Symmetry oscillations in strongly interacting one-dimensional mixtures

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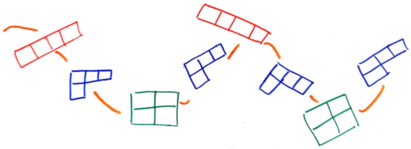

ICTS, 16th December 2024

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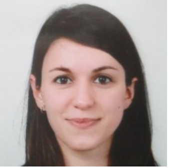
Symmetry oscillating people



and symmetry people


Outline

- Introduction
  - strongly repulsive bosons and spinless fermions in 1D

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- Exact solution for strongly interacting trapped mixtures
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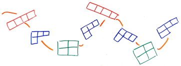
  - Ground-state properties for bosonic mixtures with SU(2) broken symmetry

- "Exact"solution for the dynamics
  - symmetry oscillations



Strongly repulsive bosons and spinless fermions in a lineland ...

The many-body Hamiltonian (for bosons)

$$\mathcal{H} = \sum_i -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x_i^2} + V(x_i) + g\sum_{i<j}\delta(x_i - x_j)$$

Lieb-Liniger model (1963) with external potential

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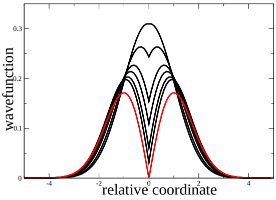
Lieb-Liniger model (1963) with external potential

- integrable if $V(x_i) = 0$
- not integrable if $V(x_i) \neq 0$ for a generic g
- integrable for a generic $V(x_i)$ if $g \to \infty$

The Tonks-Girardeau regime

. . . in the strongly repulsive regime



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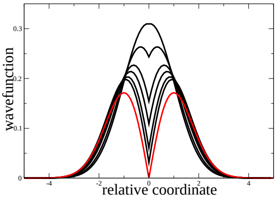

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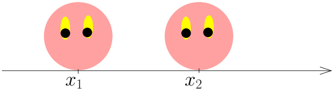

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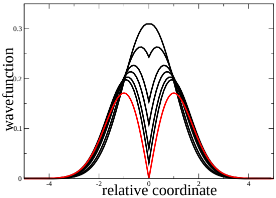
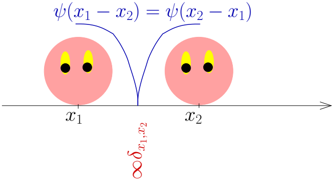
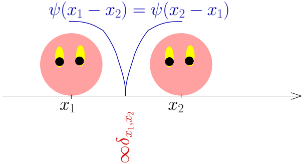




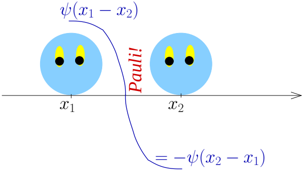
2 TG-bosons cannot be at the same place (at the same time) in their lineworld...

The boson-fermion mapping

[M. Girardeau, J. Math. Phys. 1, 516 (1960)]



$\leftrightarrow$



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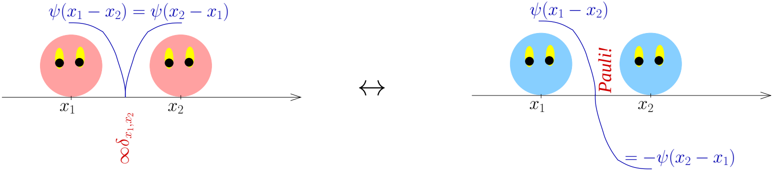

B-F mapping : $\psi_B(x_1, x_2, \ldots, x_N) = \mathcal{A}\psi_F(x_1, x_2, \ldots, x_N)$

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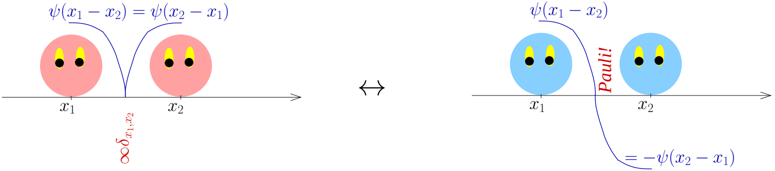

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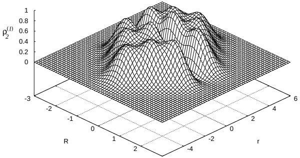
Consequences:

- $n_B(x) = n_F(x)$
- $\rho_{2,B}(x, x') = \rho_{2,F}(x, x')$
- $S_B(k, \omega) = S_F(k, \omega)$
- ...



The boson-fermion mapping (at $T = 0$)

The two particles example in a harmonic trap

The free fermions many-body wavefunction is

$$\psi_F(x_1, x_2) = \frac{1}{\sqrt{2}} \begin{vmatrix} \phi_1(x_1) & \phi_1(x_2) \\ \phi_2(x_1) & \phi_2(x_2) \end{vmatrix} = \frac{1}{\sqrt{\pi a_{ho}}} e^{-(x_1^2 + x_2^2)/2a_{ho}^2} (x_2 - x_1)$$

thus the TG many-body wavefunction is

$$\psi_B(x_1, x_2) = (\pi a_{ho})^{-1/2} e^{-(x_1^2 + x_2^2)/2a_{ho}^2} |x_2 - x_1|$$

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namely

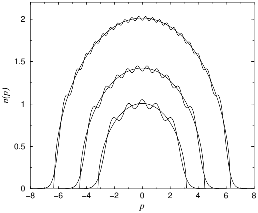
$$\psi_B(x_1, x_2) = \begin{cases} +\theta(x_1 < x_2)\psi_F(x_1, x_2) \\ -\theta(x_2 < x_1)\psi_F(x_1, x_2) \end{cases}$$

$\psi_F(x_2, x_1) = -\psi_F(x_1, x_2)$ and $\psi_B(x_2, x_1) = \psi_B(x_1, x_2)$

Spinless fermions vs strongly interacting spinless bosons (TG)

(in a harmonic trap)

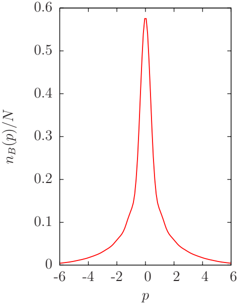
- $n_F(p)$
  - a “potato”shape (a step-function in a ring)



- $n_B(p)$
  - a large pic $n_B(p=0) \propto \sqrt{N}$
  - large tails!
    $\lim_{p\to\infty} n_B(p) = \mathcal{C} p^{-4}$

    A. Minguzzi, P.V., M. Tosi, PLA **294**, 222 (2002)

    M. Olshanii, V. Dunjko, PRL **91**, 090401(2003)



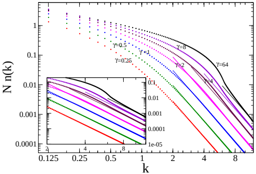
The contact $\mathcal{C}$

interplay of interactions & symmetry!

- $n(p)_{p\to\infty} \to \mathcal{C}p^{-4}$

  - in 1D for any value of γ

    [JS Caux, P Calabrese, NA Slavnov (2007)]



- in 3D too! for particles in the unitary regime *[S. Tan (2008); Debby Jin's experiment]* and in the weak interaction limit *[David Clément experiment]*

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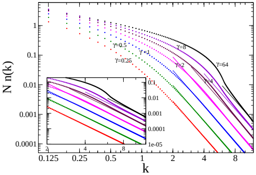

- in 3D too! for particles in the unitary regime *[S. Tan (2008); Debby Jin's experiment]* and in the weak interaction limit *[David Clément experiment]*

... not only in the momentum distribution!

- $\mathcal{C} \propto gE_{int}$ with $E_{int} = g\int dR\langle\Psi^\dagger\Psi^\dagger\Psi\Psi(R)\rangle$

- $\mathcal{C} \propto -\dfrac{\mathrm{d}E}{\mathrm{d}(1/g)}$, E being the total energy

  Tan's relations in 1D *[S. Tan (2008), M. Barth, W. Zwerger (2011)]*

The contact $\mathcal{C}$ in the strongly interacting limit

ATTENTION: $g \to \infty$, $E_{int} \to 0$ BUT $gE_{int} = \mathcal{C} \neq 0$

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can be evaluatued by exploiting the cusp condition:

$$\lim_{g\to\infty} g\psi(x_i = x_j) = -\frac{\hbar^2}{2m} \left(\frac{\partial\psi}{\partial x_i}\bigg|_{xi=x_j+0^+} - \frac{\partial\psi}{\partial x_i}\bigg|_{xi=x_j+0^-} \right) \qquad (1)$$

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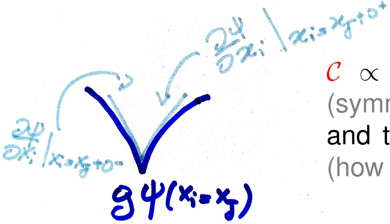


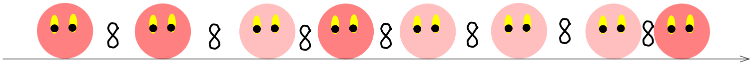
$\mathcal{C}$ $\propto$ to the number of cusps (symmetric exchanges) and to the slopes of the cusps (how particles brush against)

Strongly repulsive mixtures

Exact solutions for mixtures & symmetry considerations

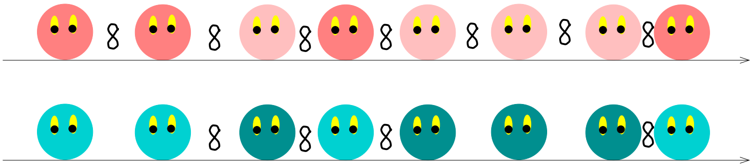
Example: 1D two-component mixtures

Bosonic mixtures



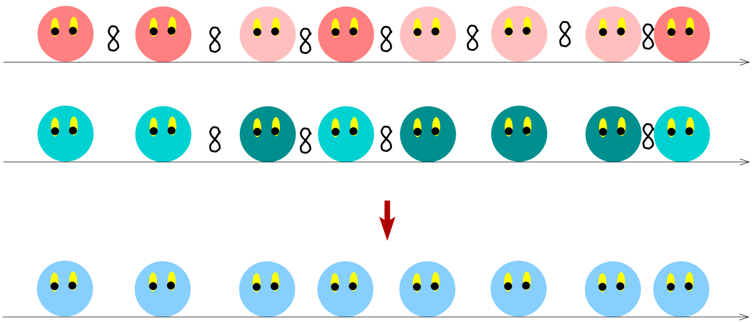
Example: 1D two-component mixtures

Bosonic mixtures or **Fermionic mixtures**



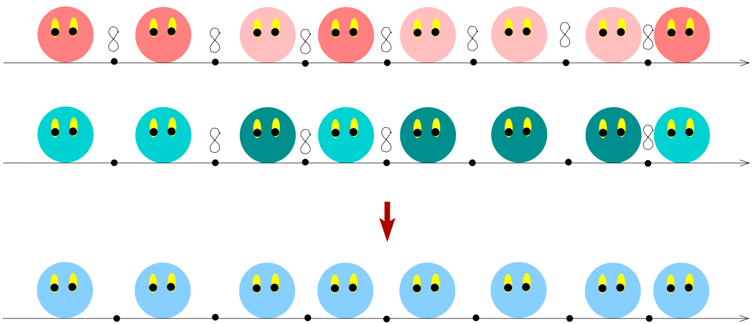
Example: 1D two-component mixtures

Bosonic mixtures or **Fermionic mixtures** ⇒ **spinless fermions**



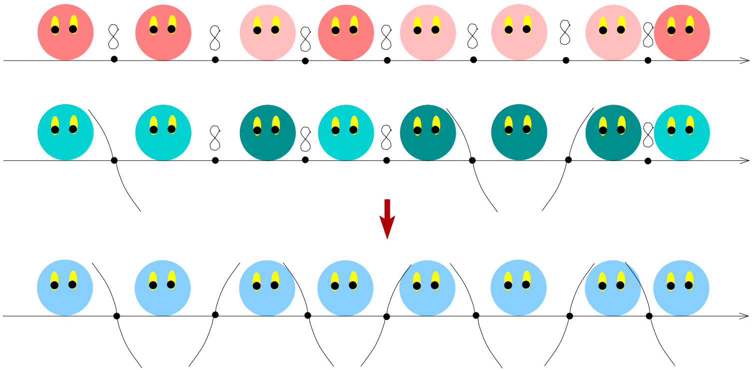
Example: 1D two-component mixtures

Mapping on **spinless fermions**: **the right nodes**



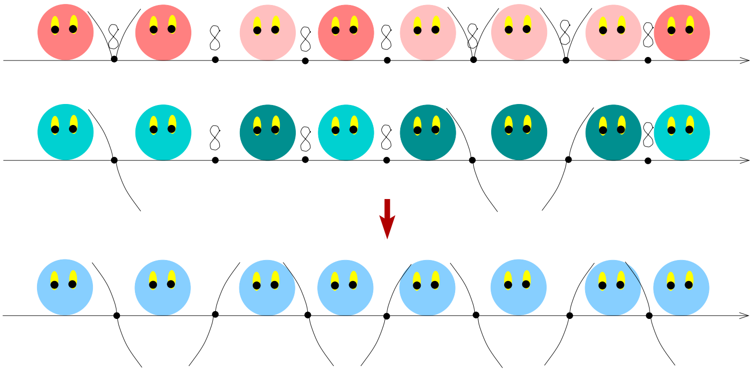
Example: 1D two-component mixtures

spinless fermions: the right exchange rules for fermions



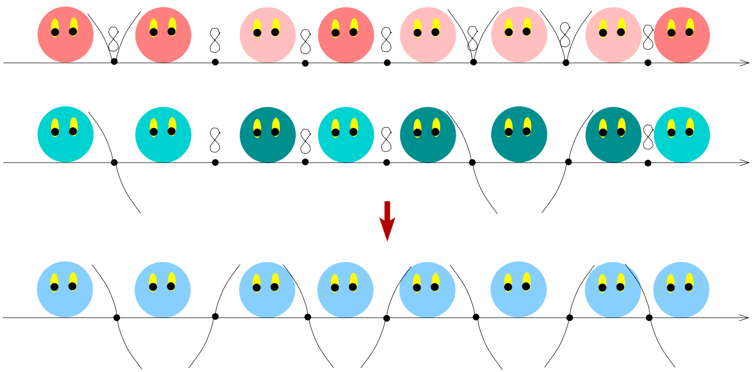
Example: 1D two-component mixtures

spinless fermions: symmetrized" exchange rules for bosons



Example: 1D two-component mixtures

spinless fermions: **What is it missing?**



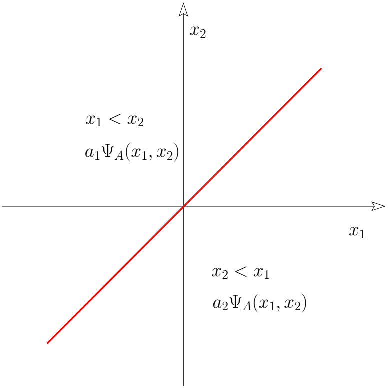
the interspecies exchange rules!

Large ground-state degeneracy: $\dfrac{N!}{N_1!N_2!\dots N_\kappa!}$ (for κ components)

Exact wavefunction in the fermionized regime

- Generalization of Girardeau's wavefunction for impenetrable bosons [Volosniev et al., Nat. Phys. 2015]

$$\Psi(x_1, \ldots, x_N) = \sum_{P \in S_N} a_P \theta(x_{P(1)} < \cdots < x_{P(N)}) \Psi_F(x_1, \ldots, x_N)$$



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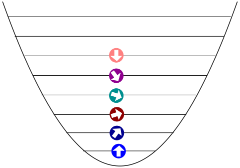
- the coefficients a_P maximize the "contact" $K \propto \mathcal{C}$

$$K = -(\partial E/\partial g^{-1})$$



Exact wavefunction in the fermionized regime

More in details, in order to obtain the a_P's for **all** the states of the lowest-energy manifold, that are **degenerate at** $g \to \infty$, but that **are not degenerate at large finite** g, we diagonalize the effective Hamiltonian

$$H_{n\ell} = E_\infty \delta_{n,\ell} - \frac{1}{g} \sum_{i<j} \lim_{g \to \infty} \int g^2 \phi_n^* \phi_\ell \delta(x_i - x_j)$$

written on the snippet (spin configurations) basis
$\{\uparrow\uparrow\downarrow\downarrow, \uparrow\downarrow\uparrow\downarrow, \uparrow\downarrow\downarrow\uparrow, \downarrow\uparrow\uparrow\downarrow, \downarrow\uparrow\downarrow\uparrow, \downarrow\downarrow\uparrow\uparrow\}$

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The effective Hamiltonian $H_{n.\ell}$ can be mapped on a spin-chain Hamiltonian and each eigenstate has a **well-defined symmetry**

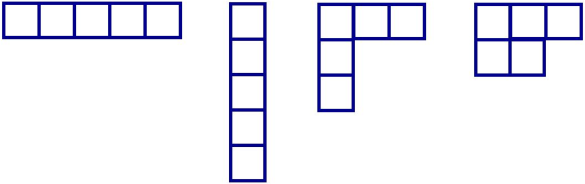
How to determine the wavefunction symmetry?

Use the class-sum operators [Katriel, J. Phys. A, 26, 135 (1993)]

$$\Gamma^{(k)} = \sum_{i_1 < \ldots i_k} (i_1 \ldots i_k)$$

$(i_1 \ldots i_k)$ being the cyclic permutation of k elements

There is a corrispondence between the eigenvalues of $\Gamma^{(k)}$ and the Young tableaux



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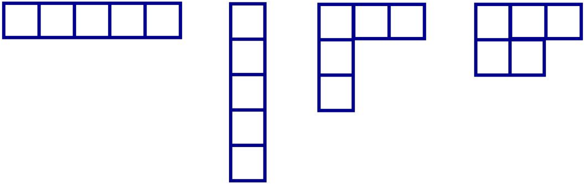
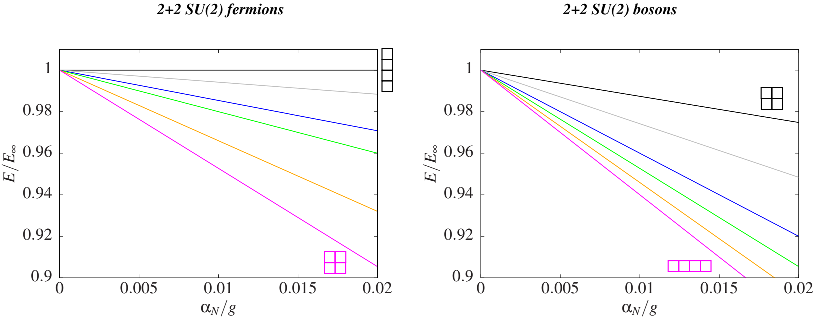


Our main symmetry witness is $\boxed{\Gamma^{(2)} = \sum_{i<j} P_{i,j}}$

[Decamp et al, New J. Phys. **18** 055011 (2016)]

Ground-state and symmetry for SU(2) Hamiltonians



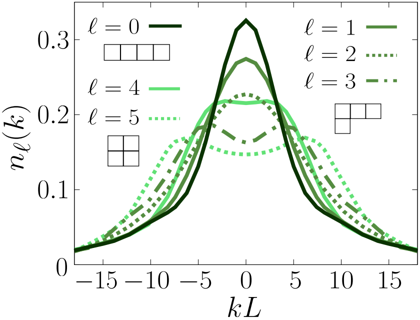
All quantum states have well defined symmetry
(are eigenstates of $\Gamma^{(2)}$) and different contacts

$$E = E_\infty - \frac{K}{g} \qquad\qquad \mathcal{C} = \frac{m^2}{\pi \hbar^4} K$$

Ground-state and symmetry for SU(2) Hamiltonians

All quantum states have well defined symmetry
(are eigenstates of $\Gamma^{(2)}$) and different momentum distributions

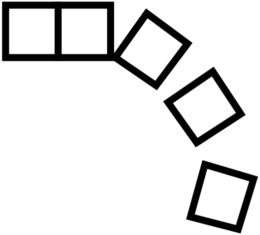
For 2+2 SU(2) bosons



$$n_\ell(k) = \langle \xi_\ell | \hat{n}(k) | \xi_\ell \rangle$$

$|\xi_\ell\rangle$ eiegenstates of $\Gamma^{(2)}$

Breaking the symmetry



Ground-state properties

[G. Aupetit-Diallo, G. Pecci, C. Pignol, F. Hébert, A. Minguzzi, M. Albert, and P.V. Phys. Rev. A 106, 033312 (2022)]

Boson-boson mixtures

$$\hat{H} = \sum_{\sigma=\uparrow,\downarrow} \sum_{i}^{N_\sigma} \left[-\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x_{i,\sigma}^2} + g_{\sigma\sigma} \sum_{j>i}^{N_\sigma} \delta(x_{i,\sigma} - x_{j,\sigma}) \right]$$

$$+ g_{\uparrow\downarrow} \sum_{i}^{N_\uparrow} \sum_{j}^{N_\downarrow} \delta(x_{i,\uparrow} - x_{j,\downarrow})$$

Boson-boson mixtures

$$\hat{H} = \sum_{\sigma=\uparrow,\downarrow} \sum_{i}^{N_\sigma} \left[-\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x_{i,\sigma}^2} + g_{\sigma\sigma} \sum_{j>i}^{N_\sigma} \delta(x_{i,\sigma} - x_{j,\sigma}) \right] + g_{\uparrow\downarrow} \sum_{i}^{N_\uparrow} \sum_{j}^{N_\downarrow} \delta(x_{i,\uparrow} - x_{j,\downarrow})$$

Let's consider two cases:

- SU(2) case

  $g_{\uparrow\uparrow} = g_{\downarrow\downarrow} = g_{\uparrow\downarrow} = g \to \infty$

  mapping on a XXX spin-chain Hamiltonian

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  $g_{\uparrow\uparrow} = g_{\downarrow\downarrow} = g_{\uparrow\downarrow} = g \to \infty$

  mapping on a XXX spin-chain Hamiltonian

- the Symmetry Breaking (SB) case

  $g_{\uparrow\uparrow} = g_{\downarrow\downarrow} = g \to \infty$, BUT $g_{\uparrow\downarrow} \neq g$, with $1/g_{\uparrow\downarrow} \ll 1$

  mapping on a XXZ spin-chain Hamiltonian

Boson-boson mixtures

Results

- ground-state symmetry
  - 2+2 SU(2) bosons: $\yng(4)$
  - 2+2 SB bosons: $\yng(4)(\frac{8}{9})$ but also $\yng(2,2)(\frac{1}{9})$

Boson-boson mixtures

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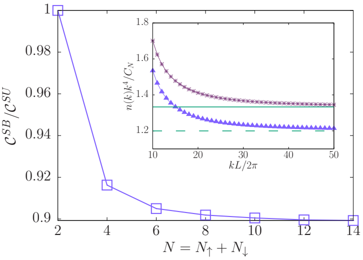
- the contact $\mathcal{C}$



very small effect: the symmetry is just slightly broken!

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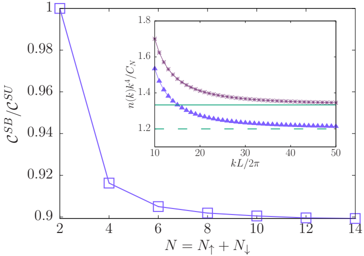
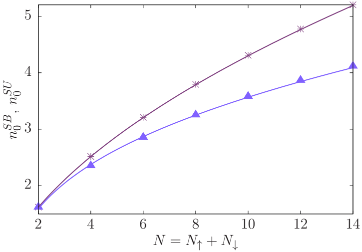


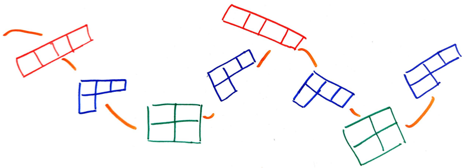
very small effect: the symmetry is just slightly broken!

- $n_{k=0}(N)$



$$\lim_{N\to\infty} \frac{n_0^{SB}}{n_0^{SU}} = \frac{1}{2}$$

Breaking the symmetry: dynamical effects



Symmetry oscillations!

[S. Musolino, M. Albert, A. Minguzzi, and P.V., Phys. Rev. Lett. **133**, 183402 (2024)]

Dynamical evolution in the strong repulsive limit

- An almost "exact" solution for the dynamics (in $1/g$)

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The $a_P(t)$ evolve under the action of the effective Hamiltonian that can be mapped on a spin-chain Hamiltonian

(XXX for the SU(2) mixture, XXZ for the SB one)

What do we expect?

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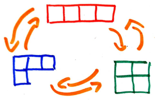
- **Symmetry oscillations in the SB case**

Particle states of given permutation symmetry are not diagonal in the basis of H_{SB}

During time evolution, the many-body wavefunction evolves from one symmetry to another



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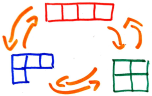
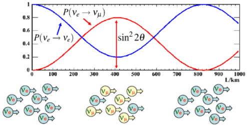


$\Updownarrow$

Neutrino oscillations

Neutrino states of given flavour are not diagonal in the basis of their dynamical evolution

During time evolution, neutrinos evolve from one flavour to another



From M.A. Thomson Part. Phys. lecture note

Permutation symmetry oscillations

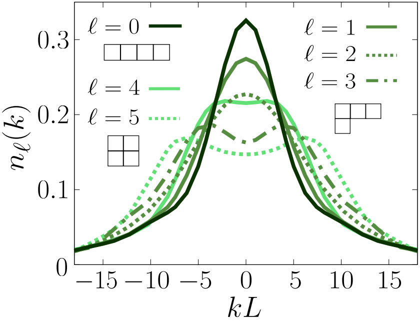
How to observe symmetry oscillations?

Permutation symmetry oscillations

How to observe symmetry oscillations?

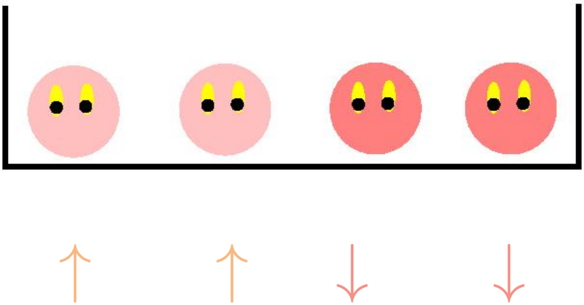
momentum distribution & symmetry

The momentum distribution depends on the symmetry state $|\xi_\ell\rangle$



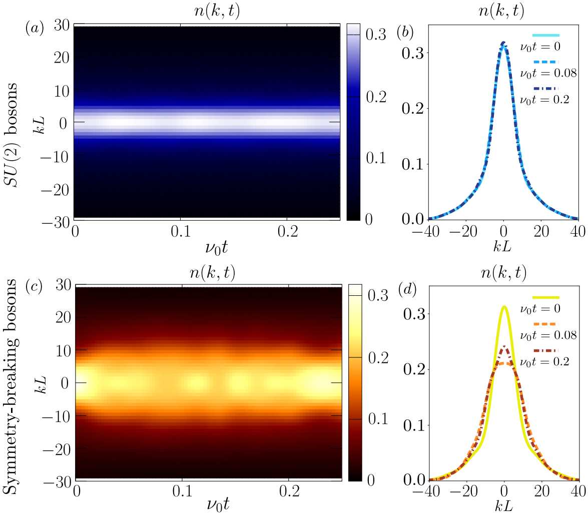
$n_\ell(k) = \langle \xi_\ell | \hat{n}(k) | \xi_\ell \rangle$

The initial state



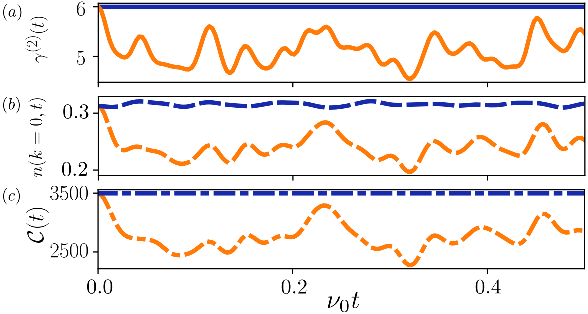
- spin oscillations in real space (but this is another story . . .)
- What about the momentum distribution?

Dynamics in momentum space



Symmetry oscillations!

Focussing on the dynamics of $n(k=0)$ and of the contact $\mathcal{C}$ ...

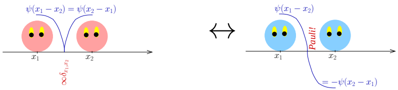


SU(2) results, SB results

Same oscillations than $\gamma^{(2)} = \langle\psi(t)|\Gamma^{(2)}|\psi(t)\rangle$,
the symmetry witness!

Conclusions

- in a lineworld ....



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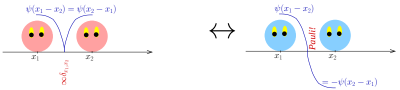

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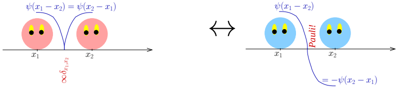

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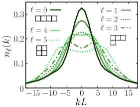
- strong signature of the symmetry in $n(k)$



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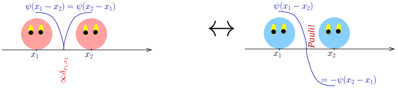

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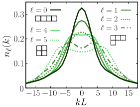

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Conclusions

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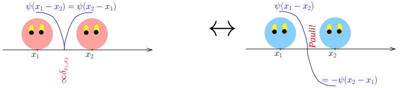

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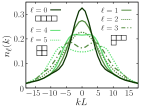
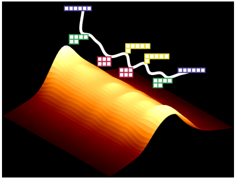
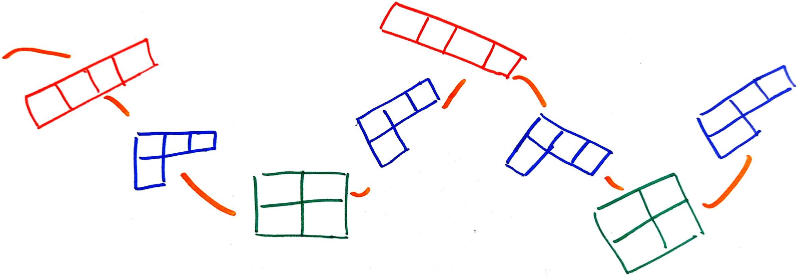
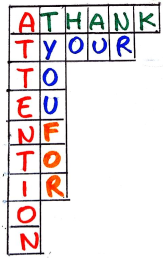


- the fermionization of the system allows to solve "exactly" (at the order $1/g$) the dynamics at zero temperature
- This has allowed us to observe that, for a case of a spin excitation,






THANK
YOUR
ATTENTION
YOU
FOR

Symmetry analysis

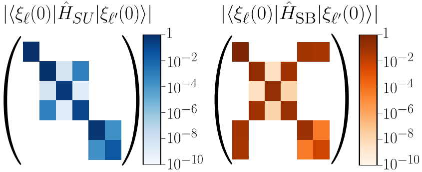
- $[\hat{\Gamma}^{(2)}, \hat{n}(k)] = 0$
- $[\hat{\Gamma}^{(2)}, \hat{H}_{SU}] = 0$

Symmetry analysis

- $[\hat{\Gamma}^{(2)}, \hat{n}(k)] = 0$
- $[\hat{\Gamma}^{(2)}, \hat{H}_{SU}] = 0$

- BUT $[\hat{n}(k), \hat{H}_{SU}] \neq 0$ in general

Let $|\xi_\ell(k)\rangle$ the basis that diagonalizes simultaneously $\hat{\Gamma}^{(2)}$ and , $\hat{n}(k)$



- SU(2): coupling within states of the same symmetry sectors
- SB: coupling within states belonging to different symmetry sectors

Boson-boson mixtures

- we start from the many-body wavefunction

$$\Psi(x_1, \ldots, x_N) = \sum_{P \in S_N} a_P \theta_P(x_1, \ldots, x_N) \Psi_B(x_1, \ldots, x_N)$$

  where $\Psi_B = A\Psi_F$

- in order to find the a_P's (the spin configurations), we minimize the energy up to the $1/g$ order: $E = E_\infty + \frac{1}{g}\frac{dE}{d(1/g)}$
- this ends up to find the eigenstates of the Hamiltonian (written on the snippet basis $\{\uparrow\uparrow\downarrow\downarrow, \uparrow\downarrow\uparrow\downarrow, \uparrow\downarrow\downarrow\uparrow, \downarrow\uparrow\uparrow\downarrow, \downarrow\uparrow\downarrow\uparrow, \downarrow\downarrow\uparrow\uparrow\}$)

$$H_{n\ell} = E_\infty \delta_{n,\ell} - \frac{1}{g} \sum_{i<j} \lim_{g \to \infty} \int g^2 \phi_n^* \phi_\ell \delta(x_i - x_j)$$

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This Hamiltonian is different for the SU(2) and the SB cases!

Boson-boson mixtures

- SU(2)

$$\hat{H}^{SU} = E_\infty - NJ - J \sum_{j=1}^{N} \hat{P}_{j,j+1}$$

$$= \boxed{E_\infty - 2J \sum_{j=1}^{N} \vec{S}^{(j)} \vec{S}^{(j+1)} - \frac{3}{2} NJ}$$

mapping on the XXX model

Boson-boson mixtures

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mapping on the XXX model

- SB

$$\hat{H}^{SB} = E_\infty - NJ - J\sum_{j=1}^{N}\hat{P}_{j,j+1} + 2J\sum_{j=1}^{N}|s\rangle\langle s|\hat{P}_{j,j+1}|s\rangle\langle s|$$

$$= \boxed{E_\infty - 2J\sum_{j=1}^{N}(S_x^{(j)}S_x^{(j+1)} + S_y^{(j)}S_y^{(j+1)} - S_z^{(j)}S_z^{(j+1)}) - \frac{1}{2}NJ}$$

mapping on the XXZ model

Dynamics of strongly interacting mixtures

Momentum distribution: decomposition in symmetry sectors

At strong interactions, spin and orbital part of the wavefunction decouple → **momentum density operator**: [Deuretzbacher et al. 2014]

$$\hat{n}_{tot}(k) = \sum_{i,j} \hat{P}_{i,i+1,i+2,\dots j} R_{i,j}(k)$$

particle permutation cycle, orbital contribution

- Crucial property: $[\hat{n}_{tot}(k), \hat{\Gamma}^{(2)}] = 0$ $(\hat{\Gamma}^{(2)} = \sum_{i<j} \hat{P}_{i,j})$

  Time-dependent momentum distribution - on the common basis of $\hat{n}(k)$ and $\hat{\Gamma}^{(2)}$

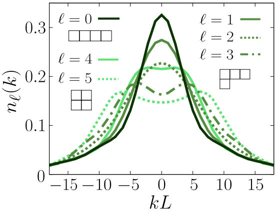
$$n(k,t) = \sum_{\ell} |\langle \psi(t) | \gamma_\ell \rangle|^2 n_\ell(k)$$

  with $n_\ell(k) = \langle \gamma_\ell | \hat{n}_{tot}(k) | \gamma_\ell \rangle$.

Symmetry-resolved momentum distribution

$$n_\ell(k) = \langle \gamma_\ell | \hat{n}_{tot}(k) | \gamma_\ell \rangle$$

The most symmetric state has the highest peak!

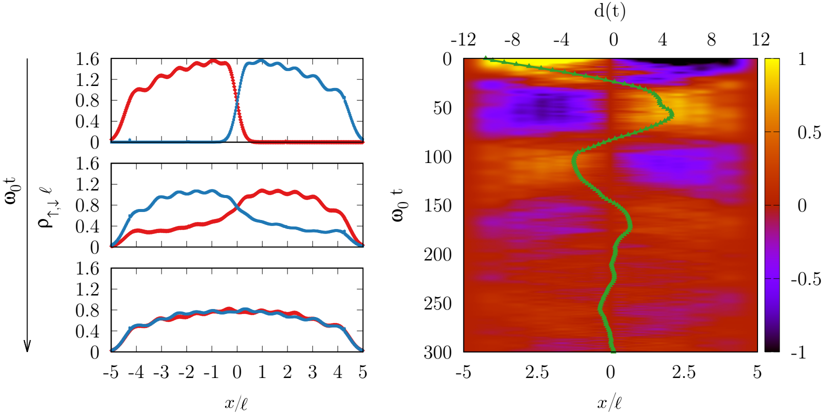


$n_\ell(k) = \sum_{i,j}(k)\langle \gamma_\ell | P_{i \to j} | \gamma_\ell \rangle$ *The momentum distribution probes particle exchange permutation cycles!*

Two importants limits:

- At large momenta, only 2-particle permutations contribute $j = i + 1$ $\rightarrow$ **Tan's contact**
- At small momenta, all permutation cycles $i \rightarrow j$ contribute: to probe large distance coherence you need to go through all particles $\rightarrow$ quasi ODLRO!

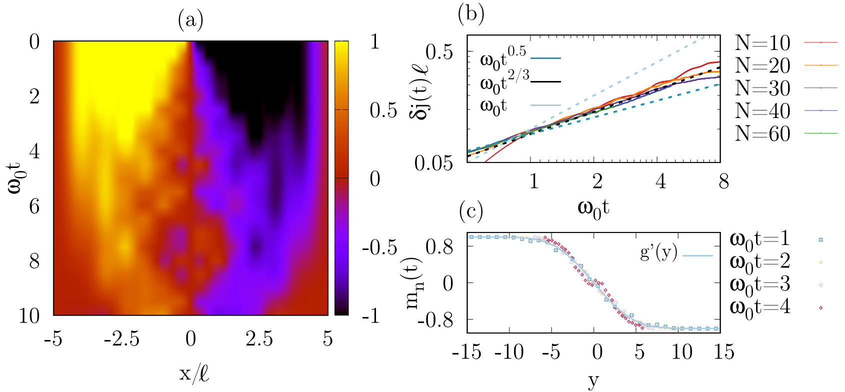
Spin-mixing dynamics



Exact magnetization dynamics at any time!

...and its barycenter $d(t) = \frac{1}{N} \int_{-\infty}^{+\infty} m(z,t) z \, dz$

Early-time dynamics



$\delta j(t) \sim t^{\eta}$ with $\eta = 0.638$

integrated spin-current density $\delta j(t) = \int_0^t dt' j(0, t')$

spin-current density $j(z,t) = \frac{1}{2}\sum_{j=1}^{N-1} J_j(\sigma_j^x \sigma_{j+1}^y - \sigma_j^y \sigma_{j+1}^x)[\rho_j(z) + \rho_{j+1}(z)]$
$y = n/(\omega_0 t)^{1/z}$, with $z = 3/2$

superdiffusion in agreement with KPZ theory

KPZ universality in magnets

What was already known

- the dynamics of the 1D **isotropic** Heisenberg model shows a superdiffusive behaviour
- the domain-wall relaxation is governed by the KPZ dynamical exponent [M. Ljubotina, M. Znidaric and T. Prosen, Phys. Rev. Lett. 122, 210602 (2019); Immanuel Bloch's group experiment, Science 376, 6594 (2022)]
- superdiffusion desappears in 2D or breaking SU(2) (the model is no more integrable) [Immanuel Bloch's group experiment, Science 376, 6594 (2022)]

What is new

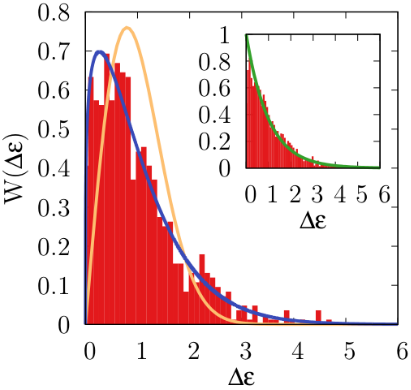
- we observe superdiffusion and domain-wall relaxation governed by a dynamical exponent in agreement with the KPZ one,

even if

our system is anisotropic (and the model is no more integrable)

Analysis of our system

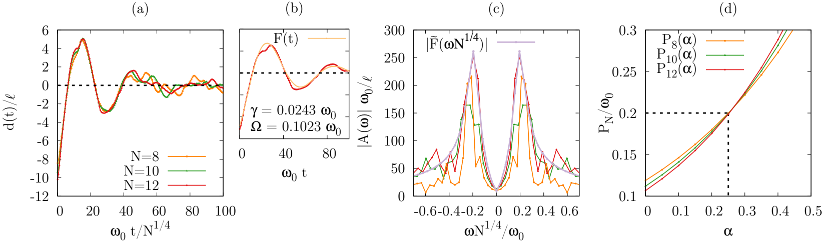
Our model is no more integrable, but . . .



$W(\Delta\epsilon)$ Level spacing distribution: not integrable system but "close" to an integrable one . . .

Intermediate-time dynamics

$$d(t) = d(t=0)e^{-\gamma t}\cos(\Omega_N t + \phi)$$



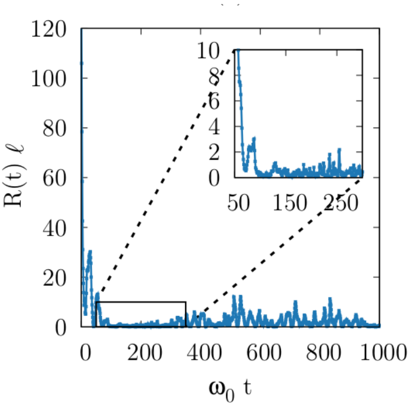
$\Omega_N = \Omega_{univ}/N^{1/4}$ ($\Omega_{univ} \simeq 0.19\omega_0$) γ does not depend on N

$$\ddot{d} + \gamma\dot{d} + \Omega_N^2 d = 0 \Rightarrow d \simeq e^{-\Gamma_{SD}t} \text{ with}$$

$$\Gamma_{SD} = \Omega_{univ}^2/(\gamma N^{1/2})$$

universal spin-drag scaling

Long-time dynamics



$R(t) = |\rho_{\uparrow}(t) - \rho_{\uparrow,MC}|$ *"thermalization" to a MC ensemble state*