Electroweak Phase Transition with pNGB Dark Matter

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Introduction

- It is generally believed that several phase transitions have taken place in the early Universe. The effects of cosmological phase transitions may well have been crucial for the evolution of the Universe, and thus for the existence of life as we know it. If these Cosmological Phase Transitions are first order in nature, they act as a source of stochastic gravitational waves which can be a good candidate for probing the early universe.
- The shape of the GW spectrum thus generated, depends on the history of evolution of the universe.

Model

- The simplest scenario for a pseudo-Nambu Goldstone (pNGB) dark matter consists of a complex gauge singlet scalar S with softly broken global $U(1)$ symmetry to provide a mass term for the pNGB dark matter. However, in this work, we extend the minimal framework of pNGB DM by another gauge singlet real scalar Φ. We also introduce $\mathcal{Z}_2 \times \mathcal{Z}_3$ discrete symmetry. The S field does not carry any $\mathcal{Z}_2$ charge while under $\mathcal{Z}_3$ it transforms as $S \to e^{\frac{2\pi i}{3}} S$. In contrast, Φ has odd charge under $\mathcal{Z}_2$, however remain invariant under $\mathcal{Z}_3$.

Model

The most general renormalizable scalar potential containing the SM Higgs doublet H, complex scalar S and the real scalar Φ is given by

$$
\begin{aligned}
V_0(H,S,\Phi) = {} & \mu_H^2\,(H^\dagger H) + \lambda_H\,(H^\dagger H)^2 + \mu_\Phi^2\,\Phi^2 + \lambda_\Phi\,\Phi^4 + \mu_S^2\,(S^\dagger S) \\
& + \lambda_S\,(S^\dagger S)^2 + \lambda_{HS}\,(H^\dagger H)(S^\dagger S) + \lambda_{S\Phi}(S^\dagger S)\Phi^2 \\
& + \lambda_{H\Phi}(H^\dagger H)\Phi^2 + \frac{\mu_3}{2}(S^3 + S^{\dagger 3}). \qquad (1)
\end{aligned}
$$

Model

In the unitary gauge, after electroweak symmetry breaking (EWSB), we parametrize the scalar fields in the following form:

$$H = \begin{bmatrix} 0 \\ \frac{v_h + h}{\sqrt{2}} \end{bmatrix}; \quad S = \frac{v_s + s + i\chi}{\sqrt{2}}; \quad \Phi = \phi + v_\phi. \tag{2}$$

where, v_h, v_s and v_ϕ represent vacuum expectation values for the H, S and Φ fields respectively. Throughout our analysis we assume these vacuum expectation values to be real.

Model

The mixing of three CP even gauge eigen states (h, s, ϕ) will yield to the following three CP even physical eigenstates:

$$\begin{bmatrix} H_1 \\ H_2 \\ H_3 \end{bmatrix} = O(\alpha_1, \alpha_2, \alpha_3) \begin{bmatrix} h \\ s \\ \phi \end{bmatrix} \tag{3}$$

where O is an orthogonal matrix and relates the physical basis (H_1, H_2, H_3) with the unphysical basis (h, s, ϕ).
In this analysis, we assume the following mass hierarchy among these CP even scalars :

$$m^2_{H_3} > m^2_{H_2} > m^2_{H_1} \tag{4}$$

where the lightest CP even scalar H_1 is identified to the Standard Model (SM) Higgs boson (h_{SM}) having mass $m_{H_1} = 125$ GeV, and $v_h = v_{\rm sm} = 246$ GeV respectively. Throughout the analysis, we ensure that the properties of H_1 remain consistent with the experimentally measured values of the Standard Model Higgs boson at the LHC.

Requirements for studying PhT

To study the dynamics of the electroweak phase transition in the early universe, we use the one-loop corrected finite temperature effective potential involving the SM and extra scalar fields of this scenario. We start the proceedings, by adding the Coleman-Weinberg potential $V_{\rm CW}$ and counterterms $V_{\rm CT}$ that encode one-loop corrections at zero temperature to the tree-level potential V_0. Finally, to incorporate the effect of temperature of the early universe, we include the finite-temperature corrections $V_{\rm T}$. The complete effective potential is given by

$$V_{\rm eff} = V_0 + V_{\rm CW} + V_{\rm CT} + V_{\rm T} \tag{6}$$

One Loop Effective Potential

We first compute the Coleman-Weinberg potential $V_{\rm CW}$ using the $\overline{\rm MS}$ scheme and taking the Landau gauge to decouple any ghost contributions, and write the $V_{\rm CW}$ at zero temperature as .

$$V_{\rm cw}(H_i) = \sum_j (-1)^{F_j} \frac{n_j m_j^4(H_i)}{64\pi^2} \left[\log \frac{m_j^2(H_i)}{4\pi\mu^2} - C_j \right] \qquad (7)$$

where, $i = 1, 2, 3$, and j runs over all particles contributing to the one-loop correction, $F_j = 0$ (1) for bosons (fermions), n_j are the degrees of freedom of the $j^{\rm th}$ particle and $m_j^2(H_i)$ is H_i field dependent mass of j-th particle. The renormalization scale μ is fixed at $\mu = m_t (= 173.5 \ {\rm GeV})$, the constant C_j is equal to $\frac{3}{2}$ for scalars and fermions and to $\frac{5}{6}$ for vector bosons.

Counter terms

It is noteworthy that, while considering Coleman-Weinberg correction, the tree-level vacuum expectation values (vevs) and masses may receive modification at zero temperature. To prevent such changes, it is necessary to incorporate counterterms into the effective potential. The counterterm potential is defined as:

$$\begin{aligned}\delta V_{\rm ct}(H_i) = &\, \delta\mu_H^2\,(H^\dagger H) + \delta\lambda_H\,(H^\dagger H)^2 + \delta\mu_\Phi^2\,\Phi^2 + \delta\lambda_\Phi\,\Phi^4 + \delta\mu_S^2\,(S^\dagger S) \\ &+ \delta\lambda_S\,(S^\dagger S)^2 + \delta\lambda_{HS}\,(H^\dagger H)(S^\dagger S) + \delta\lambda_{S\Phi}(S^\dagger S)\Phi^2 \\ &+ \delta\lambda_{H\Phi}(H^\dagger H)\Phi^2 + \frac{\delta\mu_3}{2}(S^3 + S^{\dagger 3}) \qquad (8)\end{aligned}$$

Counter Terms

To determine the expressions for the counterterms corresponding to each parameter, we use the following conditions:

$$\partial_{h_a}(\delta V_{\rm ct} + \Delta V) = 0$$
$$\partial_{h_a}\partial_{h_b}(\delta V_{\rm ct} + \Delta V) = 0, \tag{9}$$

where the partial derivatives are taken with respect to h , s and ϕ fields expressed as $h_{a/b}$. The derivatives are evaluated at the vacuum expectation values of the respective fields, $\langle H \rangle = v_h$, $\langle S \rangle = v_s$, $\langle \Phi \rangle = v_\phi$ and ΔV is the effective potential at zero temperature excluding the tree level part of the potential.

One Loop Finite Temperature

Taking into thermal effects, the temperature-dependent part of the effective potential at finite temperatures is given by

$$V_T(H_i, T) = \frac{T^4}{2\pi^2}\left(\sum_B n_B J_B(\frac{m_B^2(H_i)}{T^2}) + \sum_F n_F J_F(\frac{m_F^2(H_i)}{T^2})\right) \quad (10)$$

The $n_{B/F}$ are the degrees of freedom of bosons/fermions respectively and the $J_{B/F}$ are Bosonic and Fermionic functions which are represented as

$$J_{B/F}(x^2) = \int_0^\infty y^2 \log[1 \mp e^{-\sqrt{x^2+y^2}}]dy \quad (11)$$

At high temperature limit, one can expand the Bosonic and Fermionic integrals in powers of $x \equiv m/T$ as

$$\begin{aligned} J_{\mathrm{B}}(x^2)\,|_{x\ll 1} &\simeq -\frac{\pi^4}{45} + \frac{\pi^2}{12}x^2 - \frac{\pi}{6}x^3 + \mathcal{O}(x^4), \\ J_{\mathrm{F}}(x^2)\,|_{x\ll 1} &\simeq \frac{7\pi^4}{360} - \frac{\pi^2}{24}x^2 + \mathcal{O}(x^4). \end{aligned} \quad (12)$$

Phase Transition

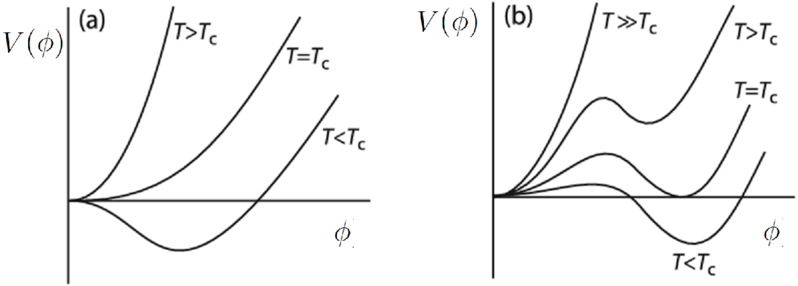


Figure: (a) Second Order Phase Transition (b) First Order Phase Transition

Phase Transition

The critical temperature T_c in a phase transition is determined by setting the potential values at two vacuum expectation values (vevs) which can be established using the following mathematical expression.

$$V(H_i^{\text{High}}, T_c) = V(H_i^{\text{Low}}, T_c) \tag{13}$$

We define the order parameter $\zeta_{c,i}$ along the i^{th} field direction as:

$$\zeta_{c,i} = \frac{\Delta H_i}{T_c} \tag{14}$$

with ΔH_i is the difference of high and low vevs of the SM/BSM scalar field. The criterion for an SFOPT is given by $\zeta_{c,i} \geq 1$.

Phase Transition

A first-order phase transition involves two key temperatures: the critical temperature (T_c) and the bubble nucleation temperature (T_n). Generally, the transition proceeds through bubble nucleation at T_n, which is typically below T_c. During this process, at a finite temperature T, the probability per unit volume of tunneling from the false vacuum to the true vacuum is given by,

$$\Gamma(T) = T^4 \left(\frac{S_3}{2\pi T} \right)^{\frac{3}{2}} e^{-\frac{S_3}{T}} \tag{15}$$

where S_3 represents the 3-dimensional Euclidean action and is represented

$$S_3 = 4\pi \int r^2 dr \left[\frac{1}{2} \left(\frac{dH_i}{dr} \right)^2 + V_{\text{eff}}(H_i, T) \right] \tag{16}$$

where the scalar field H_i follows the differential equation given by

$$\frac{d^2 H_i}{dr^2} + \frac{2}{r}\frac{dH_i}{dr} = \frac{dV_{\rm eff}(H_i, T)}{dH_i} \tag{17}$$

with the boundary conditions $H_i = 0$ as $r \to \infty$ and $\frac{dH_i}{dr} = 0$ at $r = 0$.

The nucleation temperature is defined as the temperature at which the following condition is satisfied .

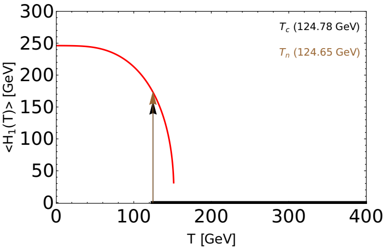
$$\frac{S_3(T_n)}{T_n} = 140 \tag{18}$$

Benchmark Points

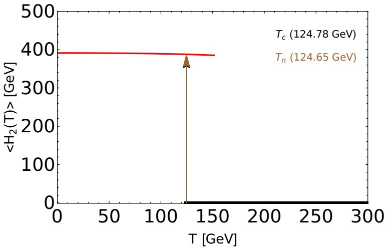
Parameters	BP 1	BP 2	BP 3	BP 4	BP 5	BP 6	BP 7
m_{H_2} (GeV)	377.15	296.93	260.50	275.64	234.94	290.09	377.22
m_{H_3} (GeV)	586.38	496.55	600.82	331.63	367.80	631.88	764.36
m_χ (GeV)	985.94	653.75	765.83	714.46	787.2	628.44	840.19
$\sin\alpha_1$	0.0016	-0.067	0.112	-0.0252	-0.0086	-0.022	0.12
$\sin\alpha_2$	0.161	0.22	0.172	0.189	0.196	0.229	0.06
$\sin\alpha_3$	0.192	0.094	0.025	0.877	0.965	0.175	-0.0074
v_s (GeV)	391.28	870.91	915.97	746.70	851.43	962.96	1013.05
v_ϕ (GeV)	152.16	121.6	184.43	156.33	227.8	238.13	624.37

Table: *Benchmark Points for the study of SFOPT.*

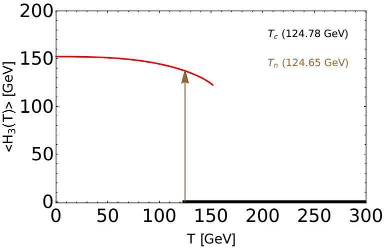
Phase Transition



(a)



(b)



(c)

Phase Transition

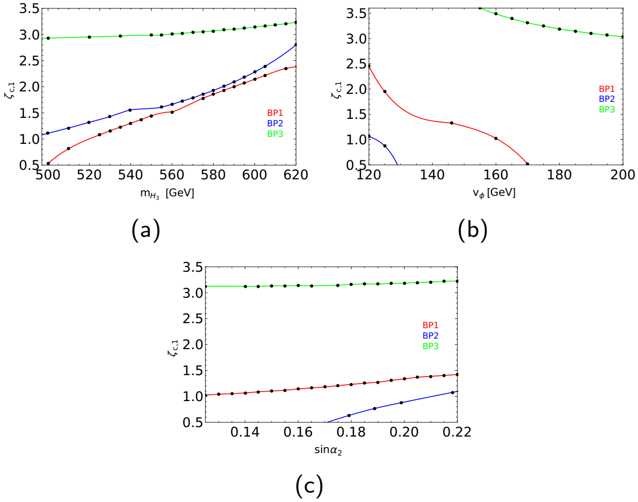
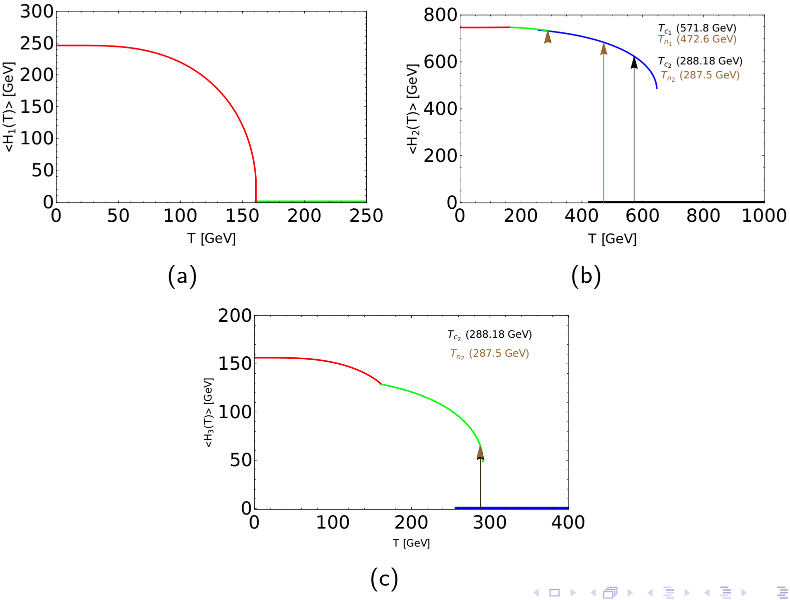


Figure: Variation of some parameters on PhT strength along SM Higgs direction

Phase Transition



DM Phenomenology

To quantify the dark matter abundance we define a ratio :

$$f_\chi = \frac{\Omega_\chi h^2}{\Omega_{\rm DM} h^2} \qquad (19)$$

where we assume that it is not mandatory for the pNGB field χ to be the only DM component. If $f_\chi < 1$, the model predicted relic density is suppressed compared to that of the observed value by f_χ which corresponds to larger values of scalar sector couplings required for the strong first-order phase transition.

Observables	BP1	BP2	BP3	BP4	BP5	BP6	BP7
$\Omega_{\rm DM} h^2$	0.00261	0.095	0.12	0.0517	0.071	0.0913	0.119
$\sigma_{\rm eff}^{\rm SI}$ (cm^2)	6.11×10^{-51}	1.9×10^{-46}	6.78×10^{-46}	3.76×10^{-47}	1.77×10^{-47}	2.46×10^{-47}	1.125×10^{-45}

Table: Relic density and Direct Detection cross section of the benchmark points. BP2,3,7 fail to satisfy latest LUX-ZEPLIN bound.

DM Phenomenology satisfying relic+DD+ID

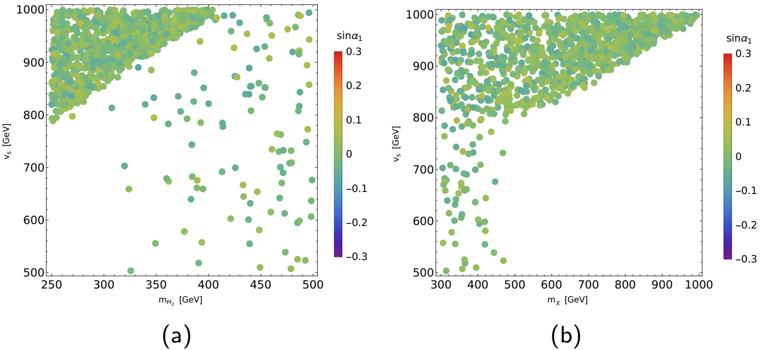


Figure: Variation of some parameters satisfying relic density, direct and indirect detection cross constraints.

Gravitational Waves

A first-order phase transition in the early universe can serve as a primordial source of gravitational wave (GW) generation. The basic characteristic of these waves are stochastic and there are three primary mechanisms for their production which are:

1. Bubble Collisions occurring in the plasma.
2. Sound Waves generated in the plasma.
3. Magnetohydrodynamic Turbulence generated in the plasma.

Gravitational Waves

The total GW energy spectrum is expressed as:

$$\Omega_{\rm GW} h^2 \simeq \Omega_{\rm col} h^2 + \Omega_{\rm sw} h^2 + \Omega_{\rm turb} h^2 \tag{20}$$

The GW Spectrum is primarily shaped by four key parameters

1. Latent heat parameter (α)
2. Inverse duration parameter (β)
3. Nucleation temperature (T_n)
4. Bubble wall velocity (v_w).

- The parameter α indicates the strength of the phase transition and is expressed as:

$$\alpha = \frac{\frac{T}{4}\frac{dV_{\rm eff}(\langle H_i \rangle, T)}{dT} - V_{\rm eff}(\langle H_i \rangle, T)}{\frac{\pi^2 g_* T^4}{30}}\Bigg|_{T=T_n} \tag{21}$$

- The parameter $\frac{\beta}{H_n}$ represents the ratio of the inverse of the time taken for the phase transition to complete to the Hubble parameter value at T_n. It can be expressed as

$$\frac{\beta}{H_n} = T_n \frac{d(S_3/T)}{dT}\Bigg|_{T=T_n} \tag{22}$$

Gravitational Waves

- The intensity of a stochastic background of gravitational waves (GWs) can be characterized by the dimensionless quantity

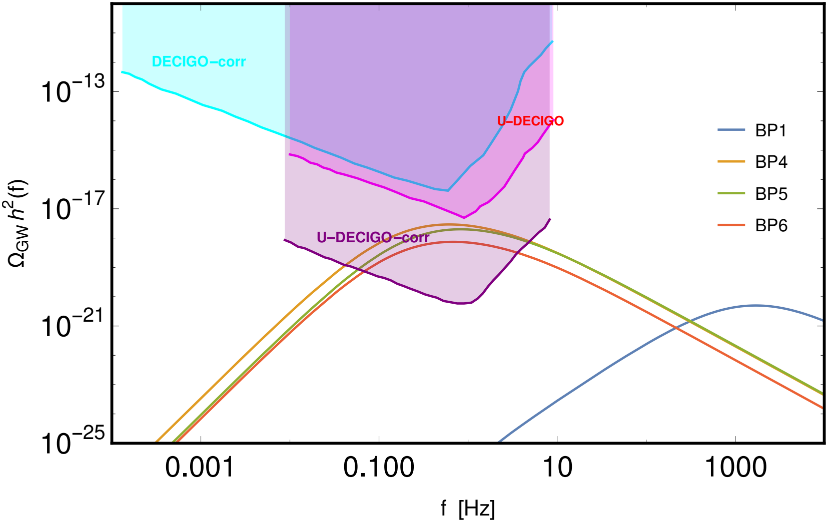
$$\Omega_{GW}(f) = \frac{1}{\rho_c} \frac{d\rho_{GW}}{d\log(f)}$$

  where ρ_{GW} is the energy density of the stochastic background of gravitational waves and ρ_c is the present value of the critical energy density of the Universe.

- The quantity ρ_{GW} is given by

$$\rho_{GW} = \frac{1}{32\pi G} < \dot{h}_{ab}\dot{h}^{ab} >$$

  where h_{ab} are the two modes of polarization of the gravitational waves in the Fourier space.


DECIGO−corr
U−DECIGO
U−DECIGO−corr
BP1
BP4
BP5
BP6
10^{-13}
10^{-17}
10^{-21}
10^{-25}
$\Omega_{GW} h^2(f)$
0.001
0.100
10
1000
f [Hz]

Conclusion

- This study investigates dark matter (DM) and phase transition dynamics in a model featuring pNGB DM within a $\mathcal{Z}_3$-symmetric complex scalar and a $\mathcal{Z}_2$-symmetric real scalar, both singlets under $SU(2)_L$ symmetry.
- Two patterns of strong first-order phase transitions (SFOPT) were identified, occurring along either both SM and BSM Higgs directions or exclusively along the BSM Higgs direction. Several benchmark scenarios illustrate different stepwise phase transitions.
- Certain benchmark points (BP2, BP3, and BP7) align with Xenon 1T limits but are ruled out by stricter LZ constraints. Additionally, SFOPT along the SM Higgs direction yields under-abundant DM relic density.

Conclusion

- The introduction of parameters such as m_{H_3}, v_ϕ, and $\sin\alpha_2$ influences phase transition strength, while a key symmetry-breaking scale (μ_3) facilitates first-order transitions within the viable parameter space, still satisfying DM detection limits.
- Analysis of gravitational wave spectra indicates the highest amplitude GW peak for BP4, followed by BP5 and BP6. While SM Higgs direction transitions yield weak GW signals, BSM Higgs transitions could be detected by upcoming GW observatories like DECIGO and U-DECIGO.
- These findings enhance our understanding of the early universe and suggest potential GW observables for testing phase transition dynamics within this theoretical model.

Thank You