Fluctuations in a chain of active particles

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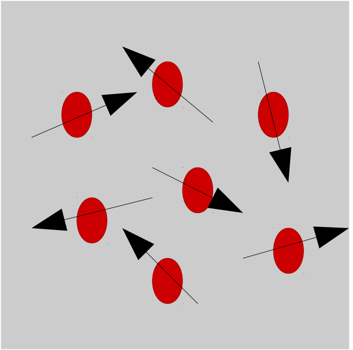
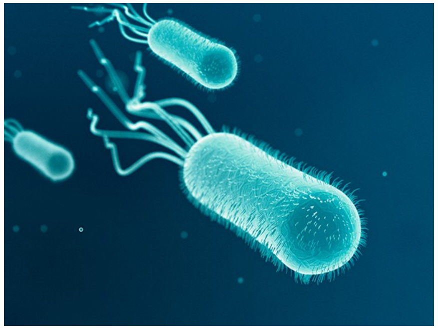
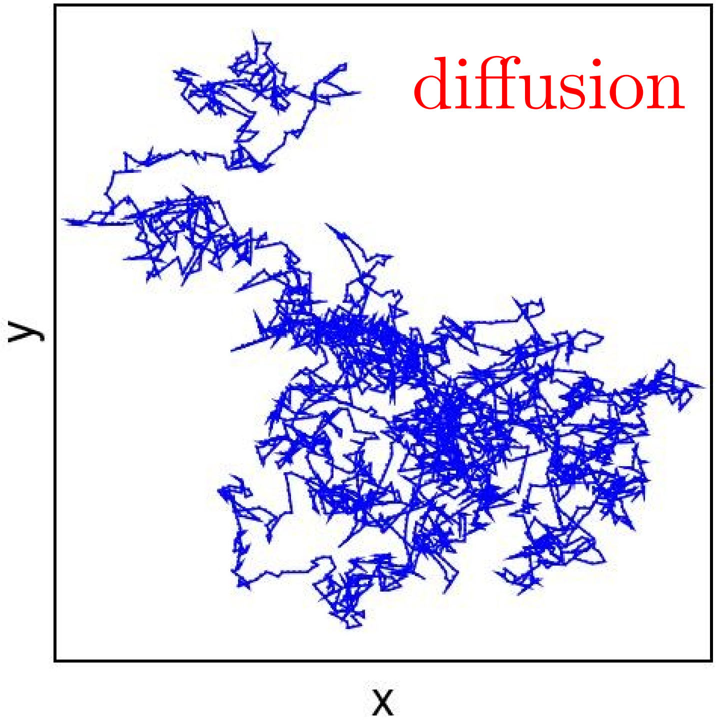


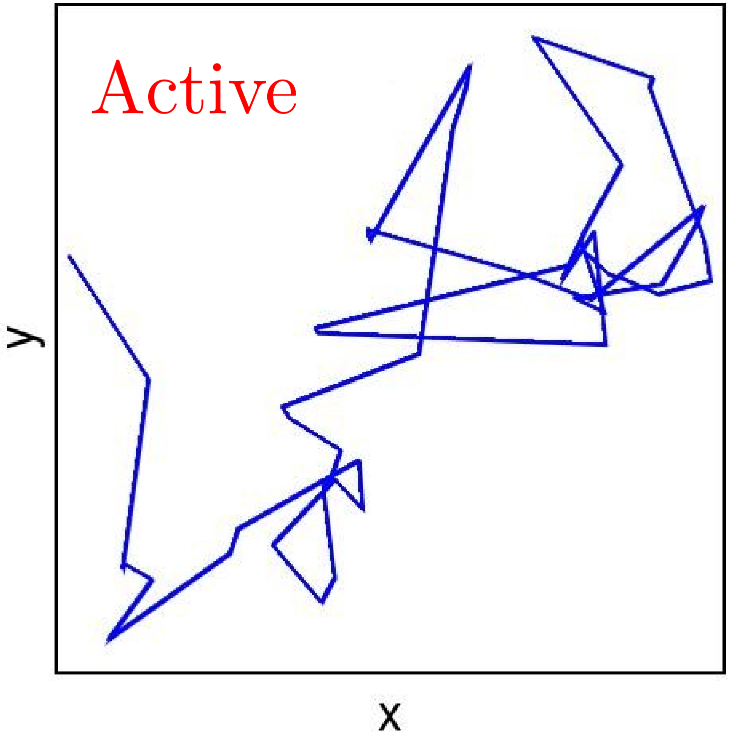
Active Matter

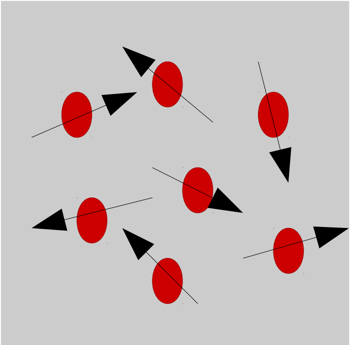


Every constituent is driven. Violation of detailed balance / time-reversal symmetry at the local scale.

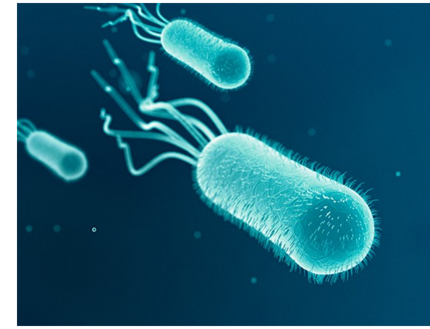
(Examples: Bacteria, Janus particles, vibrating granules....)







Active Matter



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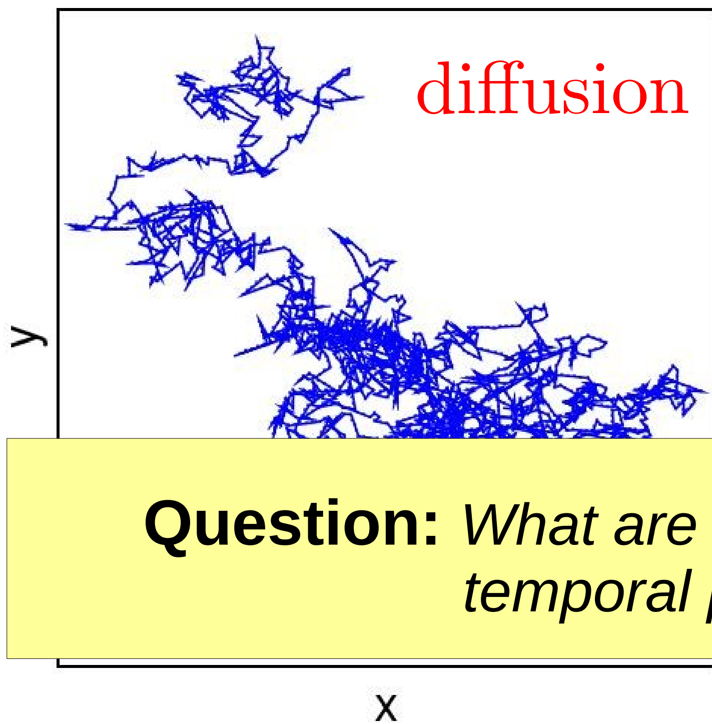
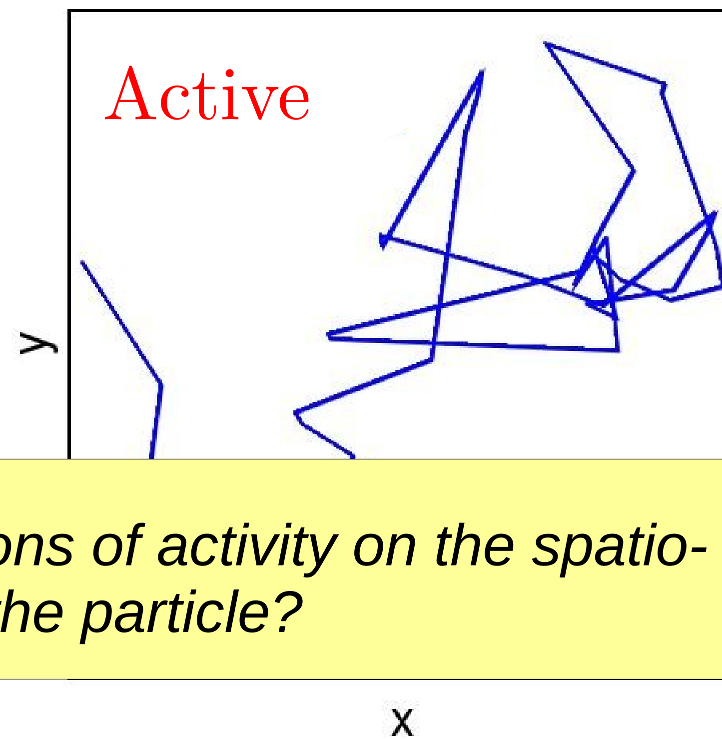
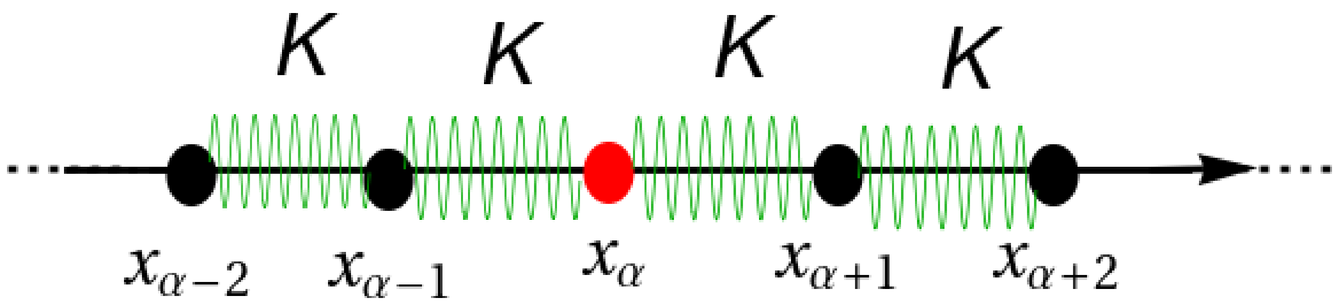




Question: *What are the ramifications of activity on the spatio-temporal properties of the particle?*

Harmonic chain of active particles



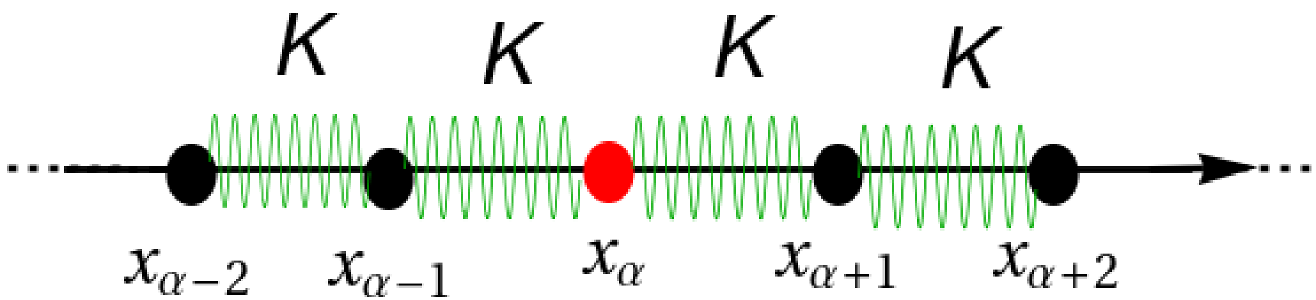
$$\frac{dx_\alpha}{dt} = -K\left(2x_\alpha - x_{\alpha+1} - x_{\alpha-1}\right) + F_\alpha^{\mathrm{a}}(t)$$

interaction

active term

Effect of interaction and activity on the properties of a tagged particle?

Harmonic chain of active particles



$$\frac{dx_\alpha}{dt} = -K\left(2x_\alpha - x_{\alpha+1} - x_{\alpha-1}\right) + F_\alpha^{\mathrm{a}}(t)$$

Three models of active particles

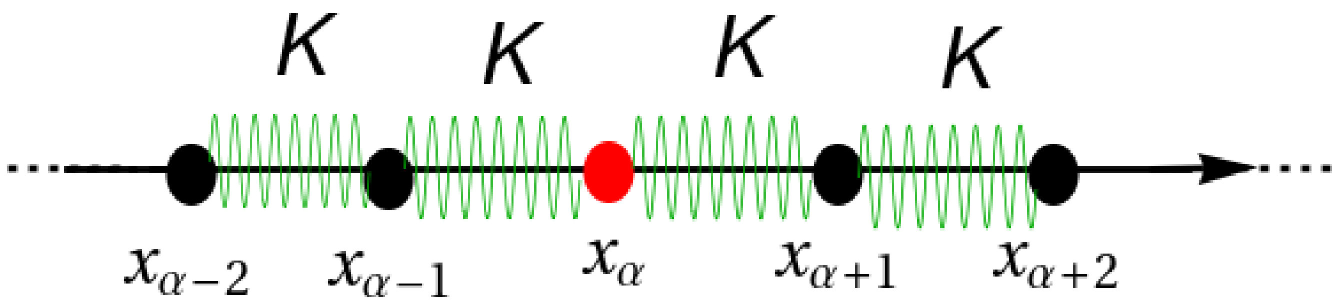
(1) run and tumble particle $F_\alpha^a(t) = v_0 \sigma_\alpha(t)$

(2) active Brownian particle $F_\alpha^a(t) = v_0 \cos\theta_\alpha \quad \dot{\theta}_\alpha = \sqrt{2D_r}\zeta_\alpha(t)$

(3) active Ornstein Uhlenbeck $F_\alpha^a(t) = \lambda_\alpha(t)$

$$\dot{\lambda}_\alpha = -\gamma\lambda_\alpha + \sqrt{2D}\eta_\alpha(t)$$

Harmonic chain of active particles



$$\frac{dx_\alpha}{dt} = -K\left(2x_\alpha - x_{\alpha+1} - x_{\alpha-1}\right) + F^{\mathrm{a}}_\alpha(t)$$

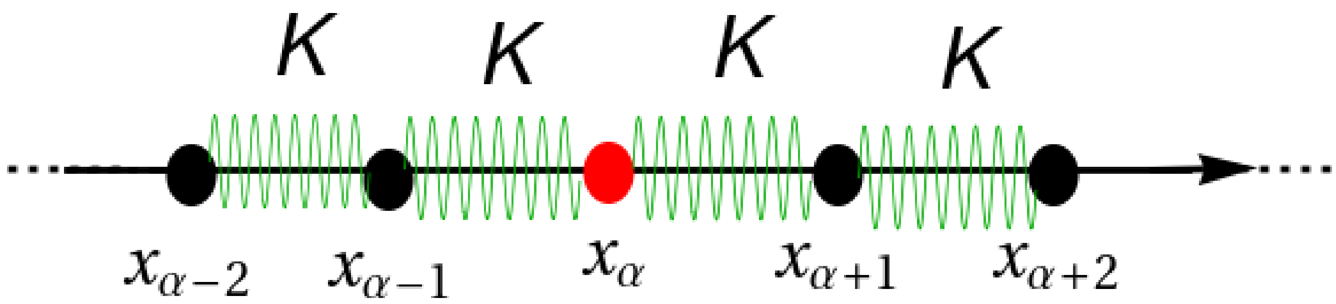
Three models of active particles

Let us analyze the variance of position of tagged particle

→ Compare for different models

→ Compare with diffusion

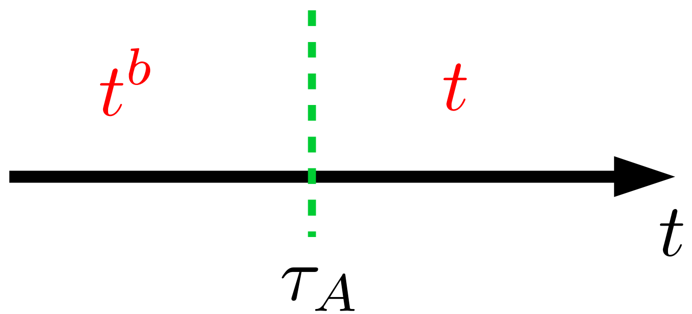
Harmonic chain of active particles



$\tau_A \rightarrow$ activity timescale

Non-interacting case $K = 0$

$$\langle x^2(t) \rangle_c \sim t^b, \quad \text{for } t \ll \tau_A$$
$$\sim t, \quad \text{for } t \gg \tau_A$$

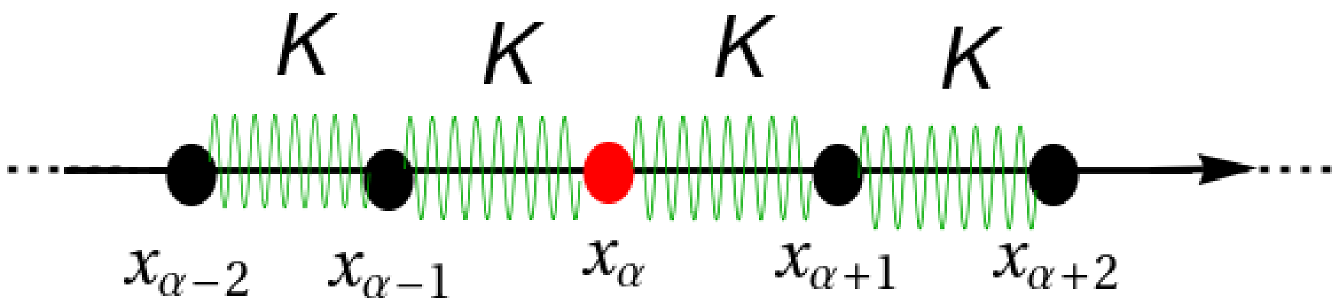


$b \rightarrow$ 2, for RTP

$b \rightarrow$ 3, for AOUP

$b \rightarrow$ 4, for ABP

Harmonic chain of active particles

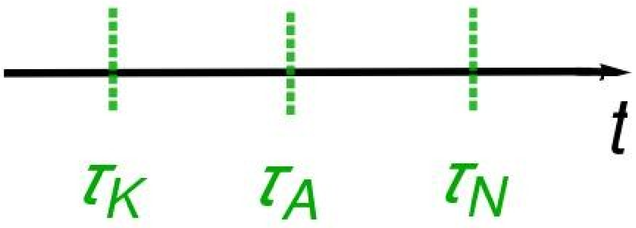


Three timescales when interacting

(i) interaction timescale $\tau_K = 1/K$

(ii) activity timescale τ_A

(iii) finite system size $\tau_N \sim N^2$

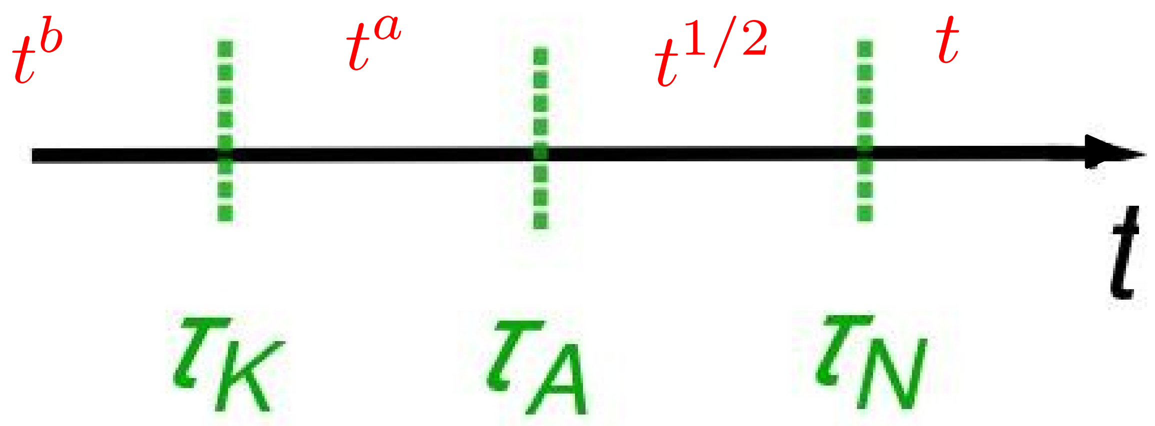


We focus in the regime $\tau_K \ll \tau_A \ll \tau_N$

Results on variance

$$\langle x_\alpha^2(t)\rangle_c \sim \begin{cases} t^b & \text{for } t \ll \tau_K \\ t^a, & \text{for } \tau_K \ll t \ll \tau_A \\ t^{1/2}, & \text{for } \tau_A \ll t \ll \tau_N \\ t, & \text{for } t \gg \tau_N \end{cases}, \quad \text{where } a = \begin{cases} 3/2 & \text{for RTP} \\ 7/2 & \text{for ABP} \\ 5/2 & \text{for AOUP} \end{cases}$$

$$\langle x_\alpha^2(t)\rangle_c \sim$$



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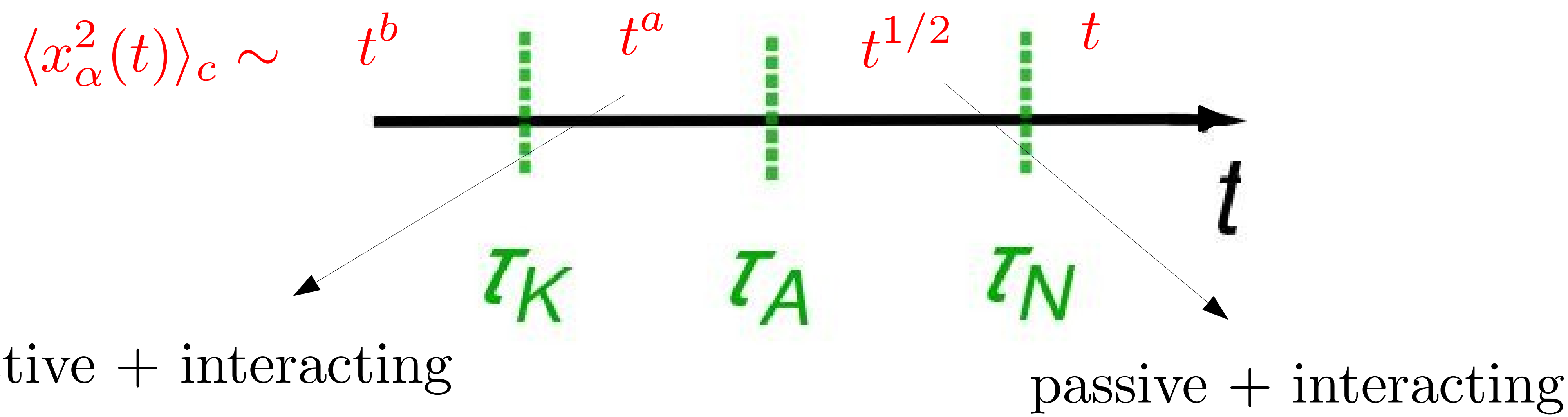
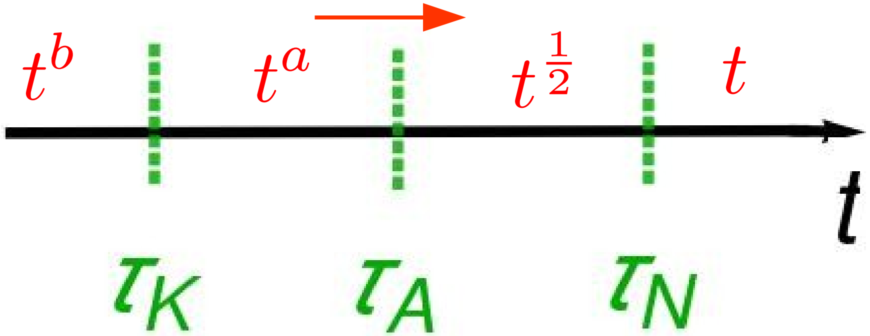
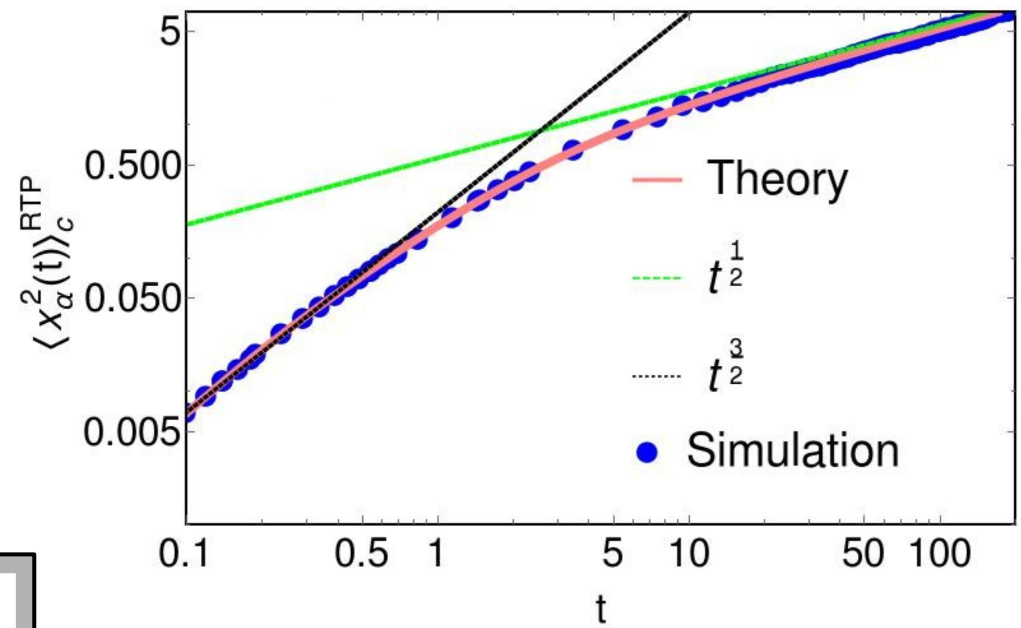
Results on variance

$$\langle x_\alpha^2(t)\rangle_c \sim \begin{cases} t^{\mathrm{b}} & \text{for } t \ll \tau_K \\ t^a, & \text{for } \tau_K \ll t \ll \tau_A \\ t^{1/2}, & \text{for } \tau_A \ll t \ll \tau_N \\ t, & \text{for } t \gg \tau_N \end{cases}, \quad \text{where } a = \begin{cases} 3/2 & \text{for RTP} \\ 7/2 & \text{for ABP} \\ 5/2 & \text{for AOUP} \end{cases}$$



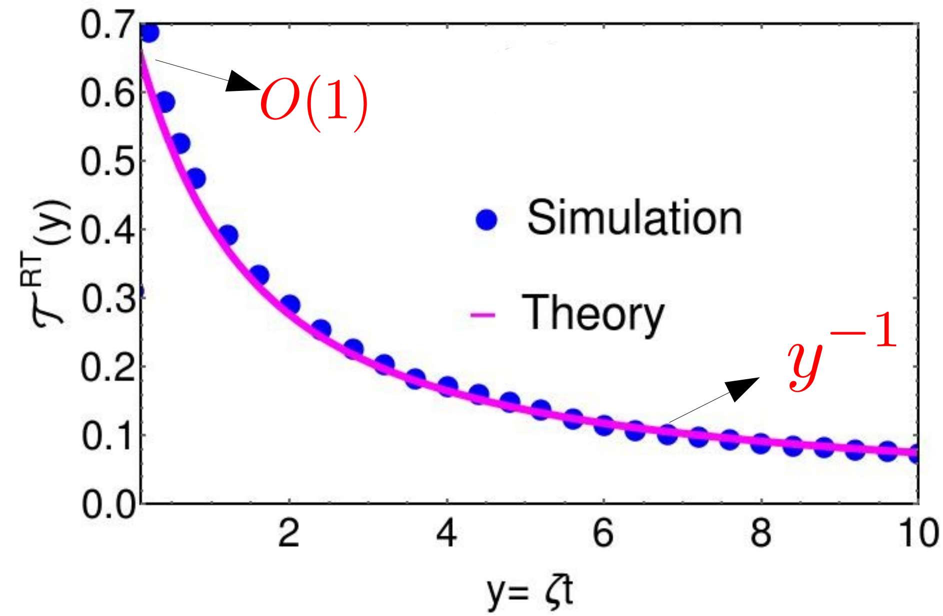
Crossover from t^a to $\sqrt{t}$



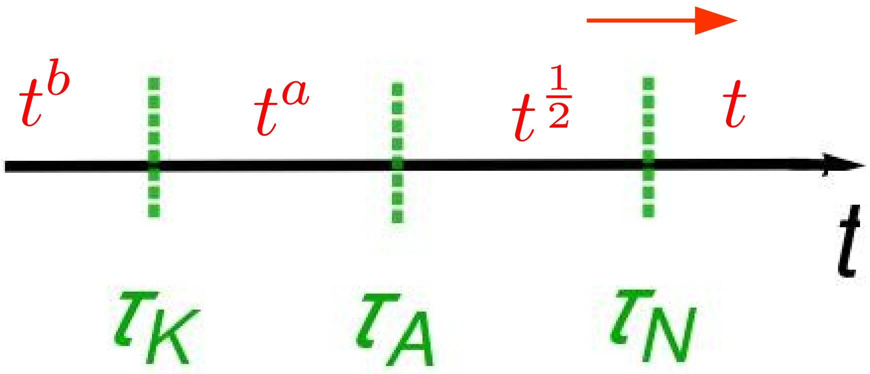


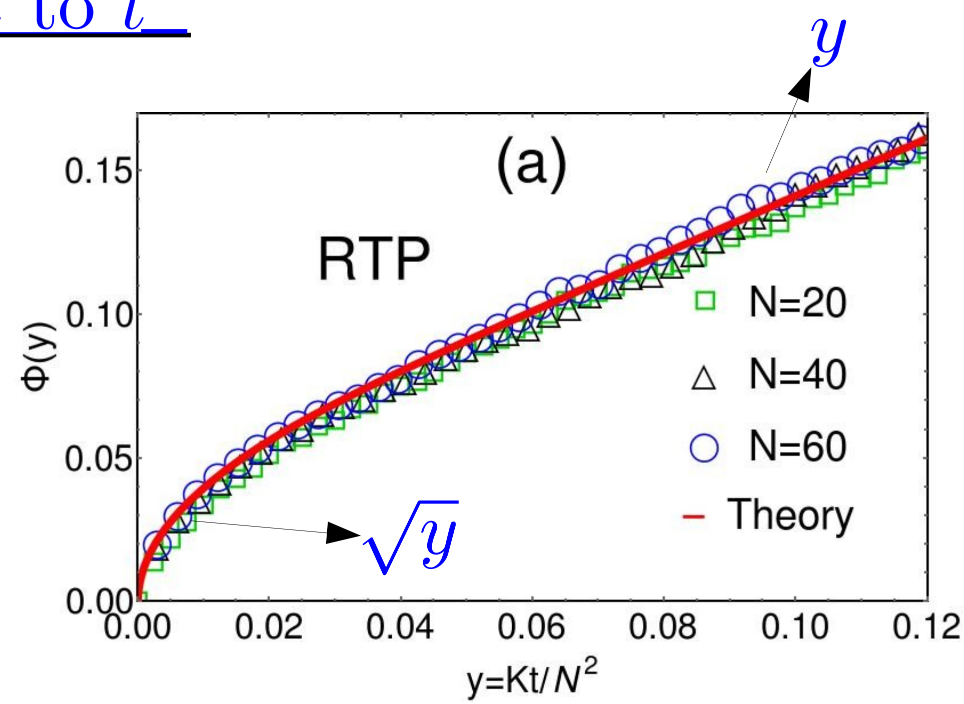
$$\langle x_\alpha^2(t)\rangle_c^{RTP} \simeq \frac{v_0^2 t^{\frac{3}{2}}}{\pi}\sqrt{\frac{2}{K}}\ \mathcal{T}^{RT}\left(t/\tau_A\right)$$

$$\mathcal{T}^{RT}(y) = \frac{1}{4}\int_{-\infty}^{\infty} dw\ \mathcal{G}\left(2y, \frac{w^2}{2}, 1\right)$$



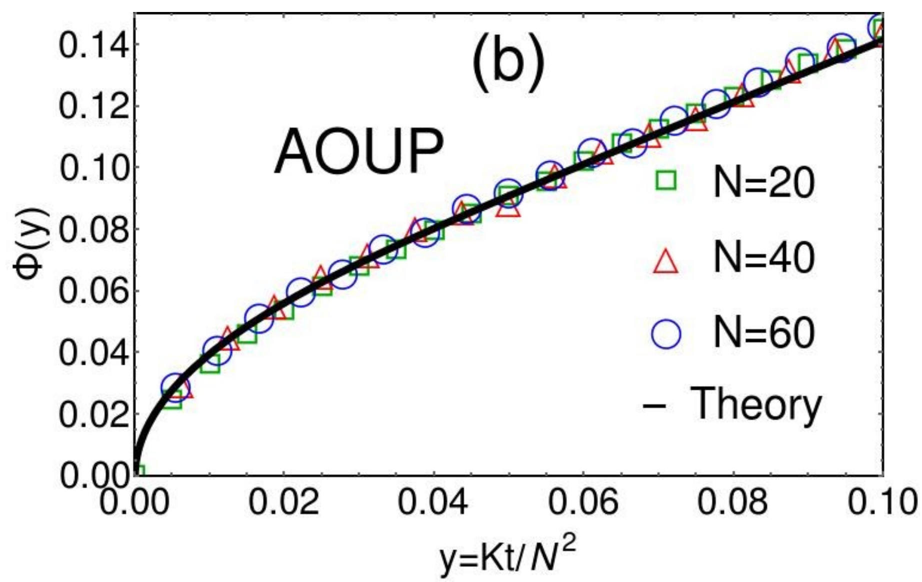
Crossover from $\sqrt{t}$ to t



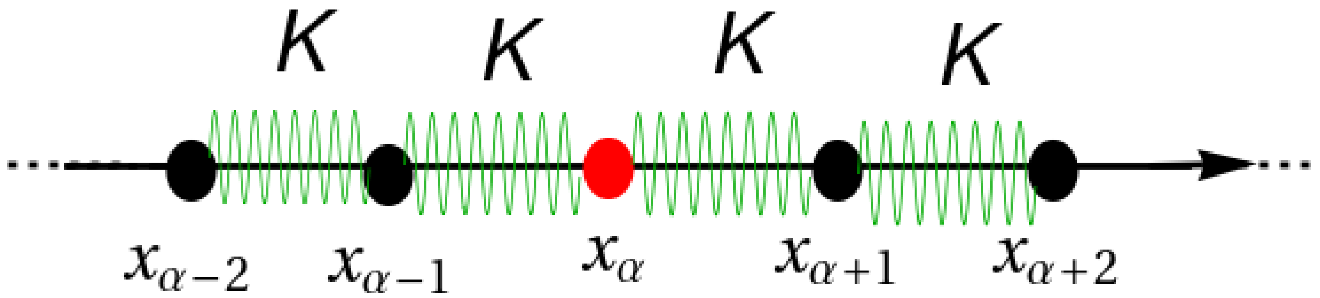


$$\langle x_\alpha^2(t)\rangle_c \simeq \frac{2D_R N}{K}\,\Phi\left(\frac{Kt}{N^2}\right)$$

$$\Phi(y) = y + \frac{1}{4\pi^2}\sum_{s=1}^{\infty}\frac{1-e^{-8\pi^2 s^2 y}}{s^2}$$



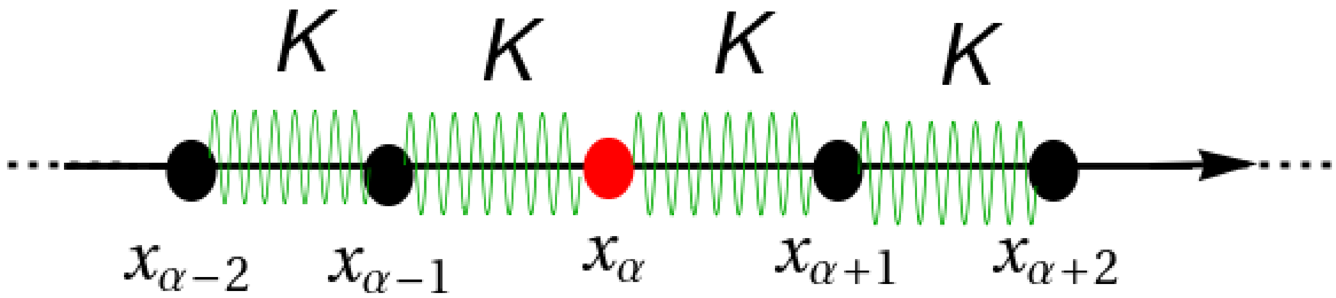
Correlation in the positions

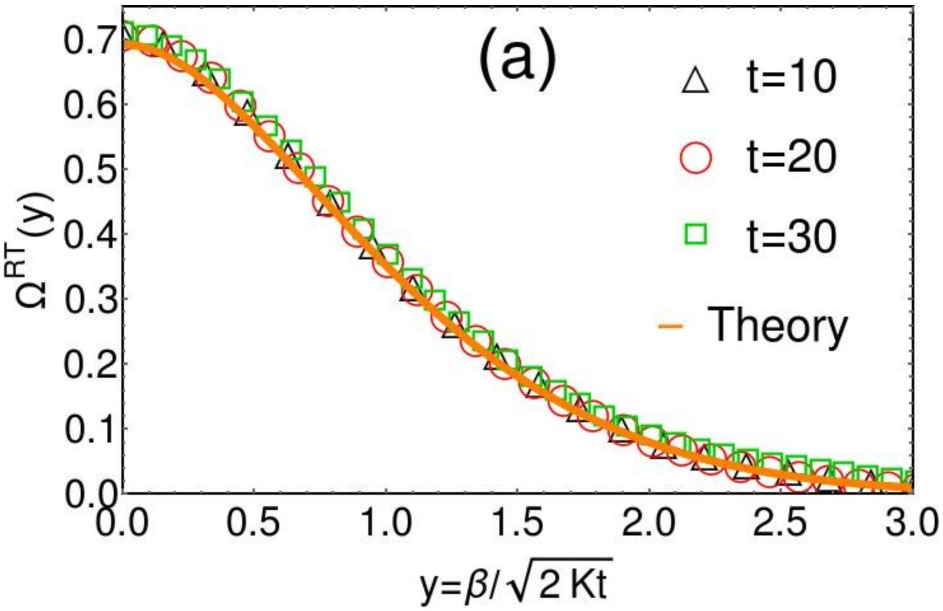


$$\langle x_0(t) x_\beta(t) \rangle_c^{RTP} \simeq \frac{v_0^2}{\pi} \sqrt{\frac{2t^3}{K}} \, \Omega^{RT}\left(\frac{\beta}{\sqrt{2Kt}} \right) , \qquad t \ll \tau_A$$

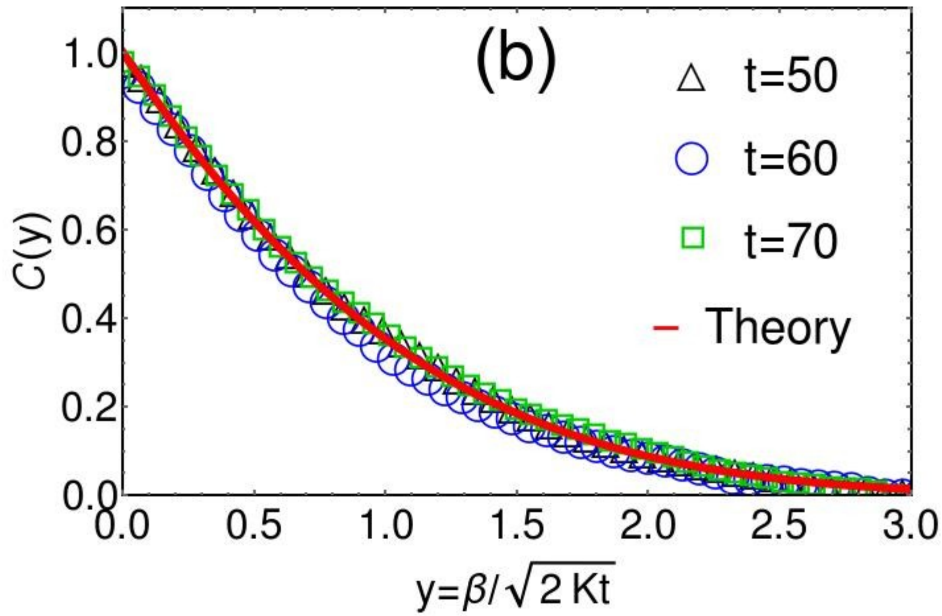
$$\langle x_0(t) x_\beta(t) \rangle_c^{RTP} \simeq D_R \sqrt{\frac{2t}{\pi K}} \, \mathcal{C}\left(\frac{\beta}{\sqrt{2Kt}} \right) , \qquad t \gg \tau_A$$

Correlation in the positions



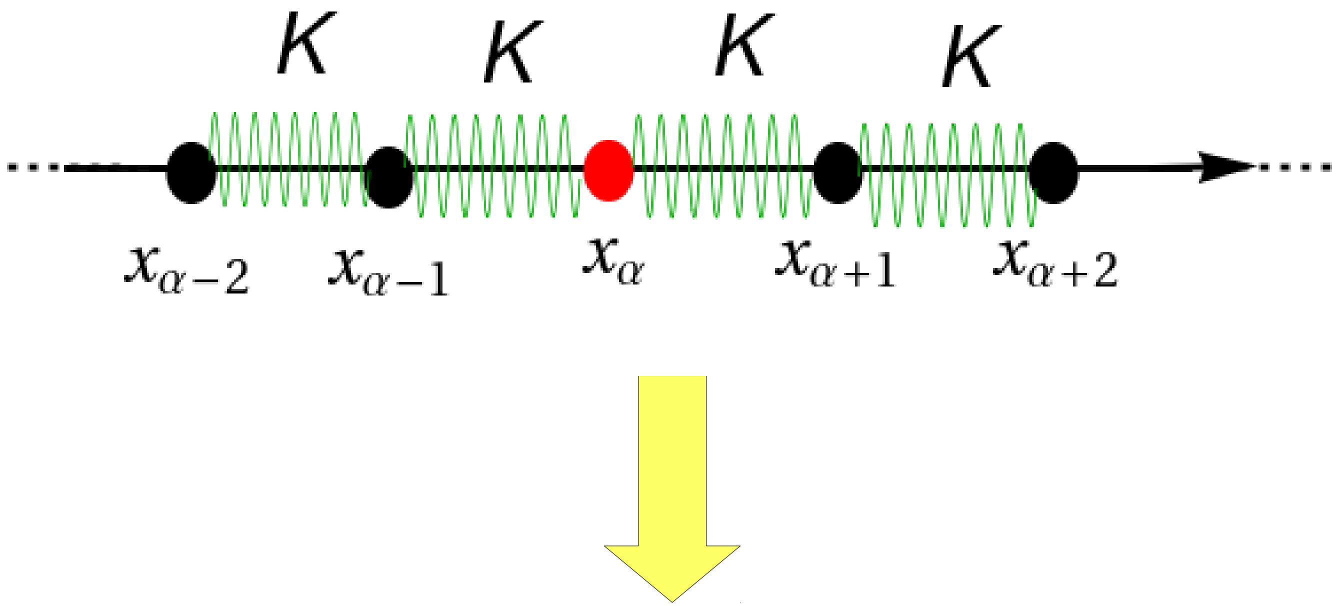


$t \ll \tau_A$

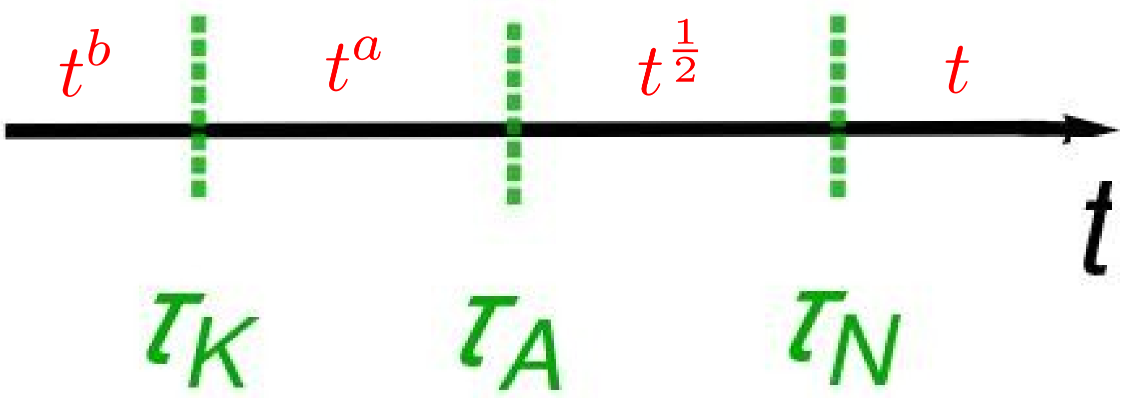


$t \gg \tau_A$

Conclusion



$$\langle x_\alpha^2(t)\rangle_c \sim$$



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