Gravitational waves from the first-order phase transition as probe of Leptogenesis

Nimmala Narendra

IACS, Kolkata,

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3 Gravitational Waves From FOPT

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- We need to look at the Cosmological prospective...

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- First order phase transition, topological defects, Cosmic strings, primordial black holes, etc.,
- We consider GW that are produced from First Order Phase Transition
- We assume after the phase transition, a heavy scalar ϕ dominates the energy budget of the Universe.



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- As the T_{RH} of the Universe passes mass scale of right-handed neutrino M_N , *i.e.*, $T_{RH} < M_N$, it behaves as a non-relativistic.
- **Sakharov conditions:(i)B (or L) number violation, (ii) C and CP violation, (iii) Departure from thermal equilibrium.

Boltzmann Equations-Leptogenesis

$$\begin{aligned} \frac{d\rho_{\phi}}{dt} + 3H\rho_{\phi} &= -\Gamma_{\phi}^{NN} \rho_{\phi} - \Gamma_{\phi}^{R} \rho_{\phi} \\ \frac{d\rho_{R}}{dt} + 4H\rho_{R} &= \Gamma_{\phi}^{R} \rho_{\phi} + \Gamma_{N}(\rho_{N} - \rho_{N}^{eq}) \\ \frac{d\rho_{N}}{dt} + 3H\rho_{N} &= \Gamma_{\phi}^{NN} \rho_{\phi} - \Gamma_{N}(\rho_{N} - \rho_{N}^{eq}) \\ \frac{dn_{B-L}}{dt} + 3H n_{B-L} &= \frac{\epsilon}{M_{N}} \Gamma_{N}(\rho_{N} - \rho_{N}^{eq}) - \Gamma_{\text{ID}} n_{B-L} \\ \Gamma_{\phi}^{NN} &= \frac{|\lambda|^{2}}{4\pi} M_{\phi} \left(1 - \frac{4M_{N}^{2}}{M_{\phi}^{2}}\right)^{\frac{3}{2}}, \\ \Gamma_{\phi}^{R} = \frac{|\lambda_{R}|^{2}}{4\pi M_{\phi}}, \\ \Gamma_{(N \to L+H)} = \frac{|Y|^{2}}{8\pi} M_{N} \end{aligned}$$

The total decay width $\Gamma_{\phi} \equiv \Gamma_{\phi}^{NN} + \Gamma_{\phi}^{R}$ The CP-asymmetry:

$$\epsilon = \sum_{i} \epsilon_{i} = \frac{3}{16\pi (Y^{\dagger}Y)_{ii}} \sum_{j \neq i} \operatorname{Im}\left[(Y^{\dagger}Y)_{ji}^{2}\right] \left(\frac{M_{i}}{M_{j}}\right).$$

Light neutrino mass-Type-I seesaw

The relevant Lagrangian for Type-I seesaw:

$$-\mathcal{L} \supset Y \overline{N} \widetilde{H}^{\dagger} L + M \overline{N^{c}} N + h.c.$$

where Y is the Yukawa coupling, H and L are SM-Higgs and lepton doublets.

The light neutrino mass matrix:

$$m_{
u} \simeq m_D M^{-1} m_D^T$$

where $m_D = Yv/\sqrt{2}$, with v = 246 GeV being the SM Higgs VEV. We can diagonalize this matrix by a unitary transformation

$$D_{\nu} = \operatorname{diag}(m_1, m_2, m_3) = U^T m_{\nu} U,$$

where U is the PMNS matrix. The Yukawa coupling matrix in terms of Casas-Ibarra parametrization

$$Y = \frac{\sqrt{2}}{v} U \sqrt{D_{\nu}} R^{T} \sqrt{M},$$

where R is a complex orthogonal matrix.

Redefinitions-Leptogenesis

Redefined variables,

$$\begin{split} E_{\phi} &= \rho_{\phi} a^3, \qquad R = \rho_R a^4, \\ E_N &= \rho_N a^3, \qquad \widetilde{N}_{B-L} = n_{B-L} a^3 \end{split}$$

Ratio of the scale factor to its initial value,

$$y = \frac{a}{a_l}$$

With redefinitions, the Hubble parameter

$$H = \sqrt{\frac{8\pi}{3M_{Pl}^2} \frac{(a_l E_{\phi} y + a_l E_N y + R)}{a_l^4 y^4}}$$

dimensionless variable,

$$z = \frac{M_{N_1}}{T} = M_{N_1} \left[\frac{\pi^2 g_*}{30 R} \right]^{1/4} a_I y$$

Re-expressed Boltzmann Equations-Leptogenesis

The equilibrium density in terms of z

$$E_{N}^{eq} = \rho_{N}^{eq} a^{3} = \frac{M_{N}^{4}}{\pi^{2}} \left[\frac{3}{z^{2}} K_{2}(z) + \frac{1}{z} K_{1}(z) \right] a_{I}^{3} y^{3}$$

In terms of newly defined rescaled quantities the set of Boltzmann equations

$$\frac{dE_{\phi}}{dy} = -\frac{\Gamma_{\phi}^{NN}}{Hy}E_{\phi} - \frac{\Gamma_{\phi}^{R}}{Hy}E_{\phi}$$
$$\frac{dR}{dy} = \frac{\Gamma_{\phi}^{R}}{H}a_{I}E_{\phi} + \frac{\Gamma_{N}}{H}a_{I}(E_{N} - E_{N}^{eq})$$
$$\frac{dE_{N}}{dy} = \frac{\Gamma_{\phi}^{NN}}{Hy}E_{\phi} - \frac{\Gamma_{N}}{Hy}(E_{N} - E_{N}^{eq})$$
$$\frac{d\widetilde{N}_{B-L}}{dy} = \frac{\Gamma_{N}}{Hy}\frac{\epsilon}{M_{N}}(E_{N} - E_{N}^{eq}) - \frac{\Gamma_{ID}}{Hy}\widetilde{N}_{B-L}$$

Initial conditions-Leptogenesis

The initial energy density of the heavy scalar field obtained from the condition $\Gamma_{\phi} = H(a_I)$:

$$\rho_{\phi_I} = \frac{3M_{PI}^2}{8\pi}M_{\phi}^2.$$

The initial conditions at $a = a_I$: $R(a_I) = 0, E_N(a_I) = 0, \widetilde{N}_{B-L}(a_I) = 0$ and $E_{\phi}(a_I) \equiv E_{\phi_I} = (3/8\pi)M_{PI}^2M_{\phi}^2a_I^3$. The \widetilde{N}_{B-I} asymmetry is related to the total N_{B-I} via,

$$N_{B-L} = \frac{n_{B-L}}{n_{\gamma}} = \left(\frac{\pi^4}{30\,\zeta(3)}\right) \left(\frac{30}{\pi^2}\right)^{1/4} \frac{g_*^{3/4}}{g_{\gamma}} R^{-3/4} \widetilde{N}_{B-L}.$$

$$g_{\gamma}\zeta(3) = \pi^3 \qquad \pi^2 g_{\gamma} = \pi^4 \qquad \pi^2 g_{\ast} = \pi^4$$

$$n_{\gamma} = \frac{\beta\gamma\varsigma(3)}{\pi^2}T^3$$
, $\rho_{\gamma} = \frac{\pi}{30}\frac{\beta\gamma}{7}T^4$, $\rho_R = \frac{\pi}{30}\frac{\beta\kappa}{7}T^4$

BAU via Leptogenesis

The predicted N_{B-L} is related to the η_B measured at recombination given as

$$\eta_B = \left(\frac{a_{sph}}{f}\right) N_{B-L}\,,$$

where $a_{sph}=28/79$, and $f=N_{\gamma}^{\rm rec}/N_{\gamma}^{*}=2387/86$.

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Gravitational Waves From FOPT

The total GW abundance come from the Bubble collision, Sound waves and Turbulence (Ellis, et.al):

$$\Omega_{col,*}h^{2}(f) = 2.3 \times 10^{3} (R_{*}H_{*})^{2} \left(\frac{\kappa_{col}\alpha}{1+\alpha}\right)^{2} \left[1 + \left(\frac{f}{f_{d}}\right)^{-1.61}\right] \times \left(\frac{f}{f_{col}}\right)^{2.54} \left[1 + 1.13 \left(\frac{f}{f_{col}}\right)^{2.08}\right]^{-2.3}$$

$$\Omega_{sw,*}h^{2}(f) = 0.384 (R_{*}H_{*}) (\tau_{sw}H_{*}) \left(\frac{\kappa_{sw}\alpha}{1+\alpha}\right)^{2} \left(\frac{f}{f_{sw}}\right)^{3} \left[1 + \frac{3}{4} \left(\frac{f}{f_{sw}}\right)^{2}\right]^{-7/2},$$

$$\Omega_{turb,*}h^{2}(f) = 6.85 \left(R_{*}H_{*}\right) \left(1 - \tau_{sw}H_{*}\right) \left(\frac{\kappa_{sw}\alpha}{1 + \alpha}\right)^{3/2} \left(\frac{f}{f_{turb}}\right)^{3} \frac{\left[1 + \left(\frac{f}{f_{turb}}\right)\right]^{-11/3}}{1 + 8\pi f/H_{*}},$$

$$\begin{split} f_d &\simeq \left(\frac{0.044}{R_*}\right), \quad f_{col} \simeq \left(\frac{0.28}{R_*}\right), \quad f_{sw} \simeq \left(\frac{3.4}{R_*}\right), \quad f_{turb} \simeq \left(\frac{5.1}{R_*}\right), \\ \tau_{sw} &= \frac{R_*}{U_f} \ , U_f \simeq \sqrt{\frac{3}{4}} \frac{\alpha_{\text{eff}}}{1 + \alpha_{\text{eff}}} \kappa_{sw}, \\ \kappa_{sw} &= \frac{\alpha_{\text{eff}}}{\alpha} \frac{\alpha_{\text{eff}}}{0.73 + 0.083 \sqrt{\alpha_{\text{eff}}} + \alpha_{\text{eff}}}, \\ \alpha_{\text{eff}} &= \alpha (1 - \kappa_{col}). \end{split}$$

Gravitational Waves-Redshift

- Once the gravitational waves produced propagates from the phase transition until today.
- The ratio of the scale factor at the transition to the scale factor today

$$f_{0} = f_{*}\left(\frac{a_{*}}{a_{0}}\right) = 1.65 \times 10^{-5} Hz\left(\frac{f_{*}}{H_{*}}\right) \left(\frac{T_{*}}{100 \text{ GeV}}\right) \left(\frac{g_{*}}{100}\right)^{1/6}$$
$$\Omega_{GW,0} = \left(\frac{a_{*}}{a_{0}}\right)^{4} \left(\frac{H_{*}}{H_{0}}\right)^{2} \Omega_{GW,*} = 1.67 \times 10^{-5} h^{-2} \left(\frac{100}{g_{*}}\right)^{1/3} \Omega_{GW,*}$$

After production the GWs redshift in the same way as radiation.

A period of matter-domination modifies the GW abundance observed today. The GW spectrum redshifts as:

$$\Omega_{\mathrm{GW},0}(f) = \begin{cases} \left(\frac{a_*}{a_0}\right)^4 \left(\frac{H_*}{H_0}\right)^2 \Omega_{\mathrm{GW},*} \left(\frac{a_0}{a_*}f\right) & \text{for } f > f_*, \\ \\ \left(\frac{a_f}{a_0}\right)^4 \left(\frac{H_f}{H_0}\right)^2 \Omega_{\mathrm{GW},*} \left(\frac{a_0}{a_*}f_*\right) \left(\frac{f}{f_*}\right)^3 & \text{for } f < f_*, \end{cases}$$

The GW amplitude redshift factor:

$$\left(\frac{a}{a_0}\right)^4 \left(\frac{H}{H_0}\right)^2 = 1.67 \times 10^{-5} h^{-2} \left(\frac{100}{g_{eff}(T_{\rm reh})}\right) \left(\frac{a}{a_{\rm reh}}\right)^4 \left(\frac{H}{H_{\rm reh}}\right)^2$$

and the frequency redshift factor:

$$\frac{a_*}{a_0} = 7.96 \times 10^{-16} \left(\frac{100 \,\text{GeV}}{T_{\text{reh}}}\right) \left(\frac{100}{g_{\text{eff}}}\right)^{\frac{1}{3}} \left(\frac{a_*}{a_{\text{reh}}}\right)$$

The evolution of the energy densities

$$\frac{d\rho_{\phi}}{dt} + 3H\rho_{\phi} = -\Gamma_{\phi}^{NN}\rho_{\phi} - \Gamma_{\phi}^{R}\rho_{\phi}$$
$$\frac{d\rho_{R}}{dt} + 4H\rho_{R} = \Gamma_{\phi}^{R}\rho_{\phi} + \Gamma_{N}(\rho_{N} - \rho_{N}^{eq})$$
$$H^{2} = \frac{\rho_{\phi} + \rho_{R}}{3M_{p}^{2}}$$

By approximating the solution of the above system,

$$H(a) = H_* \left(\frac{a_*}{a_{reh}}\right)^{\frac{3}{2}} \left(\frac{a_{reh}}{a}\right)^2 \left[1 + \left(\frac{a_{reh}}{a}\right)^2\right]^{-\frac{1}{4}}$$

Expansion is dominated by ϕ field from H_* ,

$$t_{reh} - t_* = \tau_{\phi} = \int_{a_*}^{a_{reh}} \frac{da}{aH} = \frac{2}{3} \frac{1}{H_*} \left[\left(\frac{a_{reh}}{a_*} \right)^{\frac{3}{2}} - 1 \right]$$

Duration of the ϕ domination $\tau_{\phi} = \frac{1}{\Gamma_{\phi}}$ and assuming $\Gamma_{\phi} \ll H_*$,

$$\Rightarrow \quad \frac{a_{reh}}{a_*} = \left(\frac{3H_*}{2\Gamma_{\phi}}\right)^{\frac{2}{3}}$$

Using Hubble expansion approximation, the evolution of the scale factor, $(2\pi f_{H_{reh}} = a_{reh}H(a_{reh}), 2\pi f_* = a_*H(a_*))$

$$rac{a}{a_{
m reh}} = \sqrt{rac{\chi(f)}{2} - rac{1}{2}}\,, \quad ext{ where } \quad \chi(f) = \sqrt{8\left(rac{f_{ extsf{H}_{ extsf{reh}}}}{f}
ight)^4 + 1}$$

The redshifting up to a_{reh} ,

$$\left(\frac{a}{a_{\rm reh}}\right)^4 \left(\frac{H}{H_{\rm reh}}\right)^2 \approx \begin{cases} 1 & \text{for } f < f_{H_{\rm reh}}, \\ \sqrt{\frac{\chi(f)-1}{\chi(f)+1}} & \text{for } f_{H_{\rm reh}} < f < f_*, \\ \sqrt{\frac{\chi(f_*)-1}{\chi(f_*)+1}} & \text{for } f > f_* \end{cases} .$$



$$\begin{aligned} \alpha &= 100, \ \beta/H_* = 10, \ \kappa_{\rm col} = 0.9, \\ H_* &= 10^{-8}, \ f_* = 10^{-3}, \\ M_\phi &= 10^{12} \ {\rm GeV}, \\ M_{N_1} &= 10^{10} \ {\rm GeV}, \ \lambda = 10^{-10} \\ & \because \ \Gamma_\phi^{NN} = \frac{|\lambda|^2}{4\pi} M_\phi \left(1 - \frac{4M_N^2}{M_\phi^2}\right)^{\frac{3}{2}} \end{aligned}$$



$$\begin{split} &\alpha = 100, \, \beta/H_* = 10, \, \kappa_{\rm col} = 0.9, \\ &H_* = 10^{-8}, \, f_* = 10^{-3}, \\ &M_\phi = 10^{13} \, {\rm GeV}, ({\rm solid}), \\ &M_\phi = 10^{12} \, {\rm GeV} \, ({\rm dashed}), \\ &M_{\phi} = 10^{11} \, {\rm GeV} \, ({\rm dotted}), \\ &M_{N_1} = 10^{10} \, {\rm GeV}, \, \lambda = 10^{-10} \end{split}$$







4 Results-GW as a probe of Leptogenesis



GW as a probe of Leptogenesis



Signal-to-Noise Ratio

 To quantify the detection probability of the GW signal, we compute the signal-to-noise ratio (SNR) for a given experimental sensitivity Ω_{sens}(f)h²:

$$\mathsf{SNR} \equiv \sqrt{ au \int_{f_{min}}^{f_{max}} \left[rac{\Omega_{\mathsf{GW}}(f)h^2}{\Omega_{\mathsf{sens}}(f)h^2}
ight]^2 df}$$

where

$$h^2 \Omega_{\text{Sens}}(f) = rac{2\pi^2}{3H_0^2} f^3 S_h(f).$$

where $\tau = 4$ years is the observation time, $H_0 = 100h \text{ km } s^{-1} \text{Mpc}^{-1}$ with $h = 0.678 \pm 0.009$, $S_h(f)$ is the power spectral density (PSD).

• The signal can be claimed to be detectable if SNR reaches a threshold SNR \geq 10 of a detector.

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Summary

- To explain the baryon asymmetry of the Universe via Leptogenesis while addressing the non-zero light neutrino masses through Type-I Seesaw mechanism, we require the mass scale of RHN to be very high.
- Such high scale of Leptogenesis challenging to probe in lab experiments, therefore we may require to see for the GW signals as an alternative pathways to test Leptogenesis.
- We observe that LISA, DECIGO, BBO and ET may hint towards the successful Leptogenesis at the high scale in the near future.

Thank you for you attention!