Dissipation induced by local non-Markovian baths

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Classical and quantum dynamics in out-of-equilibrium systems
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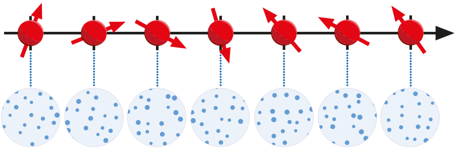

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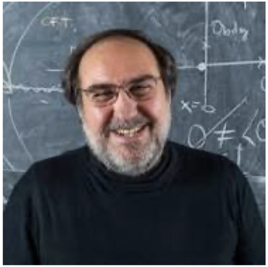


Thierry Giamarchi

Where do we stand

Time scales of the ``environment”

→

Fast	Slow	Quenched disorder
Lindblad approach for Markovian evolution	Caldeira and Leggett type of bath Power law correlated	Interaction with static impurities
High T regime	Low T regime	

The initial motivation

Two types of localisations

- with quenched disorder (Anderson insulator or Bose glass/MBL phase with interactions)
- spin-boson model: single particle with slow bath

We want to study

Many-body system with slow bath ≡ annealed disorder

Can we talk about “localisation"? (at $T = 0$)

The system

Spin chain ($S = 1/2$):

$$H = J_{xy} \sum_i \left[S_i^x S_{i+1}^x + S_i^y S_{i+1}^y \right] + J_z \sum_i S_i^z S_{i+1}^z$$

Equivalent to a model of interacting fermions:

$$H = -J_{xy} \sum_i \left[c_i^\dagger c_{i+1} + c_{i+1}^\dagger c_i \right] + J_z \sum_i (n_i - 1/2)(n_{i+1} - 1/2)$$

Two cases:

- Half filling $\langle n_i \rangle = \pi q_F = \frac{1}{2}$ $(\langle S^z \rangle = 0)$ (commensurate case)
- Doped system $1/2 < \langle n_i \rangle = \pi q_F < 1$ $(0 < \langle S^z \rangle < 1/2)$ (incommensurate case)

The system

Without the bath the system is in a *Luttinger liquid* phase (LL)

$$S_{LL} = \frac{1}{2\pi K} \int \mathrm{d}x \int \mathrm{d}\tau \left[u \left(\partial_x \phi\right)^2 + \frac{1}{u} \left(\partial_\tau \phi\right)^2 \right]$$

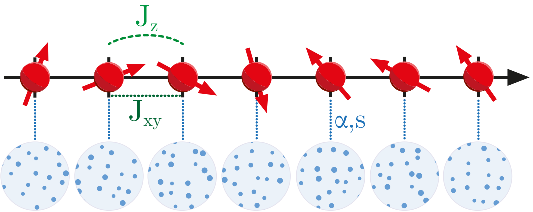
$$S^z \simeq -\frac{1}{\pi} \nabla\phi + \cos(\phi - 2q_F x)$$

- Critical phase with power law correlations
- Finite compressibility (susceptibility) and spin stiffness
- Perfectly conducting phase

Coupling to *local* baths

Caldeira Leggett bath

$$H_B = \sum_i \sum_k \left[\frac{1}{2} P_{i,k}^2 + \frac{1}{2} \Omega_k X_{i,k}^2 \right]$$



System-bath interaction

Coupling via the *density*

$$H_{SB} = \sum_i S_i^z \sum_k \lambda_k X_{i,k} = \sum_i S_i^z h_i(t)$$

In the limit of static bath

$$H_{tot} = J_{xy} \sum_i \left[S_i^x S_{i+1}^x + S_i^y S_{i+1}^y \right] + J_z \sum_i S_i^z S_{i+1}^z + \sum_i S_i^z h_i$$

paradigmatic model to study localisation with interaction

Nature of the bath

Spectral function

$$\mathrm{Im} \int e^{i\omega t} \langle [h_i(t), h_j(0)] \rangle = \delta_{ij} J(\omega)$$

$$J(\omega) = \pi \sum_j \frac{\lambda_k^2}{\Omega_k} \delta(\omega - \Omega_k) = \alpha \omega^s \quad \text{for } \omega < \Omega_D$$

- $s < 1$ sub Ohmic
- $s = 1$ Ohmic
- $s > 1$ super Ohmic

Caldeira, Leggett, Phys. Rev. Lett. 46, 211 (1981)

Leggett, Chakravarty, Dorsey, Fisher, Garg, Zwerger, Rev. Mod. Phys. (1987)

Total action

… integrating out the bath (annealed average)

$$S_{tot} = S_{LL} + S_{Dis}$$

$$S_{Dis} = -\alpha \int \mathrm{d}x \int \mathrm{d}\tau \int \mathrm{d}\tau' \, \frac{\cos(\phi(x,\tau) - \phi(x,\tau'))}{|\tau - \tau'|^{1+s}}$$

Bath with long-range correlations in time and uncorrelated in space

$2D$ classical field theory studied with

- Gaussian variational method
- RG
- Numerical simulations

Quantities of interest

Propagator $G(q,\omega_n) = \langle \phi(q,\omega_n)\phi(-q,-\omega_n)\rangle$ $\omega_n = \dfrac{2\pi n}{\beta}$

- Compressibility $\chi = \lim_{q\to 0} \dfrac{q^2}{\pi^2} G(q,\omega_n = 0)$
- Spin stiffness $\rho_s = \lim_{\omega_n \to 0} \dfrac{\omega_n^2}{\pi^2} G(q=0,\omega_n)$
- Conductivity $\sigma(\omega) = \dfrac{e^2}{\pi^2 \hbar^2} Re[\omega_n G(q=0,\omega_n)]_{i\omega_n \to \omega + i\epsilon}$

RG

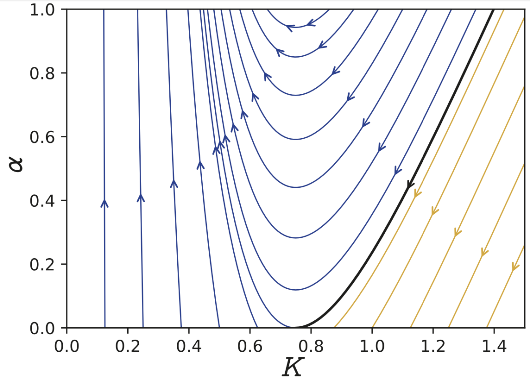
$$\partial_l K(l) = -2K^2(l)\alpha(l)$$

$$\partial_l \alpha(l) = (2 - s - 2K(l))\alpha(l)$$

Critical point

$$K_c = 1 - \frac{s}{2} \quad \alpha = 0$$

BKT transition



Cazalilla et al., Phys. Rev. Lett. 97, 076401 (2006)

Gaussian variational method

Luttinger liquid phase

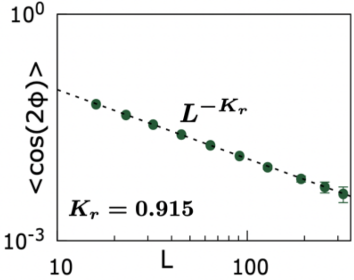
$$G_{LL}^{-1} \simeq \frac{1}{2\pi}\left[\frac{u_r}{K_r}q^2 + \frac{1}{u_r K_r}\omega_n^2\right]$$

Dissipative phase

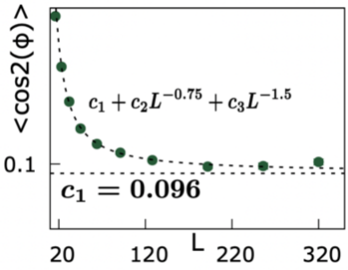
$$G_{Dis}^{-1} \simeq \frac{1}{2\pi}\left[\frac{u_r}{K_r}q^2 + \eta\omega_n^s\right]$$

Order parameter

$$\cos 2\phi \simeq \begin{cases} \frac{1}{L^{Kr}} & \text{LL phase} \\ const - \frac{1}{L^{1-s/2}} & \text{Dissipative phase} \end{cases}$$



LL phase

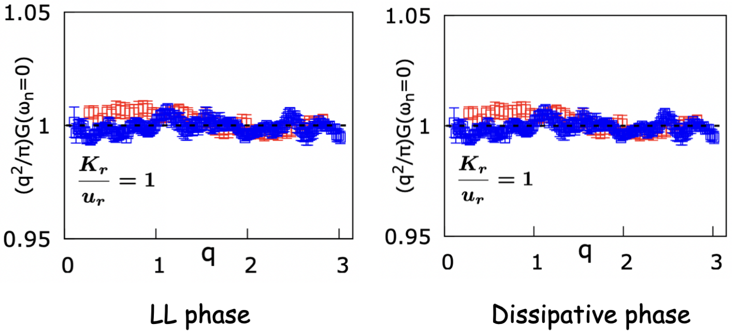


Dissipative phase

Statistical tilt symmetry

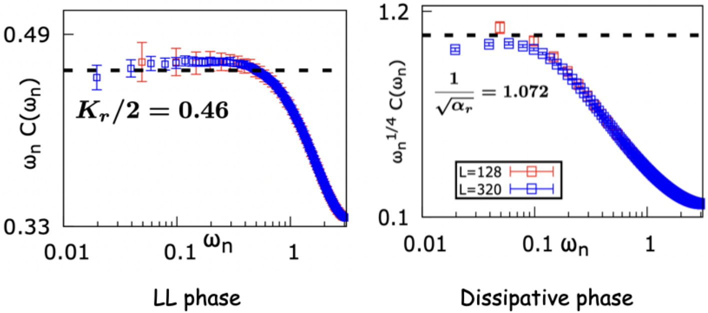
$$S_{Dis}[\phi + hx] = S_{Dis}[\phi]$$

Constant compressibility $\chi = u/K$



Small ω_n behaviour

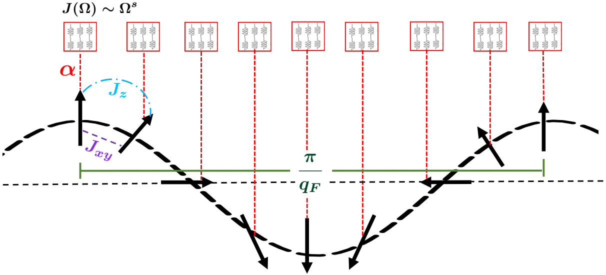
$$C(\omega_n) = \sum_q G(q,\omega_n) \simeq \begin{cases} \omega_n^{-1} & \text{LL phase} \\ \omega_n^{-s/2} & \text{Dissipative phase} \end{cases}$$



Dissipative phase

Gapless spin density wave

$$S^z(x) \simeq \cos(2q_F x)\langle \cos 2\phi \rangle$$



Transport properties

DC Conductivity

$$\sigma_{DC} = \lim_{\epsilon \to 0} \left(\frac{e^2}{\pi^2 \hbar} \epsilon^{1-s} \right) = \begin{cases} \infty & \text{Super Ohmic} \\ Const & \text{Ohmic} \\ 0 & \text{Sub Ohmic} \end{cases}$$

Bath induced localisation !

Majumdar et al., Phys Rev, B 107, 165113 (2023)

Majumdar et al., Phys. Rev. B 108, 205138 (2023)

Commensurate case

Dissipative phase

$$G_{Dis}^{-1} \simeq \frac{1}{2\pi}\left[\frac{u}{K}q^2 + \frac{1}{uK}\omega_n^2 + \Delta^2\right]$$

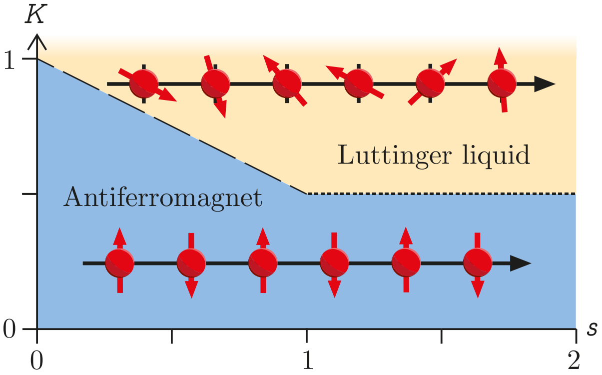
Gapped antiferromagnetic phase

Antiferromagnetic order enhanced by the bath !

Malatsetxebarria et al., Phys. Rev. A 88, 063630 (2013)

Bouverot-Dupuis et al., Phys. Rev. B 109, 205148 (2024)

Phase diagram



Order induced by on site dissipation

Other examples

Werner, Troyer, Sachdev, J. Phys. Soc. Jpn (2005)

Cai, Schollwöck, Pollet, PRL (2014)

Weber, Luitz, Assad, PRL (2022)

Kuklov, Prokof'ev, Radzihovsky, Svistunov, PRL, PRB, PRA (2023-2024)

Ribeiro, McClarty, Ribeiro, Weber, PRB (2024)

Where we would like to go

Time scales of the ``environment"

Fast		Slow		Quenched disorder
Lindblad approach for Markovian evolution	⟷	Caldeira and Leggett type of bath Power law correlated	⟷	Interaction with static impurities
High T regime		Low T regime		

Conclusions

- Order induced or enhanced by the bath
- The bath can induce a gapless insulating phase

- Whole phase diagram as a function of the magnetic field? (commensurate/incommensurate phase transition)
- Entanglement entropy between the system and the bath?
- Can we draw some link with other open quantum systems?

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Thank you!