr}(A_0)^2 - \mathrm{tr}(A_i)^2 \right\}} \qquad \gamma > 0$$

$$e^{i(S_{\mathrm{b}}(A)+S_{\mathrm{m}}(A))} = e^{-S(\tilde{A})} \qquad \begin{cases} A_0 = \mathrm{e}^{-i\frac{3}{8}\pi u}\tilde{A}_0 \\ A_i = \mathrm{e}^{i\frac{1}{8}\pi u}\tilde{A}_i \end{cases}$$

$$\tilde{F}_{\mu\nu} = -i[\tilde{A}_\mu, \tilde{A}_\nu]$$

positive real part for $0 < u \le 1$

$$S(\tilde{A}) \;\; \sim \;\; 2e^{i\frac{\pi}{2}(1-u)} \mathrm{tr}\,(\tilde{F}_{0i})^2 + e^{-i\frac{\pi}{2}(1-u)} \mathrm{tr}\,(\tilde{F}_{ij})^2$$
$$+\gamma e^{-i\frac{\pi}{2}(1+\frac{3}{2}u)} \mathrm{tr}\,(\tilde{A}_0)^2 + \gamma e^{i\frac{\pi}{2}(1+\frac{1}{2}u)} \mathrm{tr}\,(\tilde{A}_i)^2$$

negative real part for $0 < u \le 1$

One cannot define the model by contour deformation any longer !

The model can be defined based on Picard-Lefschetz theory. (cf. Fresnel integral)

Equivalence to the Euclidean model can be violated.

$$\int_{-\infty}^{\infty} dx \, e^{ix^2} = \sqrt{\frac{\pi}{2}}(1+i)$$

Classical solutions

$$\begin{aligned} Z &= \int dA\, e^{i(A^4+\gamma A^2)} \\ &= \int dA\, e^{i\gamma^2(\tilde{A}^4+\tilde{A}^2)} \end{aligned}$$

$$A_\mu = \sqrt{\gamma}\tilde{A}_\mu$$

$$\gamma^2 \Leftrightarrow \frac{1}{\hbar}$$

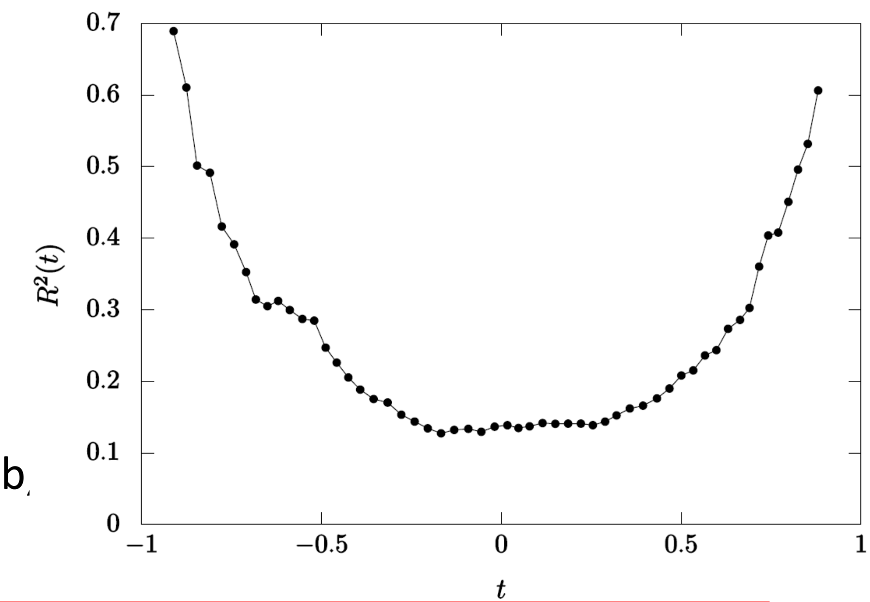
Classical solutions dominate at large γ.

$$[A^\nu, [A_\nu, A_\mu]] - \gamma A_\mu = 0$$

typical Hermitian A_μ solutions show expanding behavior for $\gamma > 0$

But not for $\gamma < 0$!

Hatakeyama-Matsumoto-J.N.-Tsuchiya-Yosprakob,
PTEP 2020 (2020) 4, 043B10



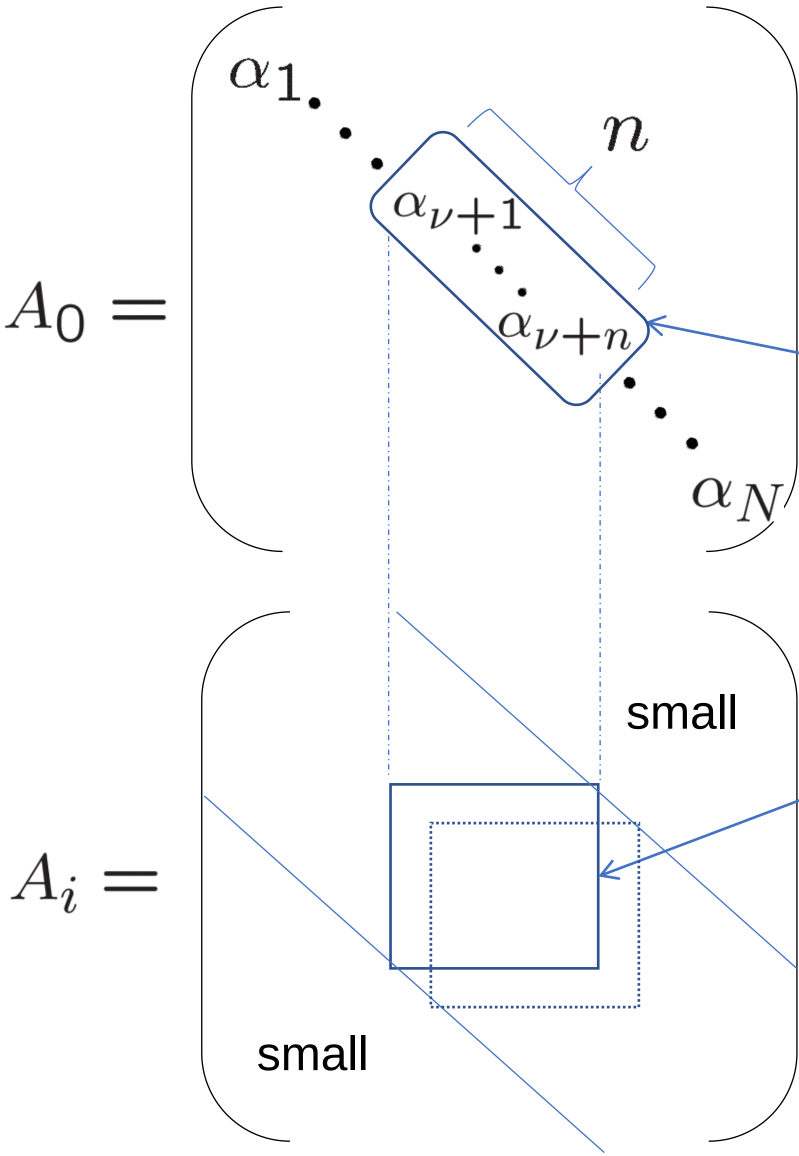
Space-time dimensionality is not fixed at the classical level.

3. How to investigate the model

Ref.) Kim-J.N.-Tsuchiya PRL 108 (2012) 011601 [arXiv:1108.1540]
J.N. and Asato Tsuchiya, JHEP 1906 (2019) 077 [arXiv:1904.05919 [hep-th]]

Extracting time-evolution from the Lorentzian model

Kim-J.N.-Tsuchiya PRL 108 (2012) 011601 [arXiv:1108.1540]



$$\alpha_1 < \cdots < \alpha_N$$

definition of "time"

$$t_\nu = \frac{1}{n} \sum_{i=1}^{n} \alpha_{\nu+i}$$

The state of the universe $\bar{A}_i(t_\nu)$ at time t_ν

A_i has a band diagonal structure

non-trivial dynamical property

cf.) Klinkhamer, arXiv: 2105.05831 [hep-th]

Defining "time" of the IKKT model in complex Langevin simulation

J.N. and Asato Tsuchiya, JHEP 1906 (2019) 077
[arXiv:1904.05919 [hep-th]]

Fixing the U(N) symmetry: $A_\mu \mapsto U A_\mu U^\dagger$

$$Z = \int dA_0 \, dA_i \, e^{-S} = \int d\alpha \, dA_i \, \Delta(\alpha) \, e^{-S}$$

$$A_0 = \mathrm{diag}(\alpha_1, \cdots, \alpha_N)$$

$$\alpha_1 < \alpha_2 < \cdots < \alpha_N$$

$$\Delta(\alpha) = \prod_{a>b} (\alpha_a - \alpha_b)^2$$: van der Monde determinant

We make the change of variables

$$\alpha_1 = 0 \ , \quad \alpha_2 = e^{\tau_1} \ , \quad \alpha_3 = e^{\tau_1} + e^{\tau_2} \ , \quad \cdots \ , \quad \alpha_N = \sum_{a=1}^{N-1} e^{\tau_a} \ ,$$

to introduce the "time ordering" respecting holomorphicity.

Complex Langevin equation

J.N. and Asato Tsuchiya, JHEP 1906 (2019) 077
[arXiv:1904.05919 [hep-th]]

The effective action

$$S_{\text{eff}} = -i\,N\left\{\frac{1}{2}\text{tr}\,[A_0, A_i]^2 - \frac{1}{4}\text{tr}\,[A_i, A_j]^2\right\} - \log\Delta(\alpha) - \sum_{a=1}^{N-1}\tau_a$$

Complex Langevin equation

$$\begin{cases} \dfrac{d\tau_a}{dt} = -\dfrac{\partial S_{\text{eff}}}{\partial \tau_a} + \eta_a \\ \dfrac{d(\mathcal{A}_i)_{ab}}{dt} = -\dfrac{\partial S_{\text{eff}}}{\partial (\mathcal{A}_i)_{ba}} + (\eta_i)_{ab} \end{cases}$$

τ_a : complex variables, $\mathcal{A}_i$: general complex matrices.

Some tricks to make the CLM work

Anagnostopoulos-Azuma-Hatakeyama-Hirasawa-J.N.-Papadoudis-Tsuchiya, work in progress

- Complex Langevin method fails
  when the Dirac operator has near-zero modes.

  To avoid this, we add a mass term to the fermionic action.

$$S_{\mathrm{f}} = \frac{1}{2}\mathrm{tr}\left(\bar{\Psi}_\alpha(\Gamma^\mu)_{\alpha\beta}[A_\mu, \Psi_\beta] + m_{\mathrm{f}}\bar{\Psi}_\alpha(\Gamma_7\Gamma_8^\dagger\Gamma_9)_{\alpha\beta}\Psi_\beta\right)$$

$$m_{\mathrm{f}} = \infty \quad : \quad \text{bosonic}$$
$$m_{\mathrm{f}} = 0 \quad : \quad \text{SUSY}$$

m_{f} should be as small as possible.

- In order to stabilize the complex Langevin simulation,
  we introduce:

c.f.) Attanasio-Jäger ('18)

$$A_i \mapsto \frac{A_i + \eta A_i^\dagger}{1+\eta}$$

after each Langevin step.

$$\eta = 0.01$$

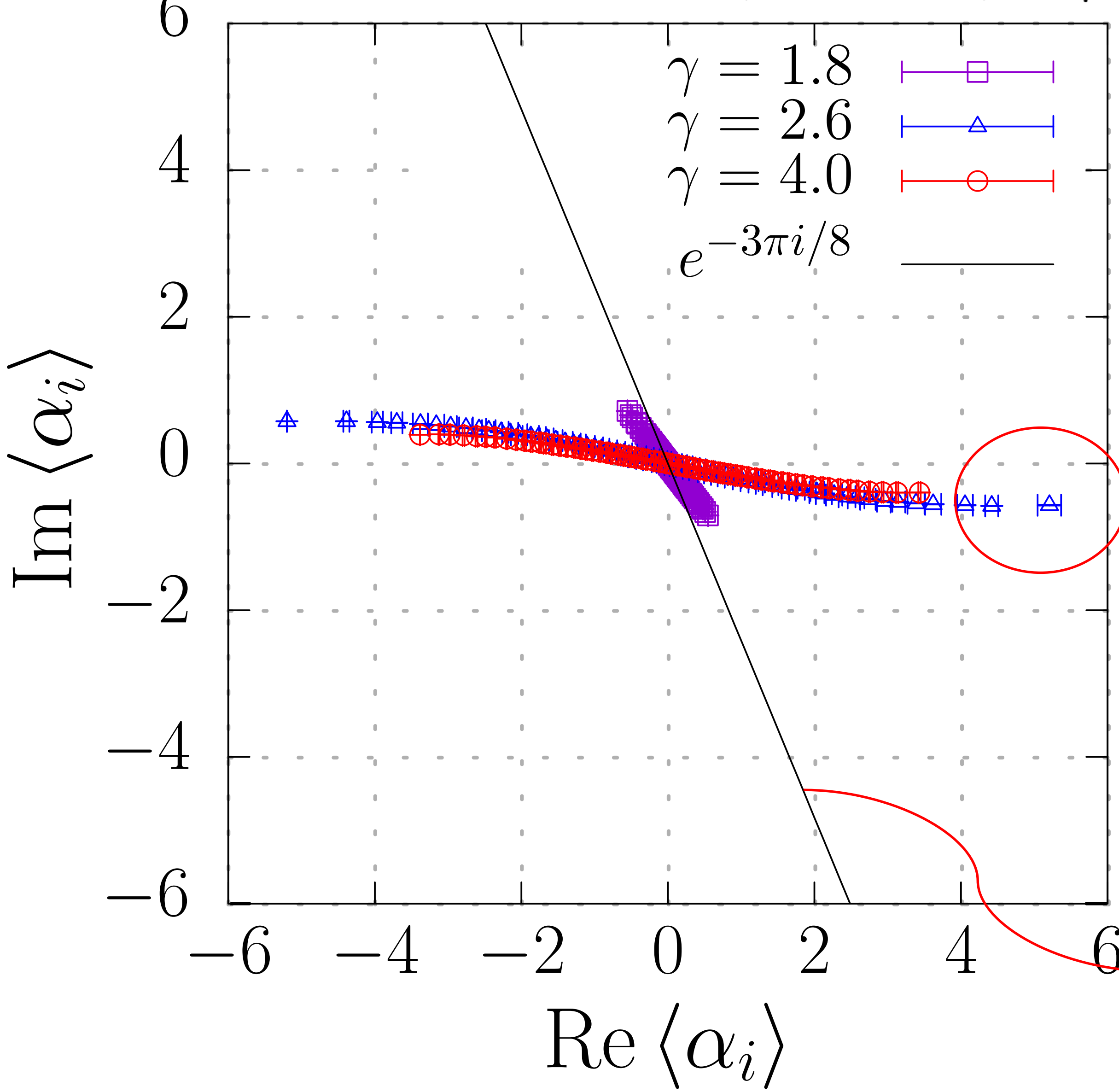
Justifiable when dominant configs.
are close to Hermitian.

4. Results of the CL simulations

Ref.) J.N., proceedings of CORFU21, e-Print: 2205.04726 [hep-th]
Anagnostopoulos-Azuma-Hatakeyama-Hirasawa-J.N.-Papadoudis-Tsuchiya, work in progress

Eigenvalues of A_0

$N = 64,\ n = 12,\ m_{\mathrm{f}} = 10,\ \eta = 0.01$



(preliminary results)

$\Delta\alpha_i \in \mathbb{R}$

(Lorentzian)

Real time emerges at late times for

$\gamma \gtrsim 2$

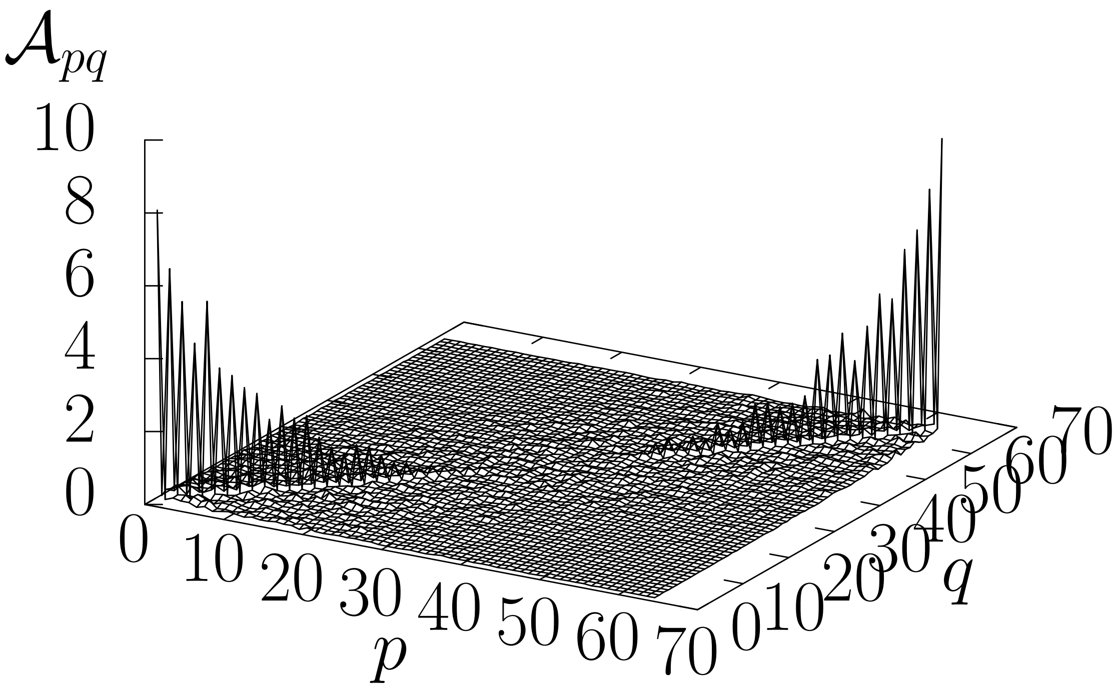
1st order phase transition(?)

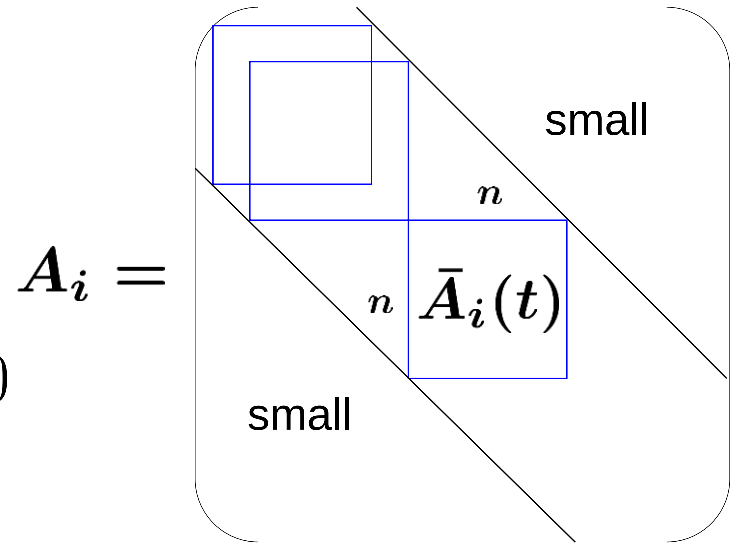
$\Delta\alpha_i \propto e^{-i\frac{3\pi}{8}}$

(Euclidean)

Band-diagonal strucure

$$\mathcal{A}_{pq} = \frac{1}{9}\sum_{i=1}^{9} |(A_i)_{pq}|^2 \qquad N = 64,\ \gamma = 4,\ m_{\mathrm{f}} = 10,\ \eta = 0.01$$





$n = 12$

Band-diagonal structure appears <u>when expansion occurs.</u>

➡ Important for extracting the time evolution from matrix configs.

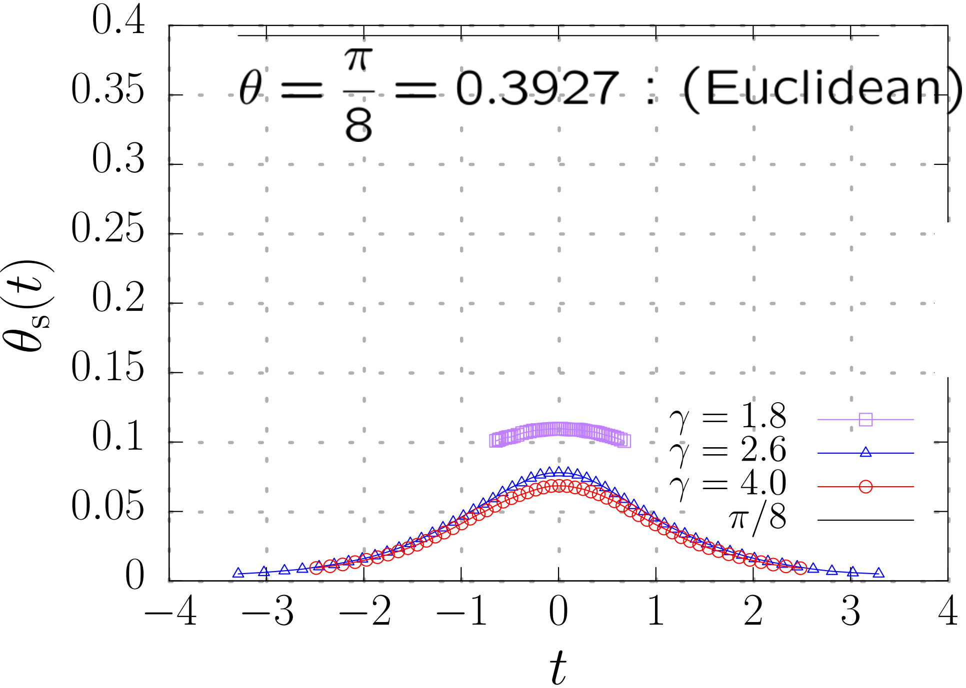
the time evolution of space

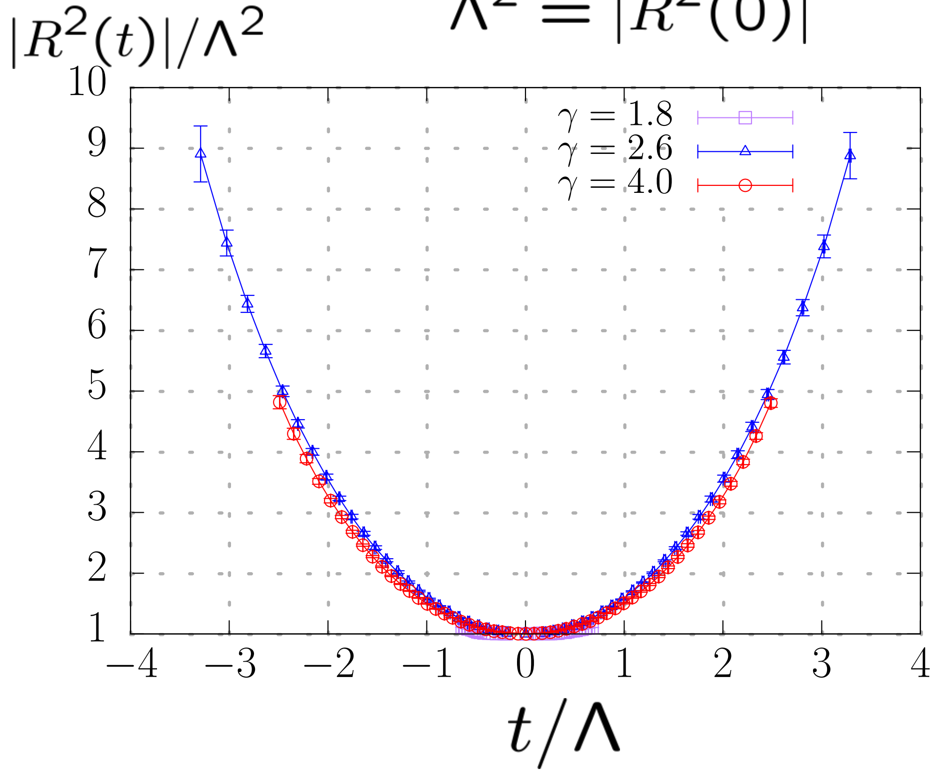
$N = 64,\ n = 12,\ m_{\mathrm{f}} = 10,\ \eta = 0.01$ (preliminary results)

$$R^2(t) = \left\langle \frac{1}{n}\mathrm{tr}\left(\bar{A}_i(t)\right)^2 \right\rangle = e^{2i\theta_{\mathrm{s}}(t)}|R^2(t)|$$

$$t_\rho = \sum_{\nu=1}^{\rho} |\bar{\alpha}_{\nu+1} - \bar{\alpha}_\nu|$$

$$\Lambda^2 = |R^2(0)|$$





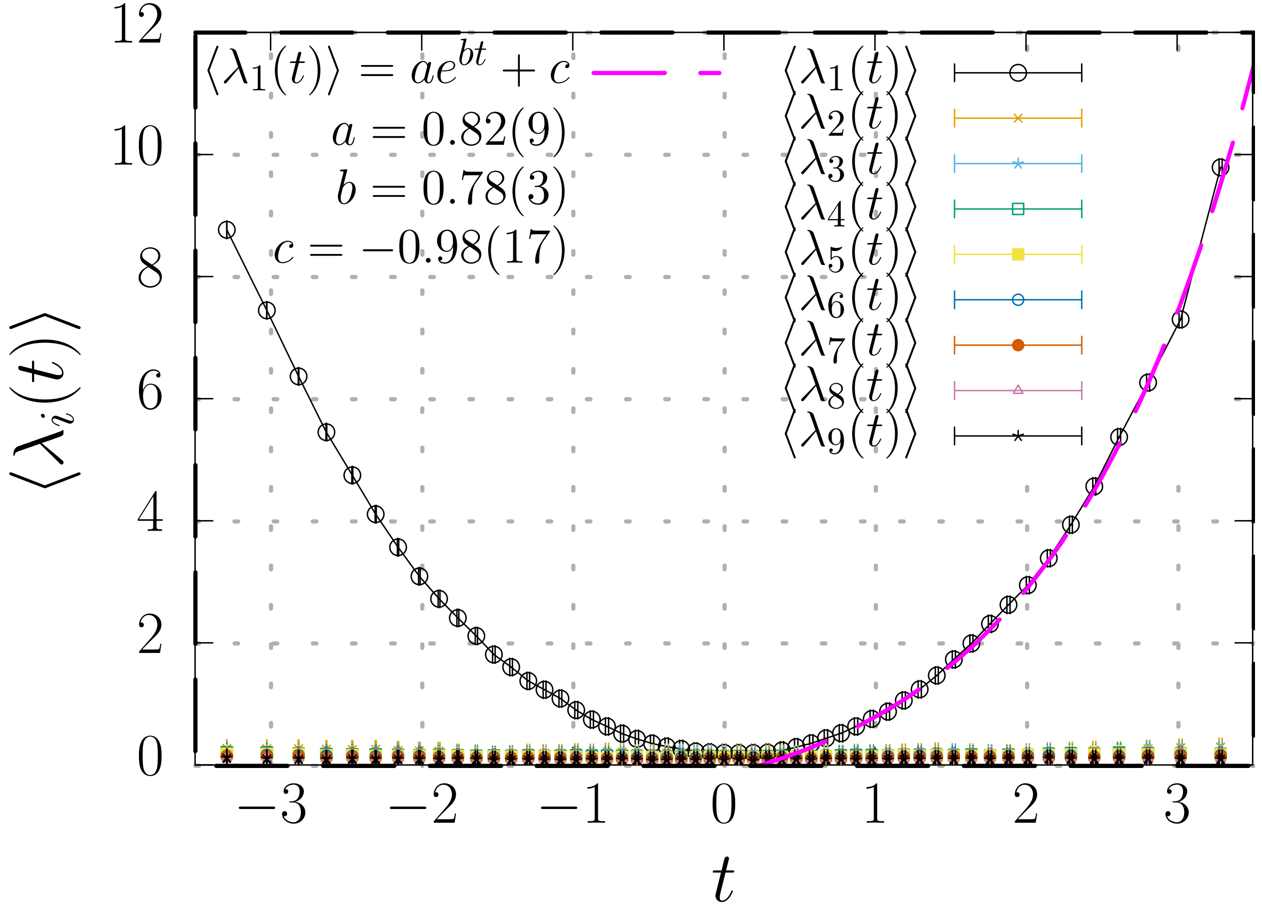
Emergence of real space at late times for $\gamma \gtrsim 2$

Expanding behavior for $\gamma \gtrsim 2$ continues longer for smaller γ

$\gamma \Leftrightarrow$ IR regulator

SSB of SO(9) rotational symmetry

$N = 64,\ n = 12,\ m_{\mathsf{f}} = 10,\ \eta = 0.01$



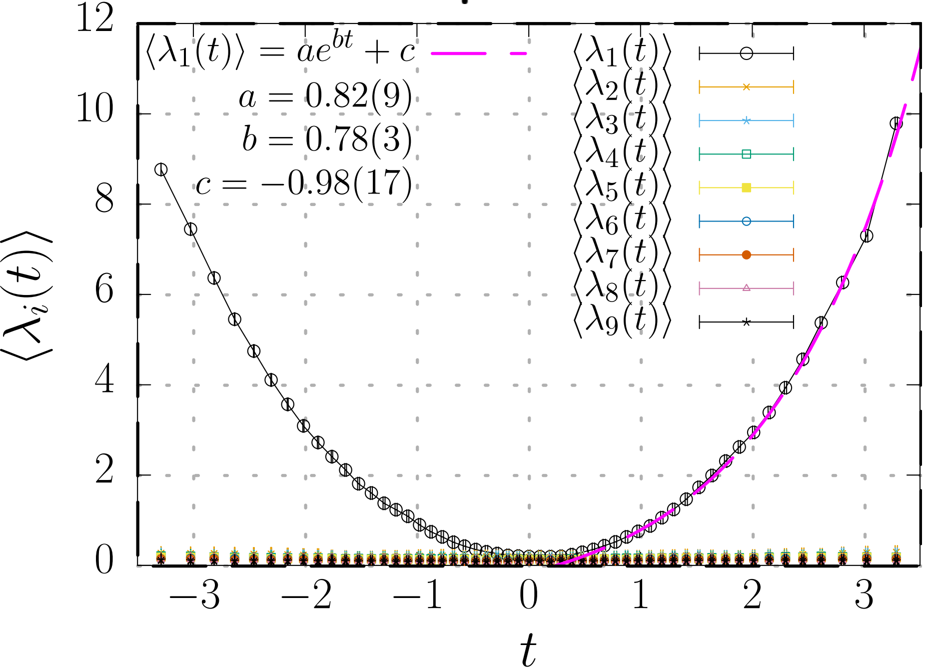
$\gamma = 2.6$

Only one direction expands. (SSB of rotational symmetry)
The expansion is fitted well by an exponential function.

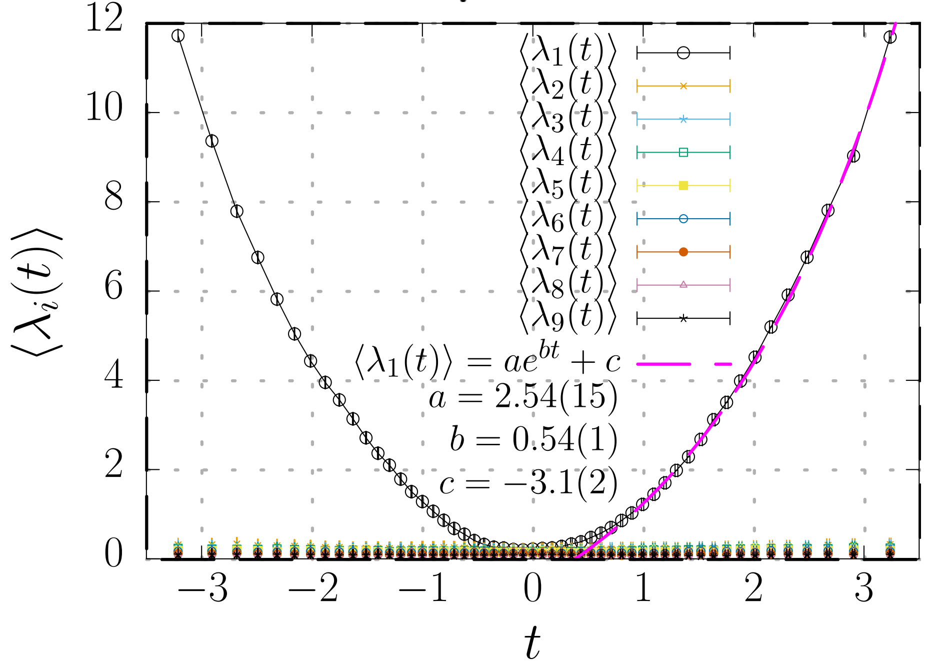
Effects of femionic matrices

$N = 64,\ n = 12,\ m_{\mathrm{f}} = 10,\ \eta = 0.01$
$\gamma = 2.6$

$m_{\mathrm{f}} = 10$



$m_{\mathrm{f}} = 5$



Expansion becomes more pronounced,
but the 1D structure remains for $m_{\mathrm{f}} = 5$.

5. Summary and Discussions

Summary

- IKKT matrix model = a nonperturbative formulation of superstring theory

- The Euclidean model exhibits SSB of SO(10) to SO(3) due to the phase of the fermion determinant (or Pfaffian).

- In fact, the Lorentzian model becomes equivalent to Euclidean model if we define it by deformation of the integration contour. Space-time becomes complex and has Euclidean signature.

- We introduce a Lorentz invariant mass term, which invalidates the contour deformation. It plays the role of IR regulator for an expanding space-time.

- Complex Langevin simulations showed the emergence of a real space-time with expanding behavior (as suggested by classical solutions.)

- 1D space (SSB of rotational symmetry) for $m_{\mathrm{f}} \gtrsim 5$

Does 3D space appear at smaller m_{f} ?

$m_{\mathrm{f}} = 0 \Leftrightarrow \mathrm{SUSY}$

Discussions

- The mechanism of SSB

  - Different from that for the Euclidean IKKT model (phase of the Pfaffian).
  - SSB in the Lorentzian IKKT model occurs even without fermionic matrices.

Classical solutions with expanding behavior appears in the presence of the mass term. But the dimensionality of the expanding space is not fixed at the classical level.

$$S_{\mathrm{b}} \sim \frac{1}{2}\mathrm{tr}\,[A_0, A_i]^2 - \cancel{\frac{1}{4}\mathrm{tr}\,[A_i, A_j]^2}$$

for 1D space

Quantum fluctuation of the action is suppressed most for 1D space.

- Important property of the Pfaffian (for $m_{\mathrm{f}} = 0$)

$\mathrm{Pf}\mathcal{M}(A_0, A_1, \cdots, A_9) = 0$ if there are only two nonzero A_μ.

Krauth-Nicolai-Staudacher ('98)

(This is the reason why SO(9)→SO(3) in the Euclidean IKKT model.)

J.N.-Venizzi ('00)

Configurations with at least 3D expanding space are enhanced by this effect for sufficiently small m_{f}.

Backup slides

The complex Langevin method

Parisi ('83), Klauder ('83)

$$Z = \int dx\, w(x) \qquad x \in \mathbb{R}$$

complex

MC methods inapplicable due to sign problem !

Complexify the dynamical variables, and consider their (fictitious) time evolution :

$$z^{(\eta)}(t) = x^{(\eta)}(t) + i\, y^{(\eta)}(t)$$

defined by the complex Langevin equation

$$\frac{d}{dt} z^{(\eta)}(t) = \boxed{v(z^{(\eta)}(t))} + \eta(t)$$

Gaussian noise (real)

probability $\propto \mathrm{e}^{-\frac{1}{4}\int dt\, \eta(t)^2}$

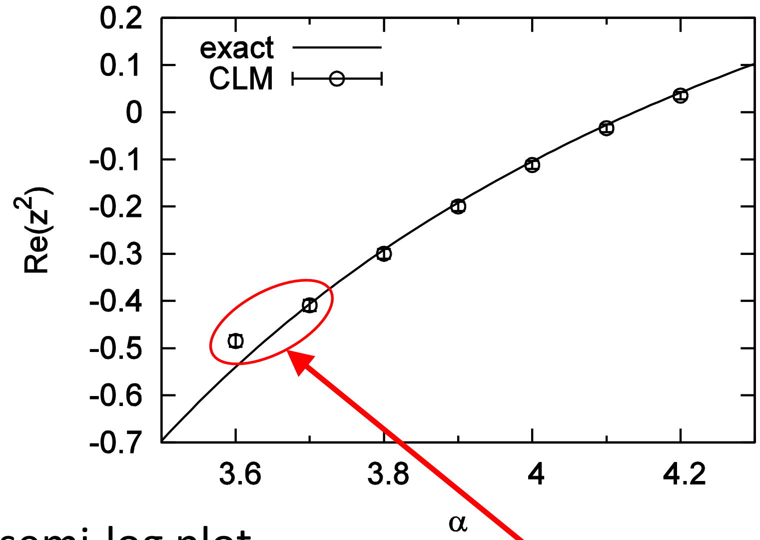
$$\langle \mathcal{O} \rangle \stackrel{?}{=} \lim_{t\to\infty} \langle \boxed{\mathcal{O}(z^{(\eta)}(t))} \rangle_\eta$$

$$v(x) \equiv \frac{1}{w(x)} \frac{\partial w(x)}{\partial x}$$

Rem 1 : When w(x) is real positive, it reduces to one of the usual MC methods.

Rem 2 : The drift term $v(x) \equiv \frac{1}{w(x)} \frac{\partial w(x)}{\partial x}$ and the observables $\mathcal{O}(x)$ should be evaluated for complexified variables by analytic continuation.

Recent development : the condition for correct convergence

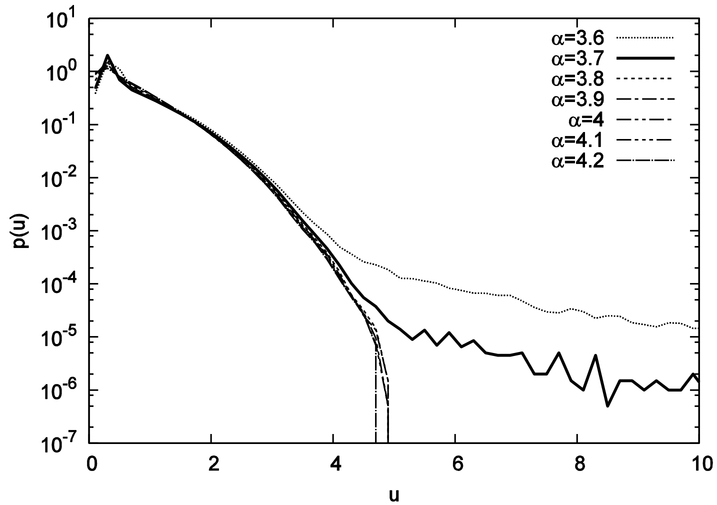


$$Z = \int dx\, w(x)$$
$$w(x) = (x + i\alpha)^p \, \mathrm{e}^{-x^2/2}$$

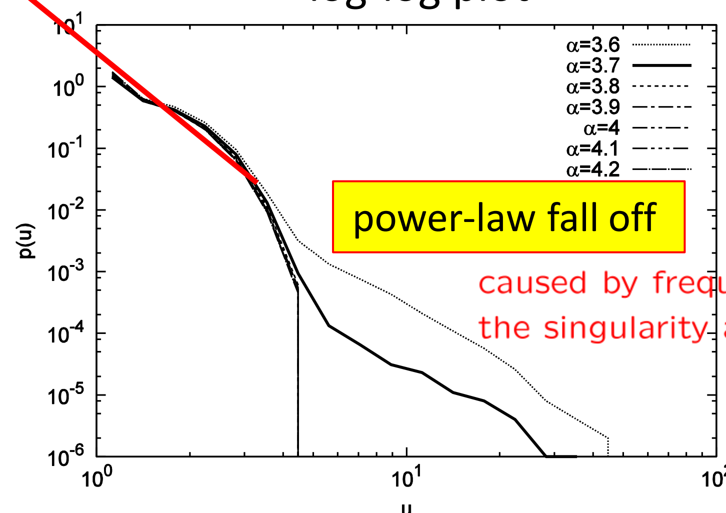
$$p = 4$$

In this model, CLM fails at $\alpha \lesssim 3.7$.

semi-log plot



log-log plot



power-law fall off

caused by frequent visits to the singularity at $z = -i\alpha$

The probability distribution of the magnitude of the drift term $u \equiv |v(z)| = \left| \frac{p}{z + i\alpha} - z \right|$ should be suppressed exponentially in order for the method to be justified.

Nagata-J.N.-Shimasaki, Phys.Rev. D94 (2016) no.11, 114515.