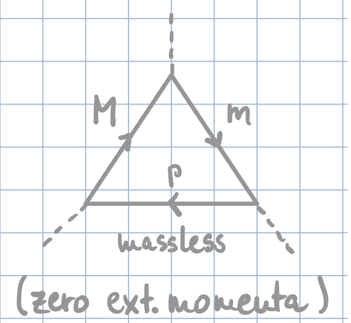
Problem Set 1 (10 April 2024)

Exercise 1:

Perform the region analysis for the integral:

$$I = \mu^{2\epsilon} \int \frac{d^D p}{(2\pi)^D} \frac{1}{p^2 (p^2 - M^2)(p^2 - m^2)} \quad ; \quad M^2 \gg m^2$$



Useful integrals:

$$\int \frac{d^D \ell}{(2\pi)^D} \frac{1}{(\ell^2 - \Delta)^n} = (-1)^n \frac{i}{(4\pi)^{D/2}} \frac{\Gamma(n - D/2)}{\Gamma(n)} \Delta^{D/2 - n}$$

$$\int \frac{d^D p}{(2\pi)^D} \frac{1}{p^2 - M^2} \frac{1}{(p^2)^a} = - \frac{i}{(4\pi)^{D/2}} \Gamma\left(1 - \frac{D}{2}\right) (M^2)^{\frac{D}{2} - 1 - a}$$

Exercise 2 :

Evaluate the collinear contribution

$$I_c = i\,\pi^{-D/2}\mu^{2\epsilon}\int d^Dk\,\frac{1}{(k^2+i0)\left[(k+p_1)^2+i0\right]\left[2k\cdot p_{2+}+i0\right]}$$

to the Sudakov form factor and give an EFT inter-pretation.

Feynman parameters for linear propagators:

$$\frac{1}{A^a\,B^b} = \frac{\Gamma(a+b)}{\Gamma(a)\,\Gamma(b)}\int_0^\infty d\lambda\,\frac{\lambda^{b-1}}{(A+\lambda B)^{a+b}}$$

linear propagators

Exercise 3 :

Evaluate the integral

$$I = \int_1^\infty dx \, \frac{x^{-\epsilon}}{x+\mu x^3} \quad ; \qquad 0<\mu \ll 1 \qquad (\epsilon > -2)$$

using the method of regions, dropping terms of $\mathcal{O}(\mu^2)$.