

Overlaps in spin glasses and models of evolution

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ICTS DISTINGUISHED LECTURE

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Outline

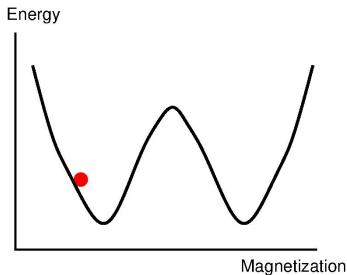
- ▶ Overlaps in spin glasses and in models of evolution
- ▶ Predictions of the Parisi theory of spin glasses and the REM
- ▶ The replica approach (for beginners)
- ▶ How to generalize the replica approach
- ▶ Models of asexual evolution : neutral versus selection
- ▶ Sexual evolution and speciation

Overlaps

Ferromagnet ($J > 0$)

$$E = -J \sum_{i,j} S_i S_j$$

2^N Configurations = $\{S_i = \pm 1\}$

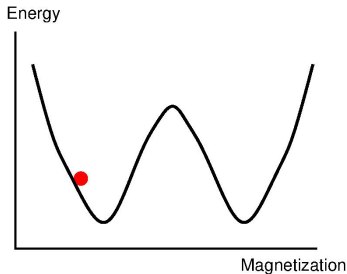


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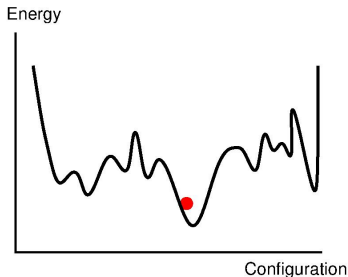
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Spin glass ($J_{ij} = \pm 1$)

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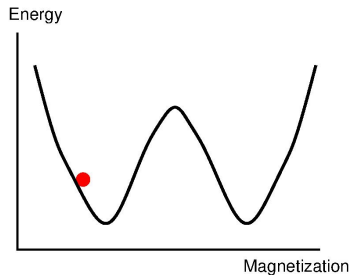
Random landscape



Overlaps

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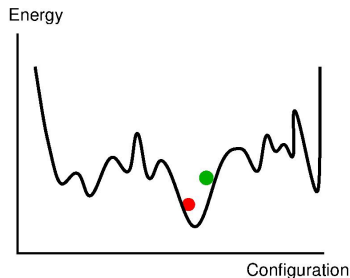
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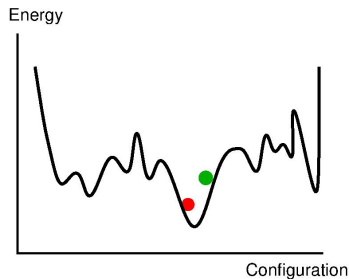
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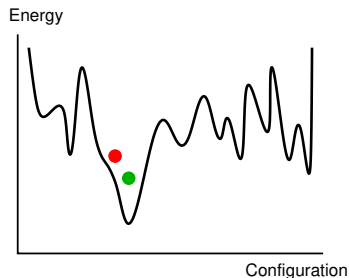


Rugged landscapes



p_i probability of finding the system in valley i

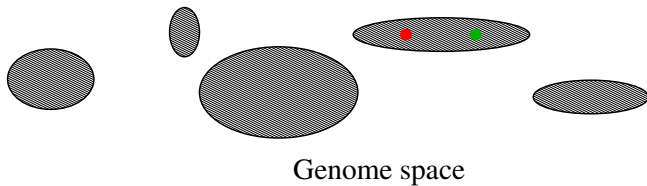
These p_i depend on the landscape



$$Y = \sum_i p_i^2$$

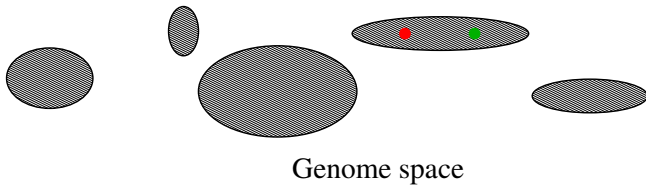
Overlaps in models of evolution

Sexual reproduction

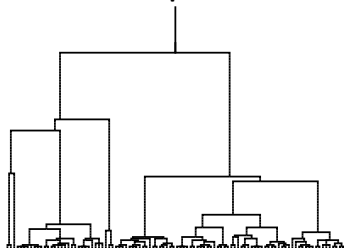


Overlaps in models of evolution

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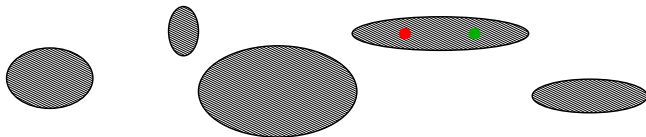


Asexual reproduction



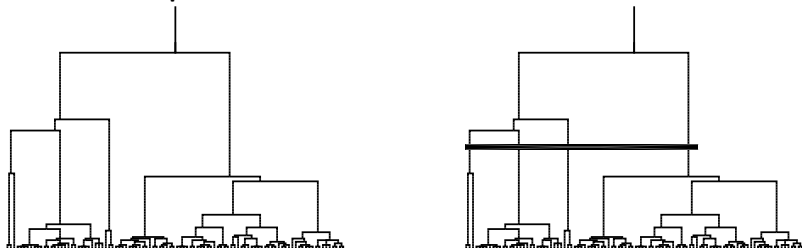
Overlaps in models of evolution

Sexual reproduction



Genome space

Asexual reproduction



Overlaps in spin glasses

A mean field spin glass model Sherrington Kirkpatrick model 1975

N Ising spins $S_i = \pm 1$, a sample $\mathcal{J} = \{J_{ij}\}$

$$\mathcal{C} = \{S_i\} \quad ; \quad E(\mathcal{C}) = - \sum_{i,j} J_{ij} S_i S_j \quad ;$$

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Overlap (Edwards Anderson 1974 order parameter)

$$q(\mathcal{C}, \mathcal{C}') = \frac{1}{N} \sum_i S_i S'_i$$

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Distribution of overlaps for mean field spin glasses Parisi 1979 $\rightarrow$

$$P(Q) = \left\langle \sum_{\mathcal{C}, \mathcal{C}'} \frac{e^{-\beta E(\mathcal{C})}}{Z(\beta)} \frac{e^{-\beta E(\mathcal{C}')}}{Z(\beta)} \delta(Q - q(\mathcal{C}, \mathcal{C}')) \right\rangle_{\mathcal{J}}$$

Overlaps in models of evolution

Population of M individuals

Each individual $\mathcal{C}$ has N nucleotides $S_i(\mathcal{C}) = \pm 1$

Overlaps in models of evolution

Population of M individuals

Each individual $\mathcal{C}$ has N nucleotides $S_i(\mathcal{C}) = \pm 1$

The overlap between two individuals $q(\mathcal{C}, \mathcal{C}') = \frac{1}{N} \sum_i S_i S'_i$

► **Asexual reproduction:**

If $\mathcal{P}$ is the parent of $\mathcal{C}$:

$$S_i(\mathcal{C}) = \begin{cases} S_i(\mathcal{P}) & \text{with probability } 1 - \epsilon \\ -S_i(\mathcal{P}) & \text{with probability } \epsilon \end{cases}$$

► **Sexual reproduction:**

For each i choose one of the two parents at random.

Non-self averaging effects in the spin glass phase

Non-selfaveraging in spin glasses

For each sample $\mathcal{J} \equiv \{J_{ij}\}$; $E(\mathcal{C}) = - \sum_{i,j} J_{ij} S_i S_j$

$Y(Q, \mathcal{J})$ = probability that 2 configurations have an overlap $> Q$

$$Y(Q, \mathcal{J}) = \sum_{\mathcal{C}, \mathcal{C}'} \Theta(q(\mathcal{C}, \mathcal{C}') - Q) \frac{e^{-\beta E(\mathcal{C})}}{Z(\beta)} \frac{e^{-\beta E(\mathcal{C}')}}{Z(\beta)}$$

Mézard, Parisi, Sourlas, Toulouse, Virasoro 1984

Y remains sample dependent in the thermodynamic limit

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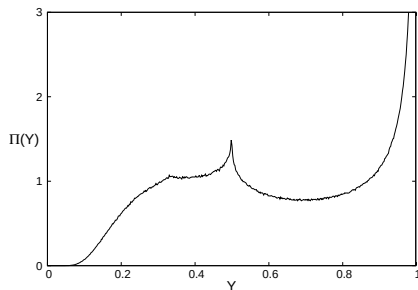
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$$\langle Y \rangle_{\mathcal{J}} = y \quad ; \quad \langle Y^2 \rangle_{\mathcal{J}} = \frac{y + 2y^2}{3} \quad ; \quad \langle Y^3 \rangle_{\mathcal{J}} = \frac{3y + 7y^2 + 5y^3}{15}$$

Distribution $\Pi(Y)$ of Y

$\Pi(Y)$ depends on a single parameter $y = \langle Y \rangle$

Mézard, Parisi, Sourlas, Toulouse, Virasoro 1984



The distribution $\Pi(Y)$ of $Y(Q)$ when $\langle Y(Q) \rangle = \frac{2}{3}$

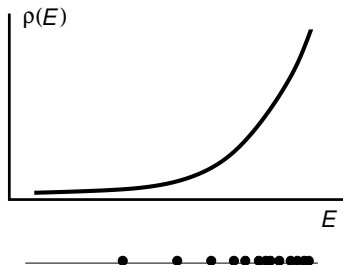
D, Flyvbjerg 1987

One way of generating the distribution $\Pi(Y)$

The Poisson REM (random energy model)

The Energies $E(\mathcal{C})$ are the points of a Poisson process* of density $\rho(E)$

Overlaps : $q(\mathcal{C}, \mathcal{C}') = \delta(\mathcal{C}, \mathcal{C}')$

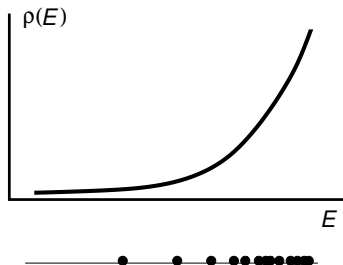


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$$Z(\beta) = \sum_{\mathcal{C}} e^{-\beta E(\mathcal{C})}$$

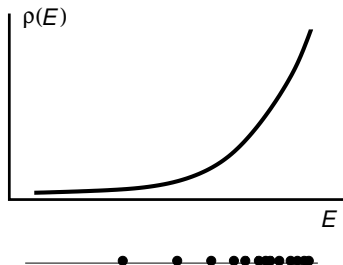
$$Y = \sum_{\mathcal{C}} \frac{e^{-2\beta E(\mathcal{C})}}{Z(\beta)^2} = \frac{Z(2\beta)}{Z(\beta)^2}$$

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$$\boxed{\rho(E) = A e^{\alpha E}}$$

$$\Rightarrow y = 1 - \frac{\alpha}{\beta}$$

(*Poisson process means that there is an energy $E(\mathcal{C})$ in $(E, E + dE)$ with probability $\rho(E)dE$)

The REM as the limit of the Ising p spin model

The SK model Sherrington Kirkpatrick 1975

$$E(C) = - \sum_{i,j} J_{ij} S_i S_j$$

The p -spin model D. 1980

$$E(C) = \sum J_{i_1 i_2 \dots i_p} S_{i_1} S_{i_2} \dots S_{i_p}$$

2^N configurations

$$P(E(C)) = \frac{1}{\sqrt{\pi N}} \exp \left[-\frac{E(C)^2}{N} \right]$$

the correlations $\langle E(C)E(C') \rangle = N(q(C, C'))^p$

The REM = $p \rightarrow \infty$ limit

The energies $E(C)$ are i.i.d. random variables

The REM

The energies are Gaussian : $P(E(C)) = \frac{1}{\sqrt{\pi} N} \exp \left[-\frac{E(C)^2}{N} \right]$

Then $Z(\beta)$ is a sum of 2^N i.i.d. random variables

$$Z(\beta) = \sum_{\mathcal{C}} e^{-\beta E(\mathcal{C})}$$

$$E_{\min} \sim -N\sqrt{\log 2}$$

The REM

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$$Z(\beta) = \sum_{\mathcal{C}} e^{-\beta E(\mathcal{C})}$$

$$E_{\min} \sim -N\sqrt{\log 2} \quad ; \quad \beta_c = 2\sqrt{\log 2}$$

For $\beta > \beta_c$, only configurations with energies close to $E_{\min}$ contribute

These configurations have zero overlaps between themselves

In the large N limit, they reduce to a Poisson REM with $\rho(E) = A e^{\beta_c E}$

► Same $\Pi(Y)$ D. Toulouse 1985 ; D. Flyvbjerg 1987

The overlaps in the replica approach
in the spin glass phase

The replica trick

For each sample $\mathcal{J} \equiv \{J_{ij}\}$; $E(\mathcal{C}) = - \sum_{i,j} J_{ij} S_i S_j$

$$Y(Q, \mathcal{J}) = \sum_{\mathcal{C}, \mathcal{C}'} \Theta(q(\mathcal{C}, \mathcal{C}') - Q) \frac{e^{-\beta E(\mathcal{C})}}{Z(\beta)} \frac{e^{-\beta E(\mathcal{C}')}}{Z(\beta)}$$

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The average over the samples $\mathcal{J}$ is difficult.

Replica trick:

$$\langle Y \rangle_{\mathcal{J}} = \lim_{n \rightarrow 0} \frac{\langle Y Z^n \rangle_{\mathcal{J}}}{\langle Z^n \rangle_{\mathcal{J}}}$$

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Taking the average over the samples is easier if $n = 2, 3, 4 \dots$.

$$\langle Y Z^n \rangle_{\mathcal{J}} = \sum_{\mathcal{C}_1} \dots \sum_{\mathcal{C}_n} \Theta(q(\mathcal{C}_i, \mathcal{C}_j) - Q) \left\langle e^{-\beta [E(\mathcal{C}_1) + \dots + E(\mathcal{C}_n)]} \right\rangle_{\mathcal{J}}$$

Parisi's RSB (Replica Symmetry Breaking)

$$\langle YZ^n \rangle_{\mathcal{J}} = \sum_{\mathcal{C}_1} \cdots \sum_{\mathcal{C}_n} \Theta(q(\mathcal{C}_i, \mathcal{C}_j) - Q) \left\langle e^{-\beta [E(\mathcal{C}_1) + \cdots + E(\mathcal{C}_n)]} \right\rangle_{\mathcal{J}}$$

In the thermodynamic limit, this is dominated by saddle points:

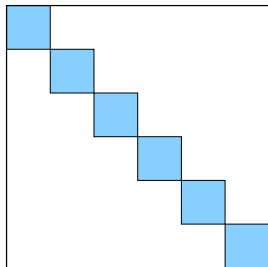
A $n \times n$ matrix $\frac{n}{\mu}$ blocks of μ replicas

- ▶ if i and j in the same block

$$q(\mathcal{C}_i, \mathcal{C}_j) > Q$$

- ▶ if they belong to different blocks

$$q(\mathcal{C}_i, \mathcal{C}_j) < Q$$



A combinatorial problem

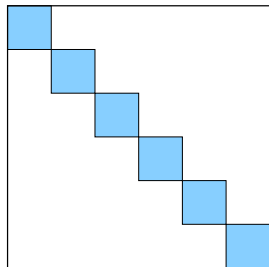
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$$\langle Y \rangle = \frac{n(\mu-1)}{n(n-1)} \quad ; \quad \langle Y^2 \rangle = \frac{n(\mu-1)(\mu-2)(\mu-3)}{n(n-1)(n-2)(n-3)} + \frac{n(\mu-1)(n-\mu)(\mu-1)}{n(n-1)(n-2)(n-3)}$$

Forget that n and μ are integers, then take the limit $n \rightarrow 0$ with $y = 1 - \mu$

General Poisson REM

$\rho(E)$ is arbitrary

The double exponential

D, Mottishaw 2024

A Poisson REM with $\rho(E) = A_1 e^{\beta_1 E} + A_2 e^{\beta_2 E}$

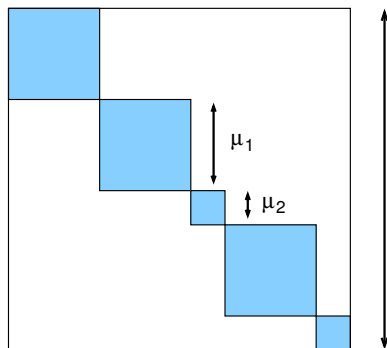
$$\frac{\langle Y Z(\beta)^n \rangle}{\langle Z(\beta)^n \rangle} = \sum_{i=1,2} \frac{n(\mu_i - 1)}{n(n-1)} W_i$$

where $\mu_i = \frac{\beta_i}{\beta}$ and

$$W_i = \frac{A_i}{\beta} \frac{\Gamma(1 - \mu_i) \Gamma(\mu_i - n)}{\Gamma(1 - n)} \frac{\langle Z(\beta)^{n - \mu_i} \rangle}{\langle Z(\beta)^n \rangle}$$

Interpretation in terms of replicas

A Poisson REM with $\rho(E) = A_1 e^{\beta_1 E} + A_2 e^{\beta_2 E}$



$$\mu_i = \frac{\beta_i}{\beta}$$

Sum of an arbitrary number of exponentials

A Poisson REM with a density $\rho(E) = \sum_i A_i e^{\beta_i E}$

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- **Finite size corrections to the REM** D, Mottishaw 2015

$$\rho(E) = \frac{2^N}{\sqrt{\pi N}} \exp \left[-\frac{E^2}{N} \right]$$

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$$\rho(E) = \frac{2^N}{\sqrt{\pi N}} \exp \left[-\frac{E^2}{N} \right]$$

can be written as

$$\rho(E) = \int \frac{dy}{\pi} e^{-Ny^2} \exp \left[(\beta_c + 2iy)(E + N\sqrt{\log 2}) \right]$$

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the box sizes become complex, their variance becomes negative !

Sum of an arbitrary number of exponentials

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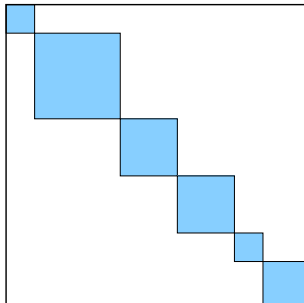
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- **Discrete energies** D, Mottishaw 2022

Energies can only take integer values

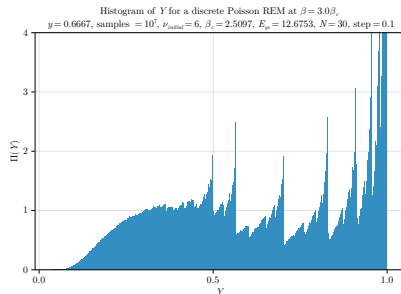
$$\rho(E) = A e^{\beta_c E} \sum_{n=-\infty}^{\infty} \delta(E - n) = \sum_{p=-\infty}^{\infty} A e^{(\beta_c + 2i\pi p)E}$$

Poisson REM with arbitrary density $\rho(E)$



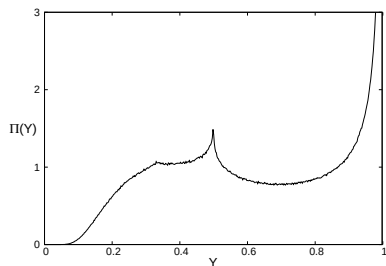
- ▶ the number r of blocks fluctuates
- ▶ the sizes μ of the blocks fluctuate

$\Pi(Y)$ of Y for discrete energies



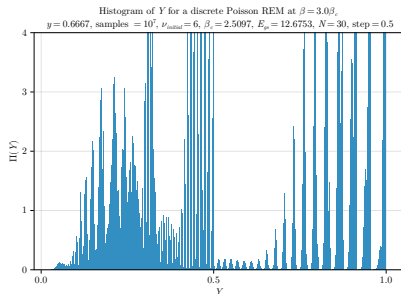
Discrete energies

Mottishaw 2026



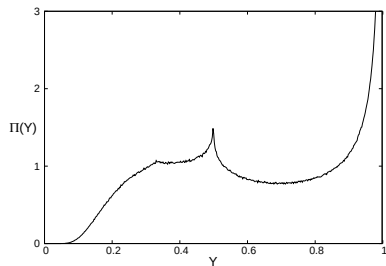
Spin glasses

$\Pi(Y)$ of Y for discrete energies



Discrete energies

Mottishaw 2026



Spin glasses

Main message on the spin glass part of this talk

"Replica-free" calculations on the Poisson REM correspond to replicas with fluctuating Parisi matrices

Is it relevant in the real world ?

Models with discrete energies:

perceptron, directed polymers, K-Sat ?

Non-self averaging effects in models of evolution

Neutral model of asexual evolution

Fisher 1922, Wright 1931, Kimura 1968, Kingman 1982 ...

- ▶ A population of M individuals.
- ▶ Each individual $\mathcal{C}$ is represented by N nucleotides

$$\mathcal{C} = \{S_1 \cdots S_N\} \quad \text{with} \quad S_i = \pm 1$$

- ▶ if $\mathcal{P}$ is the parent of $\mathcal{C}$, then

$$S_i(\mathcal{C}) = \begin{cases} S_i(\mathcal{P}) & \text{with probability } 1 - \epsilon \\ -S_i(\mathcal{P}) & \text{with probability } \epsilon \end{cases}$$

- ▶ **neutral**

For each $\mathcal{C}$, the parent $\mathcal{P}$ is chosen at random in the previous generation $\Rightarrow$ for large N

$$q(\mathcal{C}, \mathcal{C}') = (1 - 2\epsilon)^2 q(\mathcal{P}, \mathcal{P}')$$

Models of evolution versus Statistical Physics

Equilibrium Statistical Physics

Competition between energy and thermal noise

Models of evolution

Competition between genealogy and mutations

Model of evolution versus spin glasses

Models of evolution

$$Y = \frac{1}{M^2} \sum_{\mathcal{C}, \mathcal{C}'} \Theta(q(\mathcal{C}, \mathcal{C}') - Q)$$

- ▶ Y is an average over the whole population
- ▶ Y depends on the generation g and it fluctuates
- ▶ neutral

$$\langle Y \rangle_g = y \quad ; \quad \langle Y^2 \rangle_g = y^2 + \frac{2}{3}y^3 - y^4 + \frac{2}{5}y^5 - \frac{1}{15}y^6$$

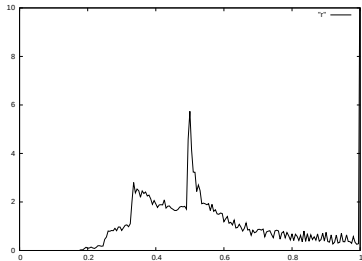
Spin Glasses

$$Y(Q, \mathcal{J}) = \sum_{\mathcal{C}, \mathcal{C}'} \Theta(q(\mathcal{C}, \mathcal{C}') - Q) \frac{e^{-\beta E(\mathcal{C})}}{Z(\beta)} \frac{e^{-\beta E(\mathcal{C}')}}{Z(\beta)}$$

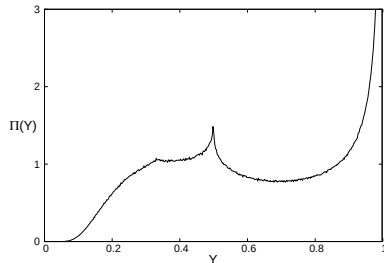
- ▶ Y is an average over all configurations
- ▶ Y depends on the sample $\mathcal{J}$ and it fluctuates

$$\langle Y \rangle_{\mathcal{J}} = y \quad ; \quad \langle Y^2 \rangle_{\mathcal{J}} = \frac{y + 2y^2}{3}$$

Asexual + neural



Asexual + neutral



Spin glasses

Asexual evolution with selection

Asexual evolution + selection

Brunet D 1997, Brunet D Mueller Munier 2006,2007 ...

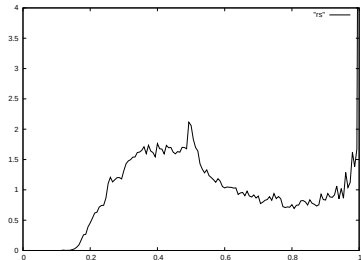
- ▶ A population of M individuals.
- ▶ Each individual $\mathcal{C}$ carries a fitness parameter $x(\mathcal{C})$ and has an arbitrary number k of offspring at the next generation (k could be fixed or random)
- ▶ if $\mathcal{P}$ is the parent of $\mathcal{C}$, then $x(\mathcal{C}) = x(\mathcal{P}) + \eta(\mathcal{C})$ where η is some noise
- ▶ for large N

$$q(\mathcal{C}, \mathcal{C}') = (1 - 2\epsilon)^2 q(\mathcal{P}, \mathcal{P}')$$

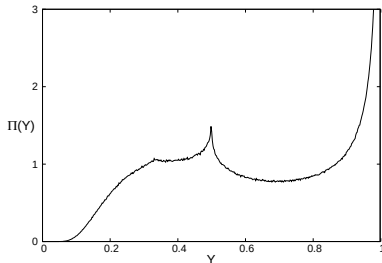
- ▶ **selection**

At each generation one keeps only the N offspring with highest $x(\mathcal{C})$

Asexual + Selection



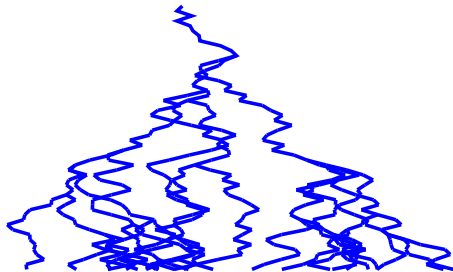
Asexual + selection



Spin glasses

The N-BBM

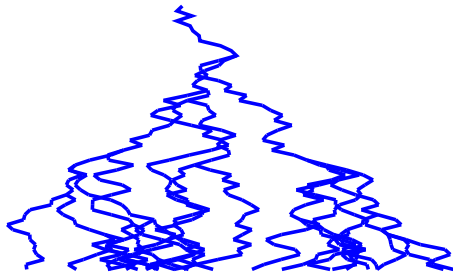
BBM= Branching Brownian Motion



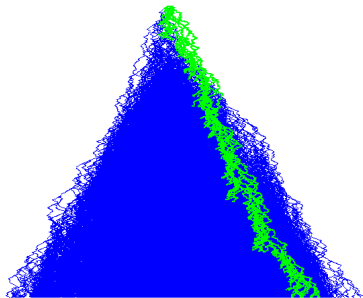
BBM

The N-BBM

BBM= Branching Brownian Motion



BBM



N-BBM

Sexual evolution

Sexual reproduction: the neutral case

- ▶ A population of M individuals.
- ▶ Each individual $\mathcal{C}$ is represented by N nucleotides

$$\mathcal{C} = \{S_1 \cdots S_N\} \quad \text{with} \quad S_i = \pm 1$$

- ▶ It has two parents $\mathcal{P}_1$ and $\mathcal{P}_2$ chosen at random in the previous generation (neutral)



$$S_i(\mathcal{C}) = \begin{cases} S_i(\mathcal{P}_1) & \text{with probability } \frac{1-\epsilon}{2} \\ -S_i(\mathcal{P}_1) & \text{with probability } \frac{\epsilon}{2} \\ S_i(\mathcal{P}_2) & \text{with probability } \frac{1-\epsilon}{2} \\ -S_i(\mathcal{P}_2) & \text{with probability } \frac{\epsilon}{2} \end{cases}$$

The population becomes homogeneous **Serva Peliti 1991**

$$q(\mathcal{C}, \mathcal{C}') \rightarrow \frac{1}{1 + 4M\epsilon}$$

Speciation

The two parents $\mathcal{P}_1, \mathcal{P}_2$ of each individual are chosen at random

Serva Peliti 1991

Y does not fluctuate

$$q(\mathcal{C}, \mathcal{C}') \rightarrow \frac{1}{1 + 4M\epsilon} \equiv q_{\text{eq}}$$

Replica Symmetry

Speciation

The two parents $\mathcal{P}_1, \mathcal{P}_2$ of each individual are chosen at random
Serva Peliti 1991

Y does not fluctuate

$$q(\mathcal{C}, \mathcal{C}') \rightarrow \frac{1}{1 + 4M\epsilon} \equiv q_{\text{eq}}$$

Replica Symmetry

Two parents are chosen at random but with $q(\mathcal{P}_1, \mathcal{P}_2) > q_{\min}$

- ▶ if $q_{\text{eq}} > q_{\min}$ then $q(\mathcal{C}, \mathcal{C}') \rightarrow q_{\text{eq}}$

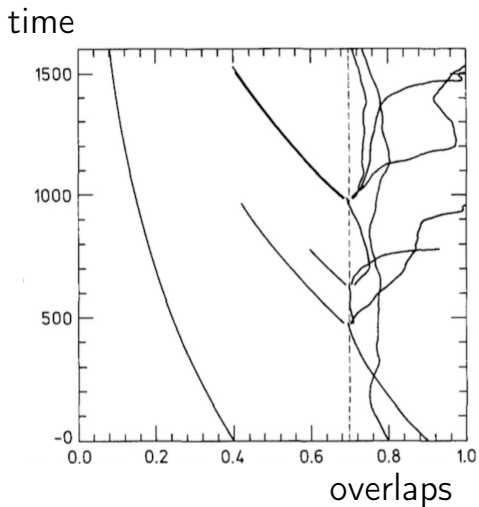
Y does not fluctuate

- ▶ if $q_{\text{eq}} < q_{\min}$ then $q(\mathcal{C}, \mathcal{C}') \not\rightarrow q_{\text{eq}}$ Higgs D 1991

Y fluctuates $\Rightarrow$ speciation

Replica Symmetry Breaking

Speciation: all the overlaps in the population



Higgs D 1992

Conclusion

- ▶ Fluctuating $Y(Q)$ or non-self-averaging effects are a very common phenomenon:
Spin glasses, models of evolution,

Conclusion

- ▶ Fluctuating $Y(Q)$ or non-self-averaging effects are a very common phenomenon:
Spin glasses, models of evolution, ferromagnets with antiperiodic boundary conditions, random networks of automata, random permutations, Poisson-Dirichlet distribution . . .
- ▶ This is only one aspect of random trees
- ▶ Poisson REM $\rightarrow$ Generalizations of the Parisi matrices
- ▶ What about systems with discrete energies:
perceptron, K-Sat, directed polymers ?
- ▶ Effect of selection

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