Planetary resonances

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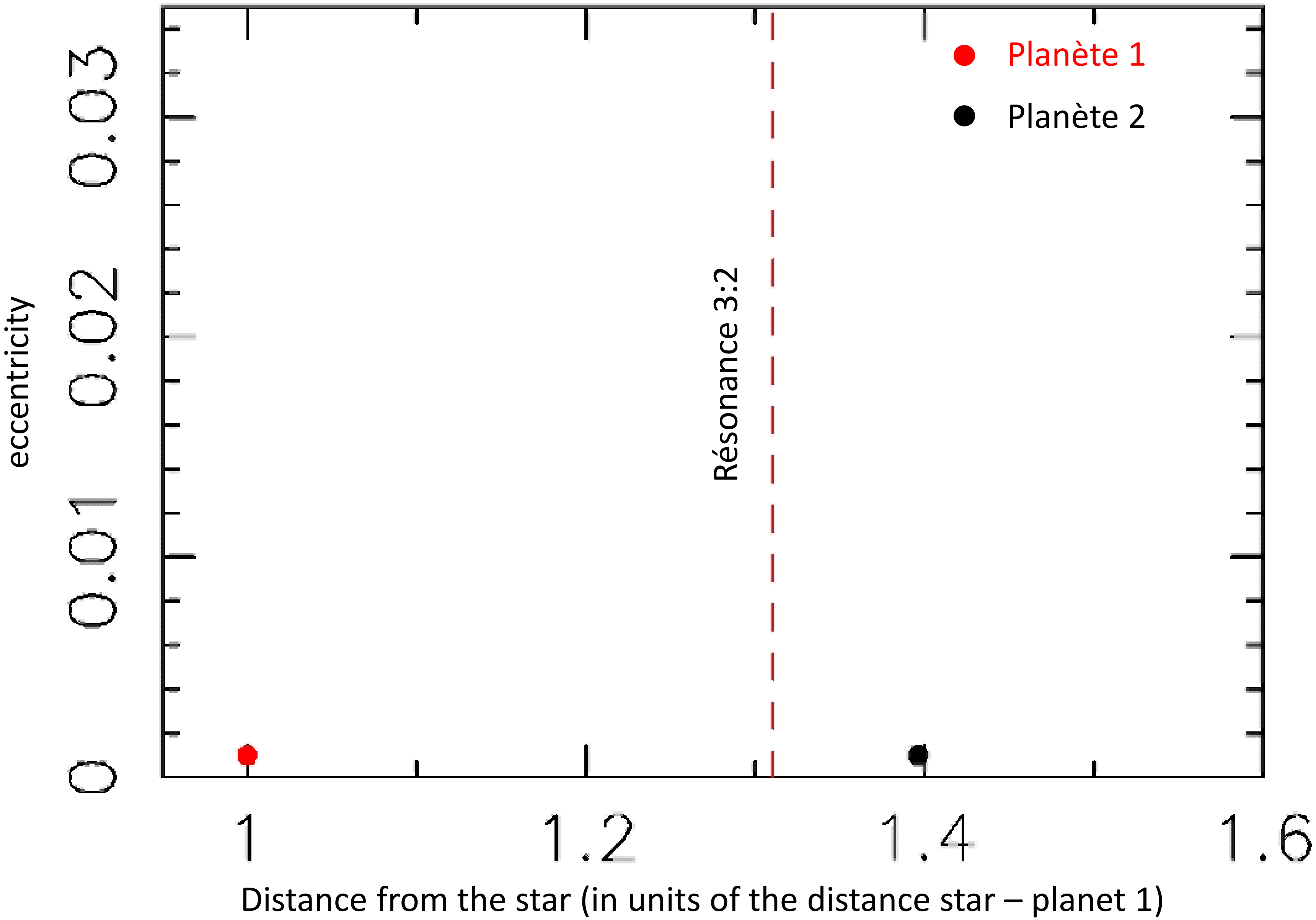
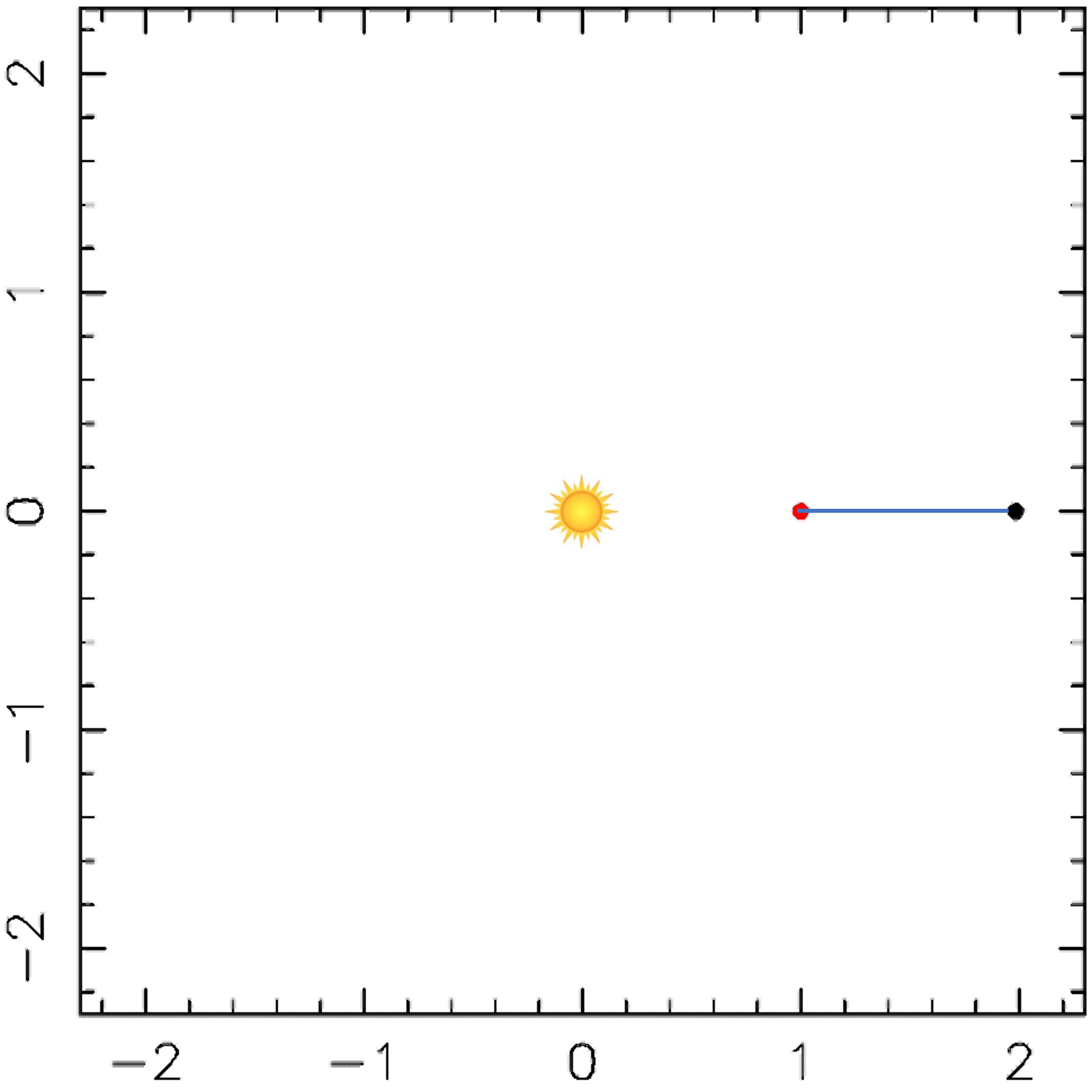
1530

Convergent migration of two planets



Example of a resonance (2:1)

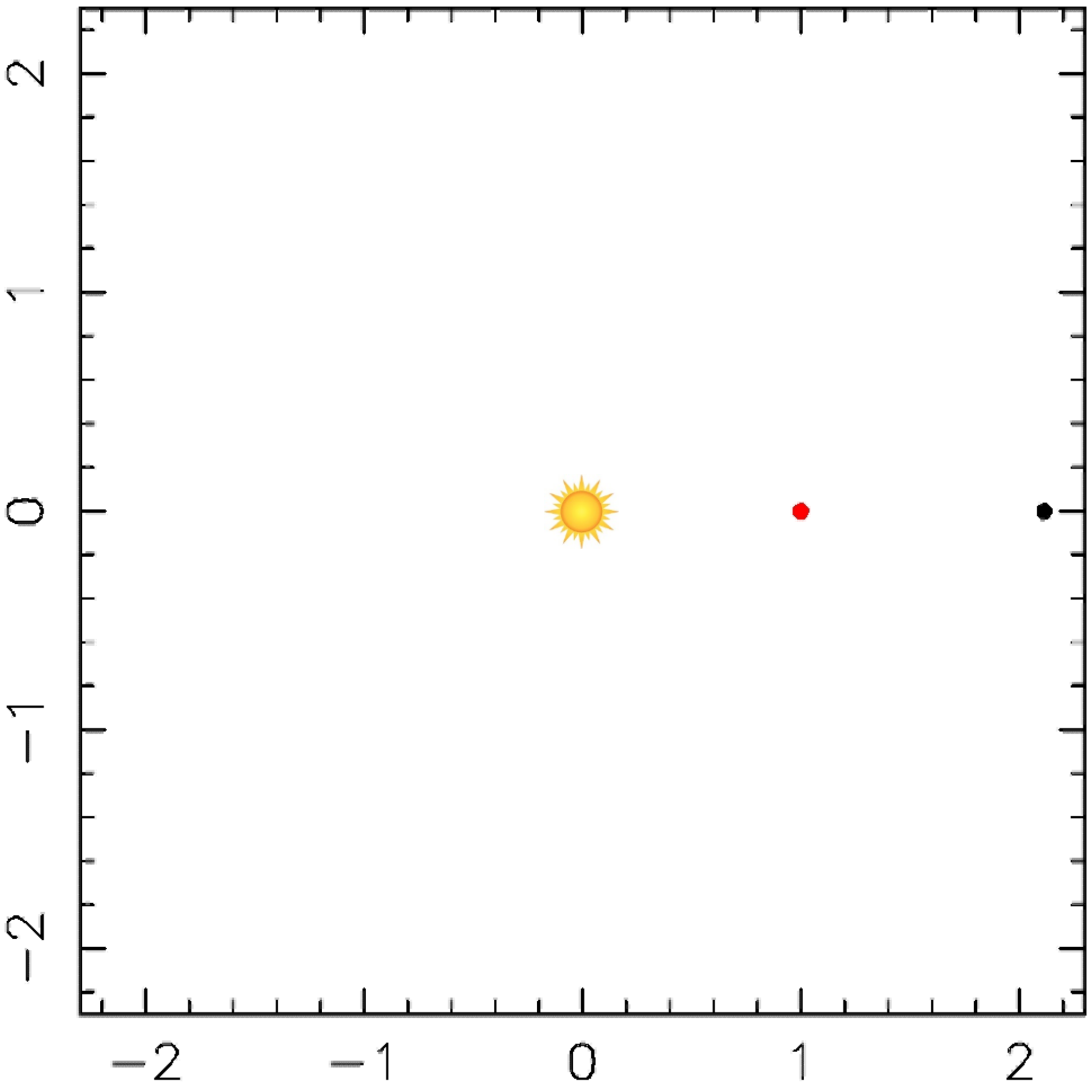


Planet 1 (circular)

Planet 2 (elliptic)

Conjunction = minimal approach distance

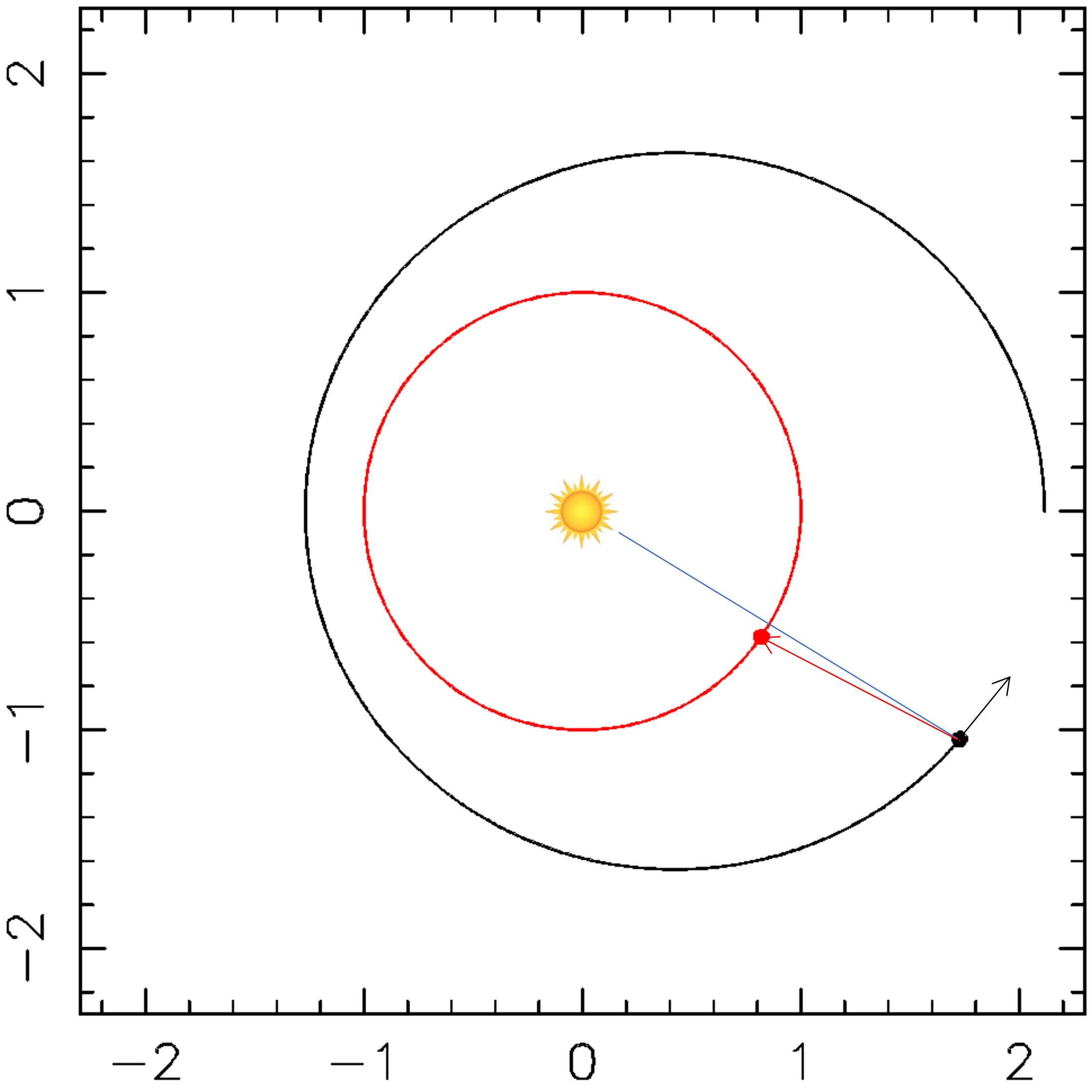
If slightly outward of resonance (2:1)



Planet 1 (circular)

Planet 2 (elliptic)

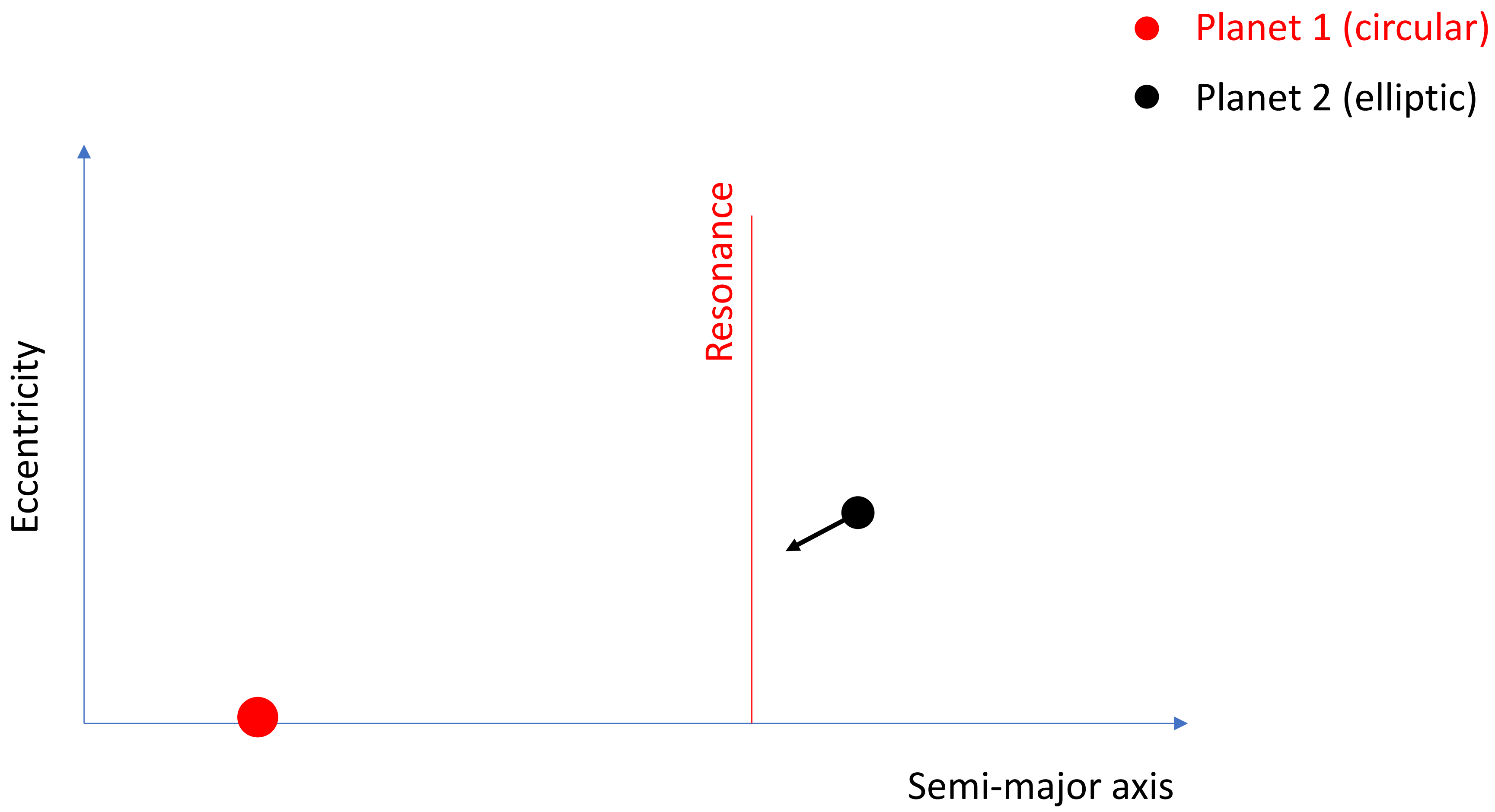
Minimal approach configuration



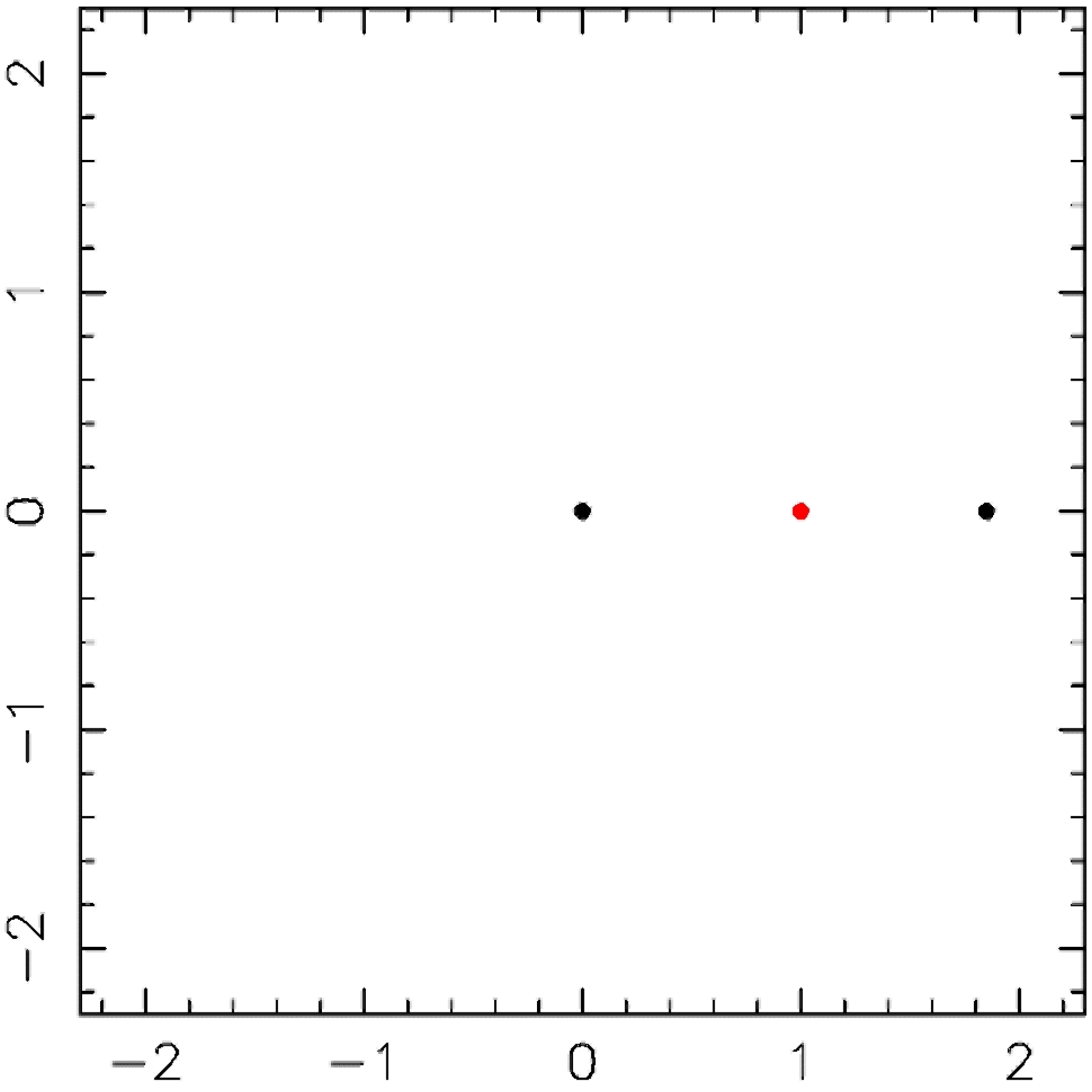
- Planet 1 (circular)
- Planet 2 (elliptic)

- The positive radial motion of the planet is slowed down(its eccentricity decreases)
- The azimuthal speed is also slowed down (the semi-major axis decreases)

Motion with respect to the resonance location

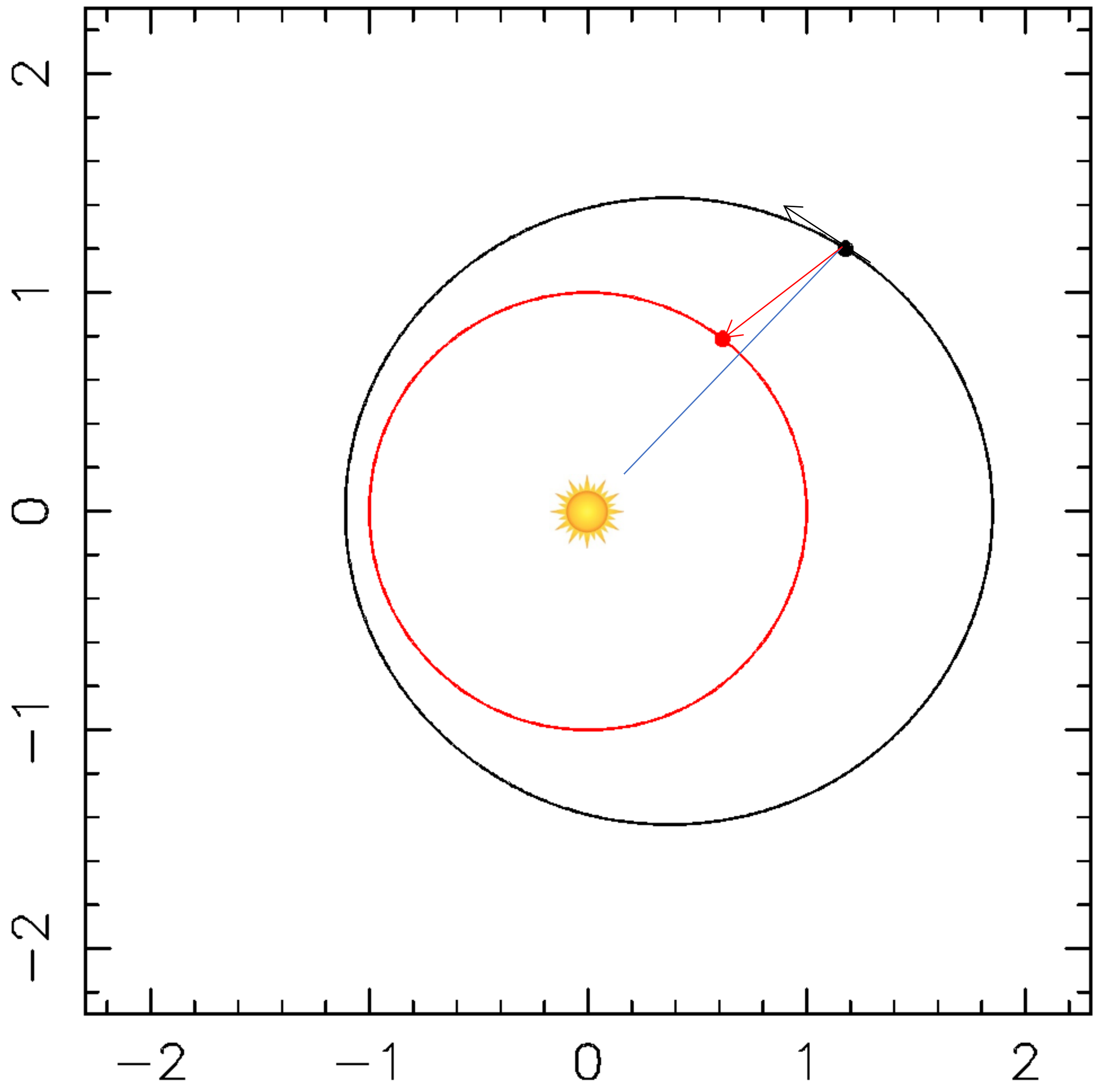


If slightly inward of the resonance (2:1)



- Planet 1 (circular)
- Planet 2 (elliptic)

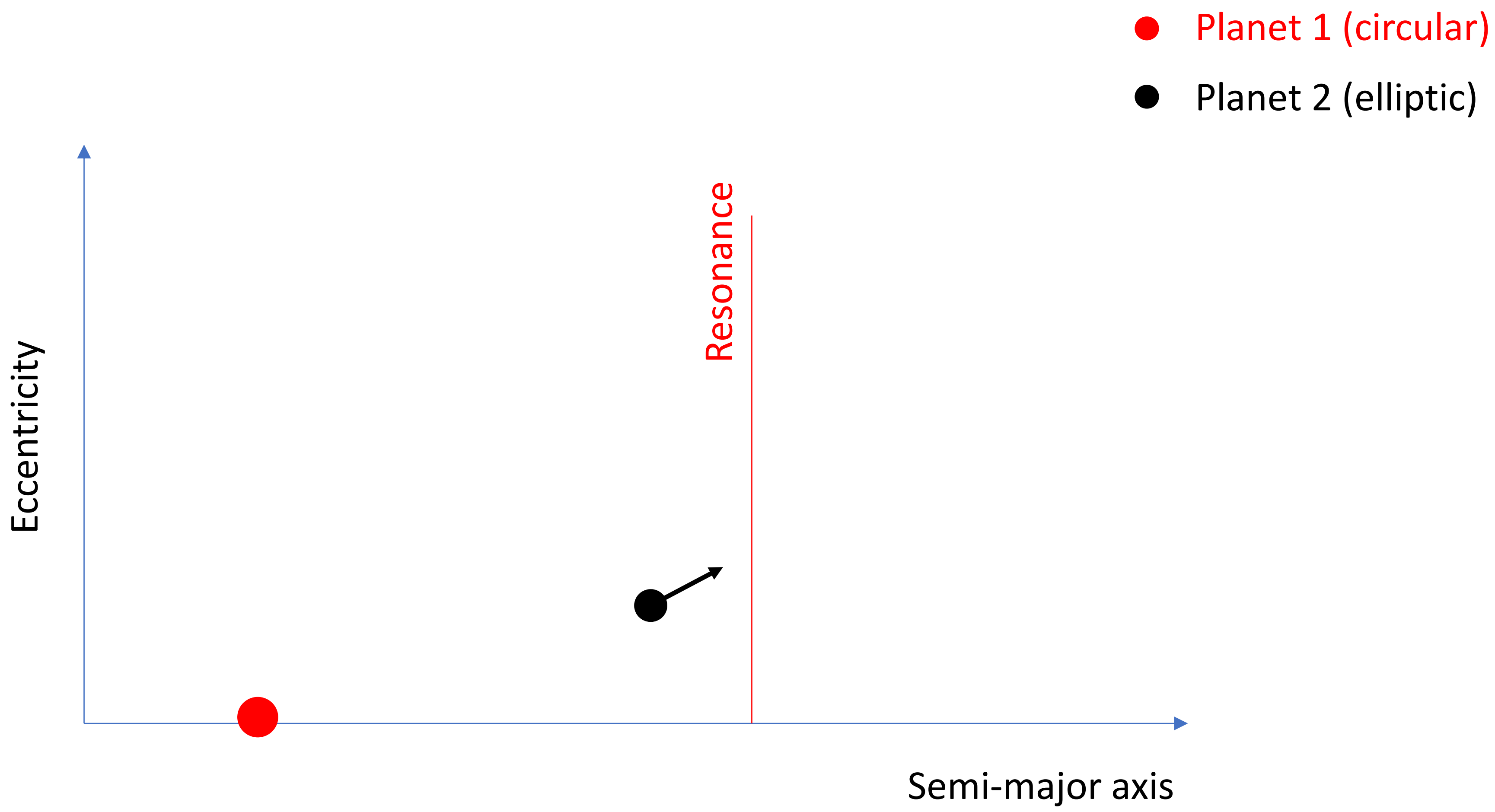
Minimal approach configuration



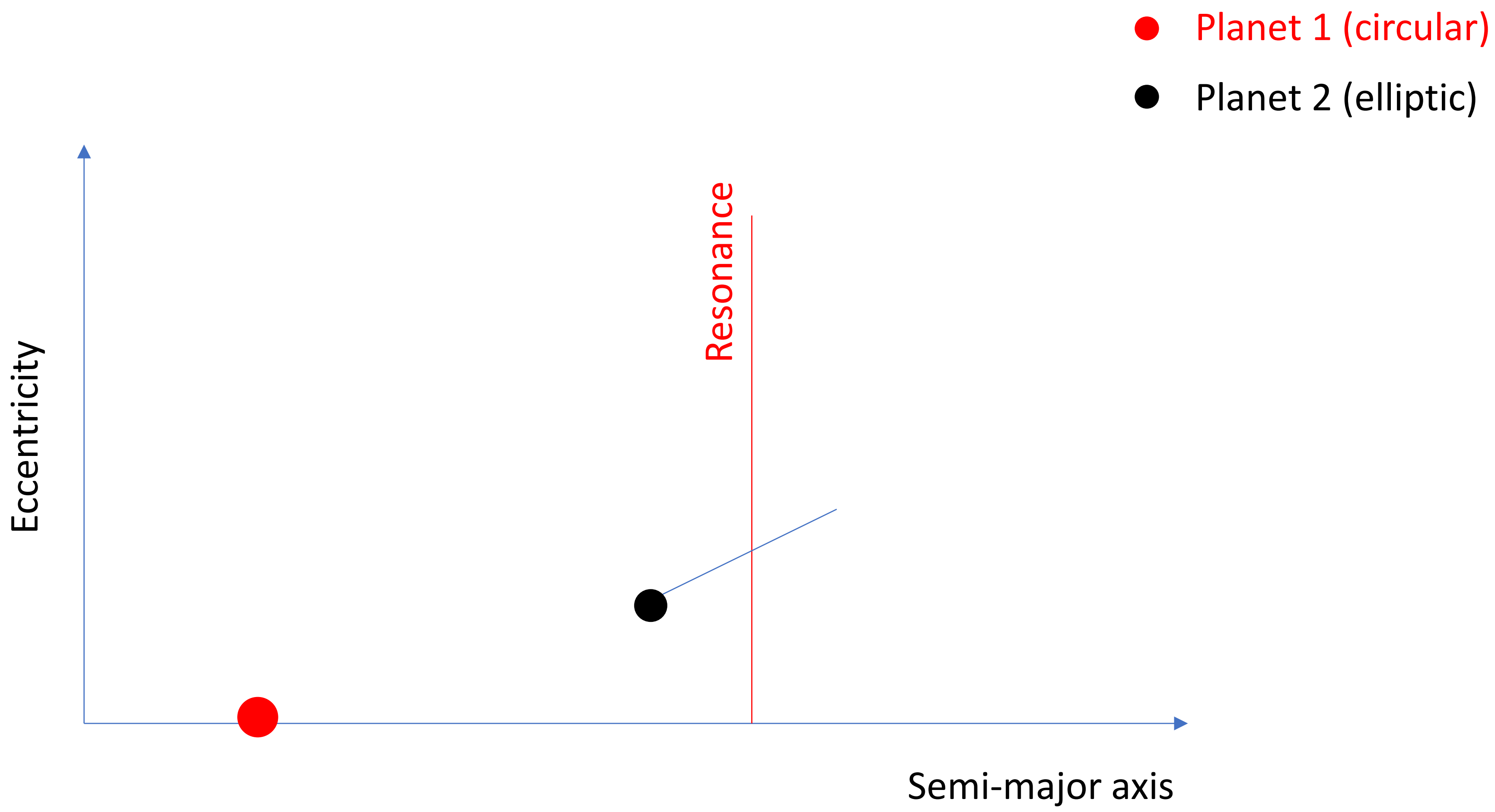
- Planet 1 (circular)
- Planet 2 (elliptic)

- The negative radial motion of the planet is accelerated (its eccentricity grows)
- The azimuthal velocity is also accelerated (the semi-major axis increases)

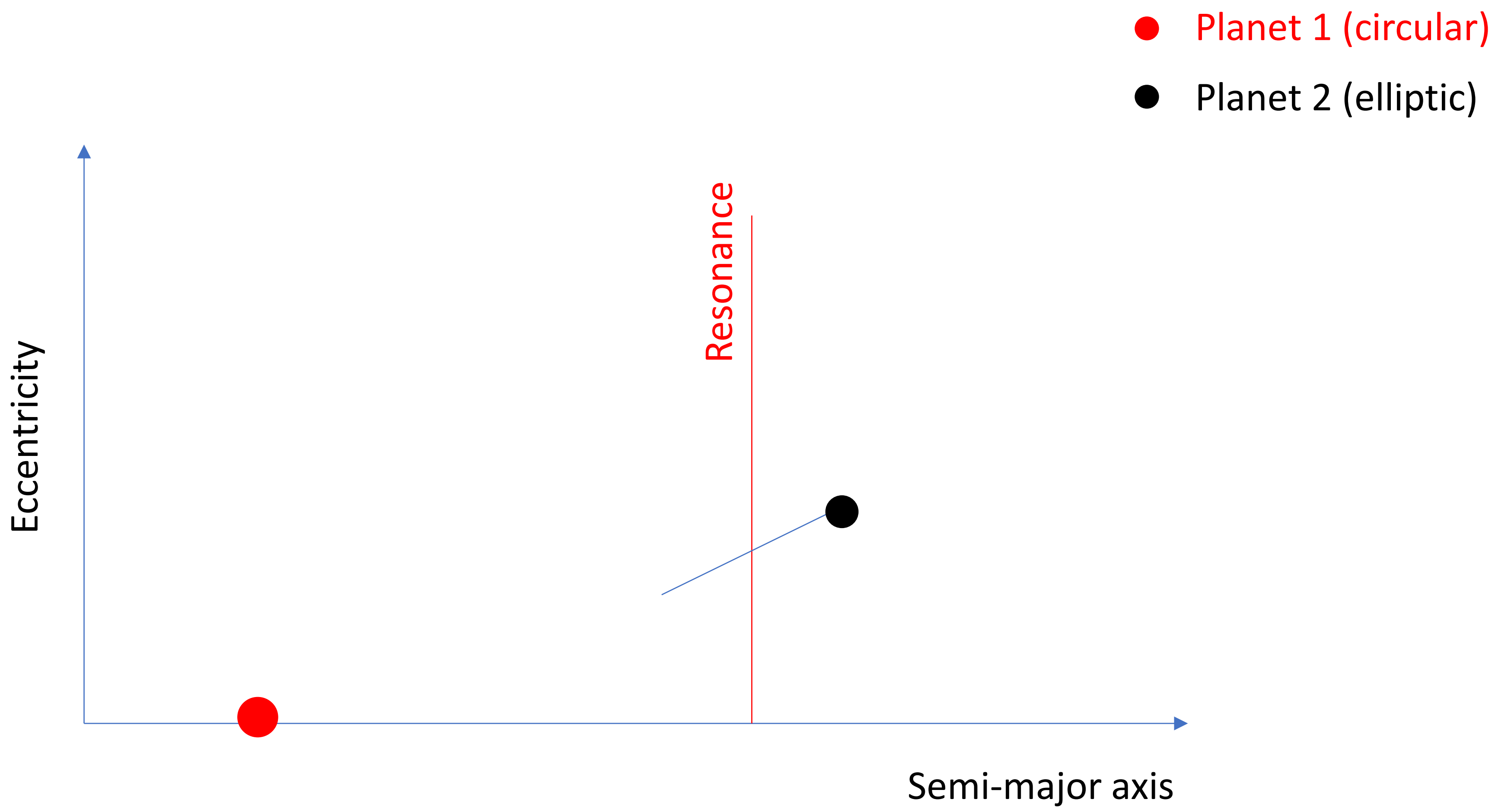
Motion with respect to the resonance location



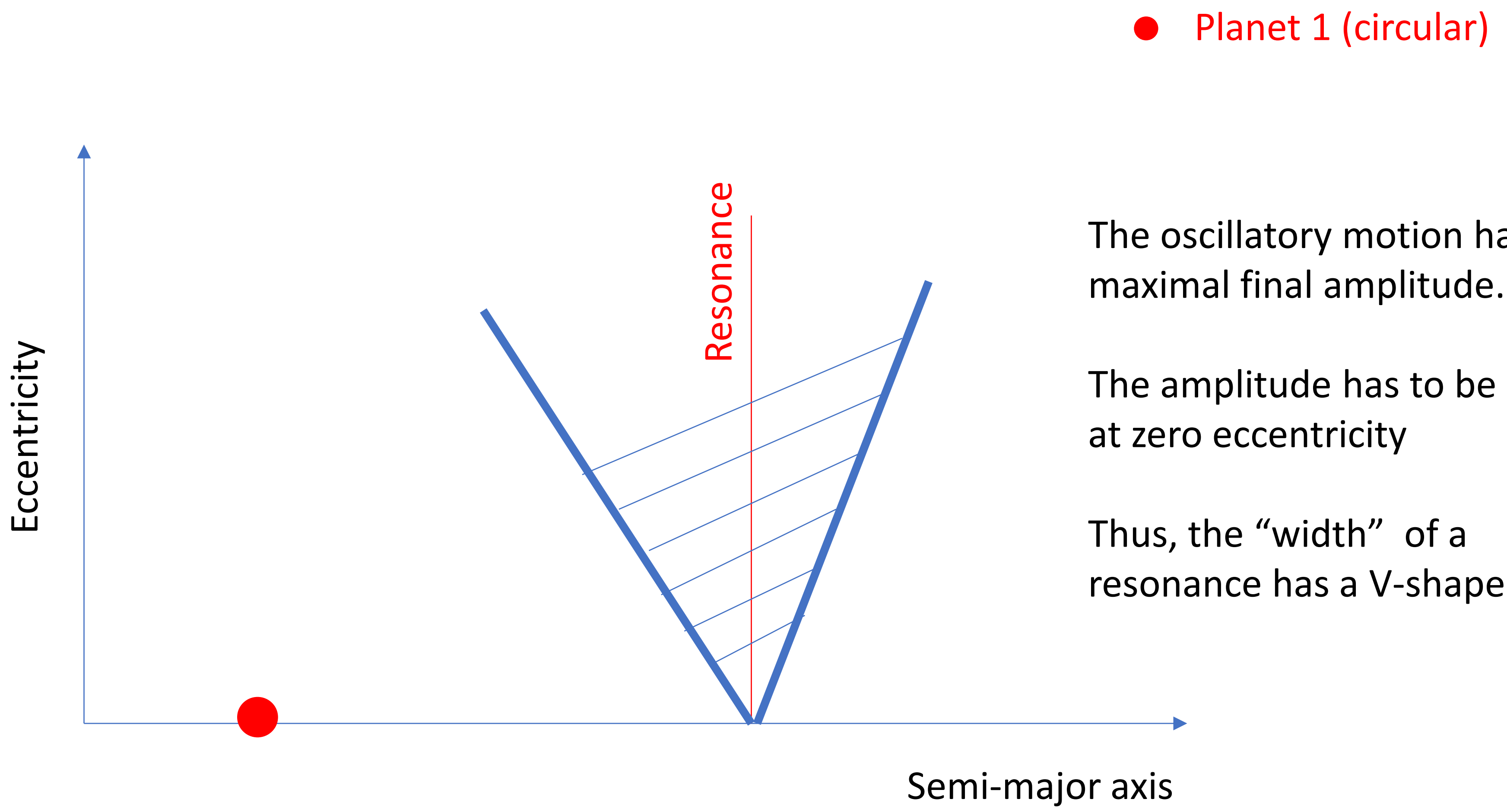
Oscillatory motion around the resonance



Oscillatory motion around the resonance



Oscillatory motion around the resonance



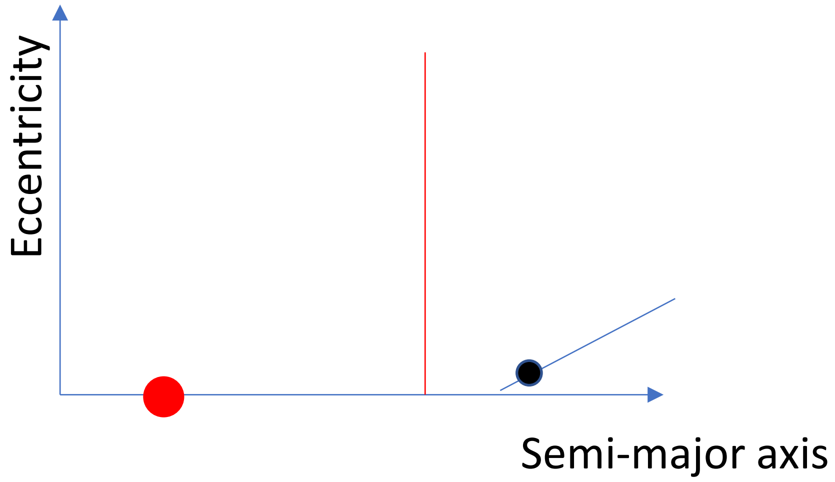
The oscillatory motion has a maximal final amplitude.

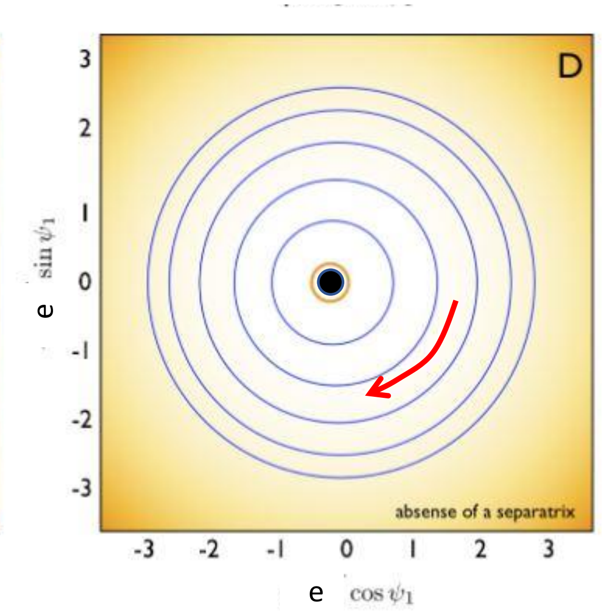
The amplitude has to be null at zero eccentricity

Thus, the “width” of a resonance has a V-shape

Phase diagrams for a k:k-1 resonance

$\Psi_1 = k\lambda - (k-1)\lambda' - \varpi$





Phase diagrams for a k:k-1 resonance

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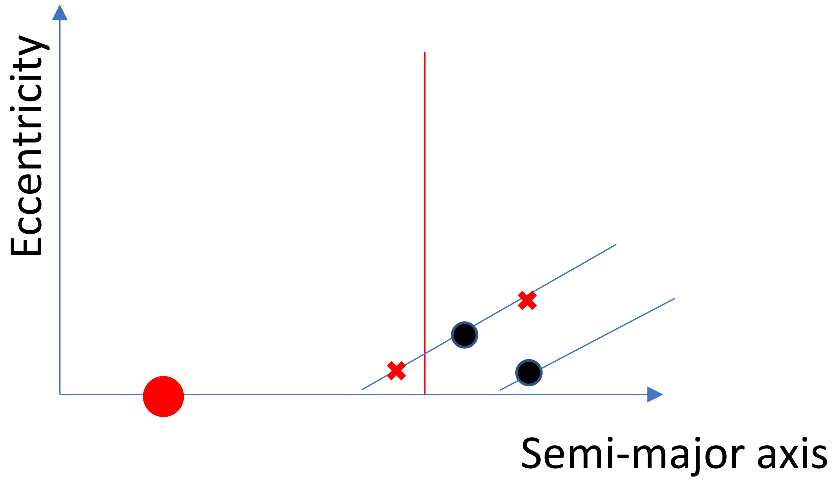
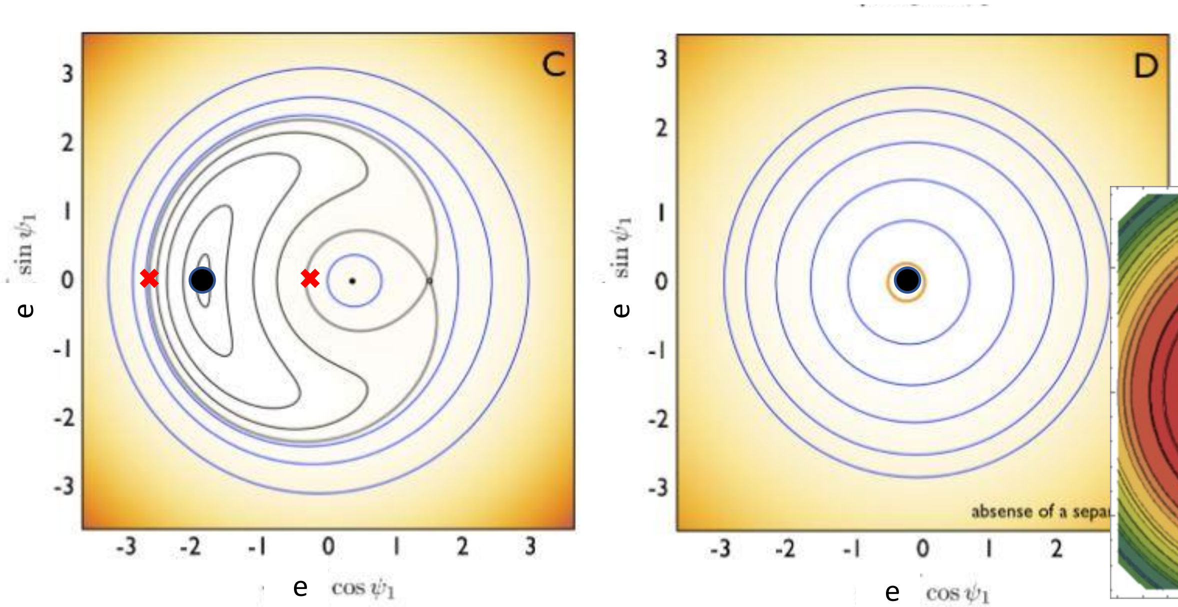
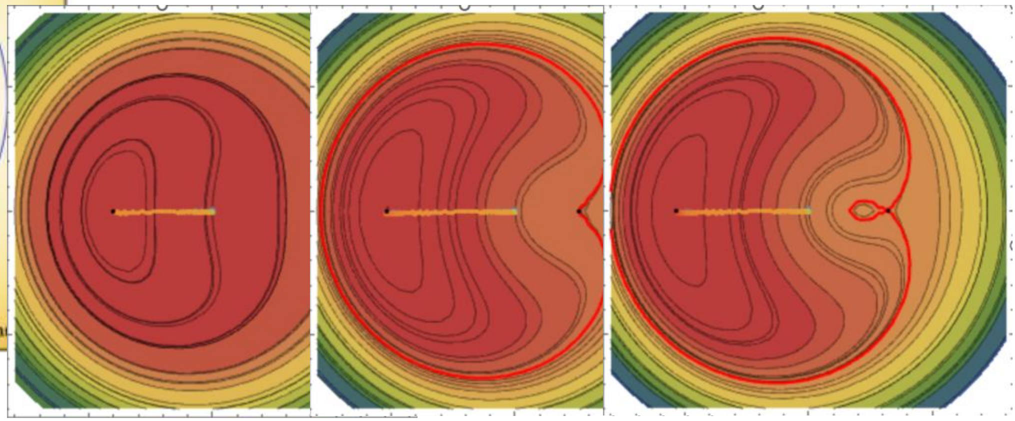
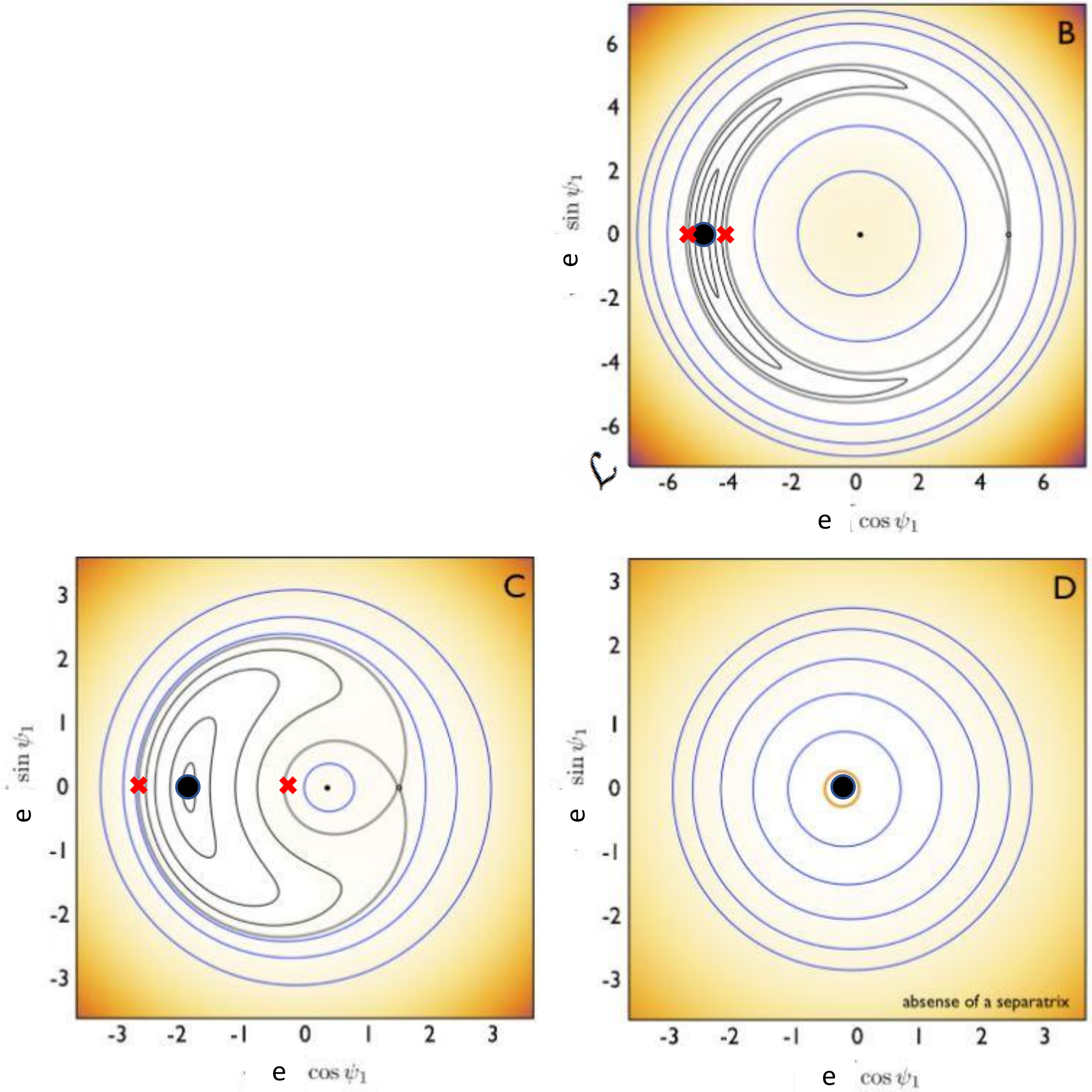


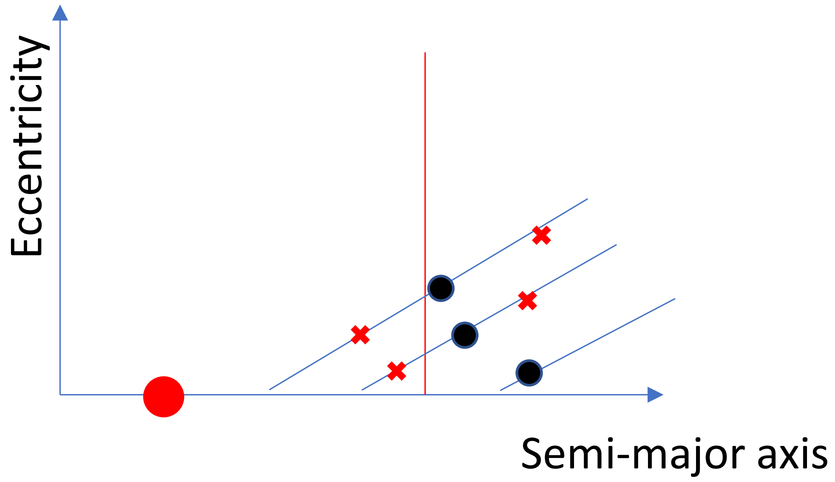




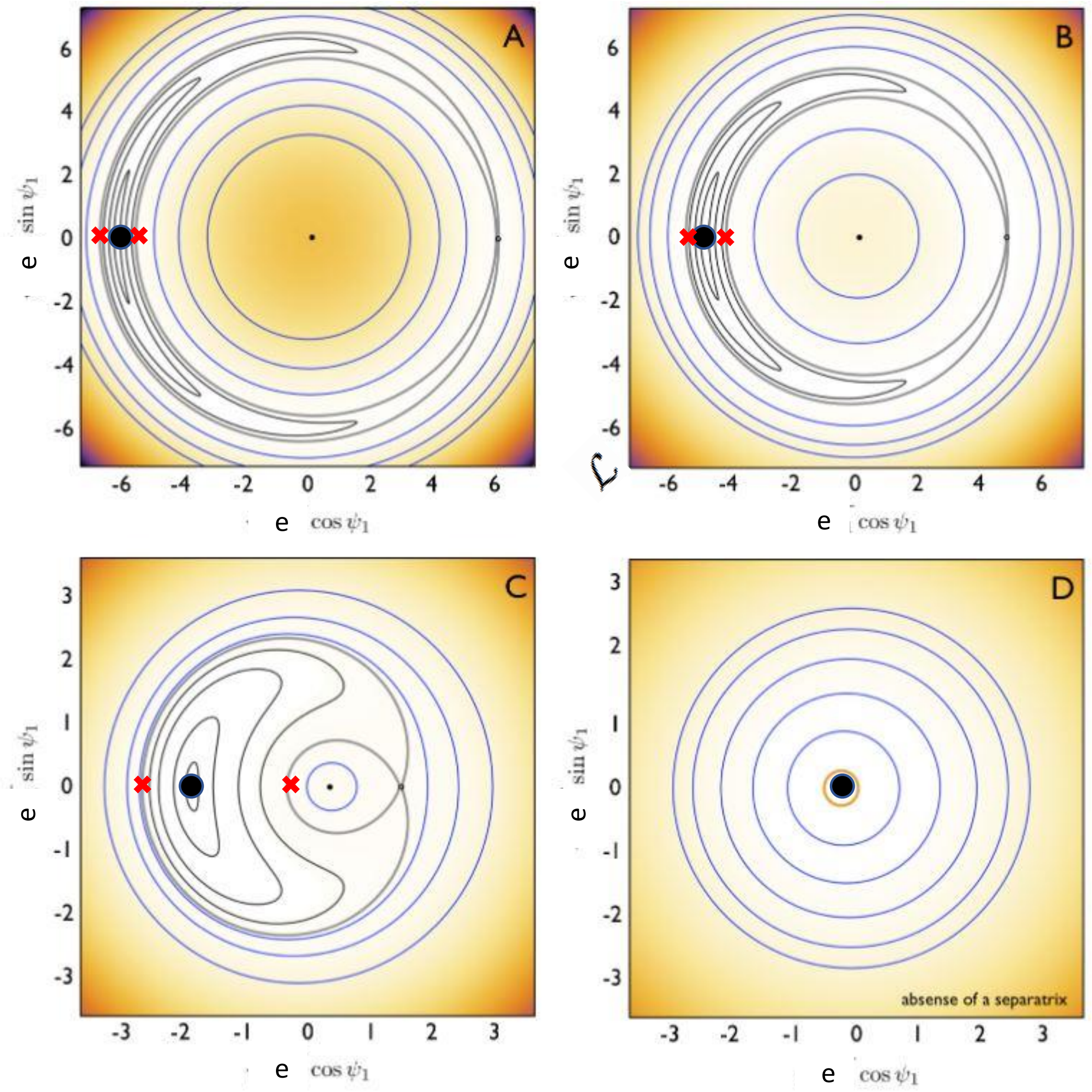
Phase diagrams for a k:k-1 resonance



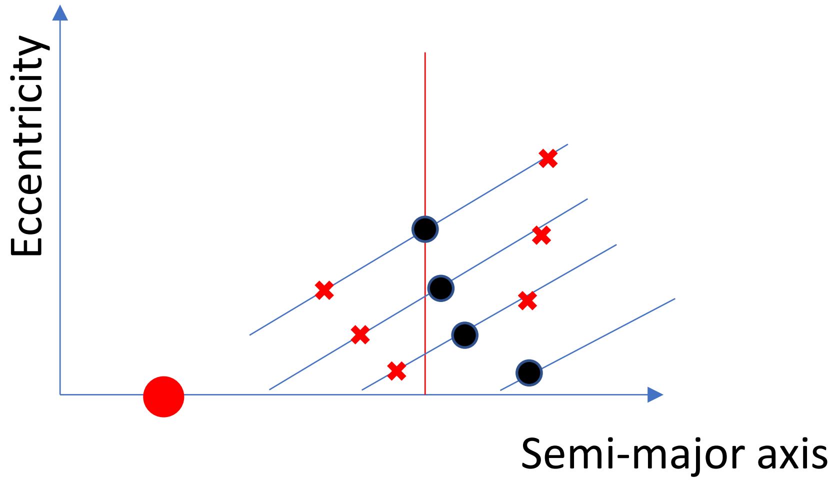
$\Psi_1 = k\,\lambda - (k\text{-}1)\,\lambda' - \varpi$



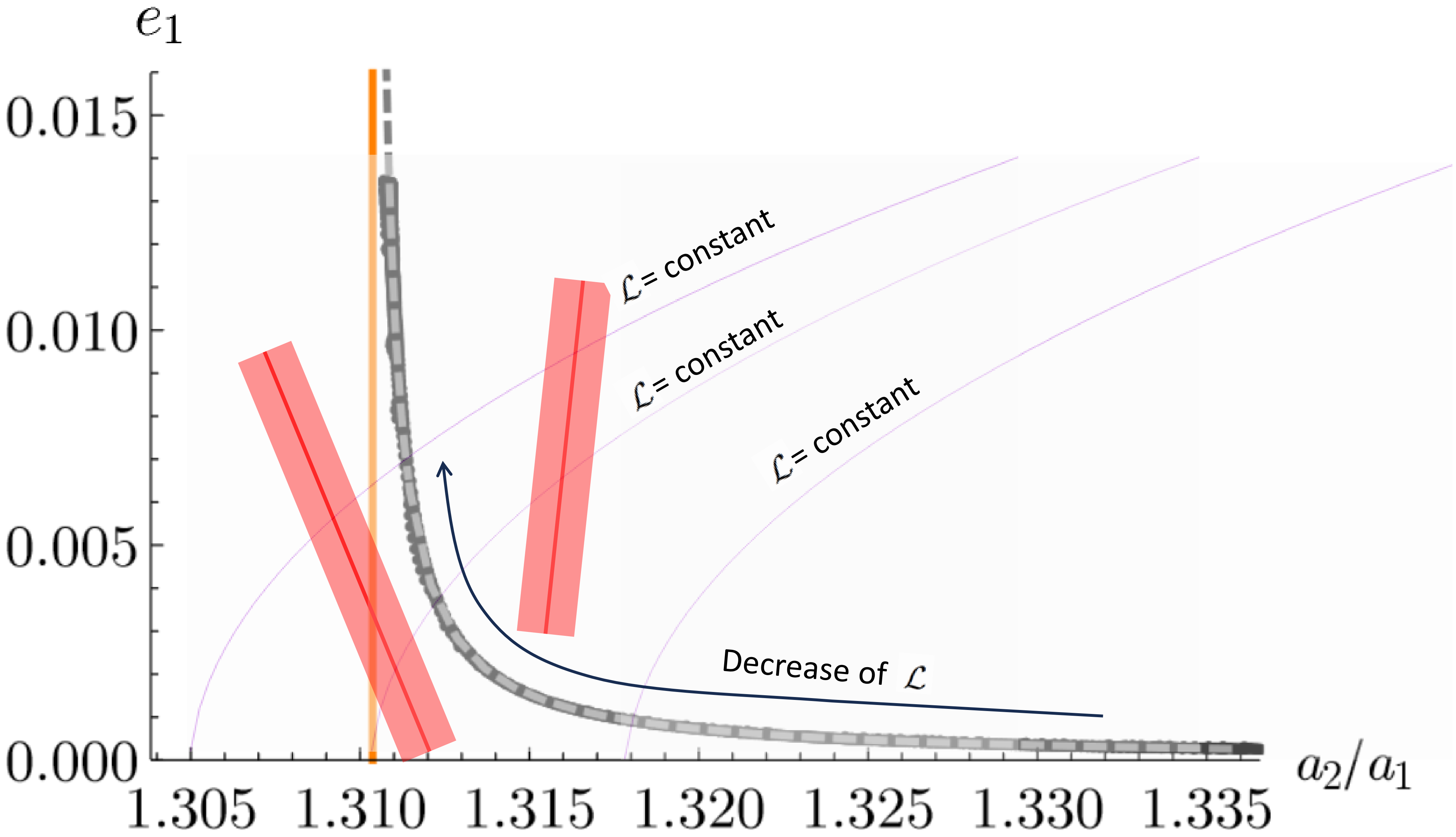
Phase diagrams for a k:k-1 resonance



$\Psi_1 = k\,\lambda - (k\text{-}1)\,\lambda' - \varpi$



k:k-1 Resonance and separatrices in a (a,e) diagram



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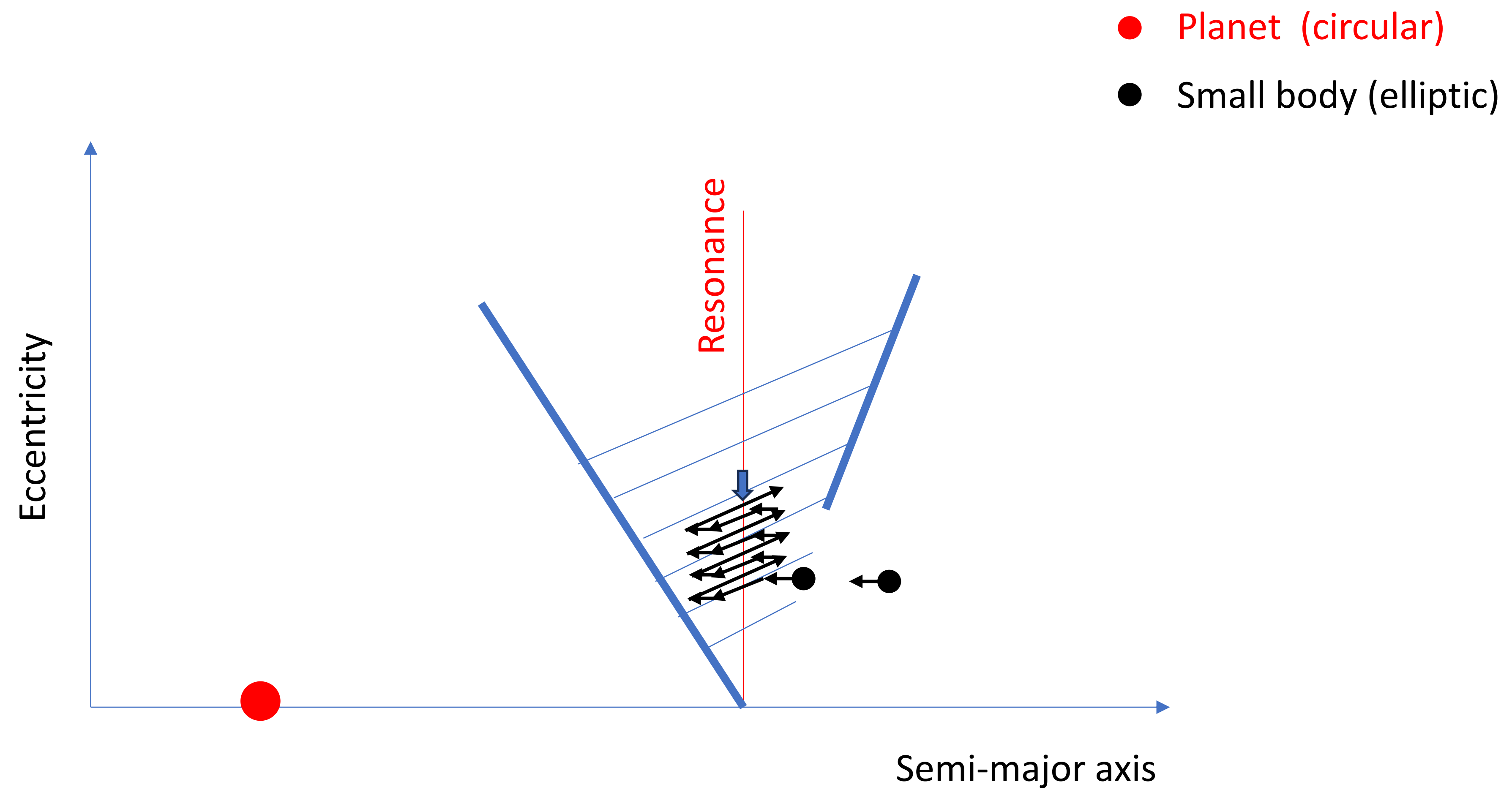
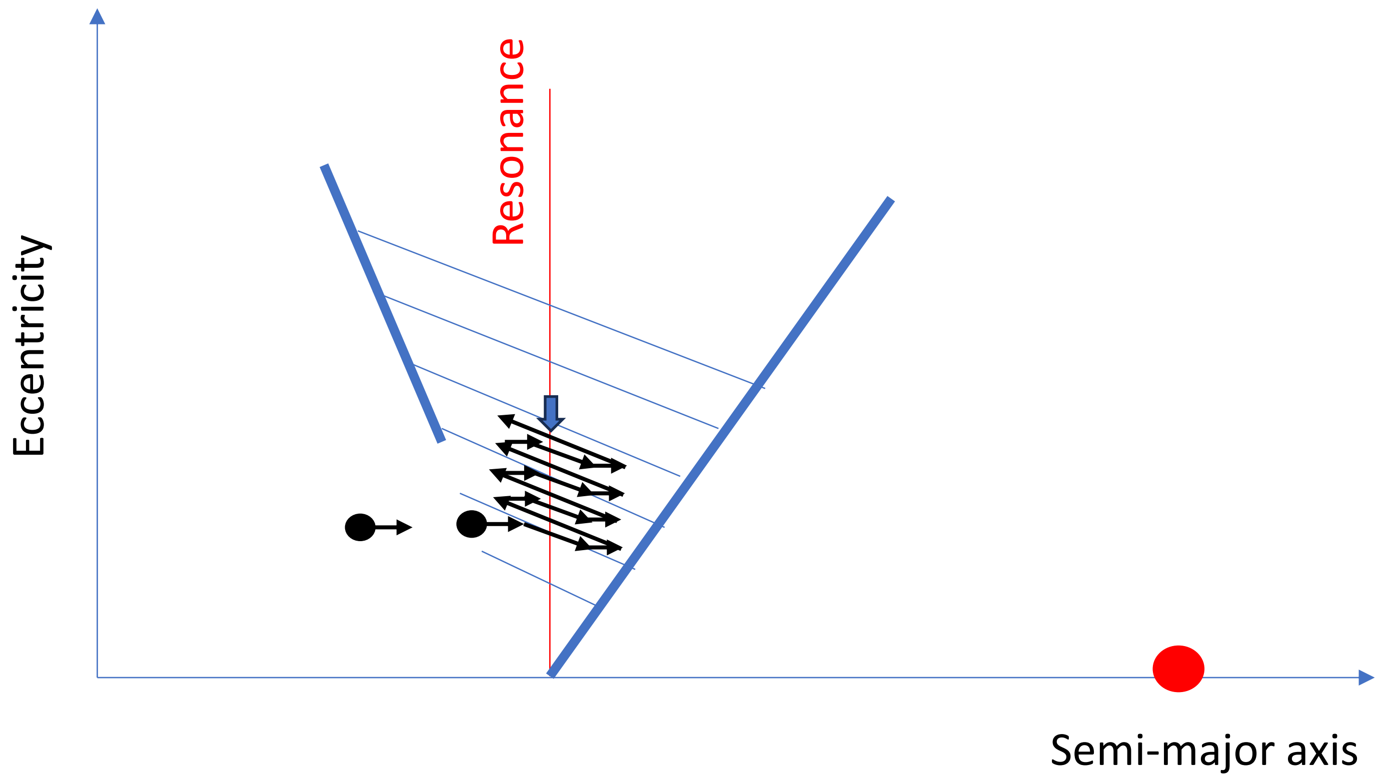
Convergent migration, capture in the resonance and growth of the eccentricity



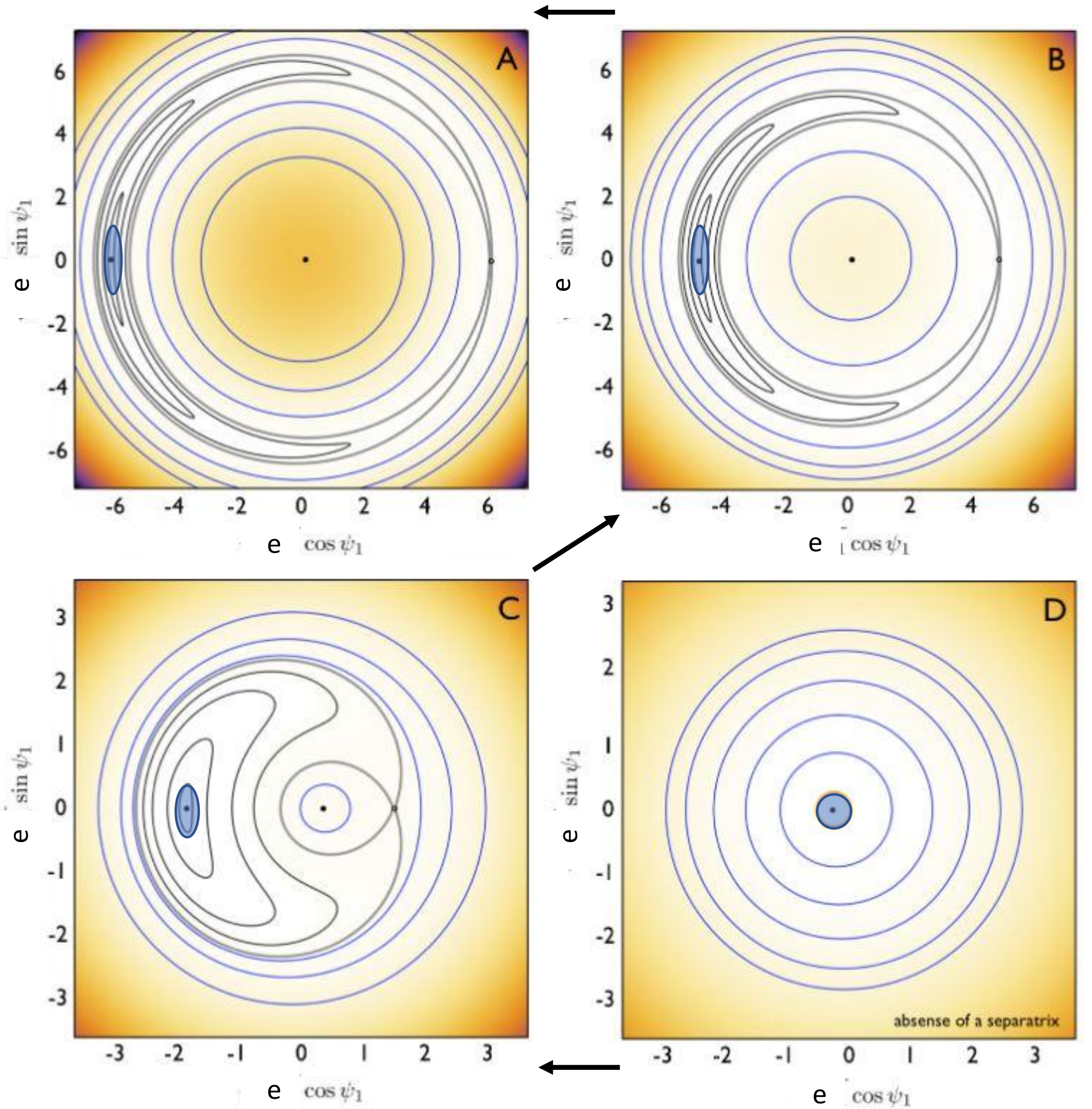
Convergent migration, capture in the resonance and growth of the eccentricity

● Plant (circular)

● Small body (elliptic)



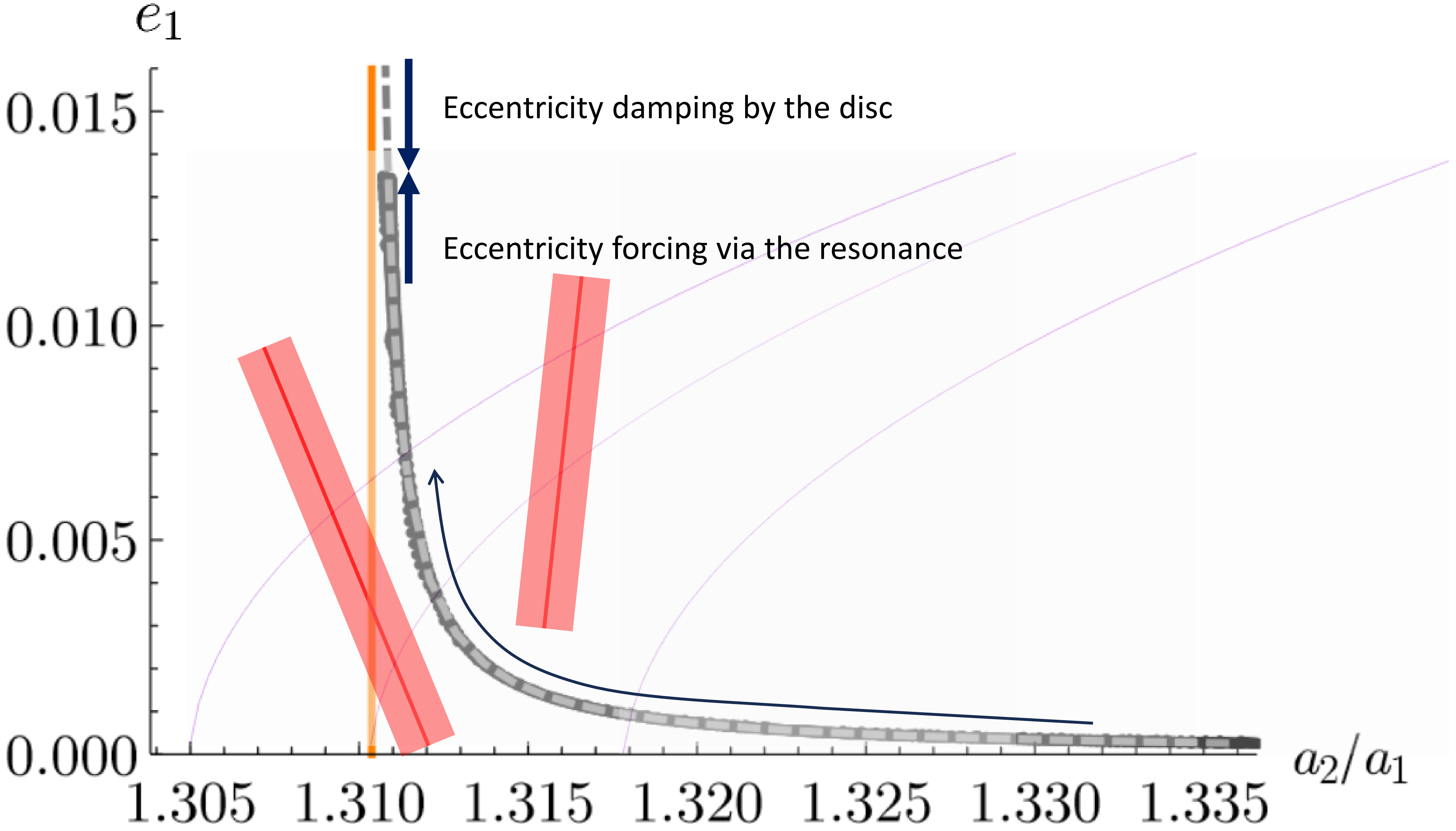
Migration and capture in a k:k-1 resonance



Adiabatic principle:

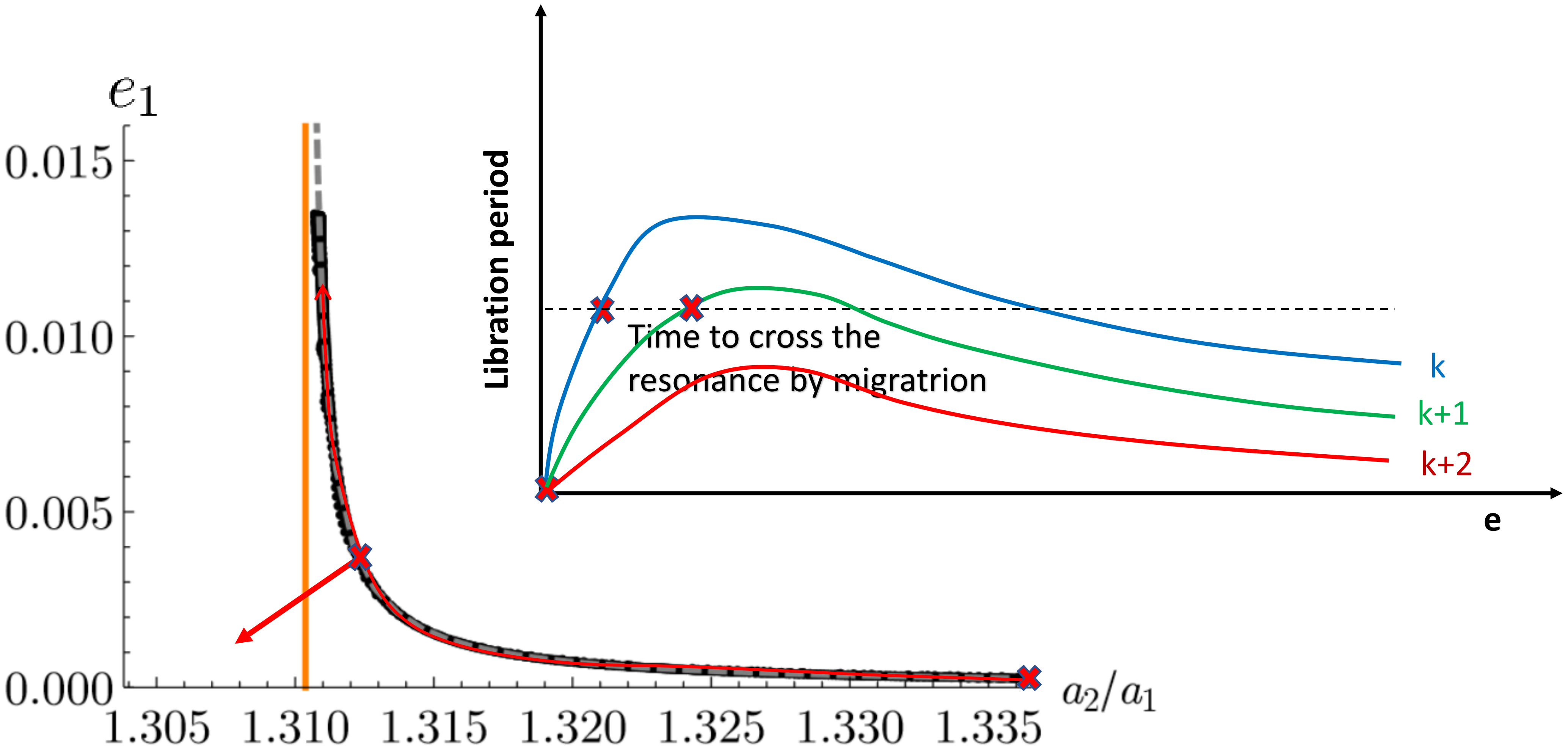
If the passage from one level to the next is slow compared to the libration period, the evolution follows the trajectory that encapsulates the same area

Equilibrium eccentricity



Failed captures: break of the adiabatic principle

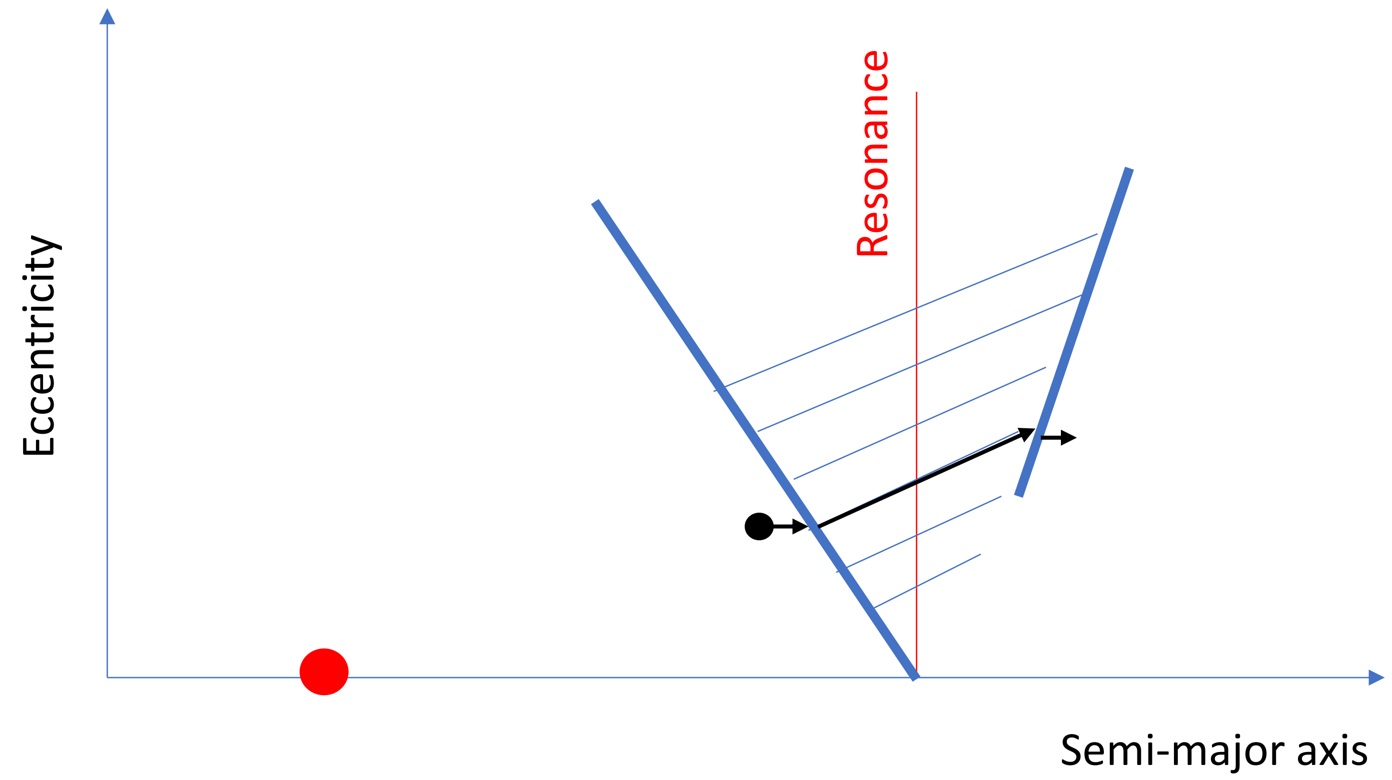
The adiabatic principle is valid only if migration is slow compared to resonant libration



Failed captures: divergent migration

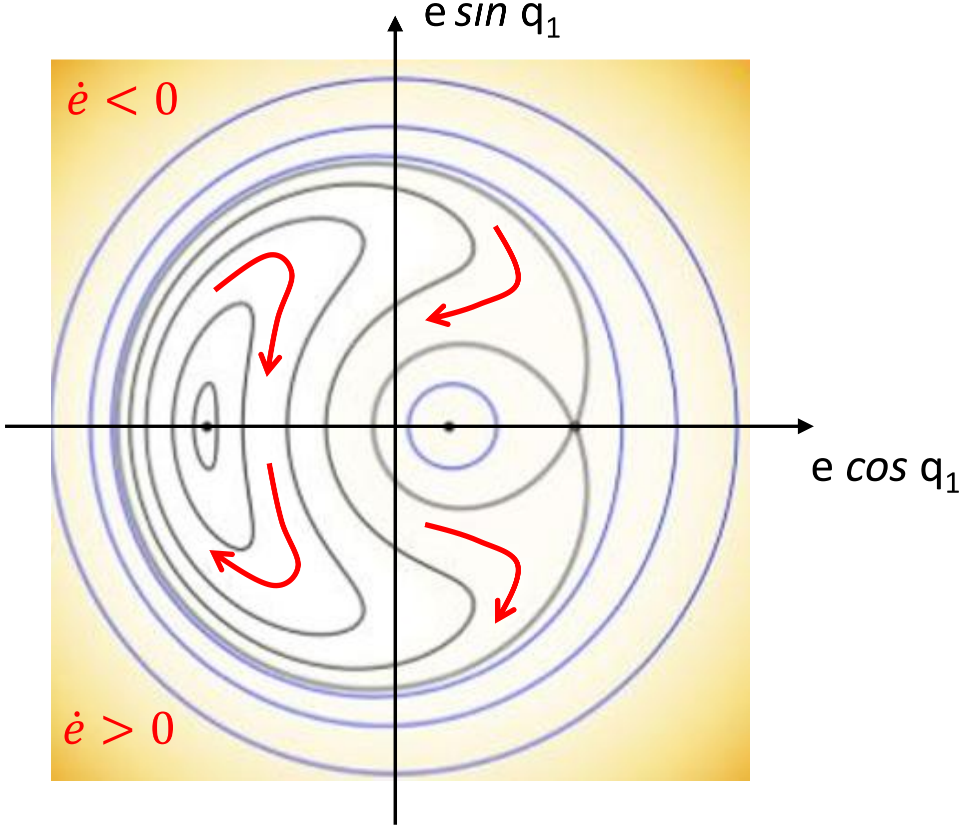
Planet (circulair)

Small body (eccentric)

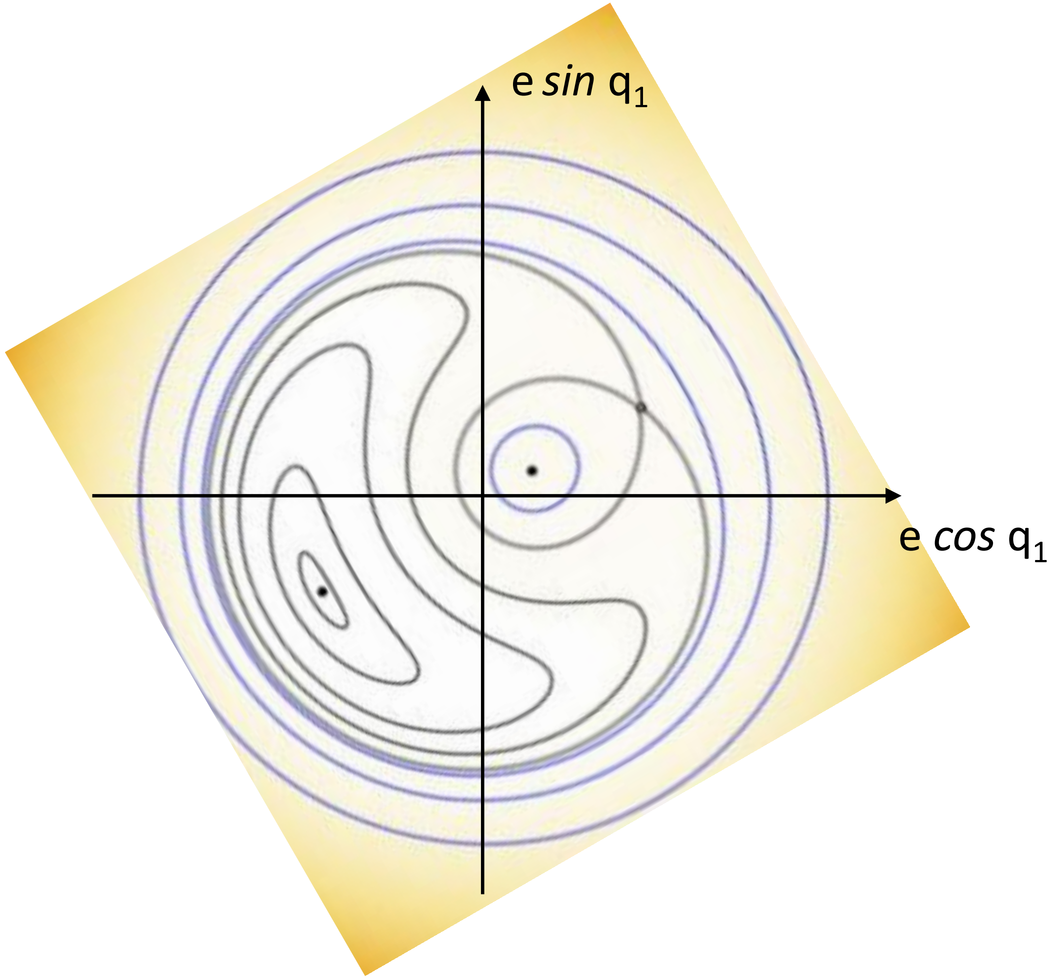


Failed captures: shifted equilibrium

$\dot{e} > 0$

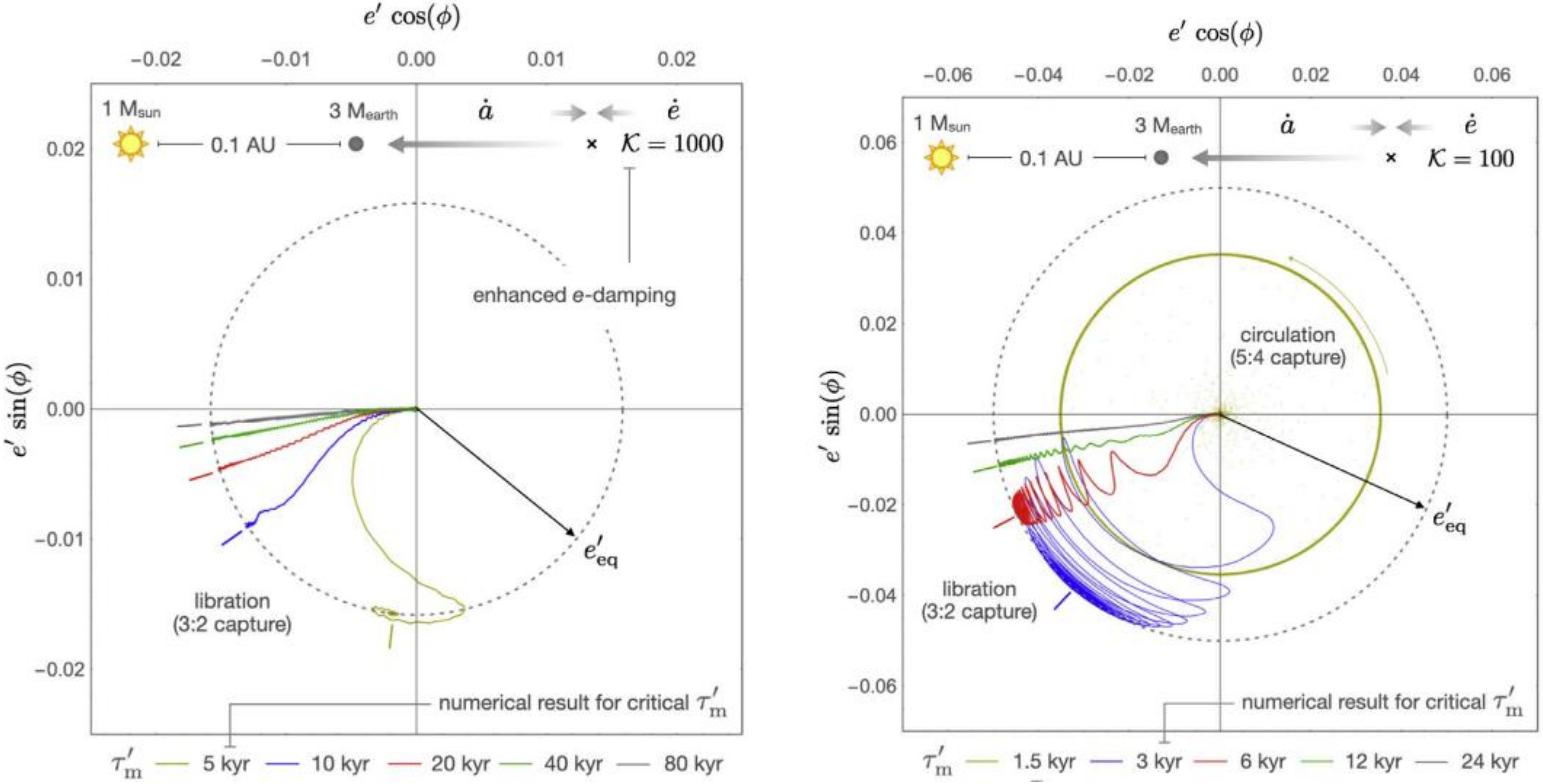


No e-damping



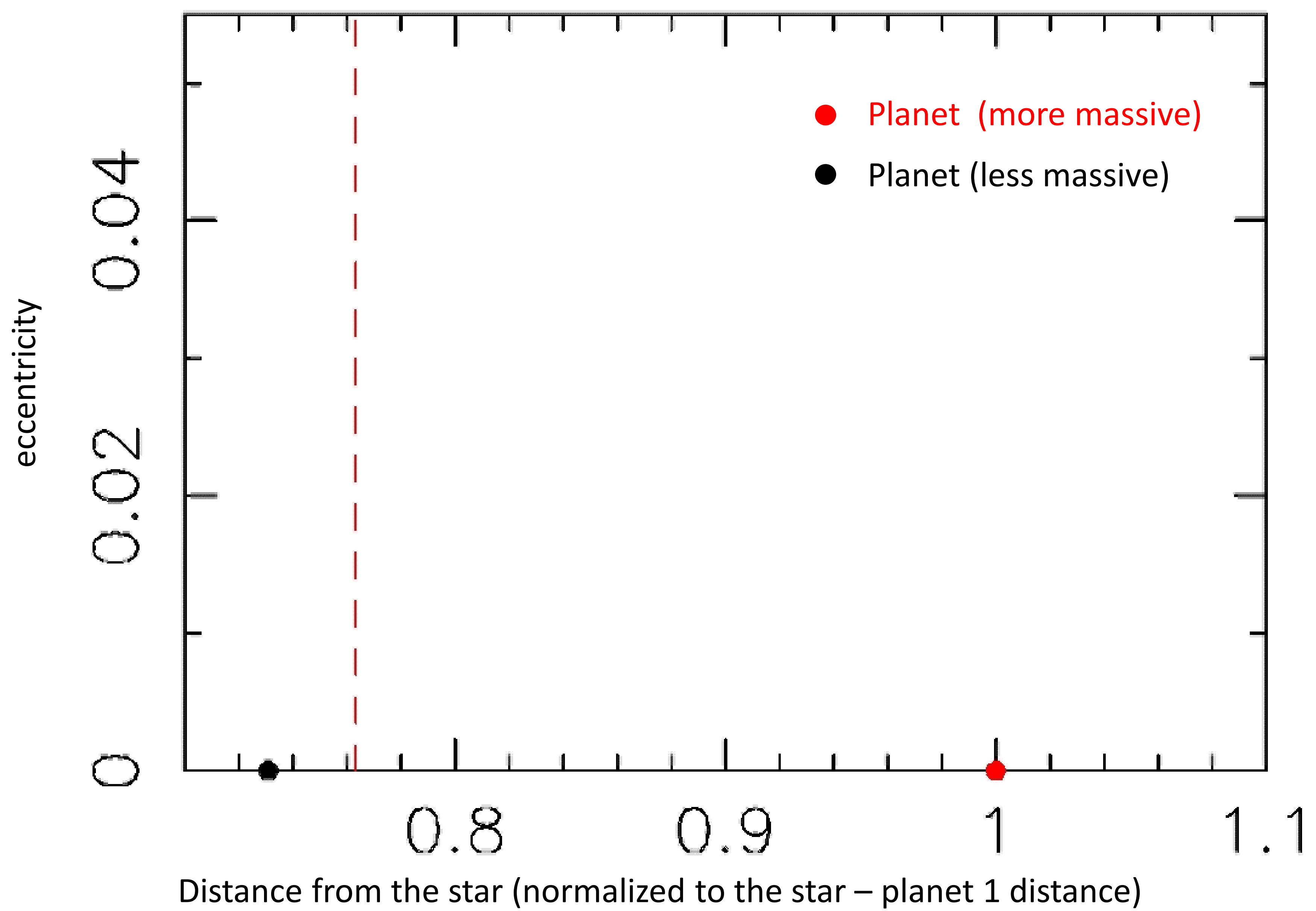
With e-damping

Failed captures: shifted equilibrium

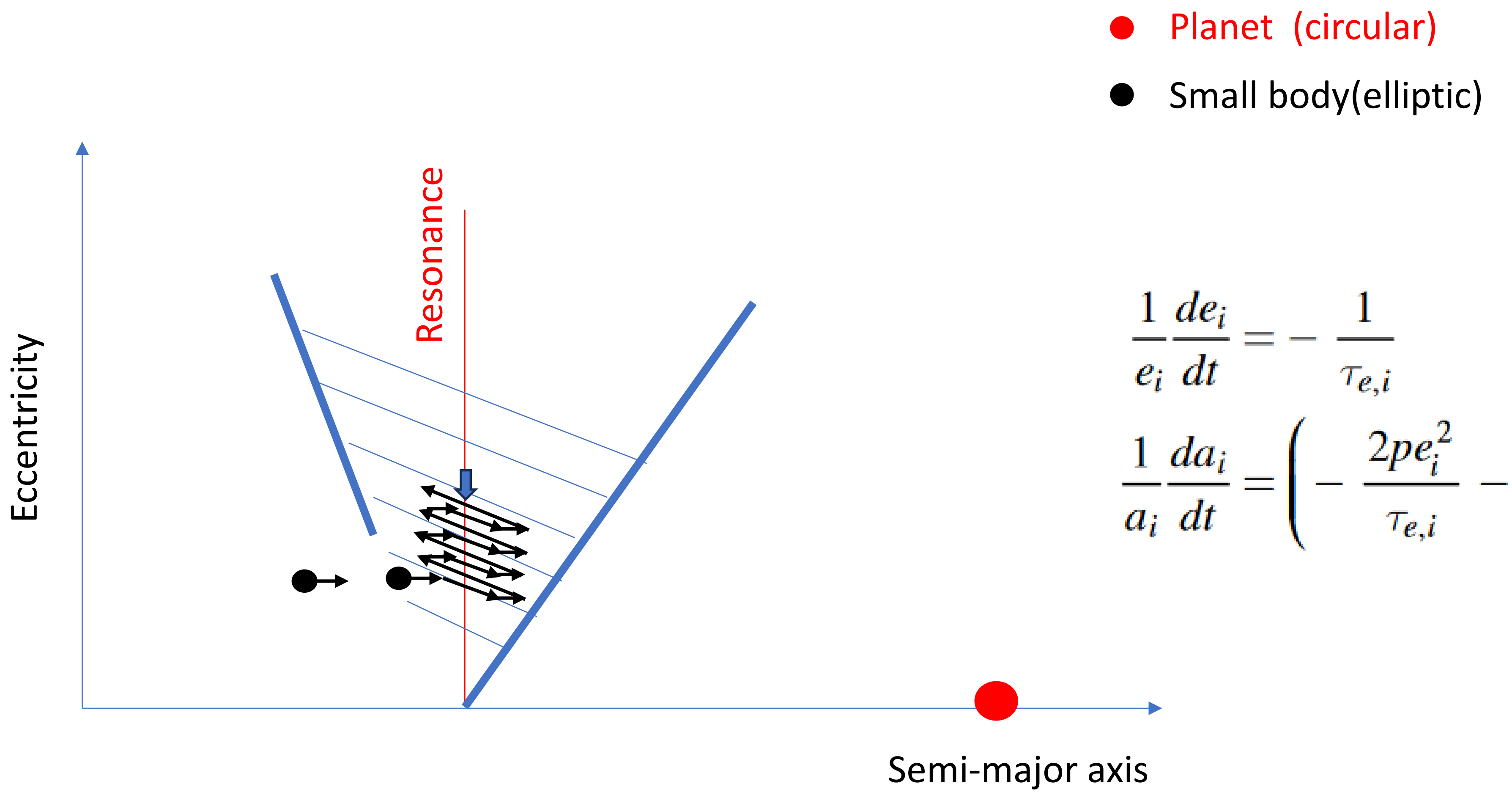


Batygin and Petit, 2023

Failed captures: overstability



Failed captures: overstability

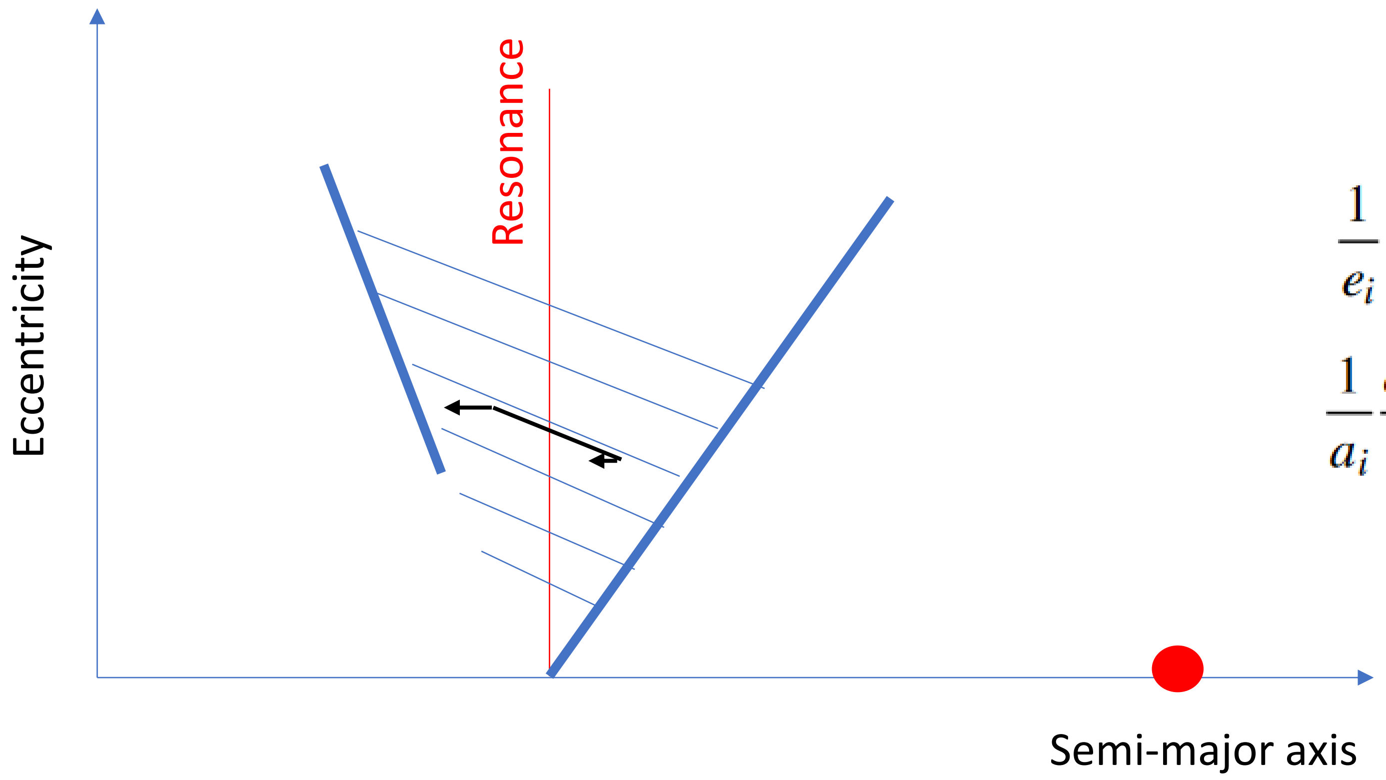


$$\frac{1}{e_i}\frac{de_i}{dt} = -\frac{1}{\tau_{e,i}}$$

$$\frac{1}{a_i}\frac{da_i}{dt} = \left(-\frac{2pe_i^2}{\tau_{e,i}} - \frac{1}{\tau_{a,i}}\right).$$

Failed captures: overstability

- Planet (circular)
- Small body(elliptic)



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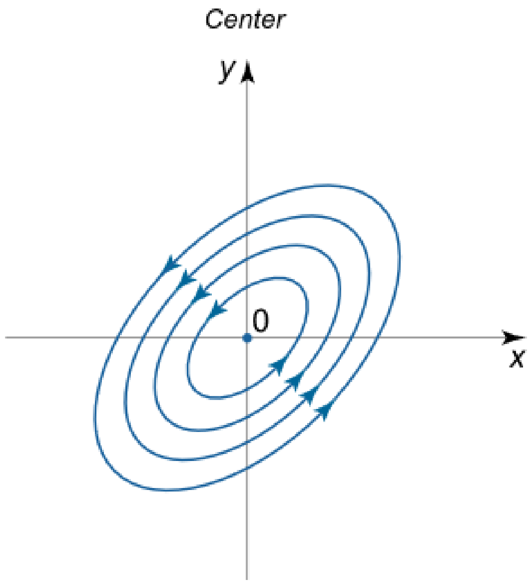
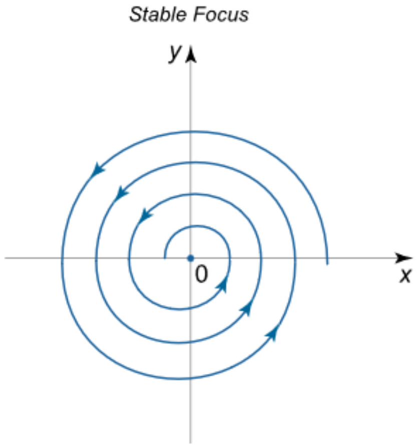
Goldreich and Schlichting, 2014
Deck and Batygin, 2015

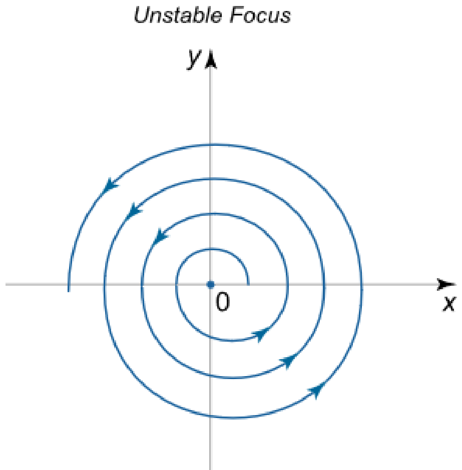
Overstability

Under the effect of damping, the equilibrium point of the dynamics is no longer a *centre*.
It becomes either a stable or an unstable *focus* :





The amplitude of libration decreases

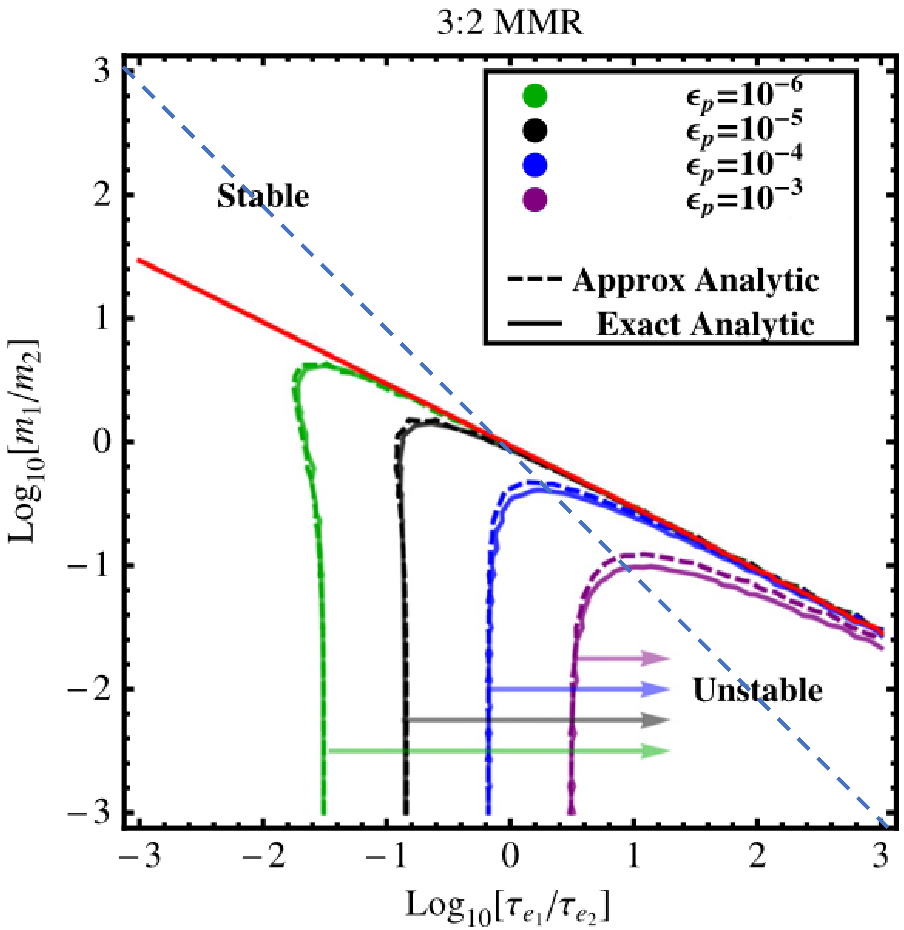


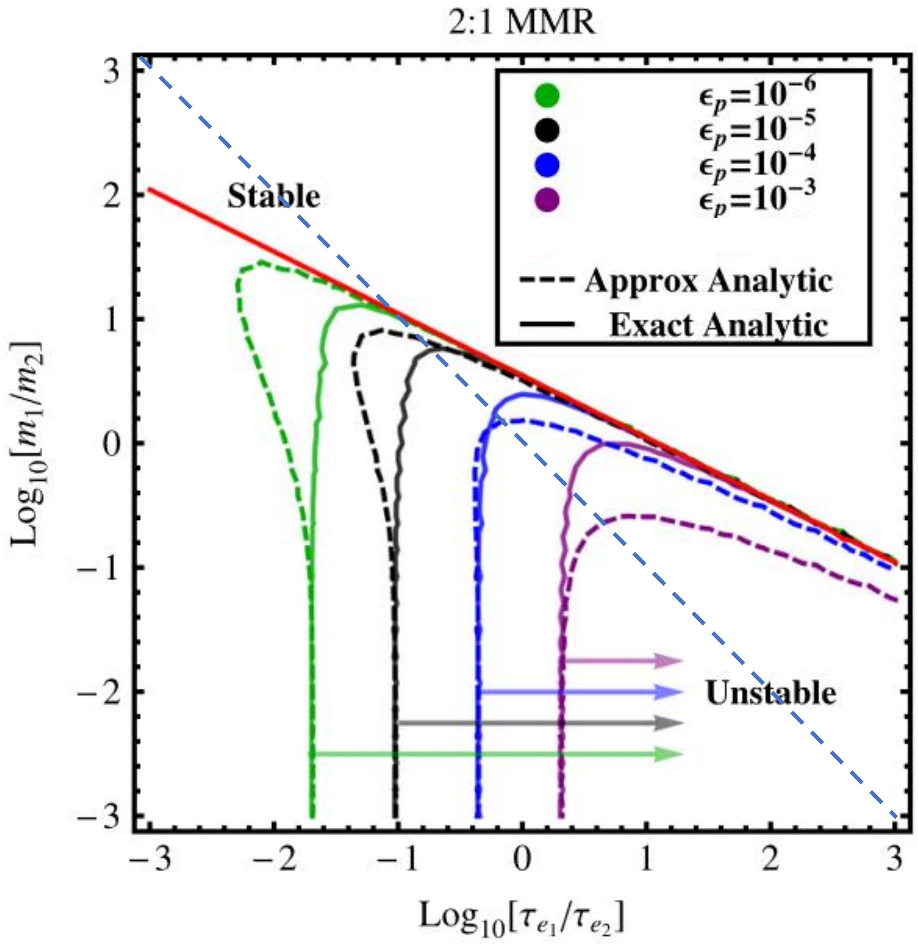
The amplitude of libration increases

Resonance Overstability (Goldreich and Schlichting, 2014)

Overstability

$\epsilon_p = (m_1 + m_2)/M_\star$

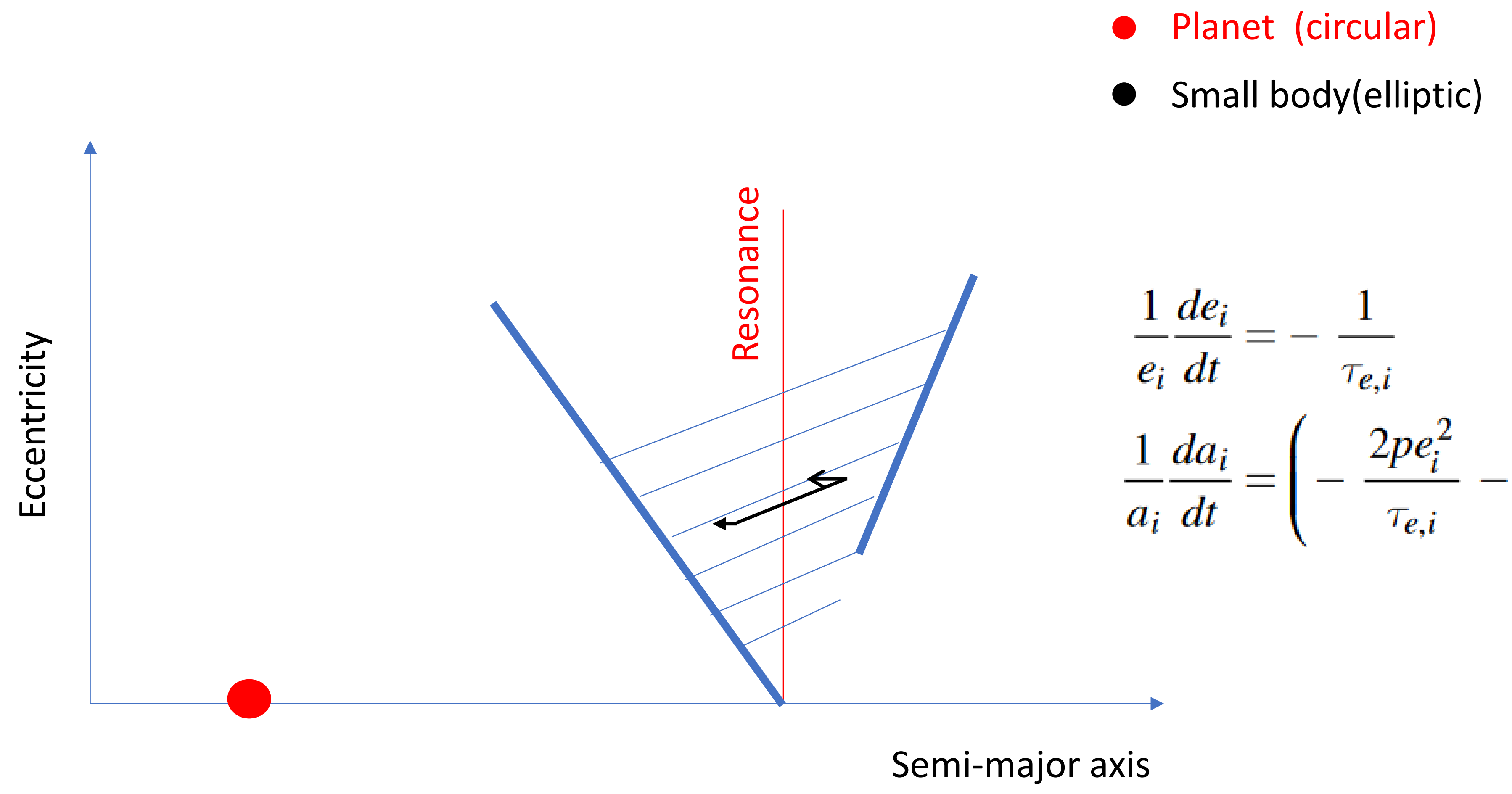




$[-1/e \, de/dt]^{-1} \equiv \tau_e \sim \frac{M_*}{m} \frac{M_*}{\Sigma a^2} \frac{h^4}{\sqrt{\mathcal{G}M_*/a^3}}$

Deck and Batygin, 2015, ApJ, 810, 119

No overstability in outer resonances (inner planet more massive)

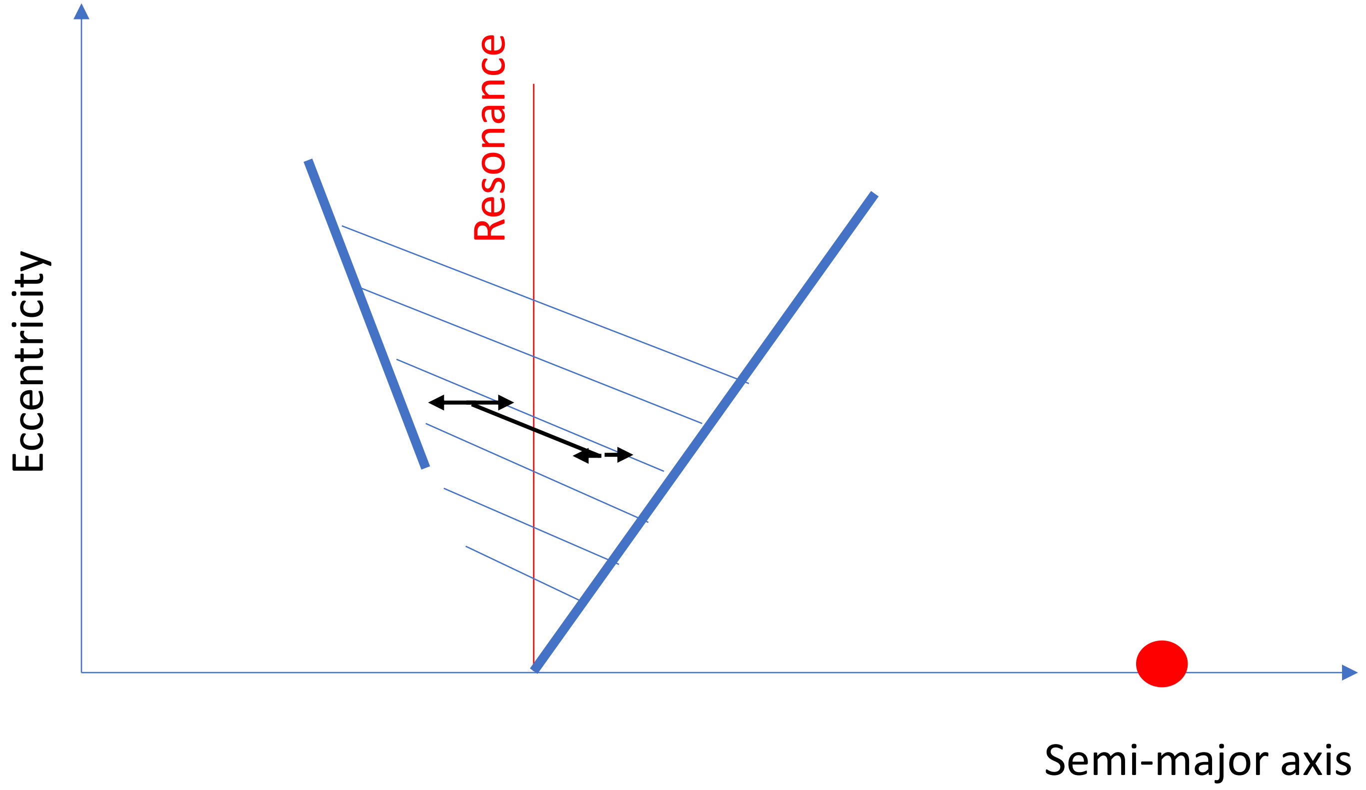


$$\frac{1}{e_i}\frac{de_i}{dt} = -\frac{1}{\tau_{e,i}}$$

$$\frac{1}{a_i}\frac{da_i}{dt} = \left(-\frac{2pe_i^2}{\tau_{e,i}} - \frac{1}{\tau_{a,i}}\right).$$

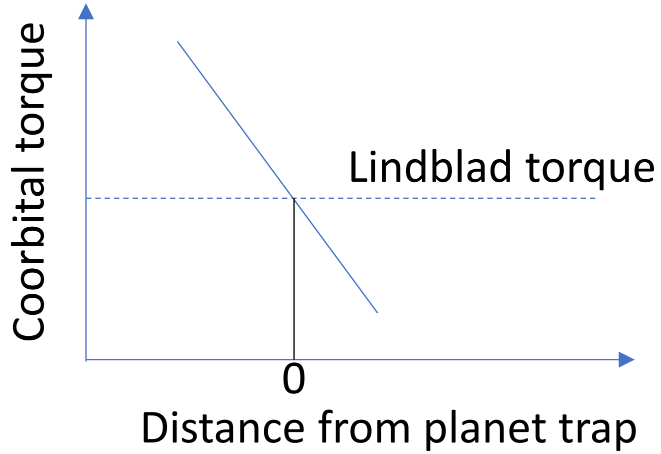
No overstability at a (steep enough) planet trap

- Planet (circular)
- Small body(elliptic)

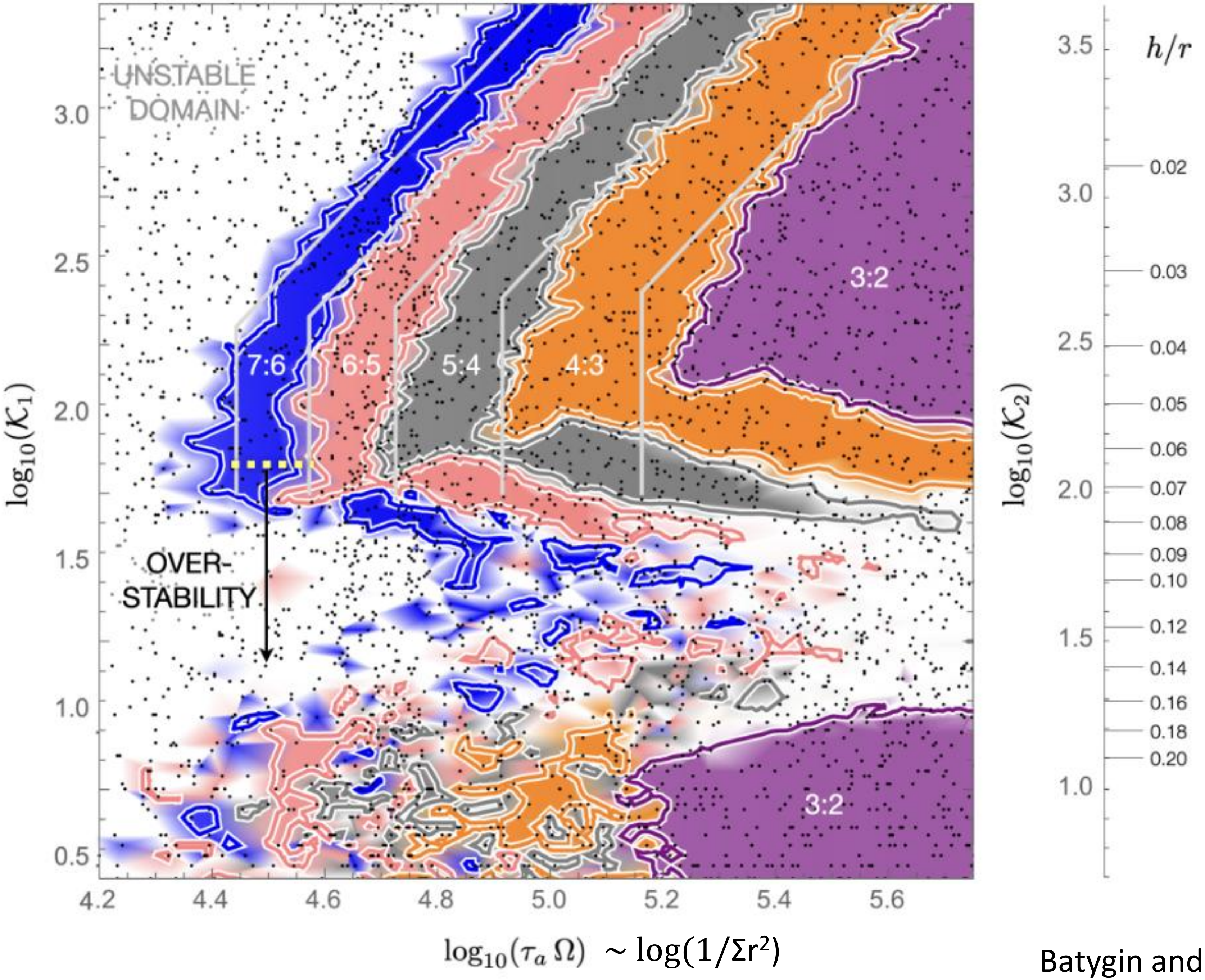


$$\frac{1}{e_i}\frac{de_i}{dt} = -\frac{1}{\tau_{e,i}}$$

$$\frac{1}{a_i}\frac{da_i}{dt} = \frac{1}{\tau_a}\left[1+\beta\frac{\delta n}{[n]}\right] - \frac{2e^2}{\tau_e}$$



Batygin et al., submitted



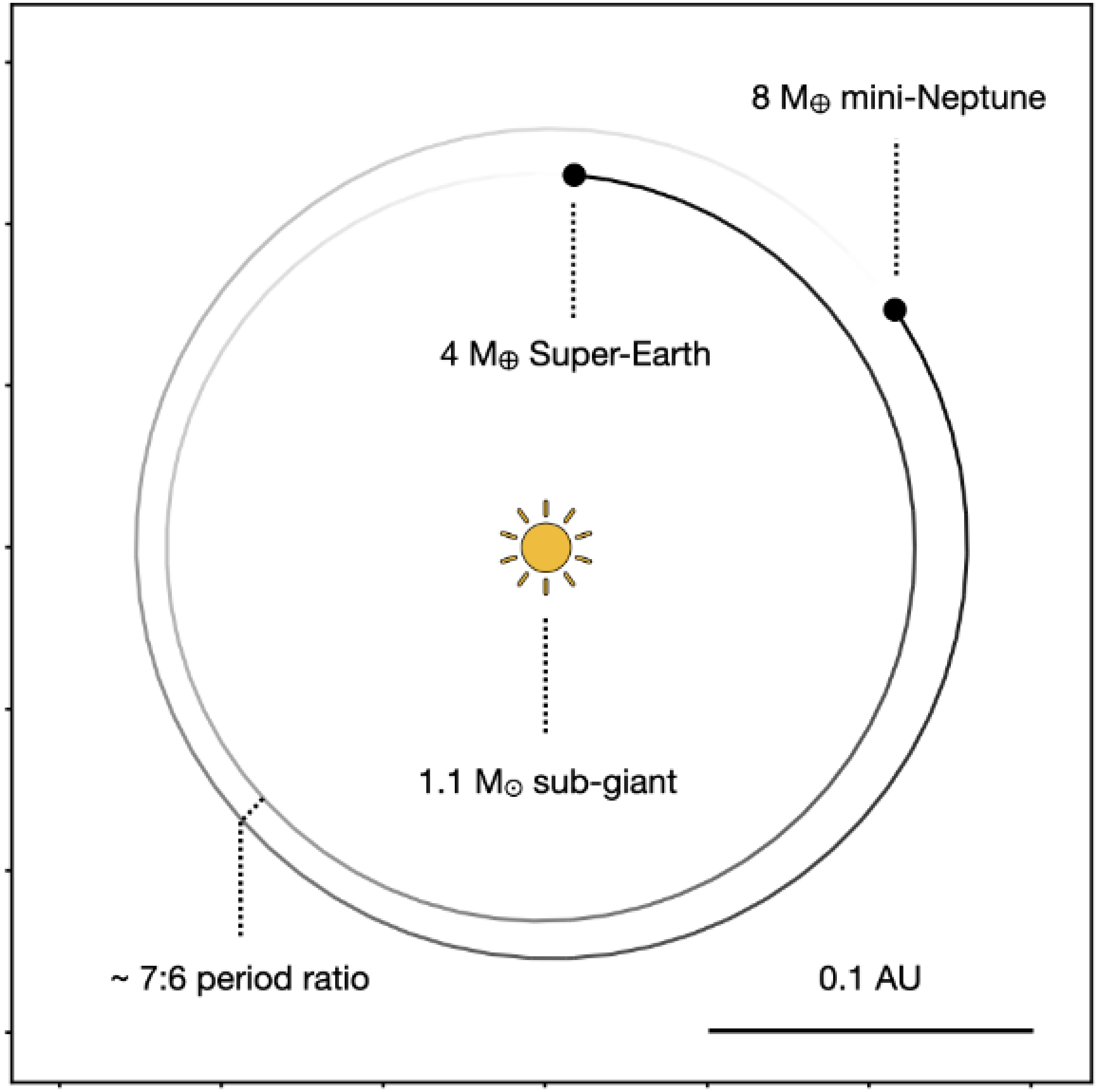
$\sim \log(1/\Sigma r^2)$

4 and 8 Earth mass planets

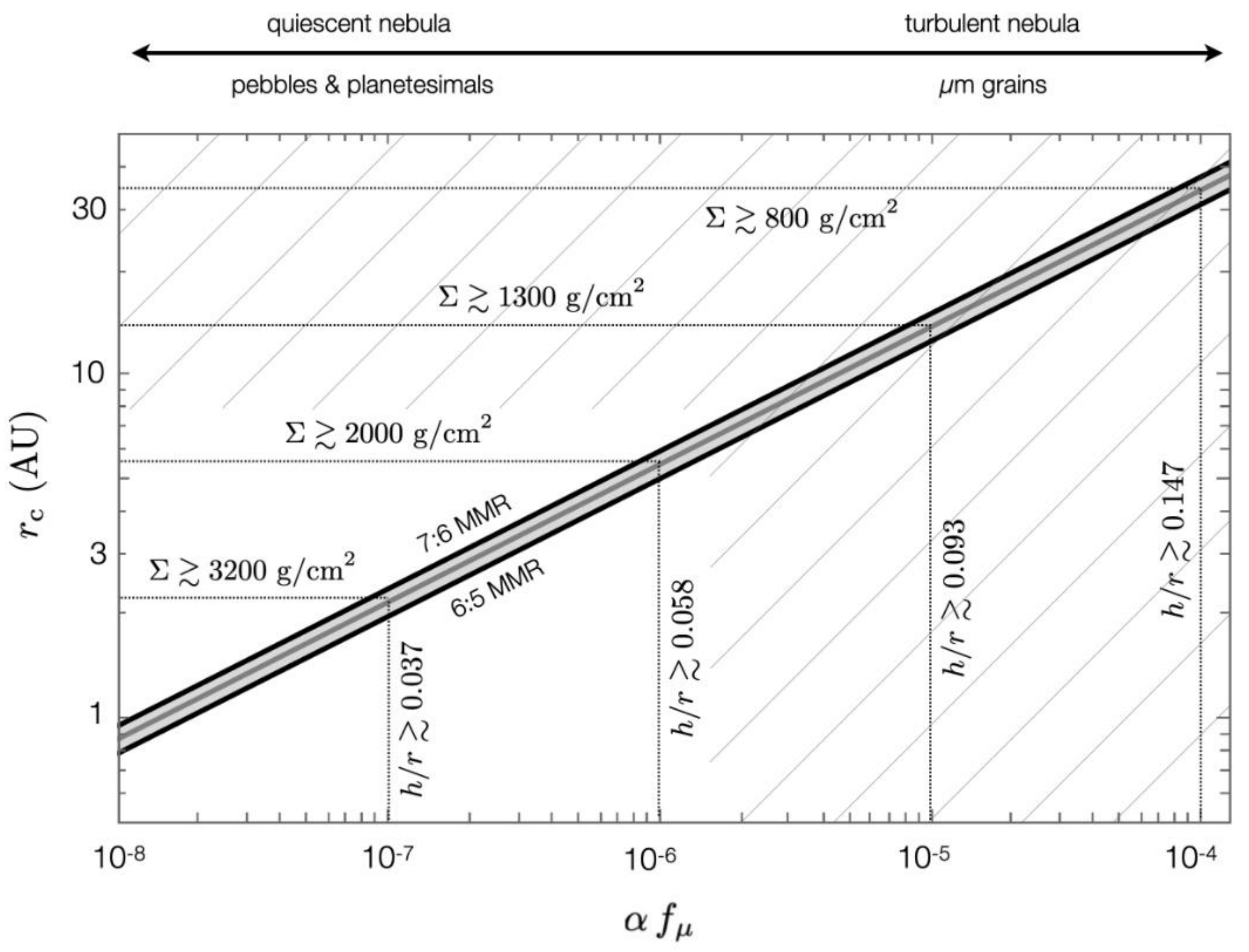
$$\mathcal{K}_j = \frac{\tau_a}{\tau_{e_j}}$$

Batygin and Morbidelli, 2026

Kepler 36: pair of planets in 7:6 resonance

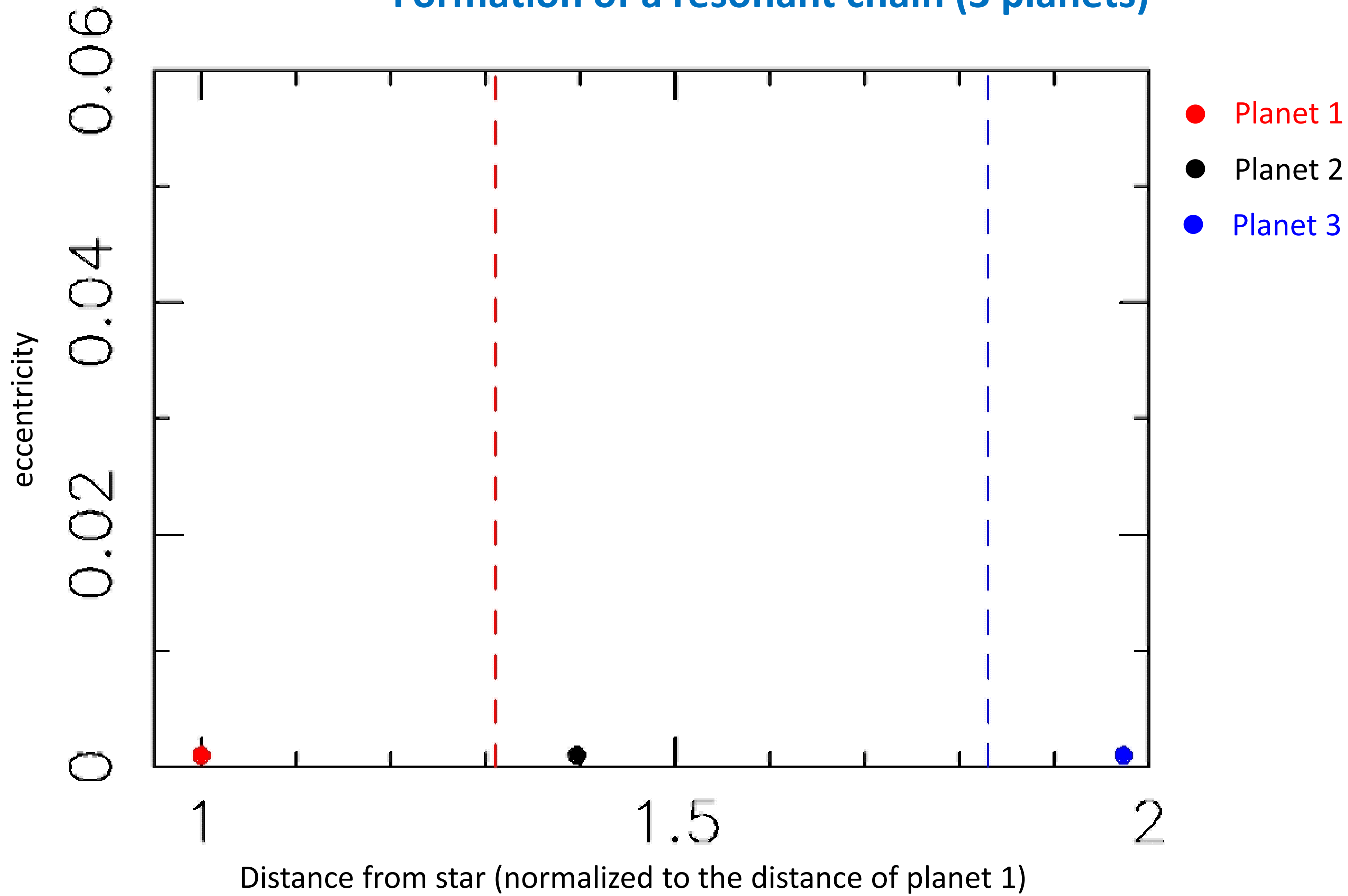


Kepler 36: pair of planets in 7:6 resonance

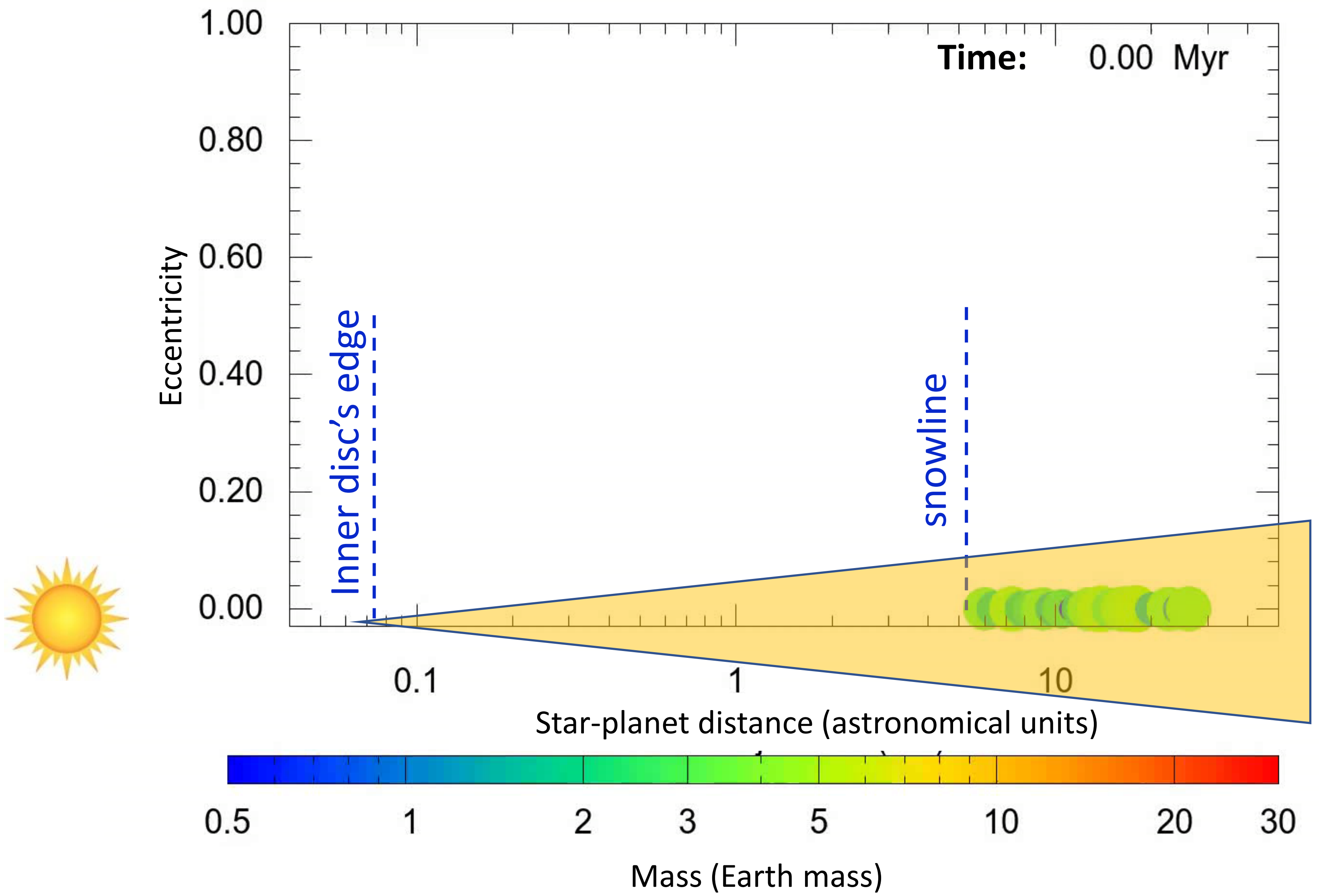


Batygin and Morbidelli, 2026

Formation of a resonant chain (3 planets)



Example of migration of a system of super-Earths



Animation de A. Izidoro (Rice U.)

Some resonant chains ARE observed…. But just a few

Table 1. Basic properties of the five well-characterized resonant chains with sub-Jovian planets.

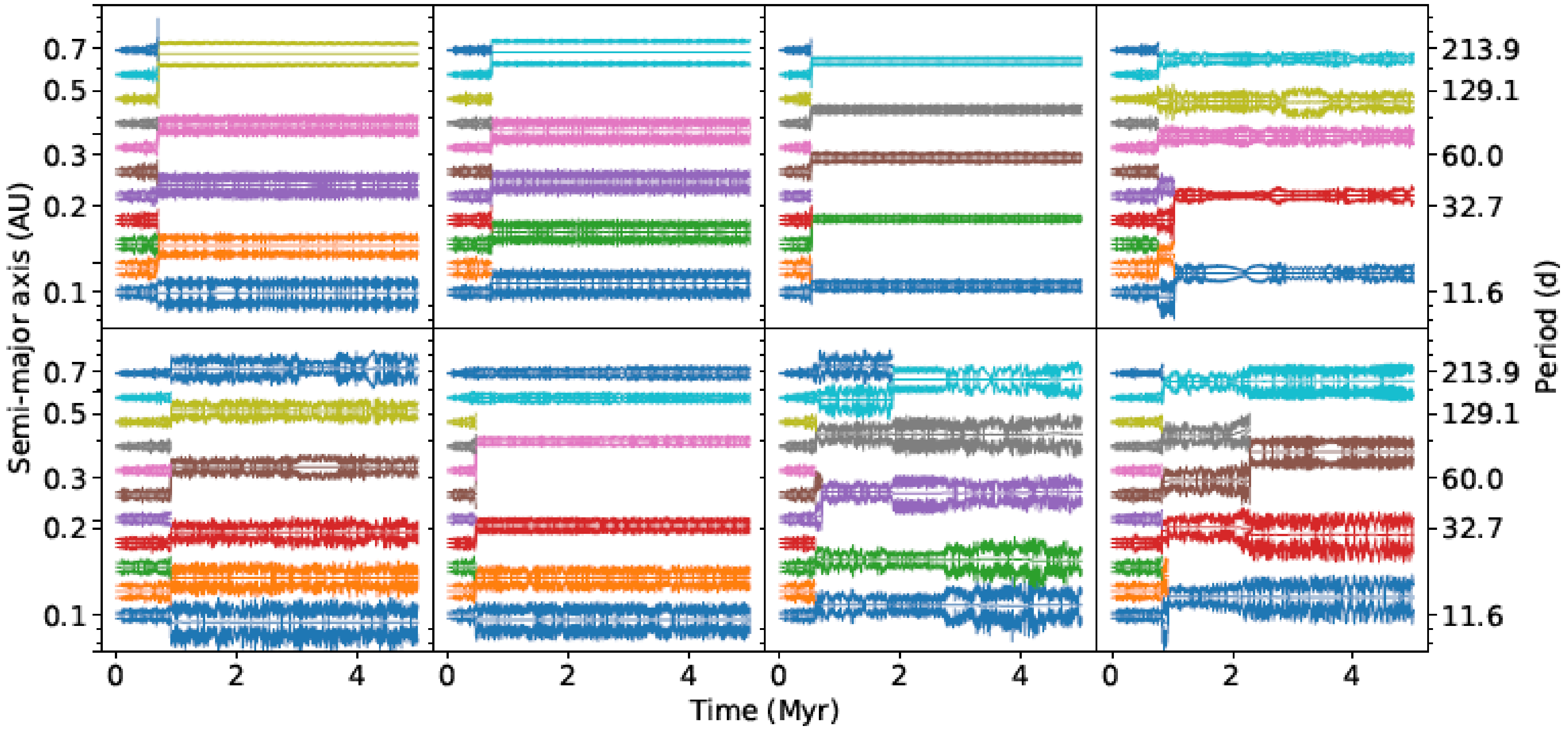
System	# planets	$\overline{m}/M_\star (M_\oplus/M_\odot)$	$\sigma_m/\overline{m}$	Resonances present	Source of mass measurements
Kepler-60	3	3.91	0.18	4:3, 5:4	Jontof-Hutter et al. (2016)
TRAPPIST-1	7	11.51	0.46	5:3, 8:5, 3:2, 4:3	Agol et al. (2020)
Kepler-223	4	5.63	0.31	3:2, 4:3	Mills et al. (2016)
Kepler-80	6[ab]	8.43	0.23	3:2, 4:3	MacDonald et al. (2016)
TOI-178	6	6.41	0.53	2:1, 5:3, 3:2	Leleu et al. (2021)
K2-138	5[b]	7.06	0.62	3:2	Christiansen et al. (2018)

[a] Only 4 planets in Kepler-80 have measured masses

[b] Kepler-80 f and K2-138 g have been excluded because they are decoupled from the resonant dynamics

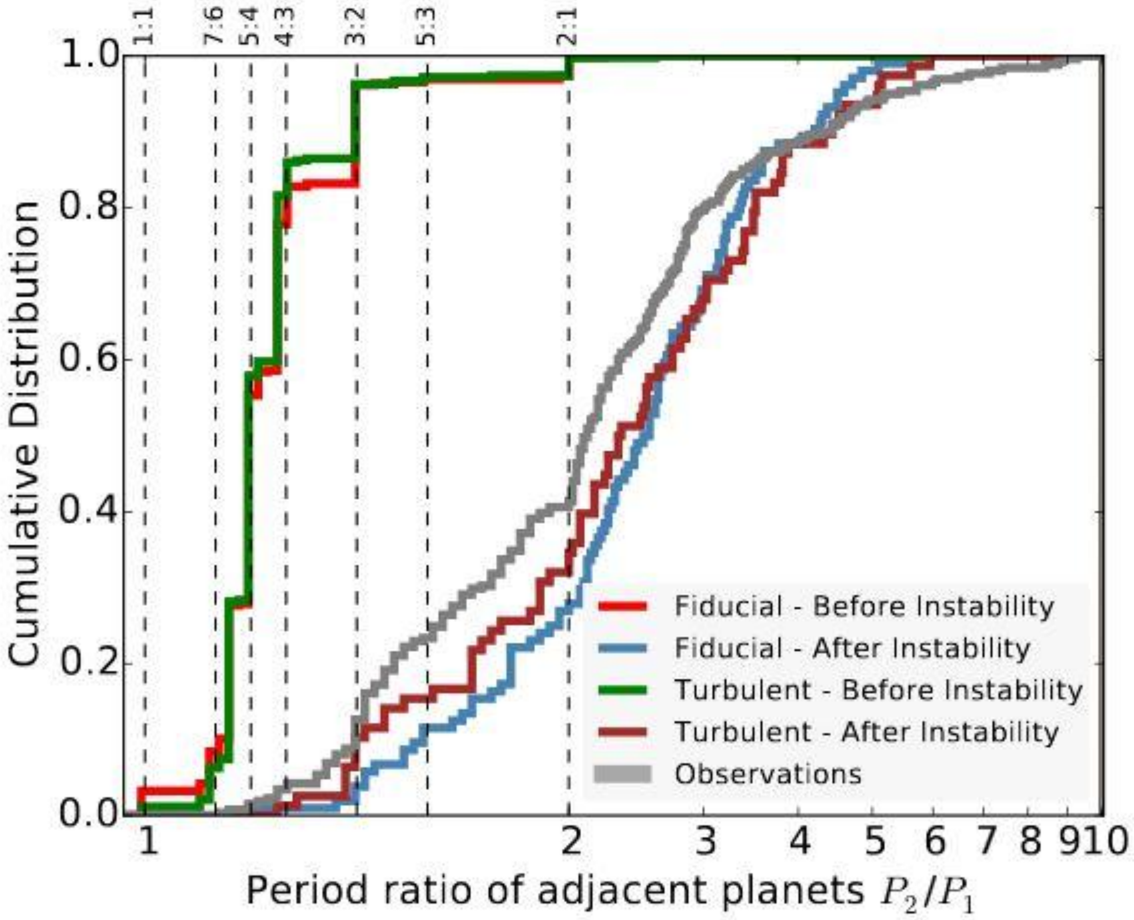
From Goldberg and Batygin, 2021

Many resonant chains become unstable after disc (damping) removal



From Goldberg and Batygin, 2021

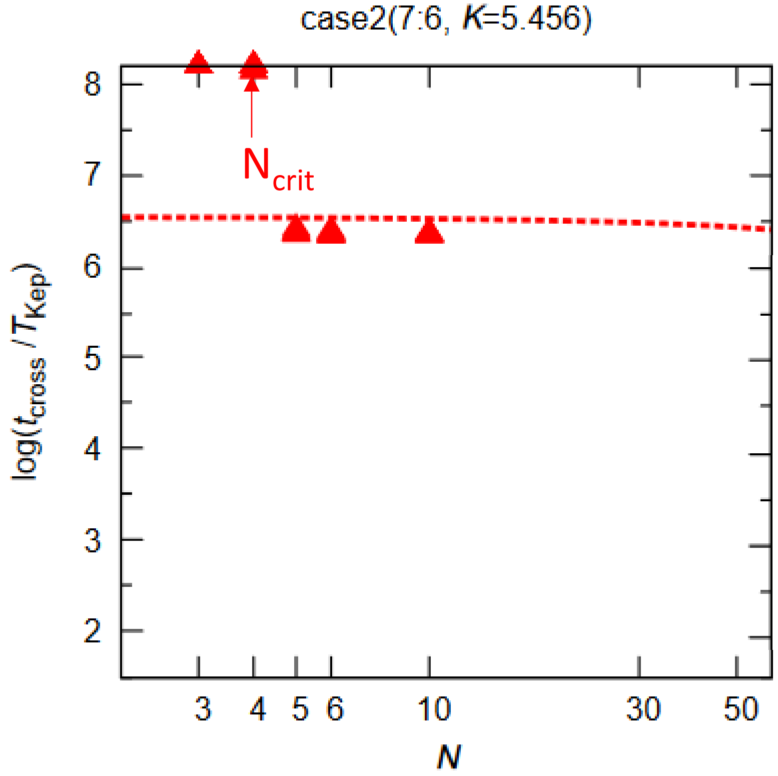
The instability breaks the resonant chains

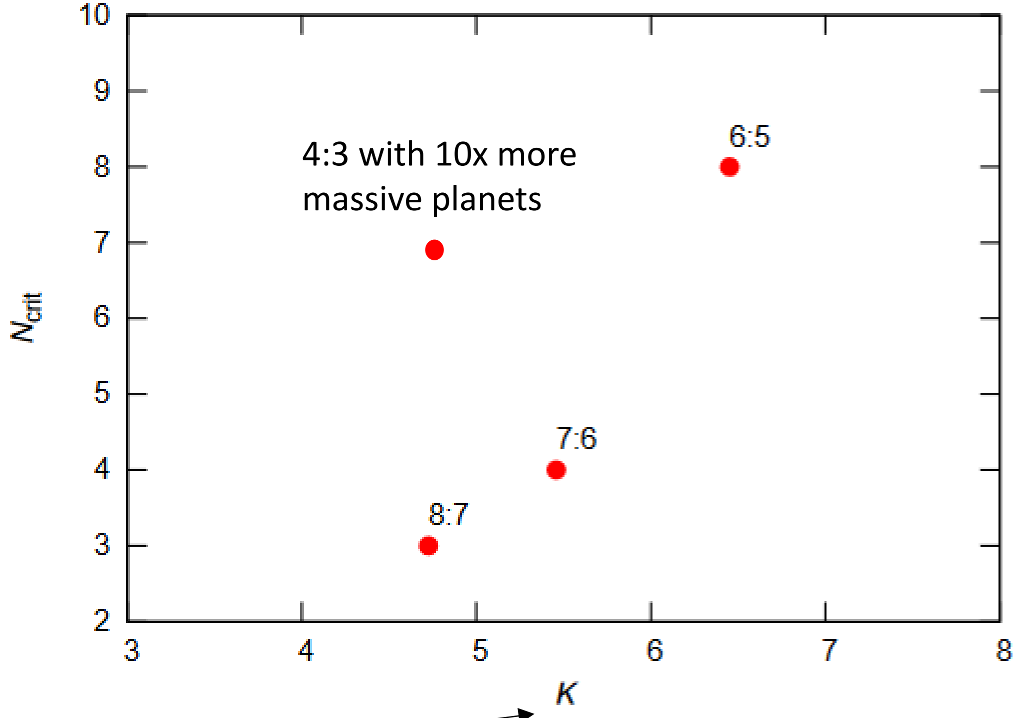


Izidoro et al., 2017

Empirical criterion for instability of resonant chains

Matsumoto et al., 2012

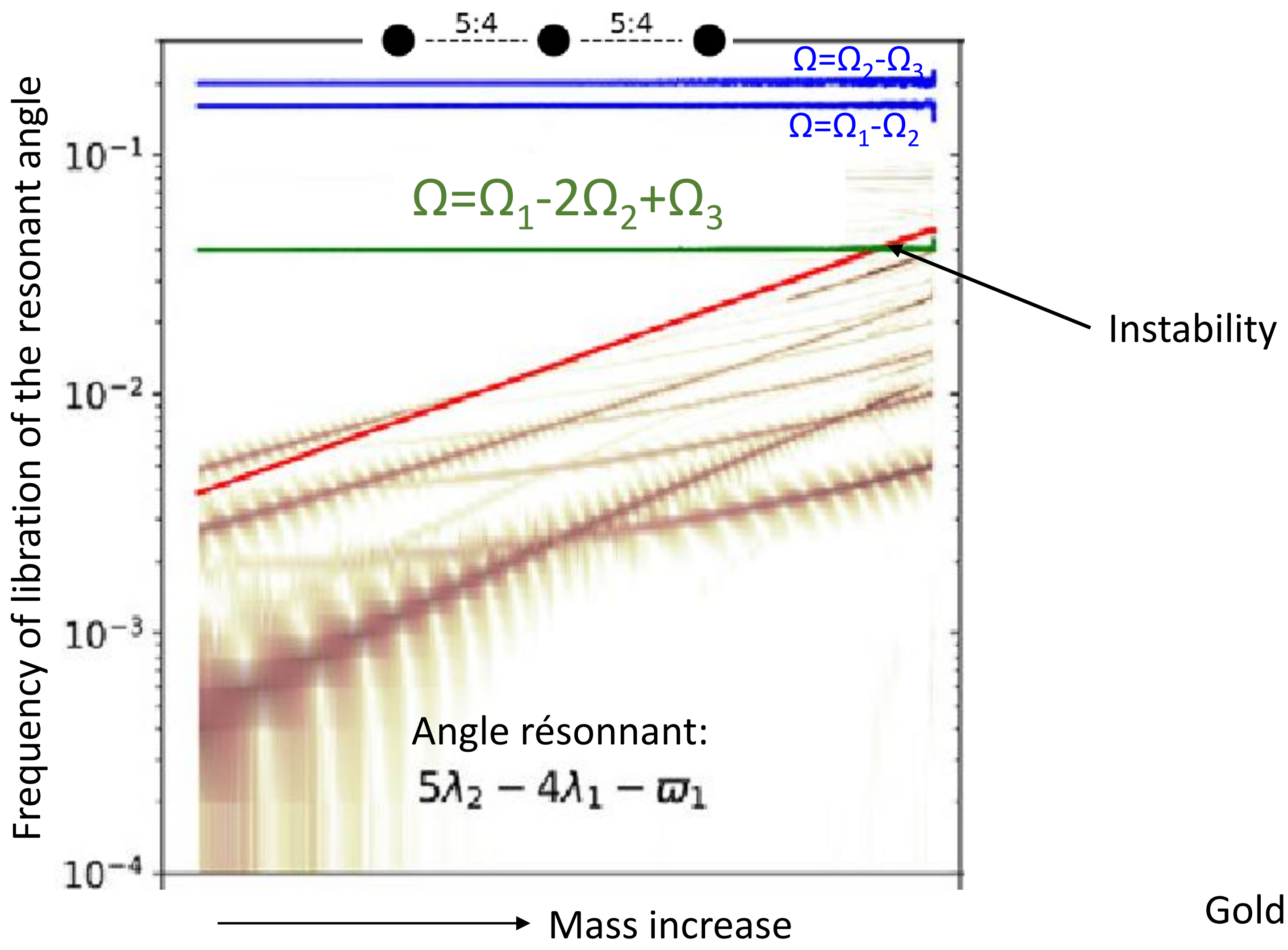




Separation in Hill radii

Instability of resonant chains

In Pichierri and Morbidelli, 2020 and Goldberg et al., 2022, we increase the masses of the planets in a given resonant chain until an instability occcurs.



This happens when a libration frequency becomes resonant with the difference of synodic periods $\Omega=\Omega_1-2\Omega_2+\Omega_3$

Goldberg et al., 2022

Instability of resonant chains

the lowest synodic frequency that appears in the $\mathcal{O}(m^2)$ term is

$$\frac{1}{k}\left(\frac{k-1}{k}\right)^{N-3}\delta\dot{\lambda}_{1,2} \simeq \frac{1}{k^2}\left(\frac{k-1}{k}\right)^{N-3} n_1 .$$

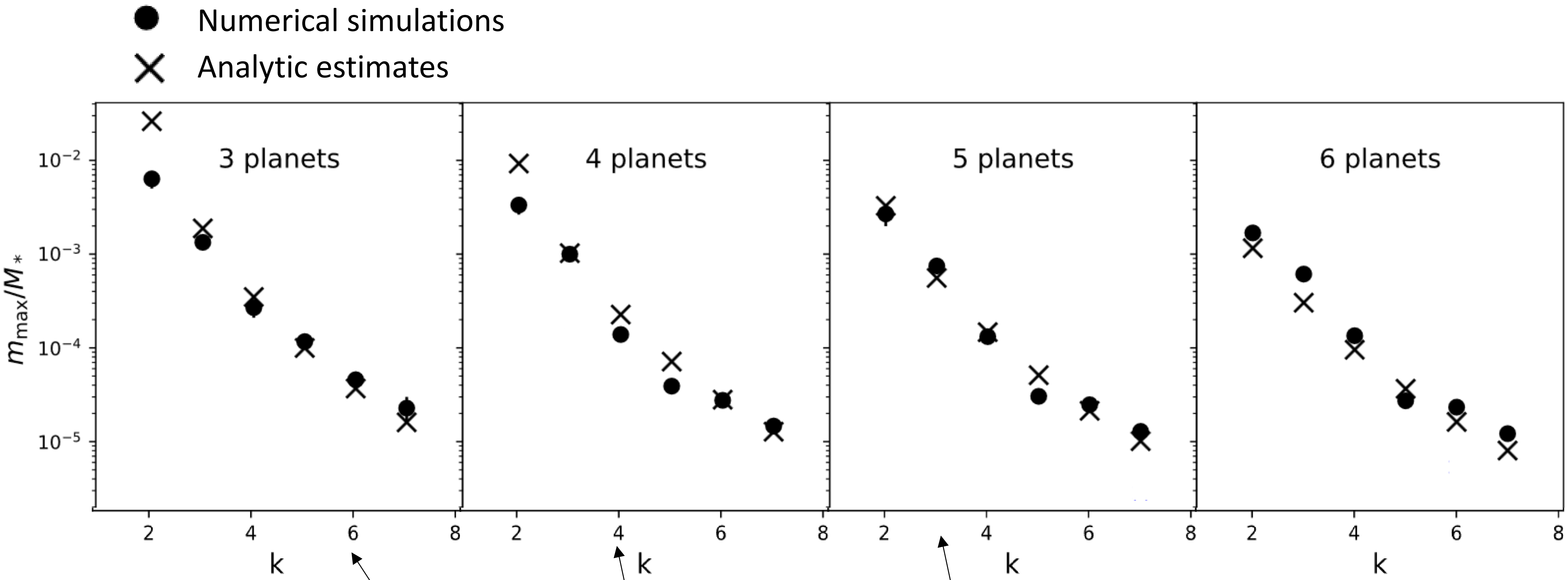
decreases with increasing N and k

$\omega \sim m_{\rm pl}^{1/2}$ or $m_{\rm pl}^{2/3}$ (depending on *e*)

the regime of secondary resonances between synodic and resonant degrees of freedom is encountered at lower masses for increasing k and/or increasing N, and therefore the critical mass $(m_{\rm pl}/M_*)_{\rm crit}$ *allowed for stability decreases with increasing N and k.*

Pichierri and Morbidelli, 2020

Instability of resonant chains



k: index of the resonant chain. Examples :

6:5 – 6:5

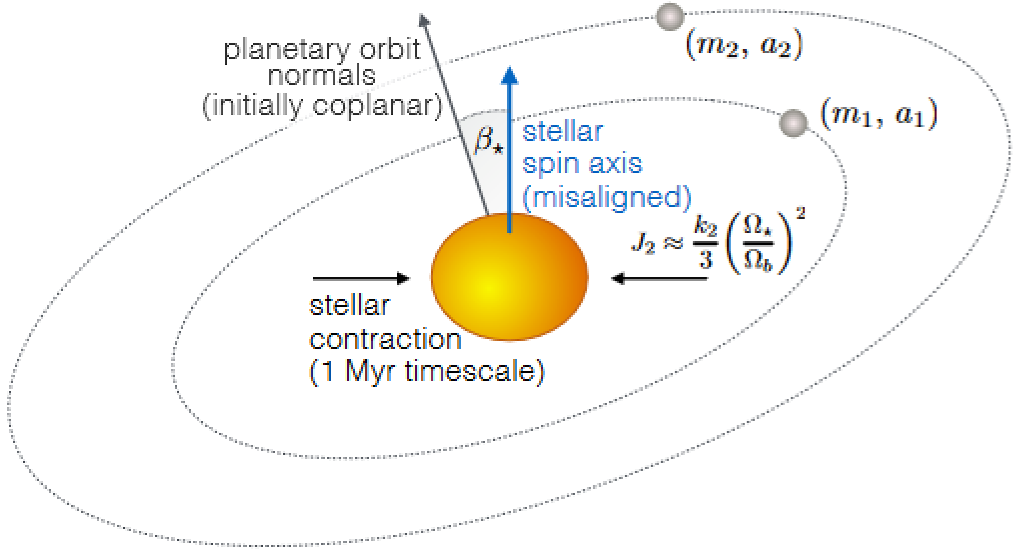
4:3 – 4:3 – 4:3

3:2 – 3:2 – 3:2 – 3:2

Goldberg et al., 2022

Disruption of resonant chains by external perturbations

- Planetesimal scattering (Chatterjee and Ford, 2015)
- Outer migration of the inner edge of the disk during photoevaporation (Liu et al., 2017)
- Tidal migration of the innermost planet (particularly if outward)
- Planet mass loss (Matsumoto and Ogihara, 2020)
- Stellar obliquity (Spalding and Batygin, 2018)



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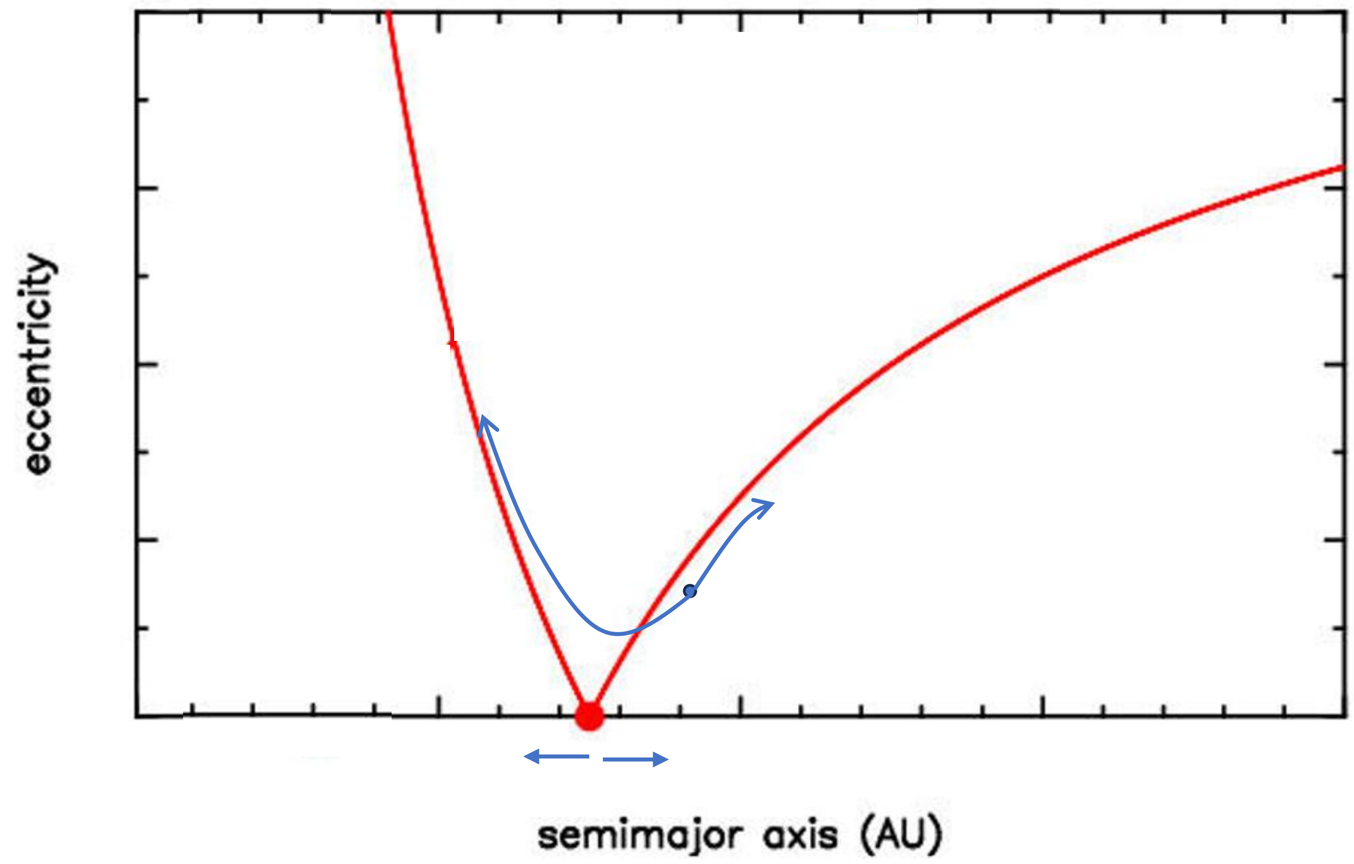
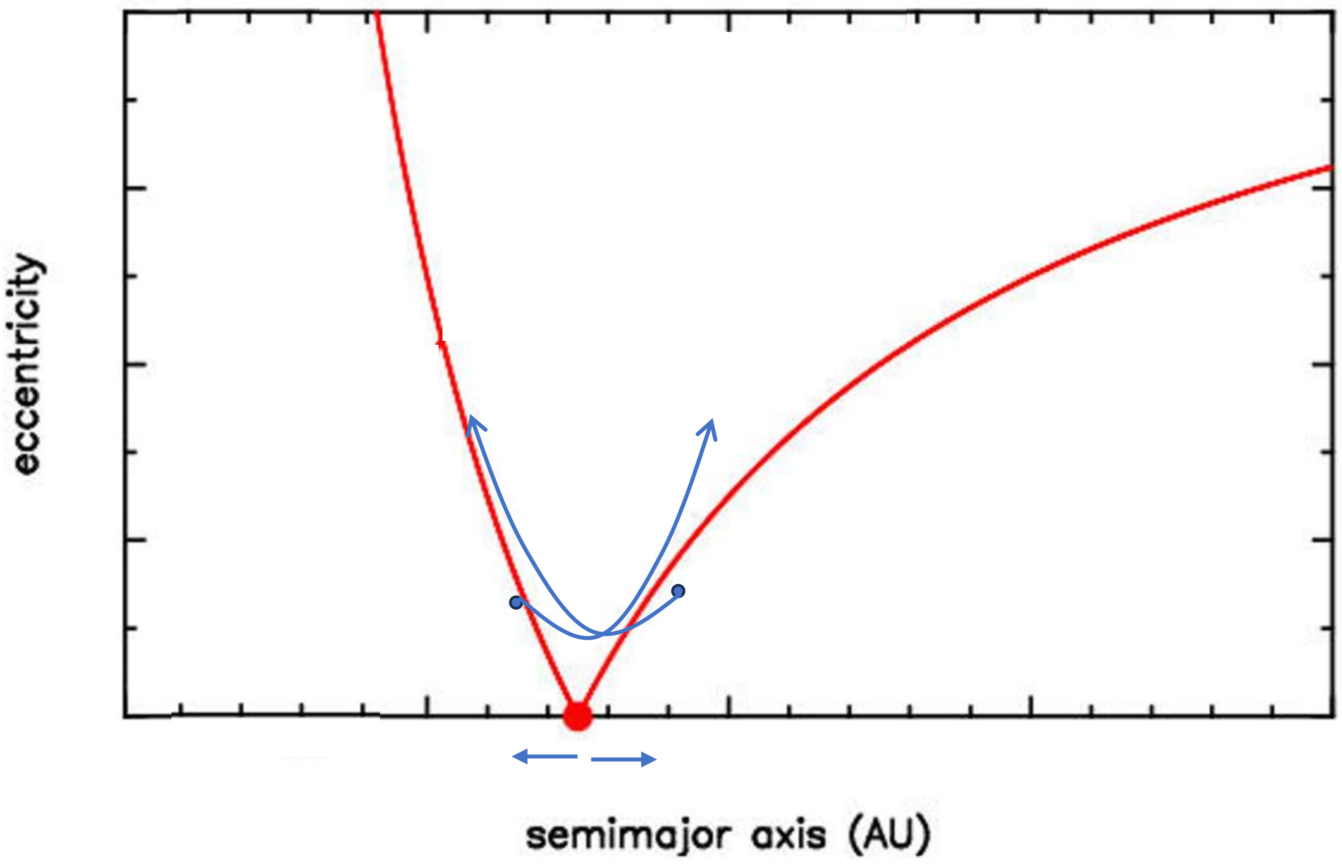
Planetesimal-driven migration

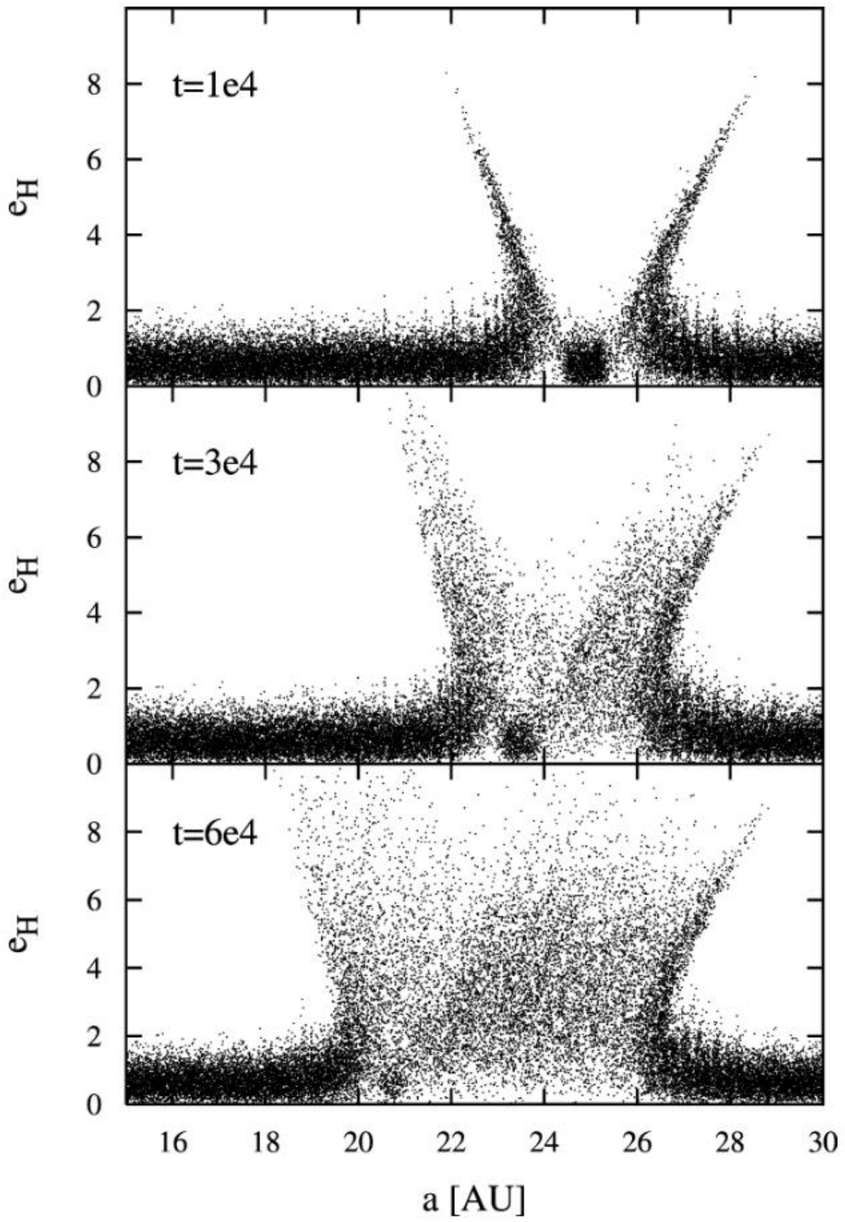


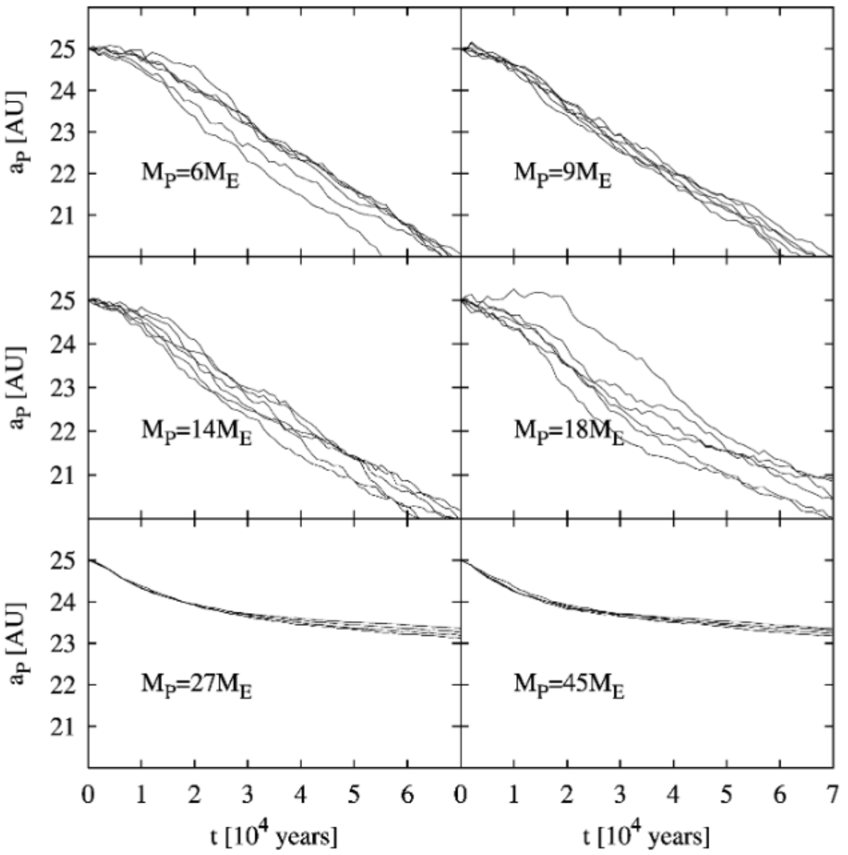
Planetesimal-driven migration



The particles that are scattered inside-out win, because their synodic period with respect to the planet is shorter (Kirsh et al. 2009)

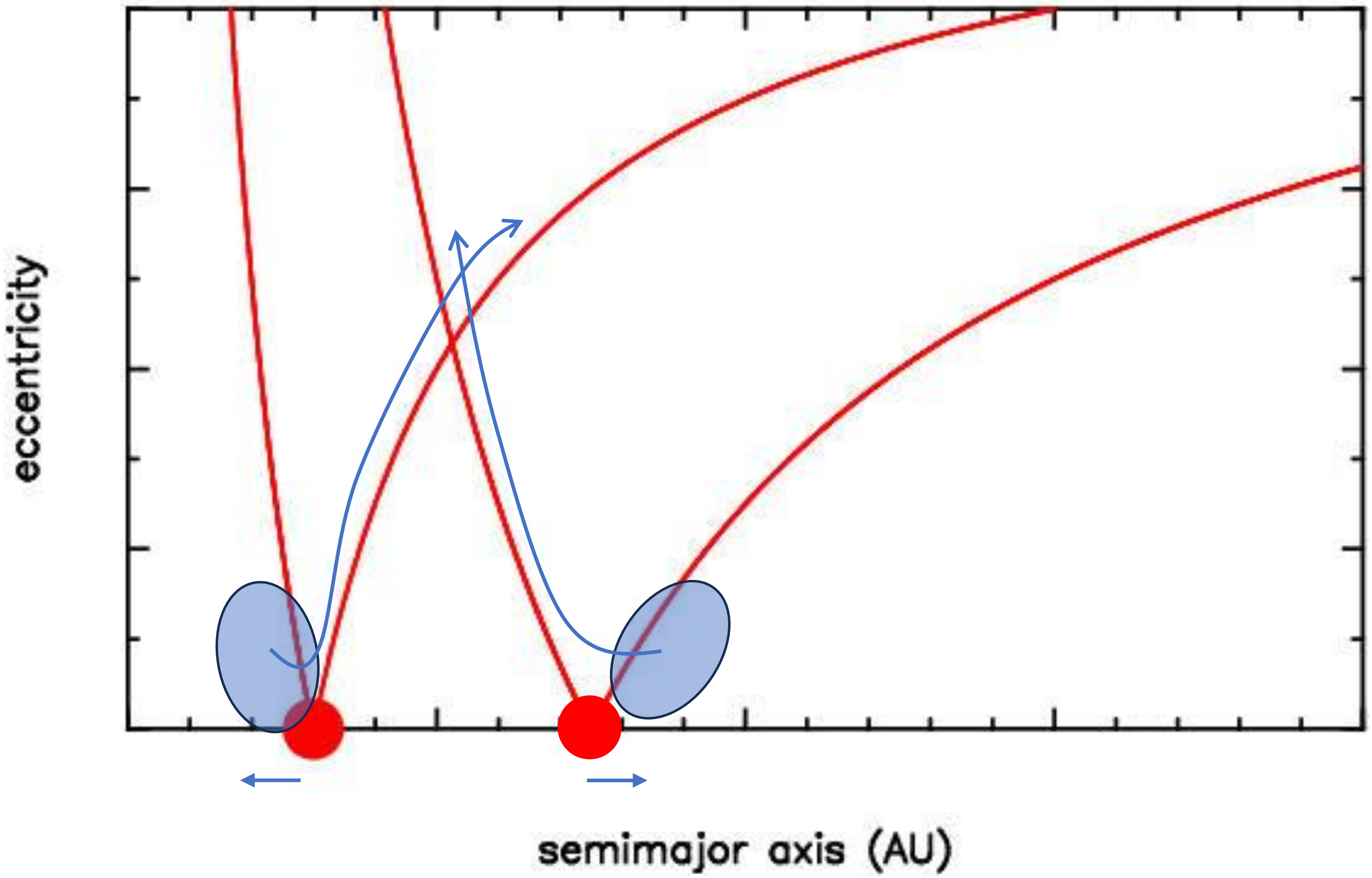
Planetesimal-driven migration



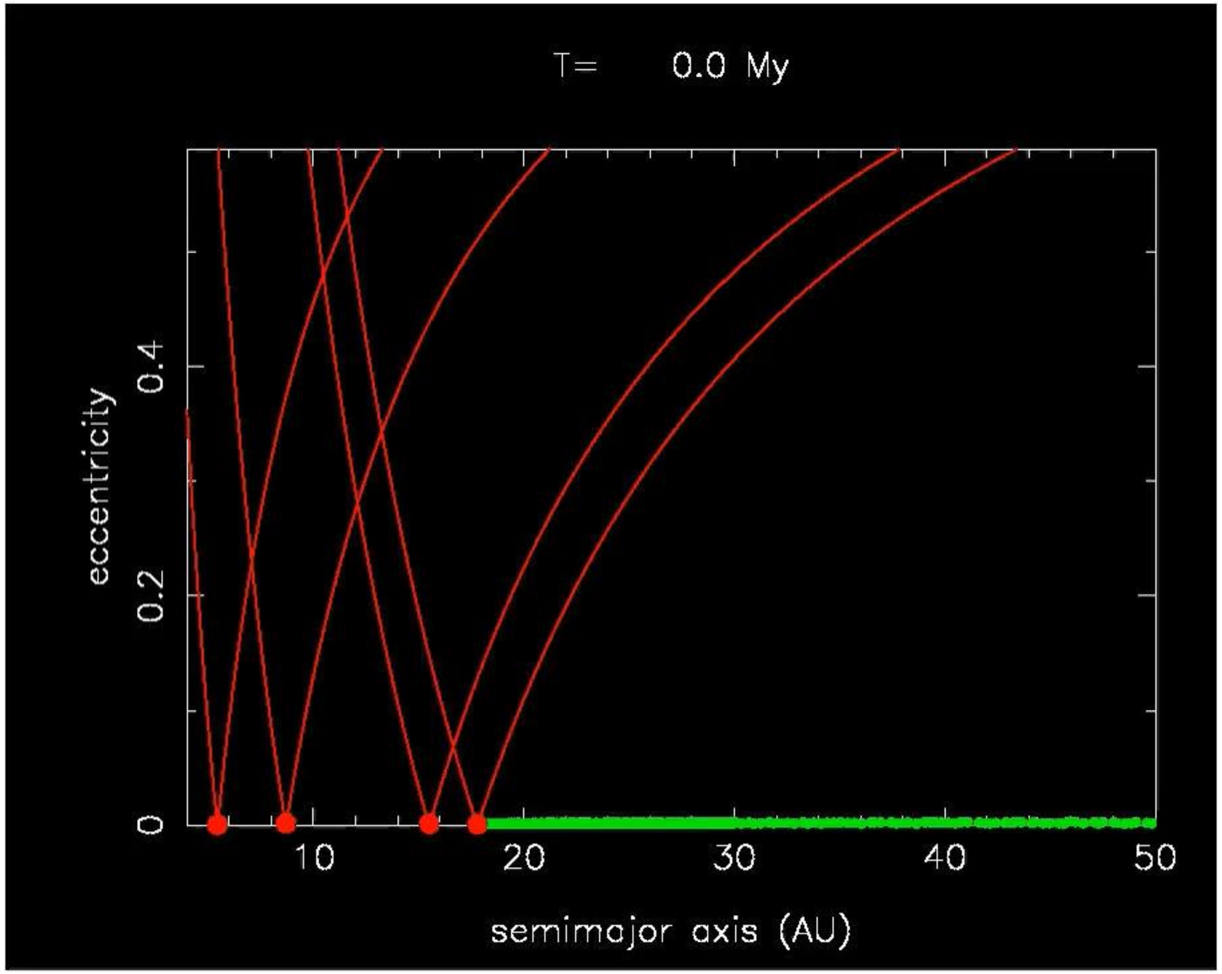


Kirsh et al., 2009

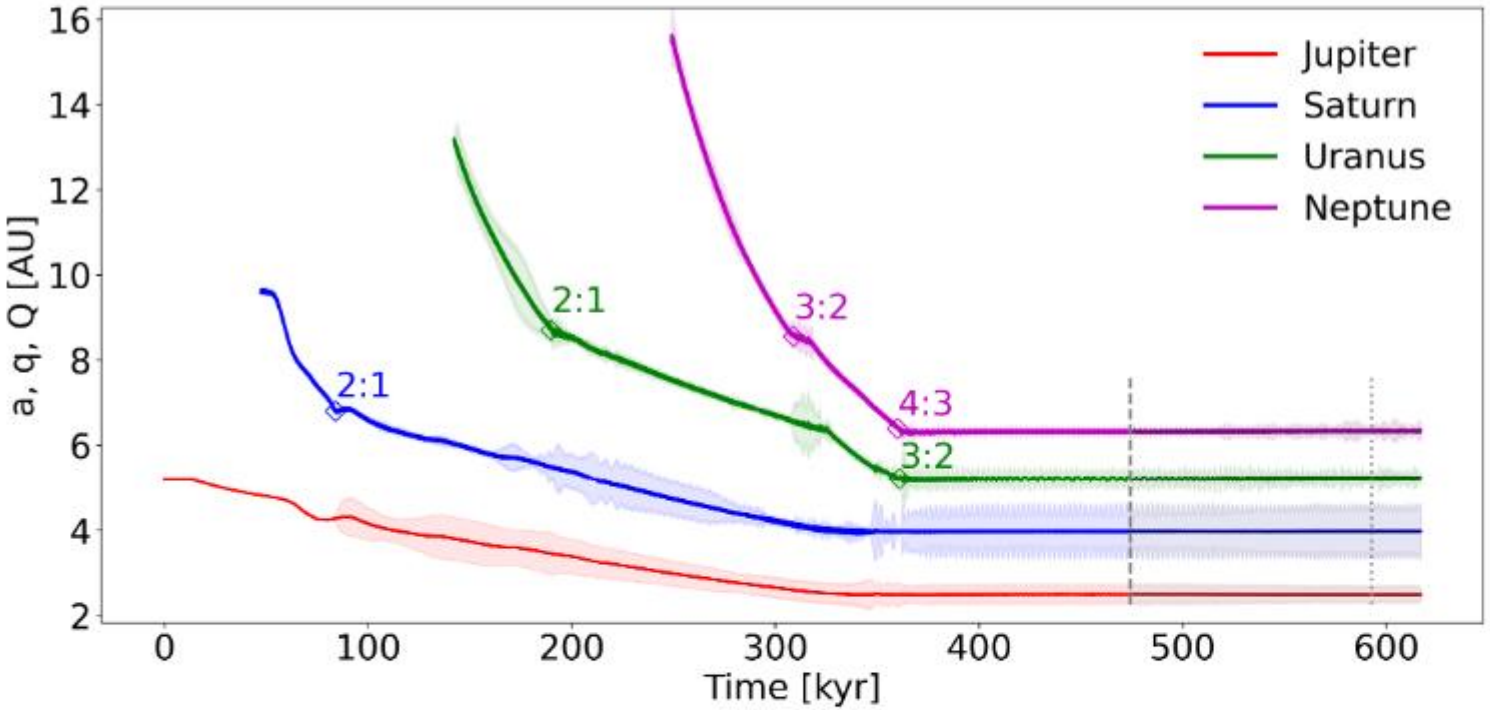
Planetesimal-driven migration



Planetesimal-driven migration



The giant planet resonant chain in the Solar system



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Breaking the Solar system giant planet chain

The Nice model

THE ASTRONOMICAL JOURNAL, 134:1790–1798, 2007 November

DYNAMICS OF THE GIANT PLANETS OF THE SOLAR SYSTEM IN THE GASEOUS PROTOPLANETARY DISK AND THEIR RELATIONSHIP TO THE CURRENT ORBITAL ARCHITECTURE

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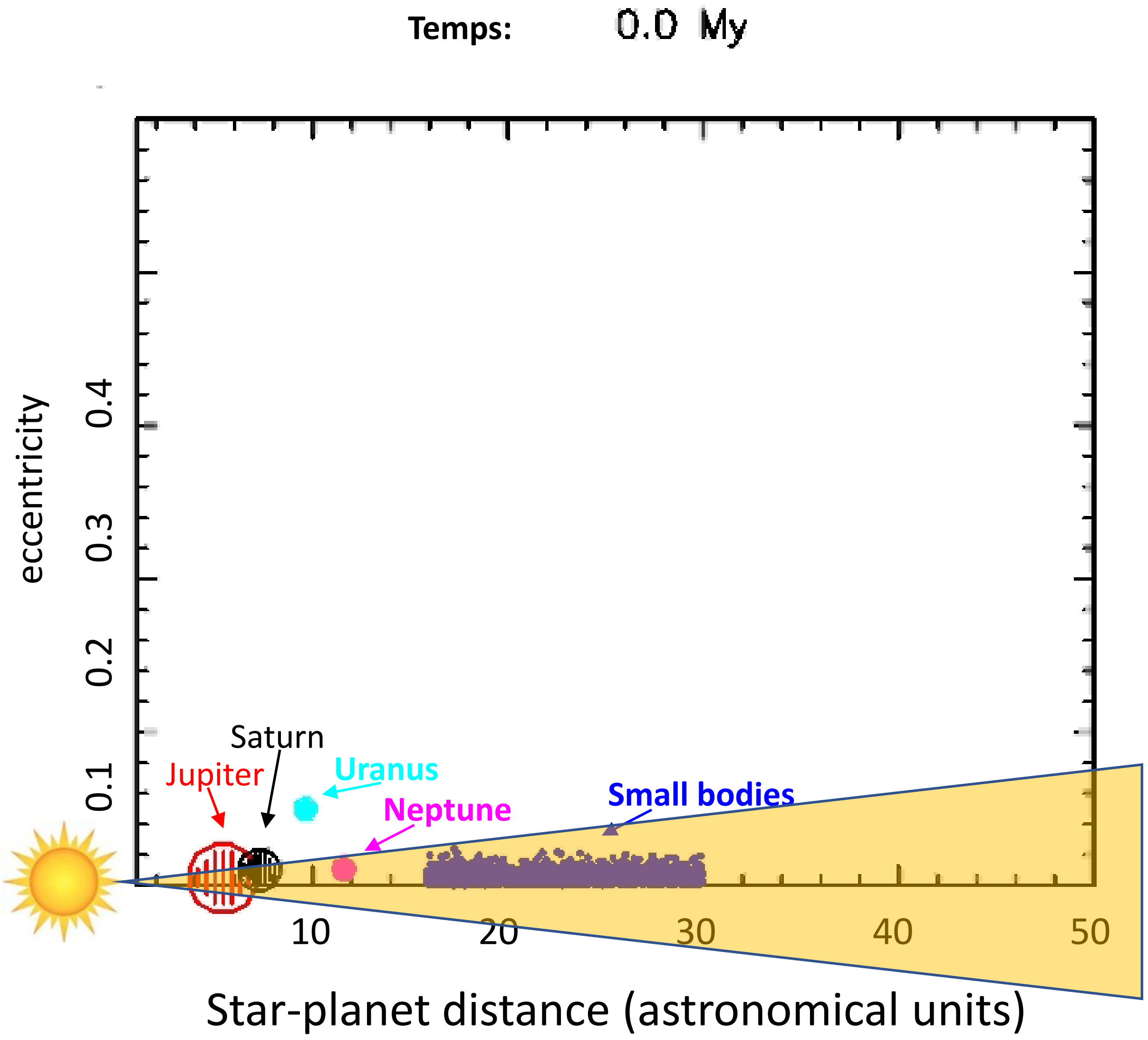

Take-away points

- Resonances behave like a pendulum. Resonant objects oscillate around an equilibrium point
- Convergent migration brings planets in resonance. Divergent migration skips resonances.
- Resonant trapping fails if the time to cross the resonance width by migration is shorter than the libration period, or if the eccentricity damping is too strong, or in case of overstability
- Eventually, planets get trapped in a resonant chain, if anything because the innermost planet stops migrating at the inner edge of the disc and therefore the migration of the second planet becomes convergent
- A resonant chain is the only configuration in which the period ratios among the planets do not change, even if the whole system remains in migration
- After the removal of the disk, resonant chains with too many planets (the number depending on their mass) become unstable
- The instability is due to secondary resonances between the libration frequencies and the synodic frequencies
- Three-body resonances are numerous. Their overlapping sets the limit of stability for three-planet systems that are not in two-body resonances.
- Other processes, such as planetesimal scattering, can move the planets out of resonance, possibly causing an instability
- This is what happened for the resonant chain of the giant planets of the Solar System