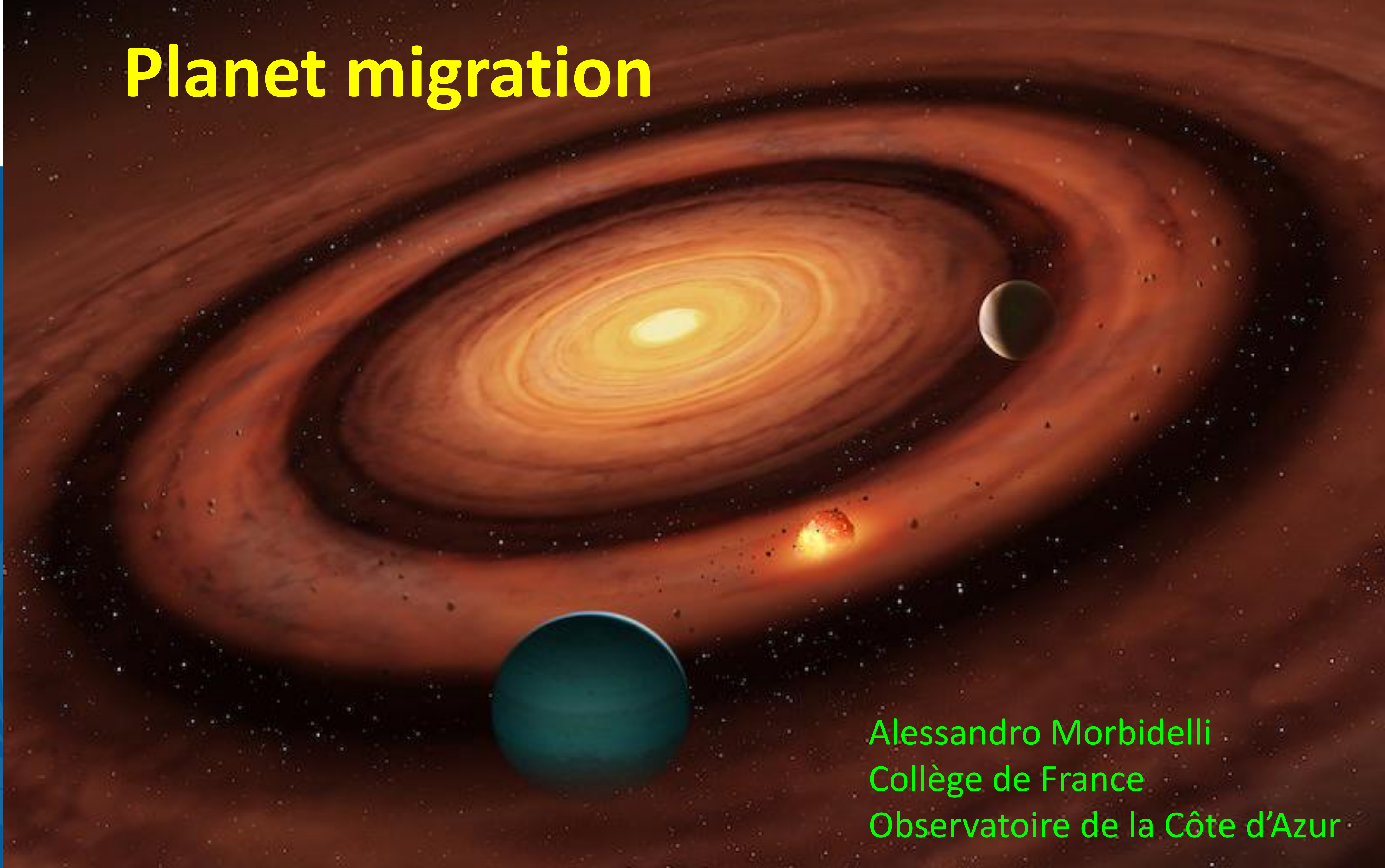
Planet migration



Alessandro Morbidelli
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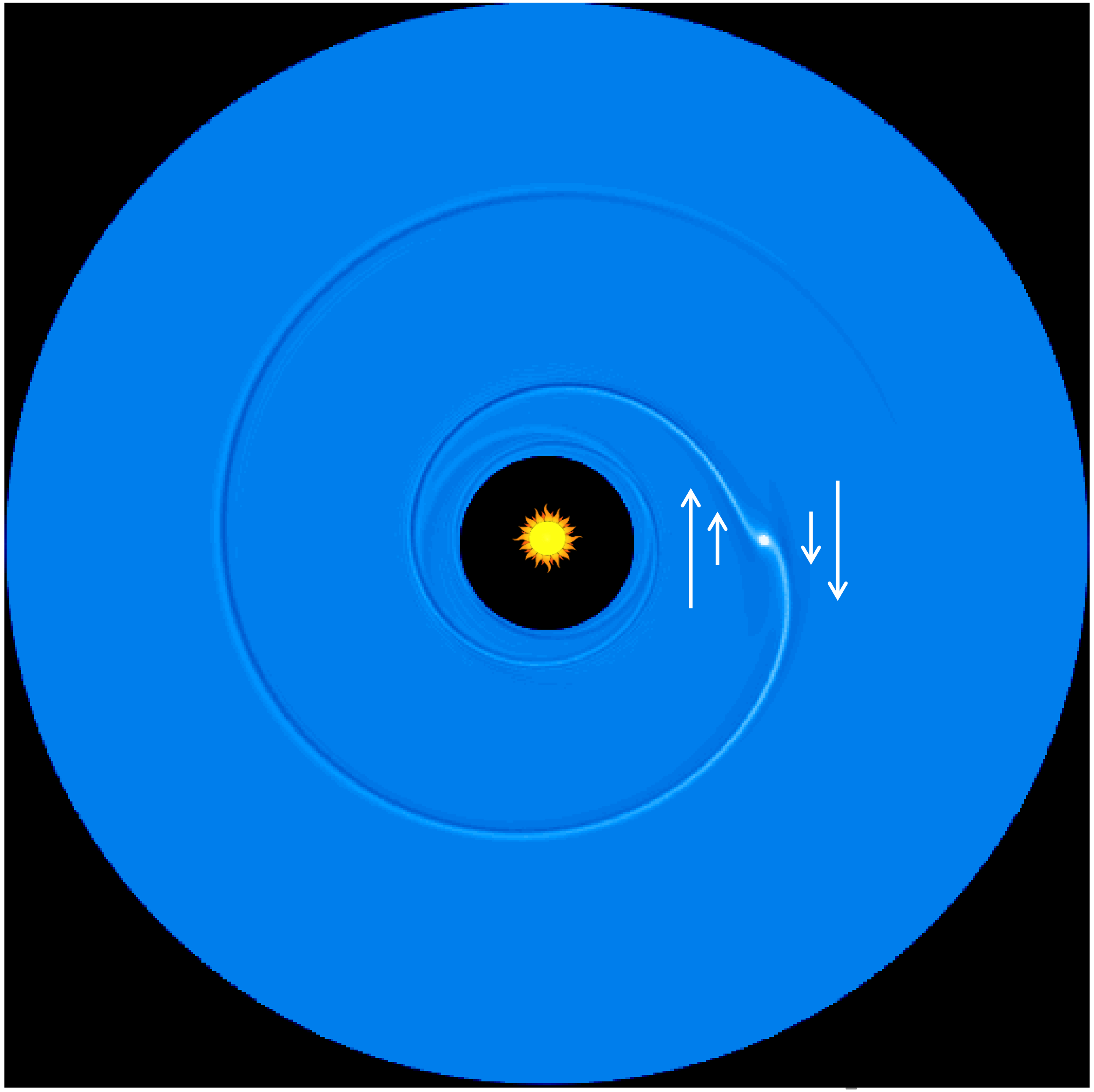
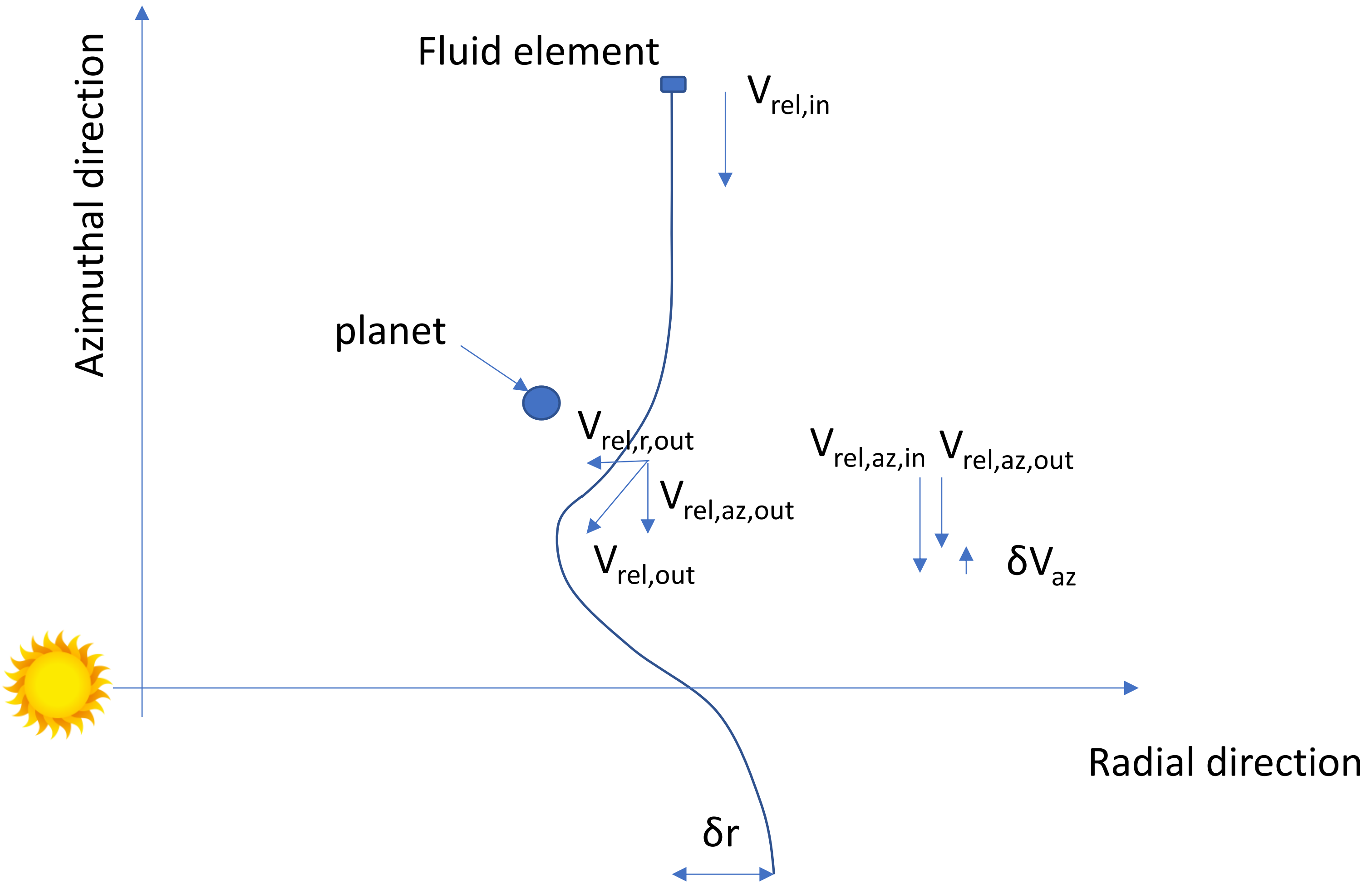
A planet embedded in a gas disk – Generation of a spiral density way

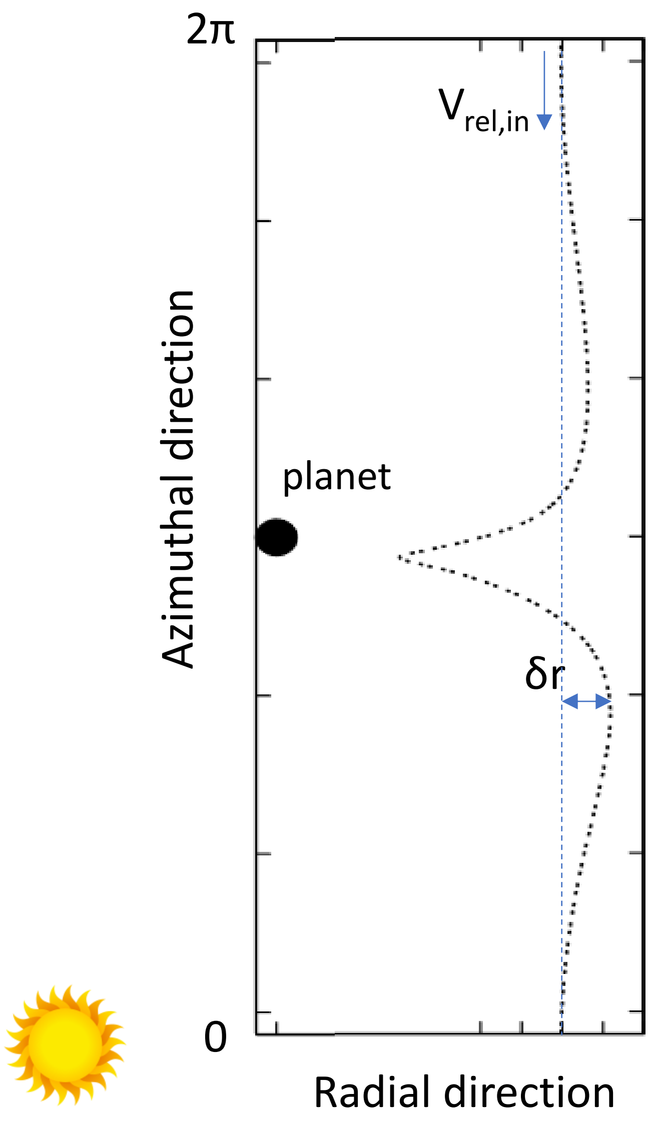


Formation of the spiral wave



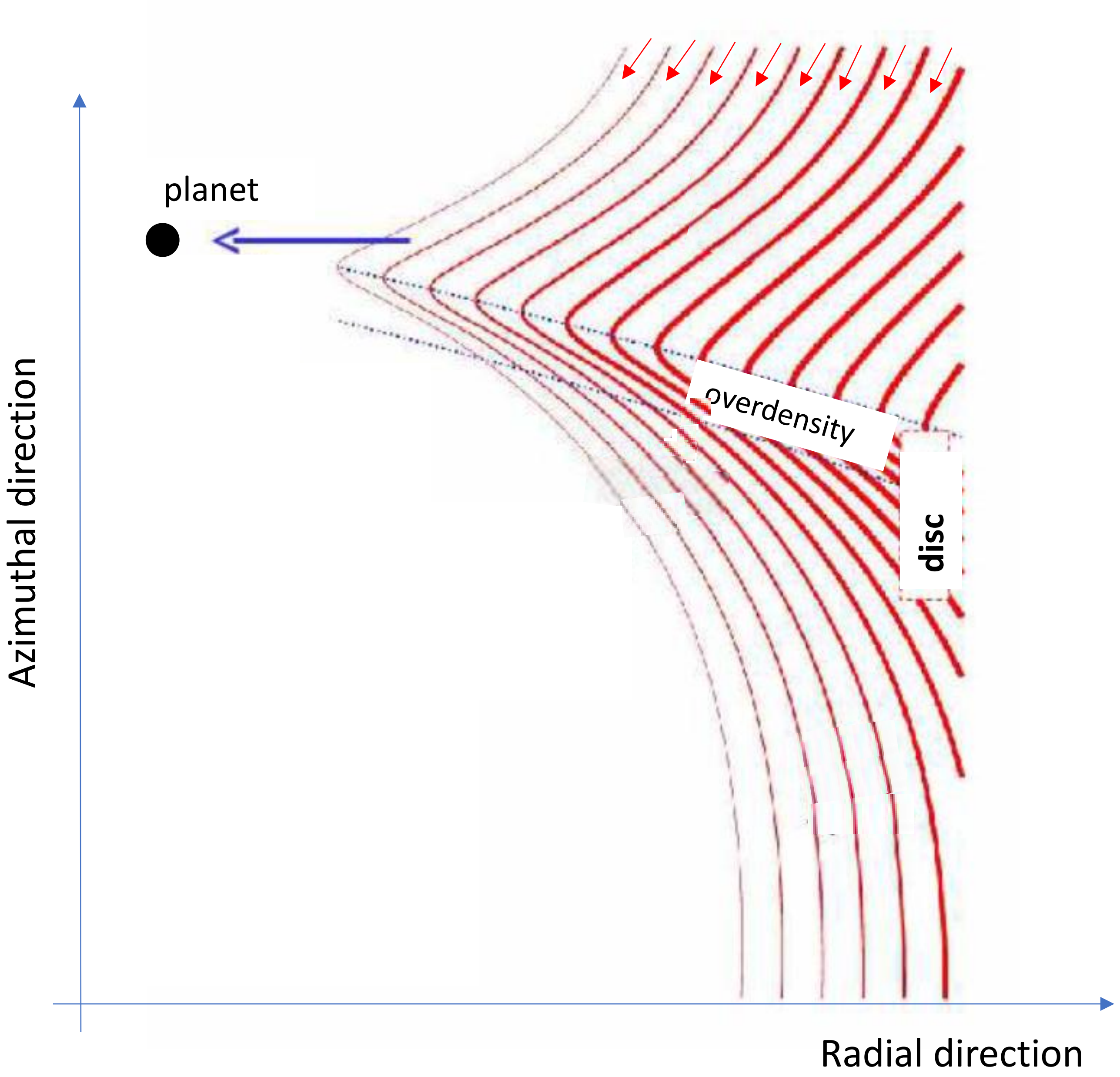
Formation of the spiral wave

Hydro-dynamical simulation of a streamline, in a frame corotating with the planet

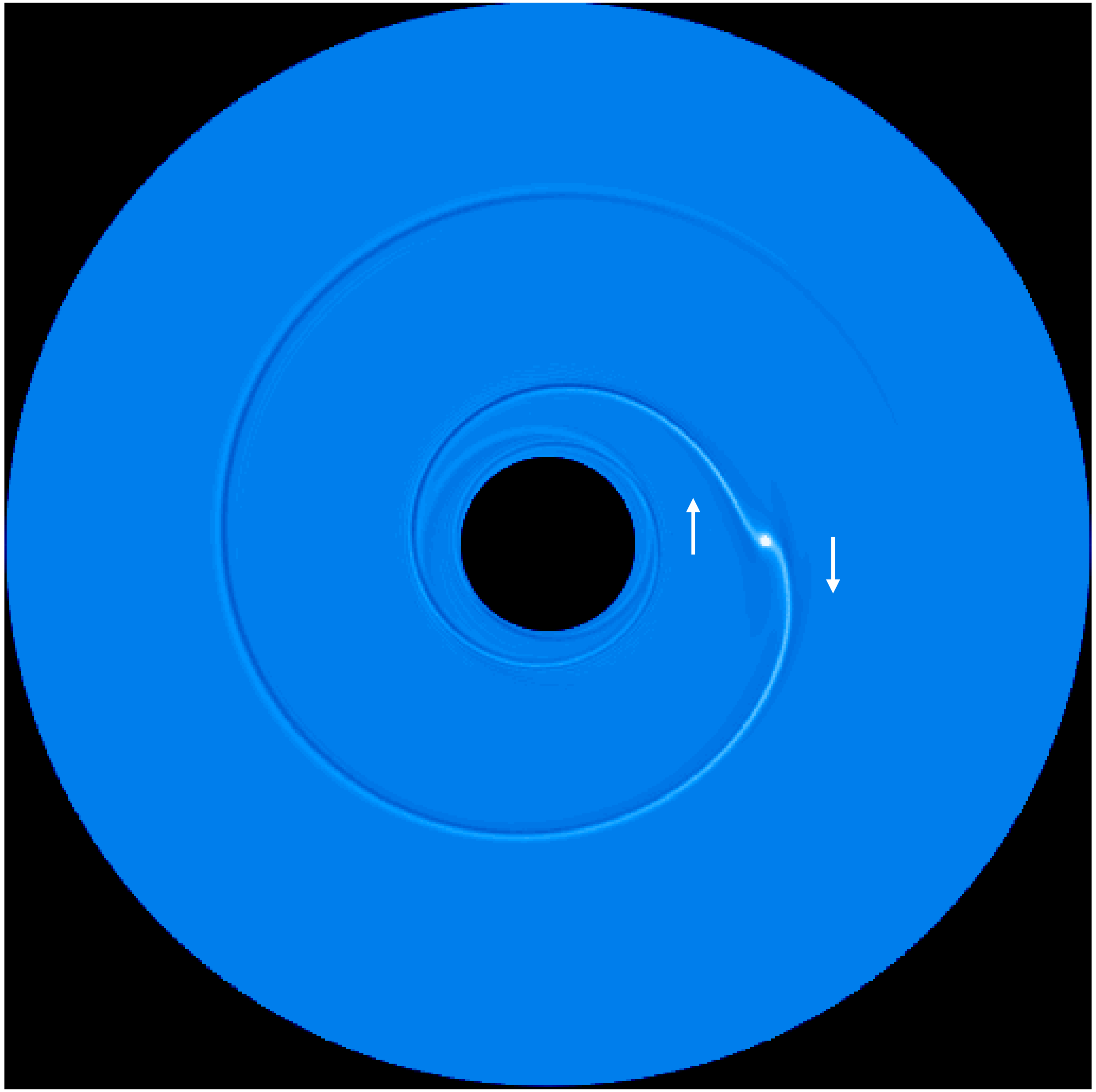


Formation of the spiral wave

The deflection fo the streamlines due to the planet encounter generates a density perturbation that propagates radially as a pressure wave at the speed of sound (Lin et Papaloizou, 1986)

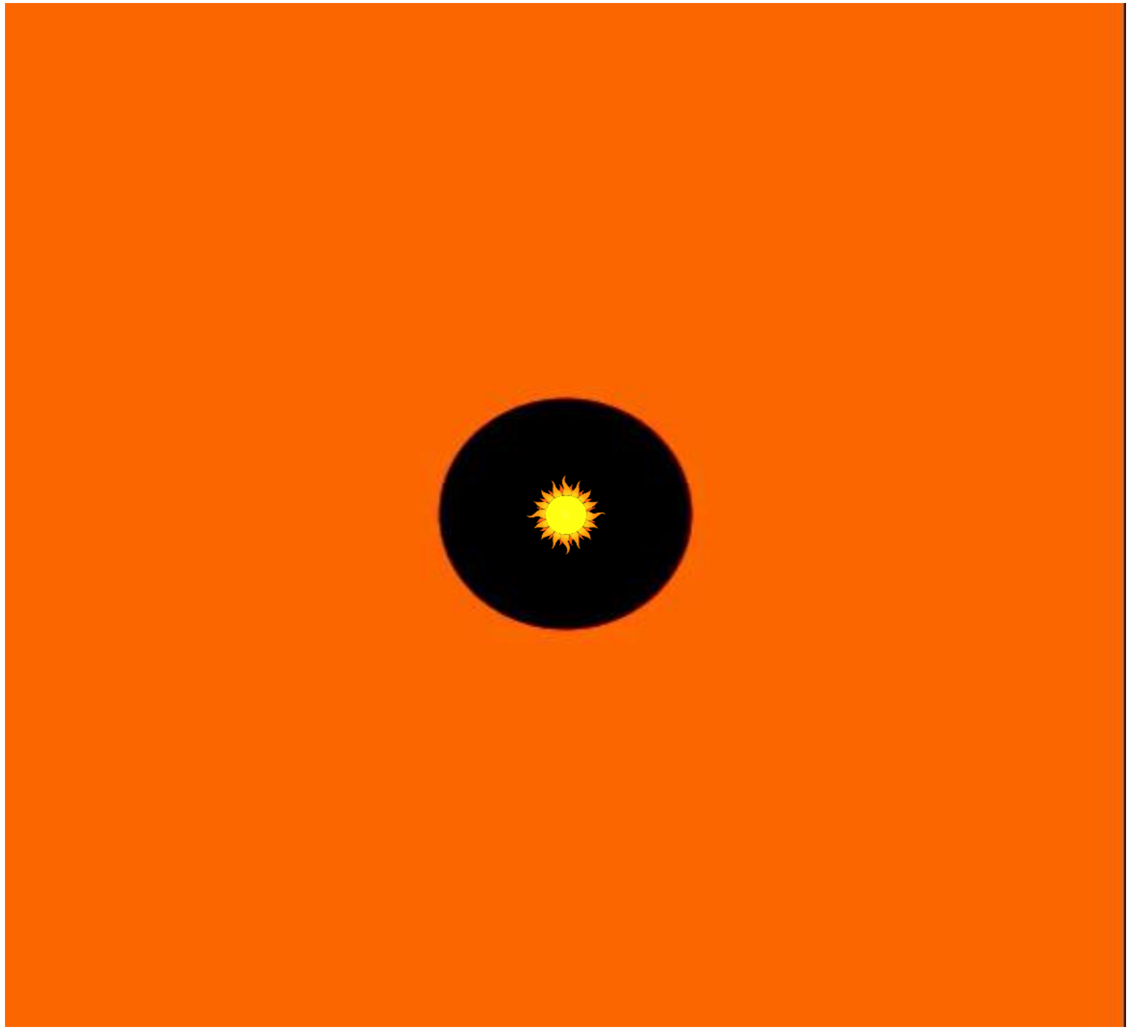


Formation of the spiral wave



While the density perturbation propagates radially, the differential rotation in the disc makes it propagate also in the azimuthal direction, generating the spiral pattern.

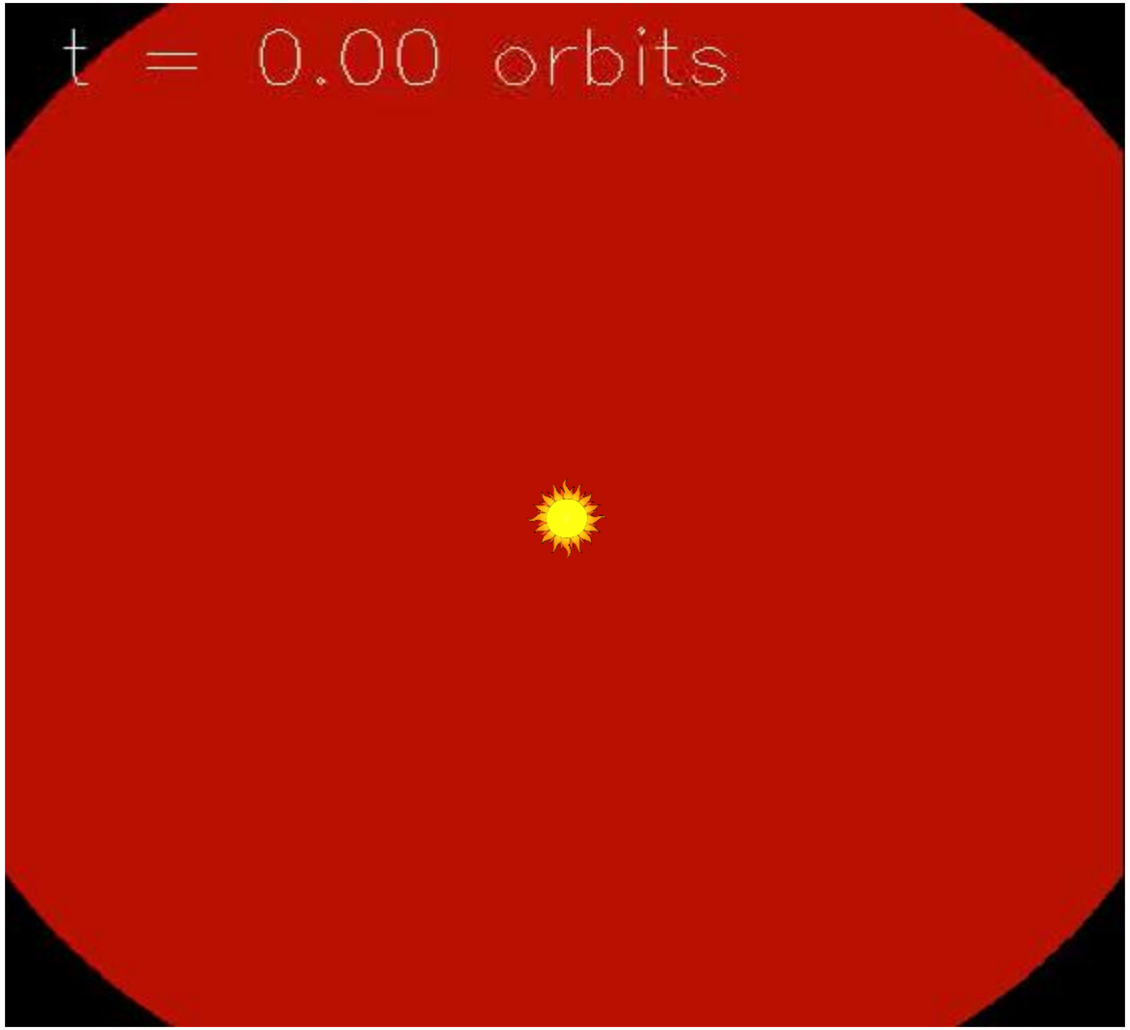
Properties of the wave



Vidéo de F. Masset

The wave is in corotation with the planet around the star ...

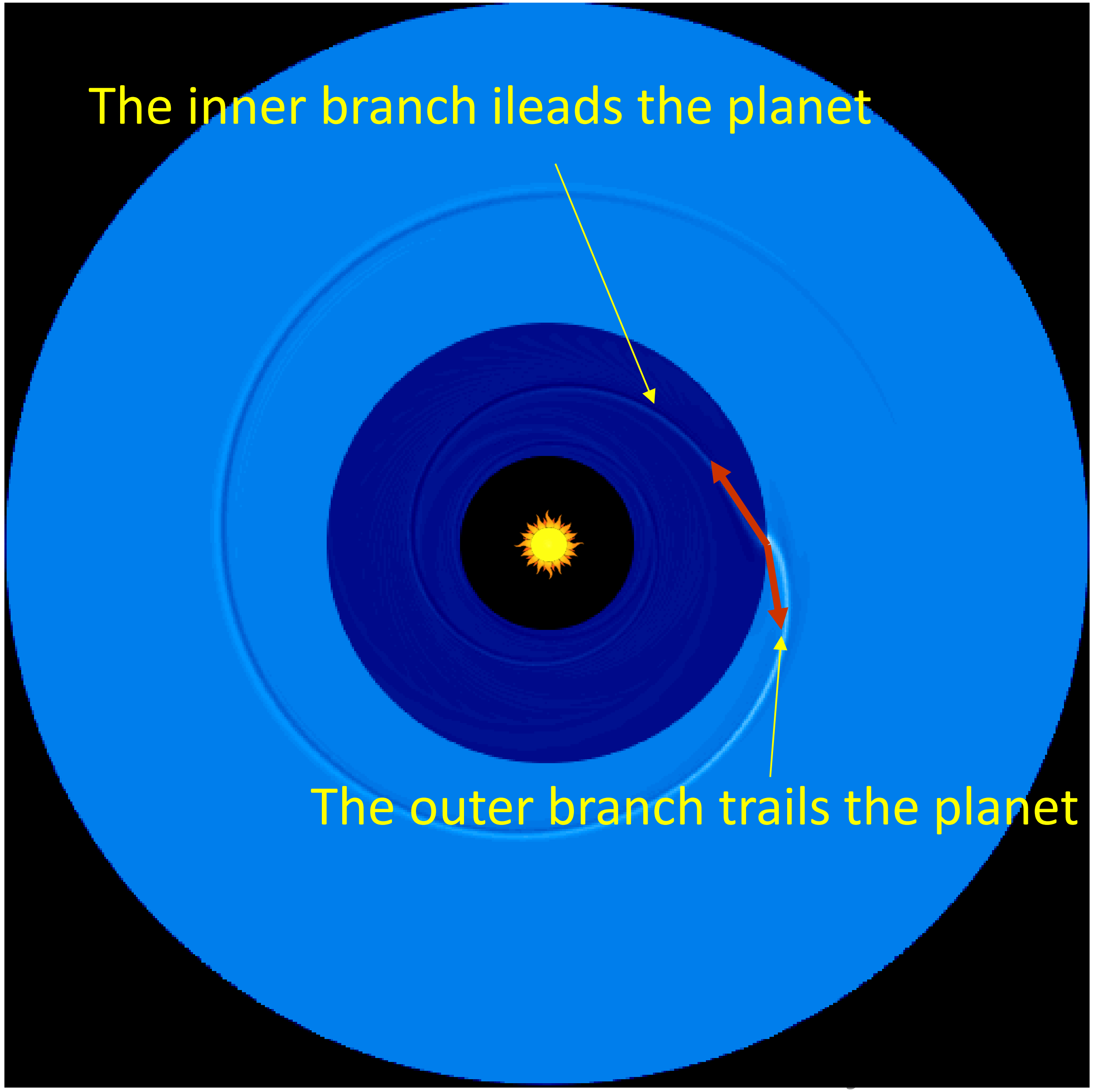
Properties of the wave



Vidéo de F. Masset

... Thus, it is stationary in a frame corotating with the planet

Torques



The outer branch exerts a negative torque on the planet

Who wins?

The inner branch exerts a positive torque on the planet

Type-I migration

Ormel , 2012

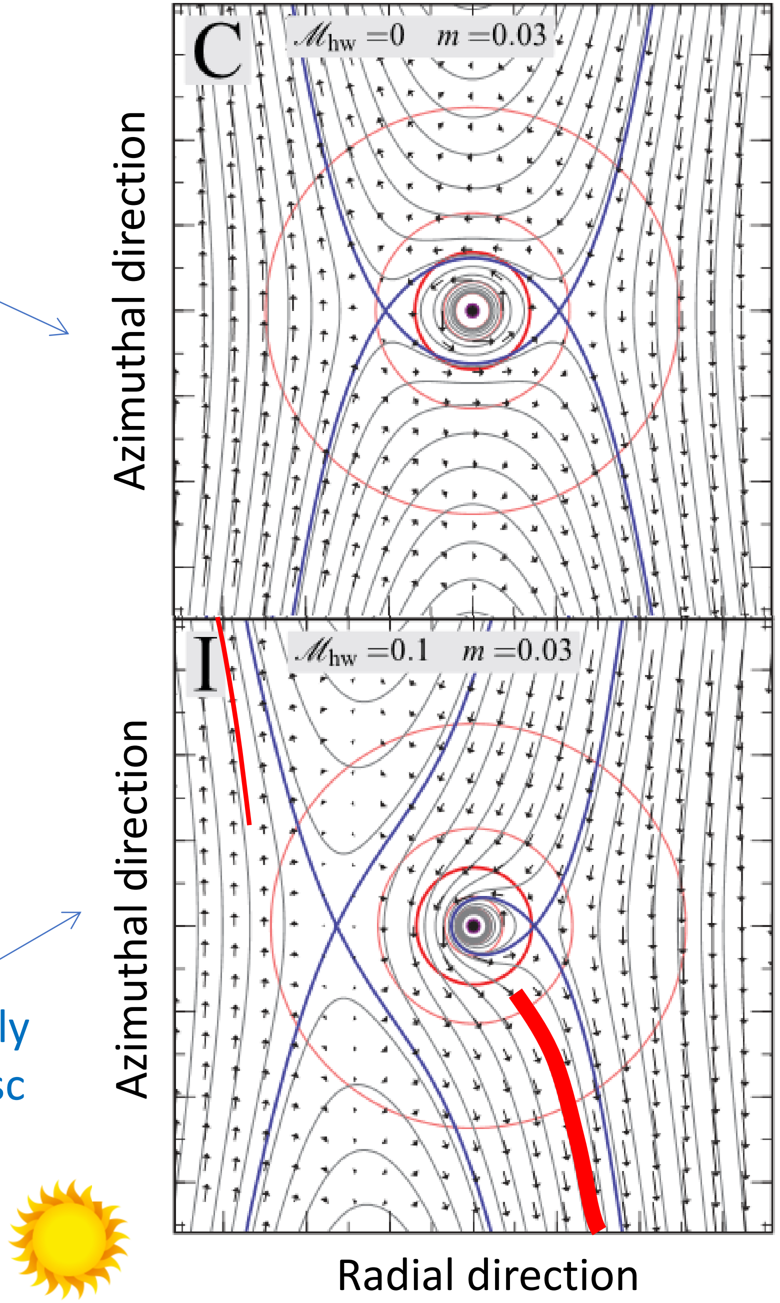
Case if the disc were in Keplerian rotation

In a “flat”disc (∑ = constant) the outer branch wins

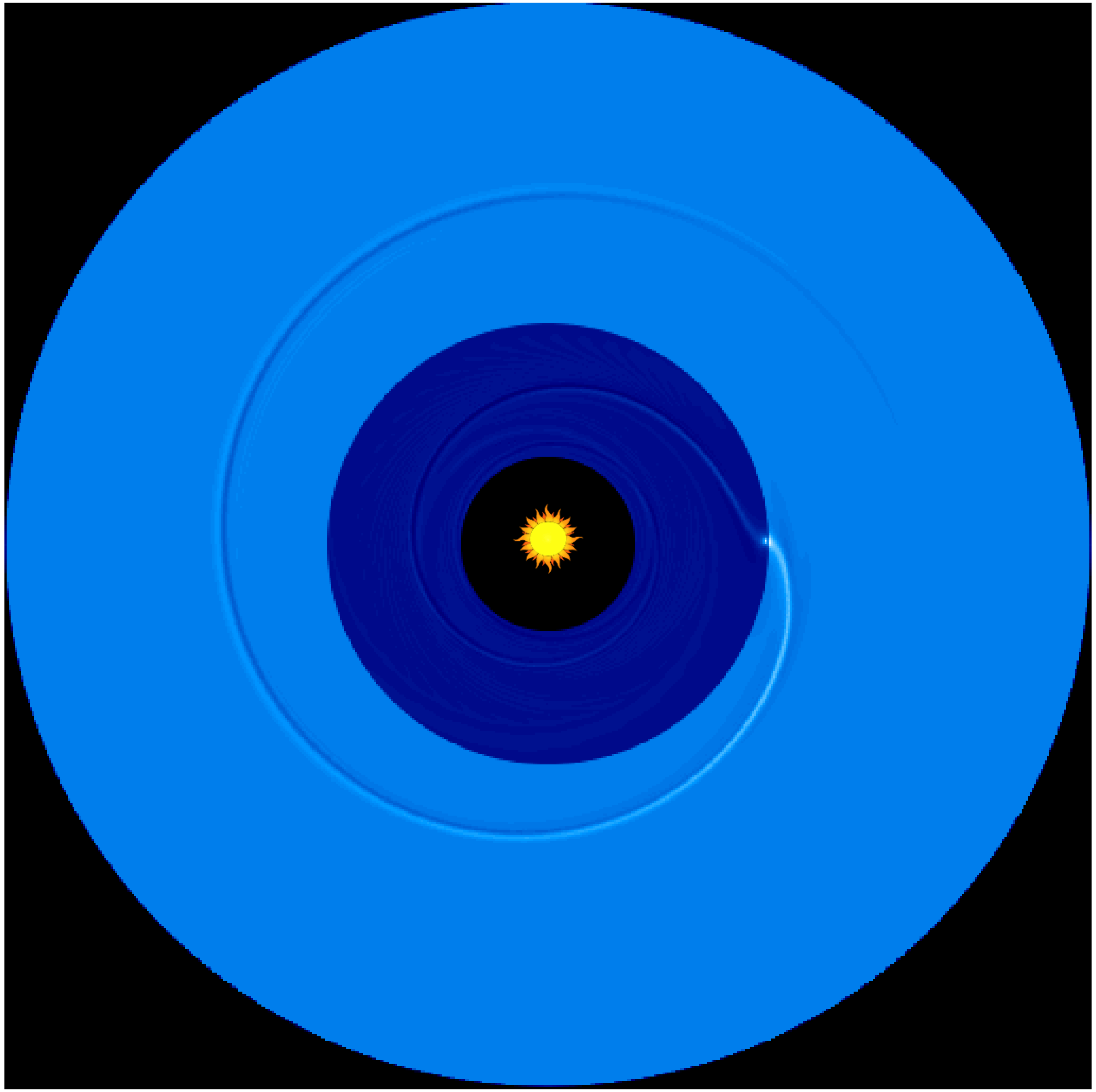
This is because the disc is in a sub-Keplerian protation due to the pressure gradient
$P=c_s^2\rho=H^2\Omega^2\rho \sim \Sigma/r^2$

The corotation region is shifted inward. Thus, the perturbation exerted by the planet on the streamlines of the outer disk is stronger. The outer branch of the wave shows a higher density contrast.

Case for a strongly sub-Keplerian disc



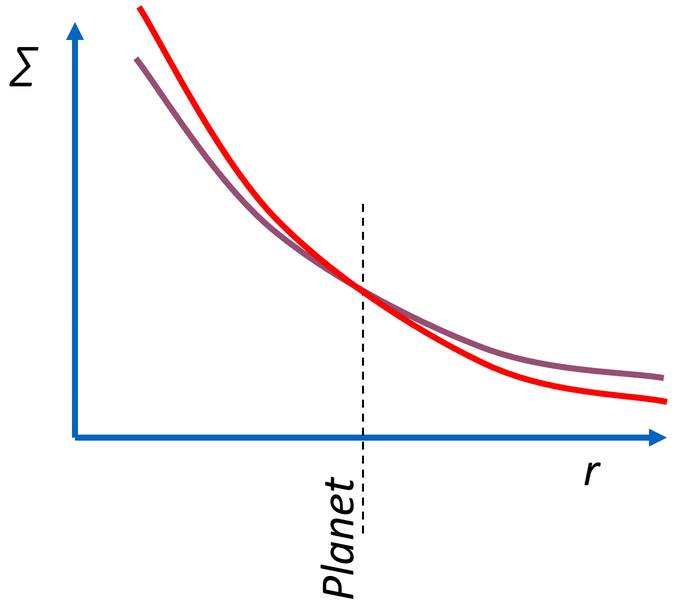
Type-I migration



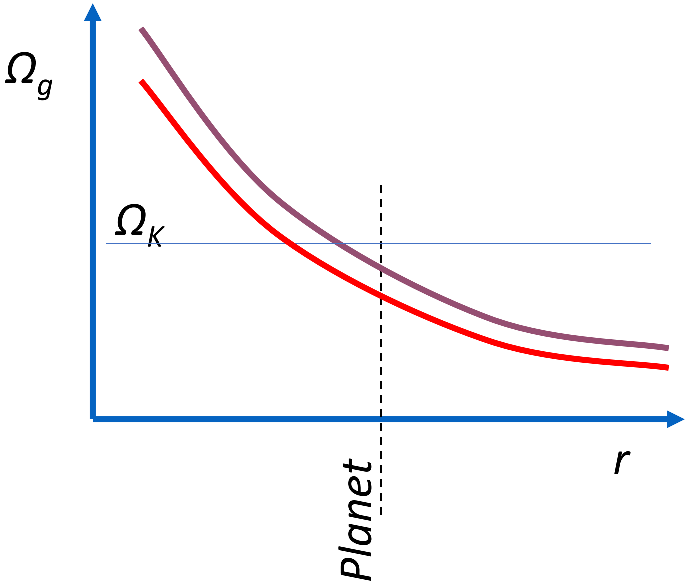
Because the overdensity in the wave is proportional to M_p and to $\sum$, the planet migration speed has to be proportional to these two quantities

One could think that in a steep disk ($\sum$ proportional to $1/r^{\alpha}$, with $\alpha>>1$) the inner branch would dominate (larger $\sum$) and reverse the migration ....

pressure buffer

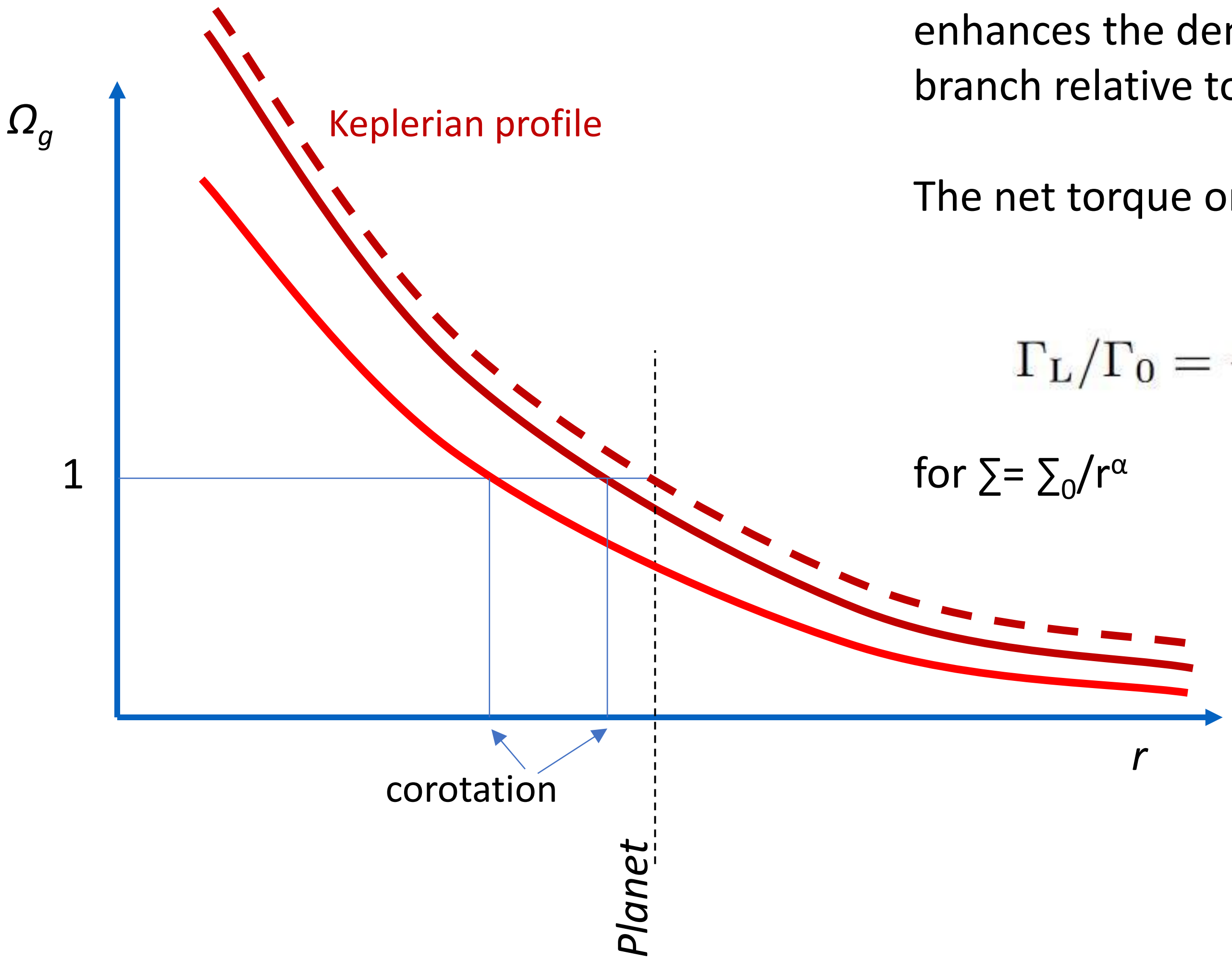


But a steeper disk enhances the pressure gradient …..



…which makes the disk even more sub-Keplerian…

pressure buffer



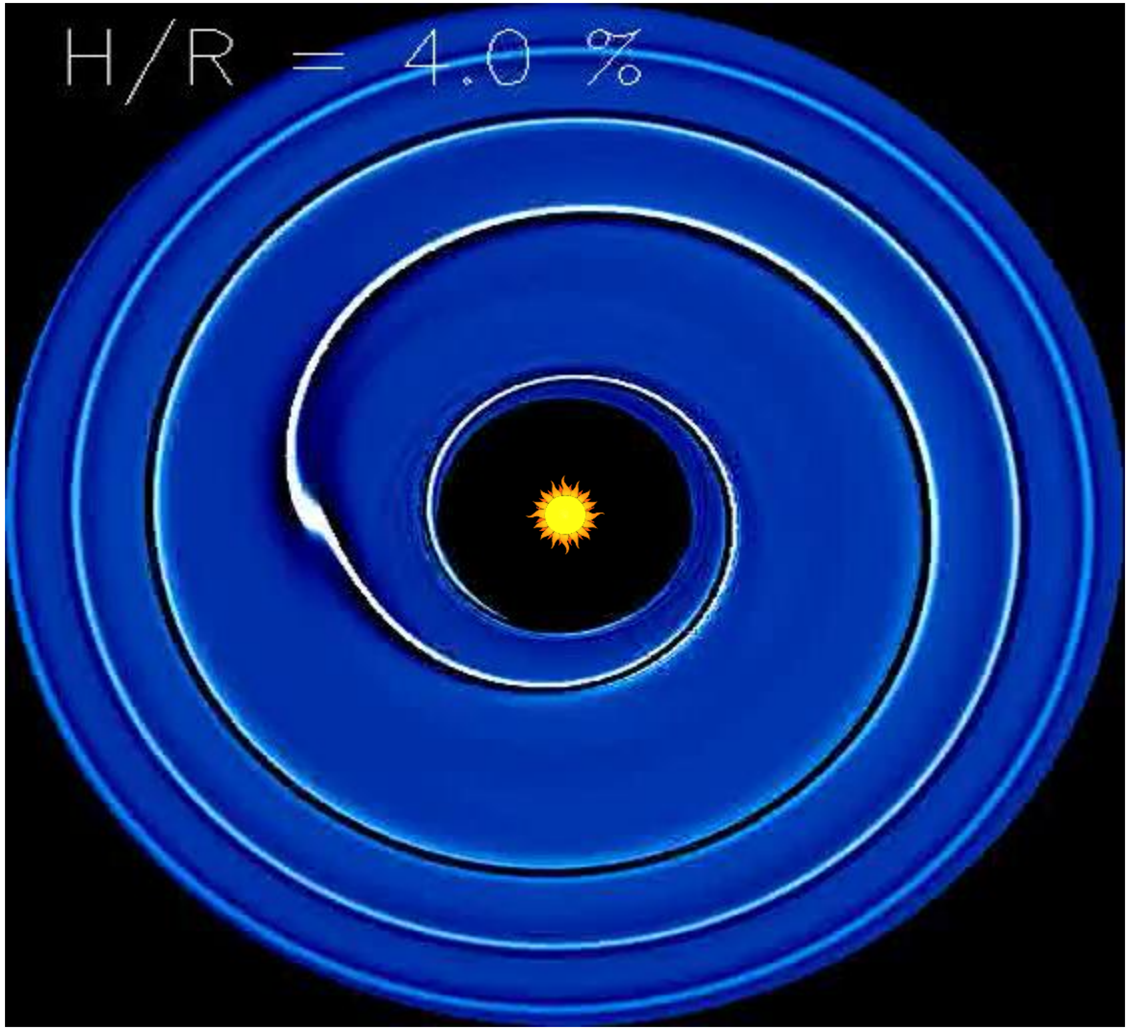
This shifts the corotation region inward and enhances the density contrast of the outer branch relative to the inner branch of the wave.

The net torque on the planet is :

$$\Gamma_L/\Gamma_0 = -3.2 - 1.468\alpha$$

for $\Sigma = \Sigma_0/r^{\alpha}$

Wrapping of the wave



Vidéo de F. Masset

With an increasing aspect ratio of the disk, the wave becomes less pronounced (pressure is stronger so it is more difficult to create an overdensity) and less wrapped (the radial propagation occurs at the sound-speed, which is faster)

Consequently, the torque felt by the planet is proportional to $(H/r)^{-2}$

Type-I migration: summary

Planet migration is proportional to:

→ the mass of the planet M_p

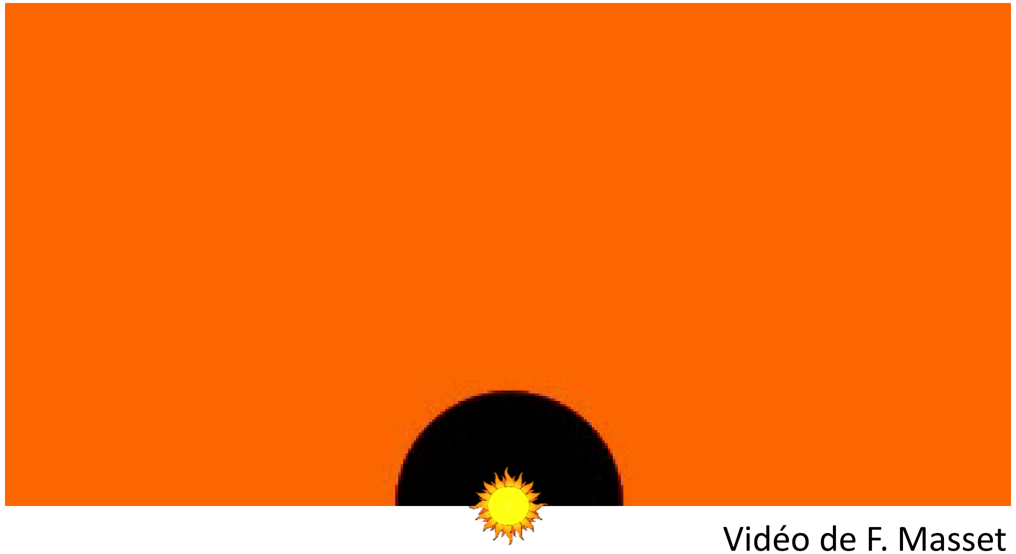
→ The surface density of the disc Σ

→ The inverse of the square of the disc's aspect ratio: $(r/H)^2$

$dh_p/dt = -(3.2+1.468\alpha)\,(M_p/M_s^2)\,(r/H)^2\,\Sigma\, r_p^4\Omega_p^2$ ($h_p=r_p^{1/2}$ is the specific angular momentum of the planet), here all quantities are evaluated at the location of the planet ($\Sigma = \Sigma_0/r^\alpha$)

- *This formula applies only to planets which don't change the global structure of the disc*
- *It's independent of the disc's viscosity*

Type-I migration: summary



Vidéo de F. Masset

For a planet of 1 Earth mass at 1 AU, in a disc with:

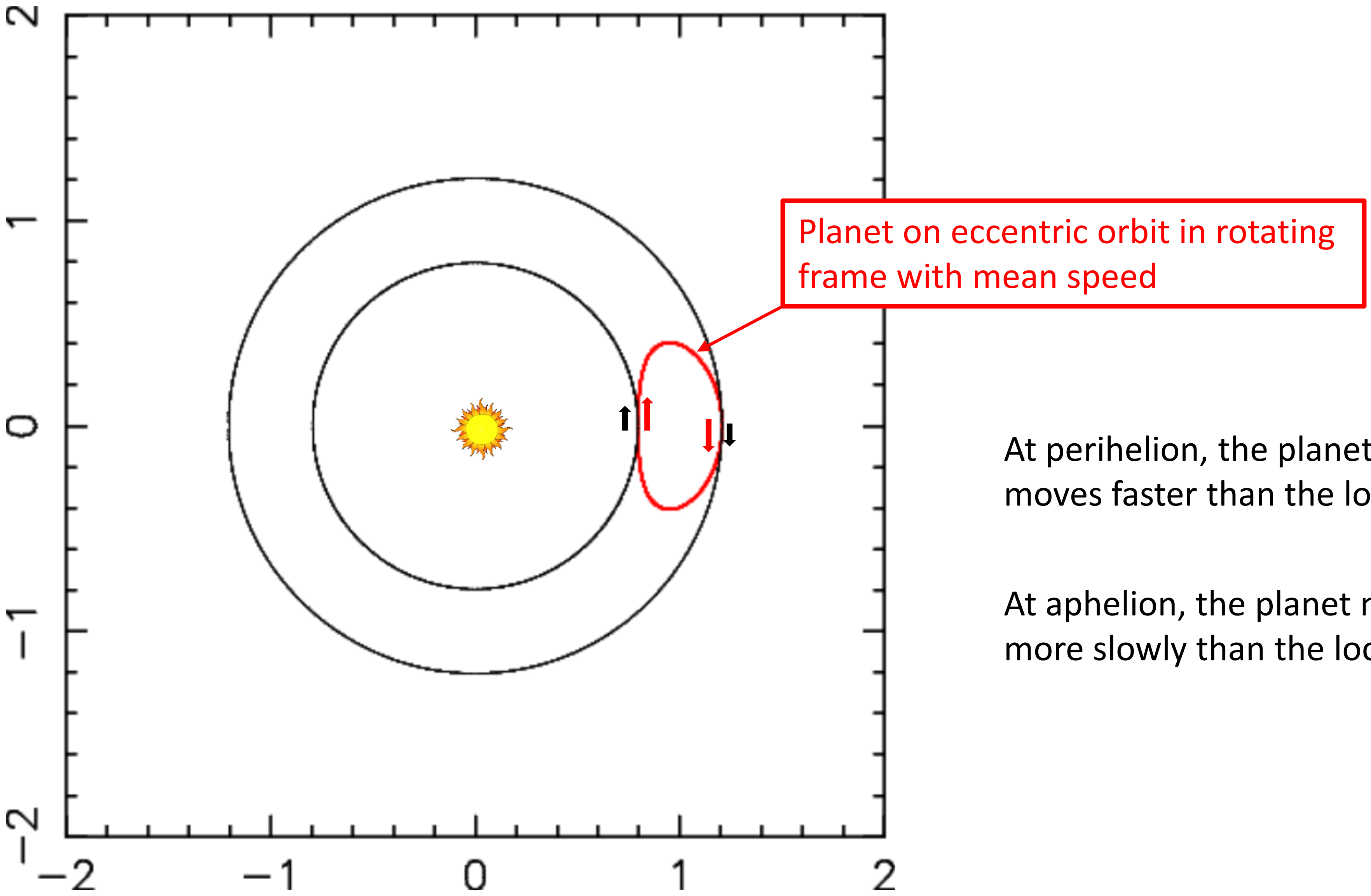
Σ = 1700 g.cm^{-2}
H/R = 0.05

around the Sun, the migration timescales is 2×10^5 y

Given:
$dh_p/dt = -(3.2+1.468\alpha)\,(M_p/M_s^2)\,(r/H)^2 \sum r_p^4 \Omega_p^2$
A planet of 5 Earth masses at 1 AU, in a disc 3x more massive and twice thicker (*H/R = 0.1*) would migrate in :

$200{,}000 / 5 / 3 * 2^2 \sim 53{,}000$ years

Eccentricity damping

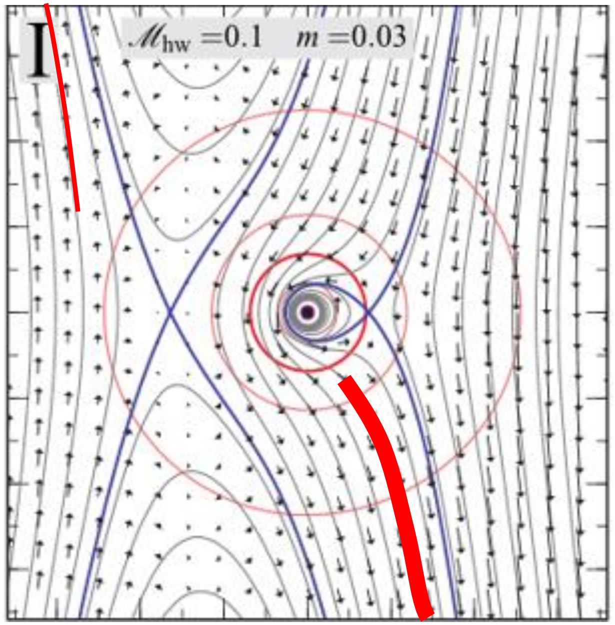


At perihelion, the planet moves faster than the local gas

At aphelion, the planet moves more slowly than the local gas

Eccentricity damping

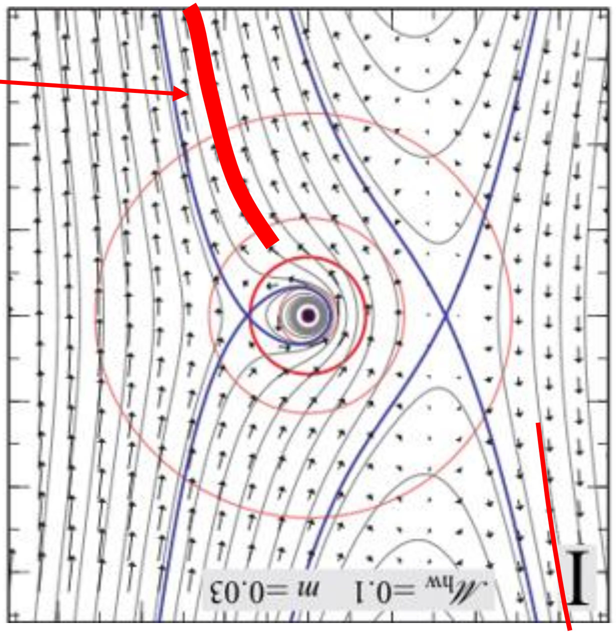
At perihelion



Slows the planet down

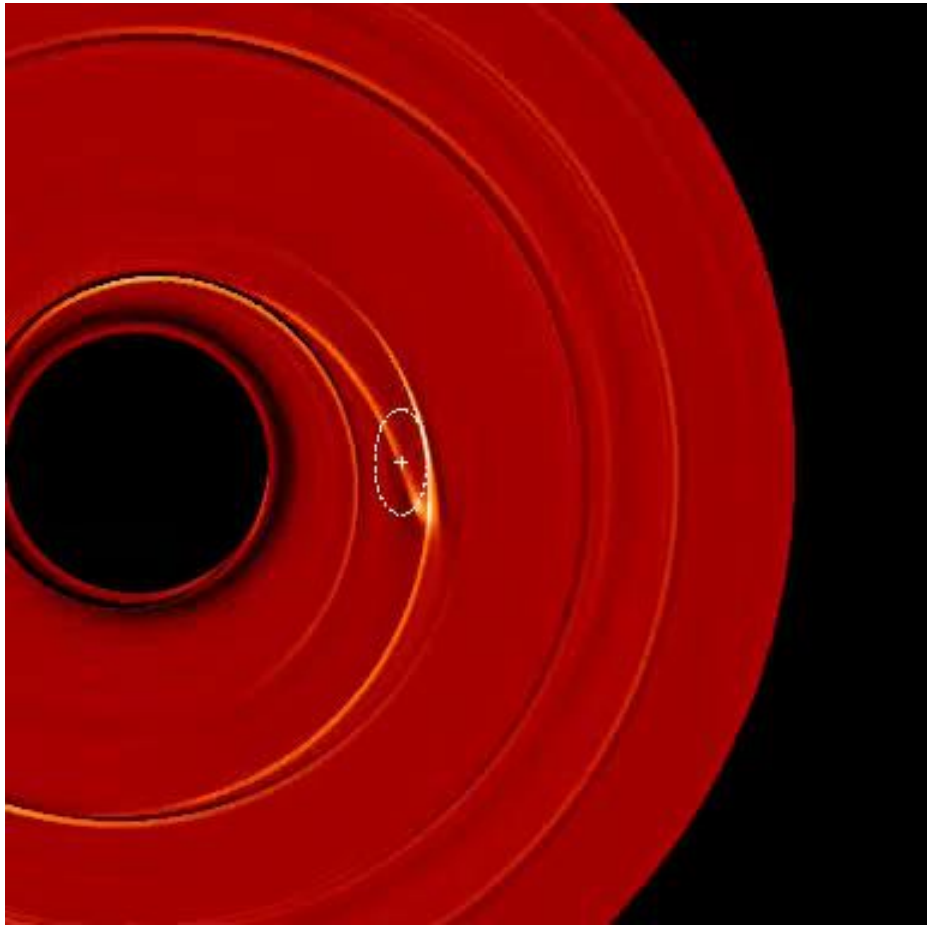
At aphelion

Speeds the planet up



Eccentricity damping!

Eccentricity damping



Eccentricity damping is proportional to the velocity of the planet relative to the disk, i.e. to the eccentricity itself.

$$\dot{e} \sim e\, M_p \Sigma \left(\frac{H}{r}\right)^{-4} = \frac{e}{\tau_e}$$

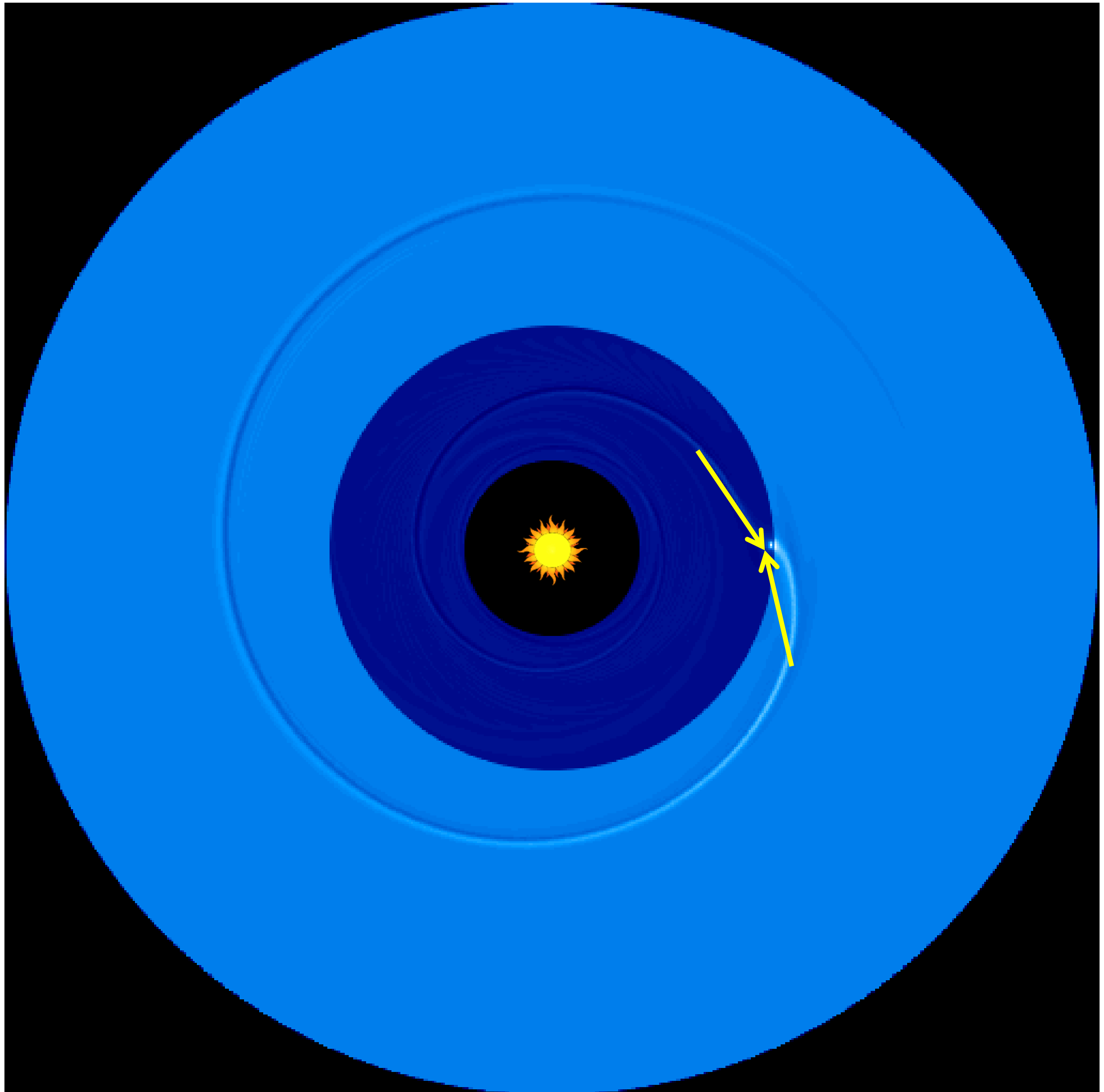
Note: migration is proportional to $\left(\frac{H}{r}\right)^{-2}$

From the equation for angular momentum :

$$\dot{\mathcal{L}} = \frac{d\mathcal{L}}{dt} = m\sqrt{\mathcal{G}(M_* + m)}\left(\frac{\dot{a}}{2\sqrt{a}}\sqrt{1-e^2} - \frac{\sqrt{a}}{\sqrt{1-e^2}}e\dot{e}\right) = -\frac{\mathcal{L}}{2\tau_a}$$

we get: $\frac{\dot{a}}{a} = \frac{1}{\tau_a} - \frac{2e^2}{\tau_e}$

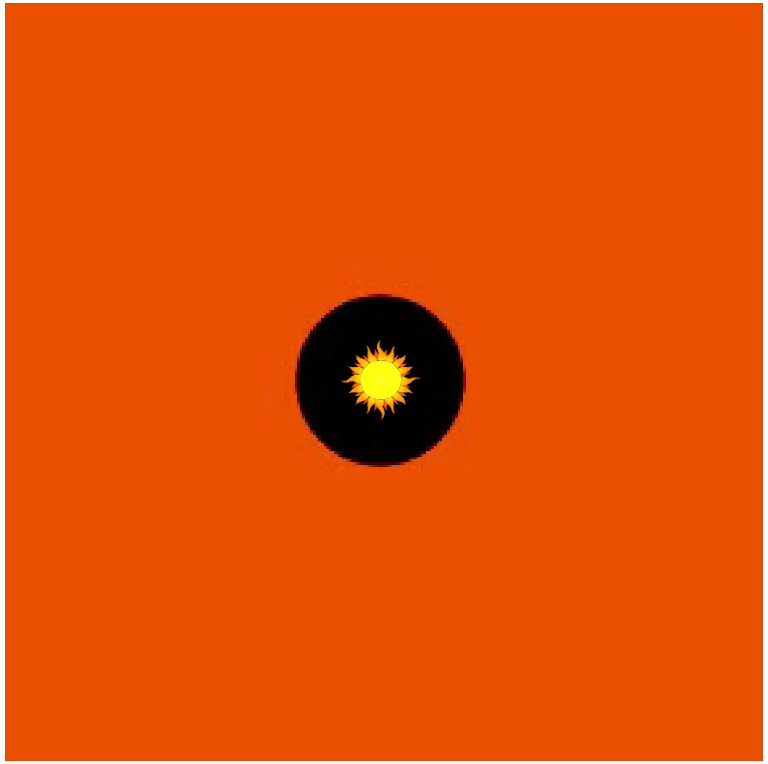
Giant planets: opening of a gap



The planet accelerates the outer disc, pushing it outward, and slows down the inner disc, pushing it inward.

If this push overcomes the viscous torque of the disc, which opposes to the establishement of density gradients, a gap opens.

Giant planets: opening of a gap



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If this push overcomes the viscous torque of the disc, which opposes to the establishement of density gradients, a gap opens.

Giant planets: opening of a gap

$$\delta T_g(r) \approx 0.4\, q^2 {r_p}^3 {\Omega_p}^2 r^{-1} \left(\frac{r_p}{\Delta}\right)^4 (2\pi r \Sigma)$$

Torque exerted by a planet of mass $q=M_p/M_s$ on a ring at $r=r_p+\Delta$

$$\delta T_\nu(r) = -\frac{3}{2}\nu\Omega\left[\frac{r}{\Sigma}\frac{\mathrm{d}\Sigma}{\mathrm{d}r} + \frac{1}{2}\right](2\pi r \Sigma)$$

Viscous torque exerted on the same ring

An equilibrium is achieved when $\delta T_\nu = \delta T_g$

In principle, this equation solved for each Δ allows to compute the gap profile ∑(Δ) (Varnière et al., 2004)

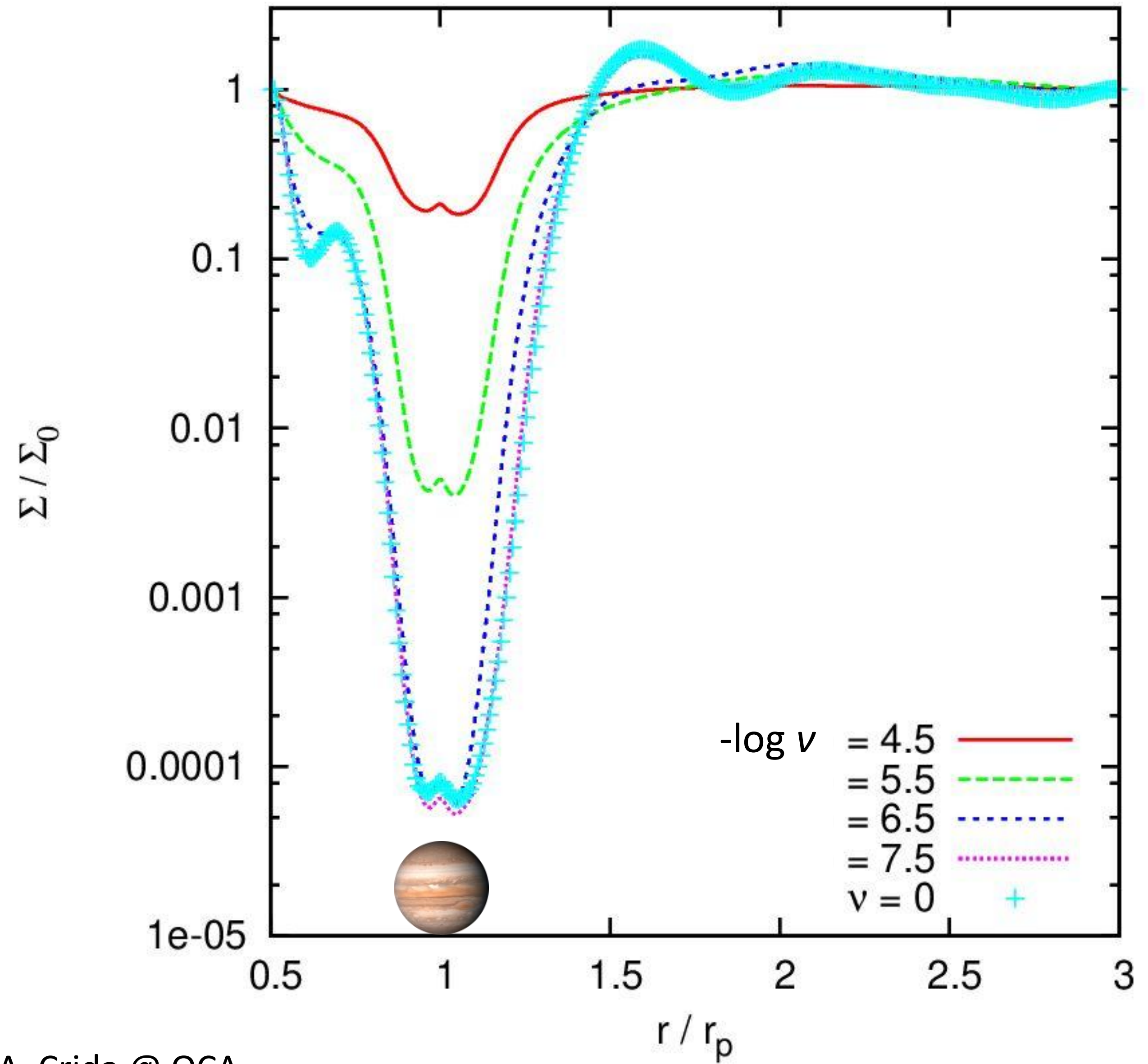
But, for $\nu \to 0$ dlog(∑)/dr ->∞ (gap of infinite depth). This is unphysical

Rayleigh condition: for $\frac{\mathrm{d}j}{\mathrm{d}R} = \frac{1}{2} R_\mathrm{p}\Omega_\mathrm{Kp}\left(1 + h_\mathrm{p}^2 \frac{\mathrm{d}^2 \ln \Sigma}{\mathrm{d}R^2}\right)$ <0 the disc is unstable: turbulence develops and ν increases.

Thus, there is a maximum slope that the gap's edges can have $h_\mathrm{p}^2 \frac{\mathrm{d}^2 \ln \Sigma}{\mathrm{d}R^2} = -1.$ (Kanagawa et al., 2015).

Giant planets: opening of a gap

Limit gap for ν -> 0



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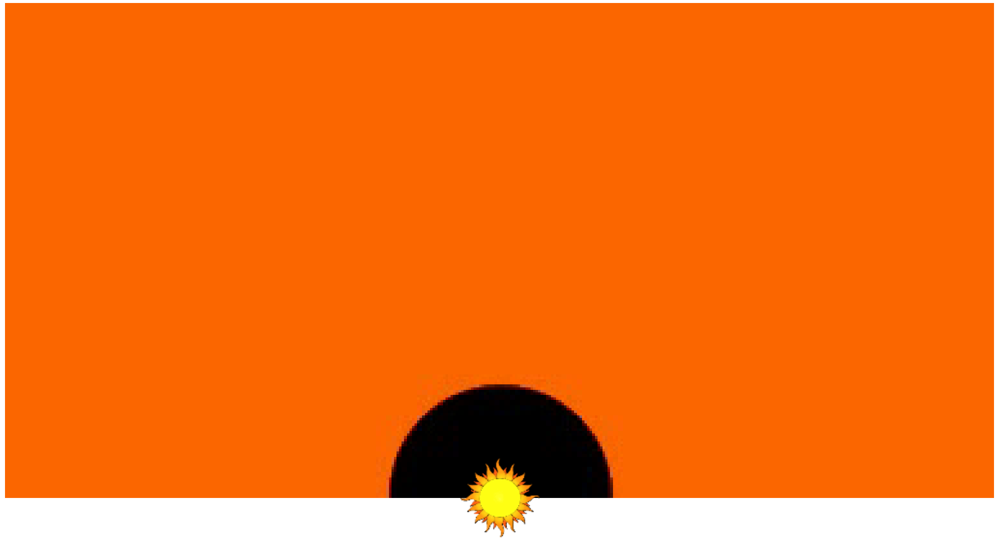
Effect of gap opening on migration



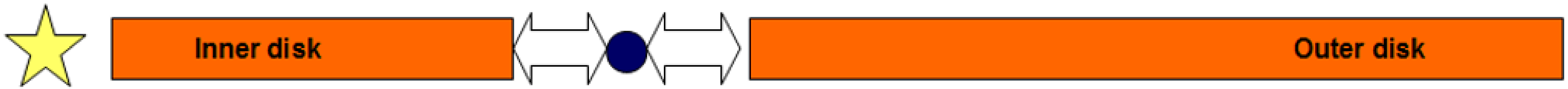
Gap opening slows down migration significantly

Type-II migration

The planet is pushed:

- Outward by the inner disc
- Inward by the outer disc

Consequently, the planet is trapped in its gap and can move only together with the gap (at the gas radial speed)

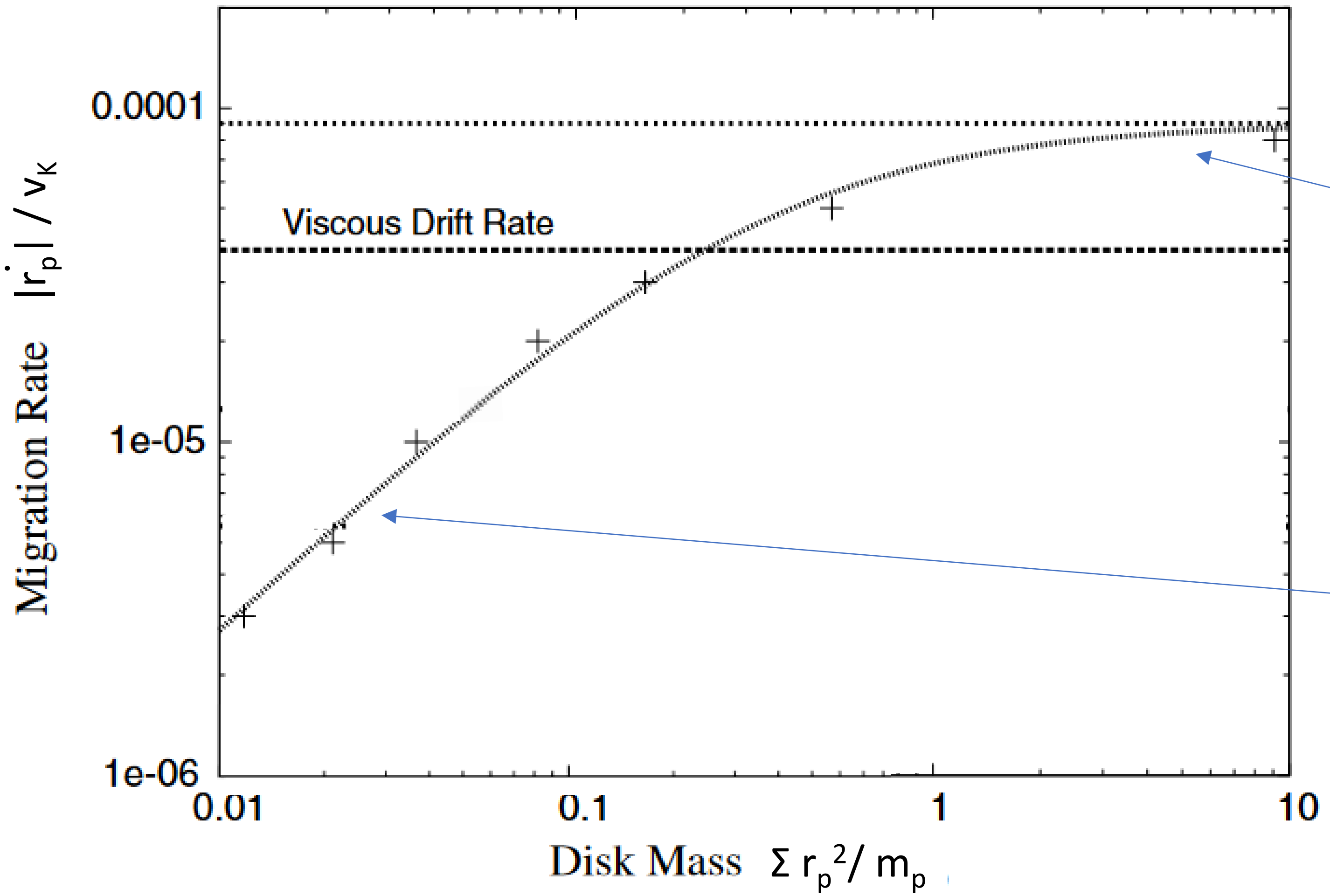


The motion of gas towards the star (accretion) occurs at viscous speed $v_r^{gas} = -3/2\ \nu/r$

Thus, the planet migration speed will also be $v_r^{pl} = -3/2\ \nu/r$
(Independent of $\sum$ et M_p; Ward, *Icarus*, 1997)

Real Type-II migration

In reality, the planet migration speed does depend on the $\Sigma\, r_p^2 / m_p$ ratio



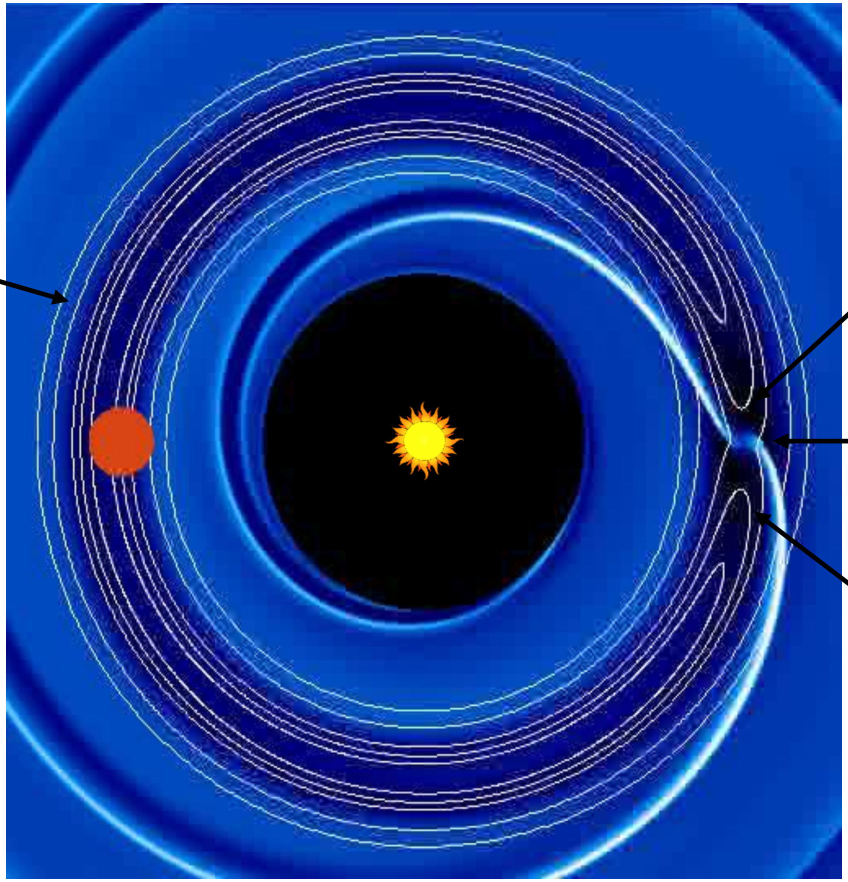
Asymptotic behaviors:

$$\frac{\mathrm{d}r_p}{\mathrm{d}t} \sim \nu$$

$$\frac{\mathrm{d}r_p}{\mathrm{d}t} \sim A(\nu)\Sigma r_p^2 / m_p$$

The coorbital (corotation) torque

Coorbital region
(Horeshoe shaped)



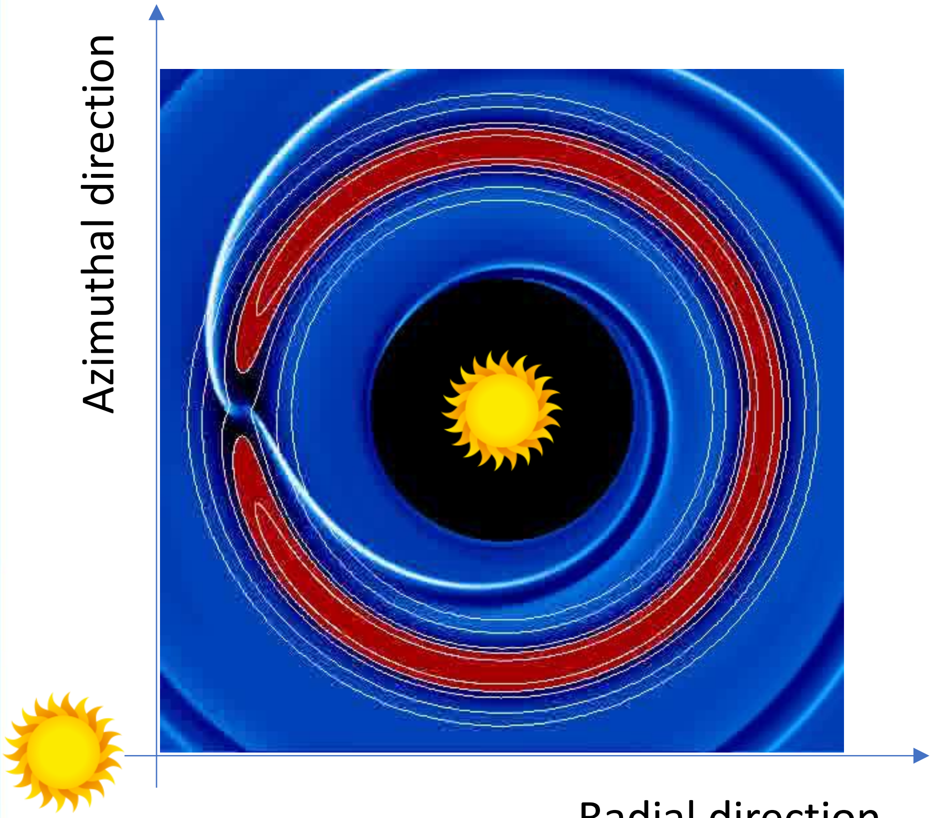
Vidéo de F. Masset

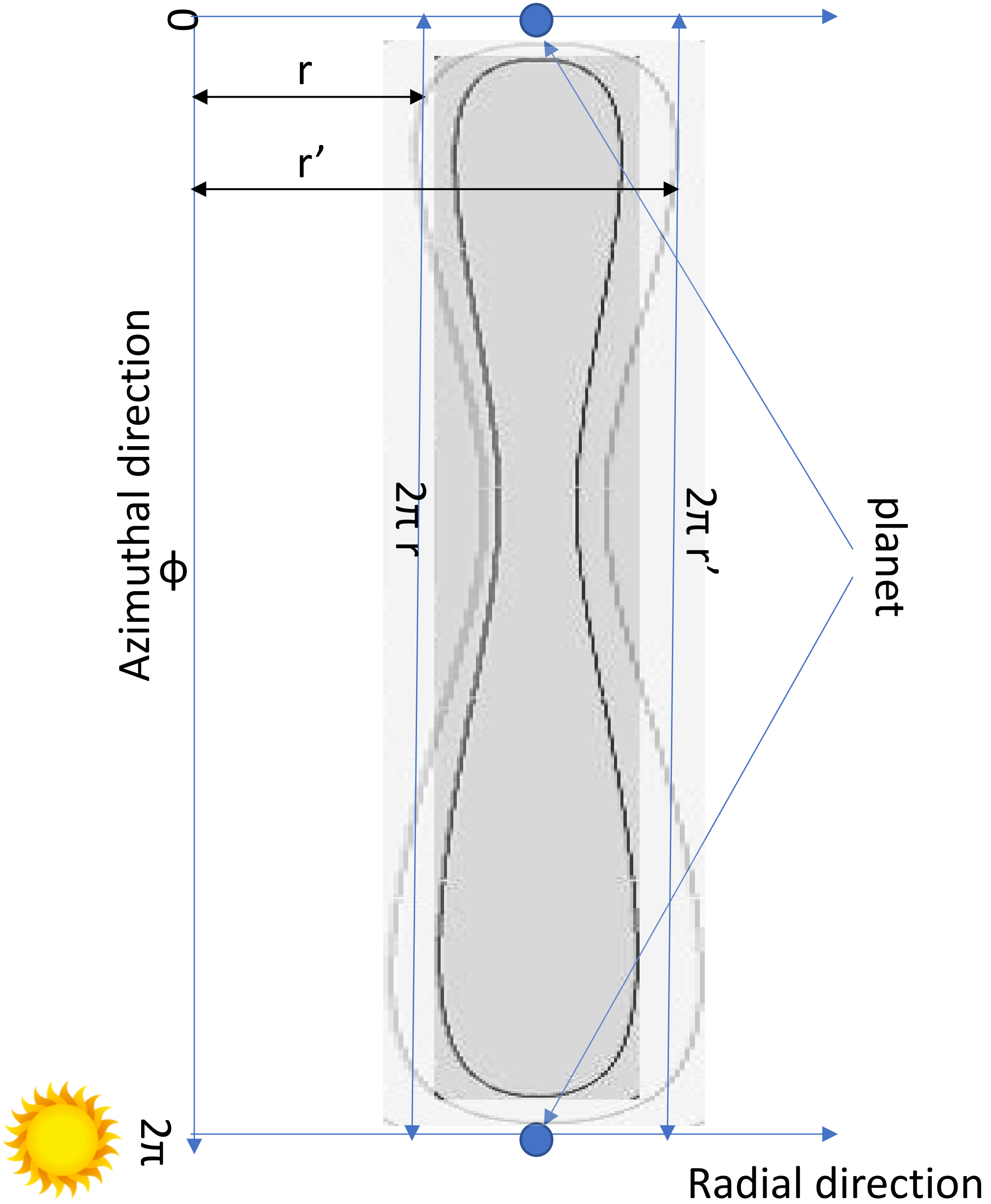
When the fluid element makes a U-turn, from outside-in, it gives a positive torque to the planet

planet

When the fluid element makes a U-turn, from inside-out, it gives a negative torque to the planet

The coorbital torque– dependence on density gradient





The coorbital torque– dependence on density gradient

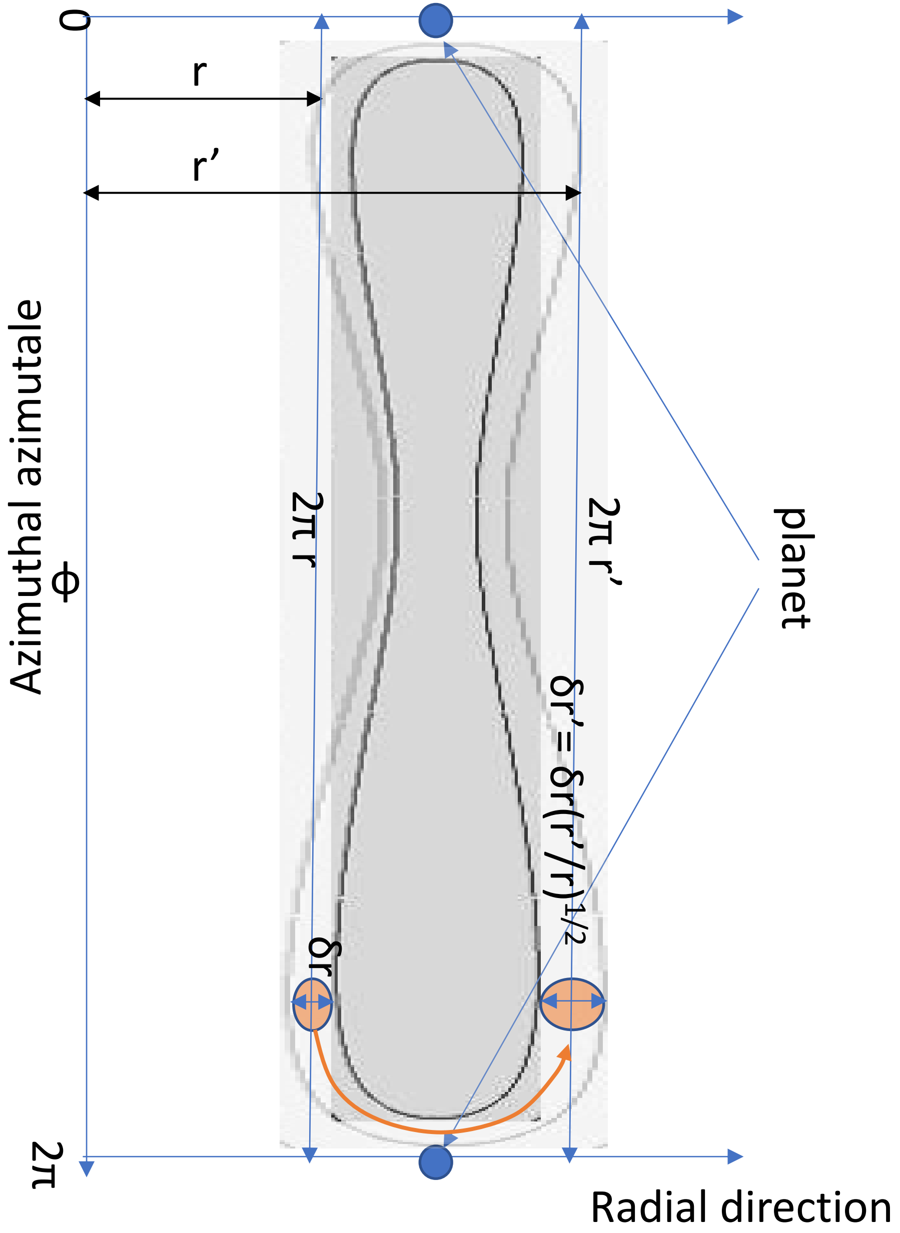
The Liouville theorem states that the canonical volume dϕ dJ (where J is the specific angular momentum) is conserved by the dynamics. dϕ is conserved, so dJ is also conserved.

Because $J=\sqrt{r}$, $dr=2\sqrt{r}dJ$

Consequently, when moving from r to r',
the fluid element expands radially by a factor $\sqrt{r'/r}$
On the other hand, the length of the circle dϕ=180° increases as r'/r.

Thus, mass conservation implies a change in the surface density proportional to $(r'/r)^{-3/2}$

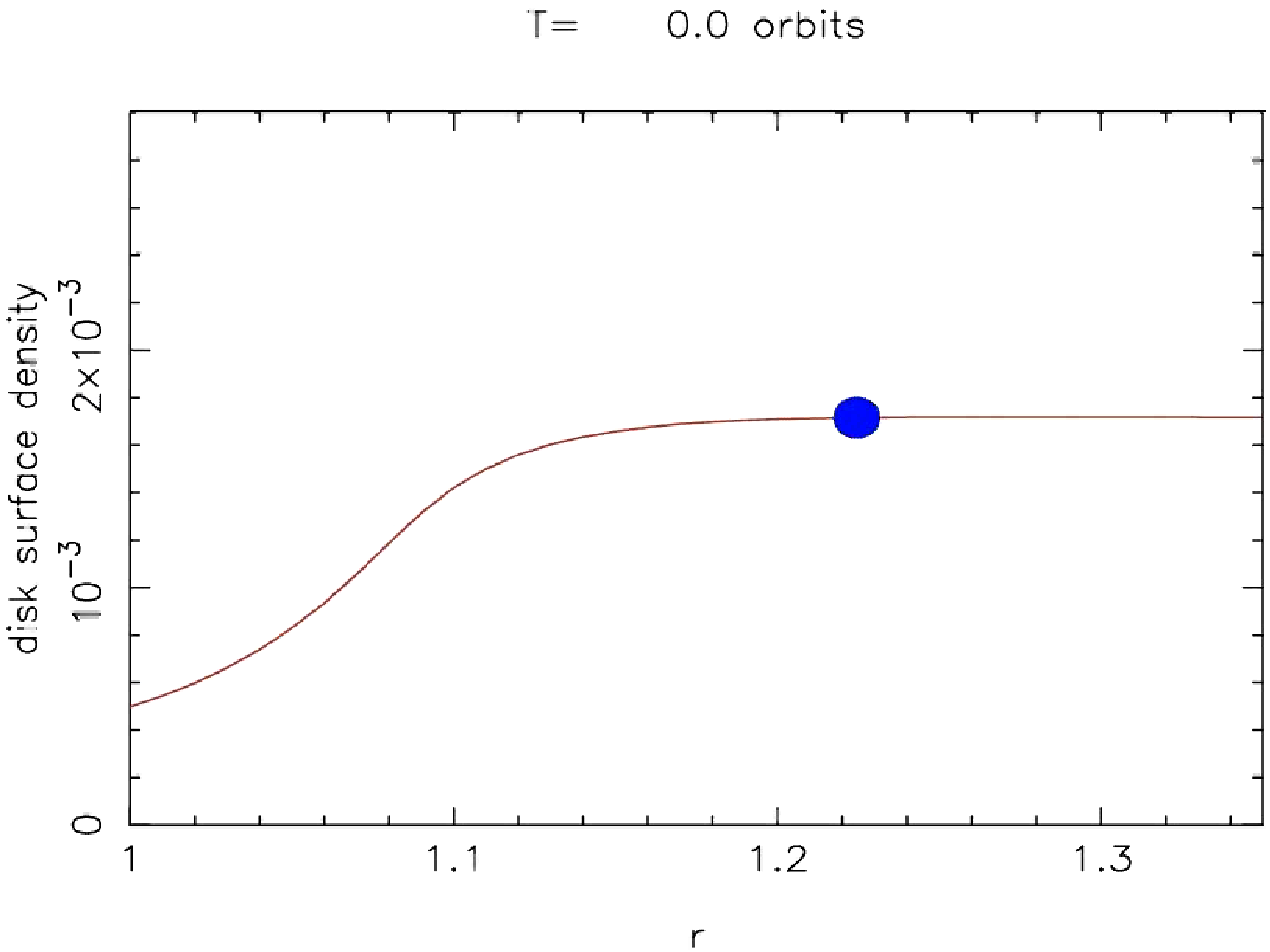
In other words, if the surface density of the disc changes as $1/r^{3/2}$ across the coorbital region, the same mass performs a U-turn out-in and in-out, per unit time. Thus the torque is nul.



The planet trap concept

If the surface density gradient is positive (ex. at the inner edge of the disc) the coorbital torque is strongly positive and can win over the negative torque exerted by the spiral density wave.

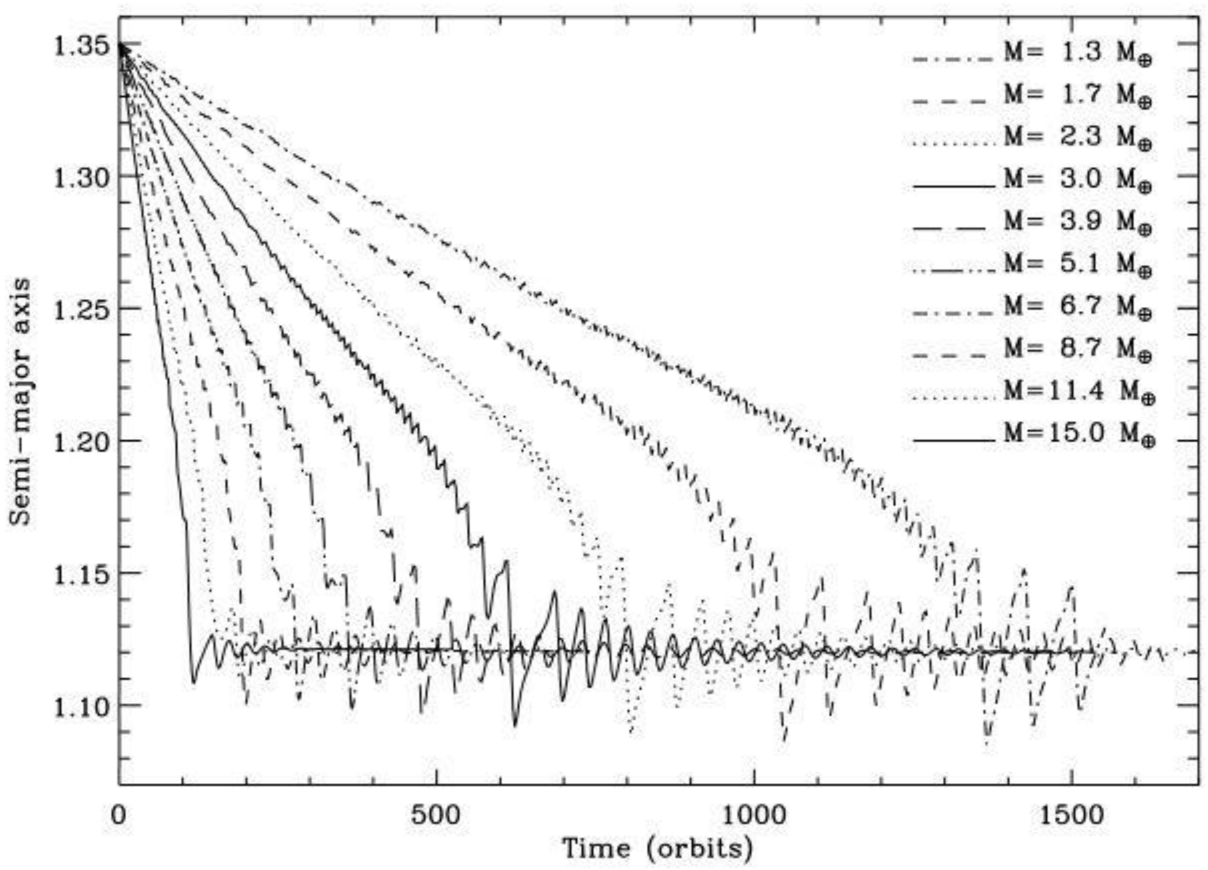
Planet Type-I migration stops (Masset et al., 2006)



The planet trap concept

All planets stop at the same location, regardless of their mass

Oscillations are due to the presence of a Rossby vortex near the surface density maximum.

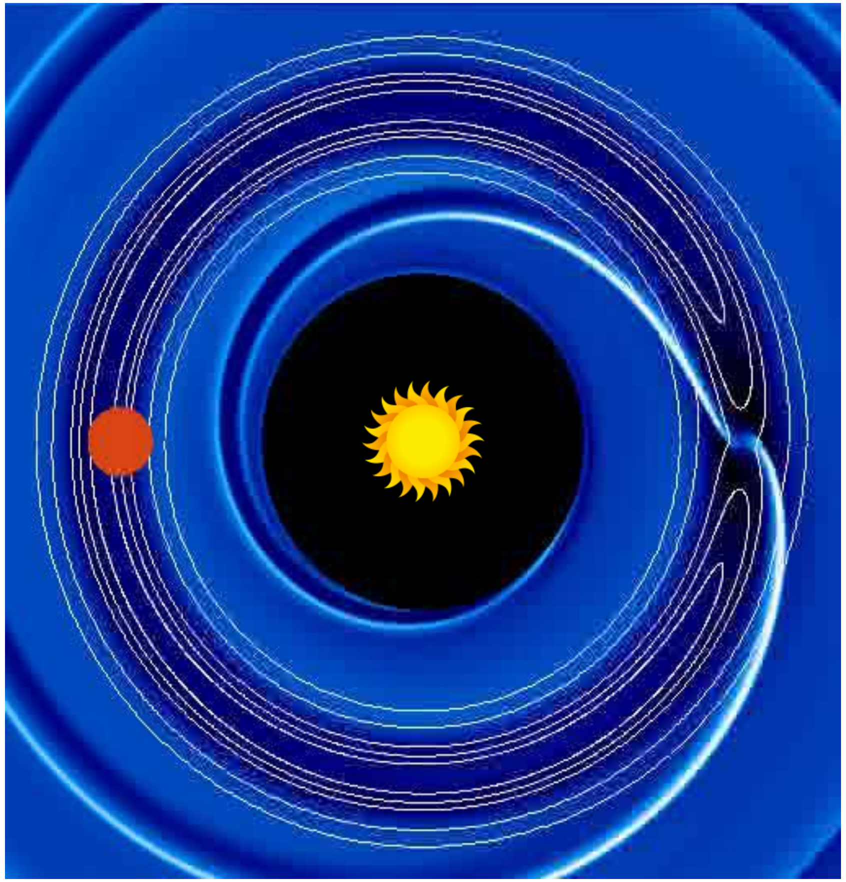


Masset et al., 2006

Saturation of the coorbital torque

The libration of the fluid éléments tends to redistribute the gas so that $\Sigma \sim 1/r^{3/2}$

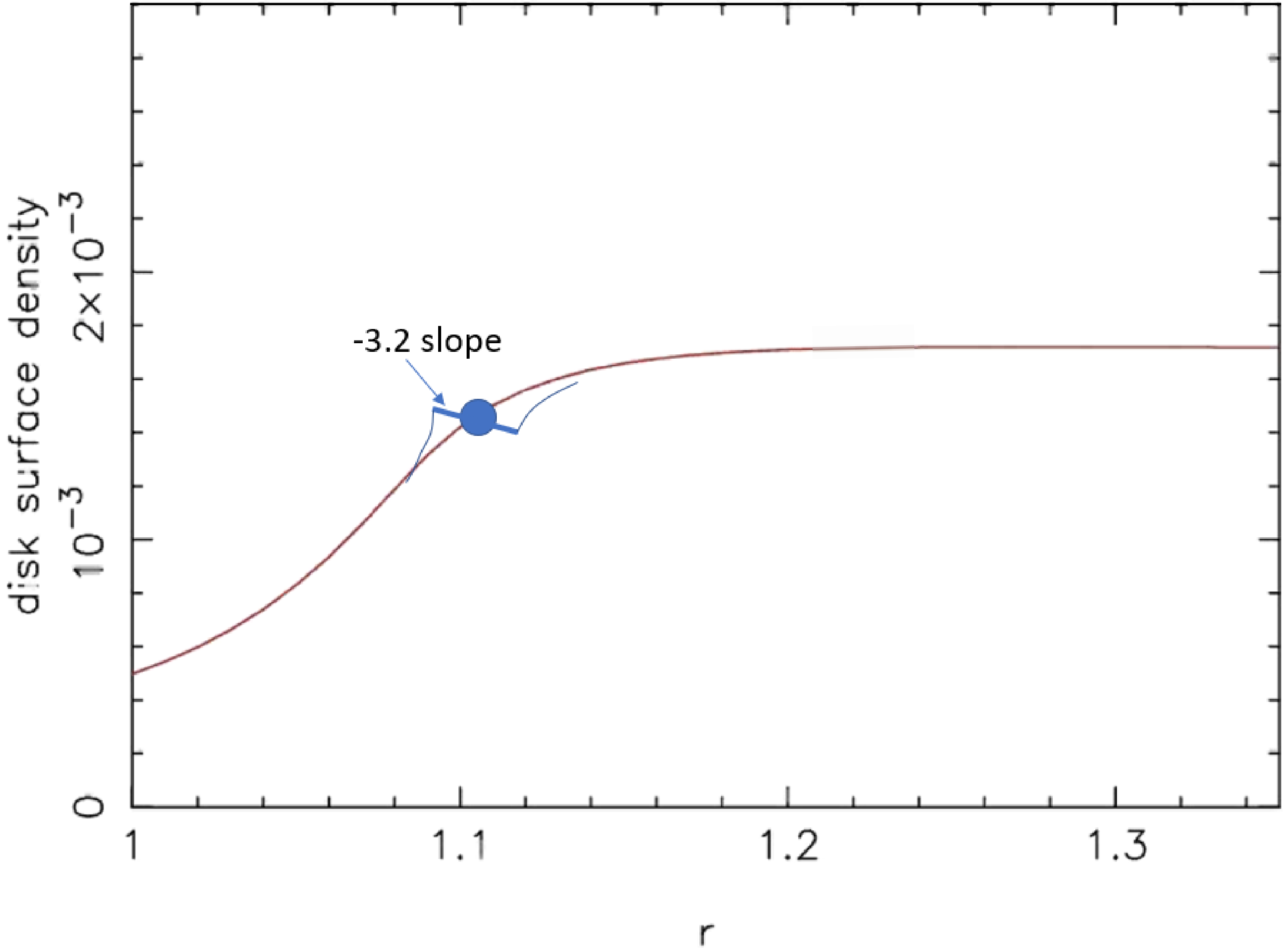
If this happens, the coorbital torque vanishes. This is called *saturation of the coorbital torque*



Saturation of the coorbital torque

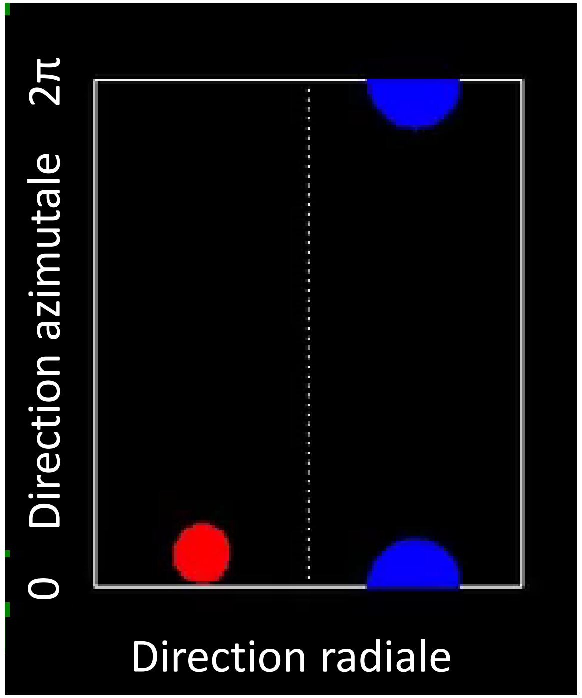
The coorbital torque can be sustained only if other forces acting on the disk (for instance the viscous torque) restore the original gradient.

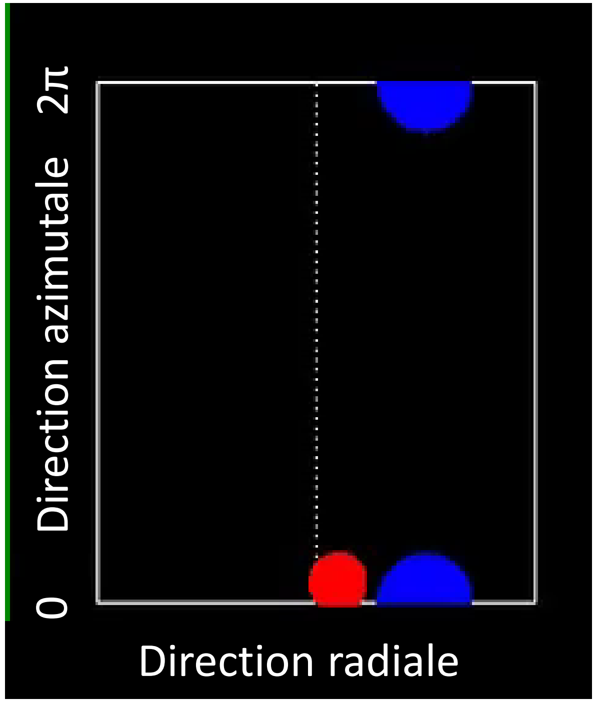
The coorbital torque can more easily saturate for more massive planets and in less viscous discs.



The dynamical coorbital torque

Acts only (under some conditions) on migrating planets





Planet migrating inward:
A negative torque is exerted by the fluid elements passing from inside to outside of the planet’s orbit.
Positive feedback on planet’s migration

A positive torque is exerted by the fluid elements that are in the coorbital region.
Negative feedback on planet’s migration

If the gas density is uniform, the two effects cancel out.
A positive/négative feedback only if the coorbital region is less/more dense

Dynamical coorbital torque for a small planet without partial gap

During migration, the canonical volume $\delta\phi\ \delta J$ of the coorbital region: $2\pi\ \delta J = \frac{\pi}{\sqrt{r}}\delta r$ (avec δr~r), shrinks as $\sqrt{r}$ and, therefore, the mass of the corotation region shrinks as well by the same factor

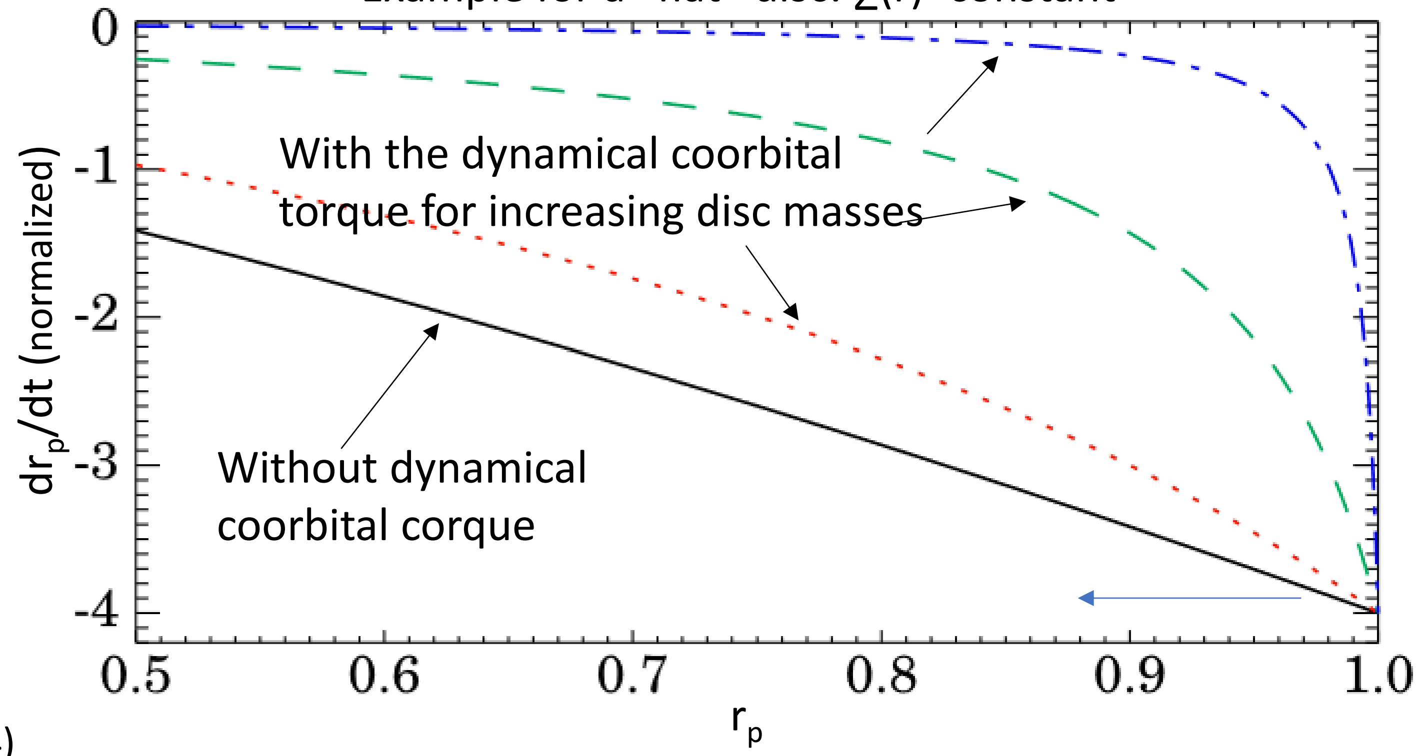
But the geometric surface of the coorbital region ($2\pi r\ \delta r$) shrinks as r^2.
-> the disc's surface density in the coorbital region increases as $1/r^{3/2}$.

Consequently, if the disc's surface density profile $\Sigma(r)$ is shallower than $1/r^{3/2}$ the feedback on planet migration is negative. The planet slows down.

The disc viscosity tends to oppose such a density contrast between the coorbital region and the adjacent disc.
Thus, the dynamical coorbital torque is important only in low-viscosity discs.

Instead, if the $\Sigma(r)$ profile is steeper than $1/r^{3/2}$ migration speeds up

Example for a "flat" disc: ∑(r)=constant

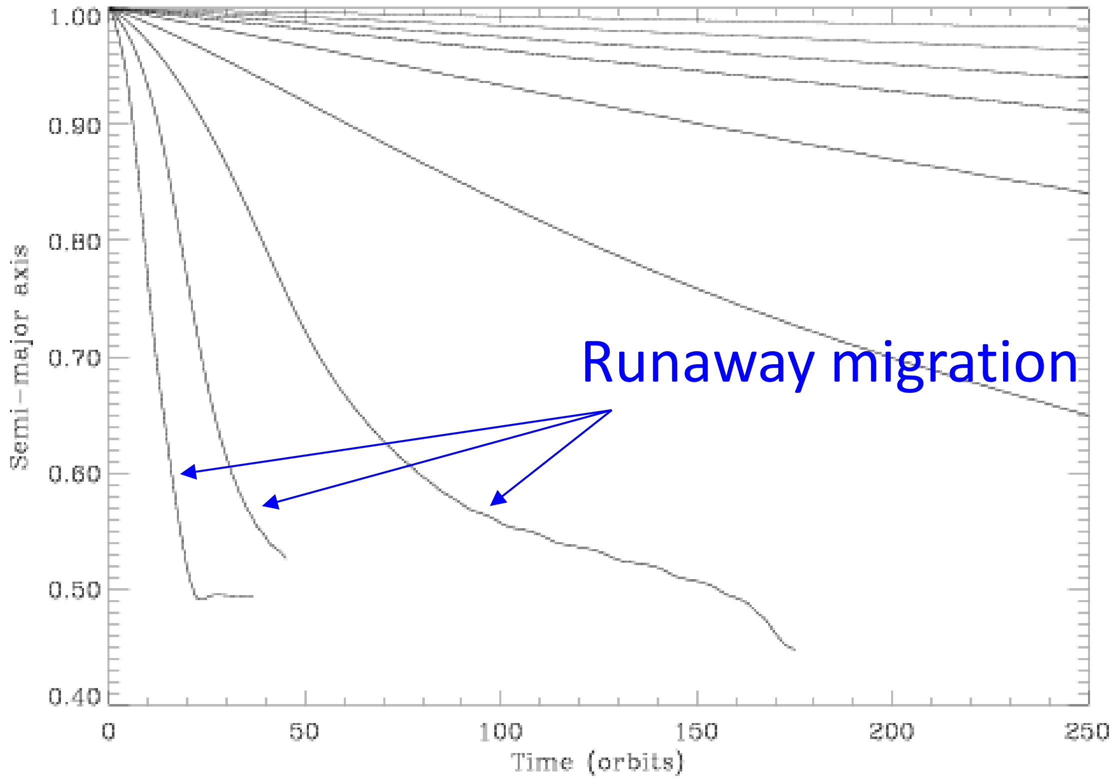


(Paardekooper, 2014)

Dynamical coorbital torque for planets with a partial gap

In this case, even if $\Sigma(r)=1/r^{3/2}$, the coorbital region is less dense. The feedback on migration is always positive. If the mass deficit of the coorbital region is larger than the planet mass, migration enters a runaway mode

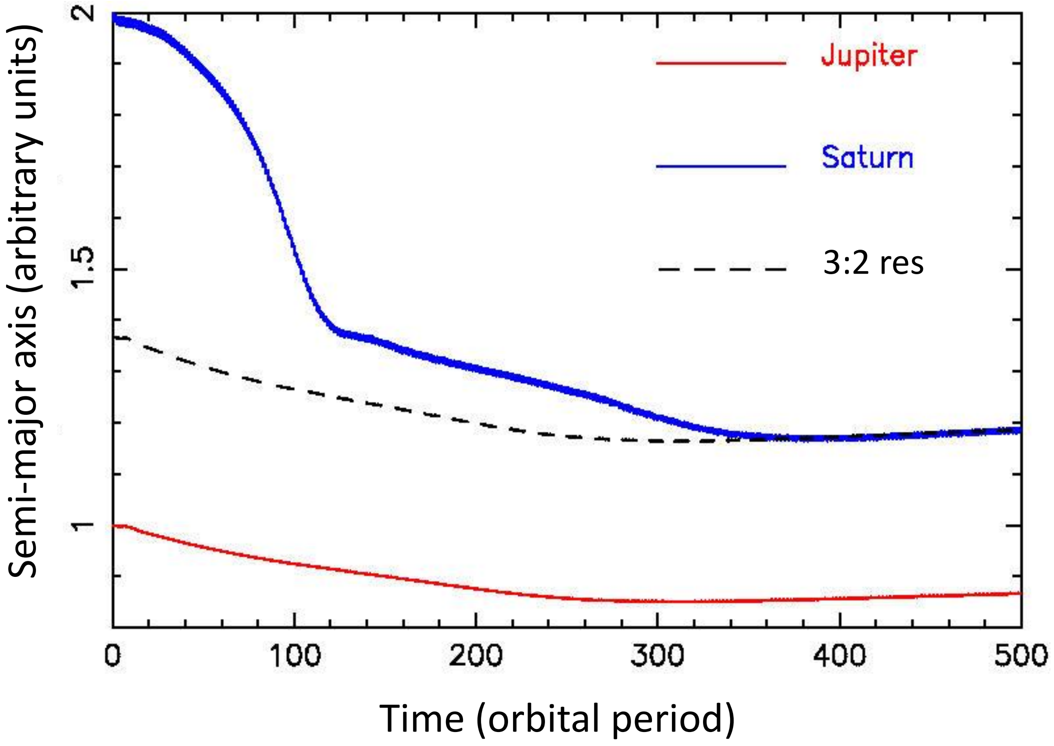
Migration of Saturn in disks with increasing density.



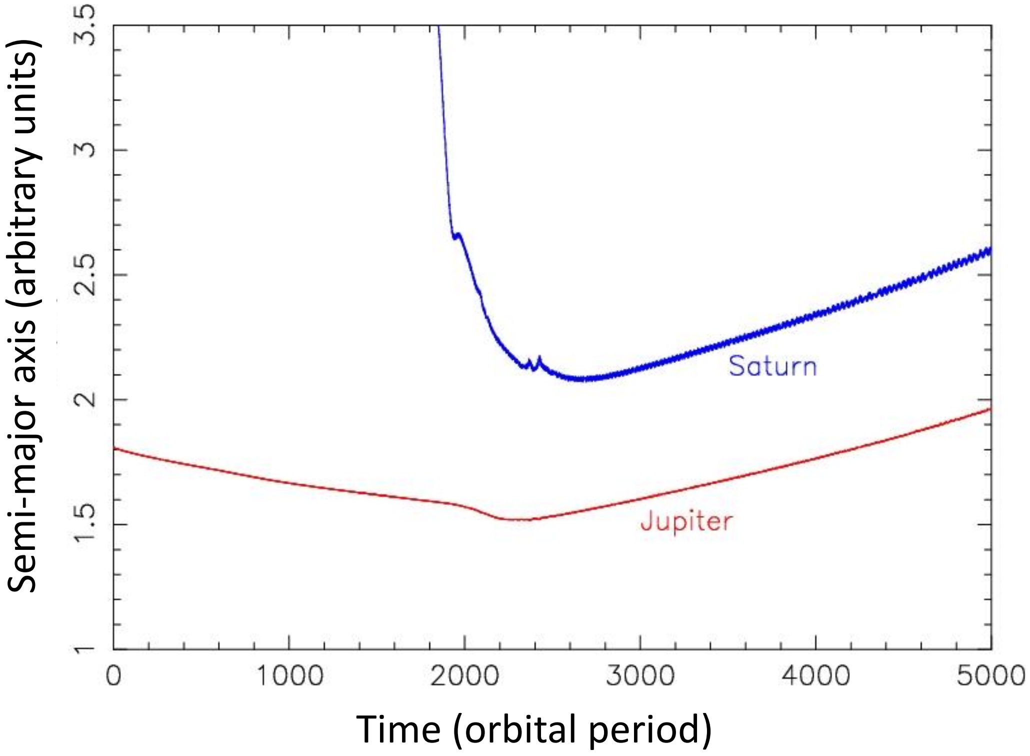
(Masset and Papaloizou 2003)

The joint migration of Jupiter and Saturn

Hot disc (large *H*/*r*)

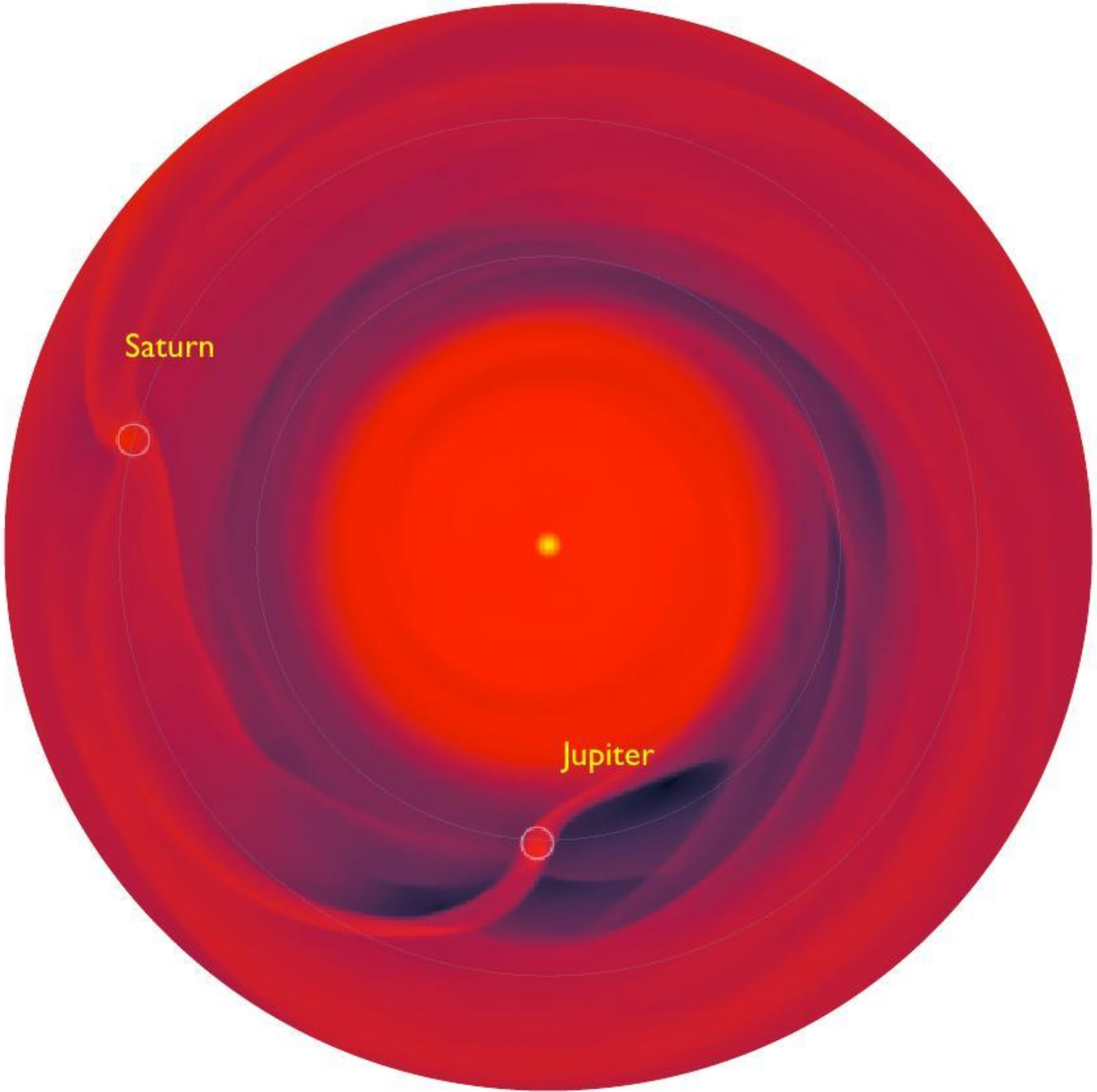


Cold disc (small *H*/*r*)



Masset and Snellgrove, 2001; Morbidelli and Crida, 2007; Pierens and Nelson 2008

The joint migration of Jupiter and Saturn



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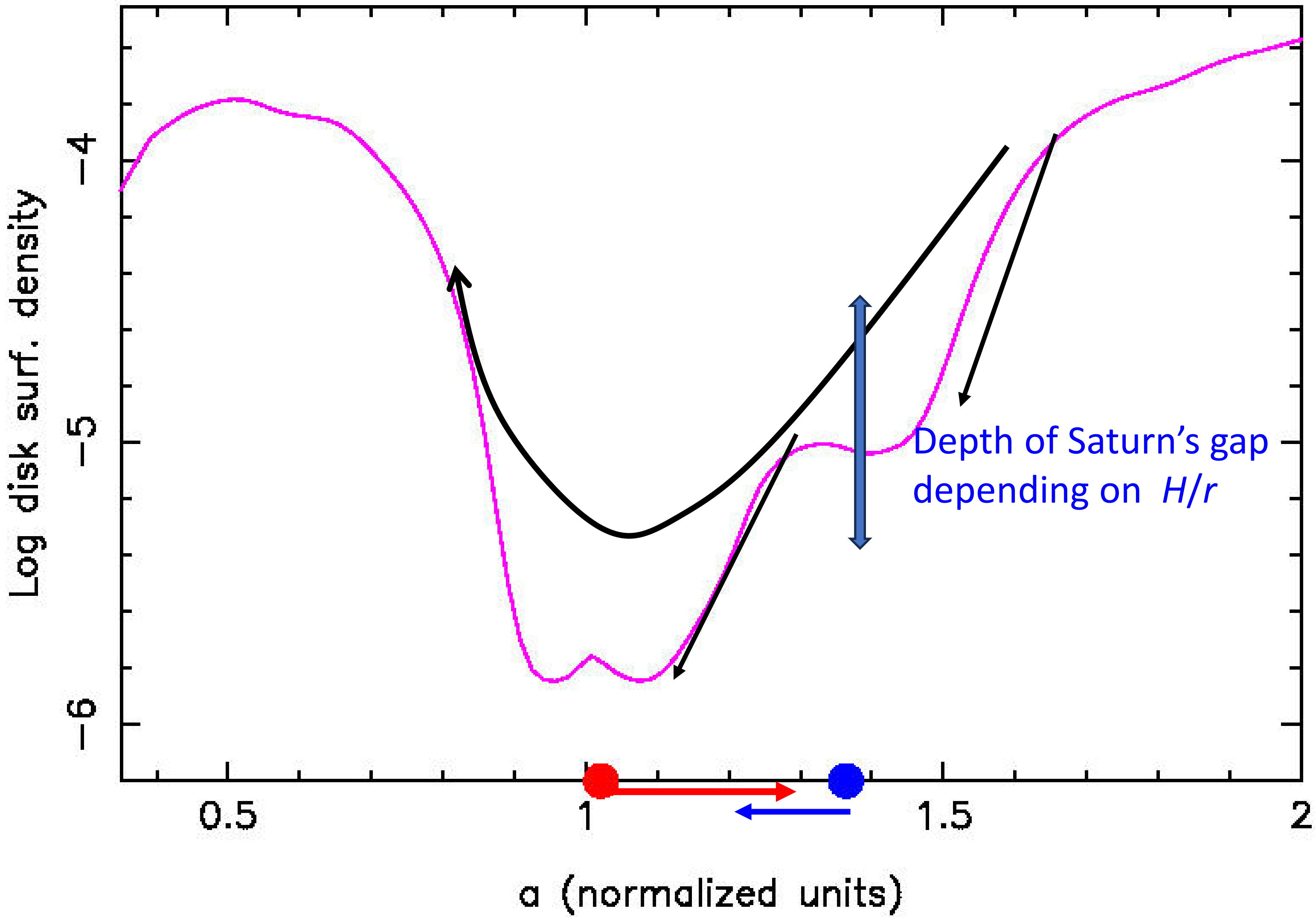
The joint migration of Jupiter and Saturn



Take-away points

- Small planets don't change the radial surface density of the disc, but break its axial symmetry by generating a spiral density wave
- The torque exerted by the wave forces the planet to migrate inward (Type-I migration)
- Type-I migration speed is proportional to the mass of the planet and the density of the disc, while it is inversely proportional to the square of the disc's aspect ratio.
- The disc also damps the planet's eccentricity, on a timescale inversely proportional to $(H/r)^4$
- By the conservation of angular momentum, eccentricity damping forces a semi-major axis damping proportional to e^2/τ_e
- The coorbital torque can block Type-I migration if the disc has a steep surface density gradient (e.g. at the inner edge of the disc)
- Type-I migration is also slowed down by the dynamical coorbital torque if the disc has (i) a low viscosity and (ii) a surface density gradient shallower than $1/r^{3/2}$.
- Giant planets open gaps in the radial surface density distribution of the disc
- Their migration has to happen together with the migration of the gap, at approximately viscous speed, (Type-II migration)
- Two giant planets, with a mass ratio comparable to that of Jupiter and Saturn, one in resonance, can halt or reverse their migration.

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