I: Non-Standard Neutrino Interactions **II** Sterile neutrinos and Short Baseline Anomalies

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"Understanding the Universe through neutrinos" ICTS, Bengaluru May 03, 2024

- Raj Gandhi
 - Allahabad

Helpful General References on Part I, NSI

Neutrino Non-Standard Interactions: A Status Report, P. S. Bhupal Dev et al, 1907.00991

Neutrino oscillations and Non-Standard Interactions, Y Farzan and M Tortola, 1710.09360

Non standard neutrino interactions: current status and future prospects, O Miranda and H Nunokawa, 1505.06254

Status of non-standard neutrino interactions, T Ohlsson, 1209.2710



 $\sin^2(\theta_{12}) = 0.307 \pm 0.013$ $\Delta m_{21}^2 = (7.53 \pm 0.18) \times 10^{-5} \text{ eV}^2$ $sin^2(\theta_{23}) = 0.539 \pm 0.022$ (S = 1.1) $sin^2(\theta_{23}) = 0.546 \pm 0.021$ (Normal order) $\Delta m_{32}^2 = (-2.536 \pm 0.034) \times 10^{-3} \text{ eV}^2$ $\Delta m_{32}^2 = (2.453 \pm 0.033) \times 10^{-3} \text{ eV}^2$ $\sin^2(\theta_{13}) = (2.20 \pm 0.07) \times 10^{-2}$ δ , CP violating phase = $1.36^{+0.20}_{-0.16} \pi$ rad

* Are neutrinos Dirac or Majorana particles? \star

★ Is there CP violation in the leptonic sector? \star

★ Is the neutrino mass hierarchy normal or inverted?

* Are there additional light sterile neutrinos?

 \star

(Inverted order)

(Inverted order) (Normal order)

★ What are the absolute masses of neutrinos?

★ Atmospheric, solar, reactor, accellerator based neutrino experiments have helped us determine neutrino mass and mixing parameters to within uncertainties.

★ While it is fairly clear that oscillations are dominantly responsible for observed excesses and deficits seen in the neutrino fluxes in these experiments, subdominant effects that have not yet been ruled out include NSI, decoherence and neutrino decay.



Mass hierarchy of neutrinos





Decoherence

One of the most essential ingredients in making the oscillation scenario work is that the spread in energy ΔE of the neutrino "beam" is not too wide.

If ΔE of the neutrino "beam" is wide then by the time the neutrinos arrive at the detector the oscillation patterns for neutrinos of different energies get sufficiently out of phase to dampen potentially observable oscillations

Decay

In transit, a neutrino may decay invisibly, e.g into sterile neutrinos which do not produce a signal in a detector.

It could decay visibly, say to another SM neutrino, which gets picked up in a downstream detector.

Both these "non-oscillation" mechanisms could modify neutrino signatures in oscillation experiments



Motivations for studying NSI.....

Generically, new interactions and couplings of neutrinos beyond those in the SM are termed and "Non-Standard Interactions" (NSI).

Such interactions could be present in the production, propagation and detection of neutrinos.

They could affect oscillation probability predictions and measurements made by existing and future experiments.

They could impact our efforts to answer the important unanswered questions about neutrinos listed earlier..



Motivations for studying NSI.....

an umbrella categorization

It is also important to realise that existing neutrino data are guite well-described by the standard 3 family oscillation picture and the currently measured mass-squared differences and mixings. Hence additional interactions are very likely small effects, sub-dominant to the present understanding of SM interactions of neutrinos.

Despite their smallness and sub-dominance, it is important to study them because they could point towards physics beyond the SM.

It is important to emphasize that NSI are not necessarily some strange new beasts, but may very well arise from known BSM physics theories, like supersymmetry, GUT models, left-right models, compositeness, extra dimensional theories......The term "NSI" is thus



Standard Neutrino oscillations.....in matter

$$i\frac{\mathrm{d}\nu}{\mathrm{d}t} = \frac{1}{2E} \left[M M^{\dagger} + \mathrm{diag}(A,0,0) \right] \nu \equiv H\nu \,,$$

$M = U Diag \{m1, m2, m3\} U^{T}$ is the neutrino mass matrix,

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = U \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix} ,$$

E is the neutrino energy,

A = 2J 2EGFNe is the effective matter potential

GF = (1.1663787 ± 0.0000006) × 10–5 GeV–2 Ne is the electron number density in matter as matrix,

> U relates the weak interaction eigenstates and the mass eigenstates through the leptonic mixing parameters θ_{12} , θ_{13} , θ_{23} , δ (the Dirac CP-violating phase)

$$\begin{aligned} & \text{Introduction of the second se$$



Standard Two flavour oscillations Standard Two flavour oscillations in matter with NSI in matter

$$P_M(\nu_e \to \nu_\mu) = \sin^2 2\theta^M \sin^2 \left(\Delta m_M^2 \frac{L}{4E}\right)$$

$$\Delta m_M^2 \equiv \Delta m^2 \sqrt{\sin^2 2\theta} + (\cos 2\theta - A)^2$$

$$\sin^2 2\theta^M \equiv \frac{\sin^2 2\theta}{\sin^2 2\theta + (\cos 2\theta - A)^2} ,$$

$$\mathcal{H}_{M} = \frac{\Delta m_{M}^{2}}{4E} \begin{pmatrix} -\cos 2\theta^{M} & \sin 2\theta^{M} \\ \sin 2\theta^{M} & \cos 2\theta^{M} \end{pmatrix} . \quad \varepsilon_{ee}$$

matter.

$$\left| P(\nu_{e} \to \nu_{\tau}) = \sin^{2} 2\theta_{M} \sin^{2} \left(\frac{\Delta m_{N}^{2}}{4E} \right) \right|$$

$$\left(\frac{\Delta m_M^2}{2EA}\right)^2 \equiv \left(\frac{\Delta m^2}{2EA}\cos 2\theta - (1 + \epsilon_{ee} - \epsilon_{\tau\tau})\right)^2 + \left(\frac{\Delta m^2}{2EA}\sin 2\theta + \frac{1}{2EA}\sin 2\theta\right)^2 + \left(\frac{1}{2EA}\cos 2\theta + \frac{1}{2EA}\cos 2\theta\right)^2 + \left(\frac{1}{2EA}\cos 2\theta\right$$

$$\sin 2\theta_{M} \equiv \frac{\Delta m^{2} \sin 2\theta + 4EA\epsilon_{e\tau}}{\Delta m_{M}^{2}}$$

 $\varepsilon_e, \varepsilon_{e\tau}, \varepsilon_{\tau\tau} \to 0$, Gives us back the standard matter oscillations

We thus see that measurements of mass-squared differences and mixing angles can be affected by the presence of NSI when we study neutrino oscillations in



Three flavour oscillations in matter with NSI

$$\begin{split} P(\nu_{\alpha} \to \nu_{\alpha}; L) &= 1 - 4 \sum_{i>j} |\tilde{U}_{\alpha i} \tilde{U}_{\alpha j}^{*}|^{2} \sin^{2} \left(\frac{\Delta \tilde{m}_{ij}^{2} L}{4E}\right) ,\\ P(\nu_{\alpha} \to \nu_{\beta}; L) &= -4 \sum_{i>j} \operatorname{Re} \left(\tilde{U}_{\alpha i}^{*} \tilde{U}_{\beta i} \tilde{U}_{\alpha j} \tilde{U}_{\beta j}^{*}\right) \sin^{2} \left(\frac{\Delta \tilde{m}_{ij}^{2} L}{4E}\right) - 8\mathcal{J} \prod_{i>j} \sin \left(\frac{\Delta \tilde{m}_{ij}^{2} L}{4E}\right) \end{split}$$

$$\begin{aligned} \mathcal{J}^2 &= |\tilde{U}_{\alpha i}|^2 |\tilde{U}_{\beta j}|^2 |\tilde{U}_{\alpha j}|^2 |\tilde{U}_{\beta i}|^2 - \frac{1}{4} \left(1 + |\tilde{U}_{\alpha i}|^2 |\tilde{U}_{\beta j}|^2 + |\tilde{U}_{\alpha j}|^2 |\tilde{U}_{\beta i}|^2 \right) \\ &- |\tilde{U}_{\alpha i}|^2 - |\tilde{U}_{\beta j}|^2 - |\tilde{U}_{\alpha j}|^2 - |\tilde{U}_{\alpha j}|^2 - |\tilde{U}_{\beta i}|^2 \right)^2. \end{aligned}$$

D. Meloni, T. Ohlsson, and H. Zhang, arXiv:0901.1784 T Ohlsson, 1209.2710



Three flavour oscillations in matter with NSI

$$\begin{split} \tilde{U}_{e2} &\simeq \frac{\alpha s_{12} c_{12}}{\hat{A}} + c_{23} \varepsilon_{e\mu} - s_{23} \varepsilon_{e\tau} ,\\ \tilde{U}_{e3} &\simeq \frac{s_{13} e^{-i\delta}}{1 - \hat{A}} + \frac{\hat{A} (s_{23} \varepsilon_{e\mu} + c_{23} \varepsilon_{e\tau})}{1 - \hat{A}} ,\\ \tilde{U}_{\mu 2} &\simeq c_{23} + \hat{A} s_{23}^2 c_{23} \left(\varepsilon_{\tau \tau} - \varepsilon_{\mu \mu} \right) + \hat{A} s_{23} \left(s_{23} \varepsilon_{\mu \tau} - c_{23}^2 \varepsilon_{\mu \tau}^* \right) ,\\ \tilde{U}_{\mu 3} &\simeq s_{23} + \hat{A} \left[c_{23} \varepsilon_{\mu \tau} + s_{23} c_{23}^2 \left(\varepsilon_{\mu \mu} - \varepsilon_{\tau \tau} \right) - s_{23}^2 c_{23} \left(\varepsilon_{\mu \tau} + \varepsilon_{\mu \tau}^* \right) \right] ,\end{split}$$

$$\begin{split} \tilde{m}_1^2 \simeq \Delta m_{31}^2 \left(\hat{A} + \alpha s_{12}^2 + \hat{A} \varepsilon_{ee} \right) , \\ \tilde{m}_2^2 \simeq \Delta m_{31}^2 \left[\alpha c_{12}^2 - \hat{A} s_{23}^2 \left(\varepsilon_{\mu\mu} - \varepsilon_{\tau\tau} \right) - \hat{A} s_{23} c_{23} \left(\varepsilon_{\mu\tau} + \varepsilon_{\mu\tau}^* \right) + \hat{A} \varepsilon_{\mu\mu} \right] , \\ \tilde{m}_3^2 \simeq \Delta m_{31}^2 \left[1 + \hat{A} \varepsilon_{\tau\tau} + \hat{A} s_{23}^2 \left(\varepsilon_{\mu\mu} - \varepsilon_{\tau\tau} \right) + \hat{A} s_{23} c_{23} \left(\varepsilon_{\mu\tau} + \varepsilon_{\mu\tau}^* \right) \right] , \end{split}$$

D. Meloni, T. Ohlsson, and H. Zhang, arXiv:0901.1784 T Ohlsson, 1209.2710



NSI at production at the neutrino source.....

$$\pi^+ \to \mu^+ + \nu_\mu, \quad \mu^+ \to e^+ + \overline{\nu}_\mu + \nu_e, \quad n$$

 $\pi^+ \to \mu^+ + \nu_e, \quad \mu^+ \to e^+ + \overline{\nu}_\mu + \nu_\mu, \quad n \to p + e^- + \overline{\nu}_\mu$

NSI at detection

$$\nu_{\mu} + n \rightarrow p + \mu^{-}$$

$$\nu_e + n \rightarrow p + \mu^-$$

$$\rightarrow p + e^- + \overline{\nu}_e$$

Examples of standard production,

Examples of non-standard production,

Example of standard detection

Example of non-standard detection



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Examining NSI parameters as stemming from an underlying gauge theory

$$H = \frac{1}{2E} \begin{bmatrix} U_{\text{PMNS}} \begin{pmatrix} 0 & & \\ & \Delta m_{21}^2 & \\ & & \Delta m_{31}^2 \end{pmatrix} U_{\text{PMNS}}^{\dagger} + a \begin{pmatrix} 1 + \varepsilon_{ee} & \varepsilon_{e\mu} & \varepsilon_{e\tau} \\ \varepsilon_{e\mu}^* & \varepsilon_{\mu\mu} & \varepsilon_{\mu\tau} \\ \varepsilon_{e\tau}^* & \varepsilon_{\mu\tau}^* & \varepsilon_{\tau\tau} \end{pmatrix} \end{bmatrix}$$

$$\varepsilon_{\alpha\beta} = |\varepsilon_{\alpha\beta}| e^{i\phi_{\alpha\beta}}.$$

While the diagonal terms are real, the off diagonal parameters can be complex, and thus interfere in CP measurements.

One would like to associate an underlying gauge structure for NSIs, as opposed to simply parametrising them effectively as we have done so far.



Example of a simple underlying gauge theory for NSI $\mathcal{L}_{\text{NC-NSI}} = -2\sqrt{2}G_F \,\epsilon_{\alpha\beta}^{fX} \left(\bar{\nu}_{\alpha}\gamma^{\mu}P_L\nu_{\beta}\right) \left(\bar{f}\gamma_{\mu}P_Xf\right)$ 7' $m_{Z'}$ Consider a new Z' with mass M_X $-2\sqrt{2}G_F \,\epsilon^{fX}_{\alpha\beta} \left(\bar{\nu}_{\alpha}\gamma^{\mu}P_L\nu_{\beta}\right) \left(\bar{f}\gamma_{\mu}P_Xf\right)$ associated with a new U(1) group

Then $\varepsilon \sim g_X^2 m_W^2 / m_X^2$. Note that if the new gauge boson is much heavier than the electroweak scale, the couplings may become unobservable small

 $m_X \ll m_W$ and $g_X \ll 1$

Where weak couplings and a low mass gauge boson are combined, one may have observability.

 $\varepsilon \sim g_X^2 m_W^2 / m_X^2.$

Hence, for new physics appearing at $\varepsilon_{\alpha\beta} \sim 10^{-2}(10^{-4})$ 1 (10) TeV, one expects

with gauge coupling q_X

 $Z'_{\mu}\bar{\nu}_{\alpha}\gamma^{\mu}P_{L}\nu_{\beta}\gamma^{\mu}\bar{\nu}_{\alpha}\gamma^{\mu}P_{L}\nu_{\beta}$

 $Z'_{\mu}\bar{f}\gamma^{\mu}P_Xf = Z'_{\mu}\bar{f}\gamma^{\mu}P_Xf$



Putting constraints on NSI.....

Requiring that the new theory preserve $SU(2)_{L} \times U(1)$ gauge invariance at high energies immediately leads to strong constraints Consider the 6-dimensional operator in effective theory which yields an effective NSI parameter leading to, say $arepsilon^{ee}_{e\mu}$

$$\frac{1}{\Lambda^2} (\bar{L}_{\alpha} \gamma^{\rho} L_{\beta}) (\bar{L}_{\gamma} \gamma_{\rho} L_{\delta})$$

 $\mu \to 3e$: BR $(\mu \to 3e) < 10^{-12}$

$$\frac{1}{\Lambda^2} (\bar{\nu}_{\alpha} \gamma^{\rho} P_L \nu_{\beta}) (\bar{\ell}_{\gamma} \gamma^{\rho} P_L \nu_{\beta}) (\bar{\ell}_{\gamma}$$

- To preserve gauge invariance, this must be part of a general $SU(2) \times U(1)$ operator
 - Applying this to an interaction with 4 charged leptons, where constraints are tight,

$$\varepsilon_{e\mu}^{ee} < 10^{-6}$$



Putting constraints on NSI......atmospheric neutrinos

important.

Further, a two flavour approximation can be made,

$$\mathrm{i}\frac{\mathrm{d}}{\mathrm{d}L} \begin{pmatrix} \nu_{\mu} \\ \nu_{\tau} \end{pmatrix} = \frac{1}{2E} \left[U \begin{pmatrix} 0 & 0 \\ 0 & \Delta m^2 \end{pmatrix} U^{\dagger} + A \begin{pmatrix} 1 + \varepsilon_{\mu\mu} & \varepsilon_{\mu\tau} \\ \varepsilon_{\mu\tau} & \varepsilon_{\tau\tau} \end{pmatrix} \right] \begin{pmatrix} \nu_{\mu} \\ \nu_{\tau} \end{pmatrix}$$

 $P(\nu_{\mu} \rightarrow \nu_{\mu}; L) = 1 - P(\nu_{\mu} \rightarrow \nu_{\tau}; L) = 1 - \sin^2(2\Theta) \mathrm{s}$

 $\sin^2(2\Theta) = \frac{1}{R^2} \left[\sin^2(2\theta) + R_0^2 \sin^2(2\xi) + 2R_0 \sin(2\theta) \right]$ $R = \sqrt{1 + R_0^2 + 2R_0 \left[\cos(2\theta)\cos(2\xi) + \sin(2\theta)\right]}$

- Atmospheric neutrinos are very sensitive to matter NSIs, since they travel over
- long distances inside the Earth before being detected, and $\nu_{\mu} \leftrightarrow \nu_{\tau}$ 'oscillations are

$$\sin^2\left(\frac{\Delta m^2 L}{4E}R\right)$$

$$\sin(2\xi)$$
],

$$)\sin(2\xi)]$$

$$R_{0} = \sqrt{2}G_{F}N_{e}\frac{4E}{\Delta m^{2}}\sqrt{|\varepsilon|^{2} + \frac{\varepsilon'^{2}}{4}}$$
$$\xi = \frac{1}{2}\arctan\left(\frac{2\varepsilon}{\varepsilon'}\right).$$

Putting constraints on NSI.....atmospheric neutrinos



Survival probabilities of $\nu_{\mu} \rightarrow \nu_{\mu}$ (upper panels) and $\bar{\nu}_{\mu} \rightarrow \bar{\nu}_{\mu}$ (lower Figure 1. panels) as a function of the neutrino energy for different zenith angle of incoming neutrinos, $\cos \theta_z = -0.3$ (left panels), -0.6 (middle panels) and -1 (right panels). The corresponding distances traveled by neutrinos are indicated in the plots. The normal mass ordering was assumed.

0.9 $\sin^2 2\theta_{23}$ Solid lines: with NSI Dashed lines: Normal

Miranda and Nunokawa, 1505.06254



Putting constraints on NSI......detection of reactor neutrinos

Nuclear reactor experiments usually have short or medium baselines, and hence effects of matter and NSI in propagation can be neglected.

Their energy is around a few MeV,

NSI may thus arise only at source (production) or in the detector.

Constraints will arise based on the deviation in the survival probability standard (known) reactor flux and standard oscillations, i.e. measurements of $\bar{\nu}_e e^- \rightarrow \bar{\nu}_e e^-$ are done at source (LHS) and detector (RHS) and compared to expectations.

Thus at reactors, the relevant NSI parameters which can be constrained are $\varepsilon_{e\mu}^f$ $\varepsilon_{e\tau}^f \varepsilon_{ee}^f$





Figure 4. Averaged differential cross section for the electron anti-neutrino scatterin off electrons for the SM case (black solid line), for a flavor changing NSI (blue dashe line), and for a flavor conserving NSI (green dashed dotted line). The reactor ant neutrino flux has been considered in order to integrate the anti-neutrino cross sectio over the entropriste neutrino energy range

 $\bar{\nu}_e e^- \rightarrow \bar{\nu}_e e^-$

give quadratic

corrections,



Flavour diagonal NSI

give linear corrections,

Miranda and Nunokawa, 1505.06254





Putting constraints on NSI......detection of reactor neutrinos



Figure 5. Allowed region, at 90 % CL, for diagonal NSI parameters, $\varepsilon_{ee}^{L,R}$, from a combined analysis of TEXONO reactor anti-neutrino and LSND neutrino electron scattering off electrons.

The NSI parameters for this reaction can be constrained by considering, for example, the data from the TEXONO collaboration, which use vee scattering as the detection signal

Usually one combines more than one experiment to get better constraints

NSI at low energies.....

The conventional formalism studied so far mainly looks at new physics in neutrinos at high energies.

It assumes heavy particles and loop contribution $\varepsilon \sim g_X^2 m_W^2/m_X^2$.

The absence of new physics discoveries at the LHC have spiked interest in searching for new physics at low energies. Such physics must be weakly (feebly) coupled . It could involve Low energy NSI of neutrinos and may show itself in anomalies at neutrino and DM detectors.

Such NSI of neutrinos may not be easily cast into the formalism studied here.....

It assumes heavy particles and loop contributions which can be integrated out and cast in the



Conclusions.....

Among anomalous observations at neutrino experiments, their flavour transitions are the most studied, using oscillations and matter effects.

At present, a fairly accurate picture of SM neutrino interactions explains all the oscillation data well across many experiments, giving us a handle on their mixings and mass-squared differences.

Given the uncertainties, however, NSI could be playing a subdominant role in oscillations and neutrino scattering. Exploring NSI thus opens a window to new physics at high energy scales.

It could play a subdominant role in the physics of production, detection and propagation of neutrinos in matter.



Conclusions.....cont'd

Analytic expressions for both 2 and 3 flavour standard oscillations in matter can be recast to include NSI. This underlines the fact that measurements made in experiments may hide subdominant effects of new physics.

Unravelling these effects would require the help of multiple experiments due to degeneracies.

Existing neutrino experiments can be used to constrain NSI parameters, e.g. data from atmospheric, reactor, coherent elastic neutrino-nucleus scattering long-baseline accelerator experiments etc.

If BSM physics resides at low energies, a somewhat different approach from the standard NSI parameters discussed here may be necessary.





Contraction of the local

Short Baseline Anomalies and Sterile Neutrinos: Status and Perspectives

Initial remarks.....

Generically, any neutral lepton that is a singlet under the gauge groups of the Standard Model (SM) is loosely referred to as a sterile neutrino. Their mass can range from light to very heavy. Heavier ones are generically called HNLs (heavy neutral leptons)

Sterile neutrinos have come to play an increasingly important role in present-day attempts to take our current understanding beyond the standard model (BSM).

Sterile neutrinos which are heavy (~>> GeV or more) have been invoked to explain the smallness of neutrino masses via the seesaw mechanism.

Over the past couple of decades, a number of anomalous results have been observed in experiments which involve the production and detection of neutrinos over short baselines (< 1 km). Sterile neutrino oscillations of mass ~ eV have been invoked to understand them and comprise a major proposed solution to resolve them.



Initial remarks..... (Contd)

Sterile neutrinos which are heavy (mass >> 100 GeV) have been invoked to explain the baryon asymmetry of the Universe.

Sterile neutrinos which have masses in ~keV or higher range have also been considered as dark matter candidates. (e.g Dodelson-Widrow mechanism)

Finally, most recently, HNLs play an important role in explaining the short baseline anomalies via new physics (non-oscillation) mechanisms.



Anomalies at Short Baselines.....1) The Gallium source Anomaly

neutrinos are captured by $v_e + {}^{71}Ga \rightarrow {}^{71}Ge + e^{-1}Ga$

Baselines over which the decay neutrinos propagate are very short, ~1 m. However, in the latest experiment (BEST) 2 target zones are created, to see evidence of oscillations.

 Radio chemistry for extraction and counting of the 71Ge was developed in SAGE solar measurements, and is well understood

Intense radioactive sources (e.g. Cr. Ar) with well-determined neutrino spectra are used. These





Anomalies at Short Baselines.....2) The Gallium source Anomaly

Earlier experiments, SAGE and Gallex, had reported a deficit in the ____ neutrino flux. (R = 0.87 \pm 0.05)









Giunti and Laveder, 1006.3224



Anomalies at Short Baselines.....1) The Gallium source Anomaly

If one were to understand the SAGE and Gallex results in terms of sterile neutrino oscillations, one would expect these results (shown adjacent) in BEST



Anomalies at Short Baselines.....1) The Gallium source Anomaly



xsecs, 2) source strength, 3) counting efficiency 4) extraction efficiency.



BEST results. The best-fit point is $\sin^2 2\theta = 0.33$, $\Delta m^2 = 1.25$ eV^2 and is indicated by a point.

No clear answer at present.

Anomalies at Short Baselines.....Reactor Antineutrino Anomaly (RAA)

Reactor antineutrinos are produced from beta decays of neutron-rich fission fragments generated by the heavy isotopes 235U, 238U, 239Pu, and 241Pu

The most important antineutrino fluxes are those produced by the fissions of 235U and 239Pu.

The flux measurement from various reactors, was, until recently, on the average, about 3.5% (~ 3σ) lower than predicted from careful calculations done by several groups.

Mueller et al. 1101.2663, Huber 1106.0687, Giunti et al. 2110.06820





CERN

Anomalies at Short Baselines.....Reactor Antineutrino Anomaly (RAA)



Figure 1. Exclusion contours of all reactor experiments in the plane of $\lfloor \sin^2(2\theta_{ee}), \Delta m_{41}^2 \rfloor$ alongside the allowed contours of the RAA and Gallium anomaly as well as Neutrino-4. KATRIN's current and expected exclusion limits are shown in addition. Reprinted from [29] under CC BY 4.0.

- Mainz 95% C.L.
 Troitsk 95% C.L.
 Prospect 95% C.L.
 DANSS 95% C.L.
 Daya Bay 90% C.L.
 Double Chooz 95% C.L.
 STEREO 95% C.L.
- RAA + GA 95% CL
 Neutrino-4 2σ
 KATRIN 95% C.L.
 Projected KATRIN final sensitivity 95% C.L.
 0νββ NH 90% C.L.
 0νββ IH 90% C.L.

Allowed oscillation regions for RAA in strong tension with many exclusion curves of reactors.



Anomalies at Short Baselines......Reactor Antineutrino Anomaly (RAA)

Nuclear databases have been improved in recent years, especially through the application of the Total Absorption Gamma-ray Spectroscopy (TAGS) technique for a better identification of the β decay branches.

This new information was used by Fallot et al [18] (EF model) (1904.09358), and Silaeva et al, 2012.09917 to obtain a 235U reactor antineutrino flux that is smaller than that of the earlier models.

This has led to improved agreement with measured fluxes, and there is now a belief in the community that the RAA has been understood to be a flux calculation/data issue (as opposed to a neutrino deficit issue).





Anomalies at Short Baselines.....MiniBooNE (2002-2017)



Mineral oil detector, 541 m baseline, 600 MeV (vµ) and 400 MeV $(v^{-}\mu)$ peak fluxes.

Three typical event signatures:

- Muon-neutrino CCQE ring on PMTS,
- ring,
- Muon-neutrino NC can -> two fuzzy rings.





Was specifically built to test the LSND anomaly



produces sharp photon - Electron-neutrino CCQE events produces fuzzy

produce π_0 : two gammas



Anomalies at Short Baselines.....MiniBooNE



- The observation of a 4.8σ excess in electron-like events for neutrino and antineutrino modes in the MiniBooNE (MB) detector is observed
- SM: 2309 events Data: 2870 Excess: 560

Excess is not small. Note it is at level of important SM backgrounds



Distinctive energy and angular distribution

Anomalies at Short Baselines.....LSND (1993-1998)



 Observation of unexplained electron-like excesses in the LSND at a level of 3.8σ above SM backgrounds.

Blue hatched region is oscillation fit.







Anomalies at Short Baselines.....LSND and MiniBooNE



• Elongated bullet shaped regions in the above are MB preferred regions. KARMEN2 and OPERA exclude much of these.



• Oscillations? or new physics?

An important point: Both are mineral oil detectors, unable to distinguish electrons from photons or e+e- pairs





Additionally, eV scale sterile neutrinos are constrained by Cosmology.....

Any relativistic neutrino species will contribute to the energy density of the Universe as radiation. Their total contribution may be parametrised by the parameter N_{eff}

Cosmology is sensitive to neutrinos in a way that is complementary to laboratory searches. It is less sensitive to individual masses and mixings, but is more directly affected by the absolute mass scale,

e ρ_r is the total radiation energy density, ρ_{γ} is the photon contribution

$$ho_
u^{
m std}~=~2$$
 >

sterile relativistic neutrino species Also, from PLANCK data,

 $\frac{\rho_r - \rho_\gamma}{\rho_u^{\text{std}}} = N_{\text{eff}} \,,$

$$\sum m_{\nu} < 0.26$$

 $\times \frac{7}{8} \frac{\pi^2}{30} \left(\frac{4}{11}\right)^{4/3} T^4.$

However, N_{eff} = 3.044 +- 0005 in the SM, leaving no space for an additional

eV (95%CL).



Hence, so far, vis a vis sterile-active oscillations of eV neutrinos,

- Situation with the Ga anomaly is somewhat unclear, since BEST confirmed earlier deficit seen by SAGE and GALLEX but did not see any variation with L.
- It is likely that RAA is resolved by the new flux calculations using improved beta decay spectra now available.
- Increasingly accurate cosmological observations put strong constraints on the mass and number of eV scale neutrinos, in tension with what is required to explain SBL anomalies by sterile active oscillations.
- MiniBooNE and LSND appear to be statistically strong and long-standing anomalies which, when we try to explain using sterile -active appearance oscillations, are in strong contradiction with disappearance data from reactors and long baseline experiments.
- This has led to a lot of efforts to explain MiniBooNE (and in some cases, LSND) by nonoscillation new physics mechanisms.

Many of these efforts also use sterile neutrinos which are significantly heavier than the eV scale.



MicroBooNE (to test MB)



80 ton LAr TPC

Excellent particle identification capabilities.

Can potentially distinguish electrons, protons and photons





14 cm

• The combined significance of the LSND and MB results is 6.1σ

Both are long-standing anomalies. The community has worked hard to check • errors and background estimates over this period. Most recent efforts in

this direction come from MicroBooNE

Search for a single-photon excess

Targeting NC Δ resonance radiative decay ($\Delta \rightarrow N\gamma$)

- Standard model process ullet
- Never been directly observed in neutrino scattering
- lacksquare



An enhancement in NC $\Delta \rightarrow N\gamma$ with a multiplicative factor of x3.18 would give good agreement with the observed MiniBooNE LEE

Phys. Rev. Lett. **128**, 111801

Previous best experimental limit at O(1 GeV) is orders of magnitude higher than the prediction





(MicroBooNE 2210.10216)

• Finally, latest results from the MicroBoone detector strongly – disfavour the sterile neutrino hypothesis of electron appearance as a solution to the LSND and MB excesses. Events Observed / Predicted (no eLEE)



MicroBooNE results.....

"These results disfavor the hypothesis that the MiniBooNE low-energy excess originates solely from an excess of ve interactions. Instead, one or more additional mechanisms [45-52] are required to explain the MiniBooNE observations."

(MicroBooNE Collab, 2210.10216)

[45] <u>A.de</u> Gouv[^]ea,O.L.G.Peres,S.Prakash,andG.V. • Stenico, arXiv:1911.01447 [hep-ph].

(Sterile to active decay)

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(Up-scattering and additional Z')

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(Up-scattering and additional Z')

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(Up-scattering and Additional scalars)

• Given the situation as summarised, it makes sense to consider new physics solutions to MB and LSND which involve sterile neutrinos, but not those at the eV scale and involved in active-sterile oscillations.

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New Physics solutions to MB and LSND

• Generic new physics process

NSI, but at low energies



New physics, LSND and MB,

Using an additional Z' and heavier sterile neutrinos, it is possible to get good fits to the MB data Bertuzzo, Jana, Machado & Funchal, 1807.09877; Ballet, Pascoli, Ross-Lonergon 1808.02915; Abdallah, RG and Roy 2006.01948)

However, it is very difficult to explain both LSND and MB simultaneously using these ingredients, because a vector mediator does not give enough events at LSND



LSND MB

Scalar mediators not only avoid HE constraints that vector mediators have difficulty avoiding, but also give enough events at LSND once you get the required number at MB.

higher E, e.g. CHARM II (E_nu ~ 20 GeV and MINERvA, E_nu ~ 4-5 GeV

One learns something new if one demands that the new physics resolve both LSND and MB, as opposed to just MB.



Vector models, given the shape of the xsec, violate constraints by experiments with

By studying the angular distribution at MB for both light and not so light scalar and vector mediators, one discerns the need for both a light and an intermediate mass mediator

(Abdallah, RG and Roy 2202.09373)

The model,

$$V = |\phi_{h}|^{2} \left(\frac{\lambda_{1}}{2} |\phi_{h}|^{2} + \lambda_{3} |\phi_{H}|^{2} + \mu_{1} \right)$$

+ $|\phi_{H}|^{2} \left(\frac{\lambda_{2}}{2} |\phi_{H}|^{2} + \mu_{2} \right) + \lambda_{4} (\phi_{h}^{\dagger} \phi_{H}) (\phi_{H}^{\dagger} \phi_{h})$
+ $\phi_{h'}^{2} \left(\lambda_{2}' \phi_{h'}^{2} + \lambda_{3}' |\phi_{h}|^{2} + \lambda_{4}' |\phi_{H}|^{2} + m' \phi_{h'} + \mu' \right)$
+ $\left[\phi_{h}^{\dagger} \phi_{H} \left(\frac{\lambda_{5}}{2} \phi_{h}^{\dagger} \phi_{H} + \lambda_{6} |\phi_{h}|^{2} + \lambda_{7} |\phi_{H}|^{2} + \lambda_{5}' \phi_{h'}^{2} - \mu_{12} \right) \right]$
+ $\phi_{h'} (m_{1} |\phi_{h}|^{2} + m_{2} |\phi_{H}|^{2} + m_{12} \phi_{h}^{\dagger} \phi_{H}) + h.c. \right], \quad (1)$



$(\phi_h, \phi_H, \phi_{h'})$

Ingredients : 2nd Higgs

doublet + one dark singlet , and 3 RH neutrinos

m_{N_1}	m_{N_2}	m_{N_3}	$y_u^{h'(H)} \times 10^6$	$y_{e(\mu)}^{h'} \times 10^4$	$y_{e(\mu)}^H \times 10^4$
$85\mathrm{MeV}$	$130\mathrm{MeV}$	$10\mathrm{GeV}$	0.8(8)	0.73(1.6)	7.25(15.9)
$m_{h'}$	m_H	$\sin\delta$	$y_d^{h'(H)} \times 10^6$	$y_{\nu_{i2}}^{h'(H)} \times 10^3$	$\lambda_{N_{12}}^{h'(H)} \! \times \! 10^3$
$15 \mathrm{MeV}$	$750\mathrm{MeV}$	0.1	0.8(8)	1.3(12.4)	7.5(74.4)

TABLE I: Benchmark parameter values used for event generation in LSND, MB and for calculating the muon g-2.

(Abdallah, RG and Roy 2010.06159)

Results.....



MB

Conclusions.....

- clarify the situation. No clear resolution has yet emerged, however.
- sterile oscillations
- the RAA.
- tension between appearance and disappearance data.

The MB and LSND anomalies persist with a high combined statistical significance of 6.1 sigma

Short baseline anomalies like the Ga source anomaly, the RAA, LSND and MB have reached a stage where a host of complementary experiments and theoretical inputs have helped gradually

• The situation with the Ga anomaly is unclear, given that the most recent experiment, BEST, verifi the presence of the deficit but could not detect any L variation, which would have signalled active

• Improved data on beta spectra and consequent improved flux calculations point to a disappearance

Attempts to understand the anomalies using oscillations with eV scale neutrinos show a very strong



Conclusions.....

MicroBooNE has recently made important (but not conclusive) strides in helping establish that SM backgrounds are unlikely to be responsible for the MB signal, strengthening the case that MB and possibly LSND could be signals for new physics.

It is significant that sterile neutrinos (much) heavier than ~eV play a role in most new physics proposals put forward to explain the anomaly (anomalies)

A definitive resolution must await results from the Fermilab Short Baseline Program, with its 3 detectors , MicroBooNE, ICARUS and SBND.



Short Baseline Neutrino Program at Fermilab



ICARUS



Three detectors sampling the *same neutrino beam* at different distances

Anne Schukraft talk at Neutrino 2022



SBN Oscillation Sensitivity

- SBND + ICARUS will test the sterile neutrino hypothesis can cover the parameter space favored by past anomalies with 5σ significance
- Observing neutrino flux at different distances from the beam target
- Effective systematics constraint through near detector (SBND) and same detector technology in near and far detector



Anne Schukraft talk at Neutrino 2022

Search for appearance of v and disappearance of v within the same experiment current results show a 4.7 σ tension between V_a appearance and V_u disappearance channels

Thank you for your attention!

the second s



Coherent neutrino Scattering and NSI.....

COHERENT collaboration

$$\frac{d\sigma}{dT} = \frac{G_F^2 Q_{SM}^2 M}{4\pi} \left(1 - \frac{T}{T_{\text{max}}} \right),$$
$$Q_{SM}^2 = \left[N - (1 - 4s_W^2) Z \right]^2, \ T_{\text{max}} = \frac{2E_\nu^2}{M + 2E_\nu},$$

Z are the neutron and proton numbers of the nucleus.



Coherent elastic neutrino-nucleus scattering (CEvNS), which has been recently observed by the

where M is the mass of the nucleus, T is the recoil energy of the nucleus, Tmax is the maximal recoil energy that can be generated for a certain value of neutrino energy, N and



$$Q_{\rm NSI}^2 \equiv 4 \left[N(-\frac{1}{2} + \varepsilon_{ee}^{uV} + 2\varepsilon_{ee}^{dV}) + Z(\frac{1}{2} - 2s_W^2 + 2\varepsilon_{ee}^{uV} + \varepsilon_{ee}^{dV}) \right]^2 + 4 \sum_{\alpha = \mu, \tau} \left[N(\varepsilon_{\alpha e}^{uV} + 2\varepsilon_{\alpha e}^{dV}) + Z(2\varepsilon_{\alpha e}^{uV} + \varepsilon_{\alpha e}^{dV}) \right]^2.$$

to the potentially high statistics.

For NSI, what CEvNS can measure is actually Q2_{NSI}, which leads to a lot of degeneracy among those NSI parameters. From studies one can summarize the current COHERENT constraints on NSIS, which are that generally around O(0.5). In the future, CEVNS experiments based on reactor sources can provide very strong constraints on NSIs, due

An example MiniBooNE anomaly



- The observation of a 4.8σ excess in electron-like events for neutrino and antineutrino modes in the MiniBooNE (MB) detector is observed
- SM: 2309 events Data: 2870 Excess: 560

Excess is not small. Note it is at level of important SM backgrounds



Distinctive energy and angular distribution

The model,

$$V = |\phi_{h}|^{2} \left(\frac{\lambda_{1}}{2} |\phi_{h}|^{2} + \lambda_{3} |\phi_{H}|^{2} + \mu_{1} \right)$$

+ $|\phi_{H}|^{2} \left(\frac{\lambda_{2}}{2} |\phi_{H}|^{2} + \mu_{2} \right) + \lambda_{4} (\phi_{h}^{\dagger} \phi_{H}) (\phi_{H}^{\dagger} \phi_{h})$
+ $\phi_{h'}^{2} \left(\lambda_{2}' \phi_{h'}^{2} + \lambda_{3}' |\phi_{h}|^{2} + \lambda_{4}' |\phi_{H}|^{2} + m' \phi_{h'} + \mu' \right)$
+ $\left[\phi_{h}^{\dagger} \phi_{H} \left(\frac{\lambda_{5}}{2} \phi_{h}^{\dagger} \phi_{H} + \lambda_{6} |\phi_{h}|^{2} + \lambda_{7} |\phi_{H}|^{2} + \lambda_{5}' \phi_{h'}^{2} - \mu_{12} \right) \right]$
+ $\phi_{h'} (m_{1} |\phi_{h}|^{2} + m_{2} |\phi_{H}|^{2} + m_{12} \phi_{h}^{\dagger} \phi_{H}) + h.c. \right], \quad (1)$



$(\phi_h, \phi_H, \phi_{h'})$

Ingredients :

2nd Higgs doublet + one dark singlet , and 3 RH neutrinos MeV energies, both for beam and particle masses

m_{N_1}	m_{N_2}	m_{N_3}	$y_u^{h'(H)} \times 10^6$	$y_{e(\mu)}^{h'} \! imes \! 10^4$	$y_{e(\mu)}^H \times 10^4$
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$15\mathrm{MeV}$	$750\mathrm{MeV}$	0.1	0.8(8)	1.3(12.4)	7.5(74.4)

TABLE I: Benchmark parameter values used for event generation in LSND, MB and for calculating the muon g-2.

(Abdallah, RG and Roy 2010.06159)

Standard Neutrino oscillations.....in the vacuum

$$P(\nu_e \to \nu_\mu; L) = \sin^2(2\theta) \sin^2\left(\frac{\Delta m^2 L}{4E}\right),$$
$$P(\nu_e \to \nu_e) = 1 - P(\nu_e \to \nu_\mu) = 1 - P(\nu_\mu \to \nu_\mu) = 1 - P(\nu_\mu \to \nu_\mu)$$

$$P(\nu_{\alpha} \to \nu_{\beta}; L) = \delta_{\alpha\beta} - 4 \sum_{i>j} \operatorname{Re}(U_{\alpha i}^{*} U_{\beta i} U_{\alpha j} U_{\beta j}^{*}) \sin^{2}\left(\frac{\Delta m_{ij}^{2} L}{4E}\right) + 2 \sum_{i>j} \operatorname{Im}(U_{\alpha i}^{*} U_{\beta i} U_{\alpha j} U_{\beta j}^{*}) \sin\left(\frac{\Delta m_{ij}^{2} L}{2E}\right) ,$$

 $\to \nu_e) = P(\nu_\mu \to \nu_\mu) \,,$

α, β = e, μ, τ.



Putting constraints on NSI.....accelerator neutrinos, (MINOS)



Figure 3. energy for the MINOS baseline, L = 730 km without the presence of NSI with and with NSI, $\varepsilon_{\mu\tau} = \pm 0.1$. The matter density was assumed to be constant, $\rho = 3.2 \text{ g/cm}^3$.

 $\nu_{\mu} \rightarrow \nu_{\mu}$ and $\bar{\nu}_{\mu} \rightarrow \bar{\nu}_{\mu}$ survival probabilities as a function of neutrino

 $-0.067 < \varepsilon_{\mu\tau} < 0.023$ at 90% CL.

Miranda and Nunokawa, 1505.06254



Standard Neutrino oscillations.....in the vacuum

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = U \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13} e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13} e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} e^{i\rho} & 0 & 0 \\ 0 & e^{i\sigma} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

where $c_{ij} \equiv \cos(\theta_{ij})$ and $s_{ij} \equiv \sin(\theta_{ij})$.

U relates the weak interaction eigenstates and the mass eigenstates through the leptonic mixing parameters θ12, θ13, θ23, δ (the Dirac CP-violating phase), as well as ρ and σ (the Majorana CP-violating phases).

,

Two flavour oscillations with NSI

$$i\frac{d}{dL}\left(\begin{array}{c}\nu_{e}\\\nu_{\tau}\end{array}\right) = \left[\frac{1}{2E}U\left(\begin{array}{c}0&0\\0&\Delta m^{2}\end{array}\right)U^{\dagger} + A\left(\begin{array}{c}1+\epsilon_{ee}&\epsilon_{e\tau}\\\epsilon_{e\tau}&\epsilon_{\tau\tau}\end{array}\right)\right]\left(\begin{array}{c}\nu_{e}\\\nu_{\tau}\end{array}\right)$$

$$P(\nu_{e} \to \nu_{\tau}) = \sin^{2} 2\theta_{M} \sin^{2} \left(\frac{\Delta m_{M}^{2} L}{4E}\right)$$

$$\left(\frac{\Delta m_M^2}{2EA}\right)^2 \equiv \left(\frac{\Delta m^2}{2EA}\cos 2\theta - (1 + \epsilon_{ee} - \epsilon_{\tau\tau})\right)^2 + \left(\frac{\Delta m^2}{2EA}\sin 2\theta + 2\epsilon_{e\tau}\right)$$

$$\sin 2\theta_{M} \equiv \frac{\Delta m^{2} \sin 2\theta + 4EA\epsilon_{e\tau}}{\Delta m_{M}^{2}}$$

T Ohlsson, 1209.2710

Two flavour oscillations with NSI

;
$$\varepsilon_{ee}, \varepsilon_{e\tau}, \varepsilon_{\tau\tau} \to 0$$
,

Note that as

$$\left(\Delta \tilde{m}_0^2\right)^2 = \left[\Delta m^2 \cos(2\theta) - A\right]^2 + \left[\Delta m^2 \sin(2\theta)\right]^2,$$
$$\sin\left(2\tilde{\theta}_0\right) = \frac{\Delta m^2 \sin(2\theta)}{\Delta \tilde{m}_0^2},$$

oscillations in matter.

, i.e. as NSI effects disappear, we get

Thus, we get back the SM MSW matter oscillations.

We thus see that measurements of mass-squared differences and mixing angles can be affected by the presence of NSI when we study neutrino



Putting constraints on NSI......detection of reactor neutrinos

In general, somewhat complicated analytical expressions which mimic the form of SM probabilities are possible

$$\varepsilon_{e\alpha}^s = \varepsilon_{\alpha e}^{d*} = |\varepsilon_{e\alpha}| \mathrm{e}^{\mathrm{i}\phi_{e\alpha}}$$

$$\begin{split} \tilde{s}_{13}^2 &= s_{13}^2 + 2s_{13}c_{13} \left[s_{23}\cos(\delta - \phi_{e\mu}) |\varepsilon_{e\mu}| + c_{23}\cos(\delta - \phi_{e\tau}) |\varepsilon_{e\tau}| \right] \\ &- s_{23}\cos(\delta - \phi_{ee} - \phi_{e\mu}) |\varepsilon_{ee}| |\varepsilon_{e\mu}| - c_{23}\cos(\delta - \phi_{ee} - \phi_{e\tau}) |\varepsilon_{ee}| |\varepsilon_{e\tau}| \\ &+ (s_{23}^2 c_{13}^2 - s_{13}^2) |\varepsilon_{e\mu}|^2 + (c_{23}^2 c_{13}^2 - s_{13}^2) |\varepsilon_{e\tau}|^2 \\ &+ 2s_{23}c_{23}c_{13}^2\cos(\phi_{e\mu} - \phi_{e\tau}) |\varepsilon_{e\mu}| |\varepsilon_{e\tau}| + \mathcal{O}(\varepsilon^3) \;, \end{split}$$

$$P(\bar{\nu}_{e}^{s} \to \bar{\nu}_{e}^{d}) = 1 - \cos^{4} \tilde{\theta}_{13} \sin^{2} 2\tilde{\theta}_{12} \sin^{2} \frac{\Delta m_{21}^{2} L}{4E} - \cos^{2} \tilde{\theta}_{12} \sin^{2} 2\tilde{\theta}_{13} \sin^{2} \frac{\Delta m_{31}^{2} L}{4E} - \sin^{2} \tilde{\theta}_{12} \sin^{2} 2\tilde{\theta}_{13} \sin^{2} \frac{\Delta m_{32}^{2} L}{4E} .$$

ection of reactor neutrinos cal expressions which mimic the form of