Positivity Constraints on EFT

B. Bellazzini (IPhT)
@ ICTS's School in Bangalore
15–19/04/2024

In this second L2 lecture, we show that the missing ingredient alluded to in L1 is: subluminality (as proxy for causality)

— U(1)-GB: $\pi \to \pi + c$ symmetric —

(1) $$\mathcal{L} = \frac{1}{2}(\partial\pi)^2 + \frac{(\partial\pi)^4}{4M^4}c_2 - \frac{(\partial\pi)^2\Box\pi}{2M^3}c_3 + \dots$$

e.o.m.

(2) $$\partial_\mu\left(-\partial^\mu\pi\left(1+\frac{(\partial\pi)^2}{M^4}c_2\right)\right) + \frac{c_3}{M^3}\left[\partial_\mu\left(\partial^\mu\pi\,\Box\pi\right) - \frac{\Box(\partial\pi)^2}{2}\right] + \dots = 0$$

(3) $\Rightarrow \begin{cases} \bar{\pi} = c + V_\mu x^\mu \\ \partial_\mu\bar{\pi} = V_\mu \end{cases}$ with $V_\mu = \text{const}_\mu$ is solution

to analize it within EFT we take $V_\mu^2/M^4 \ll 1$

Let's consider now small perturbations: $\pi = \bar{\pi} + \phi$ at $O(\phi^2)$:

(4) $$\mathcal{L}^{(2)}_\phi = \frac{1}{2}(\partial\phi)^2 + \frac{c_2}{2M^4}\left[(\partial\phi)^2V_\rho^2 + 2(V_\mu\partial^\mu\phi)^2\right] - \frac{c_3}{M^3}\Big[\underbrace{(V_\mu\partial^\mu\phi)\Box\phi}_{\text{total }\partial} + \underbrace{(\partial\phi)^2\cancel{\Box V}}_{\text{vanish}}\Big]$$

$$= \frac{1}{2}\phi\left[-\eta^{\mu\nu}\Big(1+\underbrace{\cancel{\frac{c_2}{M^4}V_\rho^2}}_{\text{small}}\Big) - 2\frac{c_2}{M^4}V^\mu V^\nu\right]\partial_\mu\partial_\nu\phi$$

where we can drop V_μ^2/M^4 corrections to terms present already, and we can drop all the $\partial\ldots\partial(\bar{\pi}) = \partial\ldots\partial V = 0$

The e.o.m. for the perturbations read

(5) $$-\left(\eta^{\mu\nu} + 2\frac{c_2}{M^4}V^\mu V^\nu\right)\partial_\mu\partial_\nu\phi = 0$$

Expanding in plane waves $\phi = e^{i k_\mu x^\mu}$

(6) $$k^2 + 2\frac{c_2}{M^4}(V\cdot k)^2 = 0 \quad \Rightarrow \quad \boxed{c_2 \geq 0}$$

to avoid superluminality

In practice, the new effective metric is

(7) $$\boxed{g_{\mu\nu} = \eta_{\mu\nu} - 2\frac{c_2}{M^4} V_\mu V_\nu}$$

Effective Light Cone

so that is broader for $c_2 < 0$ and narrower for $c_2 > 0$

Explicitly: $V_\mu = \partial_\mu \bar{\pi} = \alpha M^2 \delta_\mu^0 \qquad \bar{\pi} = \alpha M^2 t \qquad$ i.e. $\dot{\bar{\pi}} = \text{const}$

(8) $$g_{\mu\nu} = \eta_{\mu\nu} - 2c_2\alpha^2 \delta_\mu^0 \delta_\nu^0 = \left(\begin{array}{c|ccc} 1-2c_2\alpha^2 & & 0 & \\ \hline 0 & -1 & & \\ & & -1 & \\ & & & -1 \end{array}\right)_{\mu\nu}$$

"null" geodesic: $dt^2(1-2c_2\alpha^2) - d\vec{x}^2 = 0$

(9) $$\vec{v}^2 = \frac{d\vec{x}^2}{dt^2} = 1 - 2c_2\alpha^2 \quad \Rightarrow \quad \boxed{\delta \vec{v}^2 = -2c_2 \frac{V_\mu^2}{M^4}}$$

Remark: The V_μ can be made arbitrarily small to be arbitrarily well inside EFT.

— What about $c_2 \ll 1$ by Galileon Symmetry? —

Can we have $c_2 \geq 0$ with $c_2 \ll |c_3|$? Can c_3 be leading?

Let's work first in the regime c_2 negligible, so the eq. of motions read

(10) $$-\Box\pi + c_3\left[(\Box\pi)^2 - (\partial_\mu\partial_\nu\pi)^2\right] = 0$$

a solution of (10) is a plane wave traveling in n_i-direct.

(11) $$\begin{cases} \bar\pi = \pi_0(t - n_i x^i) \\ n_i n_i = 1 \end{cases}$$

$$\boxed{\begin{aligned} &\partial_\mu\bar\pi = \pi_0'(\delta_\mu^0 - n_i\delta_\mu^i);\ (\partial\bar\pi)^2 = 0 \\ &\partial_\mu\partial_\nu\bar\pi = \pi_0''(\delta_\mu^0 - n_i\delta_\mu^i)(\delta_\nu^0 - n_j\delta_\nu^j) \\ &\Box\bar\pi = \pi_0''(\eta^{00} + n_i n_j\eta^{ij}) = 0 \\ &(\partial_\mu\partial_\nu\bar\pi)^2 = \pi_0''^2(\eta^{00} + n_i n_j\eta^{ij})^2 = 0 \end{aligned}}$$

so that a perturbation around it $\pi = \bar\pi + \phi$

(12) $$\mathcal{L}^{(2)} = \frac{1}{2}(\partial\phi)^2\left(1 - \frac{c_3}{M^3}\cancel{\Box\bar\pi}\right) - \frac{c_3}{M^3}2\partial_\mu\bar\pi\,\partial^\mu\phi\,\Box\phi$$

$\downarrow$ e.o.m.

(13) $$-\Box\phi + \frac{2c_3}{M^3}\left(\underbrace{\partial_\mu(\partial^\mu\bar\pi\,\Box\phi)} - \underbrace{\Box(\partial_\mu\bar\pi\,\partial^\mu\phi)}\right) = 0$$

$$\underset{0}{\Box\bar\pi}\,\cancel{\Box\phi} + \cancel{\partial^\mu\bar\pi\,\partial_\mu\Box\phi} - (\cancel{\Box\partial_\mu\bar\pi})\partial^\mu\phi - \cancel{\partial_\mu\bar\pi\,\Box\partial^\mu\phi}$$

$$-2\partial_\mu\partial_\nu\bar\pi\,\partial^\mu\partial^\nu\phi$$

$\Updownarrow$

(14) $$\boxed{\left[\eta^{\mu\nu} + \frac{4c_3}{M^3}(\partial^\mu\partial^\nu\bar\pi)\right]\partial_\mu\partial_\nu\phi = 0}$$

so that looking for solutions $\phi = \phi_0(\underline{v}t - n_i x^i)$ we get

(15) $$\phi_0''\cdot\left[(v^2 - 1) + \frac{4c_3}{M^3}\pi_0''(v+1)^2\right] = 0 \implies \boxed{v = \frac{1 - \frac{4c_3}{M^3}\pi_0''}{1 + \frac{4c_3}{M^3}\pi_0''}}$$

Remark: regardless of sign of c_3, can always choose π_0'' such that $c_3\pi_0'' \lessgtr 0$, i.e. the sign isn't definite:

(16) $$\boxed{\delta v \simeq -\frac{8c_3\pi_0''}{M^3} \lessgtr 0}$$ the $\phi = \phi_0(vt - n_i x^i)$ can "rides" superluminally the bkg solution $\pi = \bar{\pi}$.

NOT sign definite!

subluminality $\Rightarrow$ Galileon symmetry $\pi \to \pi + c_\mu x^\mu$ can't be exact.

Question: How good an approximate symmetry galileon can be, consistently with subluminality?

Answer: it's never a good symmetry!

Indeed, let's turn on a c_2 – Galileon breaking – less irrelevant term: since $\pi = \bar{\pi} = \pi_0(t - n_i x^i)$ is still solution of new eq. of motion, we can look at the e.o.m. for the perturbations:

(16) $$\left[\eta^{\mu\nu} + \frac{4c_3}{M^3}(\partial^\mu\partial^\nu\bar{\pi}) + 2\frac{c_2}{M^4}\partial^\mu\bar{\pi}\,\partial^\nu\bar{\pi}\right]\partial_\mu\partial_\nu\phi = 0$$

$\phi = \phi_0(vt - n_i x^i)$

(17) $$(v^2 - 1) + \left[\frac{4c_3}{M^3}\pi_0'' + 2\frac{c_2}{M^4}\pi_0'^2\right](v+1)^2 = 0$$

(18) $$\boxed{v = \frac{1 - \left(\frac{4c_3}{M^3}\pi_0'' + 2\frac{c_2}{M^4}\pi_0'^2\right)}{1 + \left(\frac{4c_3}{M^3}\pi_0'' + 2\frac{c_2}{M^4}\pi_0'^2\right)}}$$

(for $\phi = \phi_0(-vt - n_i x^i)$ travelling opposite than π_0-wave the solution is $v = -1\,\frac{1+(\dots)}{1-(\dots)}$)

$\Downarrow$

always subluminal if $c_2 > 0$ & $\left|\frac{4c_3\pi_0''}{M^3}\right| < 2\frac{c_2\pi_0'^2}{M^2} \Rightarrow$ $\mathcal{L}$ in (1) not dominated by Galileon term

Lesson: Subluminality gives a new consistency condition on EFT "orthogonal" to power counting that a priori could be designed to make leading operator very irrelevant ones

Remarks: adding more irrelevant operators does not change the story since they are subleading corrections to the ones considered, as long as Π_0 is chosen such that $\Pi_0^{(n\geq 3)} < M\,\Pi_0^{(n-1)}$ and $\Pi_0''/M^3 \ll 1$, and one look for $\phi=\phi_0$ with $\phi_0^{(n\geq 2)} < \phi_0^{(n-1)} M$

__Example:__ $c_2\frac{(\partial\phi)^6}{M^8} \to \delta v^2 \sim \left(\frac{\Pi_0'^2}{M^4}\right)^2 c_2$, $\frac{(\partial\partial\phi)^4}{M^8} \to \delta v^2 \sim \left(\frac{\Pi_0''}{M^3}\right)\left(\frac{\phi_0''}{\phi_0' M}\right)^2$, ...

— superluminality resolvable? —

Yes! Take $\Pi_0'' \sim$ const over a region of space of size L where $\delta v|_{c_3} \sim c_3\frac{\Pi_0''}{M^3}$ wins over $\delta v|_{c_2} \sim c_2\frac{\Pi_0'^2}{M^4} \sim c_2\frac{(\Pi_0'' L)^2}{M^4}$.

$\Pi_0' \sim \Pi_0''\cdot L$

This L can't be larger than

$$(19)\qquad L^2 M^2 < \frac{c_3}{c_2}\left(\frac{M^3}{\Pi_0''}\right)$$

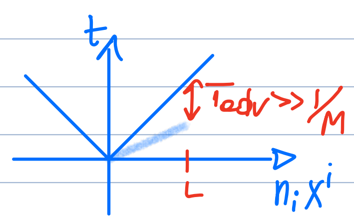
(clearly! c_2 less irrelevant than c_3!)

In that region (19) the time advance in units of cutoff $1/M$ is

$$(20)\qquad M\,T_{adv} \sim ML\cdot\delta v = \sqrt{\frac{c_3}{c_2}\left(\frac{M^3}{\Pi_0''}\right)}\cdot\left(\frac{c_3\,\Pi_0''}{M^3}\right) \sim |c_3|\left(\frac{c_3}{c_2}\cdot\frac{\Pi_0''}{M^3}\right)^{1/2}$$

which gives

$$(21)\qquad \boxed{T_{adv} \gg 1/M \quad \text{if} \quad c_3 \gg c_2}$$



The hierarchy $c_3 \gg c_2$ enforced by a symmetry is at odds with subluminality, resolvably so.

Remarks

Superluminality is here understood as bad, although it's not necessarily so, as long as one does not run in conflict with experience and/or form causal paradox via Time machines (mathematically, closed time- or light-like curves). This is hard however to analyze in a controlled way because boosting a signal to send it back in time deforms the bkg (to be contrasted with superluminality in Minkowski which is trivially inconsistent).
(for instance if waves travels with or against flow in (15), the net time delay vanishes)

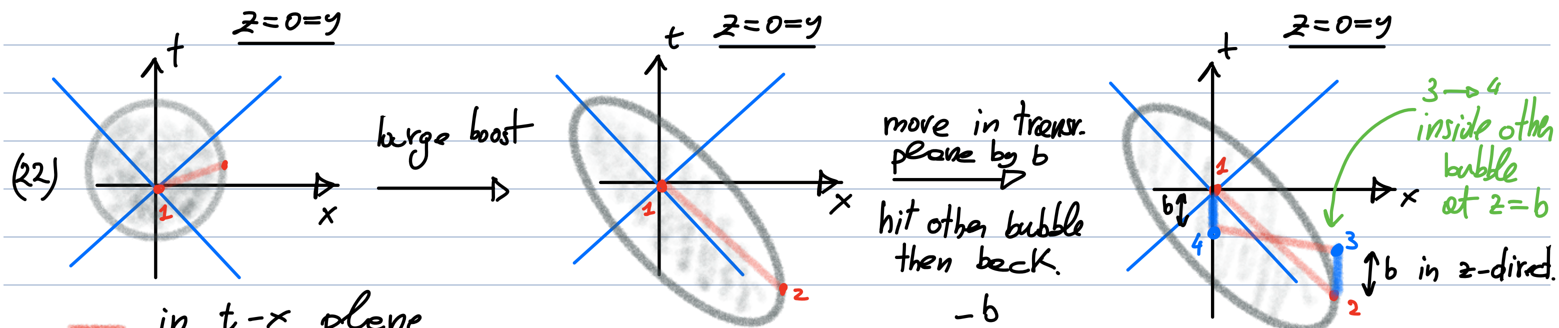
Time Machines: classic setup

two bubbles in two planes separated by impact parameter b

- bubble 1 sits in $(t, x, 0, 0)$-plane
- bubble 2 sits in $(t, x, 0, b)$-plane

] bubbles allows velocities $|v|, |v'| > 1$

- projecting everything on the first bubble plane:

(22) 

$1^\mu = (0, \vec{0}) \leftarrow 4^\mu = (\overbrace{-T+b+T'}^{-b}, 0, 0, b)$

$\downarrow \qquad\qquad\qquad \uparrow$

$2^\mu = (-T, vT, 0, 0) \rightarrow 3^\mu = (-T+b, vT, 0, b)$

$v > 1$

at $v' = -\dfrac{vT}{T-2b}$ in $\hat{x}$

$(|v'| > v > 1)$

While it's cool, it is also cumbersome to check on a case by case study if time machines are possible within EFT.

— Euler - Heisenberg —

The lesson from previous analysis on scalars is completely general. Let's repeat it quickly for the EFT of massless spin-1.

(23) $$\mathcal{L}_\gamma = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{a}{8M^4}(F_{\mu\nu}F^{\mu\nu})^2 + \dots$$ ← for simplicity we d

We could literally repeat the story, i.e. solve the e.o.m

(24) $$\partial_\mu \left(F^{\mu\nu}\left(1 - \frac{a}{M^4} F^2 + \dots\right)\right) = 0 \qquad F^2 = F_{\mu\nu}F^{\mu\nu}$$

for some bkg $\bar{F}_{\mu\nu}$, and then look for perturbations

(25) $$\begin{cases} F_{\mu\nu} = f_{\mu\nu} + \bar{F}_{\mu\nu} \\ \mathcal{L}^{(2)} = -\frac{1}{4} f_{\mu\nu}^2 + \frac{a}{4M^2}\left(f_{\mu\nu}^2 \bar{F}^2 + 2 f_{\mu\nu} f_{\rho\sigma} \bar{F}^{\mu\nu}\bar{F}^{\rho\sigma}\right) + \dots \end{cases}$$

Exercise: show that by suitable choice of bkg $\bar{F}_{\mu\nu}$ (e.g. $\bar{F}_{\mu\nu}$ constant) one must have $a > 0$ to avoid superluminality

Comment: In lower D, D = 3, the $a > 0$ and the positivity of $(\partial\pi)^4$ coefficient are literally the same condition, via Hodge duality. Specifically, let's enforce $F = dA$ i.e. $dF = 0$ as δ-function $\delta(\varepsilon^{\mu\nu\rho}\partial_\mu F_{\nu\rho}) = \int[d\pi]\, e^{i\pi\varepsilon^{\mu\nu\rho}\partial_\mu F_{\nu\rho}}$ to be inserted in path integral over F now:

(26) $$-F_{\mu\nu}^2/4 + \frac{a}{8M^3}(F_{\mu\nu}^2)^2 + \dots \xleftrightarrow{D=3} \mathcal{L}^* = -\frac{F_{\mu\nu}^2}{2} + A\,\partial_\mu\pi\,\varepsilon^{\mu\nu\rho}F_{\nu\rho} + \frac{a}{8M^3}(F_{\mu\nu}^2)^2$$

(27)
$$\begin{cases} \dfrac{\delta S}{\delta F_{\mu\nu}}=0 : -F_{\mu\nu}\left(1-\dfrac{a}{2M^3}F^2\right)+A\,\varepsilon_{\mu\nu\rho}\partial^\rho\pi = 0 \\ F_{\mu\nu} = A\,\varepsilon_{\mu\nu\rho}\partial^\rho\pi\left(1+\dfrac{a}{2M^3}A^2\underbrace{(\varepsilon_{\mu\nu\rho}\partial^\rho\pi)^2}_{(D-1)(\partial\pi)^2=(2\partial\pi)^2}+\dots\right) \\ F_{\mu\nu}^2 = 2A^2(\partial\pi)^2\left(1+\dfrac{2a}{M^3}A^2(\partial\pi)^2+\dots\right) \end{cases}$$

plug back-in

(28) $$\mathcal{L}^* = -A^2(\partial\pi)^2\left(1+\frac{2a}{M^3}A^2(\partial\pi)^2+\dots\right)+2A^2(\partial\pi)^2\left(1+\frac{a}{M^3}A^2(\partial\pi)^2+\dots\right)$$
$$+\frac{a}{8M^3}4A^4(\partial\pi)^4+\dots$$

(29) $$= A^2(\partial\pi)^2+a\frac{A^4}{M^3}(\partial\pi)^4\left(-2+2+\frac{1}{2}\right)\underset{A^2=1/2}{=}\frac{(\partial\pi)^2}{2}+\frac{a}{8M^3}(\partial\pi)^4$$

i.e. putting everything together:

(30) $$\mathcal{L}_\gamma^{D=3} - \frac{F_{\mu\nu}^2}{2}+\frac{(F_{\mu\nu}^2)^2 a}{8M^3}+\dots \quad \text{dual to} \quad \boxed{\mathcal{L}^*_{D=3} = \frac{(\partial\pi)^2}{2}+\frac{a}{8M^3}(\partial\pi)^4+\dots}$$

(31) $$\boxed{a \geq 0}$$

Comment In $D=4$: $\frac{(F\tilde{F})^2}{8M^4}\tilde{a}$ with $\tilde{a}\geq 0$ too, by (now familiar) causality arguments. In $D=3$ it is missing because $\tilde{F}$ is 1-form. The positivity of $\tilde{a}$, via $D=3$ argument, can be obtained via dimensional reduction by compactifying one dimension, because $A_\mu=(A_{\mu=0,1,2},\varphi)$ so the system is dual to 2 derivative coupled scalars (+ KK modes loops), see e.g. 1902.03250.

— Photons + Gravity —

Let's consider now an example with gravity in $D=4$:

(32) $$\mathcal{L}^{\text{Einstein-Maxwell}} = -\frac{F_{\mu\nu}^2}{2} + \frac{a}{8M^4}(F^2)^2 + \frac{\tilde{a}}{8M^4}(F\tilde{F})^2 + \dots$$

$$+\frac{M_{pl}^2}{2}\left(R + \frac{\alpha_3}{M^4}\,Rie^3 + \dots\right)$$

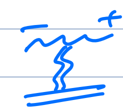
$$-\frac{\beta}{4M^2}F_{\mu\nu}F_{\rho\sigma}R^{\mu\nu\rho\sigma} + \dots$$

We are interested, say, on the size of β.

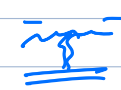
Remarks:

- From perturbative unitarity: we may naively expect that large β is ok. Explicitly, adding a neutral scalar (e.g. Schw. Black hole seen from far away) to source gravity

(33) $$\mathcal{M}(1^- 2^0 3^+ 4^0) \propto -\langle 12\rangle^2[23]^2\,\frac{1}{M_{pl}^2}\,\frac{1}{t} \sim \left(\frac{E}{M_{pl}}\right)^2$$

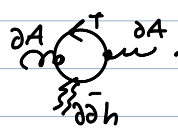


(34) $$\mathcal{M}(1^- 2^0 3^- 4^0) \propto \frac{\langle 13\rangle^2}{M_{pl}^2}\,\frac{\beta}{M^2}\,\frac{(s-u)^2}{t} + \dots \sim \left(\frac{E}{M_{pl}}\right)^2\left(\frac{E}{M}\right)^2\beta$$



so that $|\beta| \lesssim (M_{pl}/M)^2$ would seem ok ?

- Bread & Butter NDA.



$$\sim \frac{e^2 N}{16\pi^2}\frac{1}{M_{pl}}\int \frac{d^4k}{k^3}\,\frac{1}{k^4}\,k$$

minimal coup to $h_{\mu\nu}$

prop. matching ∂'s external

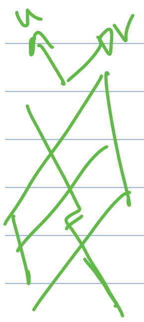
(35) $$\longrightarrow Rie\,FF\cdot\left(\frac{e^2N}{16\pi^2}\right)\cdot\frac{1}{M^2} \implies |\beta| \lesssim O(1) \text{ at best}$$

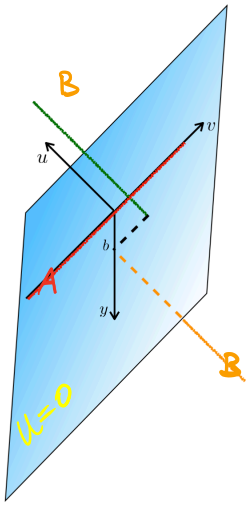
The (35) is confirmed by studying the propagation of light in the gravitational bkg generated by the scalar/black hole. If one takes a very boosted black hole (while sending its mass to zero to keep energy finite), the geometry seen by the photon is the one of a shock wave

(36) shock-wave in Mink.

$$ds^2 = \underbrace{\left(-du\,dv + d\vec{x}_\perp^{\,2}\right)}_{\text{Mink in Lightcone coordinate}} + du^2\,\delta(u)\,\Delta(\vec{x}_\perp), \qquad d\vec{x}_\perp^{\,2} = dy^2 + dz^2$$

jump at $u=0$





sources by $T_{\mu\nu}$ with only non-vanishing entry

$T_{uu} \propto E_A\,\delta(u)\,\delta(y)\,\delta(z)$ ← particle A localized on worldline $u=0=y=z$

$\Downarrow$

$$\boxed{\nabla_\perp^2 \Delta(\vec{x}_\perp) \propto \frac{E_A}{M_{pl}^2}\,\delta^2(\vec{x}_\perp)}$$

Poisson equation in $D=2$ (Lorentz contractions!)

$\Downarrow$

(37)
$$\boxed{\Delta(\vec{x}_\perp) = -\int \frac{d^2q}{(2\pi)^2}\, e^{-i\vec{q}_\perp\cdot\vec{x}_\perp}\, \frac{1}{\vec{q}_\perp^{\,2}}\, \frac{E_A}{M_{pl}^2} = -\frac{E_A}{\pi M_{pl}^2}\log\left(b/L_{IR}\right)}$$

where $b = |\vec{x}_\perp|$ is the impact parameter of the scattering

Comment: in $D=4$ (37) is IR-divergent: regulating it by putting the theory on AdS with $L_{AdS}\to\infty$ gives $\log\frac{b}{L_{IR}} \to \log b/L_{AdS}$

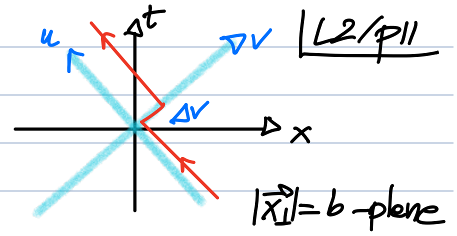
Geodesic: $ds^2=0$ for $\vec{x}_\perp = \vec{b}$ fixed and v fixed

"u" used as affine parameter

$$du\,dv = du^2\,\Delta(b)\,\delta(u) \Rightarrow \frac{dv}{du} = \Delta(b)\,\delta(u) \rightarrow \Delta V = \Delta(b)$$

(38) $$\boxed{\Delta V = \Delta(b) = -\frac{EA}{\pi M_{Pl}^2}\log(b/L_{IR}) > 0}$$

Time Delay



Comment:

- This can be seen as proof of attractiveness of gravity $\frac{E}{M_{Pl}^2} > 0$ from causality

- We could have guess it based on dim. analysis (R_s only relevant scale!)

(39) $$\boxed{\Delta T_{delay} = \Delta V \propto G_N M_{source} \sim R_s}$$

↳ boost it ΔT get γ-factor $M_{source} \to \gamma M \xrightarrow[M\to 0]{} E$-fixed

↳ missing only the $\log b$

Remark: adding now $\frac{\beta}{M^4} F_{\mu\nu} F_{\rho\sigma} R^{\mu\nu\rho\sigma}$ the shock wave is still a solution of the new e.o.m.

The difference is that one breaks the equivalence principle: different photon polarizations fall differently

(40) $$\nabla_\mu \left(F_{\mu\nu} + \frac{\beta}{M^2} R_{\mu\nu\rho\sigma} F^{\rho\sigma} \right) = 0$$

$$\Downarrow$$

~ again, basically dimensional analysis up to $O(1)$

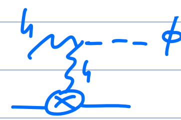
(41) $$\boxed{\Delta T^{\pm}_{delay} = \Delta V_{\pm} = -\frac{\overbrace{EA}^{R_s}}{\pi M_{Pl}^2} \left(\log b/L_{IR} \pm \frac{\beta}{M^2 b^2} \right)}$$

Subluminality: Since one can lower b up to $b \gtrsim 1/M$ at best there is no net (resolvable) time advance as long as $\beta \lesssim O(1)$

If $\beta >> 1$ instead, even for $b >> 1/M$ one could build time machines.

Let me end this L2 saying that this strategy of bounding operators in gravity is quite general, so that one can bound as well things like $\frac{\alpha_3 \mathrm{Rie}^3}{M^4}$ in (32) (1407.5597, 2211.00085,...)

$$\text{(42)} \qquad \boxed{\Delta T^{\pm}_{delay} = \Delta V_{\pm} = \frac{-E_A}{\pi M_{pl}^2}\left(\log b/L_{IR} \pm \frac{\alpha_3}{b^4 M^4}\right)}$$

and working harder also Rie^4 (2211.00085, altough easier in 1509.00851, 0612015). Even in the context of e.g. modified gravity, operators like ϕ Gauss-Bonnet are bounded by looking at  corresponding to bkg that allow graviton-to-scalar conversion. In this case the Time delay is a matrix in "flavor" space that one can diagonalize to extract the bound (e.g. 2205.08551).

Remark: that time delay is $\propto R_S \propto E_A$ is important to make it resolvable within EFT when e.g. $|\beta|$ in (41) or $|\alpha_3|$ in (42) are $\gg 1$. Time delay not growing with energy, like in QED, ϕ^4, ... is not resolvable instead within EFT.