Neutrino Scattering -Why and How

Jonathan Paley Fermilab Neutrino Division

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Jonathan M. Paley

Please Note...

- I sincerely apologize that I cannot be there in person to meet and talk with you all!
- Since this is a school:
 - I have some questions throughout my slides that I would like you to answer afterward.
 - Please send me at least 1 follow-up question after each lecture. I will answer at least 1 question from each of you and post them in Slack (#flux-and-xsec-exp).
 - You may send me questions either via email (jpaley@fnal.gov) or Slack (@Jon Paley)



Outline

- Why do we care about neutrino scattering?
- The role of neutrino event generators
- What goes into a cross section measurement?
- How to avoid cross section model dependency and model bias?





• Remember, we "see" neutrinos because they scatter off nuclei, producing charged particles that deposit energy in our detectors.





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- The probability that a neutrino scatters is the cross-section.





- We have to select our signal interactions (eg, $v_{\mu}CC$ interactions), but our selection is imperfect. The rate at which we select signal events is our *efficiency*.
- The efficiency depends on the differential cross section for producing all the finalstate particles for all interactions at a given energy.





• We don't know the energy of the neutrino coming in, so we have to reconstruct it based on the measurements of the final-state particles we see.

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• Smearing matrix accounts for unobserved particles and detector resolution.



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Event Generators



- Current neutrino experiments cover nearly two orders of magnitude of neutrino energies.
- Life is made more interesting because over this range, there are several types of scattering modes.





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| | Initial State | QE | 2p2h | Res | DIS | FSI |
|-------------------|------------------|-----------------------------|-----------------------------|-------|----------|--------------------------------------|
| GENIE v3.00.06 | LFG | Valencia (Nieves, et al) | Valencia (Nieves, et al) | B-S | PYTHIA 6 | hN |
| NEUT 5.4.0 | LFG | Valencia (Nieves, et al) | Valencia (Nieves, et al) | B-S | PYTHIA 5 | Oset (low mom. pions) + ext. data |
| NuWro 2019 | LFG | L-S + RPA | Valencia (Nieves, et al) | NuWro | PYTHIA 6 | Oset (pions) + NuWro (nucleons) |
| GiBUU 2019 | LFG | GiBUU Model | | | | BUU equations |

- Generators use very similar (often the same) models for exclusive differential cross sections. However, their implementation can be quite different.
- The models then have to be stitched together:

$$\sigma_{\rm CC}^{\rm inclusive}(E_{\nu}) = \sigma_{\rm CC}^{\rm QE} + \sigma_{\rm CC}^{\rm MEC} + \sigma_{\rm CC}^{\rm Res} + \sigma_{\rm CC}^{\rm DIS} + \sigma_{\rm CC}^{\rm Coh}$$



C. Bronner, NuSTEC 2018 Workshop Presentation



- Implementation and stitching differences between the generators is reflected in the spread of inclusive predictions from various generators.
- Cross section measurements are critical to improve our understanding of the individual processes and how all the pieces fit together.
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- Elastic scattering is a measure of the strength of a field
- Inelastic scattering is a measure of the internal structure of the target



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Electron scattering from carbon atom









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- Elastic scattering is a measure of the strength of a field
- Inelastic scattering is a measure of the internal structure of the target
- For a single target (eg, nucleon):

$$\sigma = \frac{N_{\rm int}}{\Phi}$$

where

 N_{int} = number of interactions Φ = number of incoming particles/unit area





Electron scattering from carbon atom





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- Elastic scattering is a measure of the strength of a field
- Inelastic scattering is a measure of the internal structure of the target
- For a "real" target made of many nuclei:

$$\sigma = \frac{N_{\rm int}}{\Phi N_{\rm tar}}$$

where

N_{int} = number of interactions

- N_{tar} = number of nuclear targets
- Φ = number of incoming particles/unit area





Electron scattering from carbon atom





In a real experiment, we have to "select" what we think are our signal interactions from the data. This is an imperfect process,

and so we have some corrections to make:

What is a cross section?

where

$$\sigma = \frac{N_{\rm int}^{\rm set} P}{\epsilon \Phi N_{\rm tar}}$$

a real m



$$\epsilon = \frac{N_{\rm int}^{\rm true, sel}}{N_{\rm int}^{\rm true}}$$

N^{sel}int = number of selected interactions

N_{tar} = number of nuclear targets

- Φ = number of incoming particles/unit area
- P = "purity" of the selection (background subtraction)
- ε = "efficiency" of the selection
- We often rely on our simulations to determine the efficiency and purity. One must never forget:

SIMULATION IS ALWAY WRONG

• The important question is "how wrong is it" (we need to quantify our uncertainty!), and can we develop a measurement that is minimally sensitive to the biases in the simulation?



 Total cross sections are nice to have, but what we really want and need in order to improve our neutrino scattering models are differential cross sections:

$$\left(\frac{d\sigma}{dx}\right)_{i} = \frac{\sum_{j} U_{ij}^{-1} (N_{j}^{\text{sel}} P_{j})}{\epsilon_{i} \Phi N_{\text{tar}} \Delta x_{i}}$$

$$P = \frac{N_{\rm int}^{\rm true, sel}}{N_{\rm int}^{\rm sel}}$$

$$\epsilon = \frac{N_{\rm int}^{\rm true, sel}}{N_{\rm int}^{\rm true}}$$

where

- x = some useful variable
- i = ith bin in "true" space
- $j = j^{th}$ bin in "reconstructed" space
- N^{sel}_j = number of selected interactions
- P_j = "purity" of the selection (background subtraction) in reco space
- U_{ij} = smearing matrix, true -> reco
- ε_i = "efficiency" of the selection in true space
- N_{tar} = number of nuclear targets
- Φ = number of incoming particles/unit area



What Variables to Report?

- First, we must define our signal.
- Theorists and model builders typically think in terms of "QE", "Resonance", "2p2h", etc. But final-state interactions (eg, pion absorption or charge exchange) and our own detector limitations (resolution), it is impossible for us to measure these processes directly! Eg: consider a case where we see only one muon and one proton in the final state. This could be:
 - a CC QE interaction or,
 - a CC Res interaction where the pion is absorbed in the nucleus or,
 - a 2p2h interaction where one proton has energy below our detection threshold (100 MeV)
- Instead, we should be honest and clear about what we are measuring, eg: "CC interactions with a single proton about 100 MeV in the final state".





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$$\left(\frac{d\sigma}{dx}\right)_{i} = \frac{\sum_{j} U_{ij}^{-1}(N_{j}^{\mathrm{sel}}P_{j})}{\epsilon_{i}\Phi N_{\mathrm{tar}}\Delta x_{i}}$$

- Again, theorists love to see cross sections reported as functions of E_v, Q² and W, but these are all cross-section modeldependent variables, which:
 - Makes them hard to interpret at facevalue
 - Can introduce potential bias
- The cleanest measurements are those that report the final-state particle kinematics, eg those that we can measure directly:
 - lepton energy and angle (or longitudinal and transverse momenta)
 - hadron energy and angle (or longitudinal and transverse momenta)



Developing and Optimizing The Event Selection

- Event selection is all about maximizing both your efficiency and purity.
- Best to use observables that characterize particles in the final-state, eg, particle-id based on dE/dx, scattering, time-of-flight, Ckov light, etc.
- Eg, in NOvA, we use dE/dx and scattering information of the reconstructed charged particle trajectories to isolate muons from other particle:



 Figure of Merit (FoM) is used to maximize sensitivity of the measurement:

$$\left(\frac{\delta\sigma}{\sigma}\right)^2 = \frac{1}{N_{\rm int}^{\rm sel}} + \left(\frac{\delta P}{P}\right)^2 + \left(\frac{\delta\epsilon}{\epsilon}\right)^2$$



Understanding and Constraining the Selection Efficiency

 Be sure to check that the selection efficiency doesn't drop too strongly for the things you are measuring. Eg, if muons cannot be identified below a certain energy (say, 400 MeV), then consider changing the phase space of your signal to include only muons above this threshold.

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- Be sure to check that the selection efficiency doesn't depend too strongly on things that you are not measuring but also have large uncertainties. Eg:
 - Muon selection efficiency as a function of hadronic energy in the final-state
 - Pion selection efficiency as a function of lepton momentum transfer (Q²)
- Whenever possible, compare your efficiency with real data (but not the data you are using to make your measurement). Eg:
 - Check muon selection with cosmic rays
 - · Check EM shower selection with bremsstrahlung showers of cosmic rays
 - Q: What are some other sources of data in neutrino detectors that we can use for this?



With $a_i = N_i - B_i$, and $b_i = \frac{1}{\epsilon_i}$

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Constraining Backgrounds

- Remember, SIMULATIONS ARE ALWAY WRONG, and in the case of neutrino interactions, the uncertainties in our backgrounds can be quite large.
- When the backgrounds are significant, a general approach adopted by most experiments is to use data "sidebands" (events that are not selected) to validate the modeling of, or even constrain the backgrounds.
- Ideally the events in the sideband have similar or overlapping kinematics as the background in the signal selection. But this can be tricky, since background events that "look" like your signal were probably already selected!
- Nevertheless, sidebands can be used to not only validate the simulation, they can be used to reduce the uncertainty associated with modeling the backgrounds.
- Some examples:
 - $v_{\mu}\,CC\,\pi\!0$ interactions when measuring NC $\pi\!0$ or $v_{e}\,CC$ interactions
 - NC π + interactions when measuring CC π +





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- Some examples:
 - $v_{\mu}\,CC\,\pi\!0$ interactions when measuring NC $\pi\!0$ or $v_{e}\,CC$ interactions
 - NC $\pi\text{+}$ interactions when measuring CC $\pi\text{+}$
 - Q: What are some other examples?





Analysis Variables and Binning

- As I mentioned earlier, measurements involving direct observables (eg, measured kinematics of final-state particles) are the least susceptible to the impact of model bias.
- But that does not mean that we should never look at derived variables! Studyir $Q^2 = -(P_{\mu} P_{\nu})^2$ behaves as a function of E that relies on some reasor $= \frac{2E_{\nu}}{c} \left(\frac{E_{\mu}}{c} p_{\mu} \cos \theta_{\mu}\right) m_{\mu}^2 c^2$ qualitatively informative.

$$W = rac{1}{c} |P_N + P_
u - P_\mu|$$

 $= rac{1}{c} \sqrt{m_N^2 c^2 - Q^2 + 2m_N (E_
u)}$

$$\left(\frac{d\sigma}{dx}\right)_{i} = \frac{\sum_{j} U_{ij}^{-1} (N_{j}^{\text{sel}} F)}{\epsilon_{i} \Phi N_{\text{tar}} \Delta x_{i}}$$



da/dW [10⁻³⁹cm²/(GeV/c²)

$$^{2} = -(P_{\mu} - P_{\nu})^{2}$$

$$= \frac{2E_{\nu}}{c} \left(\frac{E_{\mu}}{c} - p_{\mu} \cos \theta_{\mu}\right) - m_{\mu}^{2}c^{2}$$

$$W = \frac{1}{c}|P_{N} + P_{\nu} - P_{\mu}|$$

$$= \frac{1}{c} \sqrt{m_{N}^{2}c^{2} - Q^{2} + 2m_{N}(E_{\nu} - E_{\mu})},$$

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- We also need to take care when deciding how to bin (discretize) our data. Bin-widths should:
 - Never be smaller than our detector resolution.
 - Consider bin-to-bin migration due to systematic uncertainties. Events in a distribution with a rapidlychanging slope will migrate asymmetrically across bins and can result in magnifying the effect!



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 - Q: Will smaller or wider bins avoid this issue?



Unfolding... A Necessary Evil?

• Our detectors have finite resolution. Furthermore, we have to reconstruct the events in our detector, and our algorithms can systematically get things wrong.

250

200

150

100

50

Reco Muon Kinematics Bin

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- We wish to report measurements that are useful to the community, therefore we need to "convert" our reconstructed observable to a "true" observable.
- We rely on our simulations to get our "smearing right", as it can be a very complicated process and often not "Gaussian" in nature.
- We can construct smearing matrix by recording the reconstructed variable as a function of its true value.
- We then have to "undo" the smearing, which is an inverse problem, and ill-posed!

NOvA Simulation

8000

6000

🚰 Fermilab





Unfolding... A Necessary Evil?

- I like to think of this as starting with a blurry image and trying to extract sharp details from it.
 - The blurred image has less information.
 - To recover, one must make some assumptions. In our case, the assumptions are our model.
- But even if the model were perfect, we can't simply "invert" the matrix. This can give disastrous results!
 - Bin-to-bin correlations and limited statistics can introduce wild oscillatory behavior in the unfolded spectrum.
 - One has to apply some kind of dampening to reduce these effects.
 - The level of dampening is often left to the discretion of the analyzer.



To Unfold or Not To Unfold? That is the question...



- Alternatively, we can simply measure our event rate and provide the community the rest of the information they need to compare predictions.
- Note, both involve unavoidable model-dependencies. Again, the challenge is to keep this to a minimum.
- In both cases, it is important to make all of the pieces that go into a measurement available, as they may be needed for future re-analysis.



Tomorrow, I will cover what we know, and what we know we don't know, about neutrino-nucleus scattering at the GeV scale.

Please send me questions via email or Slack!

