ICTS | INTERNATIONAL CENTRE for THEORETICAL SCIENCES

TATA INSTITUTE OF FUNDAMENTAL RESEARCH

ICTS Lecture Series

Title : A Short Course on the Atiyah-Singer Index Theorem

Speaker : Madabusi Raghunathan (Honorary Fellow, TIFR)

Date : 24, 29 and 31 July 2024

Time : 10:30 AM (IST)

Abstract : In these lectures I will outline a proof of the Atiyah-Singer Index theorem. I will begin with the definition of the symbol σlσl of a linear differential operator LL and the notion of ellipticity of LL. I go on to define the analytic index ia(L)$ia(L)$ of a linear elliptic differential operator LL on a compact manifold MM.Then I proceed to the definition of the KK-theoretic element κ(L)∈K(M)$\kappa(L)\in K(M)$ associated to the elliptic operator LL via its symbol and the cohomology class τL(M)∈H∗(M)$\tau L(M)\in H*(M)$. The topological index it(L)$it(L)$ is defined as <τL(M),[M]><$\tau L(M),[M]>$ where [M][M] is the fundamental class of the (connected) compact (oriented) manifold. The index theorem assets that ia(L)=it(L)$ia(L)=it(L)$. I then give a proof following essentially the ideas in the announcement of the theorem by Atiyah and Singer. The proof however differs significantly from those in the Palais seminar which carries out in detail the outline given in the announcement. Unlike the Palais seminar I will not be using the sophisticated machinery of Pseudo differential operators but stay in the confines of Differential operators. A second point of difference is in the proof of the 'bordism invariance' where I will be using the ideas developed by McKean and Singer rather than results about coercive boundary value problems. The proof as outlined by Atiyah and Singer is along the following lines. They show that ia(L)$ia(L)$ depends only on τ(L)$\tau(L)$ and conclude from that it suffices to prove the theorem for the special class 'twisted index' operators on even dimensional manifolds. Then using the bordism in variance they further reduce the problem to special manifolds where the equality of the analytic and topological indices are well known theorems. This is also essentially what I will be doing. The Atiyah-Bott-Patodi proof of the index theorem follows a different path *after* reducing the problem to the case of special operators, following a method proposed by McKean and Singer. I will say something about this proof towards the end of these lectures.

I will begin with the definition it(L)$it(L)$ of the analytic index of a Linear Elliptic Differential operator LL on a smooth compact manifold. I will then describe the symbol of LL and the KK-theoretic element κ(L)$\kappa(L)$ associated to it and then the cohomology class τ(L)$\tau(L)$ on MM defined using it

Venue : Madhava Lecture Hall

ZoomLink: https://icts-res-in.zoom.us/j/91833525623?pwd=kgRqG7GGzqA810Lvib3gfVUqxXbt9d.1
Meeting ID: 918 3352 5623
Passcode: 242526