Quantum Scars in Rydberg ladders with staggered detuning

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References

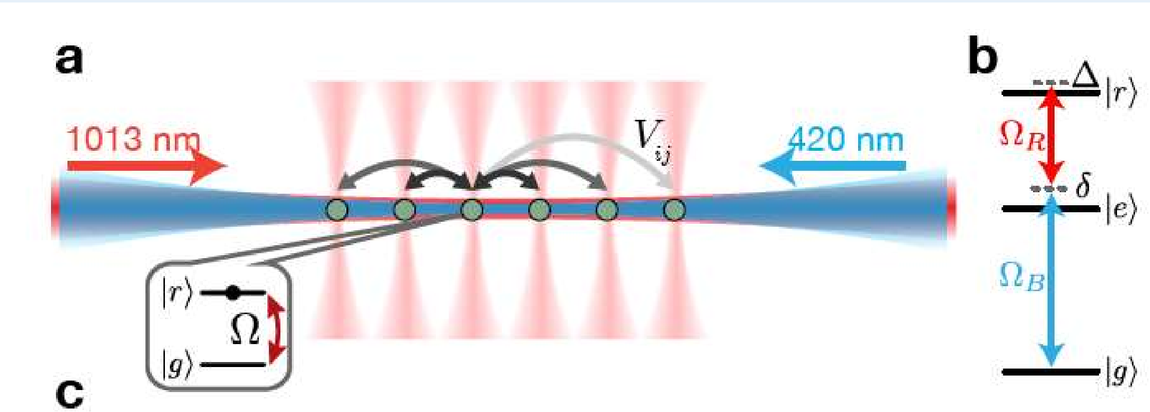
arXiv:2411.02500

Outline of the talk

1. *Rydberg chains: Experiments and models*
2. *Rydberg ladder with staggered detuning*
3. *ETH and Quantum scars*
4. *Scars in staggered ladders*
5. *Site-dependent dynamics and long-time imbalance*
6. *Chirality operators*
7. *Zero-energy states*

Rydberg chains

Rydberg atom arrays



$$\delta \sim 2\pi \times 560\,\mathrm{MHz}$$

$$\Omega_B, \Omega_R \sim 2\pi \times 60, 36\,\mathrm{MHz}$$

$$\Omega = \Omega_B \Omega_R/(2\delta) \sim 2\pi \times 2\,\mathrm{MHz}.$$

$$|g\rangle = |5S_{1/2}, F = 2, m_F = -2\rangle$$

$$|r\rangle = |70S_{1/2}, J = 1/2, m_J = -1/2\rangle$$

System of ^{87}Rb atoms controllably coupled to their Rydberg excited state.

The van dar Walls interaction between two atoms in their excited (Rydberg) states is denoted by V and is a tunable parameter.

One can vary the detuning parameter Δ which allows one to preferentially put the atom in a Rydberg or ground state

Effective low-energy description

$$\frac{\mathcal{H}}{\hbar} = \sum_i \frac{\Omega_i}{2}\sigma_x^i - \sum_i \Delta_i n_i + \sum_{i<j} V_{ij} n_i n_j,$$

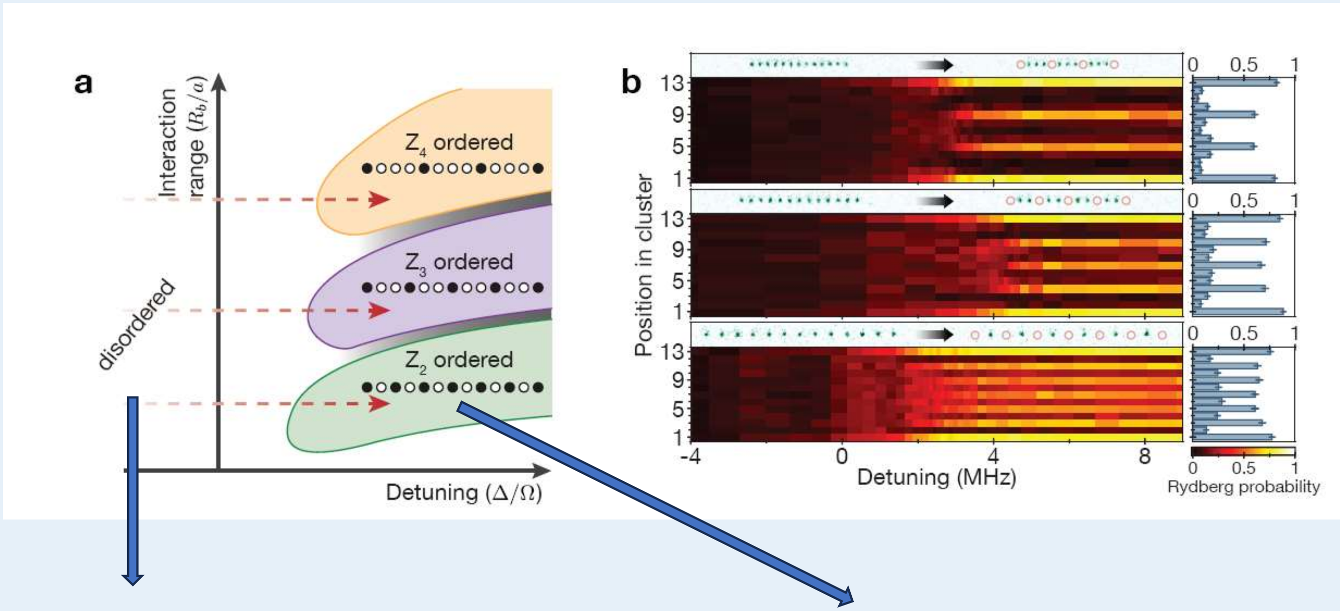
$n = (1+\sigma^z)/2$

$V_{ij} = V_0 / |r_{ij}|^6$

V_0 can be tuned so that Rydberg excitations in neighbouring sites are forbidden.

H. Bernien at al. Nature 2017

H. Bernien at al. Nature 2017



Rydberg vacuum (|0>) for $\Delta >> \Omega$ and $\Delta < 0$

Z_2 (|Z_2>) state for $\Delta >> \Omega$ and $\Delta > 0$.

These states are separated by an Ising transition

Similar to the transition found in tilted optical lattice

S. Sachdev et al, PRB 66, 075128 (2002).

Mapping to a constrained model

$$\frac{\mathcal{H}}{\hbar} = \sum_i \frac{\Omega_i}{2}\sigma_x^i - \sum_i \Delta_i n_i + \sum_{i<j} V_{ij} n_i n_j,$$

Two states per site: Natural spin ½ representation

$$\hat{n}_j = (1+\sigma_j^z)/2. \quad \sigma_j^x = (|g_j\rangle\langle r_j| + \text{h.c.})$$

Rydberg blockade on neighboring sites: $V_{i,i+1} >> \Delta, \Omega >> V_{i,i+2}$

$$P_\ell = (1-\sigma_\ell^z)/2$$

A up-spin (Rydberg excitation) can be created on a site if and only if there are no up-spins (excitations) on the neighboring sites

$$\frac{\mathcal{H}}{\hbar} = \sum_i \frac{\Omega_i}{2}\sigma_x^i - \sum_i \Delta_i n_i + \sum_{i<j} V_{ij} n_i n_j,$$

→

$$\begin{aligned} H_{\text{spin}} &= -w\sum_\ell P_{\ell-1}\sigma_\ell^x P_{\ell+1} + \lambda/2\sum_\ell \sigma_\ell^z \\ &= \sum_\ell(-w\tilde{\sigma}_\ell^x + \lambda/2\sigma_\ell^z) \end{aligned}$$

Introduction to ETH

Consider a generic state of quantum non-integrable many-body system

$$|\psi(t)\rangle = \sum_m C_m \mathrm{e}^{-iE_m t}|m\rangle,$$

D'Alessio et. al
Adv. Phys. 65, 239 (2016)

The time evolution of a generic operator for this state is given by

$$O(t) \equiv \langle\psi(t)|\hat{O}|\psi(t)\rangle = \sum_{m,n} C_m^* C_n e^{i(E_m-E_n)t} O_{mn}$$
$$= \sum_m |C_m|^2 O_{mm} + \sum_{m,n\neq m} C_m^* C_n e^{i(E_m-E_n)t} O_{mn}$$

$$O_{mn} = \langle m|\hat{O}|n\rangle.$$

Issues with long-time behavior:

a) *The steady state value of O(t) depends on the overlap coefficients: no thermalization (in the sense that the value does not agree with standard ME prediction)*

a) *It takes an incredibly long time to reach the steady state (predicts a very large relaxation time).*

Invoking random matrix theory remedies these problems since within RMT O_{mm}= O' and O_{mn}=0. However, it provides an energy independent answer which does not agree with standard numerical results.

Eigenstate Thermalization Hypothesis

Generalization of the RMT result for the matrix elements of a "typical" operator

$$O_{mn} = O(\bar{E})\delta_{mn} + e^{-S(\bar{E})/2} f_O(\bar{E},\omega) R_{mn},$$

$$\bar{E} \equiv (E_m + E_n)/2,$$

Both O and f_O are smooth functions of their arguments, S is the entropy, and R is a gaussian random number drawn from a normal distribution.

It states that for a large-enough system, the answer is nearly identical to that obtained using a microcanonical ensemble at the average energy.

$$\bar{O} \equiv \lim_{t_0\to\infty} \frac{1}{t_0}\int_0^{t_0} \mathrm{d}t\, O(t) = \sum_m |C_m|^2 O_{mm} = \mathrm{Tr}[\hat{\rho}_{\mathrm{DE}}\hat{O}],$$

$$O_{\mathrm{ME}} = \mathrm{Tr}\left[\hat{\rho}_{\mathrm{ME}}\hat{O}\right]$$

$$\bar{O} \simeq O(\langle E\rangle) \simeq O_{\mathrm{ME}}.$$

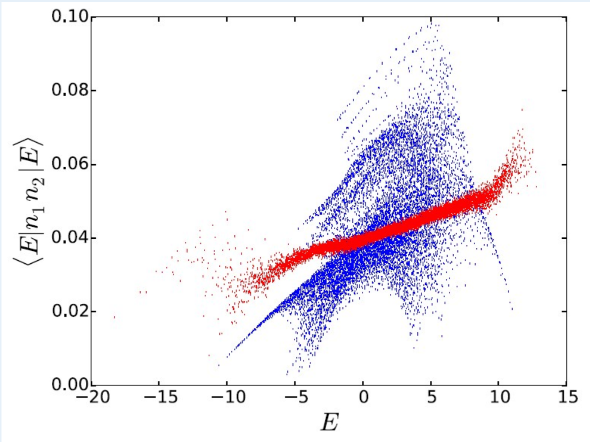
This relies on the fact that energy fluctuations in a many-body system are subextensive.

$$O_{mm} \approx O(\langle E\rangle) + (E_m - \langle E\rangle)\left.\frac{dO}{dE}\right|_{\langle E\rangle} + \frac{1}{2}(E_m - \langle E\rangle)^2 \left.\frac{d^2O}{dE^2}\right|_{\langle E\rangle}$$

$$\overline{O} \approx O(\langle E\rangle) + \frac{1}{2}(\delta E)^2 O''(\langle E\rangle) \approx O_{\mathrm{ME}} + \frac{1}{2}\left[(\delta E)^2 - (\delta E_{\mathrm{ME}})^2\right] O''(\langle E\rangle),$$

Violation of ETH

1. ***Integrable models: Presence of large number of conserved quantities lead to loss of ergodicity and prevents realization of long-time thermal steady states.***



2. ***Many-body localization: The system becomes non-ergodic due to strong disorder leading to localization of all states in its Hilbert space.***

3. ***Violation of ETH due to presence of a special class of eigenstates in its Hilbert space leading to long-time coherent oscillations: Quantum scars.***

4. ***Violation of ETH due to fragmentation of Hilbert space leading to loss of ergodicity: Strong Hilbert space fragmentation.***

Quantum scars

Scars from two-magnon states in a S=1 spin chain

Chandran et. al
Ann. Rev. Con. Mat. 14, 443 (2022)

$$H = J\,H_{\mathrm{XY}} + h\,S^z = J\sum_r (S_r^x S_{r+1}^x + S_r^y S_{r+1}^y) + h\sum_r S_r^z,$$

The local basis is denoted by |+>, |-> and |0> which are eigenstates of S_z with eigenvalues 1,-1 and 0.

The two polarized states are eigenstates of the model. They can be used as starting point of creating other states.

$$|\Omega\rangle = \bigotimes_{r=1}^{L} |-_r\rangle$$
$$|\Omega'\rangle = \bigotimes_{r=1}^{L} |+_r\rangle$$

One can create single and two-magnon states in this model which are eigenstates of H

$$|+,k\rangle = \frac{1}{\sqrt{2L}}\sum_{r=1}^{L} e^{ikr} S_r^+ |\Omega\rangle$$
$$|-,k\rangle = \frac{1}{\sqrt{2L}}\sum_{r=1}^{L} e^{ikr} S_r^- |\Omega'\rangle,$$

$E_{+(-)}$= +(-)h(L-1) + 2J cos k

←

$$h_{r,r+1}^{\mathrm{XY}}|--\rangle = h_{r,r+1}^{\mathrm{XY}}|++\rangle = 0,$$
$$h_{r,r+1}^{\mathrm{XY}}|+-\rangle = h_{r,r+1}^{\mathrm{XY}}|-+\rangle = |00\rangle.$$

→

$$|\mathcal{S}_n\rangle = \mathcal{N}(n)(J^+)^n|\Omega\rangle,$$
$$J^{\pm} = \frac{1}{2}\sum_{r=1}^{L} e^{i\pi r}(S_r^{\pm})^2.$$
$$|\mathcal{S}_1\rangle \propto \sum_{r=1}^{L}(-1)^r|\cdots - +_r - \ldots\rangle,$$

E_n= h(2n-L)

These bimagnon states yield a separate tower of states which violate ETH

→

Example of spectrum generating algebra
$[H, Q^+] = \omega\, Q^+$

The bimagnon operators form a spectrum generating algebra

$$[J^+, J^-] = 2J^z, \qquad [J^z, J^\pm] = \pm J^\pm,$$

$$J^z = \frac{1}{2}\sum_{r=1}^{L} S_r^z = \frac{1}{2}S^z.$$

$$J^\pm |\mathcal{S}_n\rangle = \sqrt{j(j+1) - m(m\pm 1)}\, |\mathcal{S}_{n\pm 1}\rangle,$$

j=L/2 and m= n-L/2

Thus they form a spin L/2 representation of a SU(2) algebra. This separates these states From the rest of the spectrum and leads to violation of ETH.

The ETH violation becomes evident from the fact that

$$\frac{4}{L^2}\langle \mathcal{S}_n|J^- J^+|\mathcal{S}_n\rangle = \left[1 - \frac{(2n-L)^2}{L^2}\right] + O\left(\frac{1}{L}\right).$$

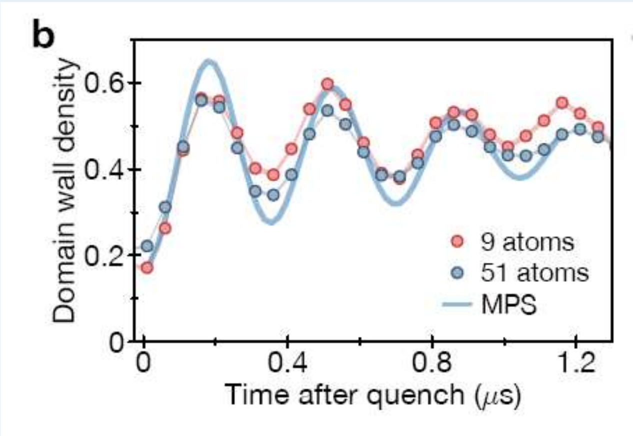
→ ***This is in contrast to the ETH predicted 1/L decay of such correlators; C ~ 4/(3L)***

$$\lim_{L\to\infty} S_A = \frac{1}{2}\left(\ln\frac{\pi L}{8} + 1\right),$$

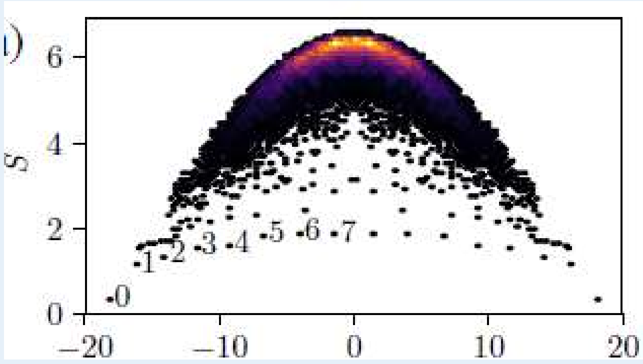
→ ***Mid-spectrum states yields sub-extensive entanglement entropy: athermal nature.***

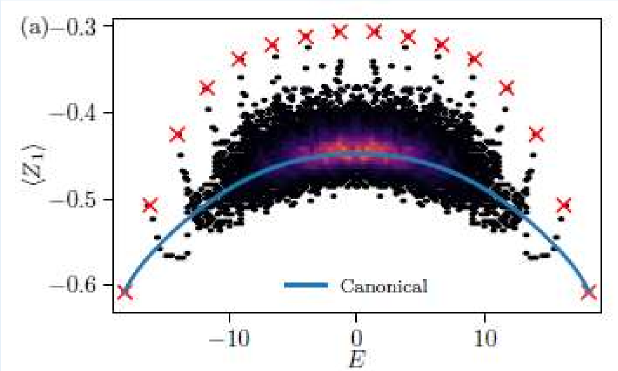
The presence of the SU(2) algebra allows one to form a closed subspace leading to loss of ergodicity

→ ***Coherent oscillatory dynamics starting from states with large overlap with states in scar subspace in Contrast to ETH predicted thermalization.***



Scars in Rydberg chains



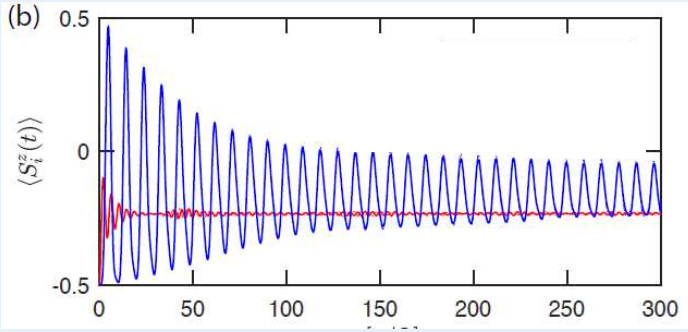


Experiments with Rydberg chains

These states are athermal; consequently the dynamics do not show signs of thermalization for very long times. This leads to long time quantum coherence and violation of ETH,

They violate ETH and have half-chain entanglement entropy which do not obey volume law : S ~ln L

They usually form a closed subspace In the Hilbert space and do not have significant overlap with other ETH obeying states.

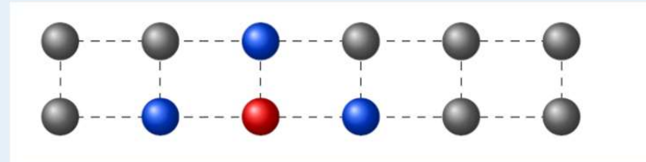


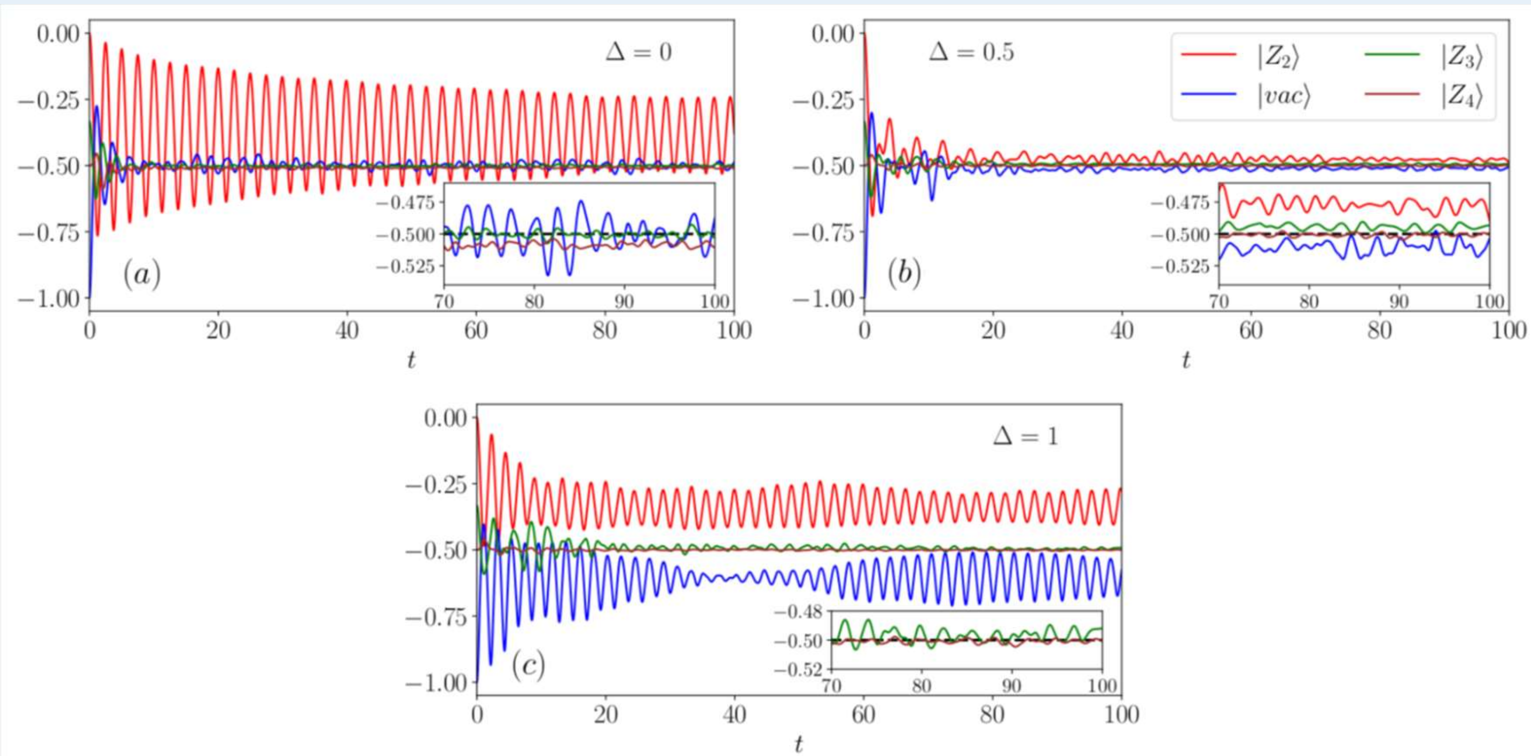
The dynamics can be approximated as coherent revivals between two Fock states; the Neel (Z_2) state and its time-reversed partner (FSA picture).

Scars in staggered ladders

Scars in Rydberg ladder: Magnetization density

$$\hat{\mathcal{H}} = -\Delta \sum_{j=1}^{L} \sum_{a=1}^{2} (-1)^j \hat{\sigma}^z_{j,a} - w \sum_{j=1}^{L} \sum_{a=1}^{2} \hat{\tilde{\sigma}}^x_{j,a},$$

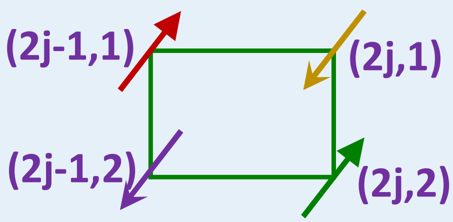




The scars at $\Delta = 1$ are qualitatively different from their $\Delta = 0$, PXP counterpart

Evolution of the Magnetization density as a function of time

Local Magnetization dynamics



The local magnetization is given by

$$M^{z[x]}_{j,a}(t) = \langle\psi(t)|\hat{\sigma}^z_{j,a}[\tilde{\sigma}^x_{j,a}]|\psi(t)\rangle$$

For the initial Z_2 state

$$M^z_{2j-1,1}(t) = M^z_{2j,2}(t) \neq M^z_{2j,1}(t) = M^z_{2j-1,2}(t)$$

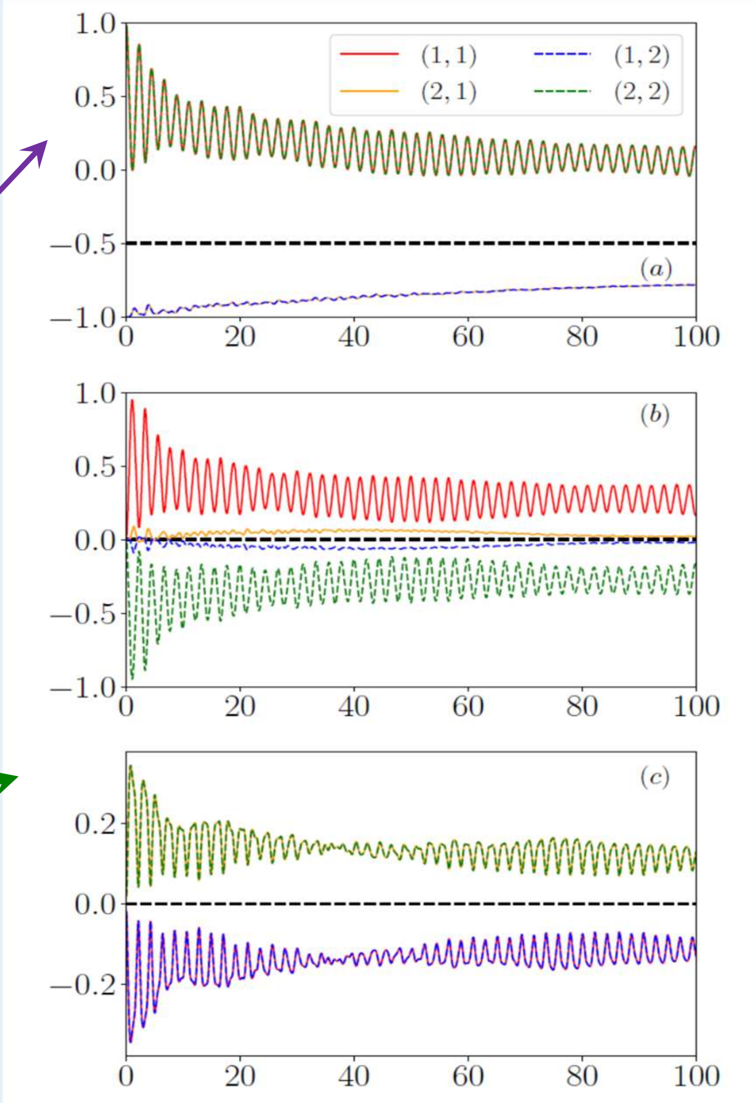
$$M^x_{2j-1,1}(t) = -M^x_{2j,2}(t) \qquad M^x_{2j,1}(t) = -M^x_{2j-1,2}(t)$$

$$|M^x_{2j,2}(t)| \ll |M^x_{2j,1}(t)|.$$

For the initial vacuum state

$$M^x_{2j-1,1}(t) = M^x_{2j-1,2}(t) = -M^x_{2j,1}(t) = -M^x_{2j,2}(t)$$

Spins at different sites converge to either superthermal or subthermal values at long time thus generating imbalance which persist at long times

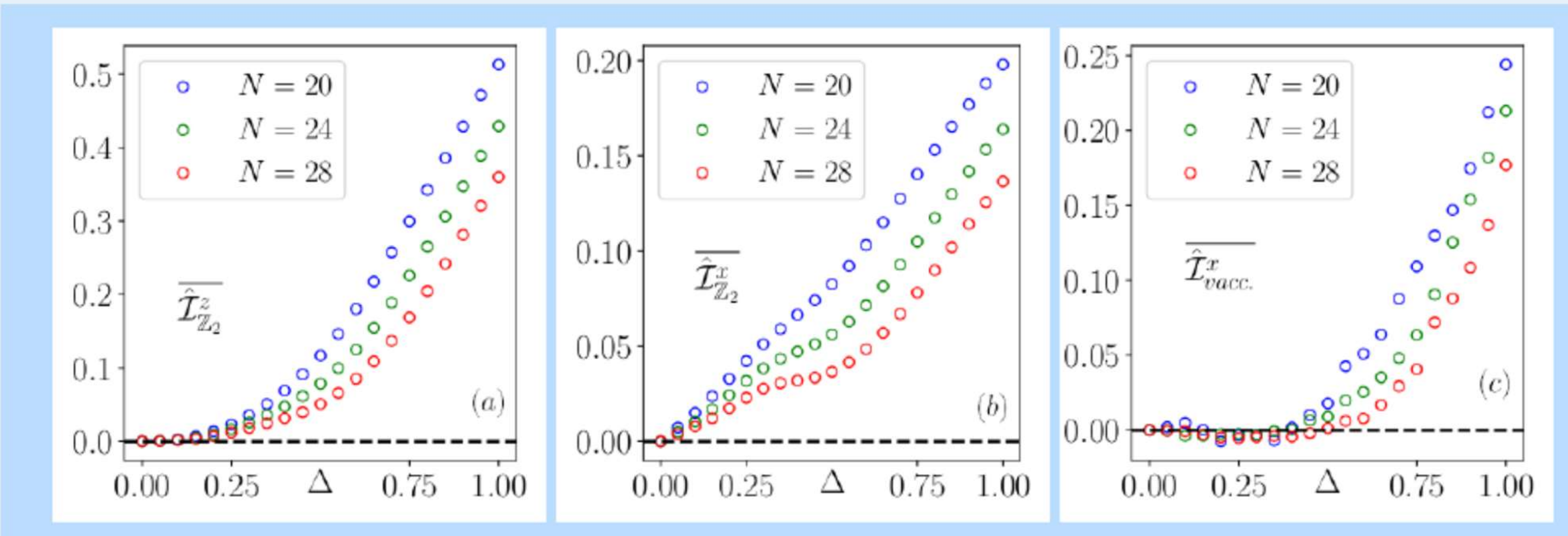


Long-time imbalance

$$\overline{\hat{\mathcal{A}}} = \lim_{T\to\infty}\frac{1}{T}\int_0^T \langle\psi(t)|\hat{\mathcal{A}}|\psi(t)\rangle\,dt = \langle\!\langle\hat{\mathcal{A}}\rangle\!\rangle_\beta,$$

β=0 →

$$\overline{\hat{\mathcal{A}}} = \sum_{\mu\notin\mathscr{H}_0} |\langle\psi(0)|E_\mu\rangle|^2\,(\hat{\mathcal{A}})_{\mu\mu} + \sum_{\mu\in\mathscr{H}_0} |\langle\psi(0)|E_\mu^{\mathcal{A}}\rangle|^2(\hat{\mathcal{A}})_{\mu\mu}.$$



$$\hat{\mathcal{I}}^z_{\mathbb{Z}_2} = \frac{1}{L}\sum_{j=1}^{L}\sum_{a=1}^{2}(s_1)_{ja}\,\hat{\sigma}^z_{j,a},$$

$$(s_1) = \begin{pmatrix} +-+-\cdots \\ -+-+\cdots \end{pmatrix},$$

$$\hat{\mathcal{I}}^x_{\mathbb{Z}_2} = \frac{1}{L}\sum_{j=1}^{L}\sum_{a=1}^{2}(s_2)_{ja}\,\hat{\tilde{\sigma}}^x_{j,a},$$

$$(s_2) = \begin{pmatrix} ----\cdots \\ ++++\cdots \end{pmatrix}$$

$$\hat{\mathcal{I}}^x_{\text{vac}} = \frac{1}{L}\sum_{j=1}^{L}\sum_{a=1}^{2}(s_3)_{ja}\,\hat{\tilde{\sigma}}^x_{j,a}.$$

$$(s_3) = \begin{pmatrix} +-+-\cdots \\ +-+-\cdots \end{pmatrix}$$

These scars lead to long-time imbalance around Δ=1 in an otherwise ergodic clean system

Dynamics of Fidelity and Shannon entropy

$$\mathcal{F}(t) = |\langle\psi(t)|\psi(0)\rangle|^2 \qquad S_1(t) = -\sum_{\alpha=1}^{\mathcal{D}_{\mathrm{H}}} |\psi_\alpha(t)|^2 \log(|\psi_\alpha(t)|^2)$$

The dynamics shows a striking difference in behavior of Shannon entropy

For $\Delta = 0$, the fidelity shows revivals at specific times t*,2t*….

The Shannon entropy dips also dips at intermediate times between the revivals at t*/2, 3t*/2 …. This indicates a FSA picture where the state oscillates between two Fock states (for example Z_2 and its time reversed partner).

In contrast for Δ =1, the Shannon entropy dips only at revival times. It remains large at intermediate times.

This seems to suggest that the scar induced oscillations do not follow a simple FSA picture of oscillation between two Fock states as in a Rydberg chain.

Nature of the scars in staggered ladder are fundamentally different compared to their chain counterparts.

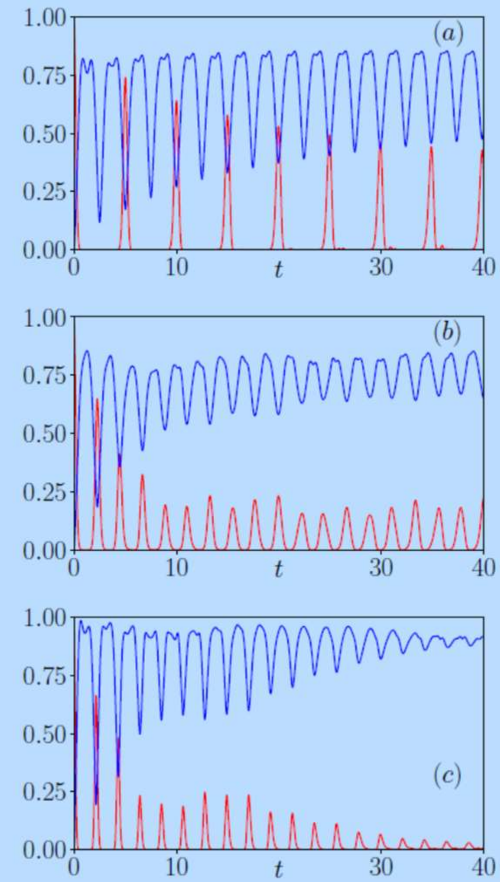
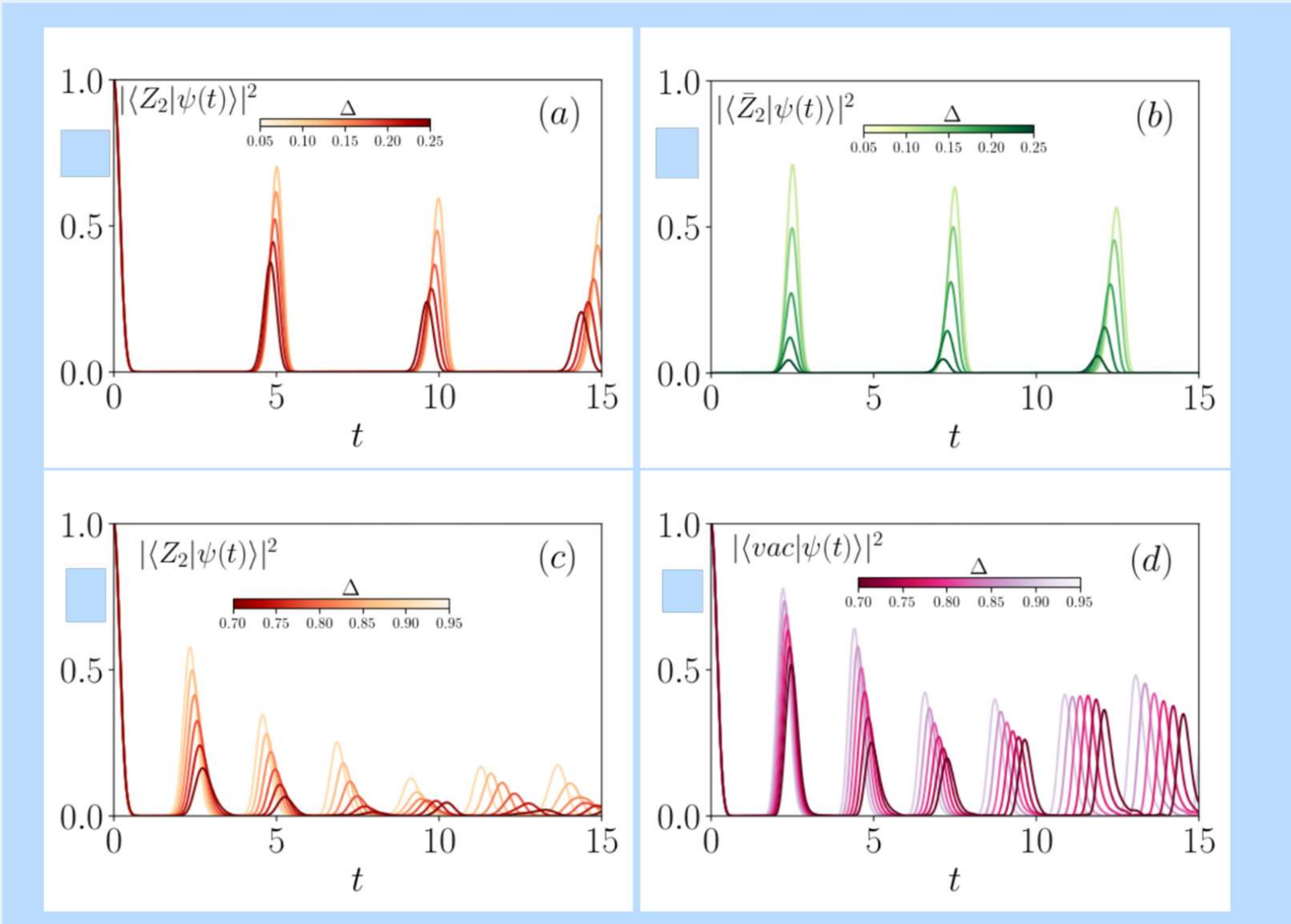


FIG. 4. Evolution of fidelity $\mathcal{F}(t)$ (in red) and (normalized) Shannon entropy $S_1/\log(\mathcal{D}_H)$ (in blue) of $N = 32$ ladder for (a) $\Delta = 0, |\psi(0)\rangle = |\mathbb{Z}_2\rangle$, (b) $\Delta = 1, |\psi(0)\rangle = |\mathbb{Z}_2\rangle$ and (c) $\Delta = 1, |\psi(0)\rangle = |\mathrm{vac}\rangle$.

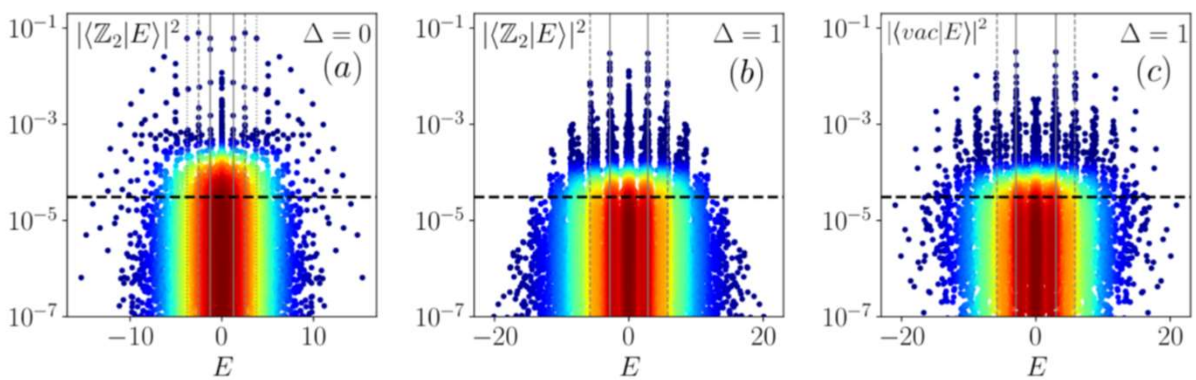
Fidelity Revival



Similar to scars in a Rydberg chain

Qualitatively different revival pattern

Fidelity oscillation and initial state overlap



Overlap with the initial state

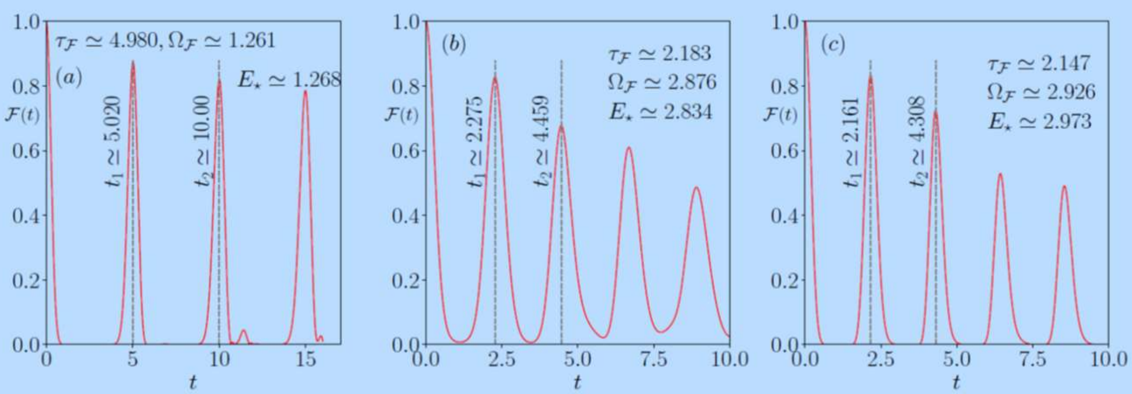


FIG. S7. Plot of the fidelity $\mathcal{F}(t)$ as a function of t showing time periods of fidelity revivals for (a) $\Delta = 0, |\psi(0)\rangle = |\mathbb{Z}_2\rangle$, (b) $\Delta = 1, |\psi(0)\rangle = |\mathbb{Z}_2\rangle$ and (c) $\Delta = 1, |\psi(0)\rangle = |\text{vac}\rangle$. For all plots $N = 28$ and $w = \hbar = 1$.

Fidelity oscillations

Chirality operators and zero-energy states

Chirality operators and Imbalance

The Rydberg ladder with staggered detuning hosts two chirality operators $\mathcal{C}_1$ and $\mathcal{C}_2$

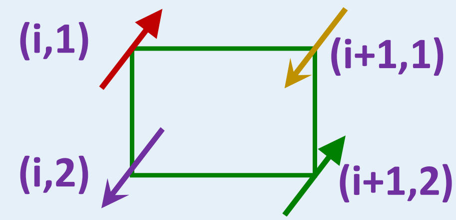
$$\hat{\mathcal{C}} = \prod_{j=1}^{L} \prod_{a=1}^{2} \hat{\sigma}^z_{j,a}, \quad \hat{\mathcal{C}}_1 = \hat{T}_x \hat{\mathcal{C}}, \quad \hat{\mathcal{C}}_2 = \hat{T}_x \hat{T}_y \hat{\mathcal{C}}.$$

$$\{\hat{\mathcal{C}}_1, \hat{\mathcal{H}}\} = \{\hat{\mathcal{C}}_2, \hat{\mathcal{H}}\} = 0.$$

For all finite eigenstates |E>, the action of these operator yield |-E>

$$\begin{aligned}\langle E_\mu | \hat{\sigma}^z_{i,a} | E_\nu \rangle &= \langle E_\mu | \hat{\mathcal{C}}_1^{-1} \hat{\mathcal{C}}_1 \hat{\sigma}^z_{i,a} \hat{\mathcal{C}}_1^{-1} \hat{\mathcal{C}}_1 | E_\nu \rangle = \langle E_\mu | \hat{\mathcal{C}}_1^\dagger \left(\hat{\mathcal{C}}_1 \hat{\sigma}^z_{i,a} \hat{\mathcal{C}}_1^{-1} \right) \hat{\mathcal{C}}_1 | E_\nu \rangle = \langle -E_\mu | \left(\hat{T}_x \hat{\mathcal{C}} \hat{\sigma}^z_{i,a} \hat{\mathcal{C}}^{-1} \hat{T}_x^{-1} \right) | -E_\nu \rangle \\ &= \langle -E_\mu | \left(\hat{T}_x \hat{\sigma}^z_{i,a} \hat{T}_x^{-1} \right) | -E_\nu \rangle = \langle -E_\mu | \hat{\sigma}^z_{i+1,a} | -E_\nu \rangle .\end{aligned}$$

$$\langle E_\mu | \hat{\sigma}^z_{i,a} | E_\nu \rangle = \langle -E_\mu | \hat{\sigma}^z_{i+1,\bar{a}} | -E_\nu \rangle .$$



For the zero modes, whose presence is guaranteed from the existence of these chirality operators via index theorem there is no such mapping; it just leads to a modifed set zero modes within the zero-energy subspace

$$\hat{\mathcal{C}}_q \, |\Phi_{\mu_0}\rangle = |\Phi_{\mu_0^*}\rangle .$$

$$\langle \Phi_{\mu_0^*} | \Phi_{\nu_0^*} \rangle = \langle \Phi_{\mu_0} | \hat{\mathcal{C}}_q^\dagger \hat{\mathcal{C}}_q | \Phi_{\nu_0} \rangle = \langle \Phi_{\mu_0} | \hat{\mathcal{C}}_q^{-1} \hat{\mathcal{C}}_q | \Phi_{\nu_0} \rangle = \delta_{\mu_0,\nu_0}$$

The matrix elements of spin operators thus satisfy

$$\langle \Phi_{\mu_0} | \hat{\sigma}^z_{i,a} | \Phi_{\nu_0} \rangle = \langle \Phi^*_{\mu_0} | \hat{\sigma}^z_{i+1,a} | \Phi^*_{\nu_0} \rangle = \langle \Phi^*_{\mu_0} | \hat{\sigma}^z_{i,\bar{a}} | \Phi^*_{\nu_0} \rangle .$$

Z imbalance

For staggered ladders both Z_2 and its time reversed partners have zero expectation values which implies

$$|\langle\psi(0)|E_\mu\rangle|^2 = |\langle\psi(0)| - E_\mu\rangle|^2,$$

$$\langle\psi(0)|\hat{\mathcal{H}}|\psi(0)\rangle = \sum_{\mu=1}^{\mathcal{D}_H} E_\mu \left|\langle\psi(0)|E_\mu\rangle\right|^2 = \sum_{\mu\notin\mathscr{H}_0} E_\mu \left|\langle\psi(0)|E_\mu\rangle\right|^2 = 0.$$

The long time z-imbalance is defined as

$$\hat{\mathcal{I}}^z_{|\mathbb{Z}_2\rangle} = \frac{1}{L}\sum_{r=1}^{L/2} \hat{T}_x^{2r}\left(\hat{\sigma}^z_{1,1} - \hat{\sigma}^z_{2,1} - \hat{\sigma}^z_{1,2} + \hat{\sigma}^z_{2,2}\right).$$

It receives contribution from expectation values of σ^z operators given by

$$\overline{\hat{\sigma}^z_{i,a}} = \lim_{T\to\infty}\frac{1}{T}\int_0^T dt\, \langle\hat{\sigma}^z_{i,a}(t)\rangle = \sum_{\mu\notin\mathscr{H}_0} |\langle\psi(0)|E_\mu\rangle|^2\, (\hat{\sigma}^z_{i,a})_{\mu\mu} + \sum_{\mu\in\mathscr{H}_0}\sum_{\nu\in\mathscr{H}_0} \langle\psi(0)|E_\mu\rangle\, \langle E_\nu|\psi(0)\rangle\, (\hat{\sigma}^z_{i,a})_{\mu\nu}.$$

The first term vanishes due to the chirality operators →

$$\sum_{\mu\notin\mathscr{H}_0} |\langle\psi(0)|E_\mu\rangle|^2\, (\hat{\sigma}^z_{i,a})_{\mu\mu} = \sum_{\mu\notin\mathscr{H}_0} |\langle\psi(0)|E_\mu\rangle|^2\, (\hat{\sigma}^z_{i+1,a})_{\mu\mu}.$$

The entire contribution to the z-imbalance for the Z_2 initial state comes from the zero-energy eigenstates → Existence of anamolous zero energy eigenstates with high overlap with an initial Fock state near Δ =1.

X imbalance

A similar analysis can be carried out for r the x imbalance

$$\begin{aligned}\langle E_\mu|\hat{\tilde{\sigma}}^x_{i,a}|E_\nu\rangle &= \langle E_\mu|\hat{\mathcal{C}}_1^{-1}\hat{\mathcal{C}}_1\hat{\tilde{\sigma}}^x_{i,a}\hat{\mathcal{C}}_1^{-1}\hat{\mathcal{C}}_1|E_\nu\rangle = \langle E_\mu|\hat{\mathcal{C}}_1^\dagger\left(\hat{T}_x\hat{\mathcal{C}}\hat{\tilde{\sigma}}^x_{i,a}\hat{\mathcal{C}}^{-1}\hat{T}_x^{\;-1}\right)\hat{\mathcal{C}}_1|E_\nu\rangle \\ &= -\langle -E_\mu|\left(\hat{T}_x\hat{\tilde{\sigma}}^x_{i,a}\hat{T}_x^{\;-1}\right)|-E_\nu\rangle = -\langle -E_\mu|\hat{\tilde{\sigma}}^x_{i+1,a}|-E_\nu\rangle.\end{aligned}$$

$$\langle E_\mu|\hat{\tilde{\sigma}}^x_{i,a}|E_\nu\rangle = -\langle -E_\mu|\hat{\tilde{\sigma}}^x_{i+1,\bar{a}}|-E_\nu\rangle.$$

For the Z2 initial state one thus have contribution only for the zero energy eigenstates. →

$$\hat{\mathcal{I}}^x_{|\mathbb{Z}_2\rangle} = \frac{1}{L}\sum_{r=1}^{L/2}\hat{T}_x^{2r}\left(-\hat{\tilde{\sigma}}^x_{1,1}-\hat{\tilde{\sigma}}^x_{2,1}+\hat{\tilde{\sigma}}^x_{1,2}+\hat{\tilde{\sigma}}^x_{2,2}\right).$$

$$\sum_{\mu\notin\mathscr{H}_0}|\langle\psi(0)|E_\mu\rangle|^2\left(\hat{\mathcal{I}}^x_{|\mathbb{Z}_2\rangle}\right)_{\mu\mu} = 0.$$

However, for the vacuum initial state the imbalance receives contribution from the finite energy states →

$$\hat{\mathcal{I}}^x_{|\mathrm{vac}\rangle} = \frac{1}{L}\sum_{r=1}^{L/2}\hat{T}_x^{2r}\left(\hat{\tilde{\sigma}}^x_{1,1}+\hat{\tilde{\sigma}}^x_{1,2}-\hat{\tilde{\sigma}}^x_{2,1}-\hat{\tilde{\sigma}}^x_{2,2}\right),$$

The imbalance starting from a vacuum state does not originate from translational symmetry breaking at t=0; this imbalance is scar induced and completely emergent.

Simultaneous zero modes in quantum many-body systems

Consider a generic many-body Hamiltonian $H = H_0 + \lambda H_1$ where $[H_0, H_1]$ does not vanish.

A generic zero-energy eigenstate (zero mode) of H satisfies $H|0\rangle = 0$

These modes are usually solutions at specific values of λ and changing λ destabilizes them.

The mid-spectrum modes are usually ETH obeying.

However, it may happen that a linear combination of zero modes of H_0 also become a zero mode of H_1

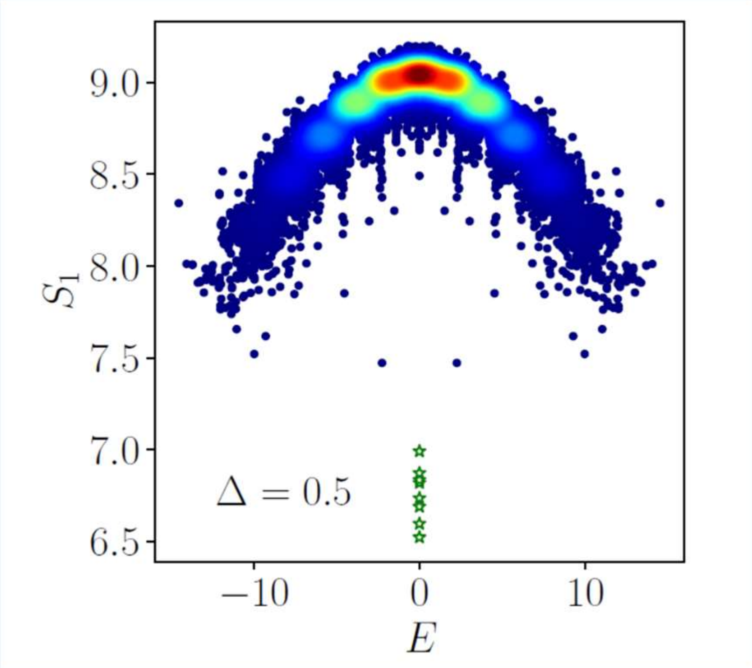
If such modes exist, they remain zero mode for all λ

The existence of such modes necessarily leads to ETH violation.

These are not generic and they have not been found in a standard Rydberg chain.

Their existence have been shown, for example, in U(1) lattice gauge theory models.
[Banerjee and Sen, PRL (2021), Biswas, Banerjee and Sen (2022)]

Presence of simultaneous zero-energy modes



These are simultaneous zero modes of both the staggered on-site and the constrained spin-flip terms

These zero modes have anomalously low Shannon entropy (localization in Hilbert space) and they necessarily violate ETH.

These modes have no analogue for the PXP chain.