ICTS International Centre for Theoretical Sciences
Tata Institute of Fundamental Research

Infosys–ICTS Chandrasekhar Lectures, 2024 Aug.
ICTS Program
"Quantum Information, Quantum Field Theory and Gravity"

Entanglement and emergence of gravitational spacetime

Tadashi Takayanagi

Yukawa Institute for Theoretical Physics
Kyoto University

YITP Yukawa Institute for Theoretical Physics

CGPQI Center for Gravitational Physics and Quantum Information
Yukawa Institute for Theoretical Physics, Kyoto University

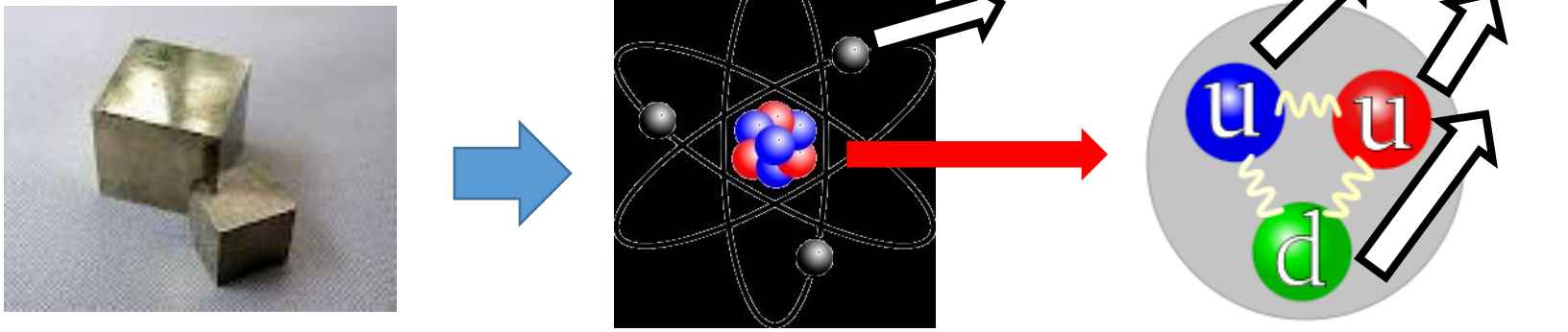
Grant-in-Aid for Transformative Research Areas A
Extreme Universe

①Introduction

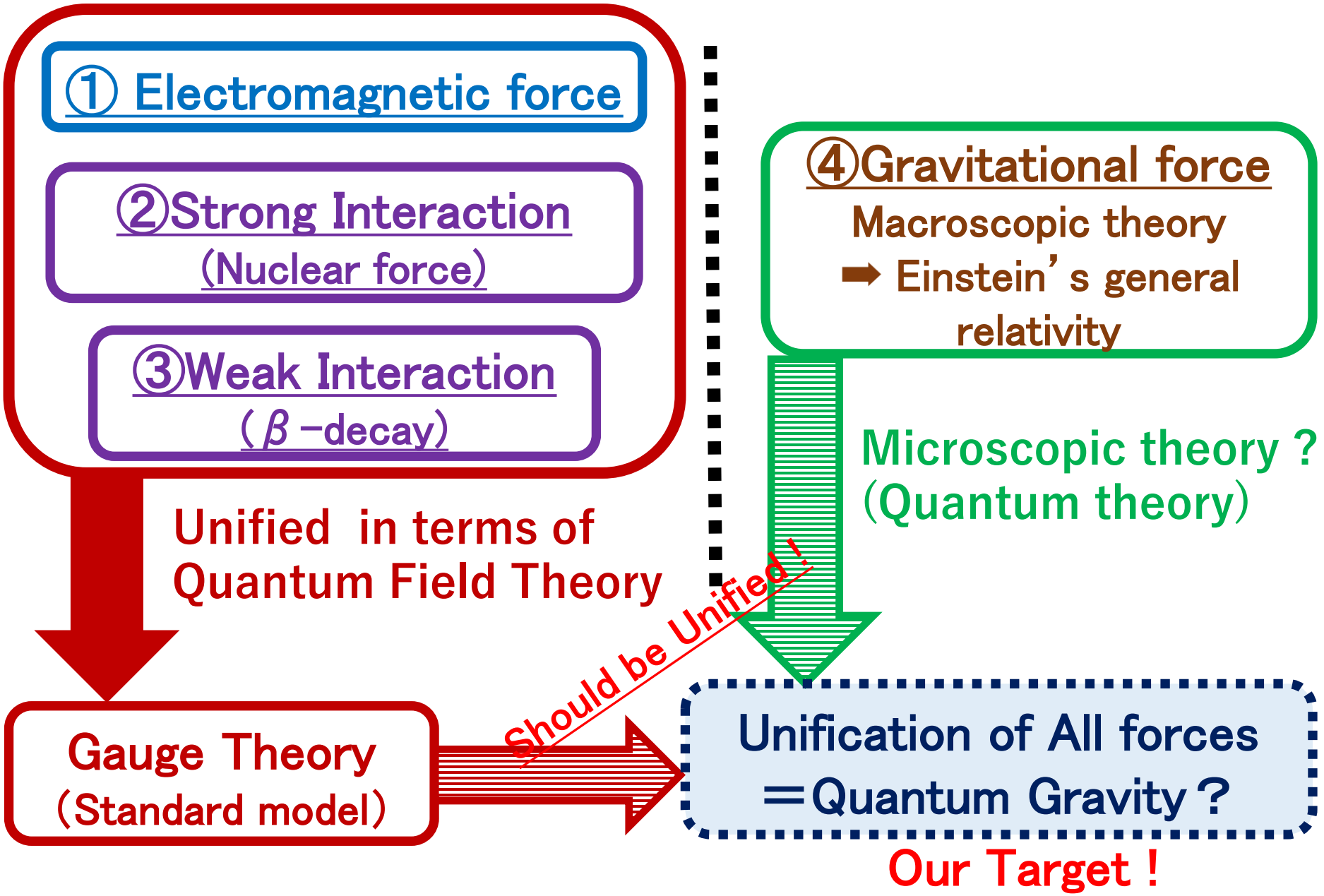
What is particle physics ?

The study of fundamental particles and forces that constitute matter and radiation.

⇒**Exploring the minimal unit of matter**



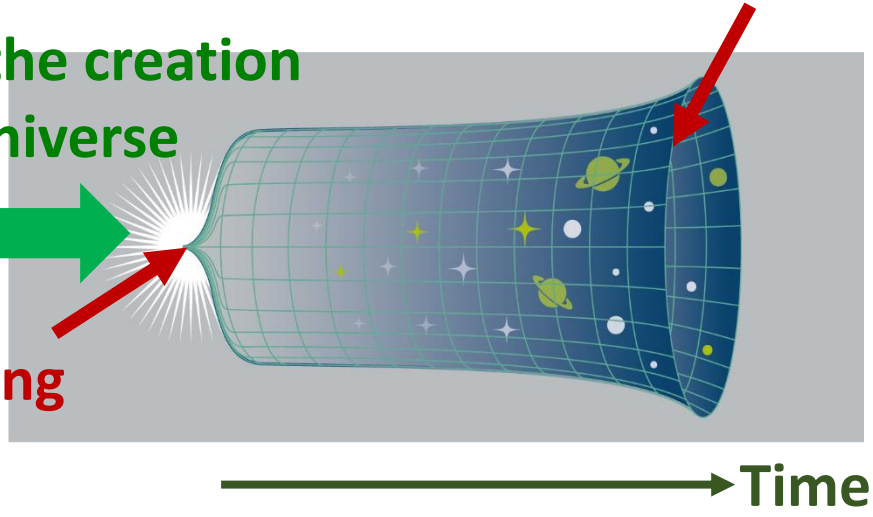
Four fundamental forces of Nature and Unification



To understand the creation of Universe, we need quantum gravity !

Quantum Gravity
= Microscopic theory of gravity

Explain the creation of the Universe



First of all, we want to magnify the Universe.

➡ We need a “good microscope” !

What is the minimal unit of spacetime ?

➔Holography plays a role of microscope in thought experiments !

Condensed matter
Biology, Chemistry

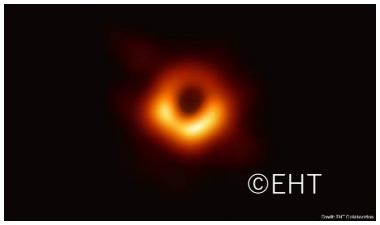
➡ Optical,
Electronic, etc
microscope

➡ Cells, Spins,
Electrons,
Crystal structure

High Energy Physics ➡ Accelerators

➡ Elementary particle
Atomic nucleus

Quantum Gravity ➡ **Holographic Principle**
(Gauge/Gravity duality) ➡ **Qunatum Information**
(Qubits with
Quantum Entanglement)



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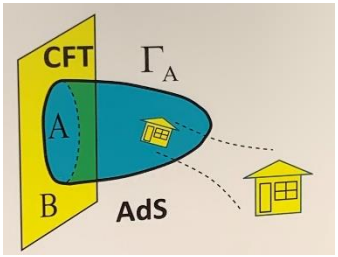

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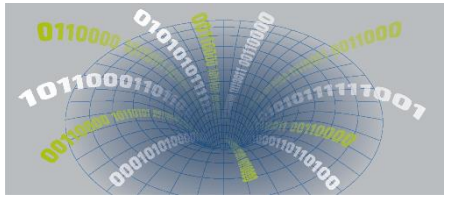


What is the minimal unit of spacetime ? ➡ Qubit ?!

Qubit = The unit of microscopic information (= a spin)

or

Head 0 Tail 1

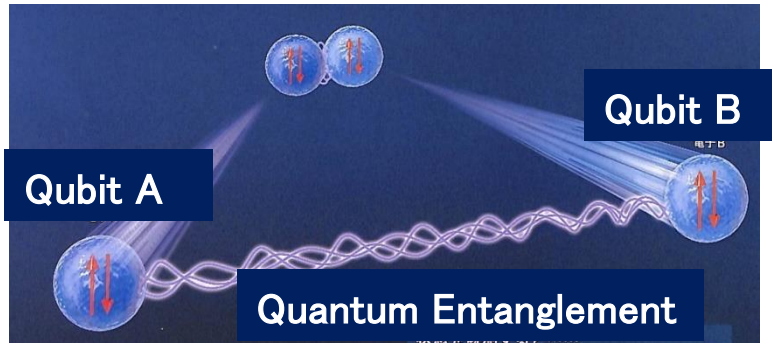
Macroscopic World

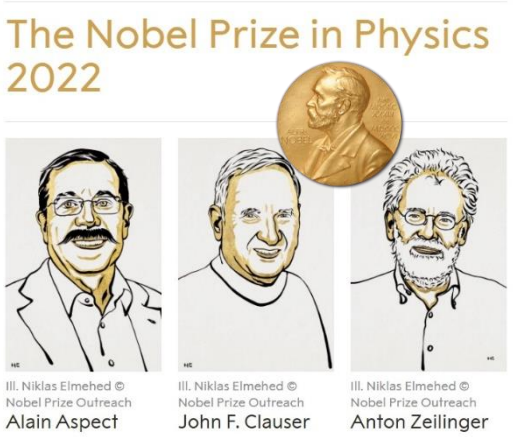
➡

$a|0\rangle+b|1\rangle$

Microscopic World

Quantum Entanglement = Correlation between Qubits





Contents

② Quantum Entanglement

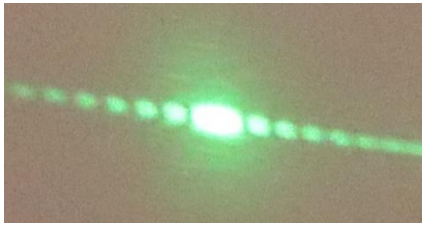
Wave-Particle Duality

What is the nature of light ?

(A) Particle (photon) [Newton,･･･]
➔Light travels straight.

(B) Wave [Huygens,･･･]
➔It shows diffraction and interference.

Laser pointer



Vernier calipers

Both are correct !

1905 Einstein's
light quantum hypothesis

Any matter such as electrons also have wave-particle duality !

Quantum Theory is based on wave-paticle duality

Particle = **Wave (wave function)**

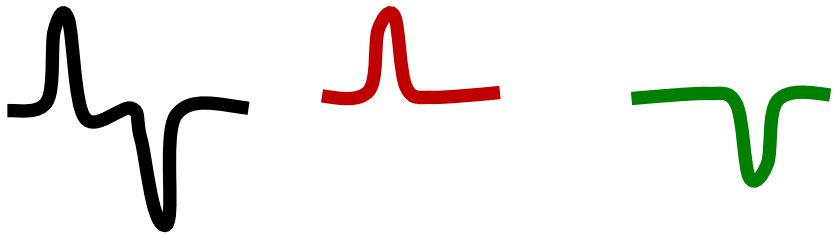
➔ **We can superpose waves !**

Ket

Quantum State $|\Psi\rangle = a|f\rangle + b|g\rangle$

Sum of functions

Wave Function $\Psi(x) = a\,f(x) + b\,g(x)$



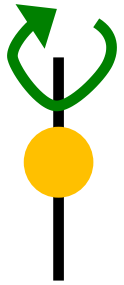
Qubit

As an example of quantum state consider **electron's spin**.

Right rotation
(Up Spin)

$|0\rangle$

Left rotation
(Down Spin)



$|1\rangle$

One qubit state: $|\Psi\rangle = a|0\rangle + b|1\rangle$

Classical Computer

Classical Information

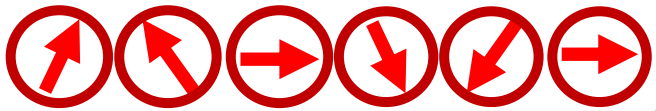
# of C info. = Bits

0 1 0 1 1 0

Quantum Computer

Quantum Information

# of Q info. = Qubits



Quantum Entanglement

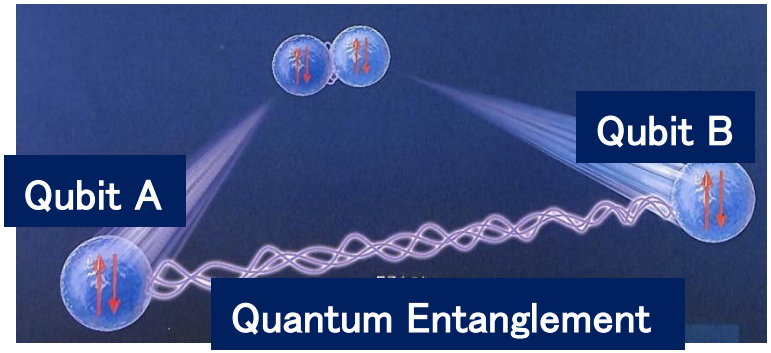
We start with a two qubit system: A and B.

Consider the Bell state :

$$|\Psi_{Bell}\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle_A|1\rangle_B + |1\rangle_A|0\rangle_B\right)$$

If we measure A spin, then that tells us B spin at the same time !

This correlation between A and B is Quantum Entanglement !



Though we know the state for the total system AB, the state for a subsystem is not definite.

Entanglement Entropy (EE)

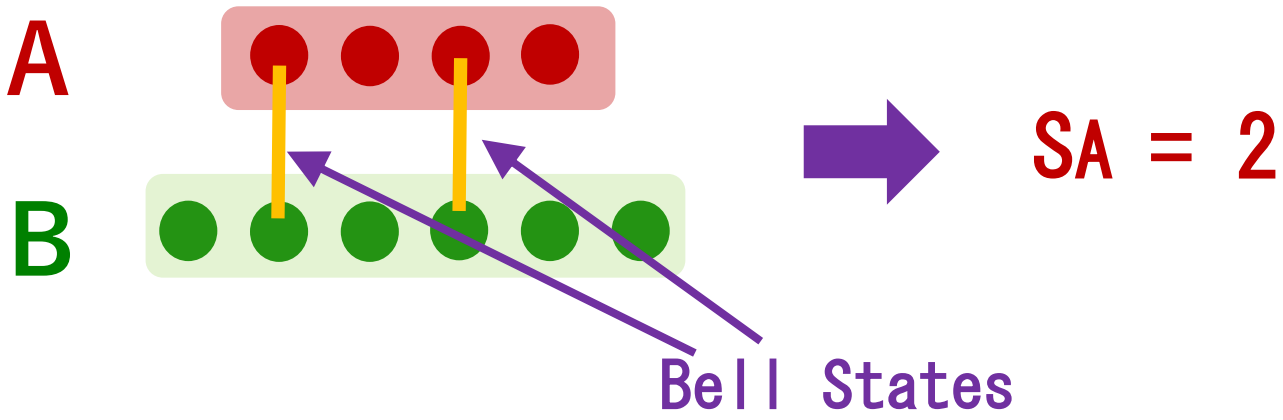
A measure of the amount of quantum entanglement

➡ **Entanglement Entropy (EE)**

Entanglement Entropy between A and B:

SA = # of Bell states between A and B

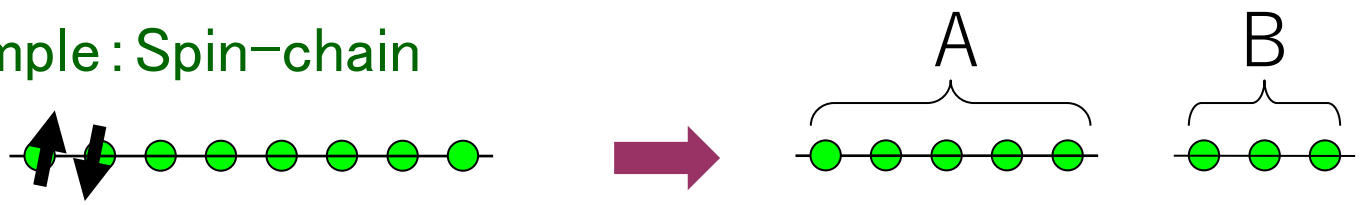
= the amount of "lost information"
if an observer can only be accessible to A.



Precise definition of Entanglement Entropy (EE)

First we decompose the Hilbert space: $H_{tot} = H_A \otimes H_B$.

Example : Spin-chain



We introduce the reduced density matrix ρ_A

by tracing out B $\rho_A = \mathrm{Tr}_B\left[\left|\Psi_{tot}\right\rangle\left\langle\Psi_{tot}\right|\right]$

The entanglement entropy (EE) S_A is defined by

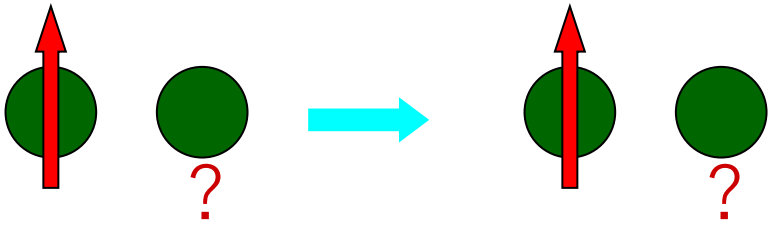
$$\boxed{S_A = -\mathrm{Tr}[\rho_A \log \rho_A]} \propto$$

# of Bell Pairs between A and B

The Simplest Example: two spins (2 qubits)

(i) $|\Psi\rangle = \frac{1}{2}\left[|0\rangle_A + |1\rangle_A\right] \otimes \left[|0\rangle_B + |1\rangle_B\right]$

$$\Rightarrow \rho_A = \mathrm{Tr}_B\left[|\Psi\rangle\langle\Psi|\right] = \frac{1}{2}\left[|0\rangle_A + |1\rangle_A\right]\cdot\left[\langle 0|_A + \langle 1|_A\right] \cong \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}.$$

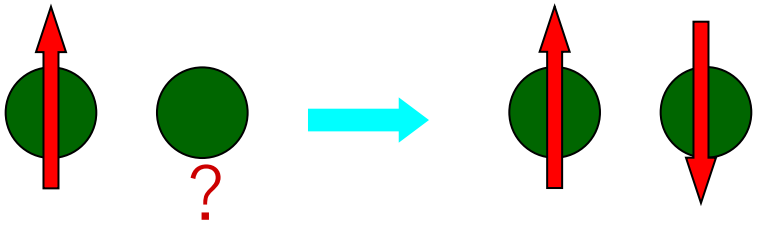


Not Entangled

$$S_A = 0$$

(ii) $|\Psi\rangle = \left[|0\rangle_A \otimes |1\rangle_B + |1\rangle_A \otimes |0\rangle_B\right] /\sqrt{2}$

$$\Rightarrow \rho_A = \mathrm{Tr}_B\left[|\Psi\rangle\langle\Psi|\right] = \frac{1}{2}\left[|0\rangle_A\langle 0|_A + |1\rangle_A\langle 1|_A\right] \cong \begin{pmatrix} 1/2 & 0 \\ 0 & 1/2 \end{pmatrix}.$$



Entangled

$$S_A = -2\cdot\log\frac{1}{2} = \log 2.$$

Measurement of EE in Cond-mat Experiments

Example1: Ultracold bosonic atoms in optical lattices

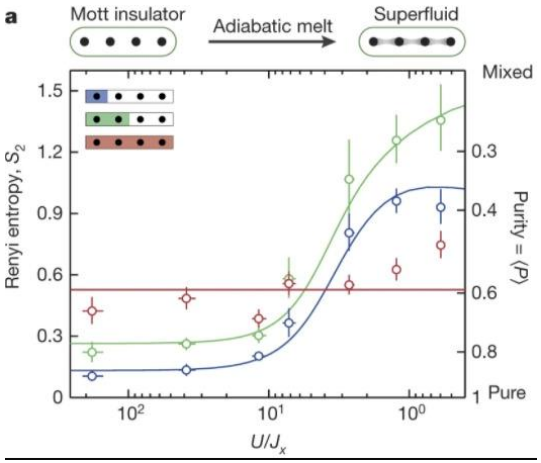
Published: 02 December 2015

Measuring entanglement entropy in a quantum many-body system

Rajibul Islam, Ruichao Ma, Philipp M. Preiss, M. Eric Tai, Alexander Lukin, Matthew Rispoli & Markus Greiner

Nature **528**, 77–83 (2015) | Cite this article

$$H = -J\sum_{\langle i,j\rangle} a_i^\dagger a_j + \frac{U}{2}\sum_i n_i(n_i-1) \qquad (4)$$



Example2: Trapped-ion quantum simulator

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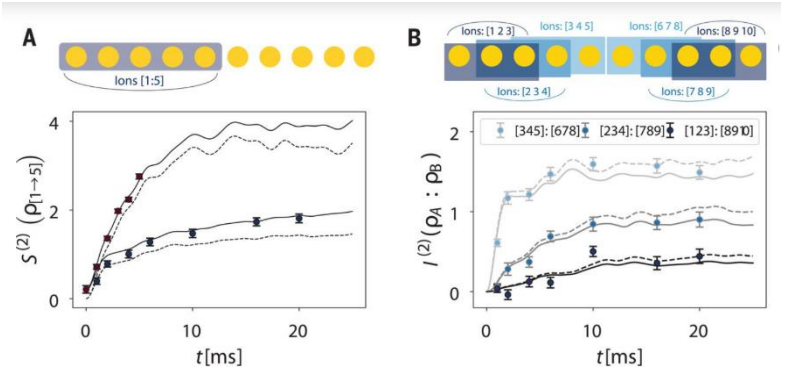
REPORT

Probing Rényi entanglement entropy via randomized measurements

TIFF BRYDGES, ANDREAS ELBEN, PETAR JURCEVIC, BENOÎT VERMERSCH, CHRISTINE MAIER, BEN P. LANYON, PETER ZOLLER, RAINER BLATT, AND CHRISTIAN F. ROOS Authors Info & Affiliations

SCIENCE • 19 Apr 2019 • Vol 364, Issue 6437 • pp. 260-263 • DOI: 10.1126/science.aau4963

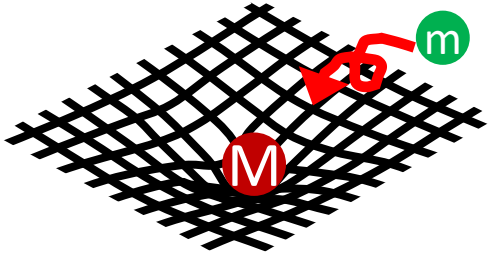
$$H_{XY} = \hbar\sum_{i<j} J_{ij}\left(\sigma_i^+\sigma_j^- + \sigma_i^-\sigma_j^+\right) + \hbar B\sum_j \sigma_j^z$$



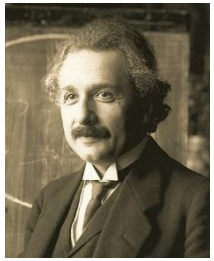
③ Black hole and Entropy

Following general relativity,
the spacetime gets distorted !

Black hole (BH)

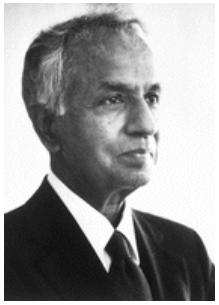


An extremely dense astrophysical object.
Due to its strong gravitational force, even light rays cannot come out from black hole.

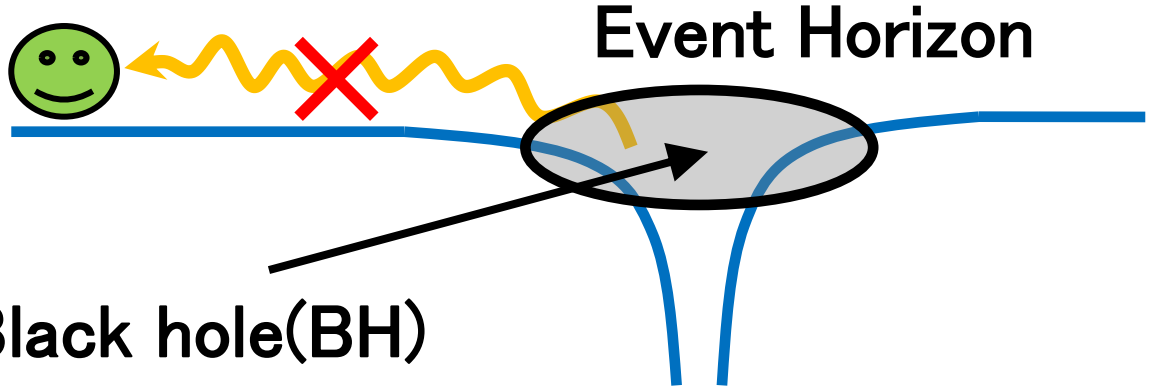


➡ Characteristic object in Einstein's general relativity

A white dwarf star with
a mass much beyond
Chandrasekhar limit

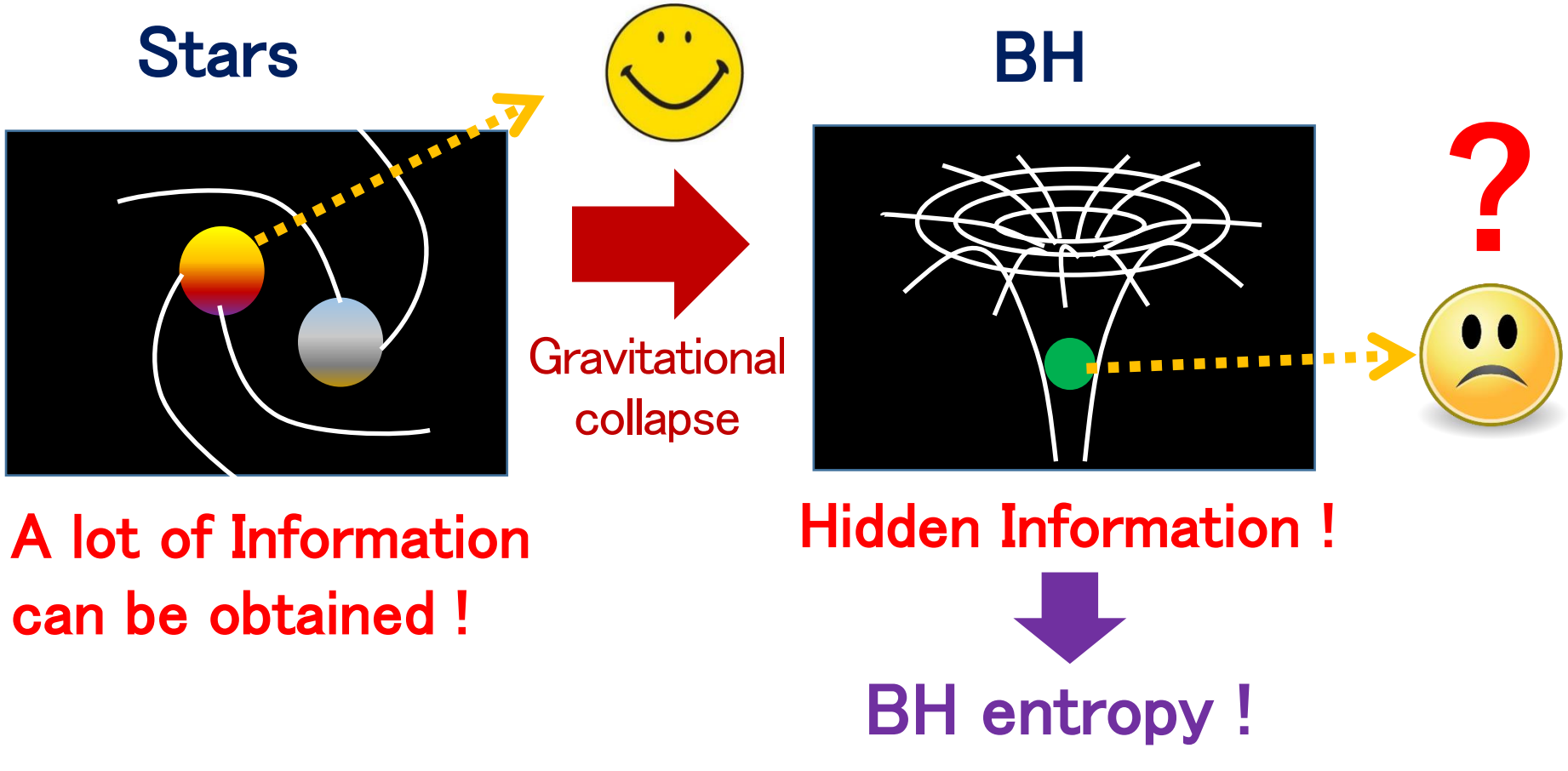


➡



BH Entropy

After stars collapsed into a BH, outside observers cannot access the information inside the BH.



Bekentein-Hawking Formula of BH Entropy [1972-1976]

Calculations in general relativity show that a BH has the following entropy:

⇒Still mysterious !

$$S_{BH} = \frac{k_B c^3}{\hbar} \times \frac{A_{BH}}{4G_N}$$

⇒ BH thermodynamics !

A_{BH}= Surface Area of Black hole ⇒ **Geometry**

G_N=Newton constant ⇒ **Gravity**

$\hbar$=Planck constant ⇒ **Quantum Mechanics**

k_B=Boltzmann const.⇒ **Stat. Mech. , Quantum Info.**

} **Quantum Gravity!**

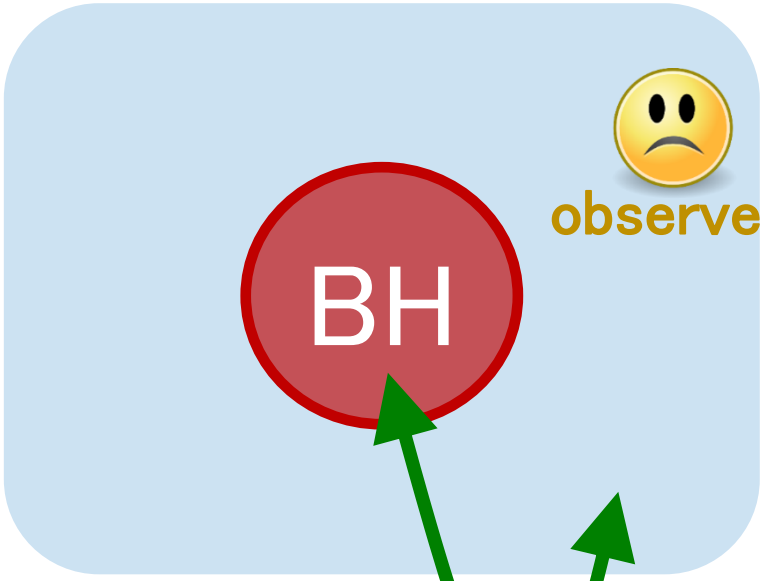
[1] BH Entropy is proportional to the **area**, not to the volume !

[2] BH has the entropy even in the **classical theory** of Gravity !

Analogy between BH and Qubits

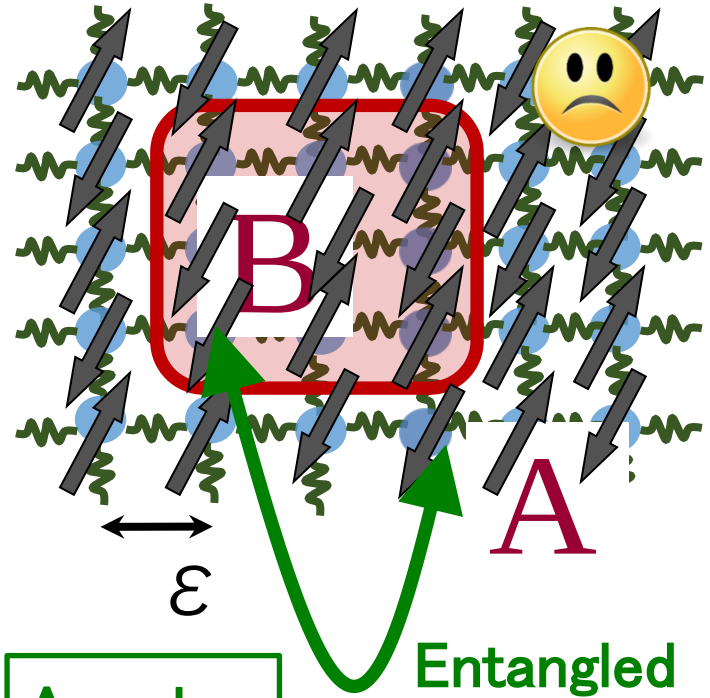
[Original motivation of studying EE in QFTs, Bombelli et.al. 1986, Srednicki 1993]

Blackhole Spacetime



Area law

Quantum Spin System



Area law

?

?

BH entropy SBH

Spacetime

Entanglement entropy SA

Matter

④ Holography and Quantum Entanglement

BH Entropy Formula

$$S_{BH} = \frac{A_{BH}}{4G_N}$$

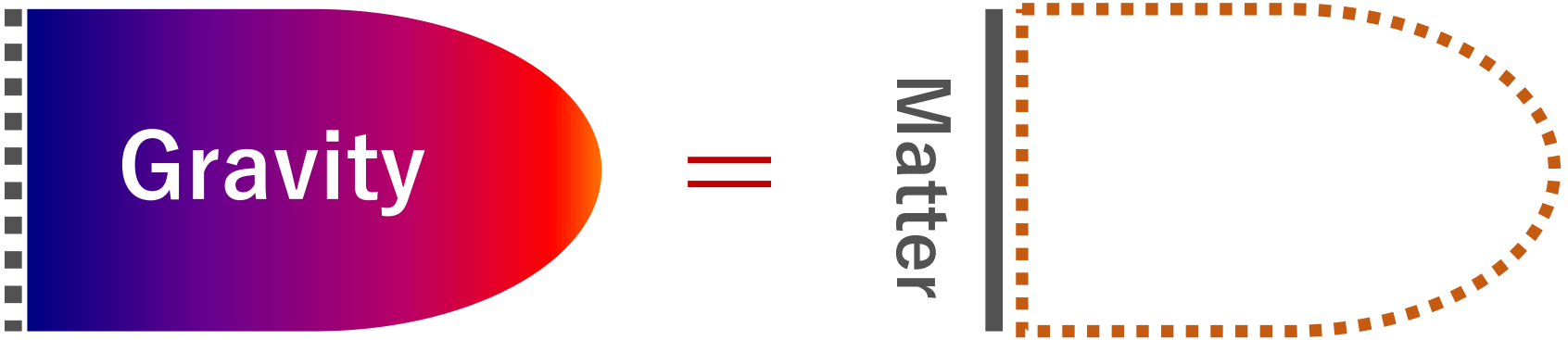
Degrees of freedom in Gravity ∝ Area

['t Hooft 1993, Susskind 1994]

Holography

Boundary of M

Gravity on M = Quantum Matter on ∂M



BH entropy(∝Area)= Thermal Entropy of Matter (∝Volume)

Gauge/Gravity Duality: best known example of holography

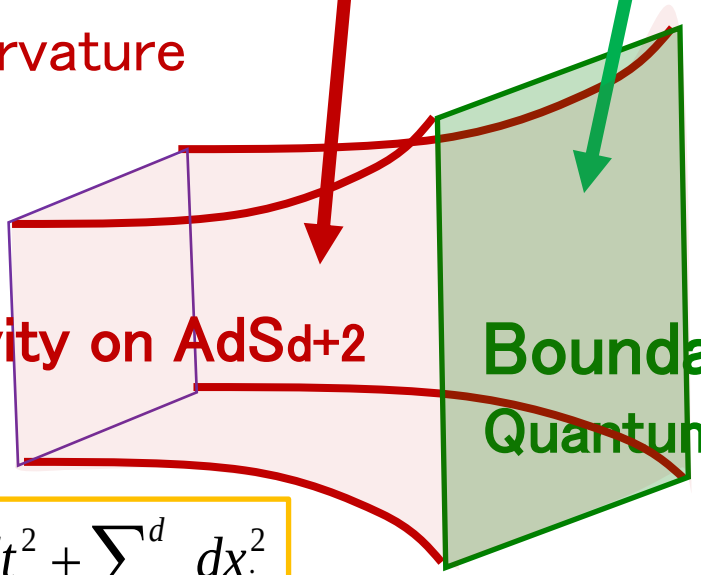
Gauge/Gravity Duality (AdS/CFT) – [Maldacena 1997]

(Quantum) Gravity on d+2 dim. Anti-de Sitter Space = **d+1 dim. Gauge theory (or Conformal field theory)**

Anti-de Sitter space (AdS)
➔ Universe with negative curvature

Conformal Field Theory(CFT)
➔ d dim. Gapless matter at quantum critical point

(quantum field theory with only massless fields)



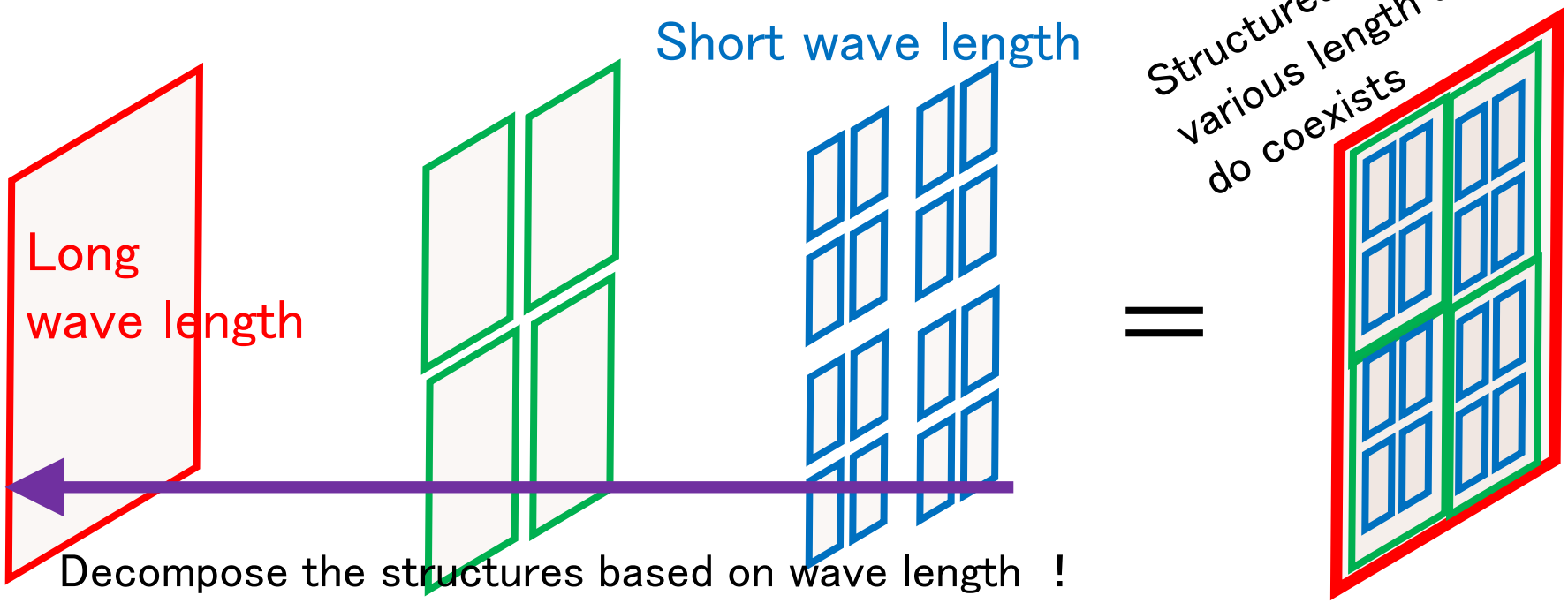
Gravity on AdS_{d+2}

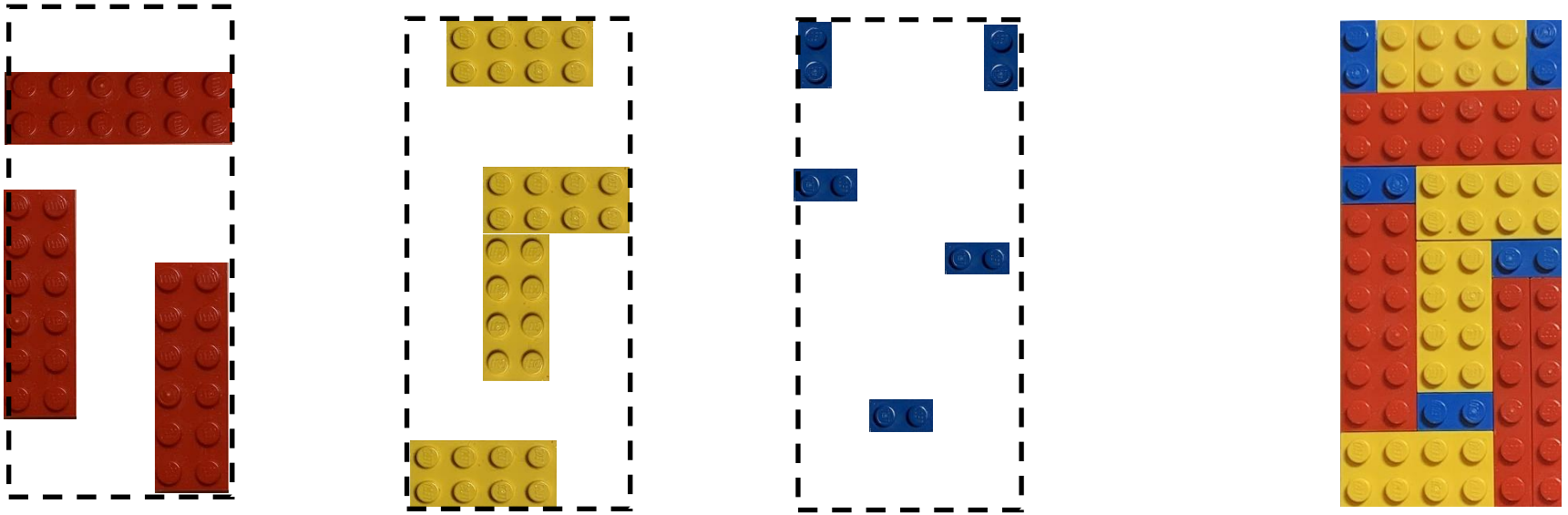
Boundary
Quantum Matter (CFT_{d+1})

AdS metric

$$ds^2 = R^2 \cdot \frac{dz^2 - dt^2 + \sum_{i=1}^{d} dx_i^2}{z^2}$$

A sketch on how AdS/CFT works





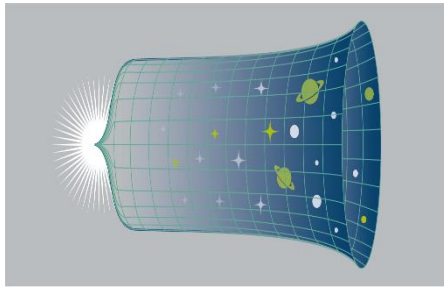
String Theory ➡ The best candidate of quantum gravity

In string theory, the most microscopic constituents are strings ! [Nambu, Goto 1970, . . Yoneya, Scherk-Schwarz 1974, . .]

magnify

Open string

Matter（gauge theory）



magnify

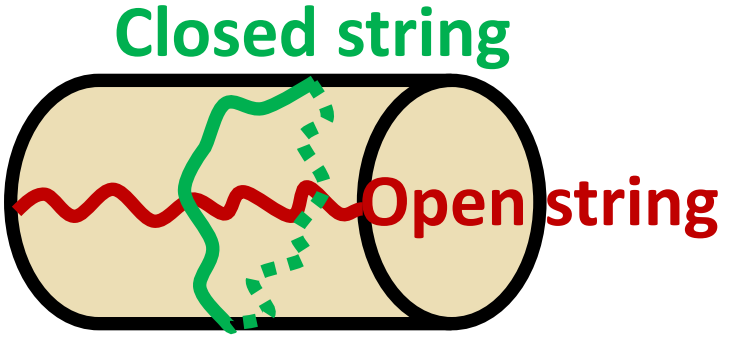
Closed string

Universe（gravity）

➡ String theory beautifully unifies gravity and matter !

String theory provides examples of holography as follows:

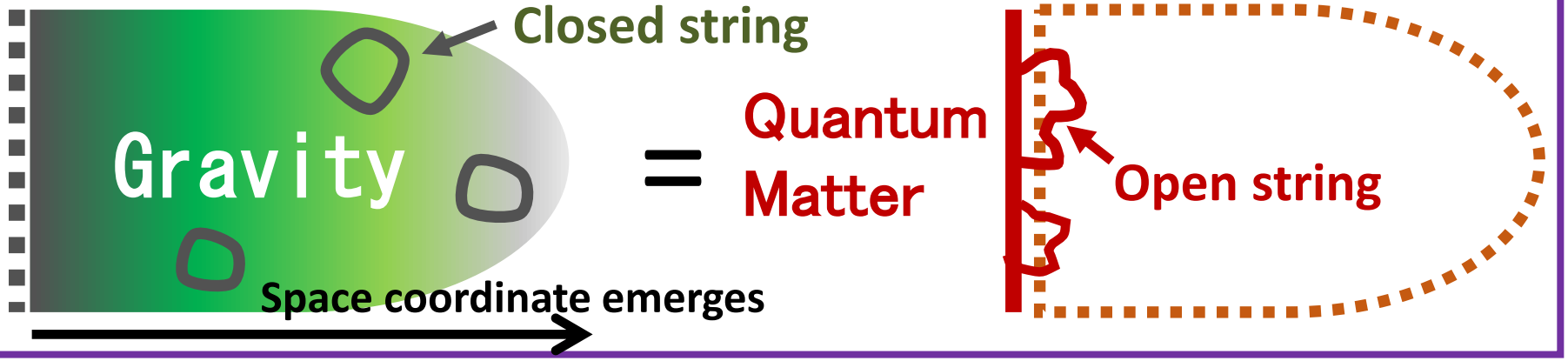
Open string (matter)
and closed string (gravity)
are equivalent !



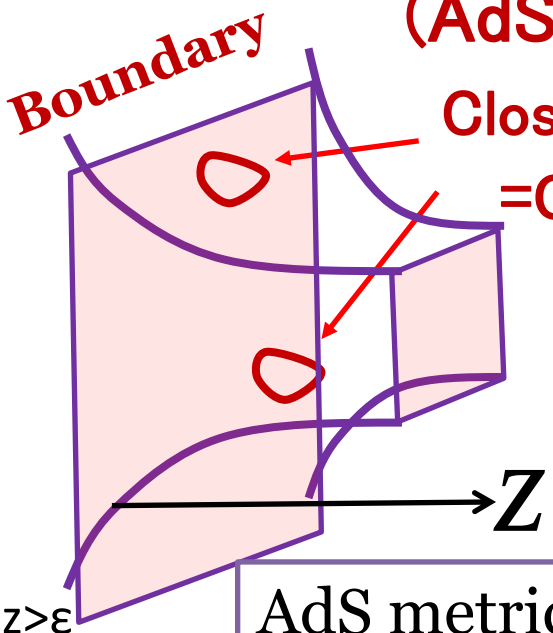
Gauge/Gravity duality（AdS/CFT）

Maldacena 1997

Gravity on a spacetime = Quantum matter on its boundary



Gravity in Anti de-Sitter space (AdS)

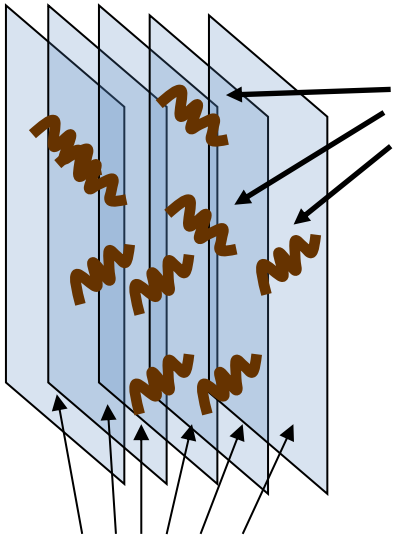


Closed Strings
=Gravitons

Equivalent

AdS/CFT

Conformal Field Theories (CFTs)



Open Strings
=Gluons in U(N) gauge theories

N D-branes

AdS metric

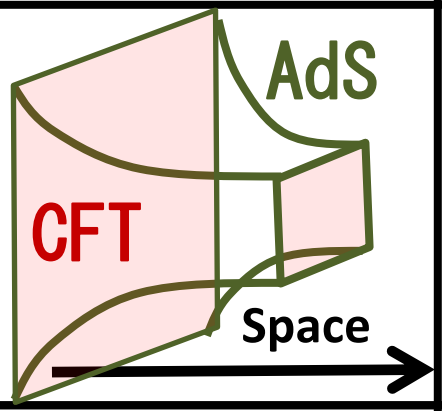
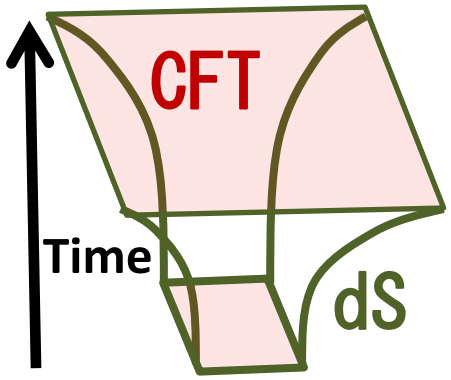
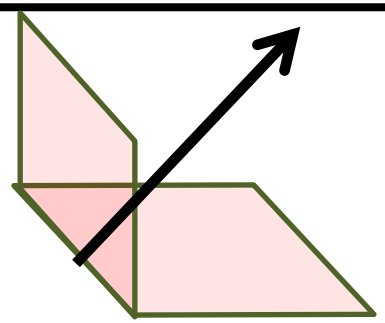
$$ds^2 = R^2 \cdot \frac{dz^2 - dt^2 + \sum_{i=1}^{d} dx_i^2}{z^2}$$

Thermodynamics of Black holes (branes)

Equivalent

Thermodynamics of various materials

Classification of Maximally Symmetric Spacetimes and Holography

Type	Geometry	Holography	Central charge
Anti de Sitter Space AdS $\Lambda<0$		**AdS/CFT** [Maldacena 1997] Gravity in d+2 dim. AdS = d+1 dim. CFT on $R^{1,d}$ ▶Emergent Space	In 3d, $C=\frac{3R_{AdS}}{2G_N}$
de Sitter Space dS $\Lambda>0$ "The Universe"		**dS/CFT** [Strominger 2001] Gravity in d+2 dim. dS $\stackrel{?}{=}$ d+1 dim. Euclid CFT on S^{d+1} ▶Emergent Time ?	In 3d, $C=i\frac{3R_{dS}}{2G_N}$ [Hikida-Nishioka-Taki-TT 2022]
Flat $\Lambda=0$		String theory can describe quantum gravity. Also celestial holography is proposed.	$C=i\infty$?

Thermodynamics

AdS3 BH $S_{AdS} = 2\pi\sqrt{\frac{cE}{3}}$

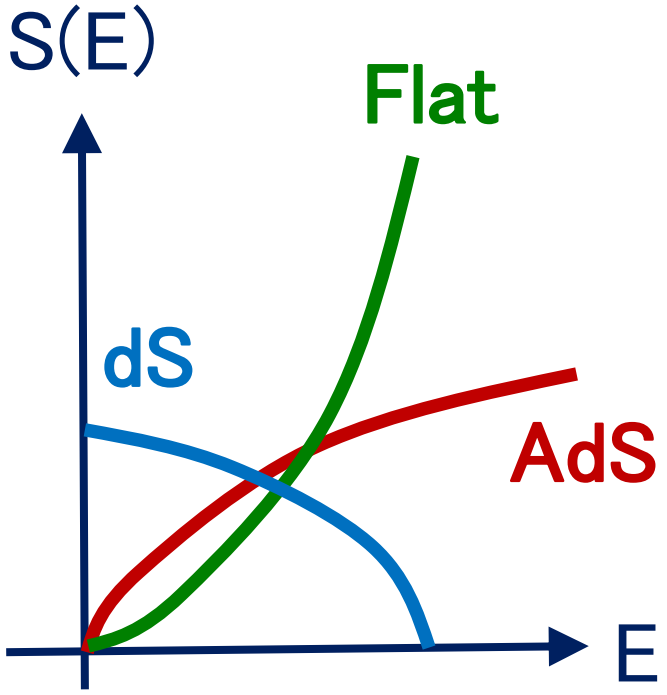
→It has a positive specific heat
and is thermodynamically stable.

dS3 BH $S_{dS} = \frac{\pi R_{dS}}{2G_N}\sqrt{1-8G_N E}$

→The vacuum E= 0, the state
is maximally entangled !

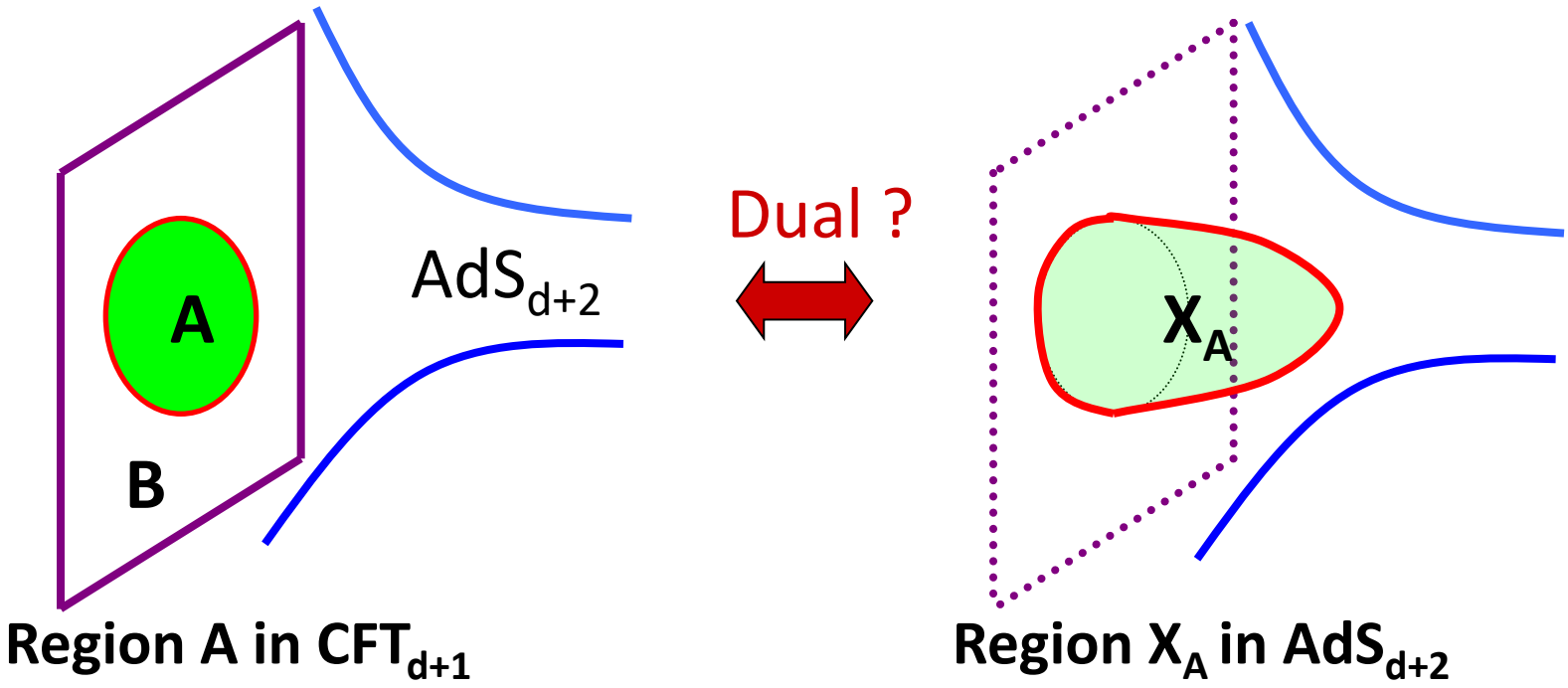
4D flat BH $S_{Flat} = 4\pi G_N E^2$

→ This leads to a negative specific heat.
It is thermodynamically, unstable.



This is one of
the main reasons
why holography
in dS/flat space
is very difficult !

A Basic Question: Which region in the AdS does encode the `information in a certain region' of the CFT ?



Consider the entanglement entropy S_A which measures the amount of information !

Holographic Entanglement Entropy [Ver.1:Static]

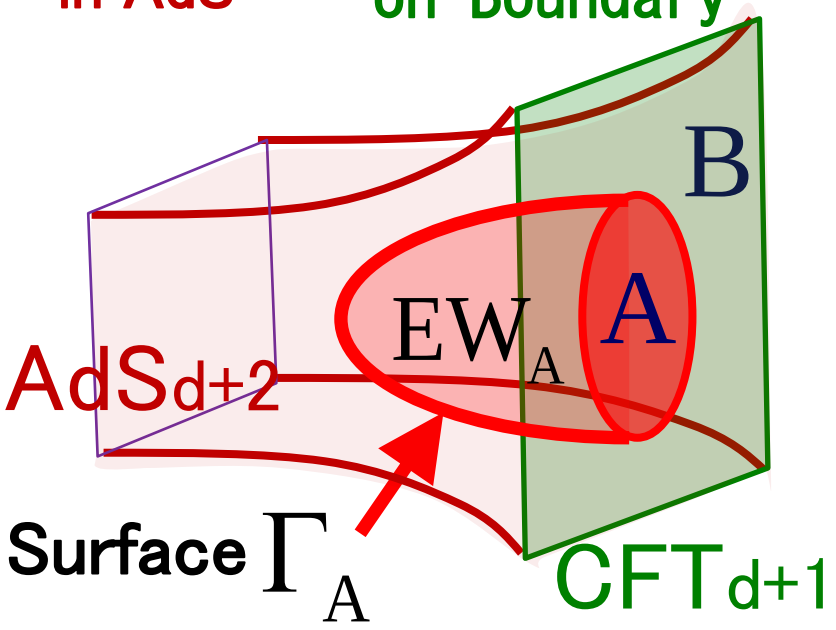
[Ryu-Takayanagi 2006]

Γ_A = Minimal Area Surface which surrounds A in AdS

$$S_A = \frac{\mathrm{Area}(\Gamma_A)}{4G_N}$$

Entanglement Entropy between A and B

Gravity in AdS = Quantum matter on Boundary

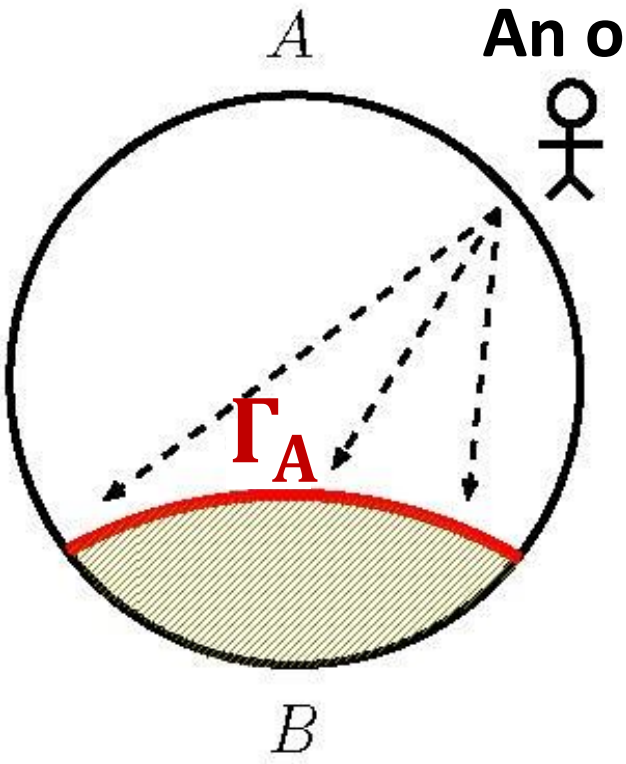


➡ A huge generalization of black hole entropy formula !

➡ Information in A is encoded in the entanglement wedge EW$_A$!

[Czech-Karczmarek-Nogueira-Raamsdonk, Wall 2012, Headrick-Hubeny-Lawrence-Rangamani 2014…]

Intuitive Understanding this formula



An observer in A

For an observer who is not accessible to B, it looks like there is a "black hole" at Γ_A .

The shaded region is hidden behind the "black hole".

⇒This BH entropy is S_A !

Holographic Entanglement Entropy [Ver.2:Time-dependent]

[Hubeny-Rangamani-TT 07]

A generic Lorentzian asymptotic AdS spacetime is dual to a time dependent state $|\Psi(t)\rangle$ in the dual CFT.

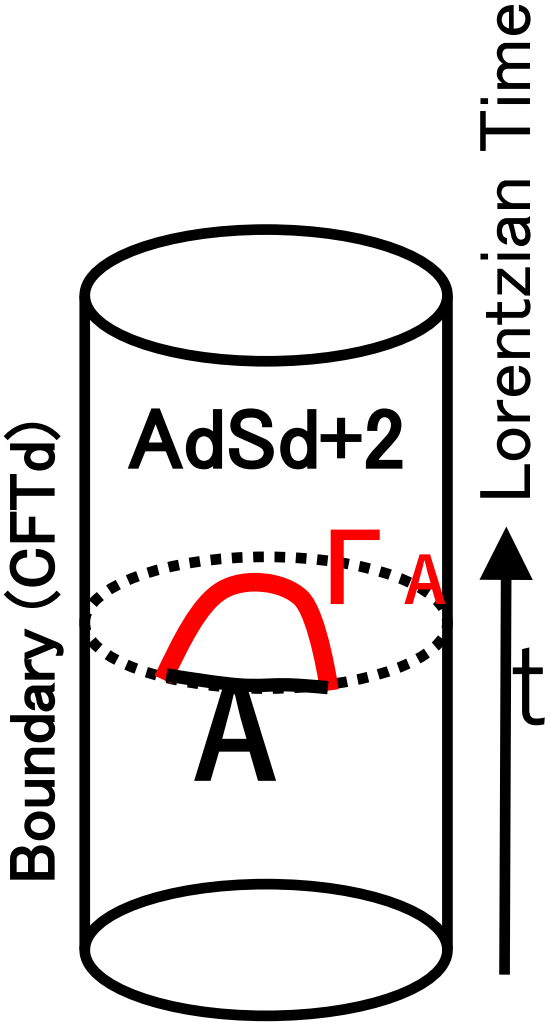
The time-dependent entanglement entropy

$$\rho_A(t) = \mathrm{Tr}_B[|\Psi(t)\rangle\langle\Psi(t)|] \Rightarrow S_A(t).$$

is computed from an extremal surface area:

$$S_A(t) = \mathrm{Min}_{\Gamma_A}\mathrm{Ext}_{\Gamma_A}\left[\frac{A(\Gamma_A)}{4G_N}\right]$$

$$\partial A = \partial \gamma_A \text{ and } A \sim \gamma_A \ .$$



These works were owing to relaxed and stimulating atmosphere.

Sequoia National Park, US
Dec.2005 @Xmas holidays

PRL 96, 181602 (2006) PHYSICAL REVIEW LETTERS week ending 12 MAY 2006

Holographic Derivation of Entanglement Entropy from the anti–de Sitter Space/Conformal Field Theory Correspondence

Shinsei Ryu and Tadashi Takayanagi
Kavli Institute for Theoretical Physics, University of California, Santa Barbara, California 93106, USA
(Received 8 March 2006; published 9 May 2006)

A holographic derivation of the entanglement entropy in quantum (conformal) field theories is proposed from anti–de Sitter/conformal field theory (AdS/CFT) correspondence. We argue that the entanglement entropy in $d+1$ dimensional conformal field theories can be obtained from the area of d dimensional minimal surfaces in AdS_{d+2}, analogous to the Bekenstein-Hawking formula for black hole entropy. We show that our proposal agrees perfectly with the entanglement entropy in 2D CFT when applied to AdS_3. We also compare the entropy computed in $AdS_5 \times S^5$ with that of the free $\mathcal{N}=4$ super Yang-Mills theory.

DOI: 10.1103/PhysRevLett.96.181602 PACS numbers: 11.25.Tq, 03.65.Ud, 04.70.Dy, 11.25.Hf

Puri, India , Dec.2006
@Indian String Meeting

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A covariant holographic entanglement entropy proposal

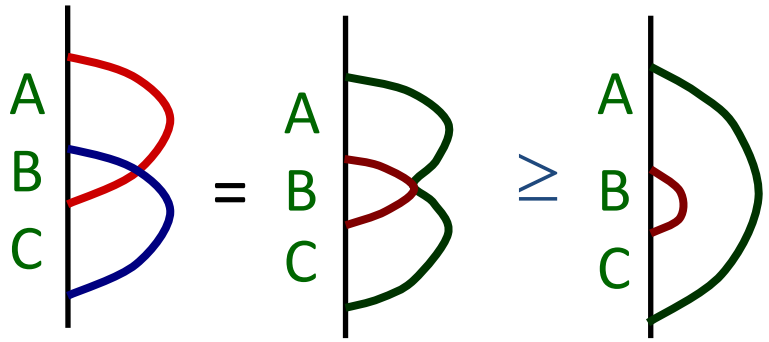
Veronika E. Hubeny and Mukund Rangamani
Centre for Particle Theory & Department of Mathematical Sciences
Science Laboratories, South Road, Durham DH1 3LE, U.K.
E-mail: veronika.hubeny@durham.ac.uk, mukund.rangamani@durham.ac.uk

Tadashi Takayanagi
Department of Physics, Kyoto University,
Kyoto, 606-8502, Japan
E-mail: takayana@gauge.scphys.kyoto-u.ac.jp

Algebraic properties in Quantum Information
⇔ **Geometric properties** in Gravity

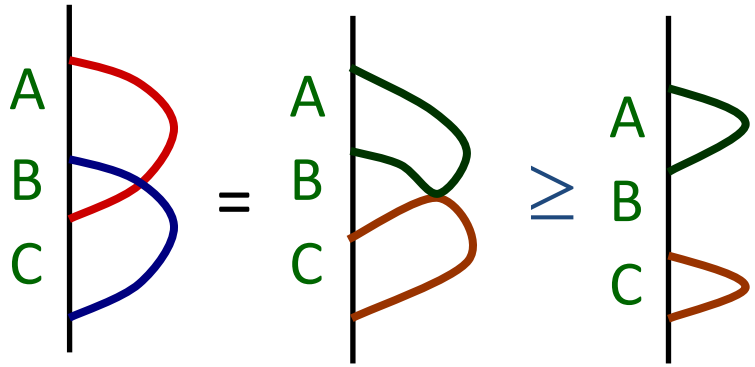
Holographic Proof of Strong Subadditivity(SSA)

[Headrick-TT 07]



$$\Rightarrow S_{AB} + S_{BC} \geq S_{ABC} + S_B$$

"Triangle inequalities in Geometry = SSA"

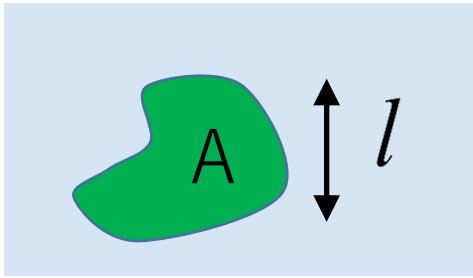


$$\Rightarrow S_{AB} + S_{BC} \geq S_A + S_C$$

$$(\text{Note}: \quad AB \equiv A \cup B)$$

In Lecture 2, we will explain the implications of SSA in quantum field theories.

General Behavior of HEE (=EE in CFT$_{d+1}$) [Ryu-TT 06,…]



$$S_A = \frac{\pi^{d/2} R^d}{2G_N^{(d+2)}\Gamma(d/2)}\left[p_1\left(\frac{l}{\varepsilon}\right)^{d-1} + p_3\left(\frac{l}{\varepsilon}\right)^{d-3} + \cdots \right.$$

$$\left. \cdots + \begin{cases} p_{d-1}\left(\frac{l}{\varepsilon}\right) + p_d & (\text{if } d+1 = \text{odd}) \\ p_{d-2}\left(\frac{l}{\varepsilon}\right)^2 + q\log\left(\frac{l}{\varepsilon}\right) & (\text{if } d+1 = \text{even}) \end{cases} \right] ,$$

where $p_1 = (d-1)^{-1}, p_3 = -(d-2)/[2(d-3)], \ldots.$

$\ldots\ldots \quad q = (-1)^{(d-1)/2}(d-2)!!/(d-1)!!$.

Area law divergence

A universal quantity (**F**) which characterizes **odd dim. CFT**.

Agrees with **conformal anomaly** (central charge) in **even dim. CFT**

Einstein Equation from Quantum Entanglement

First Law of EE

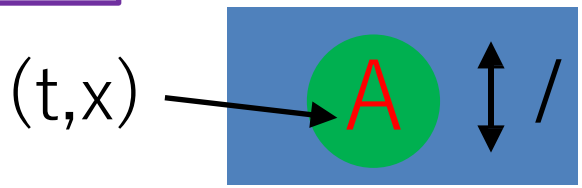
First law of thermodynamics

$$T\Delta S = \Delta E$$

$$\Delta S_A \cong \Delta E_A$$

[$E_A = -\log \rho_A$: Modular Hamiltonian]

[Casini-Huerta-Myers 13, Bhattachrrya-Nozaki-Ugajin-TT 13]



$$\left(\partial_l^2 - \partial_l - \partial_x^2 - \frac{3}{l^2}\right)\Delta S_A(t,\vec{x},l) = \langle O\rangle\langle O\rangle$$

[Nozaki-Numasawa-Prudenziati-TT 13]

[de Sitter space: de Boer-Haehl-Heller-Myers 16]

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + \Lambda g_{\mu\nu} = T_{\mu\nu}$$

Kinetic term C.C. Matter

➡ The 1st law of EE explains the perturbative Einstein eq.

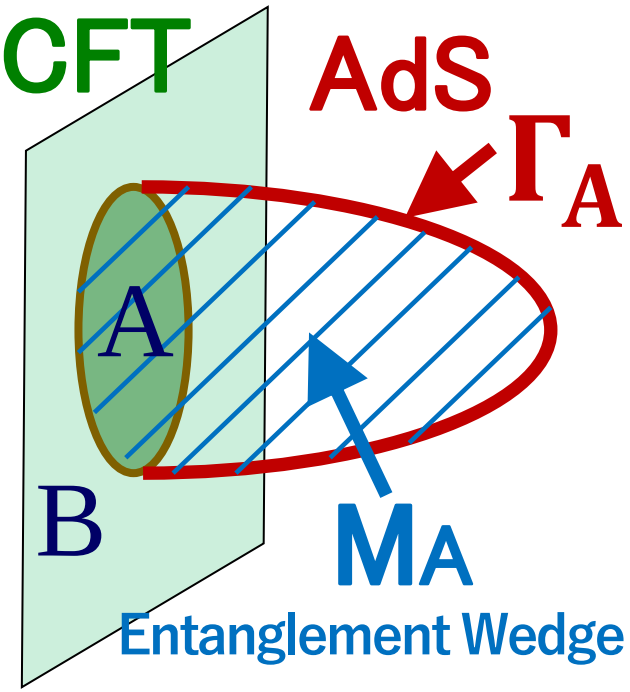
[Linear: Lashkari-McDermott-Raamsdonk, Faulkner-Guica-Hartman-Myers-Raamsdonk 13, Non-linear: Faulkner-Haehl-Hijano-Parrikar-Rabideau-Raamsdonk 17, Sarosi-Ugajin 17]

Quantum Corrections to HEE formula

HEE for classical gravity

[Ryu–TT 2006, Hubeny–Rangamani–TT 2007]

$$S_A = \underset{\Gamma_A}{\text{Min Ext}} \left[\frac{\text{Area}(\Gamma_A)}{4G_N} \right]$$



Loop corrections $1/G_N+1+G_N+\cdots$

$\leftrightarrow$ 1/N corrections in CFT

HEE with quantum corrections

$$S_A = \underset{\Gamma_A}{\text{Min Ext}} \left[\frac{\text{Area}(\Gamma_A)}{4G_N} + \underbrace{S_{bulk}(M_A)}_{\text{Bulk EE}} \right]$$

Quantum Extremal Surface

[Faulkner–Lewkowycz–Maldacena 2013, Engelhardt–Wall 2014]

Pseudo Entropy and Holography [Nakata-Taki-Tamaoka-Wei-TT, 2020]

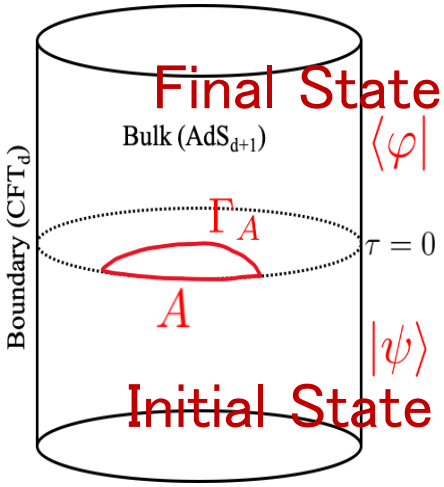
Q. What is the meaning of the minimal surface area
in an Euclidean time-dependent asymptotically AdS geometry ?

➤Answer is pseudo entropy !

Holographic Pseudo Entropy

$$S(\mathcal{T}_A^{\psi|\varphi}) = \min_{\Gamma_A} \frac{\mathrm{Area}(\Gamma_A)}{4G_N}$$

➡ The first half of Lecture 3

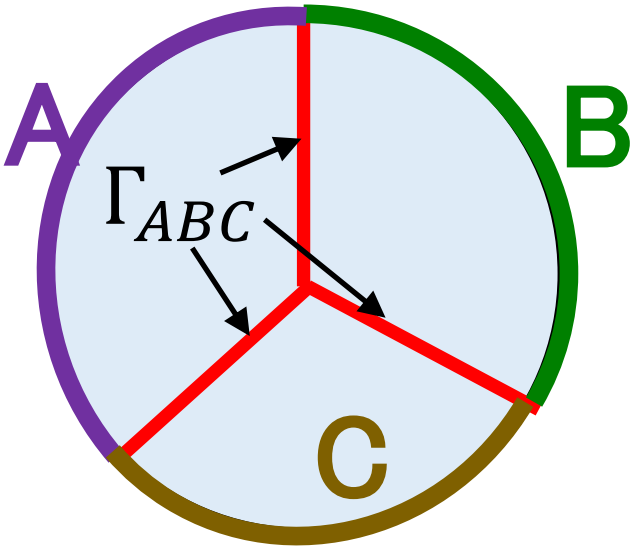


Multi-entropy

[Gadde-Krishna-Sharma 22, Penington-Walter-Witteveen 22 ; thanks to Jonathan Harper for many useful conversations]

Holographic Multi-entropy

$$S^{(3)}_{n=1} \stackrel{?}{=} \mathbf{Min}\left[\frac{\mathbf{Area}(\Gamma_{ABC})}{4G_N}\right]$$



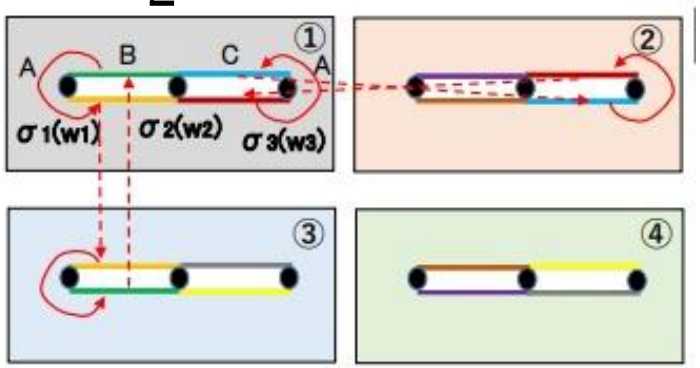
(n-th Renyi) q-partite entropy

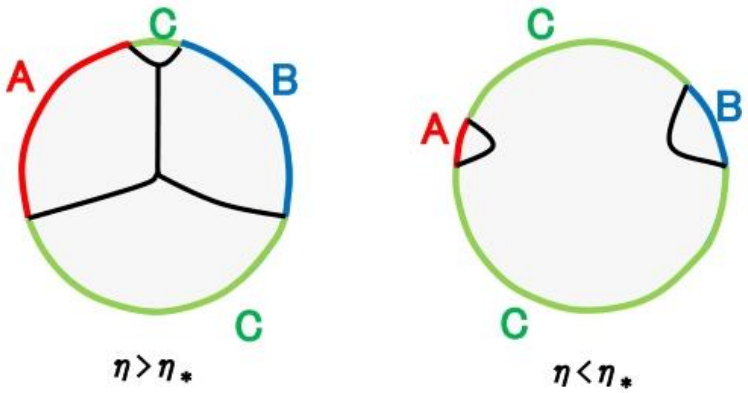
$$S_n^{(q)} \equiv -\frac{1}{(n-1)n^{q-2}}\log\left[\mathrm{Mtr}[\rho^{n^{q-1}}]\right]$$

➔Quantify tripartite entanglement ?

[2D CFT analysis: Harper-Tsuda-TT 24]

e.g. n=2 $S_2^{(3)} = -\frac{1}{2}\log\left[\rho_{abc}^{aBR}\rho_{ABC}^{AbN}\rho_{PQR}^{PMc}\rho_{LMN}^{LQC}\right]$





➡ **Large c phase transition**

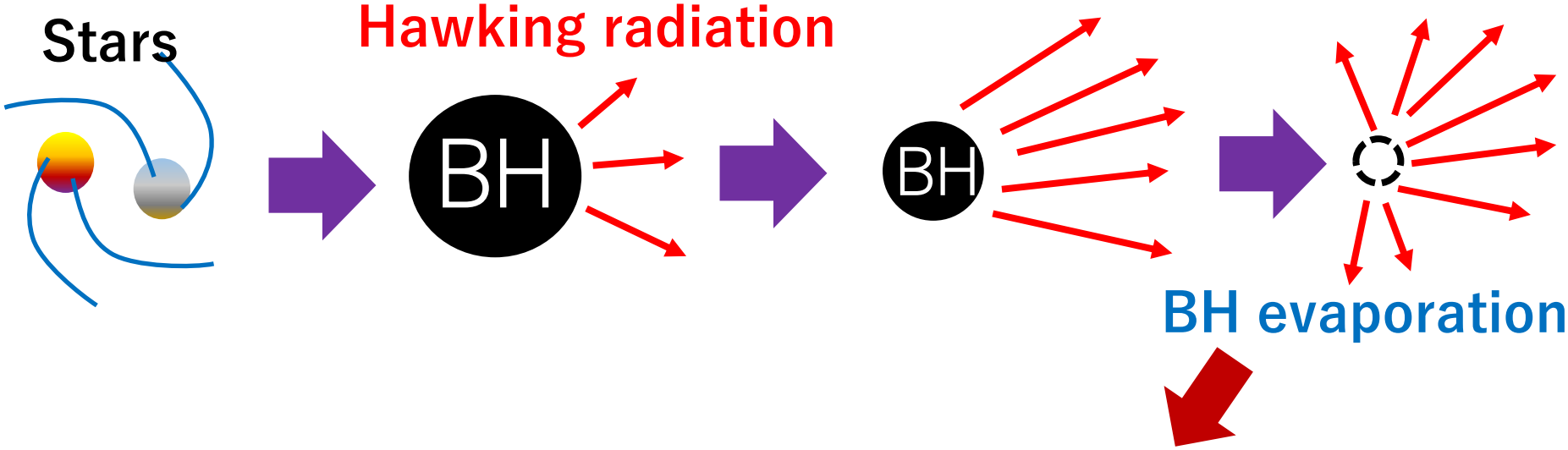
⑤ Applications to Black hole Information Problem

Black hole (BH) information Problem

A BH has a temperature and is a thermal object.

Thus, a BH radiates (Hawking radiation) and loses its energy.

Eventually it evaporates and disappears, called BH evaporation.



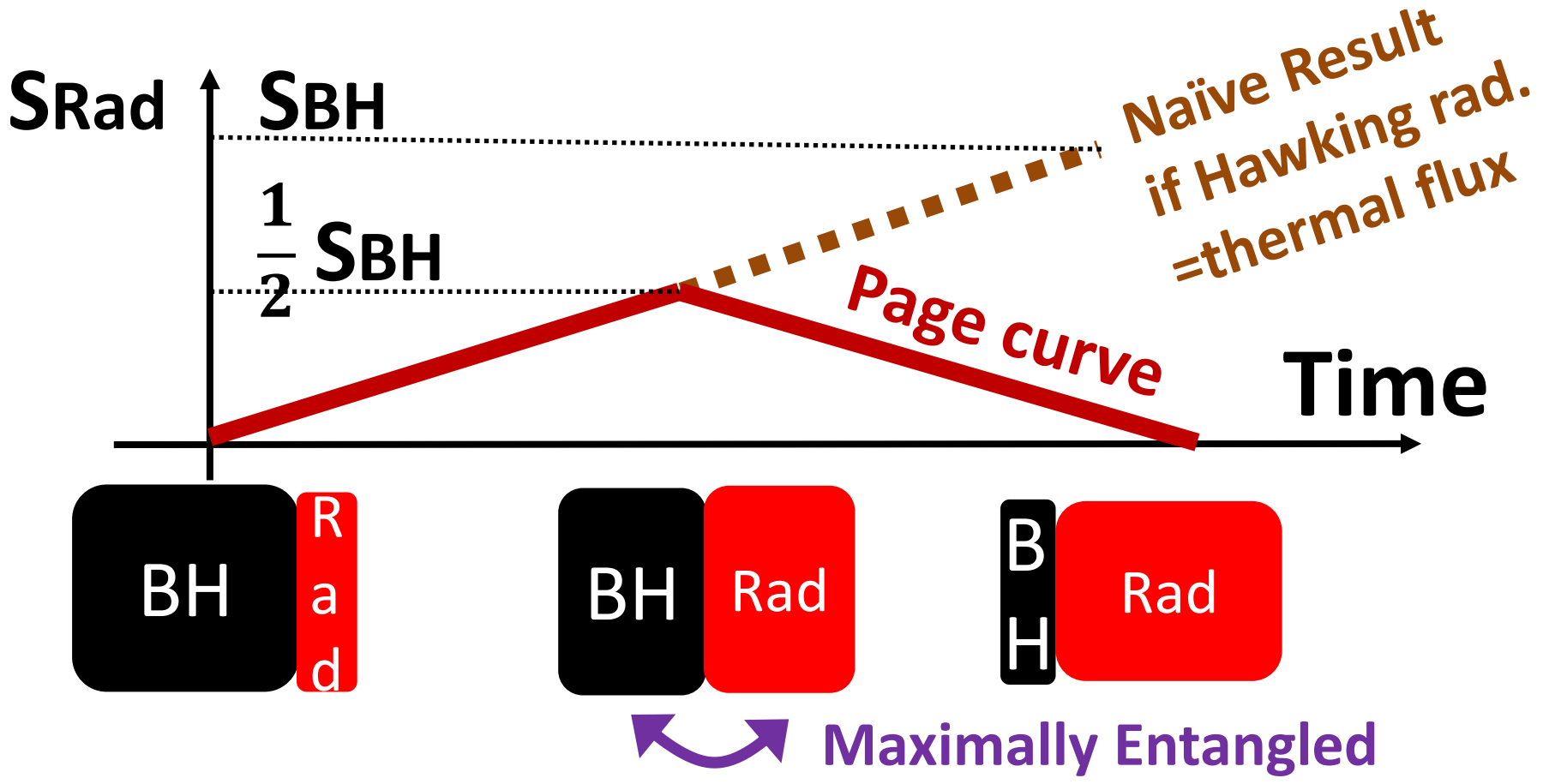
The information inside BH seems to disappear !

➔If so, this contradicts with the unitarity of quantum mechanics !

Page curve [A quantification of BH information problem]

Model: $H_{tot} = H_{BH} \otimes H_{Rad}$ $\quad |\psi(t)\rangle = e^{-iHt}|\psi_0\rangle \in H_{tot}$

$$\rho_{\mathrm{Rad}}(\mathrm{t}) = \mathrm{Tr}_{\mathrm{BH}}\left[|\psi(t)\rangle\langle\psi(t)|\right] \Rightarrow \boxed{S_{Rad}(t) = -\mathrm{Tr}\left[\rho_{\mathrm{Rad}}(\mathrm{t})\log\rho_{\mathrm{Rad}}(\mathrm{t})\right]}$$



Black hole information problem and Island Formula

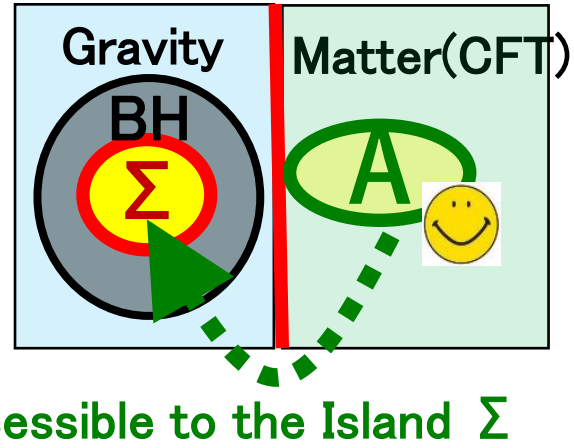
Recently, the Page curved was derive by generalizing the HEE formula to CFTs which is coupled to gravity, so called the Island formula ！

Island Formula:

[Penington 2019, Almheiri et.al. 2019]

$$S_A = \mathbf{Min}\left[\frac{\mathbf{Area}(\partial\Sigma)}{4G_N} + S_{A\cup\Sigma}\right]$$

EE for Radiations　BH entropy　Bulk EE



This explains the Page curve !

➔ Unitarity of BH evaporation!

As the BH evaporation proceeds,
a secret hole (called Island) appears inside BH.
Via this secret tunnel, we recover BH information.

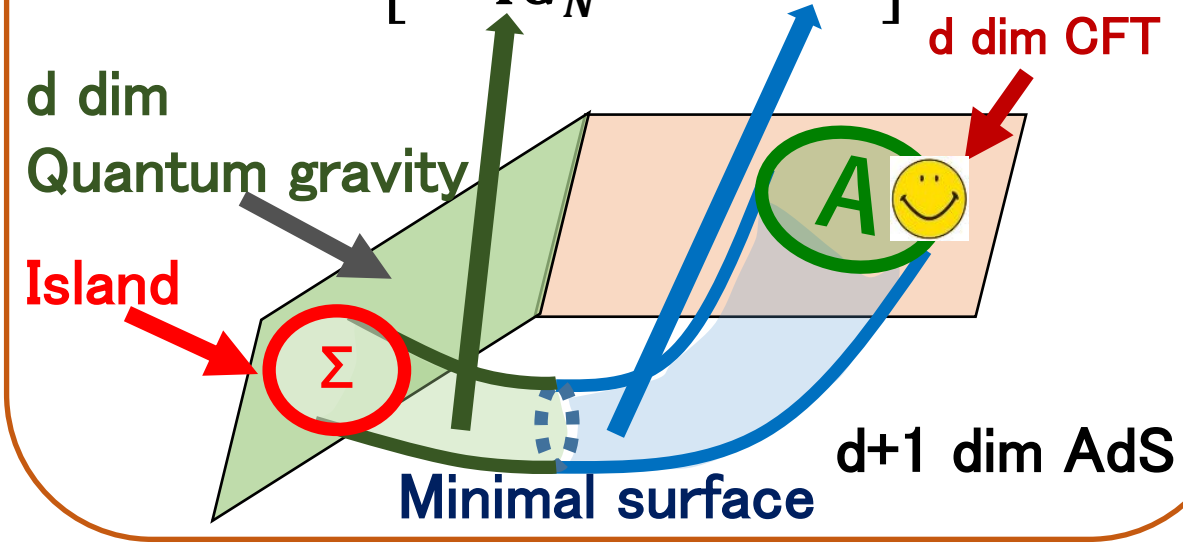
Mechanism of Resolving the BH information Problem

- ♦ A "wormhole" which connects the outside observer and the BH interior emerges in the middle of evaporation !
- ♠ However, the decoding of BH information from radiation is extremely hard (computational complexity is exponential) !

Higher dimension picture via Holography

[AdS/BCFT, Double holography]

$$S_A = \mathrm{Min}\left[\frac{\mathrm{Area}(\partial\Sigma)}{4G_N} + S_{A\cup\Sigma}\right]$$

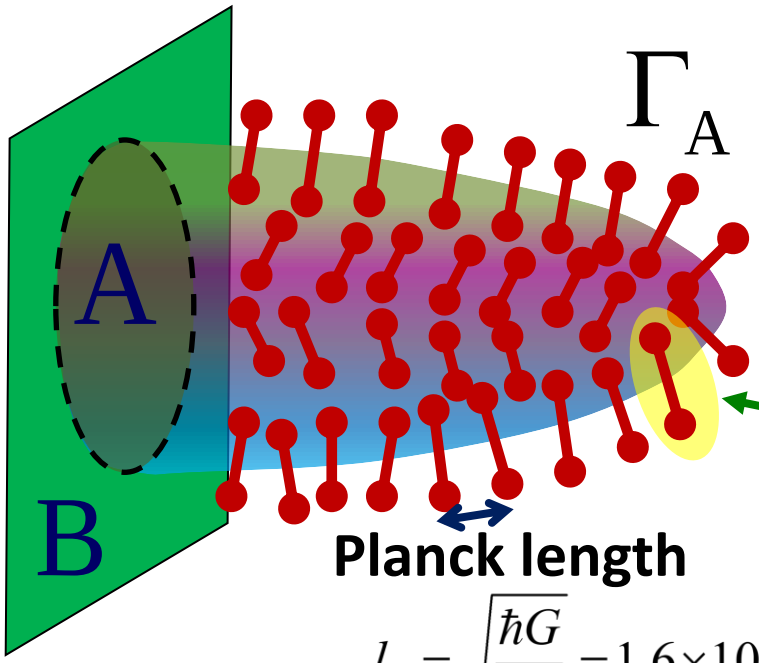


This will be briefly explained in Lecture 2.

[cf. Hilbert space structure of BH interior: Papadodimas-Raju 2012,… Raju 2021,…]

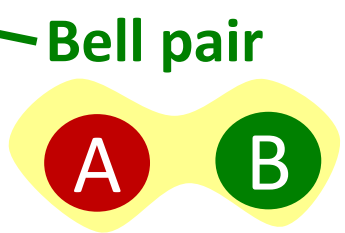
⑥ Emergence of Universe from Quantum Entanglement

The HEE suggests that there is one qubit of entanglement for each Planck length area !



$$S_A = \frac{\mathbf{Area}(\Gamma_A)}{4l_{pl}^{D-1}}$$

~10^{65} qubits per 1cm^2 !



Bell pair

= Planck scale mini Universe

$$l_P = \sqrt{\frac{\hbar G}{c^3}} = 1.6\times10^{-35}\,m$$

Spacetime may emerge from entangled Qubits !

➔ Tensor Network (TN) realizes this idea !

Tensor Network (TN)

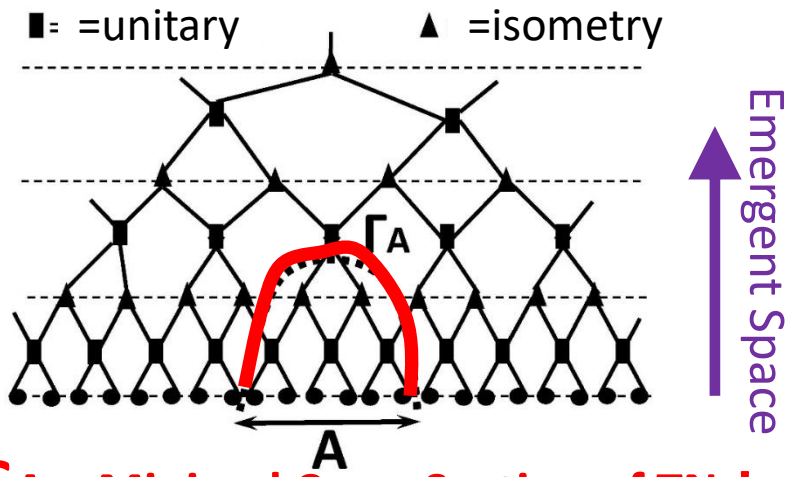
[DMRG: White 92,.. CTM: Nishino-Okunishi 96, PEPS: Verstraete-Cirac 04, ….]

TN = Graphical description of quantum states

Quantum State = Network of quantum entanglement

[Ex. 1] MERA TN [Vidal 2005]

➔ **Describe CFT vacuum**

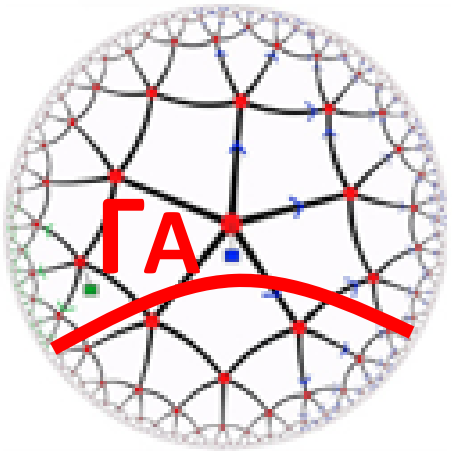


S_A = Minimal Cross Section of TN !

[Ex. 2] HaPPY/RTN model

[Patawski-Yoshida-Harlow-Preskill 2015]
[Hayden-Nezami-Qi-Thomas-Walter 2016]

➔ Use quantum error correcting code



Tensor Networks = AdS

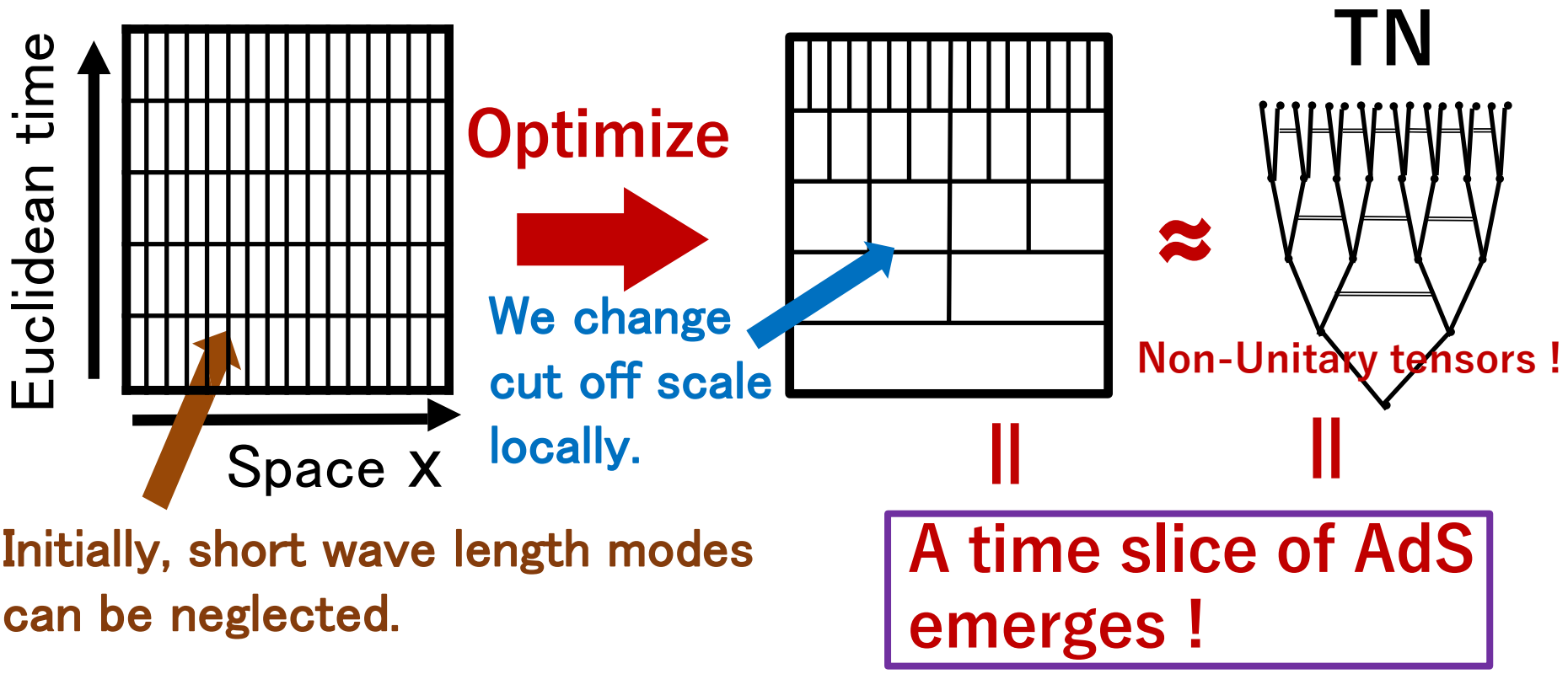
[Swingle 2009, Nozaki-Ryu-TT 2012, Czech-Lamprou-McCandlish-Sully 2015,…]

[Ex.3 Path-integral Optimization] [Caputa-Kundu-Miyaji-Watanabe-TT 2017]

Q. Can we describe CFT as a tensor network ? ⇒ **Path-integral**

Basic Principle

Minimize the computational cost of (discretized) path-integral.



How to optimize path-integral (in 2 dim. CFT)

Idea: Local change of UV cut off scale = Metric change

$$ds^2 = e^{2\omega(x,z)}(dx^2 + dz^2).$$

Owing to conformal symmetry, the wave function behaves as

$$\Psi[\phi, \omega] = e^{C[\omega]} \cdot \Psi[\phi, \omega = 0]$$

Optimization ⇒ Minimize the cost C[ω] !

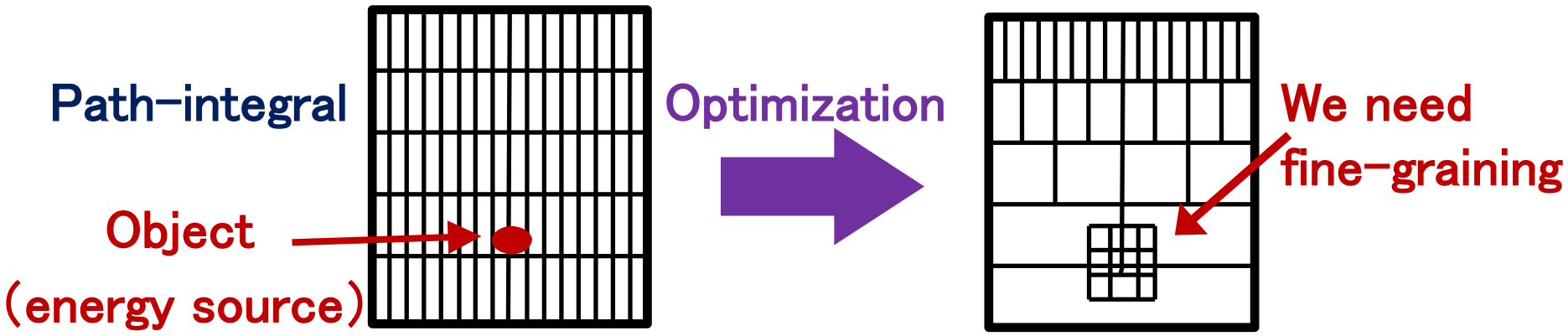
[C[ω]≈Computational Complexity]

In two dim. CFT, C[ω] is given by Liouville action:

$$C_{2D}[\omega] = \frac{c}{24\pi}\int dxdz\left[(\partial_x\omega)^2 + (\partial_z\omega)^2 + e^{2\omega}\right]$$

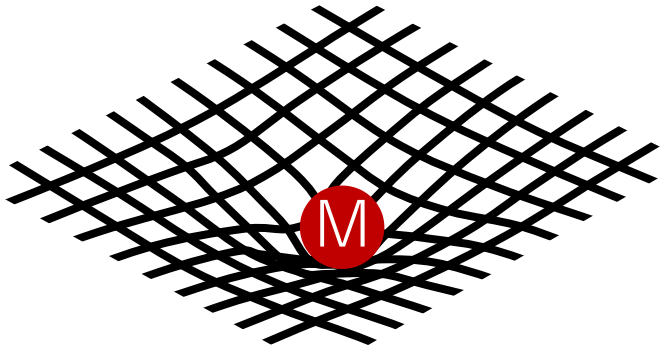
Minimization leads to AdS metric !

Upshot : Minimizing computational costs leads to gravity !



Energetic source (=information source)
distorts the spacetime

➔ The essence of general relativity !



Holographic Perspective [Boruch-Caputa-Ge-TT 2021]

Path-integral Optimization
= Maximization of Hartle-Hawking wave function for *AdS Universe*

Gravity might be an ideal "quantum computer" ?

How our universe emerges from quantum information ?

How about de Sitter space instead of AdS ?

We will discuss in the latter half of Lecture 3.

⑦ Conclusions

Our contributions
and recent developments

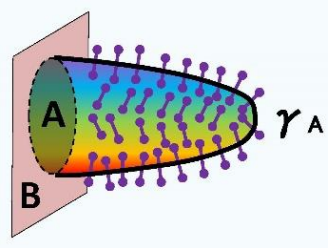
Traditional Physics

Microscopes
Accelerators

Matter = Collection of Particles

Crystallization

The formula
"Entropy＝Area"
is robust for general
spacetimes



A New direction in Physics

Holography

Is gravity the fastest "quantum computer" ?
➔New insights into quantum matter, quantum computation
and quantum cryptography

Universe = Collection of Qubits

Emergence

Does gravitational spacetime emerge from qubits ?
➔New approach to quantum gravity

Future problems

- Generalization of AdS/CFT to more general spacetimes (e.g dS)

  ➔Does the time coordinate emerge from entanglement ?

- Internal space in AdS/CFT and entanglement entropy

  [Earlier discussions: Mollabashi-Shiba-TT 2014, …, Das-Kaushal-Mandal-Trivedi 2020, .., Bohra-Das-Mandal-Nanda-Radwan 2024]

- Multi-partite entanglement entropy in AdS/CFT [Multi-entropy etc.]

- Use of quantum information to prove the full AdS/CFT

  [cf. Proof of AdS3/Sym.Prod.CFT2: Eberhardt-Gaberdiel-Gopakumar 2019]

- Understanding the microscopic origin of BH entropy from QG

- Table top experiments which model holographic spacetimes

  :

- Quantum gravity explanation of creation of the Universe

Thank you very much !