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Precision bounds for multiple currents in OQS

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Quantum Trajectories, ICTS-TIFR, 04/02/2025

Classical uncertainty relations

TUR

$$\frac{\mathrm{Var}(\Phi)}{\langle \Phi \rangle^2} \geq \frac{2}{\Sigma}$$

KUR

$$\frac{\mathrm{Var}(\Phi)}{\langle \Phi \rangle^2} \geq \frac{1}{\mathscr{A}}$$

Φ time-integrated current

Σ entropy production

$\mathscr{A}$ dynamical activity

Classical uncertainty relations

TUR

$$\frac{\mathrm{Var}(\Phi)}{\langle\Phi\rangle^2} \geq \frac{2}{\Sigma}$$

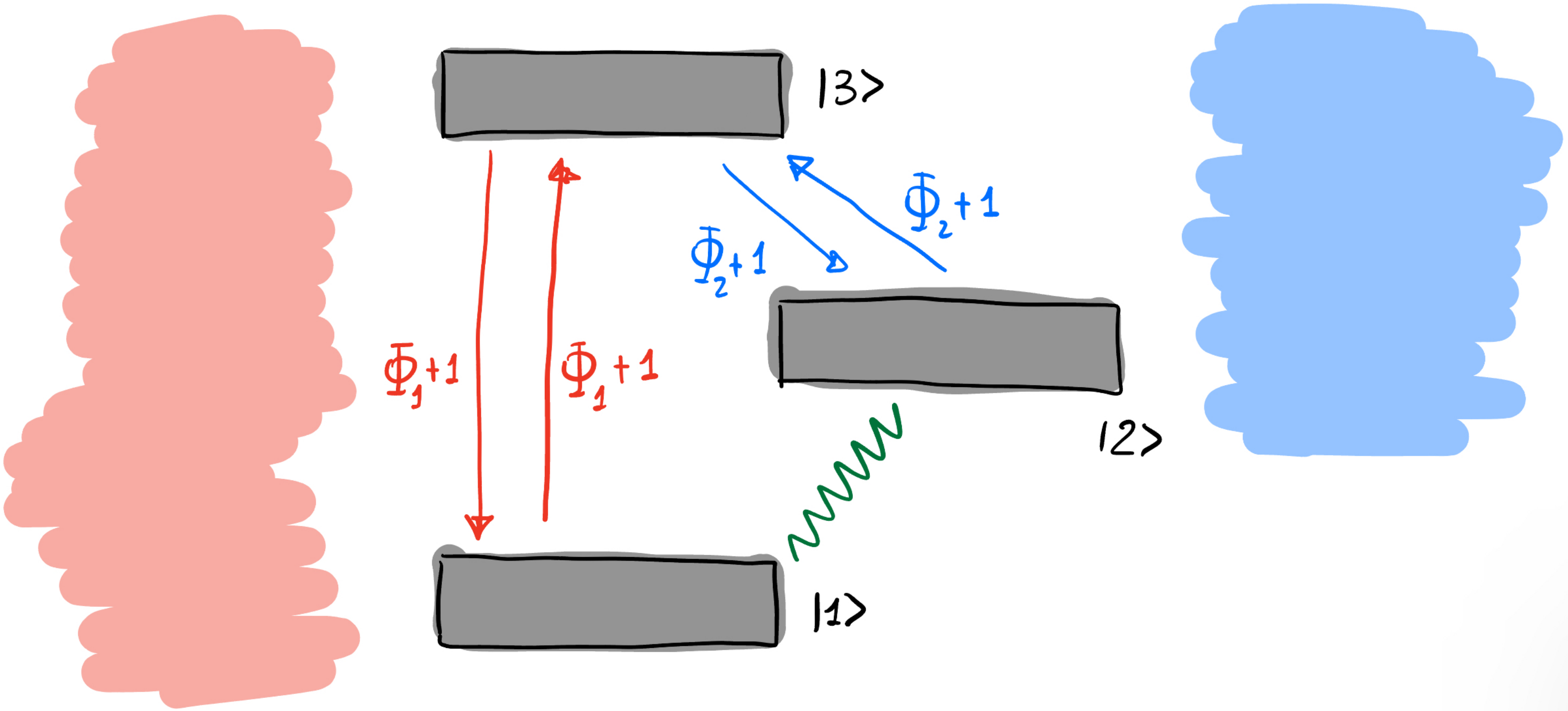
KUR

$$\frac{\mathrm{Var}(\Phi)}{\langle\Phi\rangle^2} \geq \frac{1}{\mathscr{A}}$$

Meaning: a trade-off between precision of currents and entropy production (TUR) or dynamical activity (KUR).

K. Ptaszyński, Phys. Rev. B 98, 085425 (2018)
B. K. Agarwalla, D. Segal, Phys. Rev. B 98, 155438 (2018)
J.Liu, D.Segal, Phys. Rev. E 99,062141 (2019)
K. Prech et al., Phys. Rev. Res. 5, 023155 (2023).
A. A. S. Kalaee et al., Phys. Rev. E 104, L012103 (2021).
A. Rignon-Bret et al., Phys. Rev. E 103, 012133 (2021).

Quantum violation of the classical TURs/KURs



$$\frac{d\rho}{dt} = \mathscr{L}\rho = -i[H,\rho] + \sum_k \gamma_k \left(L_k \rho L_k^\dagger - \frac{1}{2}\left\{ L_k^\dagger L_k, \rho \right\} \right)$$

Fictitious encoding of the parameters

Add a dependence on a parameter ϕ:

$$H_\phi = (1 + \phi)H$$

$$L_{k,\phi} = \sqrt{1 + \phi}L_k$$

The original dynamics is recovered for $\phi = 0$.

A Fisher information approach to KURs and TURs

$$\omega_{1:N} = \left\{k_0, (k_1, \tau_1), \ldots, (k_N, \tau_N)\right\}$$

Counting observable $\Phi(\omega_{1:N})$ accumulates dependence on ϕ: use it as an **estimator**

$$\text{Var}\left[\Theta\right] \geq \frac{1}{F(\phi)}$$

$$\frac{\text{Var}\left[\Phi(\omega_{1:N})\right]}{\left[\partial_\phi \langle \Phi(\omega_{1:N}) \rangle\right]^2} \geq \frac{1}{F(\phi)}$$

Y. Hasegawa, T. Van Vu, Phys. Rev. E 99, 062126 (2019)
T. Van Vu, K. Saito, Phys. Rev. Lett. 128, 14602 (2022)

A Fisher information approach to KURs and TURs

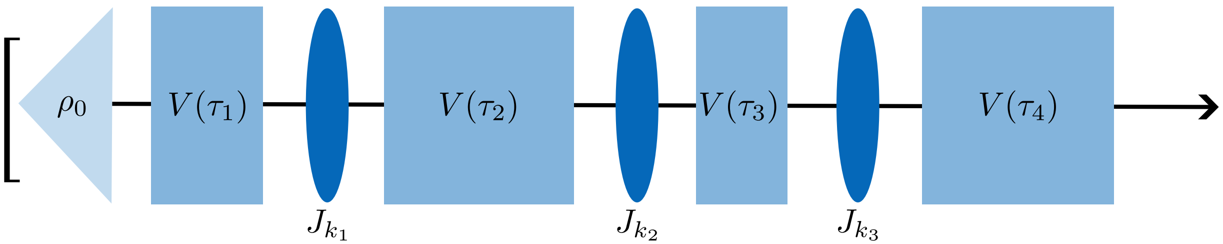
Can we **rewrite** the CR bound as a KUR?

$$\frac{\text{Var}\left[\Phi(\omega_{1:N})\right]}{\left[\partial_\phi \langle \Phi(\omega_{1:N}) \rangle\right]^2} \geq \frac{1}{F(\phi)} \quad \overset{?}{\Longrightarrow} \quad \frac{\text{Var}(\Phi)}{\langle \Phi \rangle^2} \geq \frac{1}{\mathscr{A}}$$

Y. Hasegawa, T. Van Vu, Phys. Rev. E 99, 062126 (2019)
T. Van Vu, K. Saito, Phys. Rev. Lett. 128, 14602 (2022)

Quantum Trajectories and Fisher information

$$\omega_{1:N} = \left\{k_0, (k_1, \tau_1), \ldots, (k_N, \tau_N)\right\}$$

$Pr(\omega_{1:N}) = p_0(1+\phi)^N \mathrm{Tr}\Big[$  $\Big]$

Fisher information for ϕ?

$$H_\phi = (1+\phi)H$$

$$L_{k,\phi} = \sqrt{1+\phi}\,L_k$$

Y. Hasegawa, T. Van Vu, Phys. Rev. E 99, 062126 (2019)
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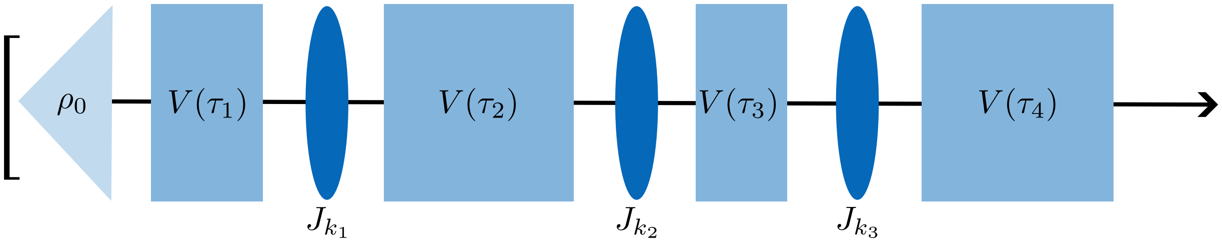
Quantum Trajectories and Fisher information

$$Pr(\omega_{1:N}) = p_0(1+\phi)^N \mathrm{Tr}\Big[\quad\Big]$$



$$F(\phi) = -\left\langle \partial^2_\phi \ln(1+\phi)^N \right\rangle\Big|_{\phi=0} - \left\langle \partial^2_\phi \ln q_\phi \right\rangle\Big|_{\phi=0}$$

$\mathscr{A}$ $\mathscr{Q}$

In general has to be computed over trajectories -> **Felix's talk**!

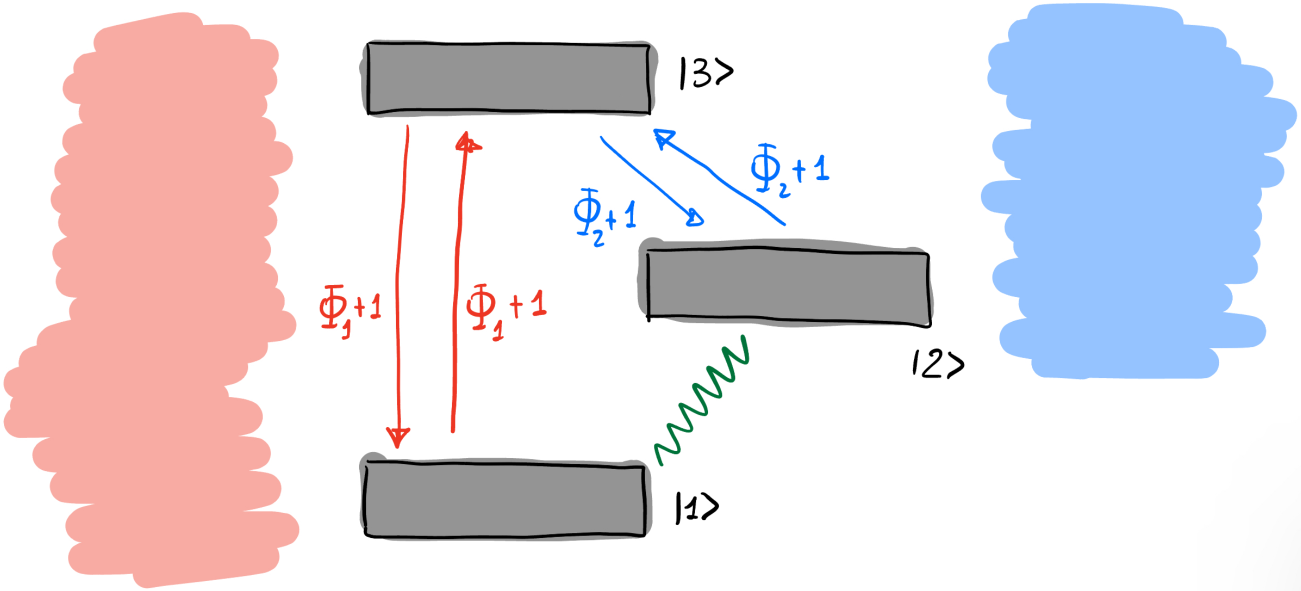
Y. Hasegawa, T. Van Vu, Phys. Rev. E 99, 062126 (2019)
T. Van Vu, K. Saito, Phys. Rev. Lett. 128, 14602 (2022)

A quantum KUR!

$$\frac{\text{Var}\left[\Phi(\omega_{1:N})\right]}{\left[\partial_\phi \langle \Phi(\omega_{1:N}) \rangle\right]^2} \geq \frac{1}{F(\phi)}$$

$$\frac{\text{Var}\left[\Phi(\omega_{1:N})\right]}{\langle \Phi \rangle^2} \geq \frac{1}{\mathscr{A} + \mathscr{Q}}$$

Multiple currents

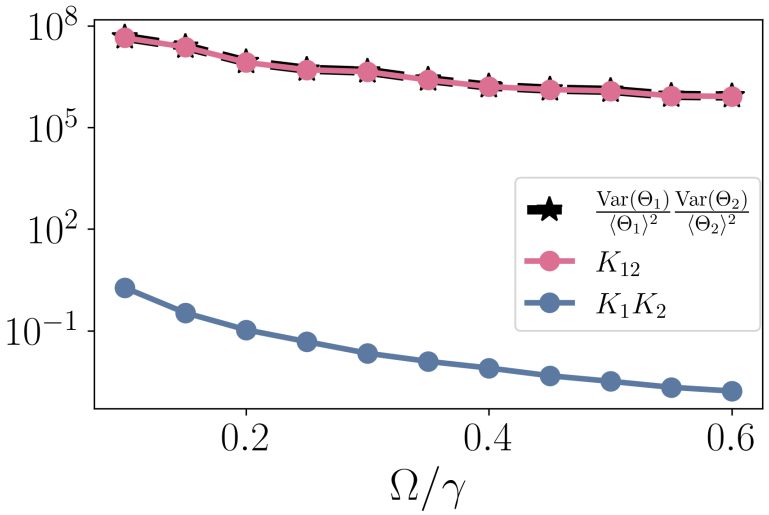


$$\frac{\text{Var}(\Phi_1)}{\langle\Phi_1\rangle^2}\frac{\text{Var}(\Phi_2)}{\langle\Phi_2\rangle^2} \geq ???$$

Multiple currents

$$\frac{\mathrm{Var}(\Phi_1)}{\langle\Phi_1\rangle^2}\frac{\mathrm{Var}(\Phi_2)}{\langle\Phi_2\rangle^2} \geq \frac{1}{\mathscr{A}_1+\mathscr{Q}_1}\frac{1}{\mathscr{A}_2+\mathscr{Q}_2}$$

But the bound obtained is not tight!
Can we consider the **correlations** between currents?



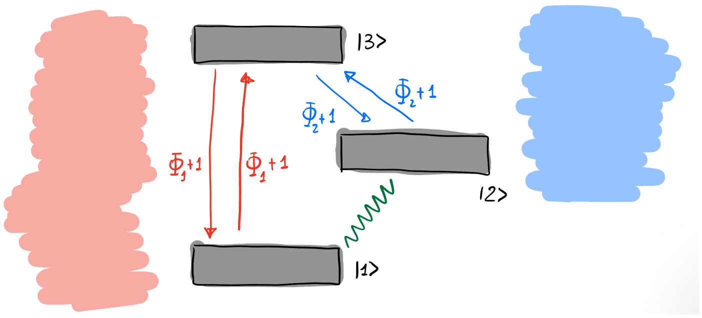
Multiple currents

$$H_\phi = (1 + \phi_1 + \phi_2)H$$

$$L^j_\phi = \sqrt{1 + \phi_j}L^j$$

A fictitious parameter ϕ for each bath.

Get the original dynamics for $\phi_1 = \phi_2 = 0$



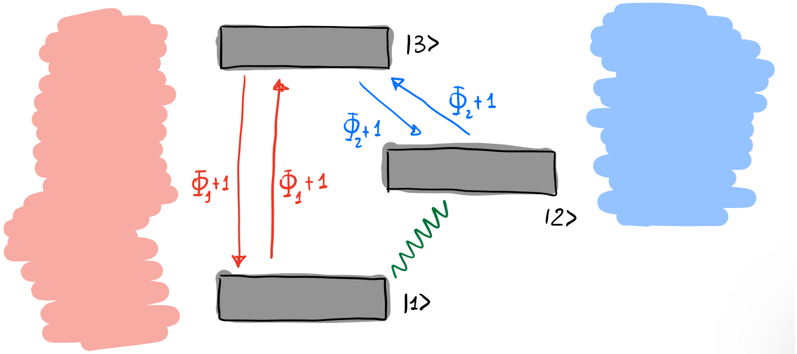
S.V. Moreira, MR et al., arXiv 2411-09088 (2024)

Multiple currents

$$J_{\vec{\Phi}}^T \Xi^{-1} J_{\vec{\Phi}} \leq \mathbb{F}$$

$$\left[J_{\vec{\Phi}}\right]_{ij} = \partial_{\phi_j} \langle \Phi_i \rangle$$

$$\Xi_{ij} = \langle \Phi_i \Phi_j \rangle - \langle \Phi_i \Phi_j \rangle$$



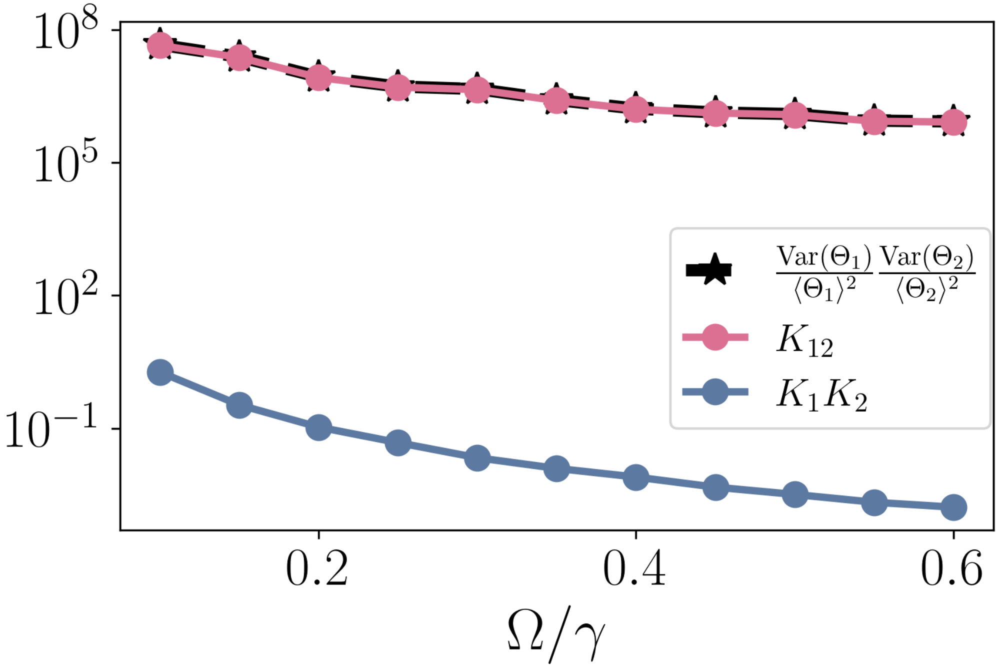
Multiparameter estimation theory

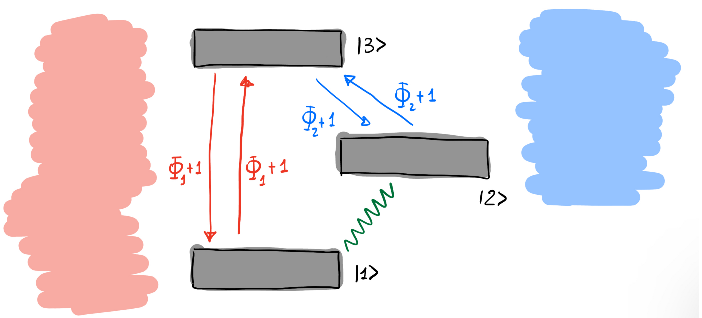
S.V. Moreira, MR et al., arXiv 2411-09088 (2024)

Multiple currents

The point: multiple current correlate with each other!

-> a tighter bound when correlations are taken into account.





S.V. Moreira, MR et al., arXiv 2411-09088 (2024)

Summary

- Quantum systems can **violate** classical KURs and TURs
- Quantum K/TURs can be obtained via the **Cramér-Rao bound** for a fictitiously encoded **parameter**
- Keeping **correlations** among currents into account allows for a tighter bound.

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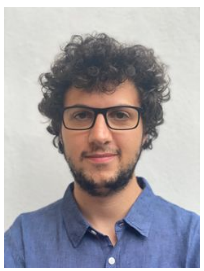

S.V. Moreira, MR et al., arXiv 2411.09088 (2024)
MR, G.T. Landi, F.C. Binder, Phys. Rev. A 110 (6), 062212 (2024)
MR, J.A. Smiga, G.T. Landi, F.C. Binder, arXiv 2402.06556 (2024)