(Anti)monomer walks in dimer models

Kedar Damle (TIFR Mumbai) December 16 2024
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Anoop Raj, Sabyasachi Chaudhury, KD; in preparation

Sabyasachi Chaudhuri, Jay Pandey, KD; in preparation

Useful background:

Rakala, Dhar, KD, PRE 103 062101 (2021)

Rakala, KD, PRE 96 023304 (2017)

Dimer model primer

Fully-packed dimers on <<regular>> bipartite graphs expected to be “critical”

Basic idea:
Divergence-free polarization field on links of lattice
Effective action in terms of “gauge field” that solves the polarization constraint

$$
\begin{aligned}
P_{AB} &= (n_{AB} - 1/z)\hat{e}_{AB} \\
\Delta \cdot P &= 0 \\
P &= \Delta \times a \\
S_{\text{eff}} &= \frac{K}{2} \int d^d x \, (\Delta \times a)^2
\end{aligned}
$$

(see e.g. Henley Ann. Rev. Cond. Mat. 2010)

General consequence:

Dipolar dimer correlations:

$$C_{\mathrm{dimer}}(r) \quad \sim \quad 1/r^{d}$$

Monomers behave as ± 1 charges (sources of divergence of P) on A/B sublattice

Test monomer-antimonomer pair on same sublattice interacts via Coulomb law potential:
(Identical to test monomer-monomer pair on opposite sublattice)

$$V(r) \quad \sim \quad 1/r^{d-2}$$

$$C_{\text{test monomers}} \sim \exp(-V(r))$$

Specifically in d=2:

Gaussian action for scalar height field

$$P_\mu = \epsilon_{\mu\nu}\partial_\nu h$$

$$S_{\text{eff}} = \pi g \int d^2x \, (\Delta h)^2$$

$$V(r) = g \log(r)$$

$$C_{\text{test monomer}} \sim 1/r^g$$

For non-interacting dimers on square and honeycomb lattices: $g{=}1/2$

Non-bipartite dimer models are very different

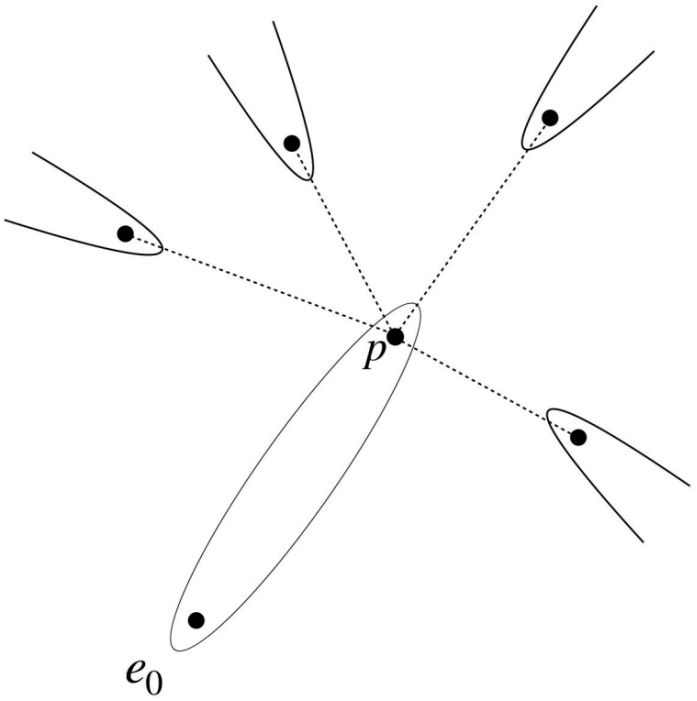
No description in terms of divergence-free polarization field

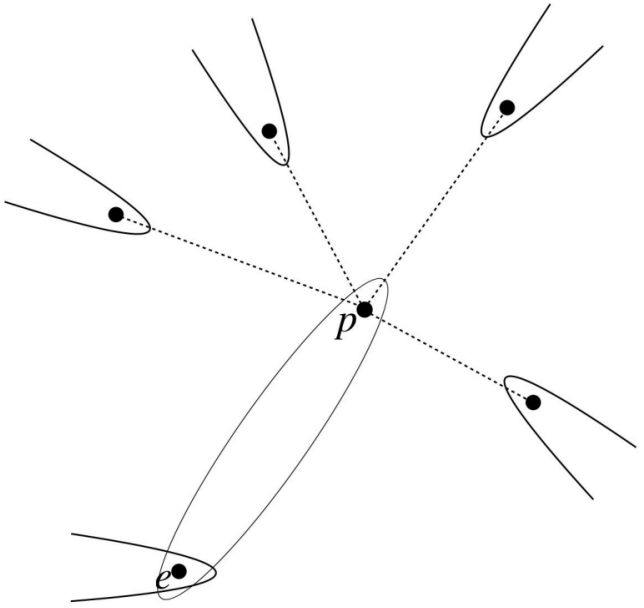
Short-range correlations between dimers

Test monomer/antimonomer pair has no long-range interaction.

Prototypical examples in 2d: Fully-packed dimers on triangular and kagome lattices

The (anti)monomer walk: Worm algorithm for fully-packed dimer models





What is the return time distribution of this monomer walk?

But first: Why is this at all interesting?

One reason:

Worm head motion affects background dimer configuration.

Each step obeys detailed balance for coupled worm-dimer system

Monte-Carlo move ends when worm head returns to “origin” (position of fixed tail)

Heuristically: Worm head moves in “Coulomb potential” of worm tail

How far is this picture valid?

Another reason:

This picture leaves out the other interesting aspect of the monomer walk:

Process is explicitly Markovian for coupled system.

But: If dimers “integrated out” of description, worm head’s motion is correlated in time

(due to correlations of the underlying dimer configuration)

(Yet) another reason:

Another interesting feature:

Process is always respects detailed balance in larger configuration space.

But: Multiple prescriptions for maintaining detailed balance

Is there universality across prescriptions (and across bipartite lattices?)

Our focus and some background:

Our focus is d=2 case.

Some background:

$$V(r) = 0 \qquad P(\tau_r) \sim \frac{1}{\tau_r \log^2(\tau_r/\tau_0)}$$

for usual random walker in d=2

$$V(r) = \eta_m \log(r/r_0) \qquad P(\tau_r) \sim \frac{1}{\tau_r^{1+\theta}}$$

$$\theta = \eta_m/2$$

(see e.g. Bray Phys. Rev. E 2000)

More background:

For simulations in a L by L box

for usual random walker in d=2

Expect/Postulate natural finite-size scaling form:

$$V(r) = 0 \qquad P(\tau_r) \log^2(\tau_r/\tau_0) = \frac{1}{cL^z} f_0(d\frac{\tau_r}{L^z})$$

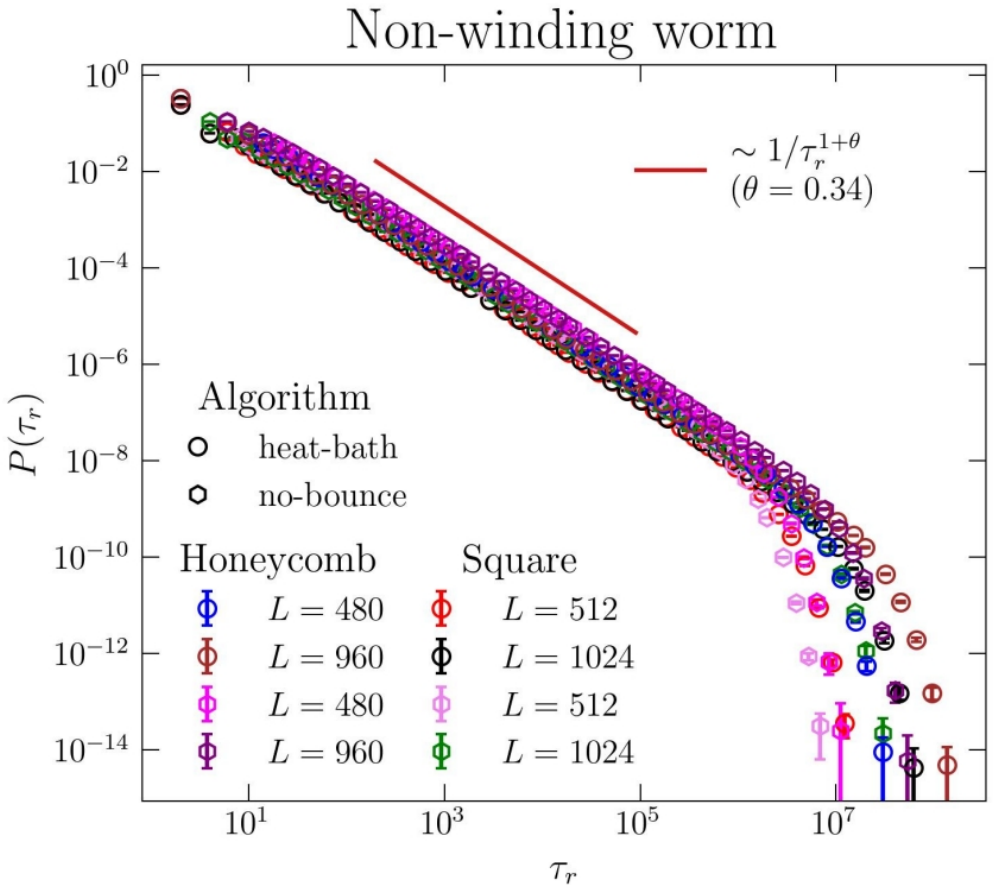
$$z = 2$$

$$V(r) = \eta_m \log(r/r_0) \qquad P(\tau_r) = \frac{1}{cL^{z(1+\theta)}} f_\theta(d\frac{\tau_r}{L^z})$$

$$\lim_{x \to 0} f(x) \sim 1/x^{1+\theta}$$

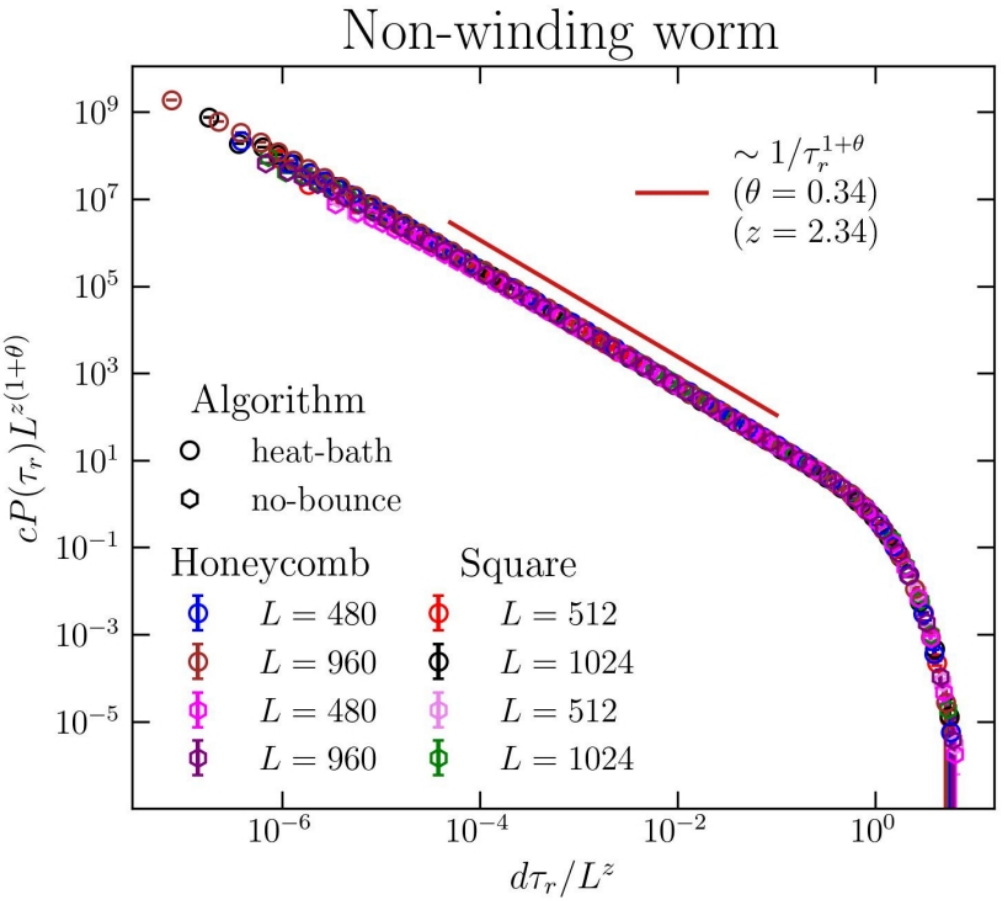
(any literature on this?)

Distribution of non-winding worms



$$\theta = 0.34(2)$$

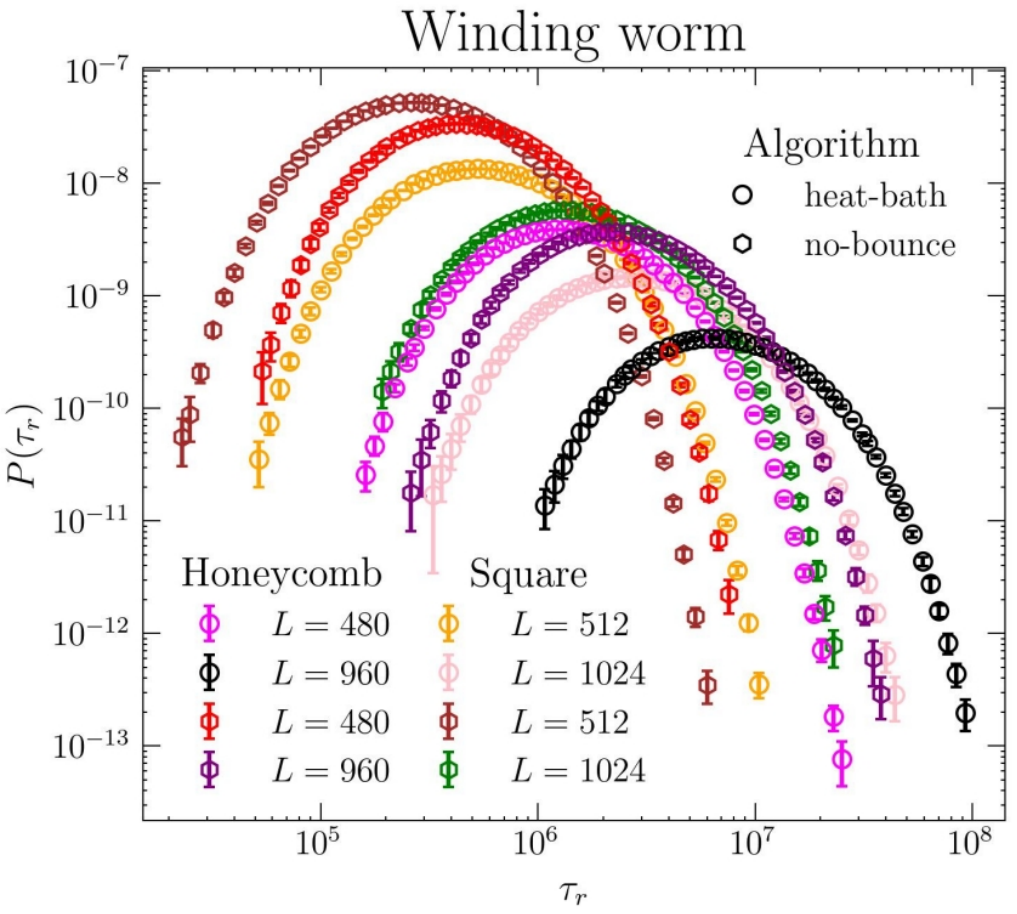
Scaling form and universality of dynamics validated



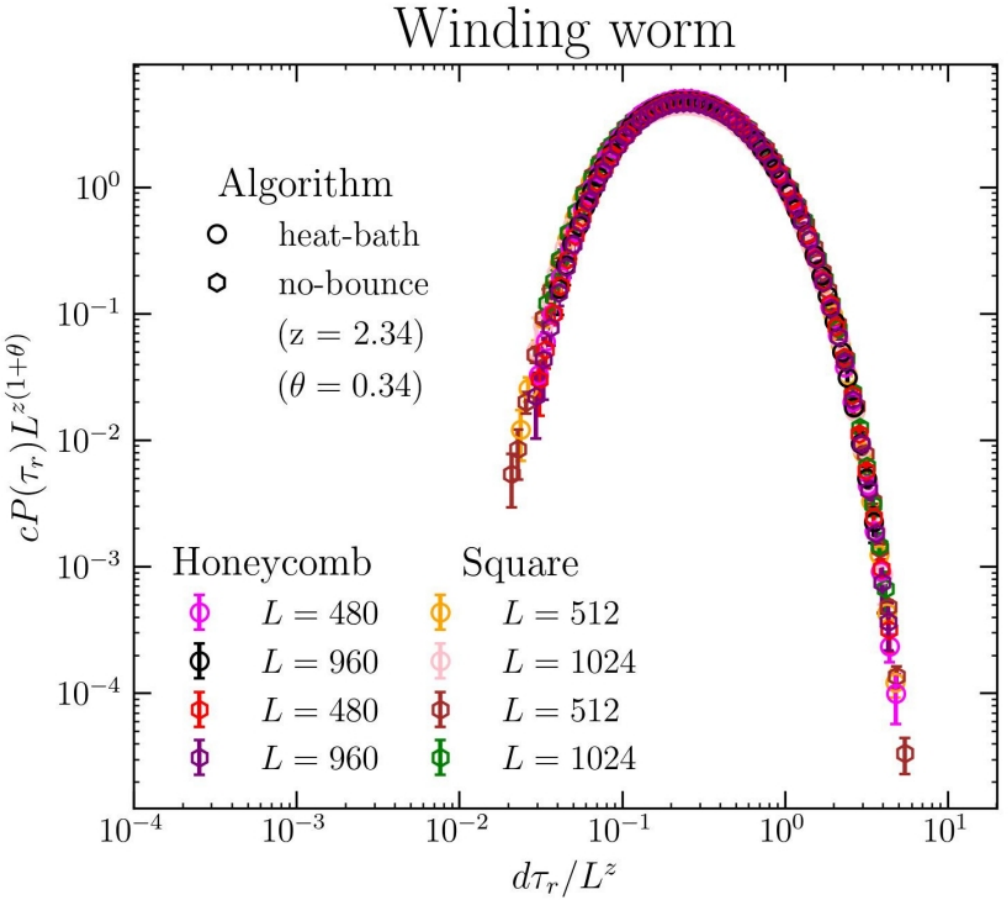
$$\theta = 0.34(3)$$

$$z = 2.34(2)$$

Distribution of winding worms



Scaling form and universality again validated



$$\theta = 0.34(3)$$

$$z = 2.34(2)$$

Fractional Brownian motion?

Value of z to be interpreted as fractional brownian motion (due to history dependence)?

note: similar earlier results for much more complicated
worm algorithms for frustrated Ising models
(Rakala, KD, Dhar PRE 2021)

Heuristic idea

Power-law history dependence controlled by dimer-dimer correlations

Persistence properties affected by “potential” between head and tail of worm

z controlled by dimer correlation exponent

θ controlled by test monomer correlation exponent

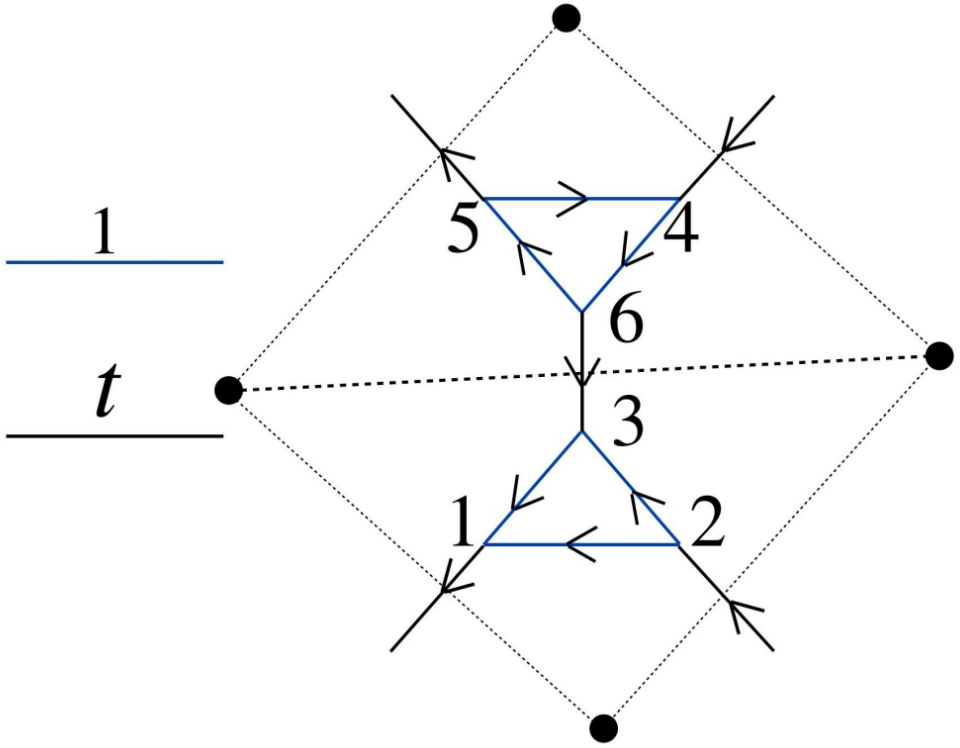
Tests with interacting dimer models?

Dimer-dimer interactions cause g to increase

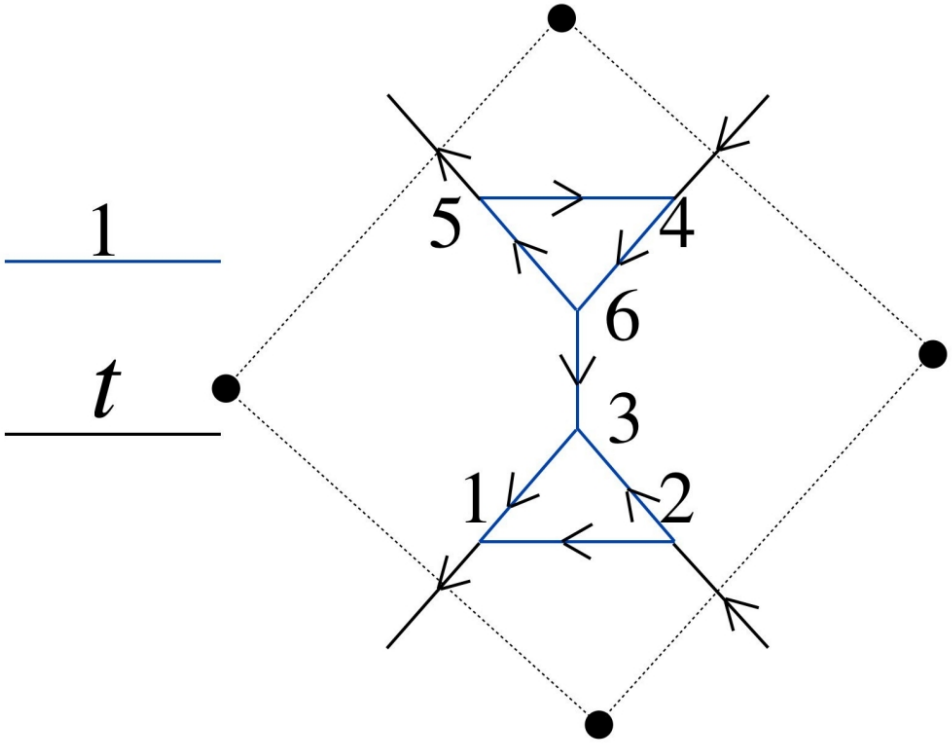
But no independent control over dimer and monomer exponents:

$$\eta_d = 1/g \qquad \eta_m = g$$

Different setting: The star (decorated honeycomb) lattice dimer model



Exactly dual to triangular lattice Ising model



Exactly dual to square lattice Ising model

Any other literature on this? (other than Thomas & Middleton 2009 & Likhosherstov, Maximov, Chertkov 2020?)

Key point:

Although non-bipartite, power-law dimer correlator at Ising critical points

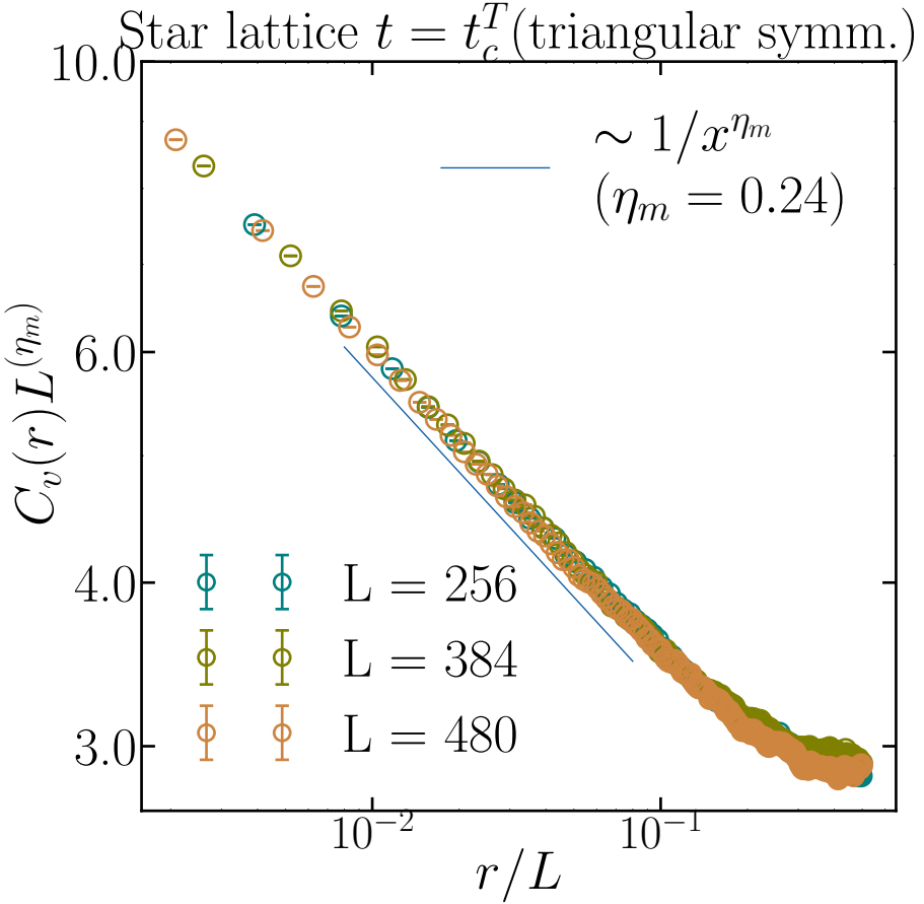
Dimer correlator maps to bond energy correlator of Ising model:

$$\eta_d = 2$$

But no height theory, so monomer exponent quite different:

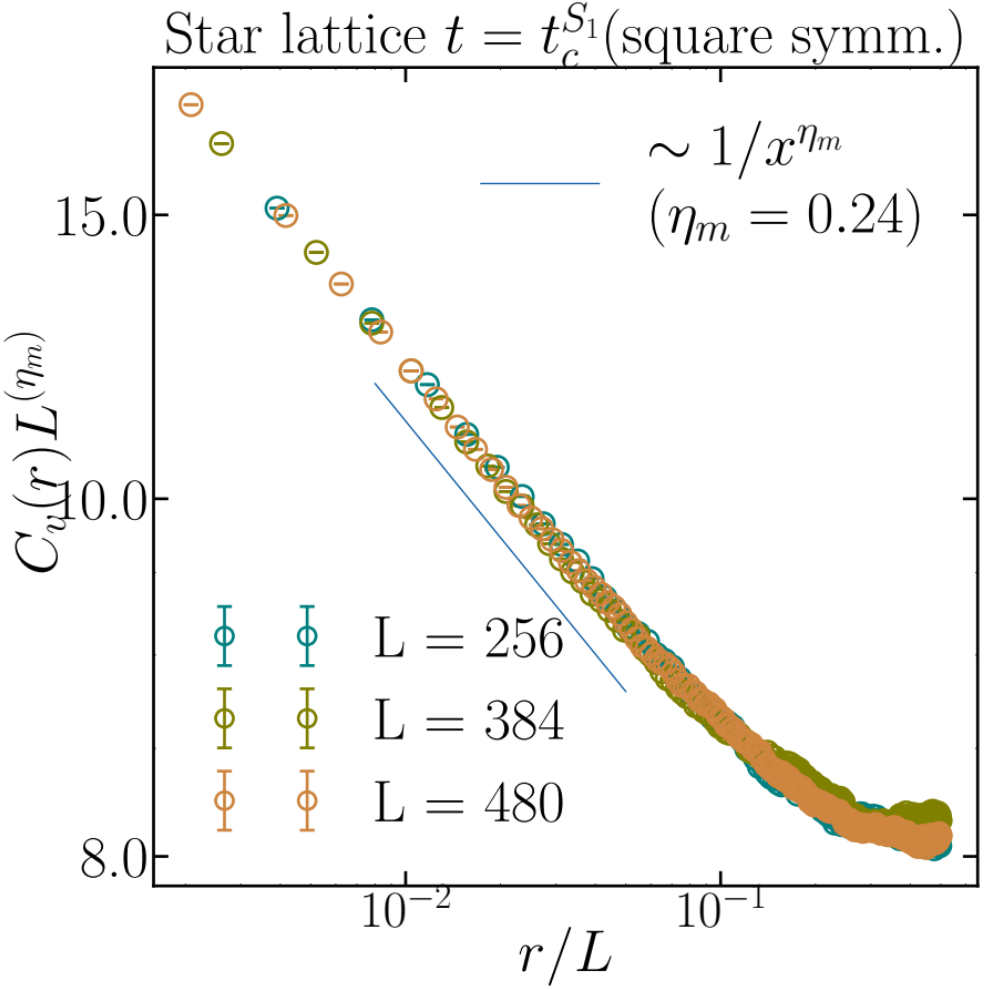
$$\eta_m \quad \neq \quad 1/\eta_d$$

Monomer exponent at triangular lattice Ising critical point



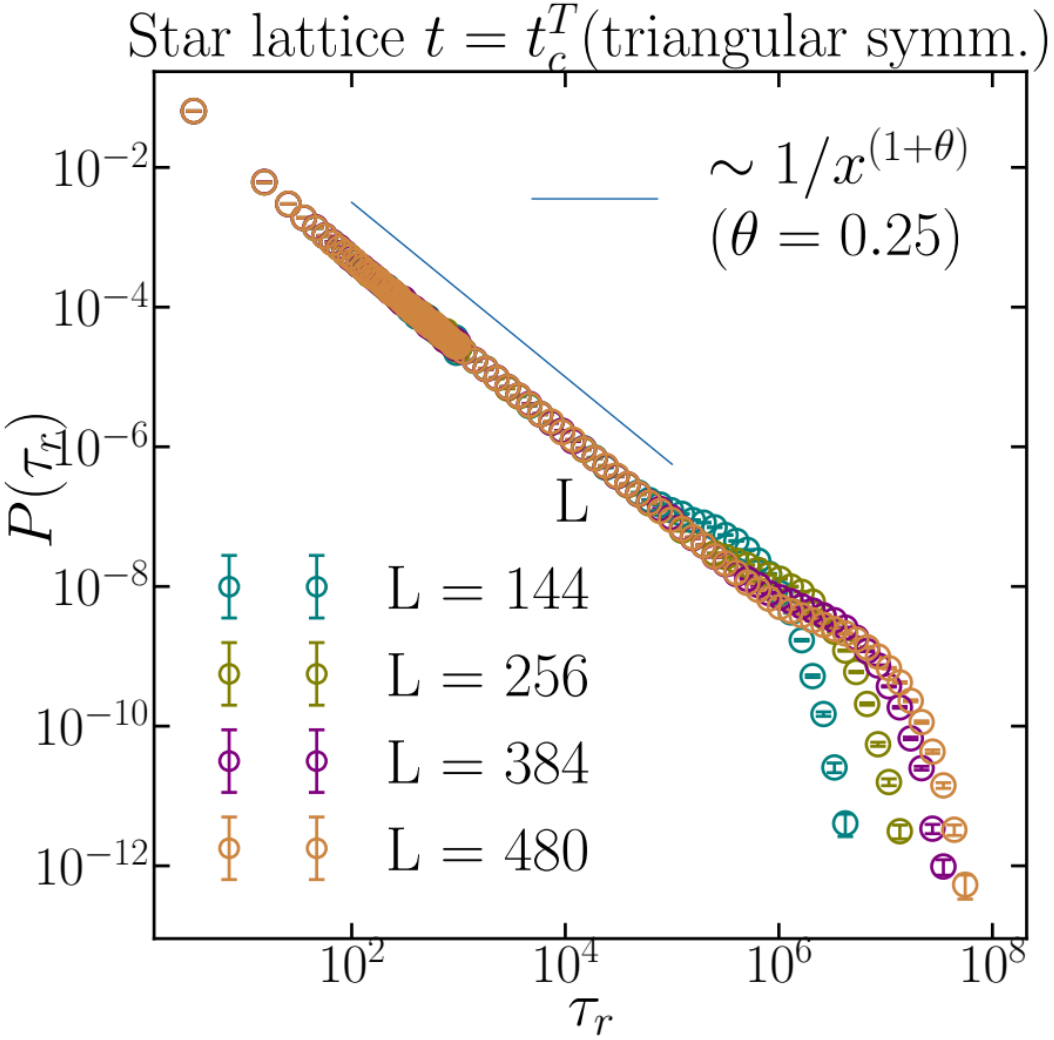
$$\eta_m = 0.24(2)$$

Monomer exponent at square lattice Ising critical point



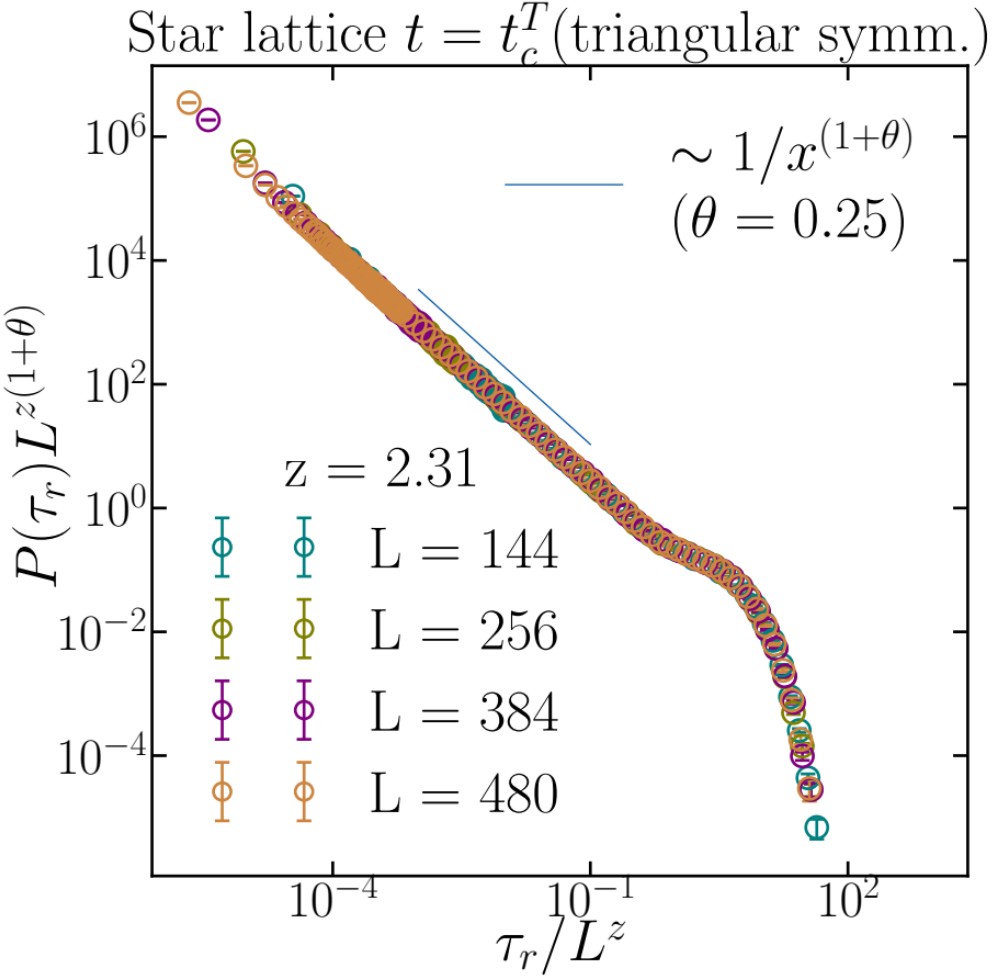
$$\eta_m = 0.24(2)$$

Distribution of all worms at triangular Ising critical point



$$\theta = 0.25(2)$$

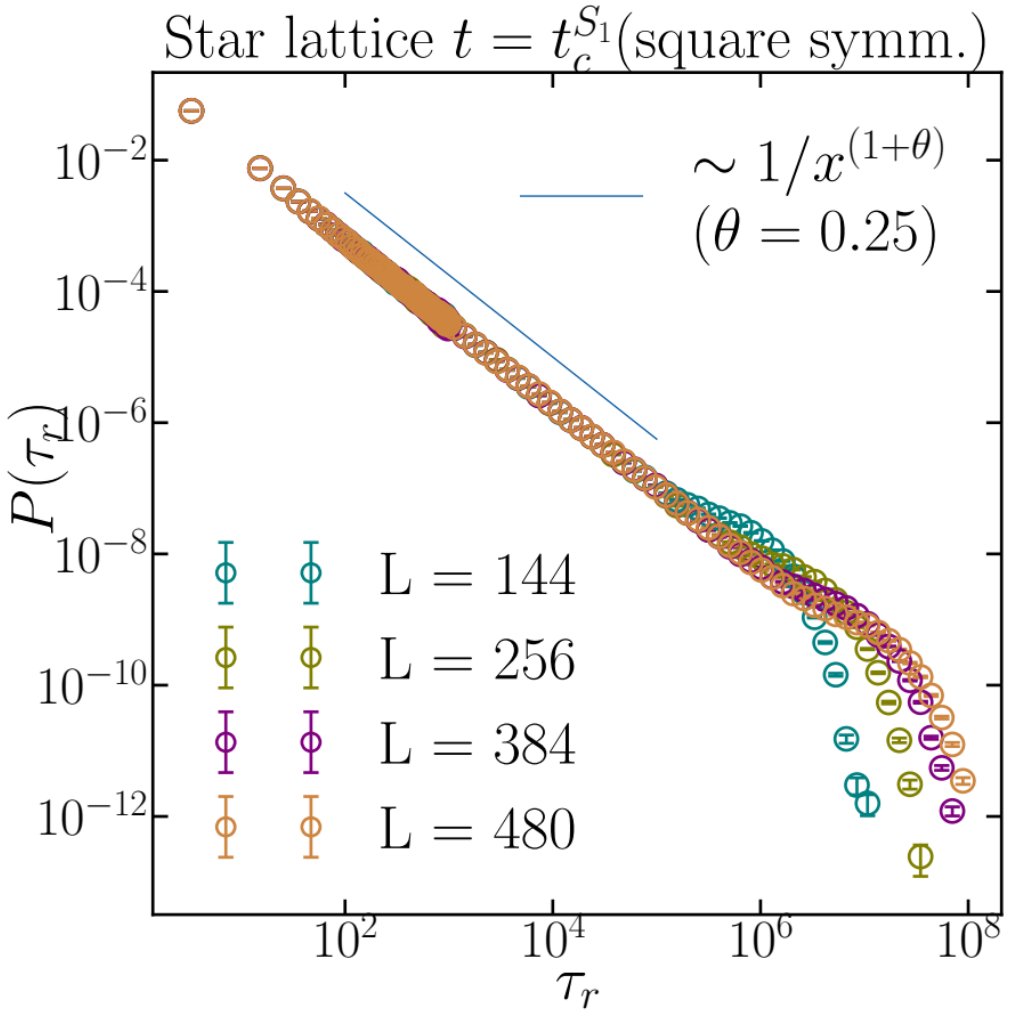
Scaling ansatz validated at triangular Ising critical point



$$\theta = 0.25(2)$$

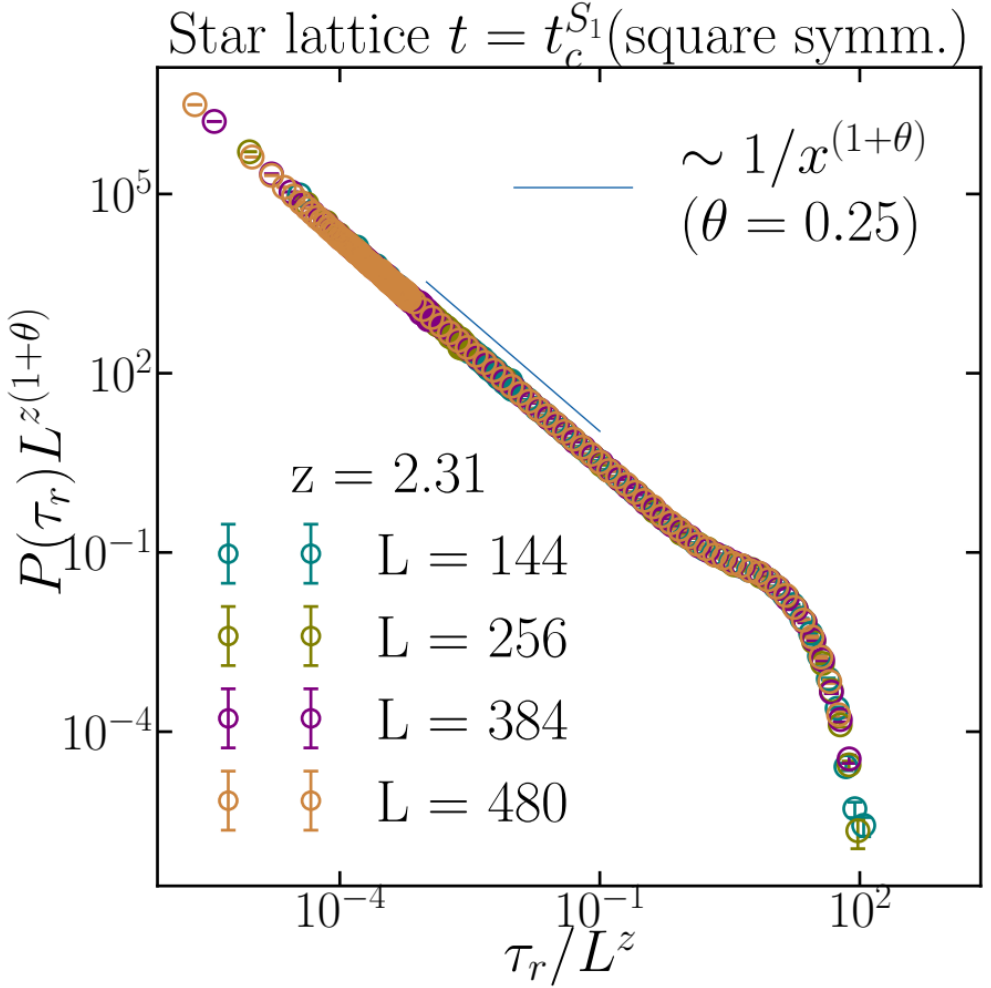
$$z = 2.31(3)$$

Distribution of all worms at square lattice Ising critical point



$$\theta = 0.25(2)$$

Scaling ansatz validated at triangular Ising critical point



$$\theta = 0.25(2)$$

$$z = 2.31(3)$$

Confirms heuristics (?)

z does not change (within errors)

But big change in θ

Acknowledgements

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