# Heavy-Quark & Soft-Collinear Effective Theory

(Matthias Neubert, 9–12 April 2024)

**Outline:**

- Motivation
- Brief Encounter with Heavy-Quark Effective Theory
- Construction of Soft-Collinear Effective Theory
- Properties of the SCET Lagrangian (Reading Assignm.)
- Matching of the 2-Jet Current
- The Sudakov Form Factor in SCET
- RG Evolution Equations
- Decoupling of Ultra-Soft Gluons & Factorization

**Suggested literature:**

Review: T. Becher, A. Broggio, A. Ferroglia, 1410.1892 (book!)

Lectures: M. Neubert, hep-ph/0512222 (TASI 2004)

T. Becher, 1803.04310 (Les Houches 2017)

Papers: C. Bauer et al., hep-ph/0011336, 0109045, 0202088

M. Beneke et al., hep-ph/0206152

Tutorials, exercises & discussions

# I. Motivation

Multi-scale problems are abundant in physics but difficult to deal with $\rightarrow$ effective theories reduce them to problems involving fewer scales

EFT: modern tool to achieve scale separation in QFT

$\rightarrow$ reduce multi-scale problems to a sequence of single-scale problems

$\rightarrow$ RGEs allow for systematic resummation of large logarithms of scale ratios; particularly important in QCD, where running of $\alpha_s(\mu)$ is significant and $\alpha_s \ln^n(Q_1/Q_2)$ can be large if $Q_1 \gg Q_2$ $(n=1,2)$

Scale separation is the basis of factorization theorems

$\rightarrow$ crucial for separation of short-distance from long-distance physics in QCD ( $\sigma_{LHC} = \sigma_{partons} \otimes$ PDFs )

$\rightarrow$ in strongly interacting theories, long-distance dofs can be different from short-distance dofs, so scale separation becomes a necessity (quarks & gluons vs. hadrons, electrons vs. Cooper pairs, etc.)

## Why should you learn about SCET?

- SCET is the EFT for high-energy processes involving light particles → relevant for collider physics and heavy-quark physics

- very powerful, but one of the most complicated EFT ever developed → full of subtleties, but physics can be subtle (e.g. "collinear anomaly")

- originally developed to understand factorization theorems in B physics, e.g.:

  $B \to X_s \gamma$, $B \to X_u \ell \bar{\nu}$ (inclusive decays to light part.)

  $B \to D\pi$, $B \to \pi\pi$ (exclusive nonleptonic decays)

  ↳ QCD factorization approach (BBNS: hep-ph/9905312, 0006124, 0104110) solved a problem that was intractable before

- Later, SCET found many applications outside flavor physics, in particular: collider physics, dense QCD matter, hadron physics, scattering amplitudes, DM phenomenology, SYM theory, BSM physics, ...

↳ series of annual workshops since 2003

Important comment:

The construction of an EFT becomes significantly more complicated in cases where the large scale $M$ remains as a parameter in the EFT, characterizing the large energies of light particles. This is precisely what happens in SCET.

In this case, as we will see, the operators in the effective Lagrangian (step 3) are non-local on large scales. These non-localities are along light-like directions. They are a characteristic feature of SCET.

* conventional EFTs : based on (Euclidean) operator product expansion (OPE)

  ↳ integrating out heavy particles ($\Delta x \sim 1/M$)

* SCET : "Minkowskian" generalization of the OPE based on the method of regions ($\Delta x$ lightlike)

  ↳ integrating out "large energies"

## II. Brief Encounter with Heavy-Quark Effective Theory

- EFT describing bound states of a heavy quark Q ($= b, c$) with light quarks and gluons ($B^{(*)}, D^{(*)}, \Lambda_{b,c}$)

- example of a two-scale problem:

$$\underbrace{m_Q}_{\text{"hard"}} \gg \underbrace{\Lambda_{QCD}}_{\text{"soft" QCD interactions (non-perturbative)}}$$

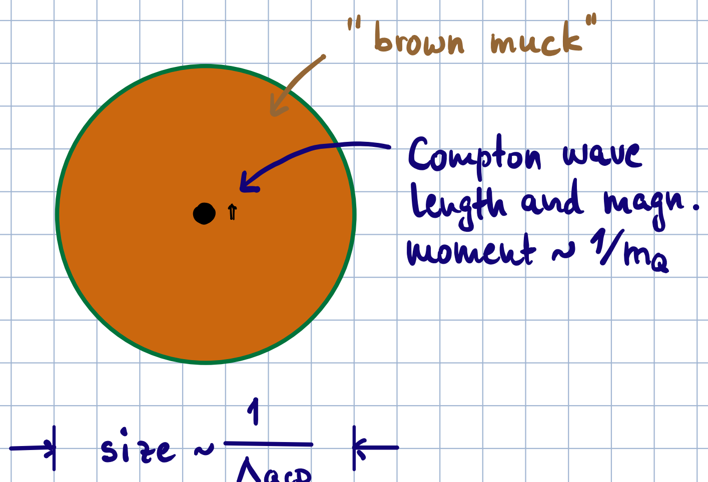
- example of an EFT where the heavy particle cannot be removed entirely from the effective Lagrangian; instead, we only integrate out its hard (far off-shell) fluctuations

  → will encounter many features that will be useful for the construction of SCET

- when a heavy quark is bound to light partons by soft interactions, new symmetries arise, which are not manifest in QCD but will be manifest in HQET at leading order

  → broken by calculable perturbative corrections (hard gluons) and power corrections $\sim (\Lambda_{QCD}/m_Q)^n$

- physical picture:




- soft gluons cannot resolve spin & flavor of heavy quark

- heavy-quark momentum is approximately conserved:

$$P_Q^\mu = m_Q v^\mu + k^\mu$$

4-velocity of bound state ($v^2=1$)

soft residual momentum

$$\Rightarrow \quad v_Q^\mu = v^\mu + \frac{k^\mu}{m_Q} = v^\mu + \mathcal{O}\left(\frac{\Lambda_{QCD}}{m_Q}\right)$$

conserved in heavy-quark limit $m_Q \to \infty$

- which dofs can be integrated out ?

$\to$ far off-shell fluctuations with $k^\mu = \mathcal{O}(m_Q)$

# Construction of HQET:

**Step 1:** pull out a phase factor corresponding to the "static" momentum $m_Q v^\mu$, and split up the 4-component Dirac spinor $\psi_Q$ into two "2-component" spinors:

carry momentum $k^\mu$

$$\psi_Q(x) = e^{-i m_Q v \cdot x} \left[ h_v(x) + H_v(x) \right]$$

initial-state heavy quark

with:

$$\not{v}\, h_v = h_v \,, \qquad \not{v}\, H_v = - H_v$$

explicitly:

$$h_v(x) = e^{i m_Q v \cdot x} \, \frac{1+\not{v}}{2} \, \psi_Q(x)$$

(field redefinition)

$$H_v(x) = e^{i m_Q v \cdot x} \, \frac{1-\not{v}}{2} \, \psi_Q(x)$$

→ hadron velocity $v^\mu$ enters as a label on the fields

**Step 2:** insert this decomposition into the Dirac Lagrangian:

$$\mathcal{L}_Q = \bar{\psi}_Q \,(i\not{D} - m_Q)\, \psi_Q$$

$$= (\bar{h}_v + \bar{H}_v)\, e^{i m_Q v \cdot x} \,(i\not{D} - m_Q)\, e^{-i m_Q v \cdot x}\, (h_v + H_v)$$

$$= (\bar{h}_v + \bar{H}_v)(i\not{D} + m_Q \not{v} - m_Q)(h_v + H_v)$$

$$= \bar{h}_v \, i\not{D} \, h_v + \bar{H}_v (i\not{D} - 2m_Q) H_v$$

$$+ \bar{h}_v \, i\not{D} \, H_v + \bar{H}_v \, i\not{D} \, h_v$$

**Step 3:** simplify the Dirac structures

$$\frac{1+\not{v}}{2} \gamma_\mu \frac{1+\not{v}}{2} = \gamma_\mu \underbrace{\frac{1-\not{v}}{2} \frac{1+\not{v}}{2}}_{=0} + \frac{\{\not{v}, \gamma_\mu\}}{2} \frac{1+\not{v}}{2} = v_\mu \frac{1+\not{v}}{2}$$

$$\frac{1-\not{v}}{2} \gamma_\mu \frac{1-\not{v}}{2} = - v_\mu \frac{1-\not{v}}{2}$$

$$\frac{1+\not{v}}{2} \gamma_\mu \frac{1-\not{v}}{2} = \frac{1+\not{v}}{2} (\gamma_\mu^\perp + v_\mu \not{v}) \frac{1-\not{v}}{2} = \frac{1+\not{v}}{2} \gamma_\mu^\perp \frac{1-\not{v}}{2}$$

$v^\mu \gamma_\mu^\perp = 0$

("spatial" projection)

in the hadron rest frame:

$$v^\mu = (1, \vec{0}) \quad , \quad \gamma_\mu^\perp = (0, \vec{\gamma})$$

$$\Rightarrow \quad \mathcal{L}_Q = \bar{h}_v \, i v \cdot D \, h_v + \bar{H}_v (-i \, v \cdot D - 2m_Q) H_v$$

$$+ \bar{h}_v \, i\not{D}^\perp H_v + \bar{H}_v \, i\not{D}^\perp h_v$$

**Step 4:** integrate out "high-frequency modes" ($\omega \sim m_Q$) corresponding to hard quantum fluctuations

→ field $H_v$ has mass $2m_Q$, while $h_v$ is massless

→ in addition, we split up $A^{\mu,a} = \underset{\text{(hard)}}{A_h^{\mu,a}} + \underset{\text{(soft)}}{A_s^{\mu,a}}$ and integrate out hard gluons

↳ integrate out $H_v$ in the functional integral:

$$\int \mathcal{D}H_v \; e^{i\int d^Dx \, \mathcal{L}_Q(x)} = e^{i\int d^Dx \, \mathcal{L}_Q^{\text{eff}}(x) + \Delta}$$

where:

$$\boxed{\begin{aligned} \mathcal{L}_Q^{\text{eff}} &= \bar{h}_v \, i v\cdot D_s \, h_v + \bar{h}_v \, i \not{D}_s^{\perp} \frac{1}{2m_Q + i v\cdot D_s} i\not{D}_s^{\perp} \, h_v \\ \Delta &= \operatorname{Tr}\ln(-2m_Q - i v\cdot D_s) = \text{irrelevant constant} \end{aligned}}$$

and:

$$i D_s^{\mu} = i\partial^{\mu} + g_s \overset{\text{soft gluon field}}{A_s^{\mu,a}} t^a$$

- when hard gluon effects are included, the functional integral is no longer gaussian and cannot be evaluated in closed form → set $A_h^{\mu,a} = 0$ and come back to hard gluon effects later

- functional determinant $\Delta$ is gauge invariant, and evaluating it in "temporal" gauge $v \cdot A_s^a = 0$ shows that:

$$\Delta = \mathrm{Tr}\ln(-2m_Q - i v \cdot \partial)$$
$$= \text{(infinite) field-independent constant, which can be dropped}$$

↳ integrate out $H_v$ using equations of motion:

$$\frac{\delta \mathcal{L}_Q}{\delta \bar{H}_v} = 0 \;\Rightarrow\; H_v = \frac{1}{2m_Q + i v \cdot D_s} \, i \not{D}_s^{\perp} \, h_v$$

pluging this solution back into the Lagrangian yields the above result for $\mathcal{L}_Q^{\text{eff}}$ ✓