CMB Constraints on Natural Inflation with Gauge Field Production

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Talk based on K. Alam, K. Dutta, and Nur jaman, Published in: JCAP 12 (2024)015

Hearing beyond the standard model with cosmic sources of Gravitational Waves at ICTS, Bengaluru

ICTS

8 January 2025

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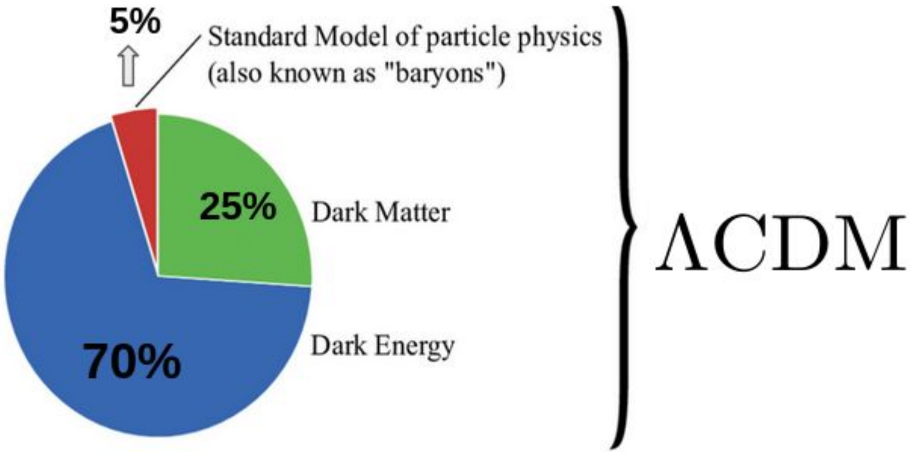
Outline

- ☆ Introduction
- ☆ Background dynamics
- ☆ Perturbation dynamics
- ☆ Observable
- ☆ Conclusion

Problems in Standard Cosmology

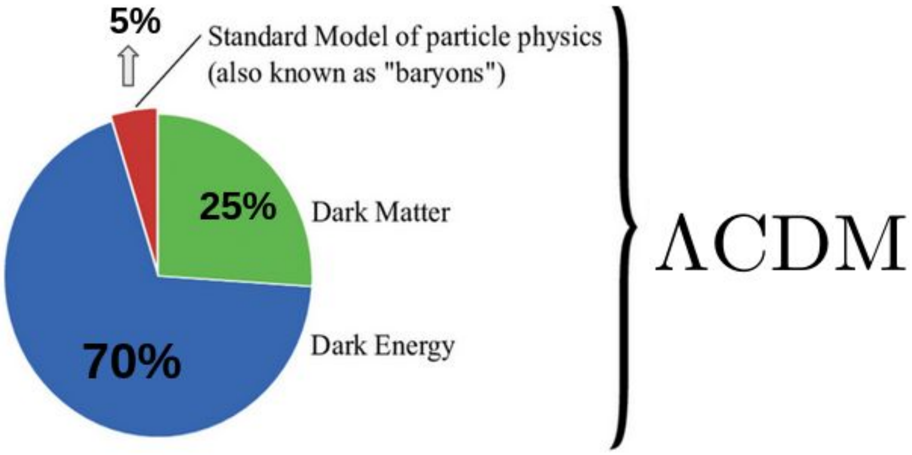


- Horizon problem
- Flatness problem
- Root of the CMB fluctuation
- etc…

$$\frac{\Delta T}{T} = 10^{-5}$$

T= 2.7 K : Background temperature

Problems in Standard Cosmology



- Horizon problem
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- etc…

$$\frac{\Delta T}{T} = 10^{-5}$$

T= 2.7 K : Background temperature

Resolution: Cosmic inflation

Observables

$$n_s - 1 \equiv \left.\frac{d\ln P_\zeta(k)}{d\ln k}\right|_{k=k_*}$$

$$r \equiv \left.\frac{P_h(k)}{P_\zeta(k)}\right|_{k=k_*} \implies$$ **Tensor to scalar ratio**

Spectral index

CMB or pivot mode ($k_* = 0.05\ Mpc^{-1}$ **)**

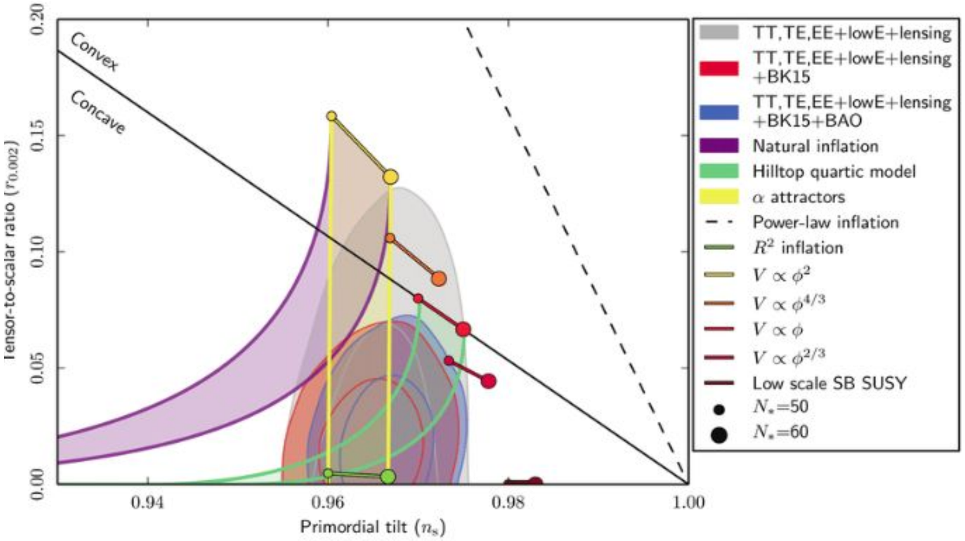
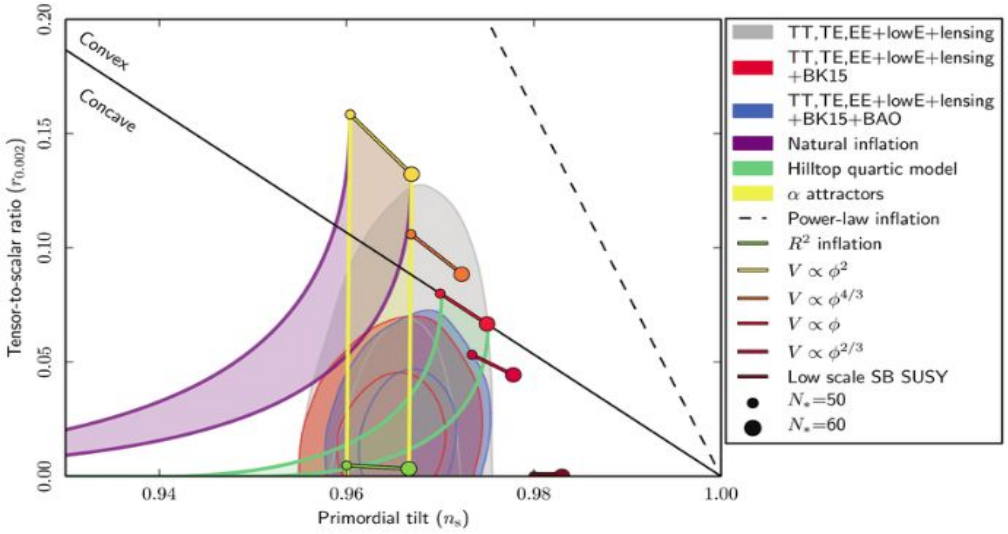


Figure: Plot represents the predictions of n_s and r based on various observation and theoretical models.

Natural Inflation model



Planck Collaboration, arXiv:1807.06211

Figure: Plot represents the predictions of n_s and r based on various observation and theoretical models.

$$V(\phi) = \Lambda^4 \left[1 + \cos\left(\frac{\phi}{f}\right)\right]$$

Freese, Frieman, Olinto arXiv:hep-ph/9207245

Note: Natural inflation model is under tension for $f \lesssim 1 m_{\rm pl}$ where $m_{\rm pl}$ is the Planck mass.

Set-up

The action for a pseudo-scalar inflaton ϕ, coupled to a massless Abelian gauge field A_μ

$$\mathcal{S} = \int d^4x \sqrt{-g}\left[\frac{R}{2} - \frac{1}{2}(\partial_\mu\phi)(\partial^\mu\phi) - V(\phi) - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{\alpha}{4f}\phi F^{\mu\nu}\tilde{F}_{\mu\nu}\right]$$

Einstein-Hilbert action

Kinetic and potential term of the inflation field

Kinetic term of gauge field

Interaction term between gauge and inflation field

$$V(\phi) = \Lambda\left[1 + \cos\left(\frac{\phi}{f}\right)\right]$$

Inflation potential

Note: Natural inflation model is under tension for $f \lesssim 1 m_{\rm pl}$ but after introducing the gauge field, would it be survive the natural inflation model.

M. M. Anber and L. Sorbo, arXiv:astro-ph/0606534, and arXiv:0908.4089 , W. D. Garretson, G. B. Field, and S. M. Carroll, arXiv:hep-ph/9209238

Field Dynamics

The action for a pseudo-scalar inflaton ϕ, coupled to a massless Abelian gauge field A_μ,

$$\mathcal{S} = \int d^4x \sqrt{-g}\left[\frac{R}{2} - \frac{1}{2}(\partial_\mu\phi)(\partial^\mu\phi) - V(\phi) - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{\alpha}{4f}\phi F^{\mu\nu}\tilde{F}_{\mu\nu}\right]$$

After varying this action

$$\phi'' + 2\mathcal{H}\phi' - \nabla^2\phi + a^2\frac{\partial V}{\partial\phi} = a^2\frac{\alpha}{f}\mathbf{E}\cdot\mathbf{B} \ ,$$

$$3\,\mathcal{H}^2 = \left[\frac{1}{2}\phi'^2 + \frac{1}{2}(\nabla\phi)^2 + a^2\,V(\phi) + \frac{a^2}{2}\left(\mathbf{E}^2 + \mathbf{B}^2\right)\right] ,$$

$$\mathbf{A}'' - \nabla^2\mathbf{A} + \nabla(\nabla.\mathbf{A}) = \frac{\alpha}{f}\phi'\,(\nabla\times\mathbf{A}) - \frac{\alpha}{f}\,(\nabla\phi)\times\mathbf{A}',$$

$$(\nabla.\mathbf{A})' = \frac{\alpha}{f}(\nabla\phi)\,.\,(\nabla\times\mathbf{A}) \ ,$$

Set of equations for the fields

$$\mathbf{E} = -\frac{1}{a^2}\mathbf{A}', \qquad \mathbf{B} = \frac{1}{a^2}\nabla\times\mathbf{A}$$

Electric and magnetic field in term of gauge field

M. M. Anber and L. Sorbo, arXiv:astro-ph/0606534, and arXiv:0908.4089 , W. D. Garretson, G. B. Field, and S. M. Carroll, arXiv:hep-ph/9209238

Background dynamics of the inflation field in the presence of the gauge field

Background dynamical equations

Dynamical equation of the gauge field in k-space is,

$$\left[\frac{\partial^2}{\partial\tau^2} + (k^2 \mp 2\, a\, H\, \xi\, k)\right] A_k^{\pm}(\tau) = 0,$$ where $\pm$ two helicity of gauge field

$$\xi = \frac{\alpha\, \dot{\phi}_0}{2\, f\, H}.$$

Mode which satisfies the condition $k/a\, H < 2|\xi|$ has an exponential solution.

Background dynamics of the inflaton field is

$$\ddot{\phi}_0 + 3H\dot{\phi}_0 + \frac{\partial V}{\partial \phi_0} = \frac{\alpha}{f}\langle \boldsymbol{E}.\boldsymbol{B}\rangle = -\frac{\alpha}{4\, f\, \pi^2\, a^4}\int dk\, k^3\, \frac{d}{d\tau}\left\{|A_k^+|^2 - |A_k^-|\right\}$$

$$3H^2 = \frac{1}{2}\dot{\phi}_0^2 + V(\phi_0) + \frac{1}{2}\langle \boldsymbol{E}^2 + \boldsymbol{B}^2\rangle = \frac{1}{2}\dot{\phi}_0^2 + V(\phi_0) + \frac{1}{8\,\pi^2\, a^4}\int dk\, k^2 \sum_{\lambda=\pm} |A_k'^{\lambda}|^2 + k^2|A_k^{\lambda}|^2$$

Source term which depend on the gauge field production

Volume average of the gauge field energy density

Time evolution of ξ and the source term

Condition for exponential growth of the gauge field modes: $k/a\,H < 2|\xi|$

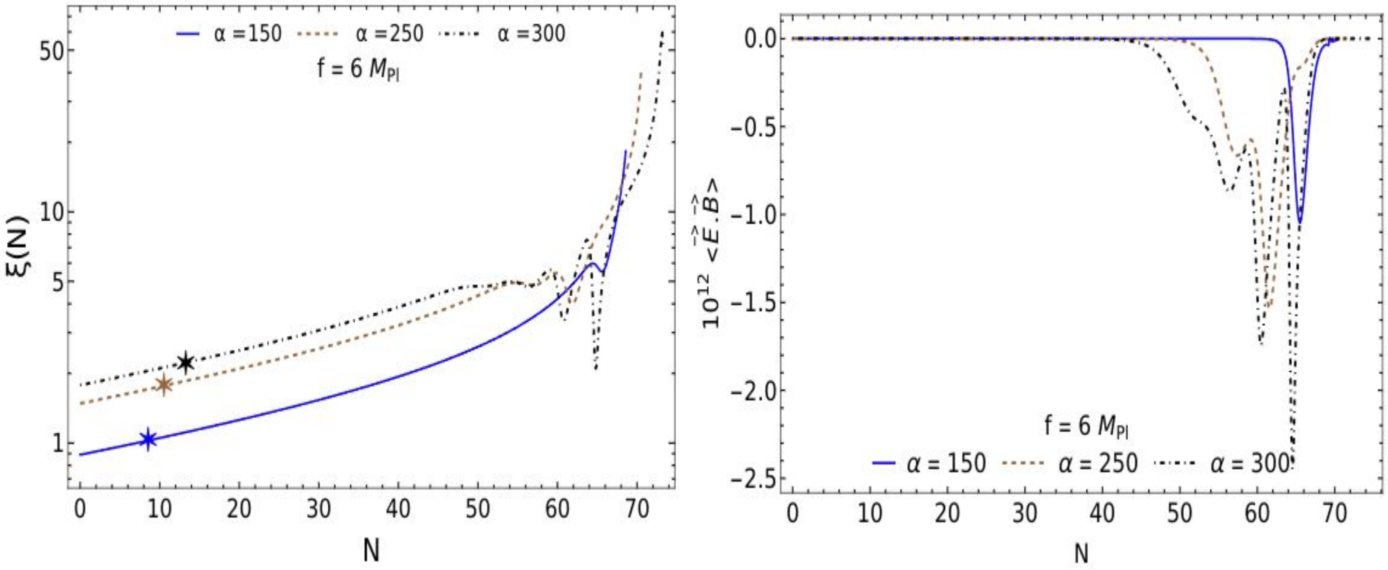
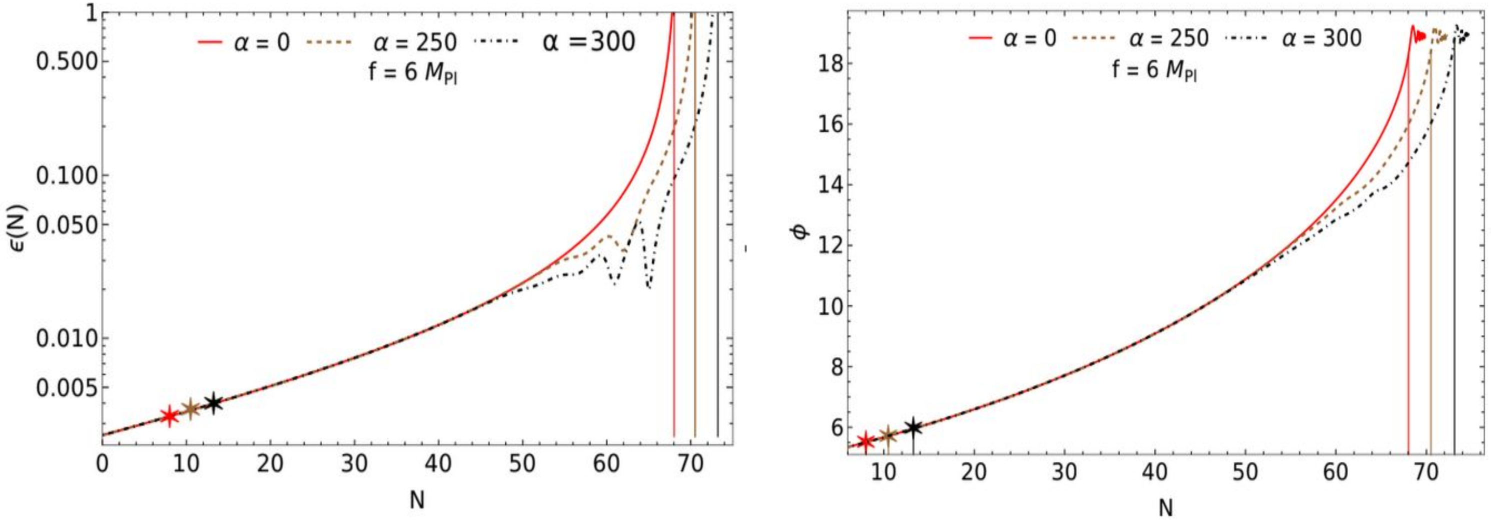


Figure: The left panel plot is time evolution of ξ and the right panel figure is time evolution of source term

Note: As time goes on, a large number of gauge fields will be excited.

Slow-roll parameter and Inflaton dynamics

$$\epsilon(N) = -\frac{\dot{H}}{H^2}$$ → Definition of the slow-roll parameter



'⋆' of different colours correspond to 60 e-foldings before the end of inflation for different choices of α

Note: The duration of inflation is extended in the presence of a gauge field, and for different choices of parameter α (for a given value of f), the CMB scales ($k = 0.05 Mpc^{-1}$) probe different parts of the axion potential.

Study the perturbation dynamics of the inflation field in the presence of the gauge field

Plot of the scalar power spectrum in the presence of the gauge field

$$P_\zeta(k) = P_\zeta(k)_{\rm vac} + P_\zeta(k)_{\rm source} = \underbrace{\left(\frac{H^2}{2\pi\dot\phi_0}\right)^2}_{\text{vaccum}} + \underbrace{\left(\frac{\alpha}{f}\frac{\langle \mathbf{E}.\mathbf{B}\rangle}{3\beta H\dot\phi_0}\right)^2}_{\text{sourced}}$$

where

$$\beta \equiv 1 - 2\pi\xi\frac{\alpha}{f}\frac{\langle \mathbf{E}.\mathbf{B}\rangle}{3H\dot\phi_0}$$

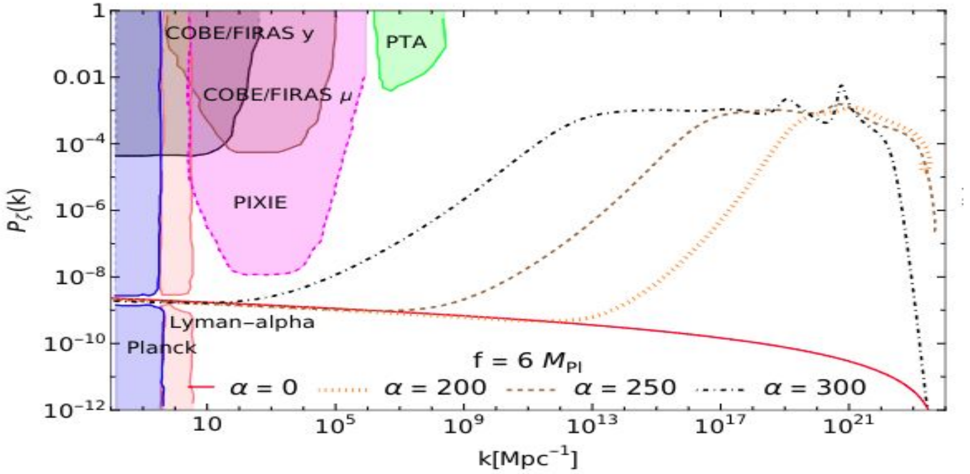


Figure: The amplitude of the scalar power spectrum is plotted against comoving wavenumbers for different values of the coupling constant α and f = 6. Several colored contours show existing (continuous) and projected future (dotted) constraints.

M. M. Anber and L. Sorbo, arXiv:0908.4089; N. Barnaby, E. Pajer, and M. Peloso, arXiv:1110.3327 ; A. Linde, S. Mooij, and E. Pajer, arXiv:1212.1693

Tensor power spectrum in the presence of the gauge field

$$P_h^{\pm} = \frac{H^2}{\pi^2}\left(\frac{k}{k_0}\right)^{n_T}\left[1 + 2\,H^2 f_h^{\pm}(\xi) e^{4\,\pi\,\xi}\right]$$

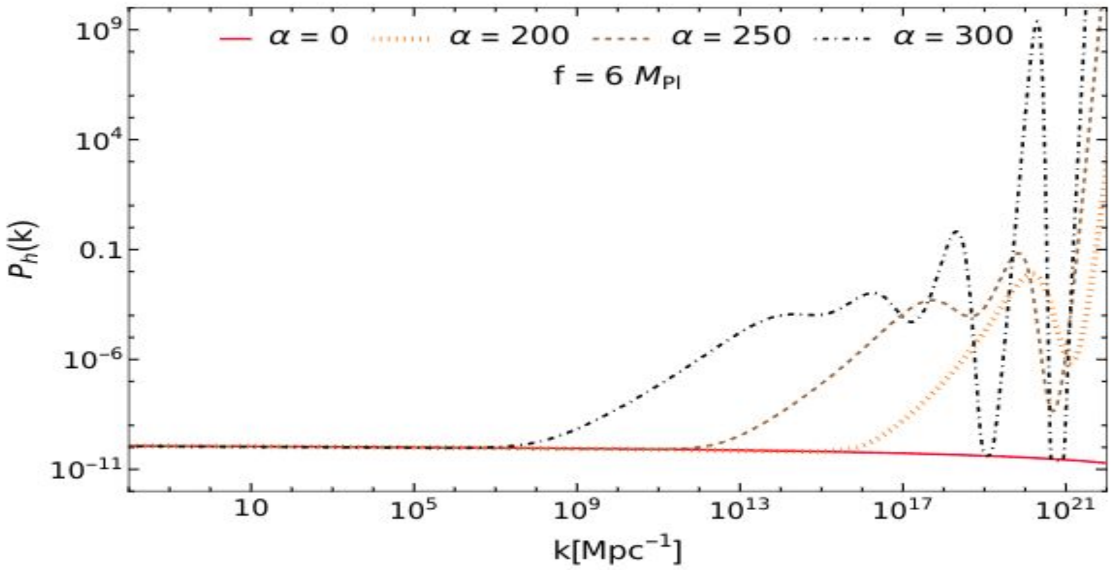
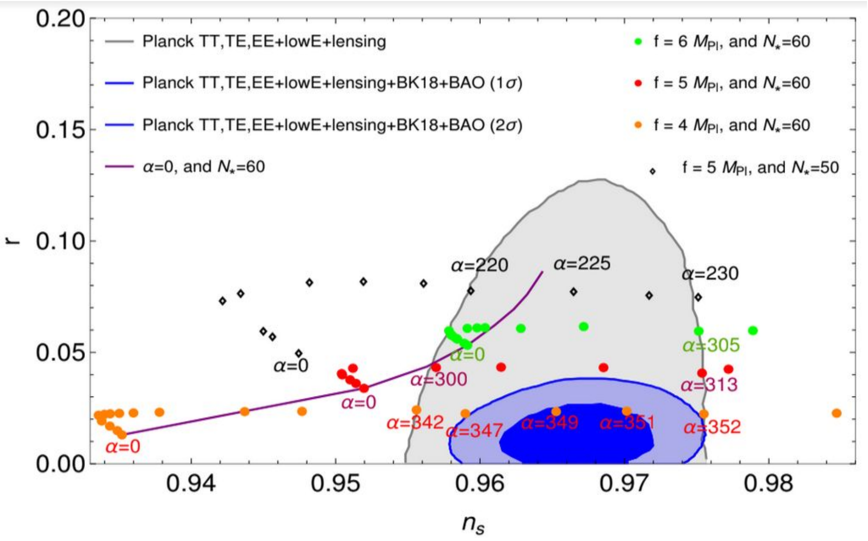
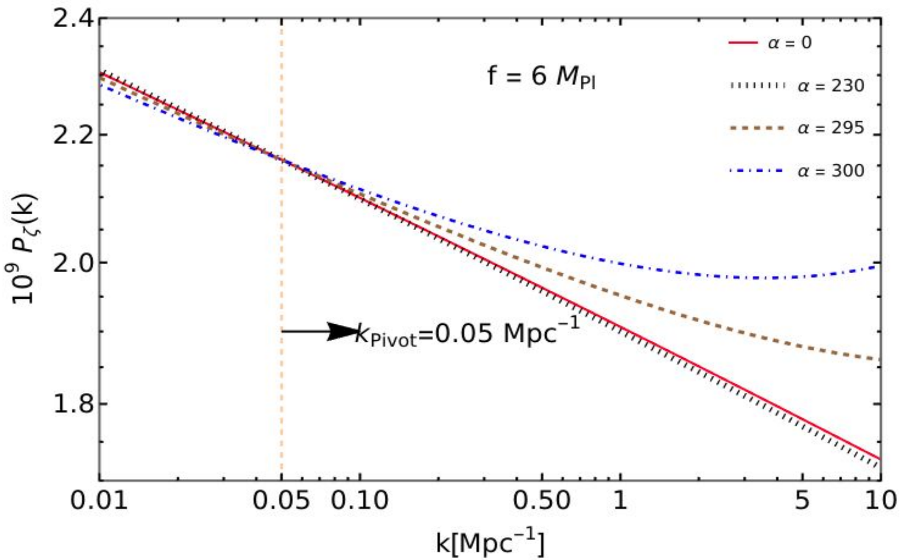


Figure: The amplitude of the tensor power spectrum is plotted against comoving wavenumbers for different values of the coupling constant α and f = 6.

Spectral index and tensor to scalar ratio





$$342 \leq \alpha \leq 352; \text{ (}2\sigma\text{ contour of Planck data)}$$
$$347 \leq \alpha \leq 352; \text{ (}2\sigma\text{ contour of Planck + BK18 + BAO data)}$$
$$349 \leq \alpha \leq 351; \text{ (}1\sigma\text{ contour of Planck + BK18 + BAO data)}$$

For a certain range of the coupling constant, the natural inflation model remains consistent with recent observational data on the n_s-r plane

Running of the spectral index and non-gaussianity

In this model, the non-gaussianity is of the equilateral type, and it can be calculated by using the formula:

$$f_{\rm NL}^{\rm equil} = \left. \frac{f_3(\xi)\,(P_\zeta(k)_{\rm vac})^3\; e^{6\pi\xi}}{(P_\zeta(k))^2} \right|_{k=k_*}$$

N. Barnaby, R. Namba, and M. Peloso, arXiv:1011.1500

N. Barnaby, R. Namba, and M. Peloso, arXiv:1102.4333

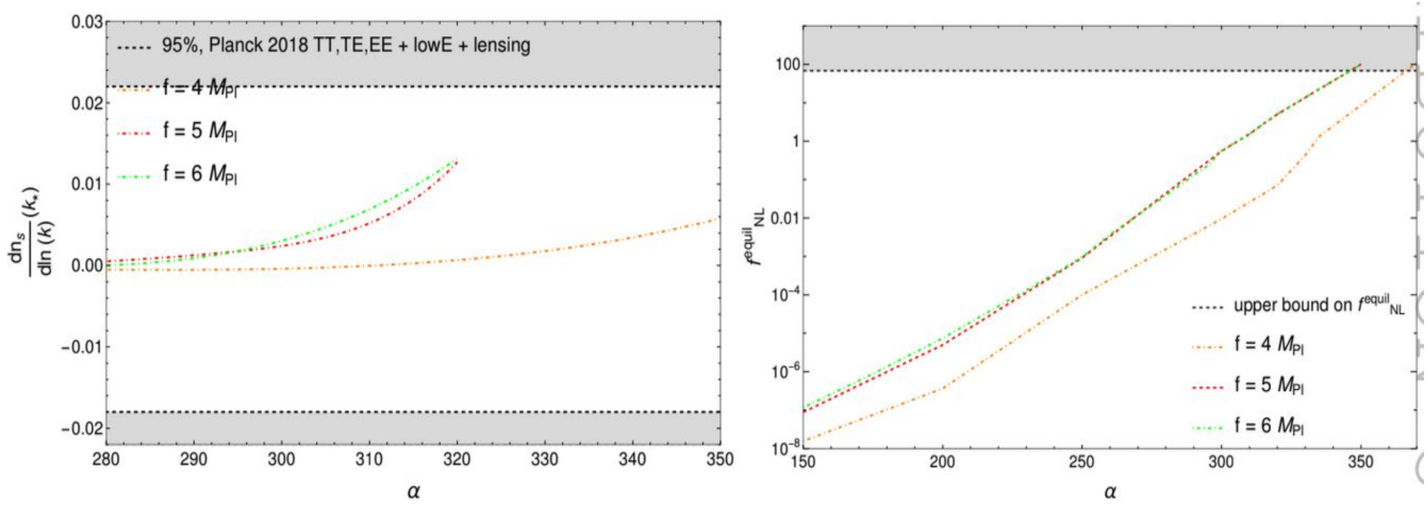


Figure: the left panel figure represents the running of the spectral index and the right panel figure represents the non-gaussianity of the system.

Note: The range of coupling constants allowed in $n_s - r$ the plane is also consistent with constraints on the running of the spectral index and non-Gaussianity.

Conclusion

- Presence of the gauge field start to influence the inflaton field dynamics later stage of inflation.

- Duration of the inflation is going to be prolonged due to the presence of the gauge field.

- The presence of a gauge field begins to affect large-scale modes when the coupling constant becomes sufficiently large.

- The extended duration of inflation helps alleviate the tension in the natural inflation model for a specific range of coupling constant.

THANK YOU ....

Future Plan

- In this work, we have considered massless abelian gauge fields. We can do similar thing by considering the non abelian gauge field.

- We observe the growth of the scalar power spectrum during the later stages of inflation which can produce the primordial black holes.

- To get the scalar and tensor power spectrum, we have used the semi analytical expression. We have not done the full numeric. To do the full numeric, we have to learnt lattice. I am now working on it.