

# Math Circle India Workshop

## Frieze Patterns: The Hidden Symmetry Party

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*“Math is like a good detective novel: full of twists, surprises, and... lots of symmetry!”*

### **Introduction: The Beauty of Symmetry**

Imagine a world where everything repeats perfectly, like dance moves on an endless loop. Welcome to the wonderful world of **frieze patterns**, where symmetry is king, and patterns know how to party!

A frieze pattern is a two-dimensional design that repeats infinitely in one direction. It's like that friend who tells the same story over and over, but you don't mind because it's just so darn interesting. These patterns have something called **translation symmetry**—a fancy way of saying that you can slide them over and they still look the same!

Now, get ready to uncover these hidden patterns, just like a math detective solving a mystery—except this case is full of footprints and Pacman.

### **Identifying Symmetries: Sherlock Mode On**

#### **Activity 1: Footprints in the Sand (No Beach Required)**

In Figure 1, we've got some funky footprints for you. Don your detective hat and describe the symmetries you spot. Does the pattern flip, slide, or maybe even twirl? Every move counts!

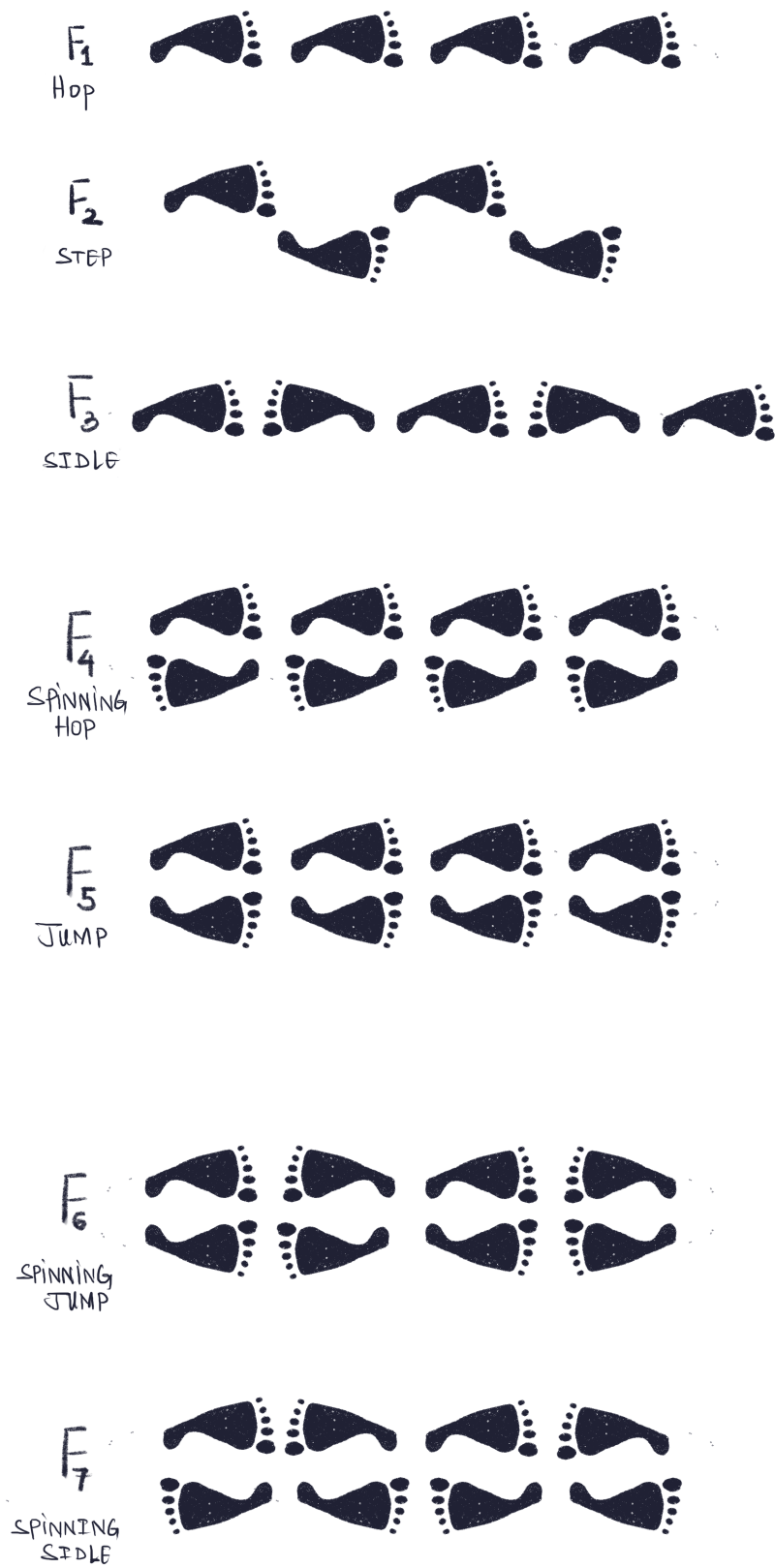


Figure 1: Conway's Footprint Symmetry Extravaganza

**Activity 2: Marking the Symmetries**

Okay, now let's grab our math magnifying glasses and mark these footprints! Can you find all the hidden symmetry moves?

1. Draw an arrow for the smallest **translation** (it's like the pattern's walk cycle).
2. Mark mirror lines wherever the pattern seems to say, "Hey, look at me, I'm symmetrical!"
3. Spot any rotation centers? Mark those dizzying points!
4. Bonus points if you spot a **glide reflection**—that's where a slide and a flip happen at the same time. It's the moonwalk of symmetries.

**Activity 3: Give them a Name!**

We've got five VIPs (Very Important Patterns) at this symmetry party:

- **T** - Translation (Sliding without slipping)
- **G** - Glide Reflection (Moonwalking... mathematically)
- **R** - Rotation (Spinning like a pro)
- **V** - Vertical Reflection (Mirror, mirror on the wall)
- **H** - Horizontal Reflection (Mirror, mirror on the floor?)

Now, assign these cool moves to the footprint patterns. Who's moonwalking? Who's twirling? Find out!

**Activity 4: Frieze Patterns in the Wild: Spot the Hidden Patterns** Frieze patterns are sneaky! They show up in places you'd never expect—like African fabrics and ancient temples. Check out these frieze patterns below and use your symmetry skills to break them down. Get ready to "frieze" and analyze!





Figure 2: Frieze Patterns in African Textiles: Cultural Symmetry Vibes!



Figure 3: Frieze patterns of Maori tribes of New Zealand

## Pacman and Symmetry: A Match Made in Geometry Heaven

### Activity 5: Let Pacman Loose

Pacman needs to run through the grid without breaking symmetry. Use the symmetry moves (translation, glide reflection, rotation, etc.) to fill the grid. Just remember, only one Pacman per square, please—no overcrowding!

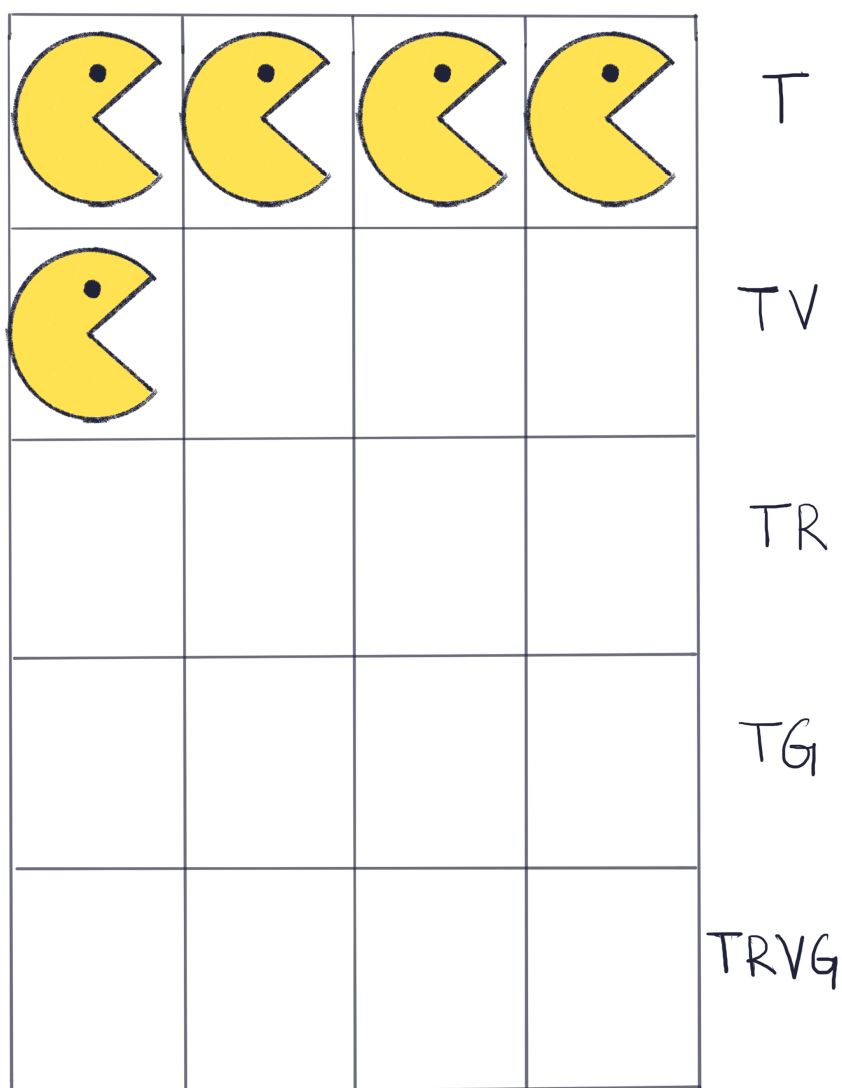


Figure 4: Pacman in Symmetry Land

### Activity 6: Recreating Frieze patterns

Consider the shape R. Can you use it to create all the seven frieze patterns you saw in the footprints pictures?

## It's All Connected (challenging exercise)

Here's where it gets interesting. Some symmetry moves are so close, they force others to tag along. For example, if you have a glide reflection (**G**), you automatically get a translation (**T**) too. Symmetry is all about teamwork!

### Activity 7: Finding relations

Let's see if you can spot these hidden relationships:

1. If a pattern has translation (**T**) and horizontal reflection (**H**), what other symmetry shows up to the party?
2. If a pattern has translation (**T**), rotation (**R**), and vertical reflection (**V**), what extra symmetry pops in unexpectedly?
3. If a pattern has translation (**T**), rotation (**R**), and glide reflection (**G**), what extra symmetry is also present?
4. Find another sneaky combination that brings along an extra symmetry friend.

Turns out that—no matter how wild and wacky the patterns get, there are only **seven** types of frieze patterns. John Conway, the symmetry superstar, gave each one a quirky nickname based on the shapes they make.

## Identifying Some More Frieze Patterns

**Activity 8: Pattern Matching** Use the labels T, G, V, H, R to identify the following frieze patterns:

1. ... DDDD ...
2. ... HHHH ...
3. ... VVVV ...
4. ... NNNN ...

**Activity 9: Graph Patterns** Determine the frieze pattern for each of the following functions:

1.  $y = \sin x$
2.  $y = |\sin x|$
3.  $|y| = \sin x$
4.  $y = \tan x$
5.  $y = \csc x$

## Fundamental domain

For any frieze pattern, a *fundamental domain* is a piece of the pattern of the smallest possible area that can be repeated to make the entire pattern, using operations that are symmetries of the entire pattern.

**Activity 10: Finding the fundamental domain** For each of Conway's footprint frieze patterns, identify a possible fundamental domain and mark it. Suggestion: as a first step, look for a translational symmetry and reduce from there. Now find fundamental domain for some of the other patterns too.

## A curiosity

Have you ever noticed automobile tire marks in mud? That's a frieze pattern! Go out and observe some tire marks from cars or even bicycles. An extensive study automobile tire marks claims that only 5 out of the 7 patterns occur. Does your study of tire marks suggest the same? Can you guess which two patterns don't occur and give a possible explanation why?

## Your Turn: Get Creative!

Time for you to join the party. Design a crazy, mind-boggling frieze pattern of your own. Use colors, shapes, whatever you like. Bring it to the next session, and we will ask others to identify the pattern! Let's see who's got the most creative symmetry skills!

## Frieze Patterns in Real Life: The Hunt Is On

Math and art go hand-in-hand, and frieze patterns are the perfect example of that. Here are some links to check out frieze patterns in real-world contexts:

- Here are some frieze patterns in music sheets  
<https://archive.bridgesmathart.org/2009/bridges2009-169.pdf>
- Here are some pretty frieze patterns  
<https://bookzoompa.wordpress.com/category/frieze-groups/>
- Here is a fun video talking about Möbius strip and a frieze pattern in the candy 'Fruit by the foot'  
<https://www.youtube.com/watch?v=Am-a5x9DGjg>
- A reference for the frieze patterns of the palaces of Alhambra in Spain  
<https://archive.bridgesmathart.org/2007/bridges2007-203.pdf>
- Proof that there are 7 frieze symmetries  
[https://issuu.com/idancarre/docs/mcguire\\_freizepatternresearchpaper](https://issuu.com/idancarre/docs/mcguire_freizepatternresearchpaper)
- A reference for frieze groups and wallpaper groups  
<https://www.d.umn.edu/~jgallian/Chapter28.pdf>