Thermal leptogenesis, dark matter and gravitational waves from an extended canonical seesaw

arXiv: 2409.08828 (with Partha K. Paul and Prashant Shukla)

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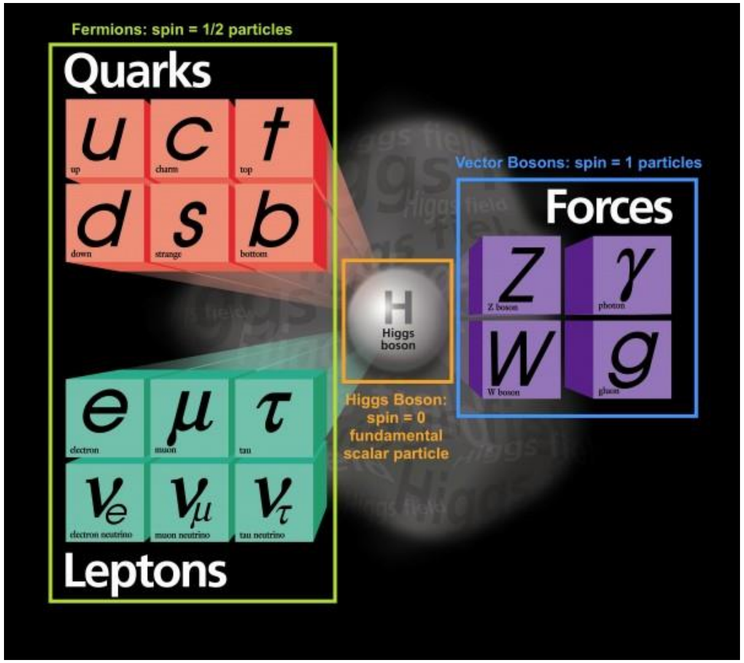
The SM in a Nutshell



Why new physics ?

Non-zero, but small masses of active neutrinos, baryon asymmetry of the Universe, dark matter conundrum ….

ν-mass: Dirac or Majorana ?

The oscillation experiments suggest that the neutrinos possess **sub-eV masses**.

Dirac ?
$\Delta L = 0$

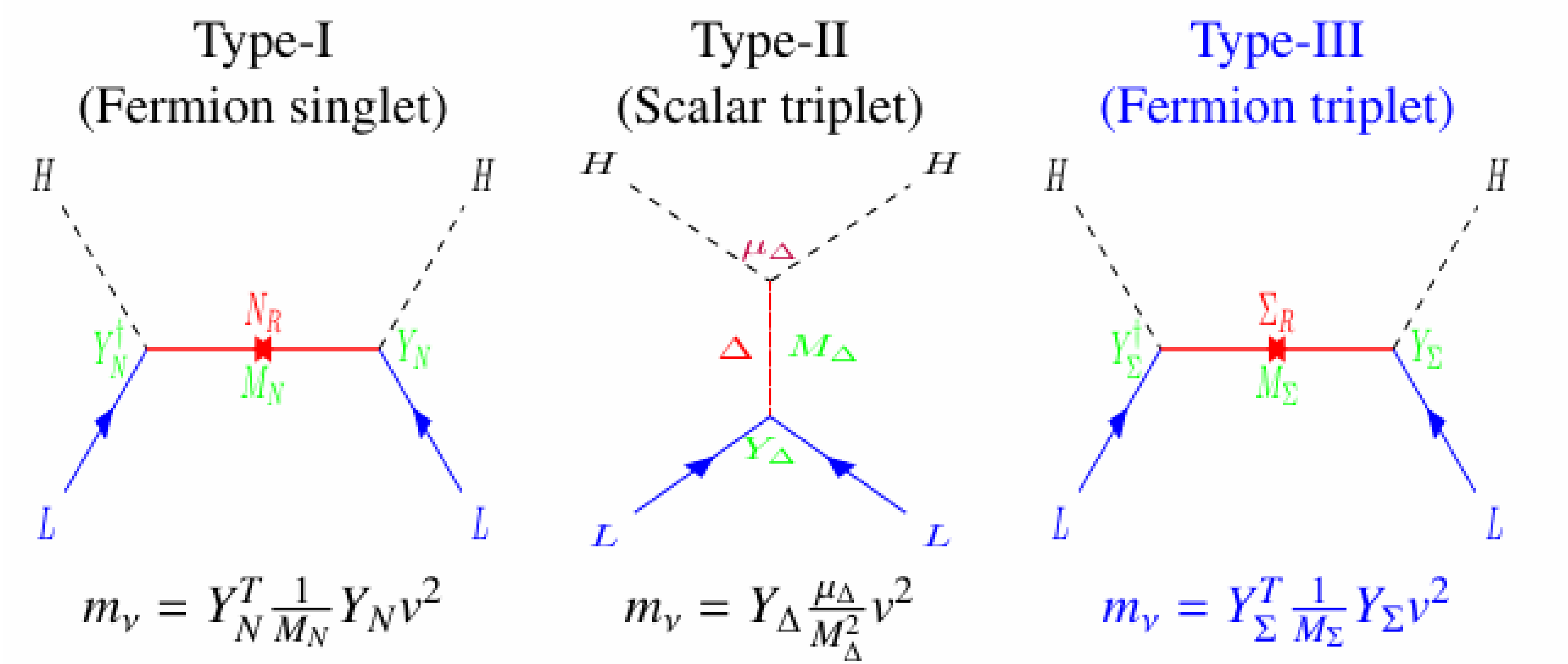
Majorana ?
$\Delta L = 2$

Assuming neutrinos are Majorana, one can introduce a dimension-5 operator using the SM fields:

Weinberg , PRL43, 1566 (1979)

$$\frac{LLHH}{\Lambda} \rightarrow m_\nu = \frac{< H >^2}{\Lambda}$$

Majorana neutrino mass in a seesaw framework



Minkowski, PLB67, 421 (1977)
Yanagida 1979
Gell-Mann, Raymond, Slansky, 1979
Mohapatra & Senjanovic, PRL44, 912 (1980)

Magg & Wetterich, PLB94, 61 (1980)
Schechter & valle, PRD22, 2227 (1980)
Cheng & Li, PRD22, 2860 (1980)
Lazarides, Shafi & Wetterich, NPB181, 287 (1981)
Mohapatra & Senjanovic, PRD23, 165 (1981)

Foot, Lew, He & Joshi, Z, Phys, C 44, 441 (1989)

Dynamical generation of Baryon asymmetry in the early Universe

Sakharov Criteria: [JETP Letter 5, 24 (1967)

(1)B–violation (or L-violation)

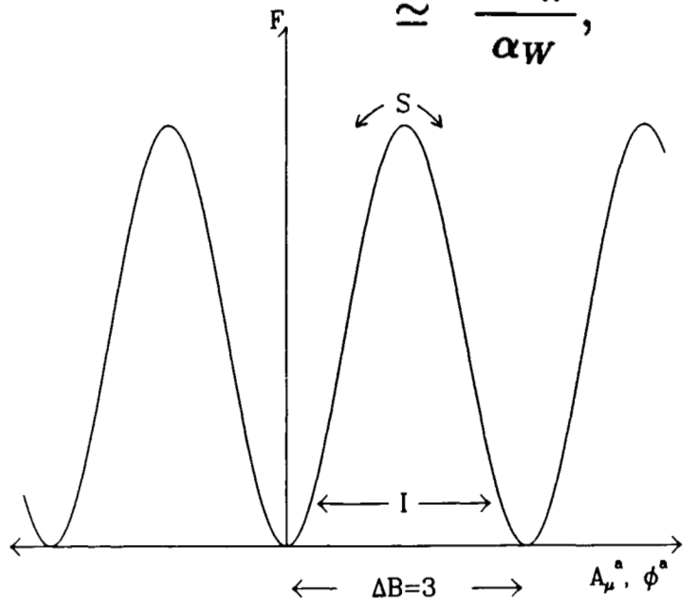
(2)C and CP-violation [CPT is conserved]

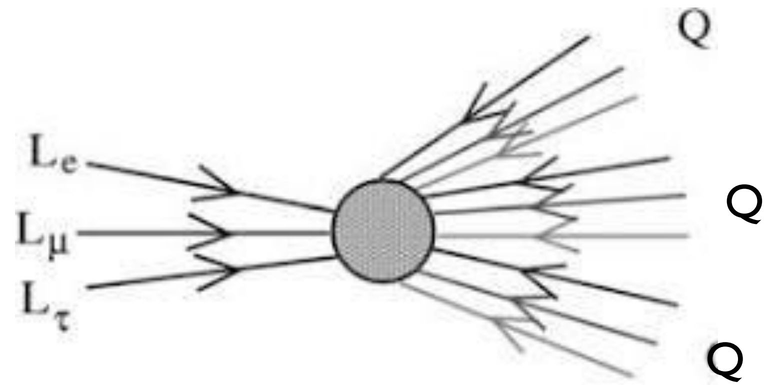
(3)Out-of-thermal equilibrium

Within the framework of the SM, baryo-lepto-genesis is not viable. Because there is not enough CP-violation. However, B+L can be violated by the non-perturbative process, called sphaleron transition, in the SM. At zero temperature this process is suppressed. But at finite temperature this process can happen rapidly. Typically above the EW-phase transition scale.

[Kuzmin, Rubakov & Shaposnikov, PLB155, 36 (1985)]

$$F = \int d^3x \left[\frac{1}{4} F^a_{\mu\nu} F^{a\mu\nu} - D_\mu \phi^a D^\mu \phi^a + \frac{1}{4}\lambda \left(\phi^a \phi^a - \sigma^2/2 \right)^2 \right]$$
$$\simeq \frac{2m_W}{\alpha_W},$$





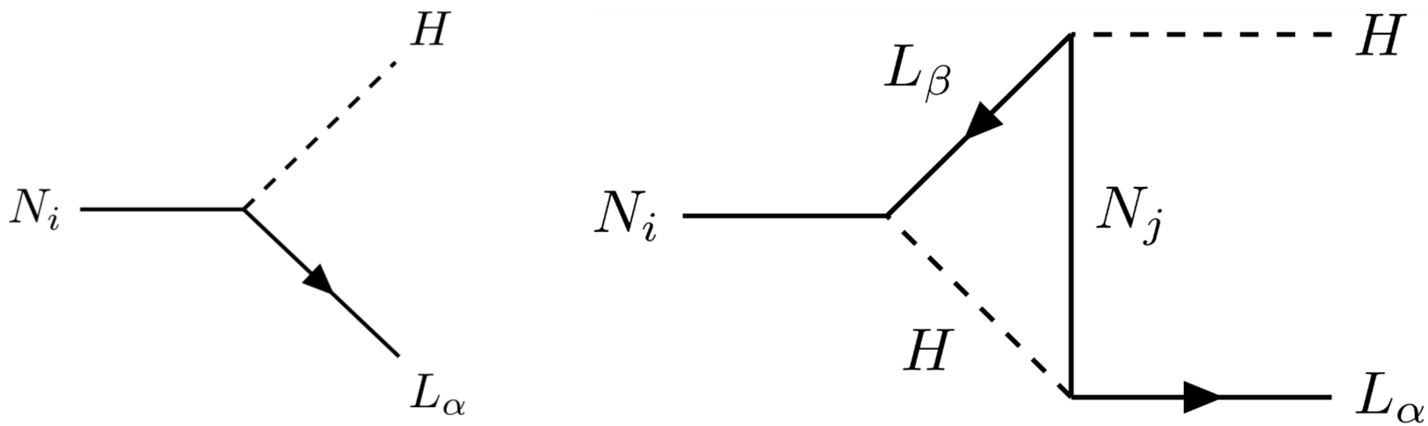
A typical sphaleron process violates B+L by 6 units. The rate of B violation

$$\dot{B} = -cBT\exp(-F/T).$$

Leptogenesis through Type-I Seesaw

SM extended with three heavy Majorana neutrinos

$$\mathcal{L} \supset M_N \bar{N}^c N + Y_N \bar{L} \tilde{H} N + H.C.$$



Fukugita & Yanagida, PLB174, 45 (1986)

The amount of CP-violation

$$\varepsilon_i = \frac{\Gamma_i - \bar{\Gamma}_i}{\Gamma_i + \bar{\Gamma}_i}$$

$$\epsilon_i(vertex) = \frac{1}{8\pi} \sum_{k \neq i} f\left(\frac{M_k^2}{M_i^2}\right) Im\left[(Y^+Y)_{ki}^2\right] / (Y^+Y)_{ii}$$

$$f(x) = \sqrt{x\left[1 - (1+x) ln \frac{1+x}{x}\right]}$$

The amount of B-asymmetry can be roughly given as

$$\boxed{Y_B = a_{sph} Y_{Ni}^{eq} \epsilon_i k_f}$$

The sphaleron conversion factor can be calculated by using the chemical potential of the particles which are in thermal equilibrium:

$$B+L = -\frac{6N+5m}{22N+13m}(B-L)\,,$$

$$B = \frac{8N+4m}{22N+13m}(B-L)\,,$$

$$L = -\frac{14N+9m}{22N+13m}(B-L)\,.$$

N=No. of generation of quarks and leptons

m=No. of Higgs doublets

Khelbnikov & Shaposhnikov, Nucl. Phys. B308, 885 (1988)

Harvey & Turner, PRD42, 3344 (1990)

Within the SM, N=3 and m=1. So we get $B = \frac{28}{79}(\mathrm{B}-\mathrm{L})$ and $B = -\frac{28}{55}L$

Detailed estimation of Lepton asymmetry by using Boltzmann equations

$$\frac{dN_{N_1}}{dz} = -(D+S)\,(N_{N_1} - N_{N_1}^{\rm eq})\,,$$

$$\frac{dN_{B-L}}{dz} = -\varepsilon_1\, D\,(N_{N_1} - N_{N_1}^{\rm eq}) - W\, N_{B-L}$$

Where, $z = M_1/T$

Denoting the Hubble expansion rate by *H* we have:

$$D = \Gamma_D/(H\,z)$$

$$S = \Gamma_S/(H\,z)$$

$$W = \Gamma_W/(H\,z)$$

Luty, PRD45, 455 (1992)
Mohapatra & Zhang, PRD46, 5331 (1992)
Plumacher, Z, Phys, C 74, 549 (1997)
W. Buchmuller and M. Plumacher, Phys. Rept. 320, 329 (1999)
Buchmuller, Bari and Plumacher, Annal of physics, 315, 305-351 (2005)
And many many authors

The decay parameter is defined as:

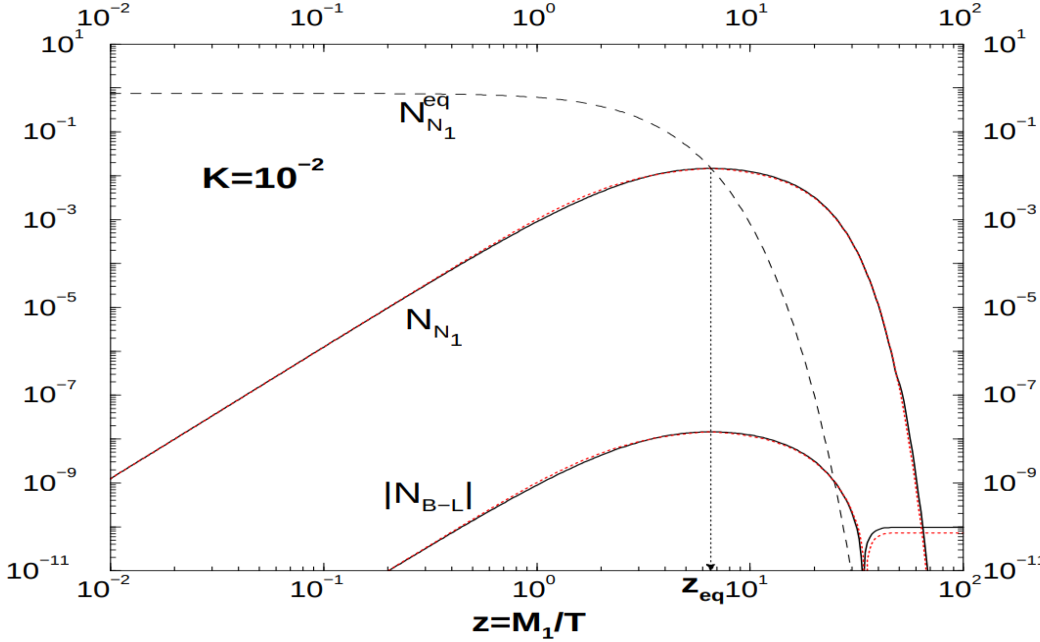
$$K = \frac{\Gamma_D(z=\infty)}{H(z=1)} = \frac{\widetilde{m}_1}{m_*}$$

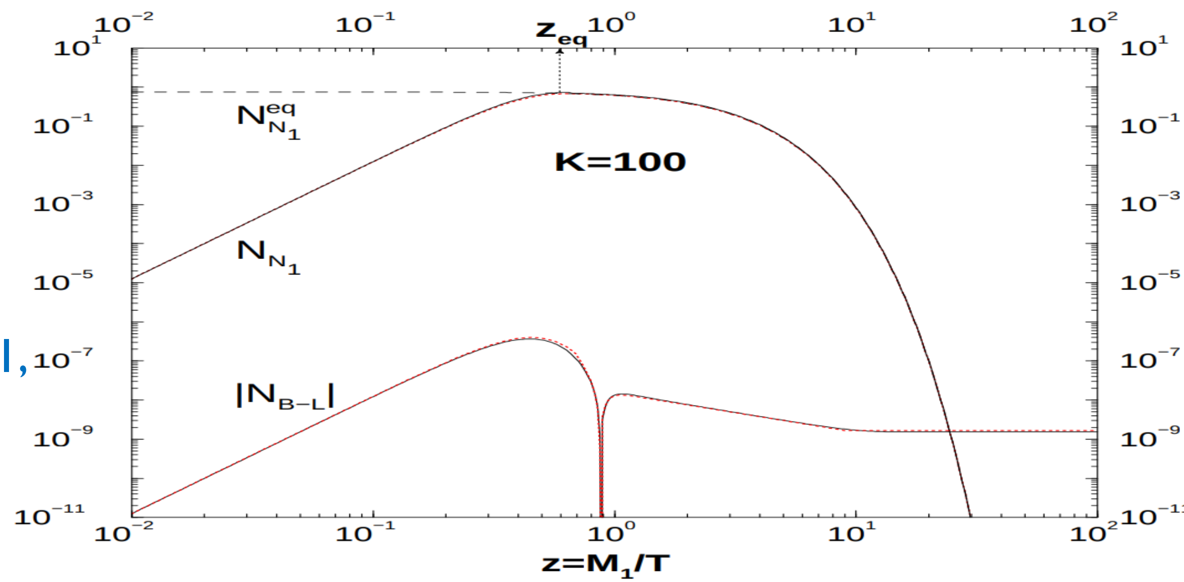
where,

$$H \simeq \sqrt{\frac{8\,\pi^3\,g_*}{90}}\frac{M_1^2}{M_{\rm Pl}}\,\frac{1}{z^2} \simeq 1.66\,g_*\,\frac{M_1^2}{M_{\rm Pl}}\,\frac{1}{z^2}$$

$$\widetilde{m}_1 = \frac{(m_D^\dagger\,m_D)_{11}}{M_1}$$

$$m_* = \frac{16\,\pi^{5/2}\,\sqrt{g_*}}{3\,\sqrt{5}}\,\frac{v^2}{M_{\rm Pl}} \simeq 1.08\times 10^{-3}\,{\rm eV}$$





Buchmuller, Bari and Plumacher, Annals of Physics, 315, 305 – 351, 2005

Upper bound on CP-asymmetry in seesaw models

$$\mathcal{L} \supset M_N \bar{N}^c N + Y_N \bar{L} \tilde{H} N + H.C.$$

Using the Casas & Ibarra parametrisation for the Yukawa coupling:

$$Y_N = \frac{1}{v} D_{\sqrt{M}} R D_{\sqrt{m}} U^{+}_{PMNS}$$

Casas & Ibarra, Nucl. Phys. B618, 171 (2001)

and using a normal hierarchy spectrum of right handed neutrinos, one gets an upper bound on the CP asymmetry

$$|\epsilon_1| \leq \frac{3 M_1}{8\,\pi\,v^2} (m_3 - m_1)$$

Davidson & Ibarra, Phys. Lett. B535, 25, (2002)
Buchmuller, Bari & Plumacher, Nucl. Phys. B643, 367 (2002)

Using the upper bound on CP-asymmetry one gets a lower bound on the lightest right handed neutrino mass:

$$M_1 \geq O(10^9)GeV \left(\frac{\eta_B}{6 \times 10^{-10}}\right)\left(\frac{0.05eV}{m_3}\right)\left(\frac{2 \times 10^{-4}}{\frac{n_N}{s}}\right)$$

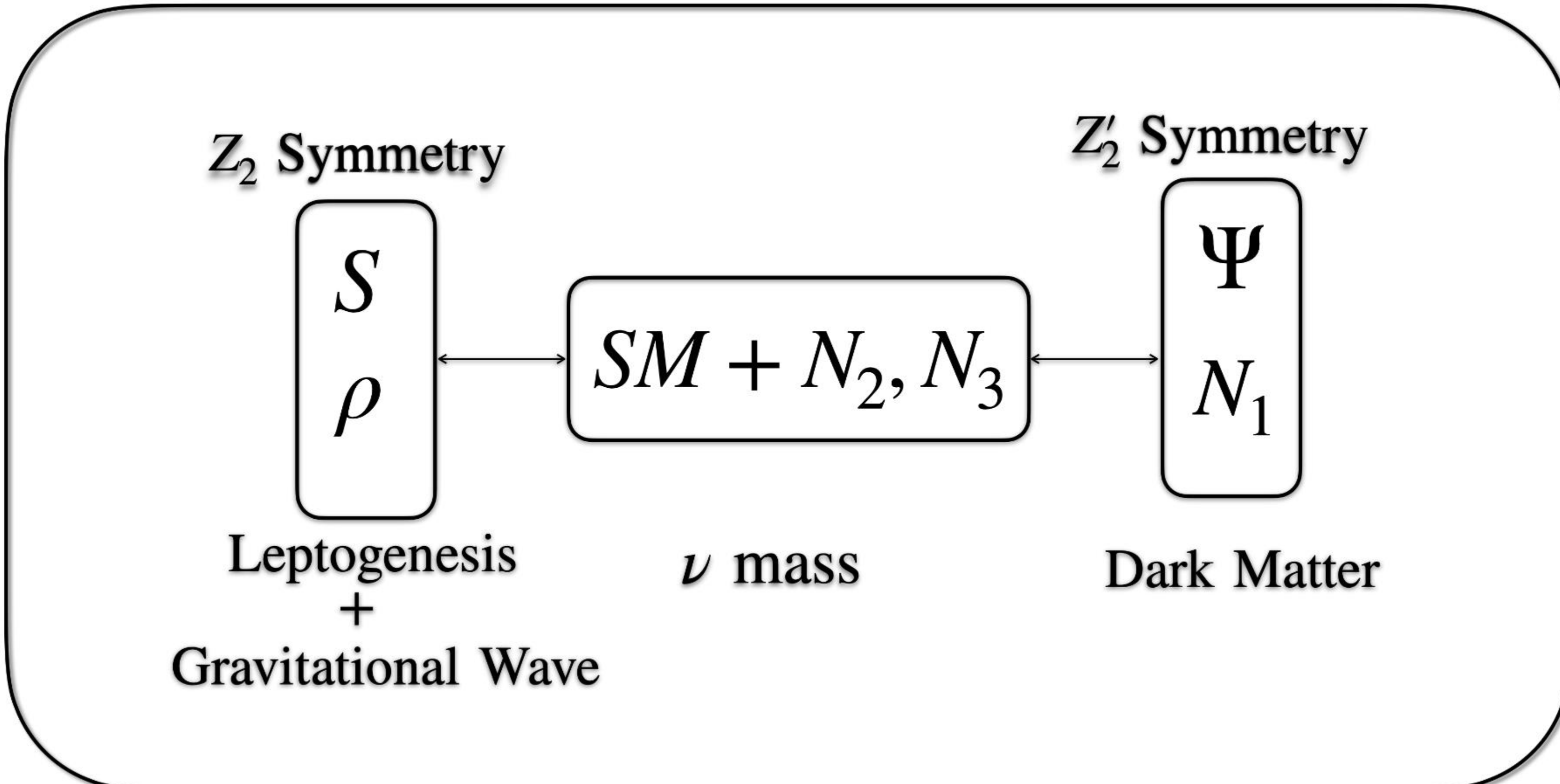
This implies that reheat temperature of the Universe should be large in order to realise thermal leptogenesis.
Q. How to reduce the scale of leptogenesis in the hierarchical RHN scenario ?

One possible solution could be to decouple the origin of neutrino mass and leptogenesis.

Ma, Sahu & Sarkar, J. Phys. G32 L65 (2006),
Ma, Sahu & Sarkar, J. Phys. G34, 741-752 (2007)

Extended canonical seesaw for leptogenesis, Gravitational waves and Dark matter

Paul. Sahu and Shukla: arXiv: 2409.08828



The Lagrangian of the model is

$$\begin{aligned}\mathcal{L} &= \bar{N} i\gamma_\mu \partial^\mu N + \bar{S} i\gamma_\mu \partial^\mu S - y_{N l} \bar{L}\tilde{H} N - y_{NS} \bar{N}\rho S \\ &- \frac{1}{2} M_S \bar{S^c} S - \frac{1}{2} M_N \bar{N^c} N + h.c + \mu_H H^\dagger H + \mu_\rho^2 \rho^2 \\ &- \lambda_H (H^\dagger H)^2 - \lambda_\rho \rho^4 - \lambda_{H\rho}(H^\dagger H)\rho^2, \qquad (7)\end{aligned}$$

Note: Here 2 and 3rd families of RHNs take part in neutrino mass generation, while the 1st generation of RHN contributes to DM.

The fermion mass matrix can be written in the the basis $[L, N, S]$ as

$$\mathcal{M} = \begin{pmatrix} 0 & m_D & 0 \\ m_D & M_N & d \\ 0 & d & M_S \end{pmatrix}$$

where $d = y_{NS} v_\rho/\sqrt{2}$, $m_D = y_{Nl} v_h/\sqrt{2}$, v_h is the vev of SM Higgs boson.

Now Diagonalising this mass matrix we obtain the heavy mass eigen values as

$$M'_N \simeq M_N + \frac{d^2}{M_N - M_S}$$
$$M'_S \simeq M_S - \frac{d^2}{M_N - M_S}$$

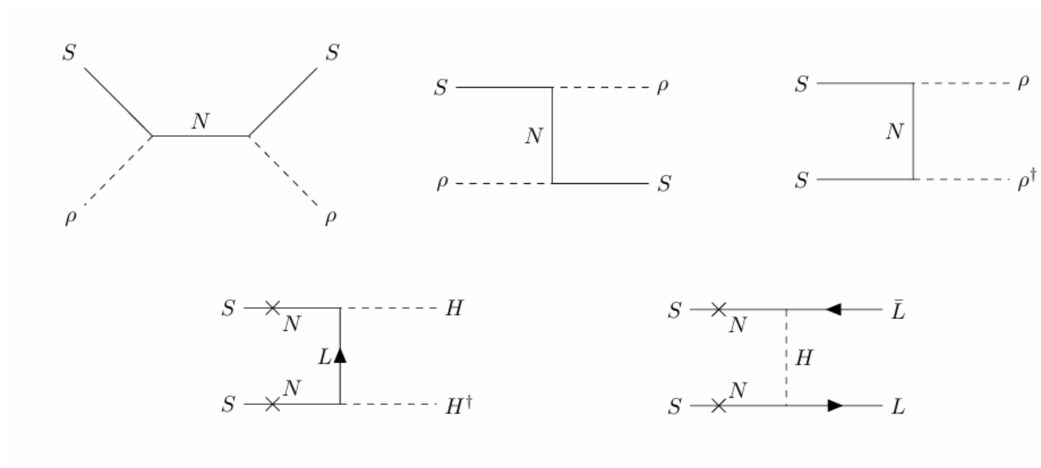
The mixing angle is given as $\theta \simeq \dfrac{d}{M_N - M_S}$

Now the light neutrino mass matrix is

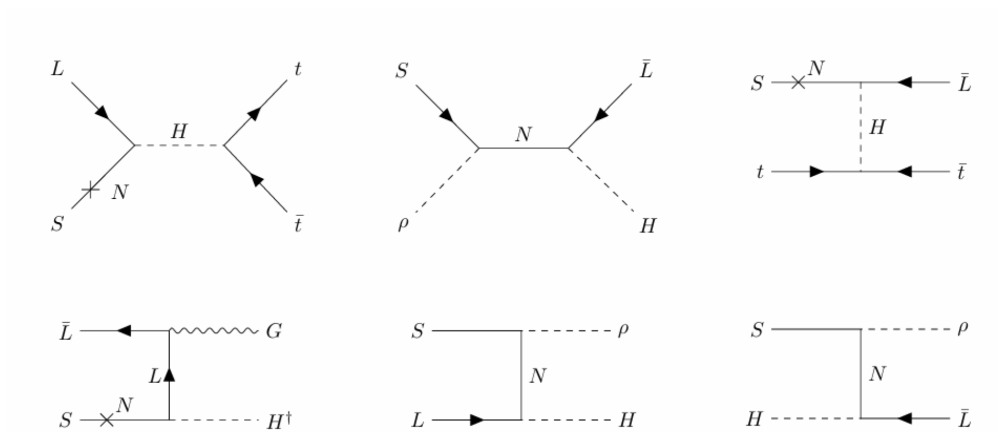
$$(m_\nu)_{ij} \simeq -\sum_k (m_D)_{ik} \left(M_k + \frac{d^2}{M_k - M_S} \right)^{-1} (m_D)_{kj}$$

Thermal Leptogenesis from S-decay

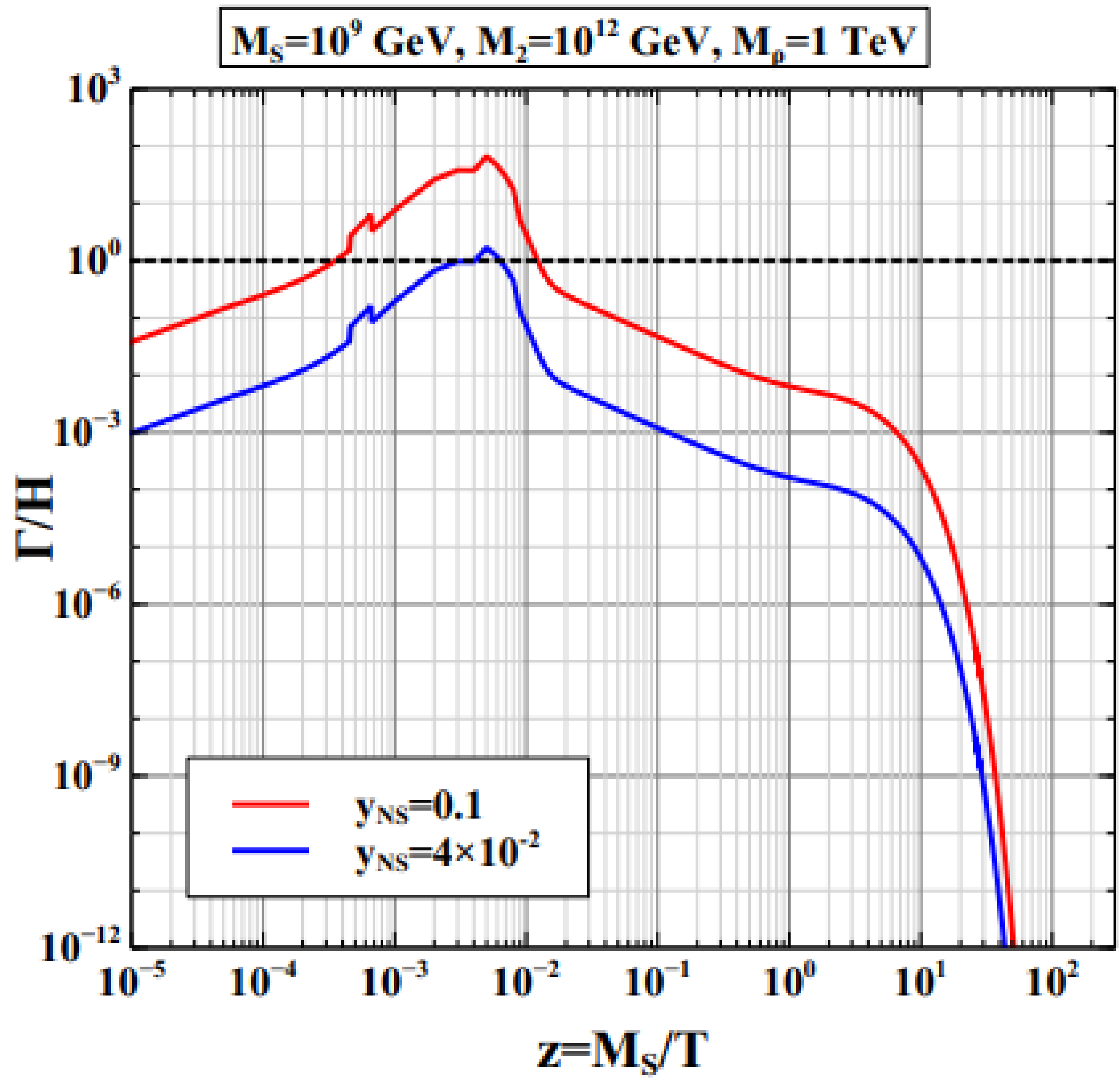
$\Delta L = 0$ Scattering processes through which S is brought to thermal equilibrium:



$\Delta L = 1$scattering processes:



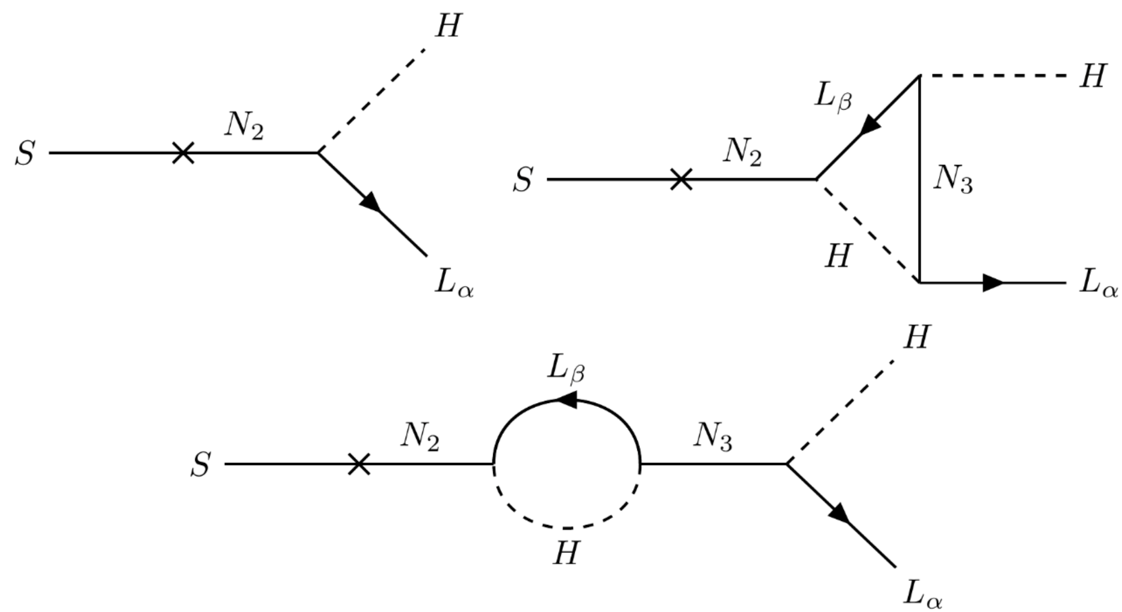
There are also $\Delta L = 2$ scattering processes.



M_S=10^9 GeV, M_2=10^12 GeV, M_ρ=1 TeV
Γ/H
z=M_S/T
10^3
10^0
10^-3
10^-6
10^-9
10^-12
10^-5
10^-4
10^-3
10^-2
10^-1
10^0
10^1
10^2
y_NS=0.1
y_NS=4×10^-2

Thermal Leptogenesis from S-decay

When ρ acquires a vev, S mixes with N_2 and N_3. In a hierarchical mass spectrum: $M_3 > M_2 > M_S$, the final lepton asymmetry is produced by the decay of S:



The CP asymmetry parameter

$$\epsilon_S = -\frac{3}{8\pi v_h^2}\frac{M_S}{M_3}\frac{Im[(m_D^\dagger m_D)_{23}]^2}{(m_D^\dagger m_D)_{22}}$$

Note that the mixing angle does not play any role here

The CP-asymmetry parameter does not suffer any suppression due to S-N mixing, but the decay rates are suppressed and the out-of-equilibrium condition can happen at low scale and hence allows a low scale leptogenesis.

$$\Gamma_S = \theta_2^2 \frac{(y_{Nl}^\dagger y_{Nl})_{22}}{8\pi} M_S$$

On the other hand, there exist an upper bound on the CP-asymmetry parameter

$$|\epsilon_S| \leq \frac{3}{8\pi v_h^2} M_S m_3$$

For small M_S the CP-asymmetry is suppressed and does not produce correct baryon asymmetry.

$$\eta_B = \frac{C_{L\to B}}{f} Y_{\Delta L} = \frac{C_{L\to B}}{f} \epsilon_S \kappa_S Y_S^{eq} = -0.0144719 \epsilon_S \kappa_S$$

We solved the relevant Boltzmann equations:

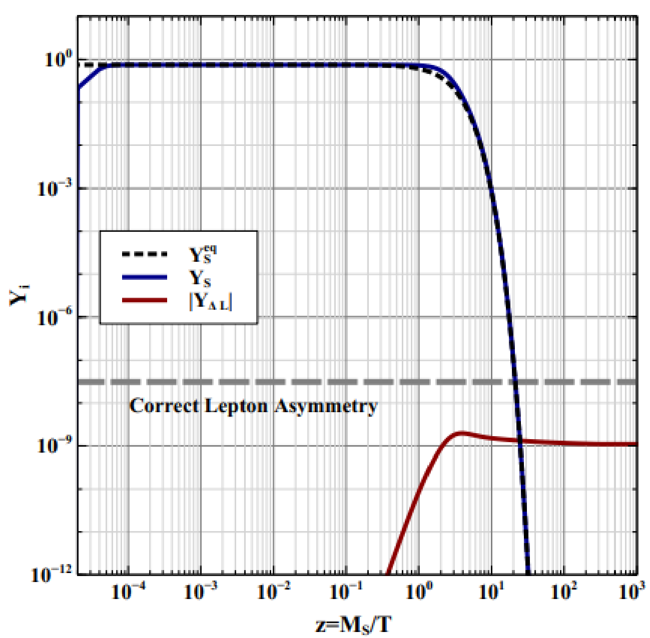
$$\frac{dY_S}{dz} = -\frac{\Gamma_D}{\mathrm{H}z}(Y_S - Y_S^{eq}) - \frac{(\Gamma_1 + \Gamma_0)}{\mathrm{H}z}(Y_S - Y_S^{eq}) - \frac{\Gamma_0'}{\mathrm{H}z}\frac{(Y_S^2 - (Y_S^{eq})^2)}{Y_S^{eq}},$$

$$\frac{dY_{\Delta L}}{dz} = \epsilon_S \frac{\Gamma_D}{\mathrm{H}z}(Y_S - Y_S^{eq}) - \left(\frac{1}{2}\frac{\Gamma_{ID}}{\mathrm{H}z} + \frac{\Gamma_1^W + \Gamma_2^W}{\mathrm{H}z}\right)Y_{\Delta L},$$

TABLE II. Benchmark points for leptogenesis

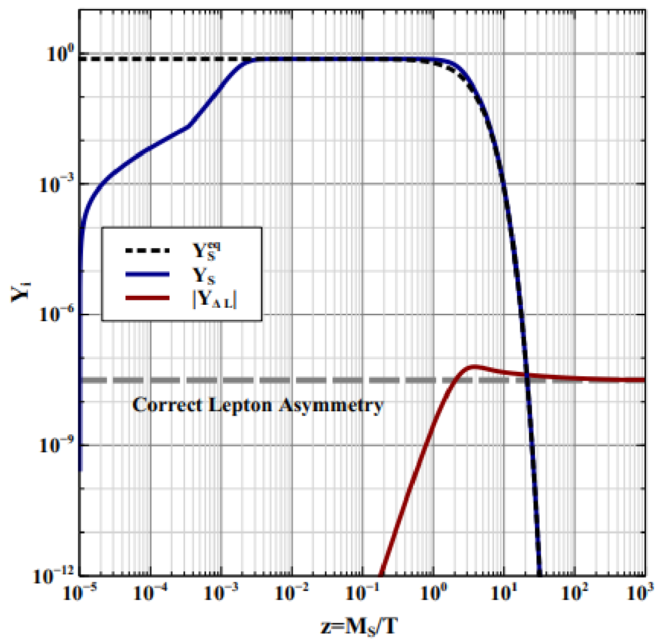
BPs	M_2(GeV)	M_S(GeV)	M_ρ(GeV)	z_a	$(y^\dagger_{NI} y_{NI})_{22}$	θ_s	y_{NS}	v_ρ(GeV)	ϵ_S
BP1	2×10^{12}	6×10^7	10^3	$1 + i0.1$	3.1348×10^{-3}	10^{-3}	0.1	2.82834×10^{10}	5.9763×10^{-9}
BP2	1.3×10^{12}	2×10^9	10^3	$1.3672 + i3.3249 \times 10^{-2}$	2.1671×10^{-3}	7×10^{-3}	5×10^{-2}	2.56991×10^{11}	1.9921×10^{-7}
BP3	10^{12}	2×10^9	10^3	$(50 + i3) \times 10^{-3}$	2.8938×10^{-4}	10^{-5}	0.42	3.36044×10^7	1.9921×10^{-7}
BP4	1.094×10^{12}	4.1×10^8	10^3	$(4.808 + i2.8955) \times 10^{-4}$	3.1306×10^{-4}	6×10^{-4}	0.15	6.18503×10^9	4.0838×10^{-8}

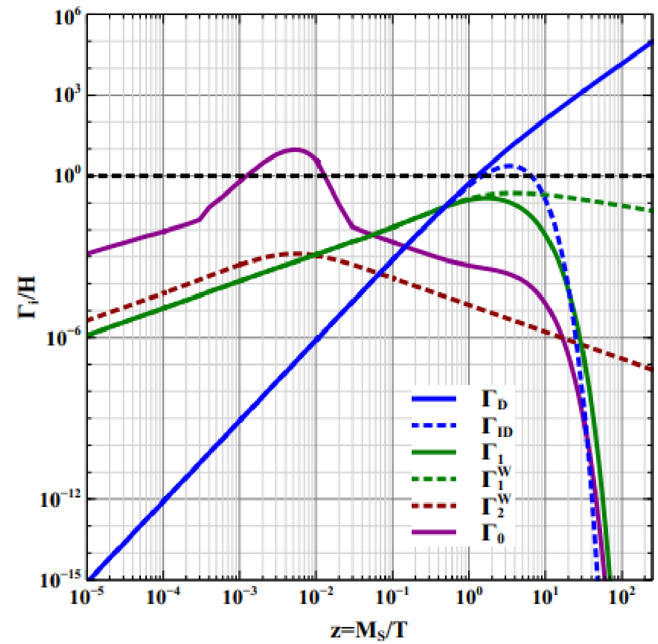
BP1



Remark: Not successful leptogenesis due to low scale of S

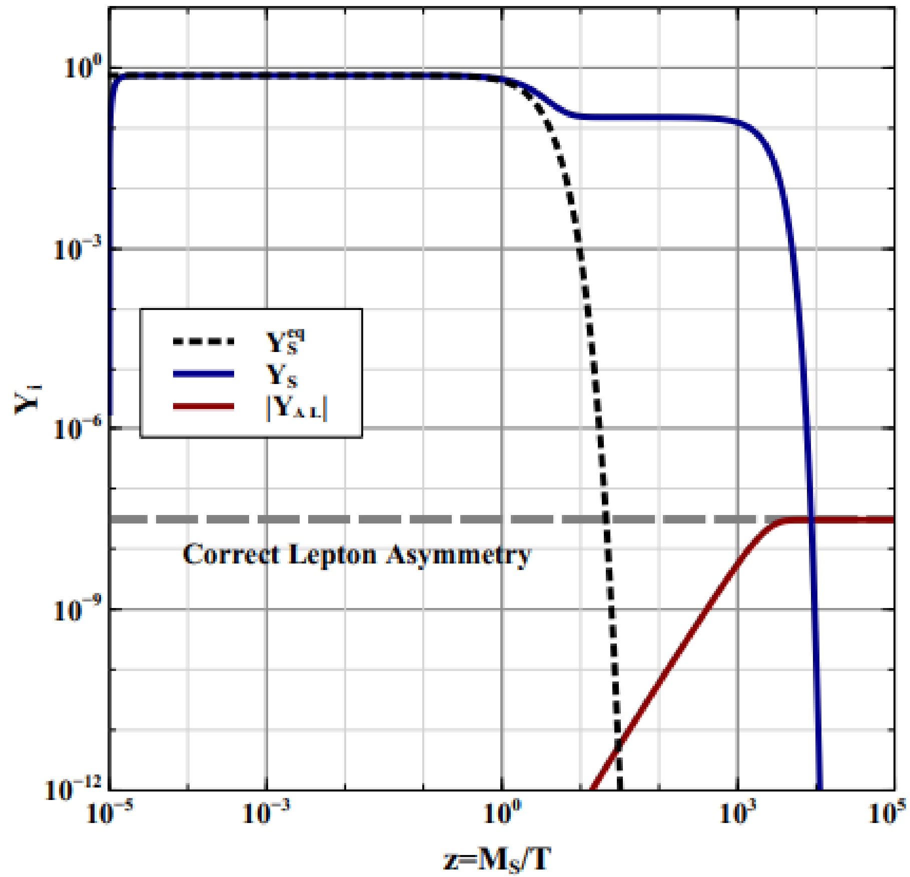
BP2





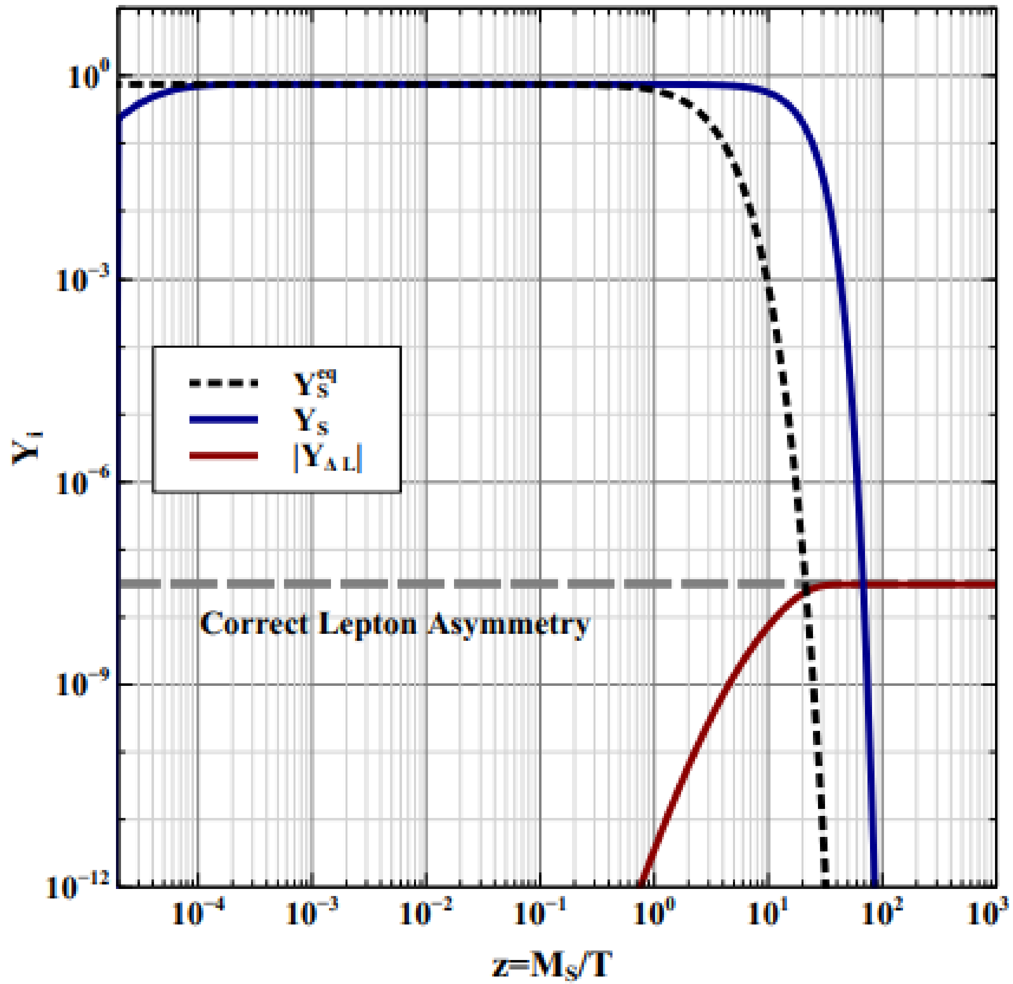
Successful leptogenesis due to appropriate mass of S

BP3

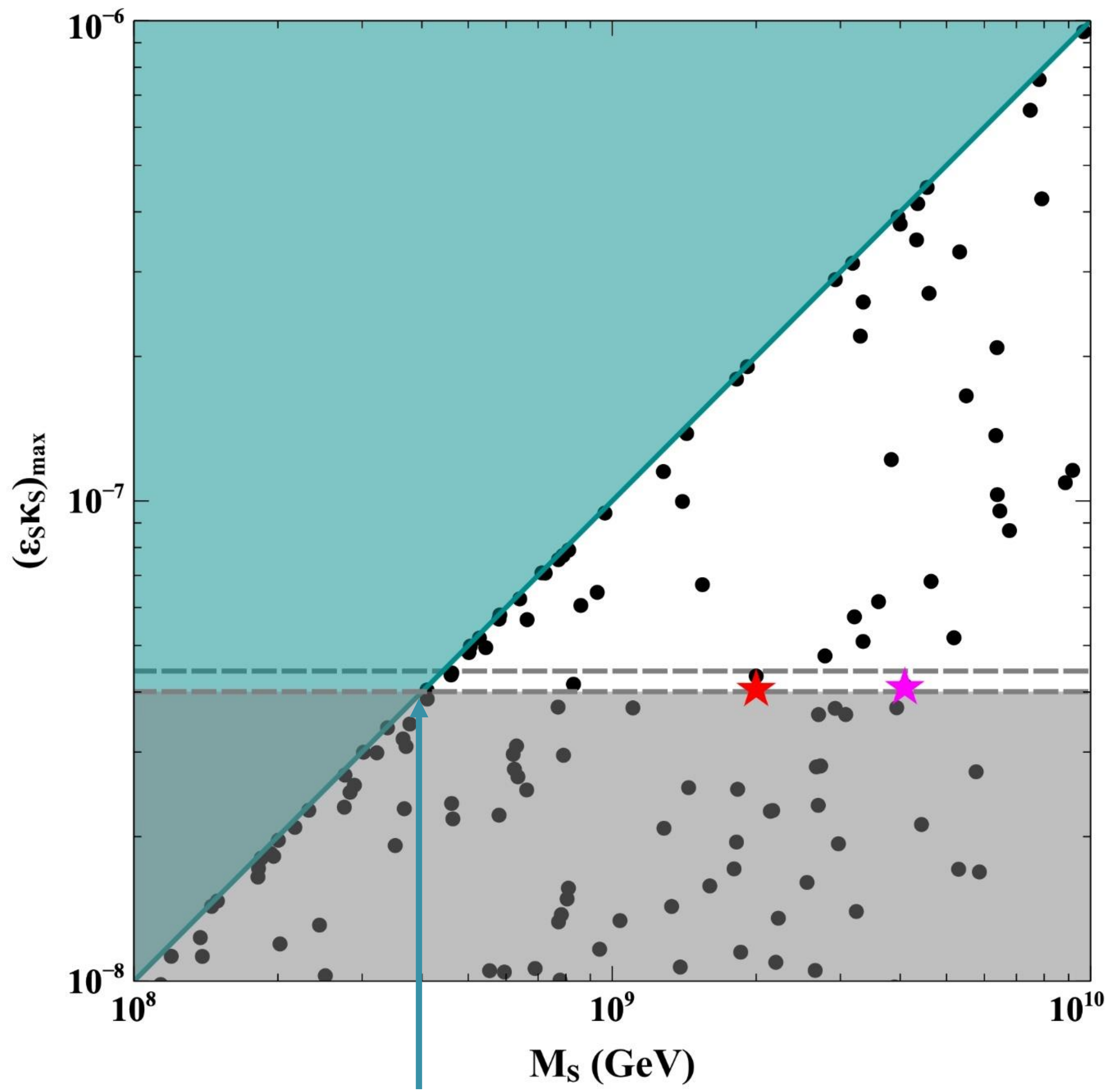


Successful leptogenesis for another set of parameters

BP4



Successful leptogenesis for another choice of parameters



Lower bound on S-mass

Gravitational waves from disappearing domain walls

Given the discrete symmetry Z_2', the potential for the scalar field ρ is

$$V(\rho) = \frac{\lambda_\rho}{4}(\rho^2 - v_\rho^2)^2.$$

When ρ gets a vev, the discrete symmetry Z_2' breaks spontaneously and leads to the formation of domain walls. Without loss of generality, the domain wall profile can be taken as:

$$\rho(z) = v_\rho \tanh(\alpha z)$$

where $\alpha \simeq \sqrt{\frac{\lambda_\rho}{2}} v_\rho$.

With the boundary condition:

$$\lim_{z \to \pm\infty} \rho(z) = \pm v_\rho$$

The surface energy density of the domain wall is

$$\sigma = \frac{4}{3}\sqrt{\frac{\lambda_\rho}{2}} v_\rho^3 \simeq \frac{2}{3} M_\rho v_\rho^2.$$

Review article by Saikawa, Universe 3, 240 (2017)

In order to destabilize the wall, we introduce an explicit Z_2' symmetry breaking term $\mu_b^3\rho$ such that the degeneracy of the minima is lifted by

$$V_{bias} \equiv |\mathcal{V}(-v_\rho) - \mathcal{V}(v_\rho)| = \sqrt{2}\mu_b^3 v_\rho$$

The energy bias has to be large enough, so that the wall disappear before the BBN epoch, I,e

$$t_{ann} = \mathcal{C}_{ann}\frac{\mathcal{A}\sigma}{V_{bias}} < t_{BBN}$$

This gives a lower bound on the Z_2' breaking parameter:

$$\mu_b > 1.45839 \times 10^{-4}\mathrm{GeV}\mathcal{C}_{ann}^{\frac{1}{3}}\mathcal{A}^{\frac{1}{3}}\left(\frac{10^{-2}\mathrm{sec}}{t_{BBN}}\right)^{\frac{1}{3}} \left(\frac{M_\rho}{1\mathrm{TeV}}\right)^{\frac{1}{3}}\left(\frac{v_\rho}{10^5\mathrm{TeV}}\right)^{\frac{1}{3}}$$

The DW has to disappear before they could start dominating the energy density of the Universe:

$$t_{ann} < t_{dom} = \frac{3}{4}\frac{M_{pl}^2}{\mathcal{A}\sigma}$$

This sets a lower bound on the annihilation temperature:

$$T_{ann} > 1.34772\mathrm{GeV}\,\mathcal{A}^{1/2}\left(\frac{g_*(T_{ann})}{10}\right)^{-\frac{1}{4}}\left(\frac{v_\rho}{10^5\mathrm{TeV}}\right)\left(\frac{M_\rho}{1\mathrm{TeV}}\right)^{\frac{1}{2}}$$

The domain walls annihilate and release their energy in the form of GWs which can be detected at present time. The peak amplitude of the spectrum at the present time is:

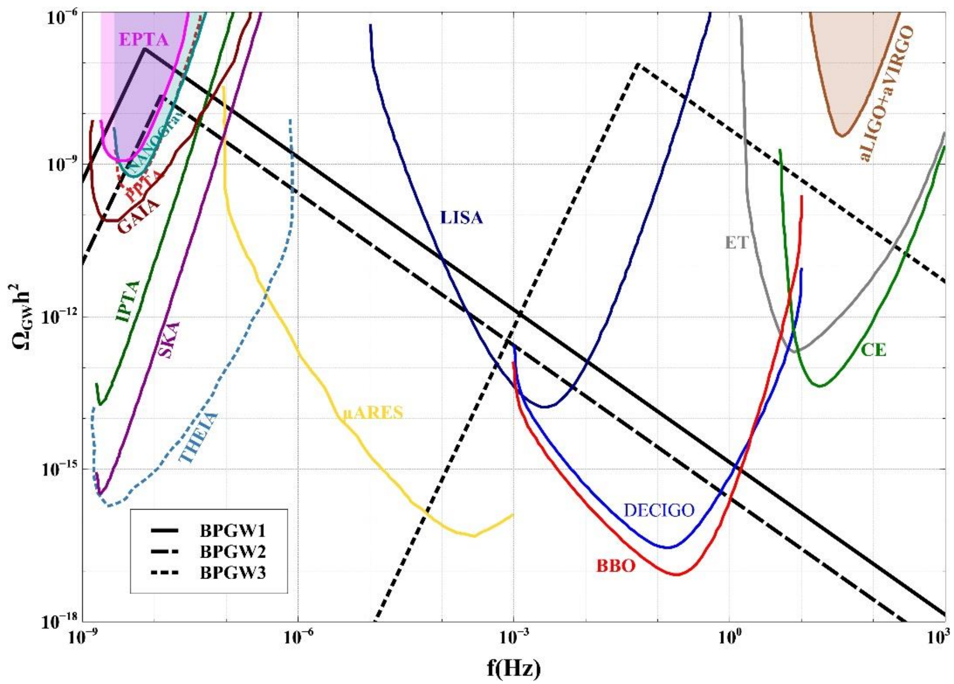
$$\Omega_{GW}h^2(t_0)|_{peak} = 7.18824\times 10^{-18}\mathcal{A}^2\tilde{\epsilon}_{GW}\left(\frac{\sigma}{1\mathrm{TeV}^3}\right)^2 \left(\frac{g_{*s}(T_{ann})}{10}\right)^{-\frac{4}{3}}\left(\frac{T_{ann}}{10^{-2}\mathrm{GeV}}\right)^{-4}$$

The peak frequency of the gravitational wave at the present time is

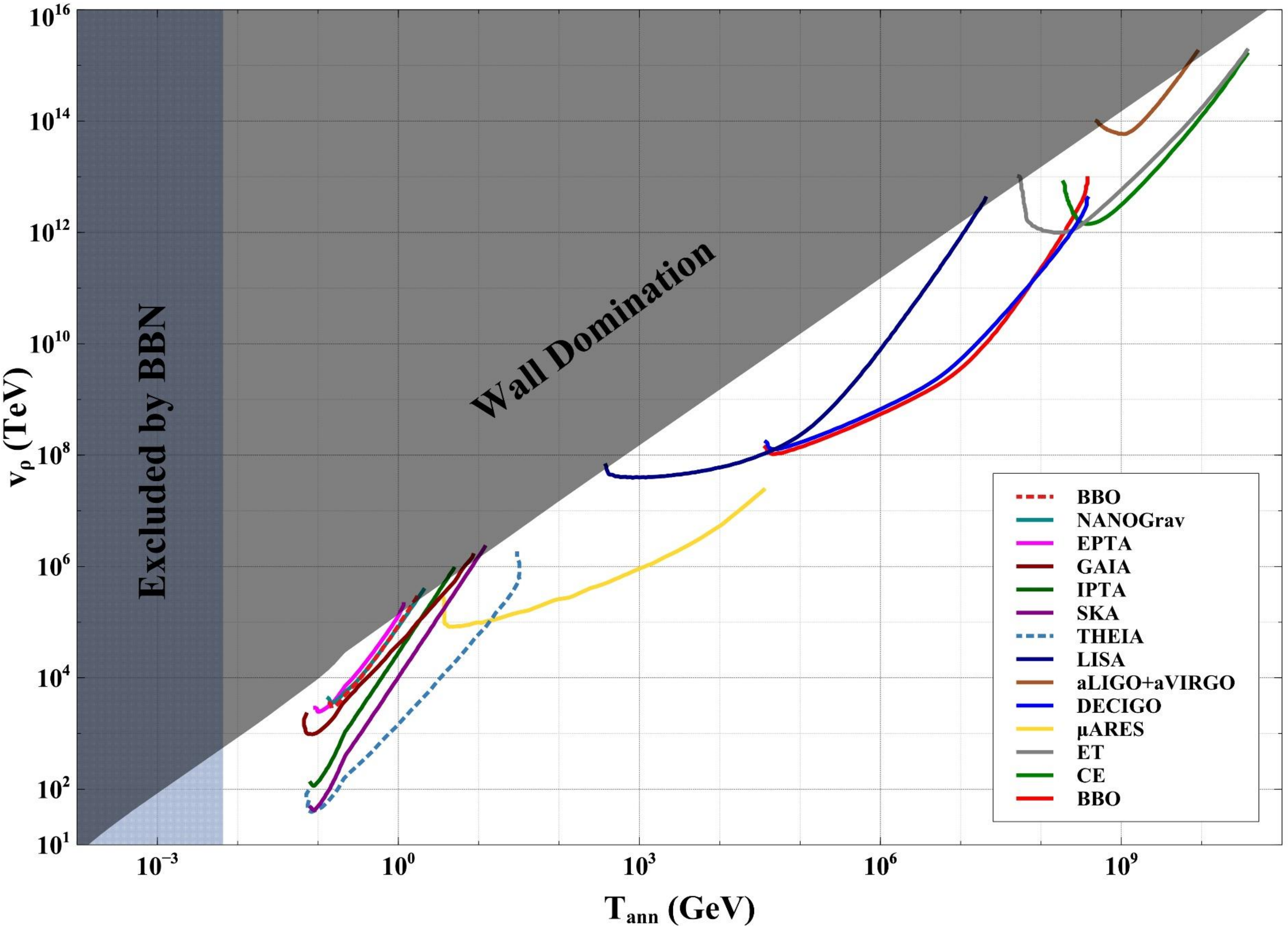
$$f_{peak}(t_0) = 1.78648\times 10^{-10}\mathrm{Hz}\left(\frac{g_{*s}(T_{ann})}{10}\right)^{-\frac{1}{3}} \left(\frac{g_*(T_{ann})}{10}\right)^{\frac{1}{2}}\left(\frac{T_{ann}}{10^{-2}\mathrm{GeV}}\right)$$

The amplitude of gravitational wave for any frequency at the present time varies as:

$$\Omega_{GW}(t_0, f) = \Omega_{GW} h^2(t_0)|_{peak} \begin{cases} \frac{f_{peak}}{f} & f > f_{peak} \\ \left(\frac{f}{f_{peak}}\right)^3 & f < f_{peak} \end{cases}$$



$BPGW1$: $T_{ann} = 0.3\ GeV\ \sigma = 7.5 \times 10^8\ (TeV)^3\ (Black\ solid)$
BPGW2: $T_{ann} = 0.5\ GeV\ \sigma = 7.5 \times 10^8\ (TeV)^3\ (Black\ dashed)$
$BPGW3$: $T_{ann} = 2 \times 10^6 GeV\ \sigma = 3.3 \times 10^{22}\ (TeV)^3\ (Black\ dotted)$



Excluded by BBN
Wall Domination
v_ρ (TeV)
T_{ann} (GeV)
10^1
10^2
10^4
10^6
10^8
10^{10}
10^{12}
10^{14}
10^{16}
10^{-3}
10^0
10^3
10^6
10^9
BBO
NANOGrav
EPTA
GAIA
IPTA
SKA
THEIA
LISA
aLIGO+aVIRGO
DECIGO
μARES
ET
CE
BBO

Singlet-Doublet DM

Due to Z_2 symmetry the singlet-doublet mixture behaves as a good candidate of dark matter. The corresponding Lagrangian is

$$\mathcal{L} = \mathcal{L}_{SM} + \overline{\Psi}\left(i\gamma^\mu D_\mu - M\right)\Psi + \overline{N_{R_i}} i\gamma^\mu \partial_\mu N_{R_i} - \left(\frac{1}{2} M_{R_i} \overline{N_{R_i}}\left(N_{R_i}\right)^c + h.c\right) + \mathcal{L}_{yuk}.$$

Where $$-\mathcal{L}_{yuk} = \left[\frac{Y_1}{\sqrt{2}}\overline{\Psi}\tilde{H}\left(N_{R_1} + (N_{R_1})^c\right) + h.c\right] + \left(Y_{j\alpha}\overline{N_{R_j}}\tilde{H}^\dagger L_\alpha + h.c.\right)$$

References:

Bhattacharya, Sahoo & Sahu, PRD93, 115040 (2016); PRD96, 035010 (2017);
Bhattacharya, Ghosh, Sahoo & Sahu, Front. In. Phys 7 (2019) 80;
Dutta, Bhattacharya, Ghosh & Sahu, JCAP03, 008 (2021)
Borah, Mahapatra, Nanda, Sahoo & Sahu, JHEP 05 (2024) 096
Borah, Mahapatra & Sahu, Phy;Lett. B831 (2022), 137196
Borah, Dutta, Mahapatra & Sahu, PRD 105 (2022), 075019
….

The electroweak symmetry breaking gives

$$-\mathcal{L}_{mass} = M\overline{\psi_L^0}\psi_R^0 + \frac{1}{2}M_{R_1}\overline{N}_{R_1}(N_{R_1})^c + \frac{m_D}{\sqrt{2}}(\overline{\psi_L^0}N_{R_1} + \overline{\psi_R^0}(N_{R_1})^c) + h.c.$$

Writing these mass terms in the basis $((\psi_R^0)^c, \psi_L^0, (N_{R_1})^c)^T$

we get the following mass matrix:

$$\mathcal{M} = \begin{pmatrix} 0 & M & \frac{m_D}{\sqrt{2}} \\ M & 0 & \frac{m_D}{\sqrt{2}} \\ \frac{m_D}{\sqrt{2}} & \frac{m_D}{\sqrt{2}} & M_{R_1} \end{pmatrix} .$$

The mass matrix can be diagonalised using the mass matrix:

$$\mathcal{U} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\pi/2} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \frac{1}{\sqrt{2}}\cos\theta & \frac{1}{\sqrt{2}}\cos\theta & \sin\theta \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{\sqrt{2}}\sin\theta & -\frac{1}{\sqrt{2}}\sin\theta & \cos\theta \end{pmatrix} .$$

Upon diagonalisation we get 3 mass eigenstates:

$$\chi_{1L} = \frac{\cos\theta}{\sqrt{2}}(\psi_L^0 + (\psi_R^0)^c) + \sin\theta (N_{R_1})^c,$$
$$\chi_{2L} = \frac{i}{\sqrt{2}}(\psi_L^0 - (\psi_R^0)^c),$$
$$\chi_{3L} = -\frac{\sin\theta}{\sqrt{2}}(\psi_L^0 + (\psi_R^0)^c) + \cos\theta (N_{R_1})^c$$

With masses

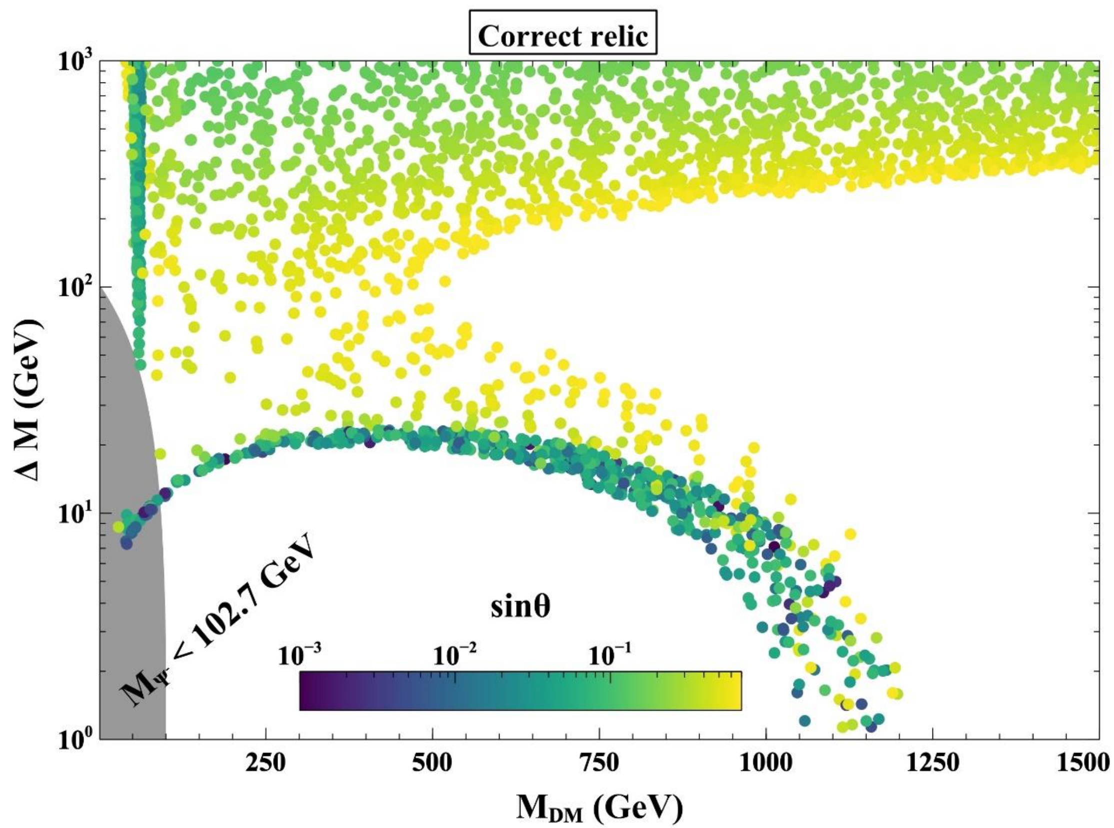
$$m_{\chi_1} = M\cos^2\theta + M_{R_1}\sin^2\theta + m_D \sin 2\theta,$$
$$m_{\chi_2} = M,$$
$$m_{\chi_3} = M_{R_1}\cos^2\theta + M\sin^2\theta - m_D \sin 2\theta$$

The mixing angle is

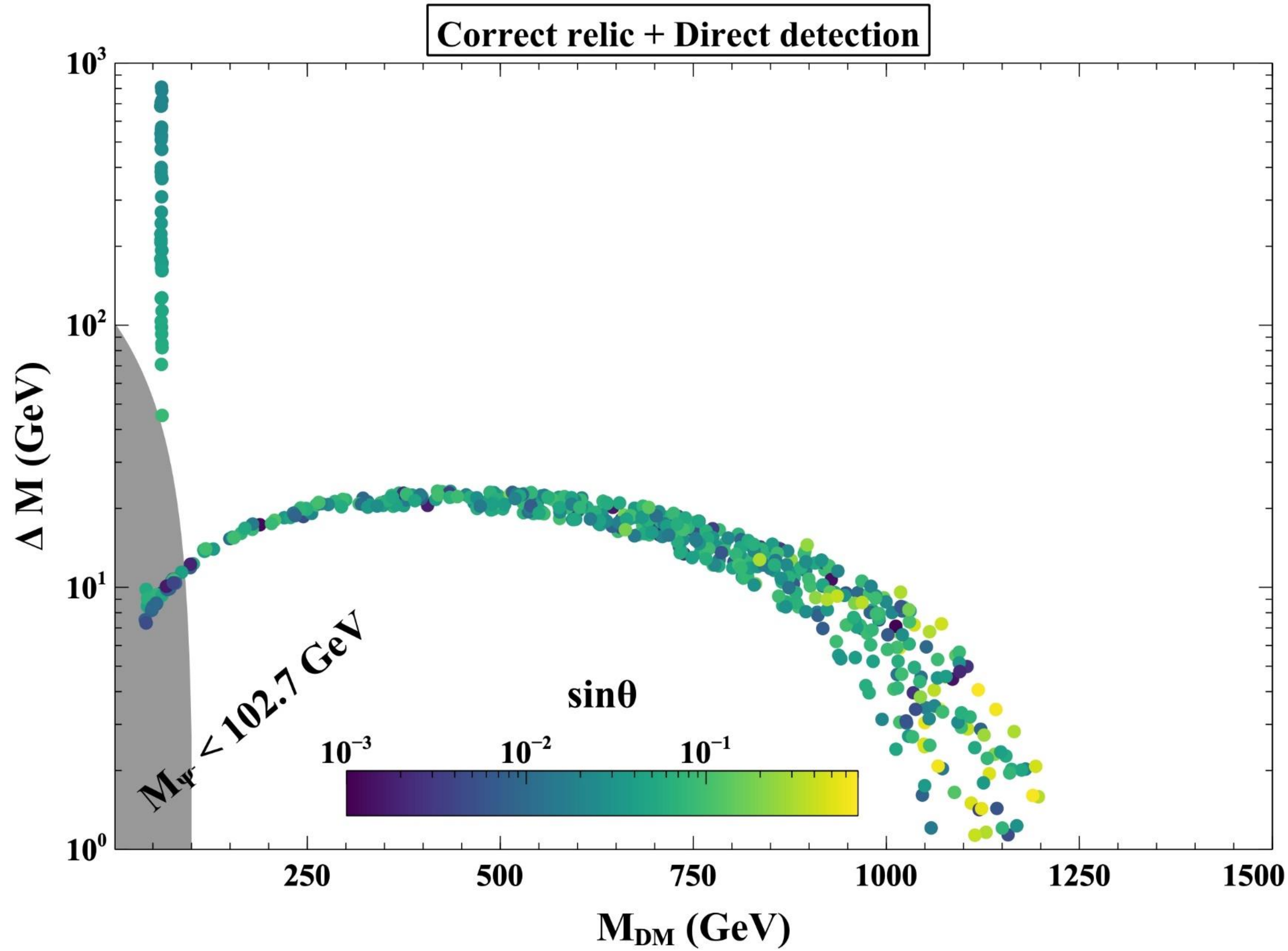
$$\tan 2\theta = \frac{2m_D}{M - M_{R_1}}$$

There are three independent parameters which decide the dark sector relic:

$$\text{Dark Parameters}: \quad \{\ m_{\chi_3}, \Delta M = (m_{\chi_1} - m_{\chi_3}) \approx (m_{\chi_2} - m_{\chi_3}),\ \sin\theta\ \}.$$



$$m_{\chi_3} \equiv M_{DM}$$

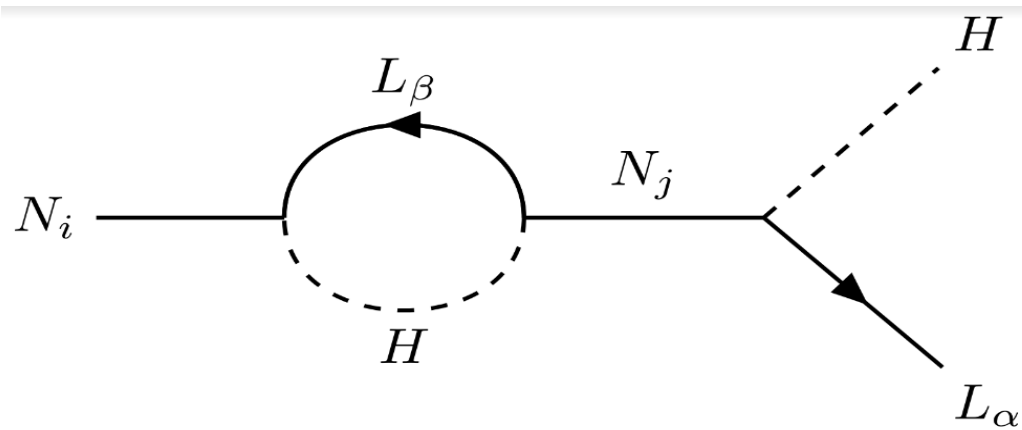


Correct relic + Direct detection
10^3
10^2
10^1
10^0
Δ M (GeV)
250
500
750
1000
1250
1500
M_DM (GeV)
M_Ψ- < 102.7 GeV
sinθ
10^−3
10^−2
10^−1


THANK
YOU

CP-violation through self-energy correction

$$\mathcal{L} \supset M_N \bar{N}^c N + Y_N \bar{L} \tilde{H} N + H.C.$$



$$\epsilon_i(\text{wave func}) = \frac{-1}{8\pi} \sum_{k \neq i} \frac{M_i}{{M_k}^2 - {M_i}^2} \frac{Im\left\{\left[M_k (Y^+Y)_{ki} + M_i (Y^+Y)_{ik}\right] {Y_{jk}}^* Y_{ji}\right\}}{(Y^+Y)_{ii}}$$

Flanz, Paschos & Sarkar, PLB345, 248 (1995)
Covi, Roulet & Vissani, PLB384, 169 (1996)

When $M_k \sim M_i$ then the CP-violation can be large.
This gives rise to the birth of resonant leptogenesis

Flanz, Paschos, Sarkar & Weiss, PLB 389, 693 (1996)

In the limit $M_k = M_i$, the conventional perturbation theory breaks down. So one can use resummation approach to calculate a correct expression:

$$\epsilon_i(wave\ function) = \frac{Im(Y^+Y)^2_{ij}}{(Y^+Y)_{ii}(Y^+Y)_{jj}} \frac{\left(M_i^2 - M_j^2\right) M_i \Gamma_j}{\left(M_i^2 - M_j^2\right)^2 + M_i^2 \Gamma_j^2}$$

In the limit $M_i = M_j$ the CP-asymmetry vanishes.

Pilaftsis, PRD56, 5431 (1997)
Pilaftsis, Nucl. Phys. B504, 61 (1997)
Pilaftsis & Underwood, Nucl. Phys. B692, 303 (2004)
Pilaftsis & Underwood, PRD72, 113001 (2005)

TeV scale leptogenesis and collider signatures