Perturbation spreading in a non-reciprocal classical isotropic magnet

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Introduction

- Classical Heisenberg spins: precessional Hamiltonian dynamics $\rightarrow$ thermalisation, spin diffusion
- This work: effect of minimal nonequilibrium dynamics via non-reciprocal exchange coupling (J. Das et al. EPL 2002)
- Time-evolution of the overlap between spin configuration and perturbed copy: propagating decorrelation front as in Hamiltonian case (A. Das et al. PRL 2018)
- Characterise nonequilibrium nature of the system through energy dissipation

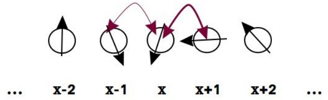
Equations of motion

- Spins of unit length precess in a local field as

$$\dot{\boldsymbol{S}}_x = \boldsymbol{S}_x \times (J_{x,x-1}\boldsymbol{S}_{x-1} + J_{x+1,x}\boldsymbol{S}_{x+1}) \tag{1}$$

- A non-reciprocal exchange coupling $J_{x,x+1} \neq J_{x+1,x}$ cannot be obtained from Hamiltonian

$$H = -\sum J_{x,x+1}\boldsymbol{S}_x \cdot \boldsymbol{S}_{x+1}$$



- Taking the simplest, extreme limit $J_{x,x+1} = -J_{x+1,x}$ we study the driven dynamics of a Heisenberg spin chain

$$\dot{\boldsymbol{S}}_x = \mu \boldsymbol{S}_x \times (\boldsymbol{S}_{x+1} - \boldsymbol{S}_{x-1}) \tag{2}$$

$x = 0, 1, ..., L-1$
and compare it with the classical Hamiltonian dynamics

$$\dot{\boldsymbol{S}}_x = \lambda \boldsymbol{S}_x \times (\boldsymbol{S}_{x+1} + \boldsymbol{S}_{x-1}) = \{\boldsymbol{S}_x, H\} \tag{3}$$

Decorrelator and OTOC analogy

- Our system is chaotic at infinite temperature. The chaos can be quantified by measuring the divergence of the dynamical trajectories.
- The classical Out-of-Time Ordered Correlator is one such quantity

$$F(t) = -\langle \{A(x,t), B(0,0)\}^2 \rangle$$

- We define the decorrelator as the deviation of a spin configuration from its perturbed copy under a time evolution, averaged over an infinite temperature thermal distribution. (A. Das *et al*)

$$D(\boldsymbol{x}, t) = \frac{1}{2} \langle \delta \boldsymbol{S}(x,t)^2 \rangle \tag{4}$$

Decorrelator and OTOC analogy

- A copy of the initial configuration (a) is perturbed at a single site

$$\delta \boldsymbol{S}_0^b(0) = \boldsymbol{S}_0^a(0) + \delta \boldsymbol{S}_0^a$$
$$\boldsymbol{S}_x^b(0) = \boldsymbol{S}_x^a(0) \quad \forall x \neq 0$$

$$\delta \boldsymbol{S}_0 = \varepsilon(\hat{\mathbf{n}} \times \boldsymbol{S}_0)$$
$$\hat{\mathbf{n}} = (\hat{\mathbf{z}} \times \boldsymbol{S}_0)/|\hat{\mathbf{z}} \times \boldsymbol{S}_0| \tag{5}$$

where $\varepsilon \rightarrow$ perturbation strength, $\hat{\mathbf{z}}$ is a unit-vector along the global spin z-axis.

- The variation of a spin at site x depends on ε, as

$$\delta S_x^\alpha(t) \approx \frac{\partial S_x^\alpha(t)}{\partial S_0^\beta}\delta S_0^\beta = \varepsilon n^\gamma \epsilon_{\beta\gamma\nu} S_0^\nu \frac{\partial S_x^\alpha(t)}{\partial S_0^\beta} = \varepsilon n^\gamma \{S_x^\alpha(t), S_0^\gamma(0)\}$$

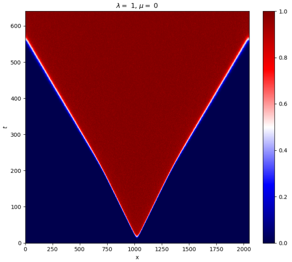
allows us to write decorrelator in a form similar to the OTOC

$$D(\boldsymbol{x}, t) = \frac{1}{2}\langle \delta \boldsymbol{S}(x,t)^2\rangle \approx \frac{\varepsilon^2}{2}\langle \{\boldsymbol{S}_x(t), \hat{\mathbf{n}} \cdot \boldsymbol{S}_0\}^2\rangle \tag{6}$$

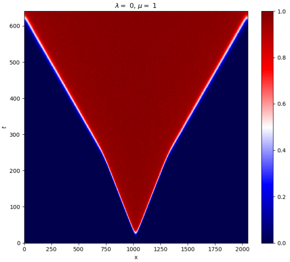
with $\boldsymbol{S}_x(t) \rightarrow A(x,t)$, $\varepsilon\hat{\mathbf{n}} \cdot \boldsymbol{S}_0 \rightarrow B(0,0)$

Numerical results

- Initial spins of unit length are drawn from a uniform random distribution, with $L = 2048$ (periodic b.c.); $\varepsilon = 0.001$. The equations are integrated via a fourth-order Runge-Kutta iteration with $\Delta t = 0.005$, and a tolerance of 10^{-5} on each spins. The system is averaged over 5000 configurations.



(a) $\lambda = 1, \mu = 0$



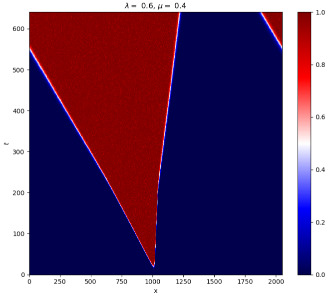
(b) $\lambda = 0, \mu = 1$

Figure: $D(x, t)$ for the pure Heisenberg and pure driven dynamics. The decorrelator for the non-conserving dynamics propagates ballistically, and symmetrically from the initial site of perturbation, quite similar to the one obtained from Hamiltonian dynamics.

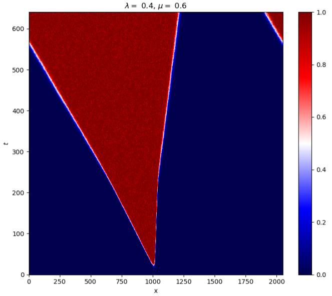
- For the generalized dynamics

$$\frac{d\boldsymbol{S}_x}{dt} = \boldsymbol{S}_x \times ((\lambda + \mu)\boldsymbol{S}_{x+1} + (\lambda - \mu)\boldsymbol{S}_{x-1})$$

the decorrelator doesn't show a left-right symmetry when $\lambda, \mu \neq 0$.



(a) $\lambda = 0.6, \mu = 0.4$



(b) $\lambda = 0.4, \mu = 0.6$

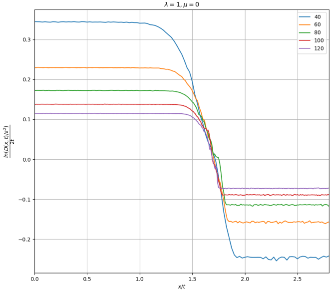
Figure: $D(x, t)$ for the hybrid dynamics

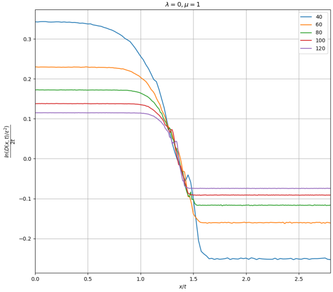
Relation with chaos spreading

- The decorrelator front represents the rate of spread of chaos, and is quantified through an empirical formula

$$D(x,t) = \varepsilon^2 e^{\left(2\kappa t(1-(x/v_B t)^2)\right)} \tag{7}$$

where $\varepsilon \to$ initial perturbation , $\kappa \to$ Lyapunov exponent, $v_B \to$ butterfly velocity. Plotting $log\left(\frac{D(x,t)}{\varepsilon^2}\right)$ against x/t, we find $v_B = 1.64$, $\kappa = 0.50$ and $v_B = 1.32$, $\kappa = 0.46$ for the Heisenberg dynamics and the pure driven dynamics respectively.





Symmetries of the equation of motion

- The generalized equation of motion

$$\dot{\boldsymbol{S}}_x = \boldsymbol{S}_x \times ((\lambda+\mu)\boldsymbol{S}_{x+1} + (\lambda-\mu)\boldsymbol{S}_{x-1}) \tag{8}$$

under spatial inversion $\mathcal{X}$, $\boldsymbol{S}_x^{\mathcal{X}} = \boldsymbol{S}_{-x}$,

$$\dot{\boldsymbol{S}}_x^{\mathcal{X}} = \boldsymbol{S}_x^{\mathcal{X}} \times ((\lambda-\mu)\boldsymbol{S}_{x+1}^{\mathcal{X}} + (\lambda+\mu)\boldsymbol{S}_{x-1}^{\mathcal{X}}) \tag{9}$$

doesn't remain invariant.

- Let a $\mathcal{O}$ be a second operation to restore the invariance such that

$$\dot{\boldsymbol{S}}_x^{\mathcal{O}\mathcal{X}} = \boldsymbol{S}_x^{\mathcal{O}\mathcal{X}} \times ((\lambda-\mu)\boldsymbol{S}_{x+1}^{\mathcal{O}\mathcal{X}} + (\lambda+\mu)\boldsymbol{S}_{x-1}^{\mathcal{O}\mathcal{X}}) \tag{10}$$

- Comparing with the original equation, this is true only for

$$\boldsymbol{S}_x^{\mathcal{O}\mathcal{X}} = \frac{\lambda+\mu}{\lambda-\mu}\boldsymbol{S}_{-x} \quad = \frac{\lambda-\mu}{\lambda+\mu}\boldsymbol{S}_{-x} \tag{11}$$

$\Rightarrow (\lambda+\mu)^2 = (\lambda-\mu)^2 \quad \Rightarrow \lambda = 1, \mu = 0$ or $\lambda = 0, \mu = 1$.

Symmetries of the decorrelator

- No transformation $\boldsymbol{S}_x \to \boldsymbol{S}_{-x}$ exists for the hybrid case $(\lambda, \mu \neq 0)$ that leaves the EOM invariant.
- When $(\lambda+\mu)^2 = (\lambda-\mu)^2$, we have $\boldsymbol{S}_x^{\mathcal{OX}}(t) = \pm \boldsymbol{S}_{-x}(t)$ as we integrate the equation from the initial condition. This means that two distinct initial configurations (a) and (c) are invariant under $\mathcal{OX}$ with (b), (d) as their perturbed copies.
- This one-one mapping translates to the definition of the decorrelator

$$\langle \boldsymbol{S}_x^a(t) \cdot \boldsymbol{S}_x^b(t) \rangle = \langle \boldsymbol{S}_{-x}^c(t) \cdot \boldsymbol{S}_{-x}^d(t) \rangle \qquad (12)$$

- The perturbed copy differs as

$$\boldsymbol{S}_0^d(0) = \boldsymbol{S}_0^c(0) \pm \varepsilon(\hat{\mathbf{n}} \times \boldsymbol{S}_0^c(0))$$

  for $\lambda = 1, \mu = 0; \lambda = 0, \mu = 1$ respectively.
- Thus

$$D_x(t) = D_{-x}(t) \qquad (13)$$

  for both cases upto $\mathcal{O}(\varepsilon^2)$.

Energy dissipation

- For the microscopic dynamics

$$\dot{\boldsymbol{S}}_x = \boldsymbol{S}_x \times ((\lambda + \mu)\boldsymbol{S}_{x+1} + (\lambda - \mu)\boldsymbol{S}_{x-1})$$

the energy dissipation is given by

$$\begin{aligned} \dot{H} &= -\sum_x \left(\boldsymbol{S}_{x+1} \cdot \dot{\boldsymbol{S}}_x + \boldsymbol{S}_x \cdot \dot{\boldsymbol{S}}_{x+1}\right) \\ &= -2\mu \sum_x \boldsymbol{S}_x \cdot (\boldsymbol{S}_{x+1} \times \boldsymbol{S}_{x-1}) \end{aligned} \quad (14)$$

- Expanding in the continuum limit

$$\boldsymbol{S}_{x\pm a} \simeq \boldsymbol{S}_x \pm a\partial_x \boldsymbol{S}_x + \frac{a^2}{2}\partial_{xx}\boldsymbol{S}_x$$

the non-zero contribution to the energy dissipation comes from

$$\dot{H} = -2\mu a^3 \boldsymbol{S}_x \cdot \left(\partial_x \boldsymbol{S} \times \partial_x^2 \boldsymbol{S}\right) \quad (15)$$

- Continuum coarse-grained 1-D Langevin dynamics,

$$\partial_x \boldsymbol{S}(x,t) = D[\partial_x^2(r-\partial_x^2)]\boldsymbol{S} + \mu(\boldsymbol{S}\times\partial_x\boldsymbol{S}) + \zeta. \tag{16}$$

  We write the dynamic action in terms of Martin-Siggia-Rose response fields,

$$\begin{aligned} A_0[\tilde{\boldsymbol{S}},\boldsymbol{S}] &= \int_{x,t}\left(\tilde{S}^\alpha\left(\partial_t S^\alpha - D\partial_x^2\left(r-\partial_x^2\right)S^\alpha\right) + D\tilde{S}^\alpha\partial_x^2\tilde{S}^\alpha\right) \\ A_\mu[\tilde{\boldsymbol{S}},\boldsymbol{S}] &= -\mu\int_{x,t}\epsilon_{\alpha\beta\gamma}\tilde{S}^\alpha S^\beta\partial_x S^\gamma \end{aligned} \tag{17}$$

  and carry out a first-order perturbative expansion at the μ-vertex .

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$$\left\langle \dot{H}\right\rangle = \frac{\int D[i\tilde{S}]D[S]\dot{H}e^{-A_0[\tilde{\boldsymbol{S}},\boldsymbol{S}]}e^{A_\mu[\tilde{\boldsymbol{S}},\boldsymbol{S}]}}{\int D[i\tilde{S}]D[S]e^{-A_0[\tilde{\boldsymbol{S}},\boldsymbol{S}]}e^{A_\mu[\tilde{\boldsymbol{S}},\boldsymbol{S}]}} \tag{18}$$

- Counting the contributions to the μ term in the Fourier space, we get per unit length

$$\mu^2 \int_{(k,k',q,q')} \left\langle (ik - iq)\tilde{\boldsymbol{S}}_{k-q} \cdot (\boldsymbol{S}_k \times \boldsymbol{S}_q)(-iq')(k' - q')^2 \boldsymbol{S}_{k'-q'} \cdot (\boldsymbol{S}_{k'} \times \boldsymbol{S}_{q'}) \right\rangle_0 \quad (19)$$

which can be split into 4 distinct contributions.
scaling the subsequent integral, the rate of energy-dissipation reduces to

$$\left\langle \dot{H} \right\rangle = \mu^2 (2\pi)^3 \int_{k,q} \frac{dkdq}{r} \frac{(k-q)^2 q(2q-k)}{D(1+q^2)(q+(k-q)^2)} \frac{1}{k^2(1+k^2)+q^2(1+q^2)+(k-q)^2(1+(k-q)^2)} \quad (20)$$

- This integral is ultraviolet and infrared convergent, which puts a finite value on the power dissipation for the driven dynamics, which is proportional to a non-zero entropy production. It is precisely zero for the Hamiltonian dynamics.

Summary and Conclusion

- We discover ballistic spreading of decorrelation in a 1-D Heisenberg chain with non-reciprocal, non-Hamiltonian dynamics.
- Spreading is left-right symmetric for purely antisymmetric exchange.
- We present a partial analytical understanding of the symmetry.
- We characterise the nonequilibrium nature of the dynamics through the rate of energy dissipation.

N. Bhatt, S. Mukerjee, S. Ramaswamy; in preparation
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[1] Union Grants Commision of India